

# Stabilization of spherically symmetric static Boson stars by Coleman-Weinberg self-interaction potential

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## Abstract

Our motivation in this work is due to the great importance of relativistic boson stars as fierce competitors of black holes. From theoretical perspective, they described by complex Klein Gordon (KG) scalar fields interacting with curved background in general form. In this work we use the Einstein-Klein Gordon (EKG) scalar tensor gravity in presence of the Coleman-Weinberg (CW) self-interaction potential to study formation of a spherically symmetric boson star and situation of its stability. We choose the CW potential because of its important role in description of cosmic inflation.

## 1 Introduction

Ordinary stars are formed by gas and dust clouds that are distributed non-uniformly throughout most galaxies in the universe. All active ordinary stars eventually reach to a finite scale object and collapses due to its own weight and it undergoes the process of stellar death [1]. This process for most stars is responsible for the formation of very dense and compact stellar remnant, which is called compact (relativistic) star. In other words, compact stars, which includes white dwarfs, neutron stars, quark stars, and black holes, are the final stages in the evolution of ordinary stars. When an ordinary star ceases its nuclear fuel supply, then its remnants takes one of these relativistic forms, by depending on its mass during the lifetime. One can follow [2] for more discussions about classification of the relativistic compact stars. Microscopically, we can separate interior matter of relativistic compact stars to two different kinds called as the bosonic and the fermionic respectively. They have integer and half-integer spins and follow different statistics laws, i.e., the Bose-Einstein and the Fermi-Dirac statistical distribution respectively. However, the temperature is a dominant effect for the equation of

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state (EOS) of ordinary stars, because of their molecular kinetic energies, but that is not dominant effect for relativistic stars. In the latter case, the super-relativistic degenerate bosons/fermions particles have energies in order of the fermi energy for which the principle of exclusion of Pauli generate degenerate pressure as dominant pressure independent of the temperature. EOS for such a super-relativistic stars is a relationship between the pressure and the density and the entropy. One can see [2] to find several kind of EOS used for relativistic stars. In summary, by 1939, investigation of researchers dedicated on the problem of what happens to a compact star core made entirely of degenerate fermions, i.e., the electrons and the neutrons. Because of these contributions, on one side, nuclear matter and particle physics were becoming essential for the description of matter at such extreme densities and on the other hand, it had become clear that such superdense objects could be described only within Einstein's theory of gravity. This caused to begin a new approach to world of relativistic astrophysics. For studies in this direction, usually the internal metric of a contracting star should be determined with the help of Einstein's gravitational equation, which is known as the Oppenheimer-Volkoff equation. This equation relates pressure changes to the mass and density of matter inside the star, which was first introduced by Robert Oppenheimer himself. For instance, white dwarfs had provided in 1915, a new test of Einstein's general theory of relativity [3].

A boson star is supported by a complex massive scalar field, coupled to gravity given by general relativity or other alternatives. There are two different approaches to investigate formation of such stars, i.e., the classical and the quantum fields theory approaches. In this work we use a classical approach of the scalar field theory. In the classical regime of the interacting matter fields to form a relativistic star, we can choose one of two different approaches, namely time-dependent evolution of collapsing bosonic cloud [4] or time-independent one. We will work here within the context of classical time-dependent field theory for which the field has a complex form (see for instance [4] [5], [6]). In summary, the boson stars are particle-like solutions of the Einstein metric equations that were found in the late 1960s. Since then, boson stars have been used in a wide variety of models as sources of dark matter, as black hole mimickers, in simple models of binary systems, and as a tool for finding black holes in higher dimensions. A full review for important varieties of boson stars, their dynamic properties, and some of their applications, concentrating on recent efforts, is collected in the ref. [7]. Boson stars for a free scalar field without any kind of self interaction potential was

studied previously to obtain boson star with maximum mass  $M \approx M_{Pl}^2/m_b$ , which is less than the Chandrasekhar mass  $M_{Ch} \approx M_{Pl}/m_f^2$  obtained from fermionic stars, and hence, they are called as ‘mini’ boson stars. In the latter formula,  $m_b$  and  $m_f$  are the masses of the candidates particles of bosons and fermions, respectively, and  $M_{Pl}$  is the Planck mass. In order to extend this limit and reach astrophysical masses comparable to the Chandrasekhar mass, some potentials were generalized to include a self-interaction term that provides an extra pressure against the gravitational collapse [8]. As another application of the EKG theory in presence of self interaction potentials as  $\phi^6$  together with  $\phi^4$  is used in ref [9] to investigate a rotating boson star. Instead of using different forms of self-interacting potentials of scalar fields other physical effects, for instance, the electric field or magnetic field effects (see for instance [10] and [11]), have also been studied for the investigation of the stability of boson stars. To follow this, one can see [12], in which authors solved the Einstein-Maxwell-Klein-Gordon equations to obtain a compact, static axially symmetric magnetized object which is electrically neutral. It is composed from two complex massive charged scalar fields and has several particular properties, including the torus form of the matter density and the expected dipolar distribution of the magnetic field, with some peculiar features in the central regions. The authors showed that their stellar model is free of divergencies in any of the field and metric functions. Also they presented a discussion about their model where the gravitational and magnetic fields in the external region are similar to those of neutron stars. To study conditions of stability of a boson star from the EKG equations interacting with a perfect fluid matter, one can see [13]. Another application of the EKG gravity model is presented in ref. [14] where its authors studied the properties of Bose-Einstein Condensate (BEC) systems consisting of two scalar fields, focusing on both scale of the stellar and the galactic objects. They showed that the presence of extra scalars and possible interactions between them can leave unique imprints on the BEC system mass profile, especially when dominance of one scalar is changed to the other. At stellar scales (non-linear regime), they presented that a repulsive interaction between the two scalars of the type  $+\phi_1^2\phi_2^2$  can stabilize the BEC system and support it up to high compactness, a phenomenon known to exist only in the  $\phi^4$  system. They provided a simple analytic understanding of this behavior and pointed out that it can lead to interesting gravitational wave signals at LIGO-Virgo. At galactic scales, on the other hand, they showed that two-scalar BECs can address the scaling problem that arises when one uses ultralight dark matter

mass profiles to fit observed galactic core mass profiles. They constructed a particle model of two ultralight scalars with the repulsive  $\phi_1^2\phi_2^2$  interaction using collective symmetry breaking.

So far, two incredibly significant experimental results have appeared in relation to the existence of boson stars: (a) The first, scalar particle is so called the Higgs particle has been found by the LHC, although its instability causes that it dose not considered as the fundamental constituent of boson stars. (b) Far from the quantum particle regime of the LHC, the LIGO-Virgo collaboration directly detected gravitational waves in 2015, which were completely consistent with the merger of a binary black hole system, as predicted previously by general relativity [8]. Hence, future work on boson stars may be experimental more, particularly if fundamental scalar fields can be observed or if evidence arises indicating that the boson stars uniquely fit galactic dark matter. But regardless of any experimental results found by these remarkable experiments, there will always be unexplored regimes by experiments where boson stars will find a natural place. Boson stars have a long history as candidates for all manner of phenomena, from fundamental particle, to galactic dark matter. A huge variety of solutions have been found and their dynamics have been studied. Mathematically, boson stars are fascinating soliton-like solutions. Astrophysically, they represent possible explanations of black hole candidates and dark matter, with observations constraining properties of boson stars. Therefore, we would like to use in this paper, an EKG scalar tensor gravity model in the presence of a CW potential [15] and we investigate that how could formed a CW boson star such that remain as stable. As we mentioned in the abstract section our motivation comes from importance of such a potential in rate of cosmic inflation. Usually Boson stars have not sharp surface same as ordinary or neutron stars which their radius are determined by setting with zero value the matter density and the radial pressure. Density and pressures asymptotically vanish at infinite distance usually. One of the merits of this paper is that it shows that the CW potential ensures that the boson particles in such a boson star do not decay quickly and thus the CW boson star have a hard surface with finite radius. Layout of the paper is organized as follows.

In section 2, we define the EKG gravity model and present the corresponding field equations. In section 3, we generate the explicit form of the equations for a spherically symmetric static curved line element. We find linear order solutions of the nonlinear equations of the fields. This is done via dynamical system approach. We determined the critical points of the fields equations

and calculated Jacobi matrix of the fields equations in these critical points. Then we solved secular equation of the Jacobi matrix and at last interpret situations where the system remain as stable. In section 4, we interpret our obtained solutions and we plotted figures of physical quantities versus the radius and density parameters of the CW boson star. Mathematical derivations show that this is the boson mass which controls values of the star radius but in presence of the CW self-interaction potential. The summary and outlook of this work are dedicated to the last section.

## 2 The gravity model

As we pointed out above, in the case of boson stars, the energy-momentum content is that of a complex valued scalar field and so let's start with EKG scalar tensor gravity with a complex form for the scalar KG field such that

$$I = \frac{1}{16\pi} \int d^4x \sqrt{g} [R - \frac{1}{2}g^{\mu\nu}(\partial_\mu\psi^*\partial_\nu\psi + \partial_\mu\psi\partial_\nu\psi^*) - m^2\psi\psi^* - V(|\psi|)] \quad (2.1)$$

where  $g$  is absolute value of determinant of the metric field  $g_{\mu\nu}$ ,  $R$  is Ricci scalar and  $m$  is mass parameter of the complex KG scalar field  $\psi$  (with complex conjugate  $\psi^*$  and norm  $|\psi| = \sqrt{\psi\psi^*}$ ).  $V(|\psi|)$  is arbitrary self-interaction potential of the KG field. We use geometric unites  $c = 1 = G$  with metric signature  $(+, -, -, -)$  in which  $\psi$  is dimensionless, but  $m$  has inverse of the length dimension and the dimensions for self interaction potential  $V(|\psi|)$  is  $(length)^{-2}$  too. To find the metric field of a boson star we should first choose a particular form for  $V(|\psi|)$ . Several types of boson stars, along with the definitions of their corresponding potentials, are presented in the article [9]. We choose here the CW potential and consider its effect on the stability of the boson star. At first, we present a short review about the historical importance of the CW potential as follows:

Historically, the Higgs potential  $V(|\psi|) = \lambda|\psi|^4$  with coupling constant  $\lambda > 0$  was used to describe the chaotic inflation in cosmology. Ordinarily,  $\lambda < 0$  is forbidden on the grounds that it leads to a potential without a lower bound but of course, this statement pertains only to the zero Feynman loop approximation of the effective potential, i.e., the loop contributions may (or may not) provide a lower bound on the effective potential even for negative  $\lambda$  [15]. It is obvious that the above action functional contains a discrete symmetry under the transformation of  $\psi \rightarrow -\psi$  for the Higgs potential  $\lambda|\psi|^4$

[16]. One may ask a question such that how this symmetry can be broken in both the classical and the quantum regimes of the KG field by modifying the potential  $\lambda|\psi|^4$ ? It is easy to check that the vacuum expectation value of the KG field is found at the extremum of the potential  $m^2|\psi|^2 + \lambda|\psi|^4$ . This means that in the vacuum state the fluctuations of the KG field should vanish. The vacuum expectation value occurs at  $\langle |\psi| \rangle_{vac} = 0$  which is symmetric for  $m^2 > 0$  (the real physical particles) but is anti-symmetric  $\langle |\psi| \rangle_{vac} = \pm\sqrt{-m^2/2\lambda}$  for  $m^2 < 0$  (the tachyonic non-physical particles). In other words, if the mass term is tachyonic,  $m^2 < 0$  there is a spontaneous breaking of the gauge symmetry at low energies, a variant of the Higgs mechanism. On the other hand, if the squared mass is  $m^2 \geq 0$  the vacuum expectation of the field  $\psi$  is zero and the symmetry breaking is not occur.

To give a real perspective about the symmetry breaking, Erick Weinberg and his supervisor Sidney Coleman demonstrated [15] that even if the re-normalized mass is zero, the spontaneous symmetry breaking still happens due to the radiative corrections from at least one-loop Feynman diagrams in the interacting KG field. In short, they showed that radiative corrections to mass re-normalized effective potential generates a logarithmic singular term in the effective potential called as the generalized CW potentials such that

$$V_{cw}(|\psi|) = \lambda \left[ |\psi|^4 \ln \left( \frac{|\psi|}{|\psi_0|} \right) - \frac{1}{4} |\psi|^4 + \frac{1}{4} |\psi_0|^4 \right] \quad (2.2)$$

where in the geometric units  $c = G = 1$  the dimensions for the coupling constant  $\lambda$  is  $(length)^{-2}$  because  $|\psi|$  is dimensionless. This model introduces a mass scale  $|\psi_0|$  for a classically conformal theory of the model with a conformal anomaly. In fact  $|\psi_0|$  is originated from ultraviolet cutoff frequency in calculating the Feynman path integrals of radiative corrections in one loop level which should be re-normalized. This in fact describes that how massive (Goldstone) gauge bosons can be created from the self-interaction of elementary massless bosons which is called as the Higgs mechanisms (related to the Goldstone's theorem [17]). The potential (2.2) is applicable in the new inflation in cosmic models because the potential is very flat and has a maximum value at  $|\psi| = 0$  (see figure 1-a). The scalar field dose not escape from classical tunneling via Sphalerons at high temperature, but due to quantum fluctuations via Instantons at low temperatures, (see chapter four in ref. [18]). This kind of potential describes self-reproduction of the universe and primordial inhomogeneities of the universe successfully but, after the end of the primordial inflation [18]. To see importance of the CW

potential more, one can follow other references for instance [19], [20], [21], [22],[23]. Despite the usefulness of this type of potential in theoretical studies of cosmic inflation, there are some inconsistencies with observational data from Planck 2018 and even DESI2024 datasets. For instance the tensor-to-scalar ratio in CW inflation dose not satisfied the observational date (see page 6 in ref. [24]). However, we like to investigate at below, the physical effects of the CW potential (2.2) on formation and stability of a boson star in presence of fluctuations of massive bosons.

By varying the above action functional with respect to the fields  $\psi^*$  and  $g^{\mu\nu}$  we obtain equations of motion for the KG field  $\psi$  and the metric field respectively such that

$$\square\psi - m^2\psi - 2\lambda(\psi\psi^*)\psi \ln\left(\frac{\psi\psi^*}{\psi_0\psi_0^*}\right) = 0, \quad \square = g^{-\frac{1}{2}}\partial_\mu(g^{\frac{1}{2}}g^{\mu\nu}\partial_\nu) \quad (2.3)$$

and

$$G_{\mu\nu} = T_{\mu\nu}(|\psi|), \quad (2.4)$$

where

$$T_{\mu\nu}(|\psi|) = \frac{1}{2}(\partial_\mu\psi^*\partial_\nu\psi + \partial_\mu\psi\partial_\nu\psi^*) - \frac{g_{\mu\nu}}{2}\left\{\frac{1}{2}g^{\alpha\beta}(\partial_\alpha\psi^*\partial_\beta\psi + \partial_\alpha\psi\partial_\beta\psi^*) + m^2\psi\psi^* + \lambda\left[\frac{(\psi\psi^*)^2}{2}\ln\left(\frac{\psi\psi^*}{\psi_0\psi_0^*}\right) - \frac{1}{4}(\psi\psi^*)^2 + \frac{1}{4}(\psi_0\psi_0^*)^2\right]\right\}. \quad (2.5)$$

We should remember that this model contains an additional conserved Noether current

$$j^\mu = -\frac{i}{2}(\psi^*\nabla^\mu\psi - \psi\nabla^\mu\psi^*), \quad \nabla_\mu j^\mu = 0 \quad (2.6)$$

due to the internal global  $U(1)$  symmetry  $\psi \rightarrow e^{i\chi}\psi$  in which  $\chi$  is a constant field. In the subsequent section, we set these equations for a spherically symmetric static curved line element to investigate formation of a boson star.

### 3 Coleman-Weinberg boson star

Let's start with the following ansatz for the spherically symmetric complex KG scalar field which makes a boson star [25]

$$\psi(t, r) = e^{i\omega t}\phi(r), \quad \omega > 0. \quad (3.1)$$

We assume that the above KG scalar field participates in the self interaction CW potential (2.2) for which the isotropic spherically symmetric curved space-time line element is given in the standard form by (see for instance [26] chapter 8)

$$ds^2 = e^{U(r)}dt^2 - e^{H(r)}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (3.2)$$

Substituting (3.1), the KG wave equation (2.3) reads

$$\phi'' + \left(\frac{2}{r} + \frac{U'}{2} - \frac{H'}{2}\right)\phi' + e^H \left[ (\omega^2 e^{-U} + m^2)\phi + 4\lambda\phi^3 \ln\left(\frac{\phi}{\mu}\right)\right] = 0 \quad (3.3)$$

where  $'$  is derivative with respect to  $r$  coordinate, the constant parameter  $\mu = |\psi_0|$  comes from ultraviolet cut off scales when we do evaluate re-normalized expectation values of the quantum KG field  $\psi$  in presence of the radiative corrections of the Feynman diagrams. Substituting the line element (3.2) into the Einstein equation (2.4) we find

$$\begin{aligned} & \frac{H'}{r} - \frac{1 - e^H}{r^2} + \frac{\phi'^2}{2} + \frac{\omega^2 \phi^2 e^{H-U}}{2} \\ & - \frac{e^H}{2} \left\{ m^2 \phi^2 + \lambda \left[ \phi^4 \ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4}\phi^4 + \frac{1}{4}\mu^4 \right] \right\} = 0 \end{aligned} \quad (3.4)$$

for tt component

$$\begin{aligned} & \frac{U'}{r} + \frac{\phi'^2}{2} + \frac{1 - e^H}{r^2} + \frac{\omega^2 \phi^2 e^{H-U}}{2} \\ & + \frac{e^H}{2} \left\{ m^2 \phi^2 + \lambda \left[ \phi^4 \ln\left(\frac{\phi}{\mu}\right) - \frac{\phi^4}{4} + \frac{\mu^4}{4} \right] \right\} = 0 \end{aligned} \quad (3.5)$$

for rr component and

$$\begin{aligned} & U'' + \frac{U'^2}{2} - \frac{H'U'}{2} + \frac{U'}{r} - \frac{H'}{r} - \phi'^2 + \omega^2 \phi^2 e^{H-U} \\ & + e^H \left[ m^2 \phi^2 + \lambda \left[ \phi^4 \ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4}\phi^4 + \frac{1}{4}\mu^4 \right] \right] = 0 \end{aligned} \quad (3.6)$$

for angular components. In the above equations we used the density  $\rho(r)$ , the radial pressure  $p_r(r)$  and the transverse pressure  $p_t(r)$  of the KG field

together with the CW potential (2.2) such that

$$\begin{aligned}\rho &= T_t^t = \frac{e^{-H}\phi'^2}{2} + \frac{e^{-U}\omega^2\phi^2}{2} - \frac{1}{2}\left\{m^2\phi^2 + \lambda\left[\phi^4\ln\left(\frac{\phi}{\mu}\right) - \frac{1}{4}\phi^4 + \frac{1}{4}\mu^4\right]\right\} \\ p_r &= T_r^r = p_t - e^{-H}\phi'^2 \\ p_t &= T_\theta^\theta = T_\varphi^\varphi = \rho + \omega^2\phi^2e^{-U}.\end{aligned}\quad (3.7)$$

In fact, to have explicit from of the fields solutions  $\phi(r)$ ,  $U(r)$  and  $H(r)$  we need three equations from (3.3), (3.4), (3.5) and (3.6) only and so one of them is a constraint condition between the solutions. This claim is proved via conservation equation of the stress tensor of the gravitational system or equivalently the Bianchi identity in which related three components of the Einstein metric equations (3.4), (3.5) and (3.6) to each other. The covariant conservation of matter stress tensor or Bianchi's identity  $\nabla_\mu G_\nu^\mu = \nabla_\mu T_\nu^\mu = 0$  defined by internal metric of the stellar object is usually called as Tolman-Oppenheimer-Volkoff (TOV) equation (see for instance [27] and [28]) which for the line element (3.2) reads to the following form

$$p'_r - \frac{U'}{2}\rho + \left(\frac{U'}{2} + \frac{2}{r}\right)p_r - \frac{2p_t}{r} = 0 \quad (3.8)$$

where  $\rho(r)$ ,  $p_r(r)$  and  $p_t(r)$  should be substituted by (3.7). If there is determined form of the equation of state of the gravitational system then one can solve the equation of TOV instead of the Einstein field equations. The equation of state is a relationship between the directional pressures  $p_{r,t}$  and the density function  $\rho$ . Although in the literature some of equation of states are presented for relativistic stars but we do not use them same as our previous work [10] and we like to be free in this work such that after to solve the metric equations then we find explicit form by eliminating the radius parameter between (3.7) (see figures 3, 4). Thus to determine explicit form of the functions  $U(r)$ ,  $H(r)$  and  $\phi(r)$  we choose (3.3) and two new generated equations  $(3.4) + (3.5) = 0$  and  $2(3.4) + (3.6) = 0$  which in a dimensionless forms they are respectively

$$\ddot{\sigma} + \dot{\sigma}^2 + \left(\frac{2}{\tau} + \frac{\dot{U}}{2} - \frac{\dot{H}}{2}\right)\dot{\sigma} + e^H\left[\bar{\omega}^2e^{-U} + \bar{m}^2 + 4\epsilon\mu^2\sigma e^{2\sigma}\right] = 0, \quad (3.9)$$

$$\dot{H} = -\dot{U} - \tau\mu^2e^{2\sigma}(\dot{\sigma}^2 + \bar{\omega}^2e^{H-U}) \quad (3.10)$$

and

$$\ddot{U} + \frac{\dot{U}^2}{2} - \frac{\dot{H}\dot{U}}{2} + \frac{\dot{H} + \dot{U}}{\tau} - \frac{2(1 - e^H)}{\tau^2} + 2\bar{\omega}^2\mu^2e^{2\sigma+H-U} = 0 \quad (3.11)$$

in which we defined dimensionless quantities as

$$\begin{aligned} \sigma &= \ln(\phi/\mu), & \tau &= \frac{r}{\ell}, & \epsilon &= \lambda\ell^2, & \bar{m} &= m\ell, \\ \bar{\omega} &= \omega\ell, & \prime &= \frac{d}{dr} = \frac{1}{\ell} \frac{d}{d\tau} = \cdot. \end{aligned} \quad (3.12)$$

The equations (3.9) , (3.10) and (3.11) are nonlinear second order differential equations and to solve them we are free to choose two different ways, i.e., the numeric or the analytic perturbation series methods. We use perturbation series method to find analytic solutions near the asymptotic surfaces  $\tau \rightarrow 0$  and  $\tau \rightarrow \infty$ . To do this, we use dynamical system approach where each of higher order differential equation transforms to several differential equations with first order by defining new fields such that

$$\dot{\sigma} = x, \quad \dot{U} = y. \quad (3.13)$$

Substituting (3.13) and  $\dot{H}$  given by (3.10) the equations (3.9), (3.10) and (3.11) can be rewritten respectively as follows

$$\begin{aligned} \dot{x} &= -x^2 - x[2/\tau + (\tau/2)\mu^2e^{2\sigma}(x^2 + \bar{\omega}^2e^{H-U})] \\ &\quad - \bar{\omega}^2e^{H-U} - \bar{m}^2e^H - 4\epsilon\mu^2\sigma e^{H+2\sigma} \\ \dot{H} &= -y - \tau\mu^2e^{2\sigma}(x^2 + \bar{\omega}^2e^{H-U}) \\ \dot{y} &= -y^2 - (\tau/2)\mu^2ye^{2\sigma}(x^2 - \bar{\omega}^2e^{H-U}) + \mu^2e^{2\sigma}(x^2 + \bar{\omega}^2e^{H-U}) \\ &\quad + (2/\tau^2)(1 - e^H) - 2\bar{\omega}^2\mu^2e^{H-U+2\sigma}. \end{aligned} \quad (3.14)$$

From point of the dynamical system approach, the set of variables  $\{x, H, y, \sigma, U\}$  make 5 dimensional phase space which satisfies the 5 nonlinear differential equations (3.13) and (3.14). To solve these equations with a dynamical system approach one usually should expand them around the critical points in phase space which are obtained from the equations

$$\dot{x} = 0, \quad \dot{H} = 0, \quad \dot{y} = 0, \quad \dot{\sigma} = 0, \quad \dot{U} = 0 \quad (3.15)$$

and then keep up linear order perturbation series expansion of the equations (3.13) and (3.14) near the critical points which their coefficients are called

usually as the Jacobi matrix. By determining sign of eigenvalues of this Jacobi matrix one can find nature of stability of the obtained solutions and also by solving the linear order equations one can obtain asymptotic solutions of the fields near the critical points (see suitable text books for instance [29] or introduction section of the ref. [30] for more descriptions about the dynamical system approach). After to present a little description about the dynamical system approach we are now ready to calculate the critical points of the equations (3.13) and (3.14). In this case, the equations (3.15) give the following values for the critical points

$$P_c = \{x_c = 0, \quad y_c = 0, \quad H_c = 0, \quad e^{-U_c} = -\frac{\bar{m}^2}{\bar{\omega}^2}, \quad \sigma_c = -\infty(\phi_c = \mu)\}. \quad (3.16)$$

Elements of the Jacobi matrix on the critical point above are obtained as

$$J_{ij}|_{P_c} = \frac{\partial \dot{x}_i}{\partial x_j}|_{P_c} = \begin{pmatrix} -2/\tau & 0 & 0 & 0 & -\bar{m}^2 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -2/\tau^2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \quad (3.17)$$

where we consider the asymptotic surface  $\tau \rightarrow 0$  because a boson star is a compact object and so its matter is localized. As an extension of the work one can seek same calculates for regimes  $\tau \rightarrow \infty$  where the boson particles make a cloud distribution. However one can solve secular equation  $\det(J_{ij} - \delta_{ij}E) = 0$  of the Jacobi matrix above to find eigenvalues  $E$  as

$$E^2(E + \frac{2}{\tau})(E^2 - \frac{2}{\tau^2}) = 0 \quad (3.18)$$

with solutions

$$E_{1,2} = 0, \quad E_3 = -\frac{2}{\tau}, \quad E_{4,5} = \pm \frac{\sqrt{2}}{\tau} \quad (3.19)$$

where  $\tau > 0$  and so positive values of the eigenvalues above show instability nature of the system but negative values show stable nature.  $E_{1,2}$  with zero values show that the system is degenerate in stabilization and to make stable more we should consider other interaction potentials such that these degeneracy break to make negative eigenvalues more than. At present form the obtained solutions show quasi-stable nature of the system because some

of the eigenvalues have negative sign but some of them have positive sign (see figure 1-b). Linear order form of the equations (3.13) and (3.14) read

$$\frac{d}{d\tau} \begin{pmatrix} x \\ H \\ y \\ \sigma \\ U \end{pmatrix} = \begin{pmatrix} -2/\tau & 0 & 0 & 0 & -\bar{m}^2 \\ 0 & 0 & -1 & 0 & 0 \\ 0 & -2/\tau^2 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ H \\ y \\ \sigma \\ U \end{pmatrix}. \quad (3.20)$$

It is not complicated to find solutions of the above matrix equation as

$$\begin{pmatrix} x \\ H \\ y \\ \sigma - \sigma_c \\ U - U_c \end{pmatrix} = \tau^\eta \begin{pmatrix} \bar{m}^2\tau/(3+\eta) \\ 1 \\ -\eta/\tau \\ \bar{m}^2\tau^2/(2+\eta)(3+\eta) \\ -1 \end{pmatrix}, \quad \eta_\pm = \frac{1 \pm 3}{2} \quad (3.21)$$

for which we obtain

$$\begin{aligned} \phi &= \mu \exp \left[ \frac{\bar{m}^2\tau^{2+\eta}}{(2+\eta)(3+\eta)} \right], \quad e^U = -\frac{\bar{\omega}^2}{\bar{m}^2} \exp(-\tau^\eta), \quad e^H = \exp(\tau^\eta) \\ \phi^+ &= \mu \exp \left[ \frac{\bar{m}^2\tau^4}{20} \right], \quad e^{U_+} = -\frac{\bar{\omega}^2}{\bar{m}^2} \exp(-\tau^2), \quad e^{H_+} = \exp(\tau^2) \\ \phi^- &= \mu \exp \left[ \frac{\bar{m}^2\tau}{2} \right], \quad e^{U_-} = -\frac{\bar{\omega}^2}{\bar{m}^2} \exp(-\tau^{-1}), \quad e^{H_-} = \exp(\tau^{-1}) \end{aligned} \quad (3.22)$$

which show two different kind of solutions. However, by substituting these two different branches of solutions into the stress tenor equations (3.7) we

find explicit form of the density and directional pressures as

$$\begin{aligned}
\rho^+ &= \frac{\mu^2}{2\ell^2} \left\{ \left( \frac{\bar{m}^2 \tau^3}{5} \right)^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} - \tau^2 \right) - \bar{m}^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} \right) \right. \\
&\quad \left. - \bar{m}^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} + \tau^2 \right) - \frac{\epsilon \mu^2}{4} \left[ 1 + \left( \frac{\bar{m}^2 \tau^4}{5} - 1 \right) \exp \left( \frac{\bar{m}^2 \tau^4}{5} \right) \right] \right\} \\
p_r^+ &= \frac{\mu^2}{2\ell^2} \left\{ 3 \left( \frac{\bar{m}^2 \tau^3}{5} \right)^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} - \tau^2 \right) - \bar{m}^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} \right) \right. \\
&\quad \left. - 3 \bar{m}^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} + \tau^2 \right) - \frac{\epsilon \mu^2}{4} \left[ 1 + \left( \frac{\bar{m}^2 \tau^4}{5} - 1 \right) \exp \left( \frac{\bar{m}^2 \tau^4}{5} \right) \right] \right\} \\
p_t^+ &= \frac{\mu^2}{2\ell^2} \left\{ 5 \left( \frac{\bar{m}^2 \tau^3}{5} \right)^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} - \tau^2 \right) - \bar{m}^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} \right) \right. \\
&\quad \left. - 3 \bar{m}^2 \exp \left( \frac{\bar{m}^2 \tau^4}{10} + \tau^2 \right) - \frac{\epsilon \mu^2}{4} \left[ 1 + \left( \frac{\bar{m}^2 \tau^4}{5} - 1 \right) \exp \left( \frac{\bar{m}^2 \tau^4}{5} \right) \right] \right\}
\end{aligned} \tag{3.23}$$

for + sign solutions and

$$\begin{aligned}
\rho^- &= \frac{\mu^2}{2\ell^2} \left[ 1 + \left( \frac{\bar{m}^2}{2} \right)^2 \exp \left( \bar{m}^2 \tau - \frac{1}{\tau} \right) \right. \\
&\quad \left. - \bar{m}^2 \exp \left( \bar{m}^2 \tau + \frac{1}{\tau} \right) + \frac{\epsilon \mu^2}{2} [2 \bar{m}^2 \tau - 1] \exp(2 \bar{m}^2 \tau) \right] \\
p_r^- &= \frac{\mu^2}{2\ell^2} \left[ 1 + 3 \left( \frac{\bar{m}^2}{2} \right)^2 \exp \left( \bar{m}^2 \tau - \frac{1}{\tau} \right) \right. \\
&\quad \left. - 3 \bar{m}^2 \exp \left( \bar{m}^2 \tau + \frac{1}{\tau} \right) + \frac{\epsilon \mu^2}{2} [2 \bar{m}^2 \tau - 1] \exp(2 \bar{m}^2 \tau) \right] \\
p_t^- &= \frac{\mu^2}{2\ell^2} \left[ 1 + 5 \left( \frac{\bar{m}^2}{2} \right)^2 \exp \left( \bar{m}^2 \tau - \frac{1}{\tau} \right) \right. \\
&\quad \left. - 3 \bar{m}^2 \exp \left( \bar{m}^2 \tau + \frac{1}{\tau} \right) + \frac{\epsilon \mu^2}{2} [2 \bar{m}^2 \tau - 1] \exp(2 \bar{m}^2 \tau) \right]
\end{aligned} \tag{3.24}$$

for - sign solutions respectively. In the following section we investigate physical properties of the obtained solutions above.

## 4 Physical interpretations of the solutions

As we see at below that our model predicts a particular CW boson star with 'hard surface' for which the matter density and radial pressure vanish at a finite radius parameter  $s$  defined by the boson mass (see Eqs. (4.3) and (4.6) at below), but in many of models, the boson stars have not a hard surface same as neutron stars and density and directional pressures vanish asymptotically at infinite distances (see for instance [25]). Looking at the figures 3-c and 4-a one infer that the directional pressures vanish just for particular value of the matter density in case of + sign of the solutions and by comparing with the figures (3-d) and (4-b) we obtain that this situation is not happened for - sign solutions. Furthermore negativity values of the density in figures (3-d) and (4-b) forces us to decide that the minus sign of the solutions are not physical or it describes anti-matter distribution of the star (say anti-star) with negative energies. However we continue our statements just for solutions with + sign solutions which at least give us some regular understandable concepts. In this case prediction of a CW boson star with finite scale can be considered an advantage and privilege of this current work which come from CW self-interaction potential between the KG bosons. In cases where the boson stars have not hard surface, one usually use an estimated radius

$$R = \frac{1}{Q} \int dx^3 \sqrt{g} r j^t = \frac{4\pi\omega}{Q} \int dr r^3 \phi^2 e^{\frac{H-U}{2}} \quad (4.1)$$

in which  $Q$  is Noether charge such that

$$Q = - \int dx^3 \sqrt{g} j^t = 4\pi\omega \int r^2 dr \phi^2 e^{\frac{H-U}{2}}, \quad j^t = \omega \phi^2 e^{-U}. \quad (4.2)$$

In the above relation  $j^t$  is the locally conserved current associated to the globally conserved Noether charge  $Q$ . In fact, in the model with un-gauged U(1) symmetry, the Noether charge  $Q$  is usually interpreted as the number of boson particles with mass ' $m$ ' that make up the boson star. Usually, the scalar field making up the boson star decays fast and so has not a hard surface. In fact presence of the CW potential in this work causes that the bosons do not decay fast and so surface of such a boson star remains stable. Hence we do not apply to calculate estimated radius (4.1) of this kind of boson star but we determine exact form of the radius parameter by solving

the equations  $\rho^+(s) = 0 = p_r^+(s)$  for (3.23) such that

$$\bar{m}_+^2 = \frac{5e^{s^2}}{s^3} \quad (4.3)$$

with particular dimensionless CW potential coupling constant

$$\epsilon^+ = \frac{4}{5\mu^2} \left[ \frac{(1-s^2)e^{s^2} - s^2}{(se^{s^2} - 1)\exp[\frac{s}{10}e^{s^2}] + \exp[-\frac{s}{10}e^{s^2}]} \right]. \quad (4.4)$$

Substituting the above conditions into the density and pressures given by the equations (3.23) we find

$$\begin{aligned} \rho^+ &= \frac{\mu^2}{2\ell^2} \left\{ \exp[s^2 + se^{s^2/2}] - \frac{5\exp[s^2 + se^{s^2/2}]}{s^3} - \frac{5\exp[2s^2 + se^{s^2/2}]}{s^3} \right. \\ &\quad \left. - \frac{1}{5} \frac{[(1-s^2)e^{s^2} - s^2][1 + (se^{s^2} - 1)\exp[se^{s^2}]]}{(se^{s^2} - 1)\exp[se^{s^2}/10] + \exp[-se^{s^2}/10]} \right\} \\ p_r^+ &= \frac{\mu^2}{2\ell^2} \left\{ 3\exp[s^2 + se^{s^2/2}] - \frac{5\exp[s^2 + se^{s^2/2}]}{s^3} - \frac{15\exp[2s^2 + se^{s^2/2}]}{s^3} \right. \\ &\quad \left. - \frac{1}{5} \frac{[(1-s^2)e^{s^2} - s^2][1 + (se^{s^2} - 1)\exp[se^{s^2}]]}{(se^{s^2} - 1)\exp[se^{s^2}/10] + \exp[-se^{s^2}/10]} \right\} \\ p_t^+(s) &= \frac{5\mu^2}{2\ell} \exp\left(\frac{s}{2}e^{s^2}\right) \left[ \left(1 - \frac{1}{s^3}\right)e^{s^2} - \frac{3e^{2s^2}}{s^3} \right. \\ &\quad \left. - \frac{\exp[-\frac{s}{2}e^{s^2}]}{25s^2} - \frac{e^{s^2+(s/2)\exp(s^2)}}{25s} + \frac{\exp[\frac{s}{2}e^{s^2}]}{25s^2} \right] \end{aligned} \quad (4.5)$$

which are defined versus the radius parameter  $s$ .

With same calculations, we can find position of radius of the CW boson stars by solving  $\rho^-(s) = 0$  and  $p_r^-(s) = 0$  such that

$$\bar{m}_- = \exp\left(\frac{1}{s}\right) \quad (4.6)$$

with CW potential coupling constant

$$\epsilon^- = \frac{2}{\mu^2} \left( \frac{\exp[-8se^{\frac{2}{s}}]}{1 - 8se^{\frac{2}{s}}} \right) \quad (4.7)$$

and transverse pressure

$$p_t^-(s) = \frac{4\mu^2}{\ell^2} \exp\left(\frac{3}{s} + 4se^{\frac{2}{s}}\right). \quad (4.8)$$

By looking at the positive sign metric solution one infer that it behaves as a Minkowski flat space time asymptotically at central regions  $\tau \rightarrow 0$  while the metric solution with negative sign treats same but at far from the central region  $\tau \rightarrow \infty$ . Signature of both of the metric solutions are Euclidian  $(-, -, -, -)$  which means that these metric solutions are for inside of the star. We plotted the mass functions (4.3) and (4.6) versus the radius parameter  $s$  in figures 2-a and 2-b respectively. Comparing them, one infer that there is a local minimum point for  $\bar{m}_+$  but not for  $\bar{m}_-$ . Minimum value of the mass  $\bar{m}_+$  is  $\bar{m}_{min}^+ = 12.198$  at particular radius  $s_p = \sqrt{1.5} = 1.2248$ . Physically, this minimum point describes that positive sign branch of the solutions tend to be remain as stable around this minimum point but the solutions with negative sign are not tend. Looking at the figure 2-b, one infer that the figure for  $\bar{m}_-$  is absolutely decreasing function and so has not a local minimum point. It is confirmed by phase space trajectories given by figure 1-b where the system is in quasi-stable (saddle) nature. Because some of eigenvalues of the secular equation of the Jacobi matrix have positive and some other have negative sign. To understand more about physical behavior of the obtained solutions we plot figures of the re-scaled CW potential coupling constant  $\bar{\epsilon}^+ = \mu^2 \epsilon^+ / 4$  and  $\bar{\epsilon}^- = 2\epsilon^- / \mu^2$  versus the radius parameter  $s$  in figures 2-c and 2-d respectively.

Comparing the figures 2-c and 2-d we find that effects of the CW self interaction of KG bosons are dominant for  $\bar{\epsilon}_+$  more than  $\bar{\epsilon}_-$  at smaller scales. Both of them have negative values. Furthermore we plotted figures of the dimensionless transverse pressures for both of the branches of the solutions  $\pm$  in figures 3-a and 3-b. Comparing them, one can infer that the  $+$  sign branch of the field solutions describe dark stars with negative transverse surface pressure but the  $-$  sign branch of the field solutions show a visible boson star with positive transverse surface pressure.

One can check other ways to prove that our obtained solutions do not describe black hole solutions but they show a star with regular metric. From geometrical point of view, one usually solve the horizon equation  $g^{\mu\nu} \partial_\mu \zeta(r) \partial_\nu \zeta(r) = 0$  to determine radius of a black hole solutions where  $\zeta(r)$  is a spherical symmetric surface and it is surface of the black hole horizon if a null vector field to be tangent to it. Using this equation for the obtained solutions above

gives us  $e^{-H_{\pm}} = 0$  which by regarding the solutions (3.22) one find solutions at  $\tau = 0$  and  $\tau \rightarrow \infty$  which have not physical meaning. This can be investigated by other way too as follows: If we calculate values of the metric fields on the surface of the CW boson star by substituting (4.3) and (4.6) into the metric field solutions (3.22), then we find

$$e^{U_+(s)} = -\frac{\bar{\omega}^2}{5}s^3 e^{-2s^2}, \quad e^{H_+(s)} = e^{s^2}, \quad \phi^+(s) = \mu \exp\left(\frac{s}{4}e^{s^2}\right) \quad (4.9)$$

and

$$e^{U_-(s)} = -\bar{\omega}^2 e^{-\frac{3}{s}}, \quad e^{H_-(s)} = e^{\frac{1}{s}}, \quad \phi^-(s) = \mu \exp\left(\frac{s}{2}e^{\frac{2}{s}}\right). \quad (4.10)$$

Obviously, one can see that none of them becomes zero or infinite for a given value of radius  $s$ , which means that the found stellar object is indeed a boson star with a non-singular metric. They do diverge to infinity just for  $s = 0$  and  $s \rightarrow \infty$  which have not physical meaning. At last to study equation of state of the system we plot  $p_r^{\pm}$  versus the density functions  $\rho^{\pm}$  given by (4.5) and (4.8) in figures 3-c and 3-d. One can infer that the figure 3-c shows a physical system in which for smaller densities the radial pressure is negative and describes a collapsing object but for larger densities it is an expanding unstable object because of positivity of the radial pressure. There is just a particular density for which the  $p_r^+$  takes a zero value describing a CW boson star with finite scale. Matter content of a star can be described also by the barotropic index defined by slop of the equation of state. In this way one can look at the figures 3-c and 4-a, then who find a minimum density as  $\rho_{min}^+ \approx 20000 \times (\frac{\mu^2}{2\ell^2})$  with corresponding pressures  $p_r^+ \approx -35000 \times (\frac{\mu^2}{2\ell^2})$  and  $p_t^+ \approx -35000 \times (\frac{5\mu^2}{2\ell^2})$  for which slope of these figures is changed from negative to positive signs. This predicts that for densities less than the minimum value  $\rho^+ < \rho_{min}^+$  the CW boson star treats as a dark star because of negativity of the slope (the barotropic index)  $\gamma_{t,r} = \frac{dp_{t,r}^+}{d\rho^+} < 0$ , while for densities larger than the critical one this star treats as visible stellar object because of positive sign of the slope  $\gamma_{t,r} = \frac{dp_{t,r}^+}{d\rho^+} > 0$ . This means that the solutions with sign of + can be both behavior as visible or invisible (dark) stars depending on whether their density is greater or less than this critical value. The right side figure 3-d has not physical content because of negativity values of the matter density. Hence we exclude solution with negative sign as

a physical solutions and claim that this model gives out a physical solution just with positive sign. We end this section by investigating the anisotropy property of the CW boson star. This is done by plotting the subtraction of the pressures  $p_t - p_r$  called as anisotropy factor given by (4.5) and (4.8) versus the corresponding density functions in figures 4-c and 4-d. They show that the anisotropy is non-vanishing in both of  $\pm$  branches of the solutions throughout the different scales of the densities.

## 5 Summary and outlook

In this work, we used a massive complex KG time dependent scalar field in the presence of self-interaction CW potential, to solve the Einstein metric equations in a spherically symmetric static form. The analysis reveals that the stability and formation of the boson star are significantly influenced by the self-interaction potential, which plays a pivotal role in the scalar field's behavior. Particularly this kind of potential makes a finite value of the radius of the star with hard surface which in usual ways a boson star has not this finite radius. We solved field equations by using dynamical systems approach in which stability of the system is investigated by determining sign of the eigenvalues of the Jacobi matrix of the field equations. Our solutions show a CW boson star which is anisotropic with finite scale. This kind of star is dark invisible or visible, depending on value of its density which whether is less or larger than the critical density. The critical density is defined by the minimum value of the CW potential and a ultraviolet cut-off length scale which we consider to remove divergencies of radiative corrections of Feynman diagrams. This statement is found by investigating the slope of the pressure-density figures in both of radial and transverse pressures. Although we solved small scale regime of the field equations because a boson star should be a compact stellar object but there is different behavior for the field equations at large scales of the boson matter distribution which we do not seek this term at the present work. As an extension of this work we like to investigate the latter problem in our future work.

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### Ethics declarations

### Conflict of interest

The authors have no competing interests to declare that are relevant to the content of this article.

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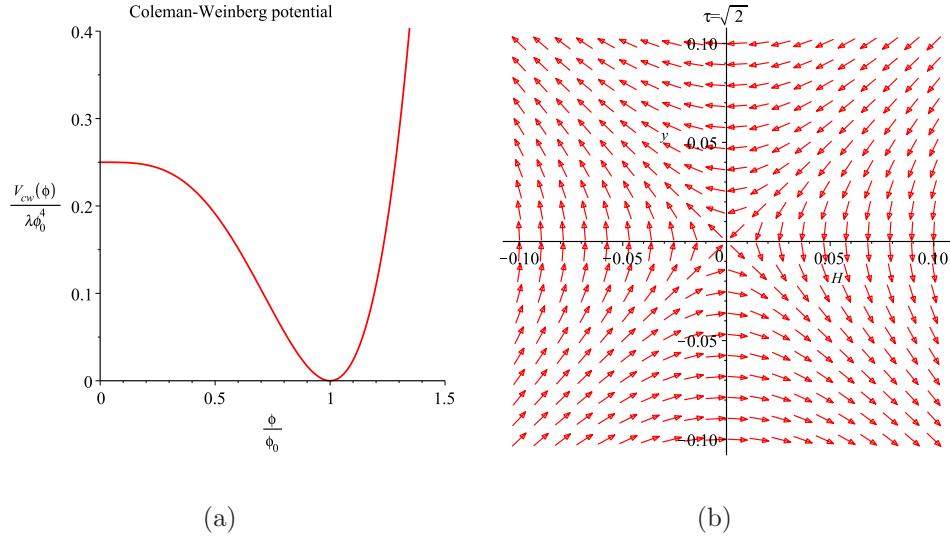


Figure 1: (a) The Coleman-Weinberg potential shows a minim value at  $\phi_0 = \mu$  which our solutions are valid near this minimum point, (b) Phase space trajectories show a quasi stable (saddle) nature for obtained solutions.

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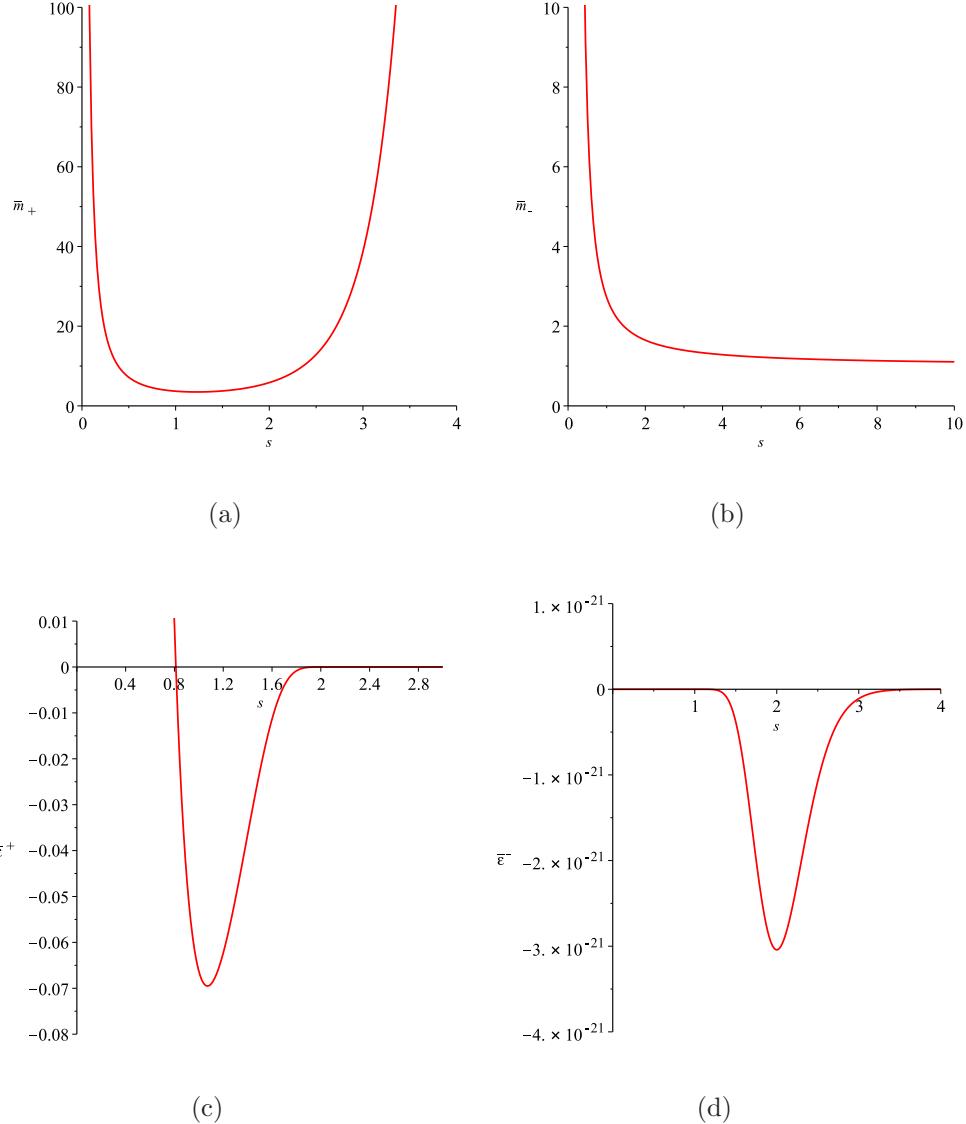


Figure 2: (a) mass parameter of the bosons is plotted versus the radius parameter for + sign solutions where the mass parameter takes a local minimum value at  $s \sim 1.3$ , (b) mass parameter of the bosons is plotted versus the radius parameter for - sign solutions. This choice of the solution has not a minimum point and so it dose not predict a stable state for finite scale CW boson star, (c) CW potential coupling constant is plotted versus the radius parameter of the CW boson star  $s$  for solutions with + sign. It shows that the CW potential coupling constant is dominant just for radiuses  $s < 2$ , (d) CW potential coupling constant is plotted versus the radius parameter of the CW boson star  $s$  for solutions with - sign. It shows that this coupling constant is dominant for radius  $s = 2$  only.

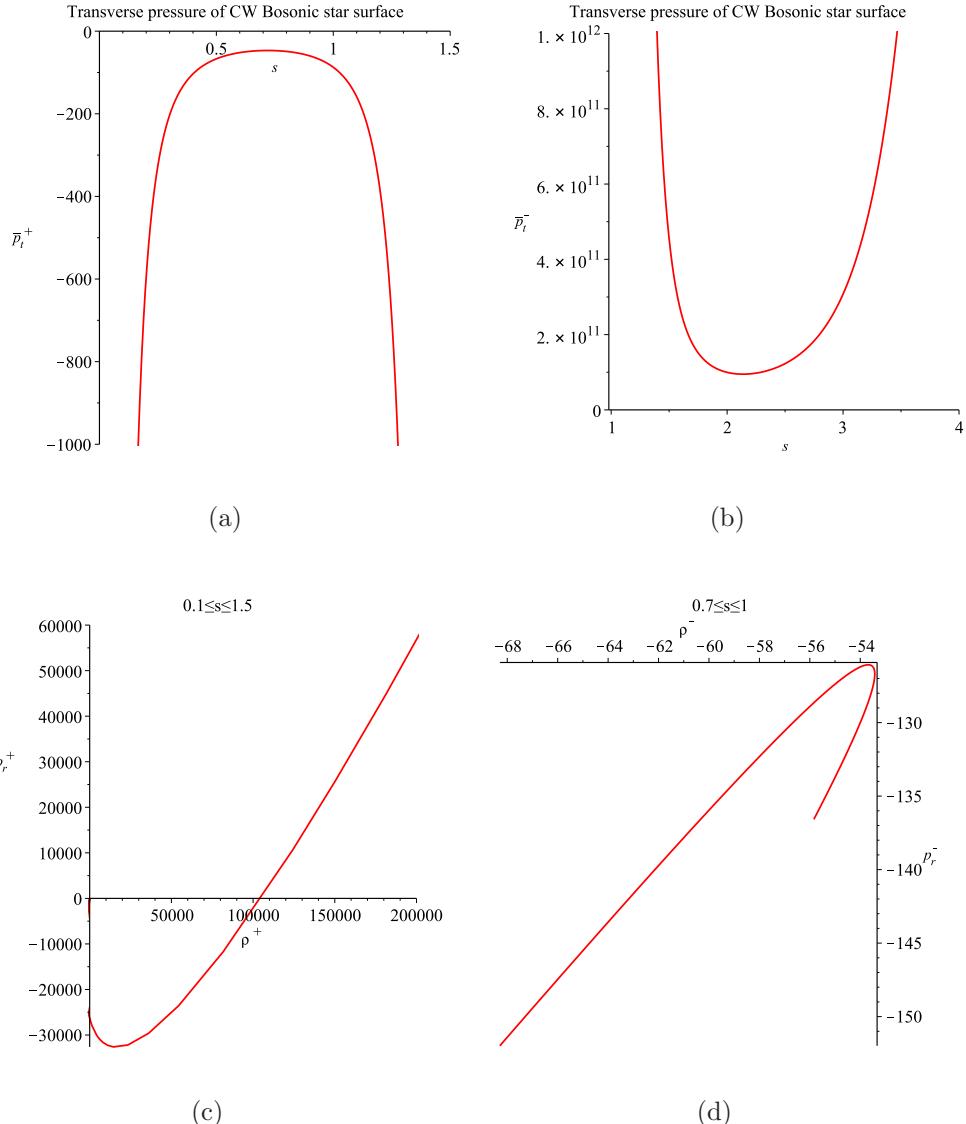


Figure 3: (a) shows variation of transverse pressure versus the radius parameter  $s$  for  $+$  sign solutions. Absolute negative values of this pressure can be able us to claim that the matter content of this star behaves as dark star, (b) shows variation of the transverse pressure versus  $s$  for negative sign solutions. Absolute positive values for this pressure shows that this kind of solution describes a visible star, (c) shows variation of the radial pressure versus the matter/energy density in case of solutions with  $+$  sign. It has a local minimum point. Slope of the figure has negative sign for densities less than the minimum value which means this regime is same as dark star but for densities larger than the minimum density, the slope has positive sign showing visible phase of the star (d) has not physical content because the density takes absolutely negative values.

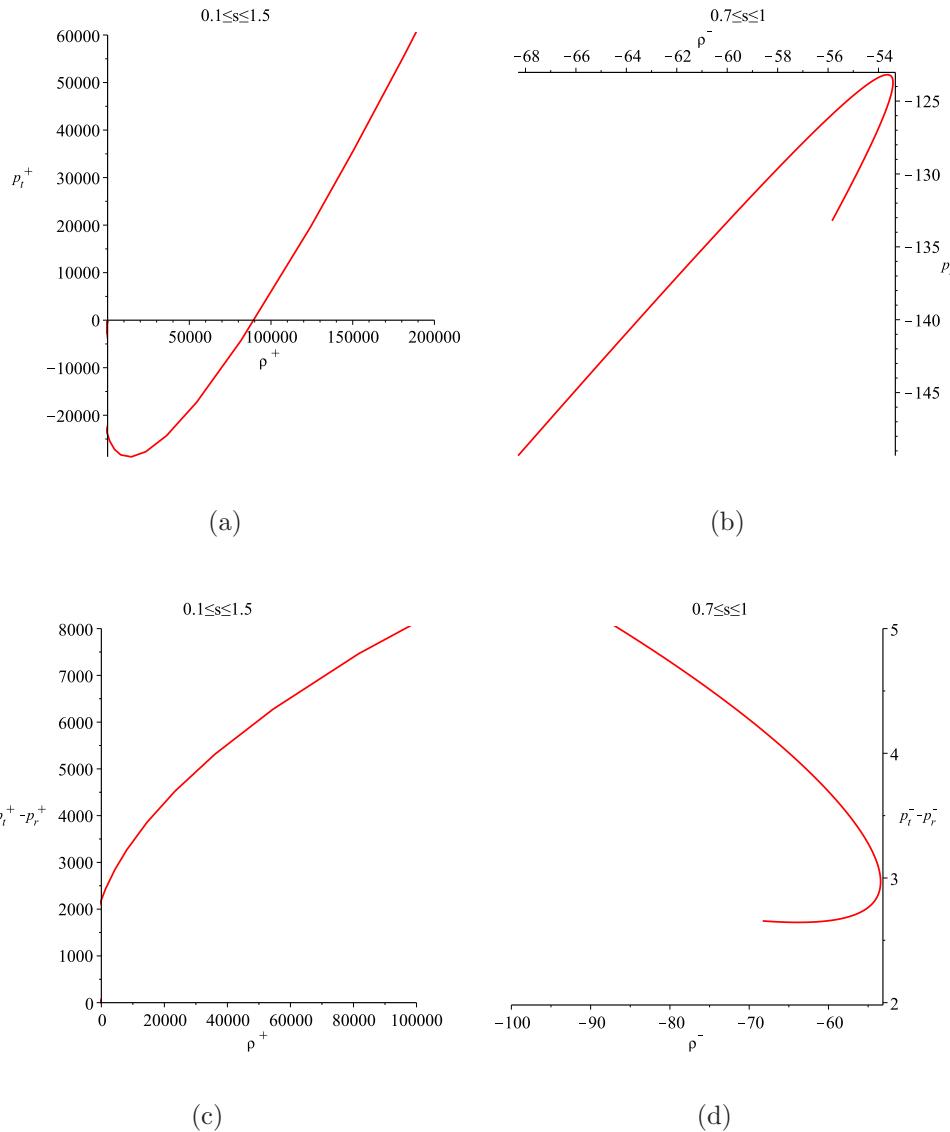


Figure 4: (a) shows variation of transverse pressure versus the matter/energy density for + sign solutions. It has a local minimum point. Slope of the figure has negative sign for densities less than the minimum value which means this regime is same as dark star but for densities larger than this minimum density the slope has positive sign showing visible phase of the star, (b) has not physical content because the density takes absolutely negative values for – sign solution, (c) shows anisotropy of the CW boson star in all scales. (d) has not physics content because of negativity of the density.