Linear Regression

I/P — x | O/P — y

Training

x	y
2	8
	20
4	14
3	11
7	23
4	14

Test

x	y
10	? 32
1	? 5

Prediction → $\hat{y}$

Model pattern

$$y = 3x + 2 + \varepsilon$$

epsilon

error

0

Minimal as possible

predicted

x = Input

y = model

$y = 3x + 2$

3 = Slope

2 = Intercept

$\hat{y}$ = prediction

$$Y = ax + b + \varepsilon$$
$$mx + b + \varepsilon$$
$$\beta_0 x + \beta_1 + \varepsilon$$
$$bx + a + \varepsilon$$

Agenda

Find slope and Intercept

Make the error as minimum as possible

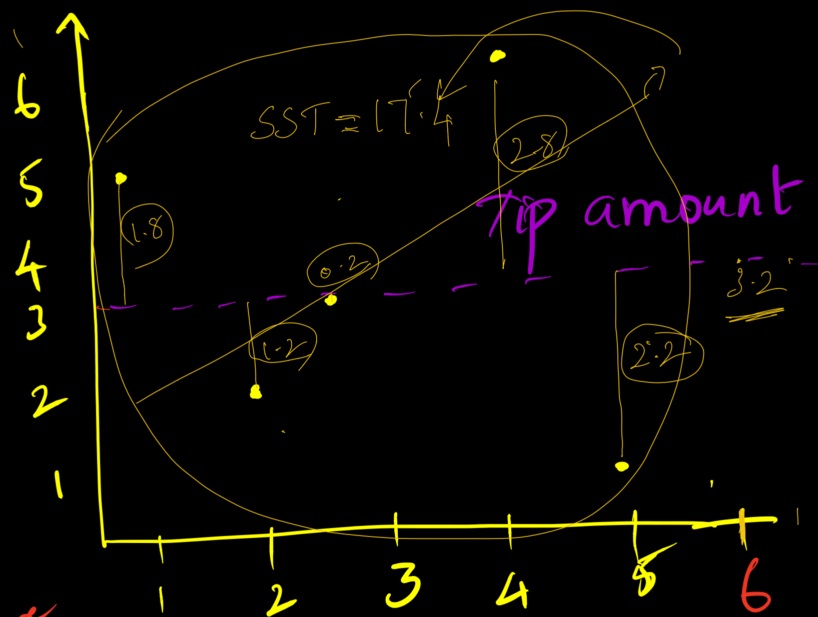
Simple / multi Linear Regression

Linear - Straight Line

Regression - Relationship between two points

Linear Regression - A straight line that attempts to predict the relationship between two points

X Customer	Y Tip amount
1	5
2	2
3	3
4	6
5	1



17/5 = 3.2

Z-Score

4 ?

Mean ± 6

Average

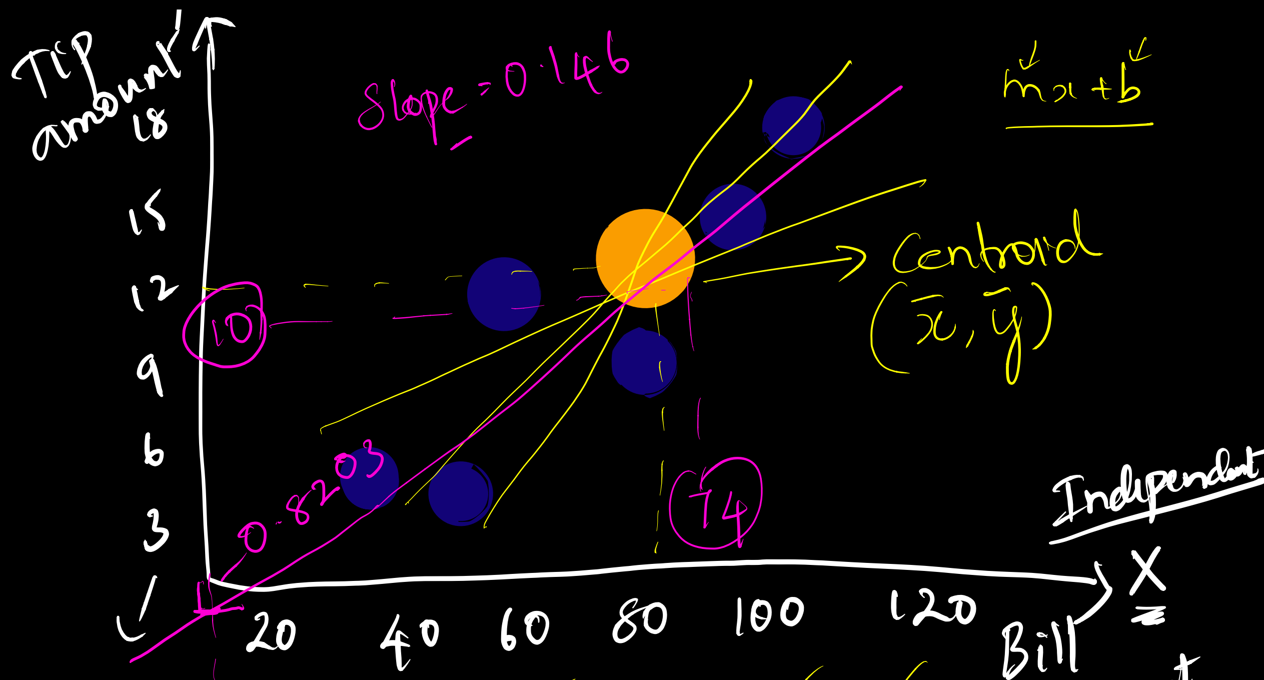
Mean - median ?

Independent

Y

Bill amount	Tip amount
34	5
108	17
64	11
88	8
99	14
51	5

Y → dependent



1. Simple equation m b
2. Linear Algebra Matrix
*3 Gradient descent ← Optimization Algorithm
   ↳ Deep Learning

Best Fit line

$$y = mx + b$$

$$m = \frac{\sum_{i=0} (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$b = \bar{y} - m x$

x	y	$\hat{y}$
2	5	4.1
3	6	3.9
4	2	3.7
1	3	4.3

	x	y	$(x_i - \bar{x})$	$(y_i - \bar{y})$	$(x_i-\bar{x})(y_i-\bar{y})$	$(x_i-\bar{x})^2$
1	2	5	−0.5	1	−0.5	0.25
2	3	6	0.5	2	1	0.25
3	4	2	1.5	−2	−3	2.25
4	1	3	−1.5	−1	1.5	2.25
					−1	5

$\hat{y} = -0.2x + 4.5$

$SSE = \Sigma(y - \hat{y})^2 = 9.8$

$SST = \Sigma(\bar{y} - y)^2 = 10$

$SSR = 0.2$

$\bar{x} = 2.5 \quad \bar{y} = 4$

$$m = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{-1}{5} = -0.2$$

2.5

$$b = \bar{y} - m\bar{x}$$
$$= 4 - (-0.2)\,2.5$$
$$= 4 + .5 = 4.5$$

$$\boxed{y = -0.2x + 4.5}$$

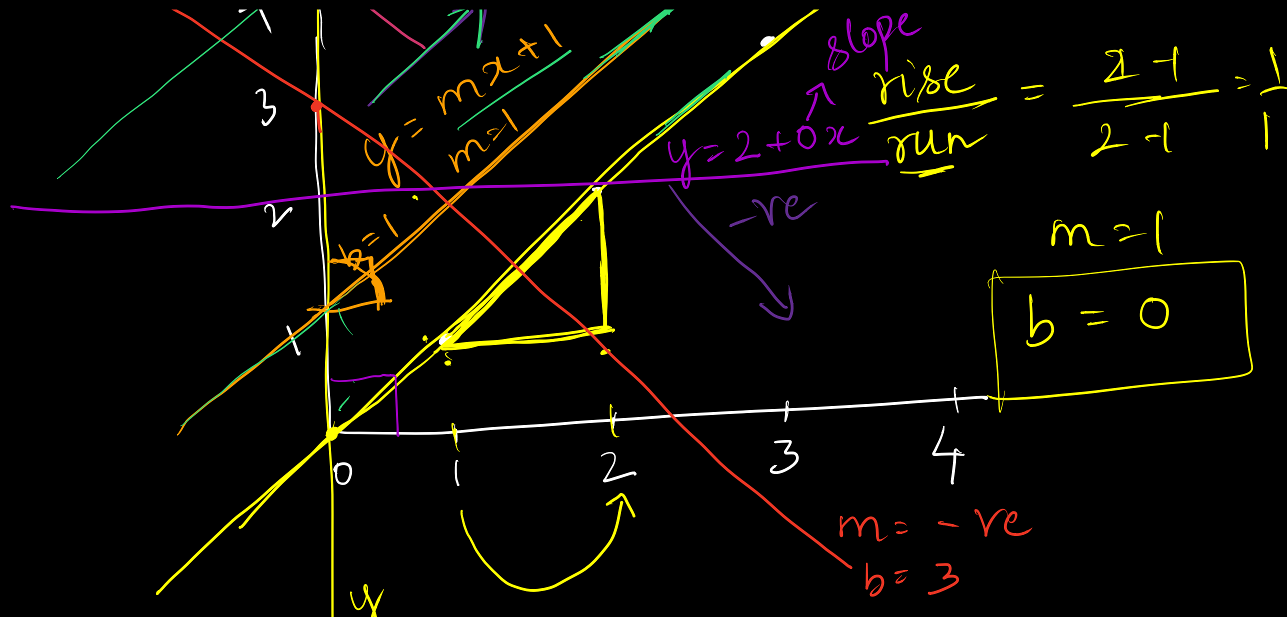
$b = 4 \rightarrow y = mx + 4$

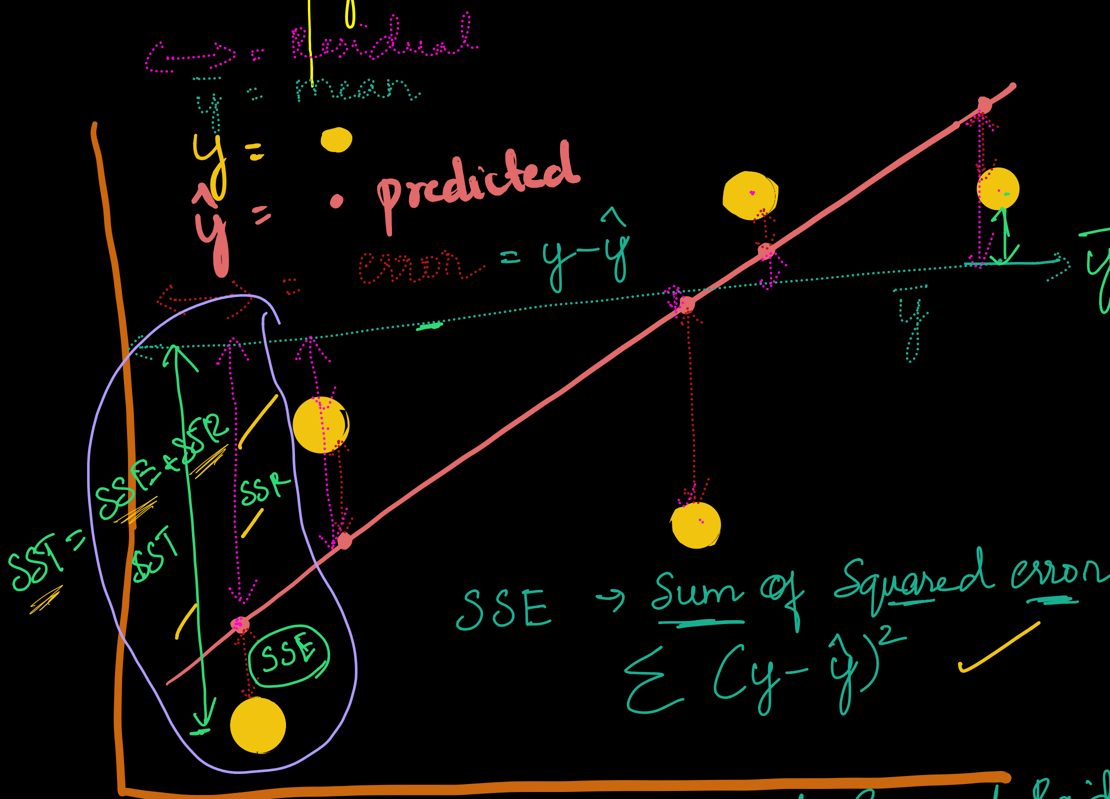
$m = 1 \rightarrow$

Tip Amount

$y = 0.146x - 0.8203$

+ve +ve $m = 1$ $m = 1$





SSE → Sum of Squared error

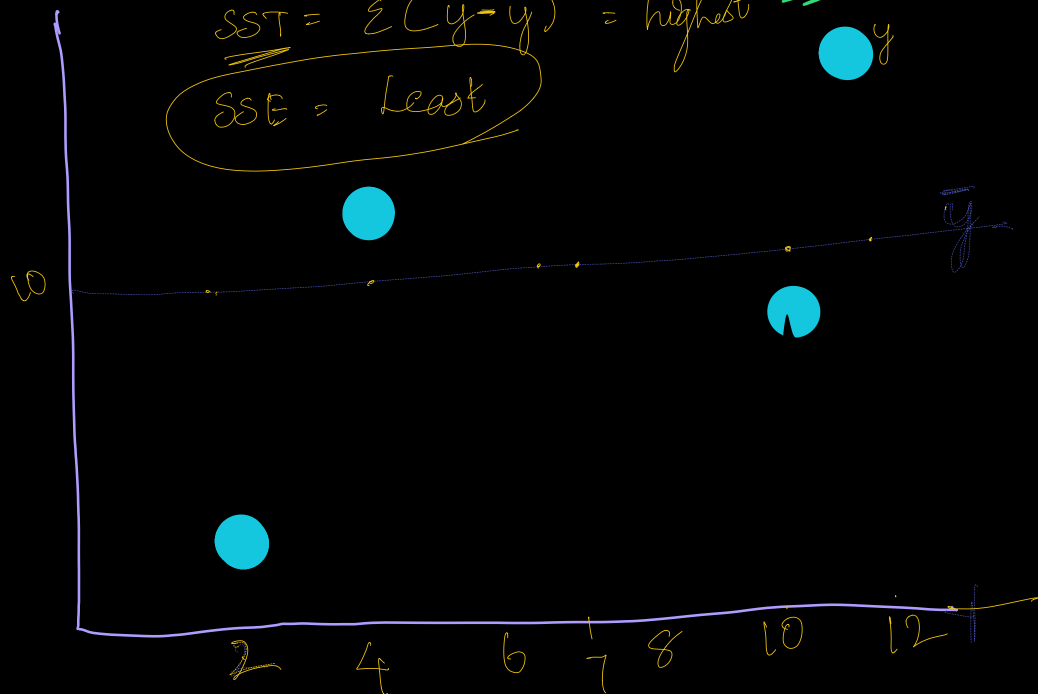
$\sum (y-\hat{y})^2$

SSR = Sum of Squared Residual

$\sum (\hat{y}-\bar{y})^2$

SST = Total Sum of Square

$\Sigma(y-\bar{y})$ ✓



Outlier

all outliers is $\boxed{\text{not bad data}}$

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300

200+

5

6

10

12

Linear Algebra

$$2x+3y=10$$
$$x+y=5 \quad \times 2$$

$$\cancel{2x}+3y=10$$
$$(-)\cancel{2x}\ (-)+2y=(-)10$$

$$y=0$$

$$\boxed{x=5 \quad y=0}$$

$$2x+3y+2z=20$$
$$x+y+z=10$$
$$x+2y+3z=30$$

$$\underset{A}{\begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 1 \\ 1 & 2 & 3 \end{pmatrix}} \underset{X}{\begin{pmatrix} x \\ y \\ z \end{pmatrix}} = \underset{B}{\begin{pmatrix} 20 \\ 10 \\ 30 \end{pmatrix}}$$

$AX = B$ y

$$\boxed{X = BA^{-1}}$$

$a + b$ $3+2$ $2+3$

$b + a$

$a \times b$

$b \times a$

$$A^{-1} = \frac{1}{A}$$

$$\boxed{\cancel{A^{-1} = \frac{1}{|A|} \underline{adj(A)}}}$$

Gauss Jordon Method

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 2 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$$

Identify Matrix A Identity A^{-1} Identity

augument

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 2 & 1 & 0 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ & & & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{\qquad}_{\text{Identity}}$ $\qquad\qquad$ $\underbrace{\qquad}_{A^{-1}}$

$R_1 \Rightarrow R_1 + R_2$

$$\left[\begin{array}{ccc|ccc} 5 & 0 & 0 & 1 & 1 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \Rightarrow R_1/5$

$$\begin{array}{c} \\ \times 2 \\ \\ \end{array} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 2 & 0 & -2 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \Rightarrow$ 2 times of $R_1 - R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 0 & +2 & 0.4 & -0.6 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \Rightarrow R_2 \times \frac{1}{2}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 0 & 1 & 0.2 & -0.3 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

Swap $R_2 \Leftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0.2 & -0.3 & 0 \end{array}\right]$$

$R_2 \Rightarrow R_2 - R_3$ Identi $\qquad A^{-1}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0.2 & 0.2 & 0 \\ 0 & 1 & 0 & -0.2 & 0.3 & 1 \\ 0 & 0 & 1 & 0.2 & -0.3 & 0 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 0.2 & 0.2 & 0 \\ -0.2 & 0.3 & 1 \\ 0.2 & -0.3 & 0 \end{bmatrix}$$

$$\underset{A}{\underset{3\times 3}{\begin{pmatrix} 2 & 3 & 0 \\ -2 & 3 & 10 \\ 2 & -3 & 0 \end{pmatrix}}} \quad \underset{B}{\underset{1\times 3}{\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}}}$$

$3\times \boxed{1 \quad 3} \times 3$

$(m \times n)(p \times q)$ $\quad$ (3×1)

$(n \times p)$ ✓

$$= \begin{pmatrix} 2\times 1 + 3\times 2 + 0\times 3 \\ -2\times 1 + 3\times 2 + 10\times 3 \\ 2\times 1 + -3\times 2 + 0\times 1 \end{pmatrix} = \begin{pmatrix} 2+6+0 \\ -2+6+30 \\ 2-6+0 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 8 \\ 34 \\ -4 \end{pmatrix}$$

X $\quad$ Y $\quad$ $mx + b(1)$

$$\begin{array}{ccc} A & \times & B \end{array}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 4 & 1 \\ 5 & 1 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 7 \\ 5 \\ 11 \end{pmatrix}$$

multiply with A' with both side

$$(\underline{5 \times 2}) \times (\underline{2 \times 5})$$

$$(2 \times 2) \leftarrow \text{Square}$$

$$\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 4 & 1 \\ 5 & 1 \\ 7 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 4 & 5 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} =$$

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 7 \\ 5 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 4 & 1 \\ 5 & 1 \\ 7 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 \times 1 + 2 \times 2 + 4 \times 4 + 5 \times 5 + 7 \times 7 & 1 + 2 + 4 + 5 + 7 \\ 1 + 2 + 4 + 5 + 7 & 1 + 1 + 1 + 1 + 1 \end{pmatrix}$$

$$\begin{pmatrix} 95 & 19 \\ 19 & 5 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} \quad \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 4 & 5 & 7 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 2 \\ 3 \\ 7 \\ 8 \\ 11 \end{pmatrix}$$

$$\begin{pmatrix} 1\times 2 + 2\times 3 + 4\times 7 + 5\times 8 + 7\times 11 \\ 2 + 3 + 7 + 5 + 11 \end{pmatrix}$$

$$\begin{pmatrix} 138 \\ 28 \end{pmatrix}$$

$$\underset{A}{\begin{pmatrix} 95 & 19 \\ 19 & 5 \end{pmatrix}} \underset{X}{\begin{pmatrix} m \\ b \end{pmatrix}} = \underset{B}{\begin{pmatrix} 138 \\ 28 \end{pmatrix}}$$

$$AX = B$$

$$X = BA^{-1}$$

$m = 1.39$ ✓

1.28 ✓

$$\left(\begin{array}{cc|cc} 95 & 19 & 1 & 0 \\ 19 & 5 & 0 & 1 \end{array}\right)^{-1}$$

$$X \left(\begin{array}{cc} 0.048 & -0.1667 \\ -0.1667 & 0.833 \end{array}\right) \begin{pmatrix} 138 \\ 28 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 95 & 19 & 1 & 0 \\ 19 & 5 & 0 & 1 \end{array}\right)$$

$R_1 \Leftarrow R_1 - 19$

$$\begin{pmatrix} 76 & 0 & -18 & -19 \\ 19 & 5 & 0 & 1 \end{pmatrix}$$

$R_1 \Rightarrow R_1 / 76$

$$\begin{pmatrix} 1 & 0 & -18/76 & -19/76 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & & \\ 19 & 5 & 0 & 1 \end{pmatrix}$$

$R_2 \Rightarrow R_2 - 19$

$$\begin{pmatrix} 1 & 0 & -18/26 & -19/26 \\ 0 & -14 & -19 & -18 \end{pmatrix}$$

$R_2 = R_2 / -14$

$$\begin{pmatrix} 1 & 0 & -18/26 & -19/26 \\ 0 & 1 & -19/-14 & -18/-14 \end{pmatrix}$$

$$\begin{pmatrix} -18/26 & -19/26 \\ 19/14 & 18/14 \end{pmatrix} \times \left(\begin{matrix} \text{or} \\ \cancel{26} \end{matrix} \right)$$

$$\begin{pmatrix} m \\ b \end{pmatrix} = \begin{pmatrix} 1.38 \\ 0.3 \end{pmatrix}$$

Gradient Descent

Iterative method of Optimization Algorithm of find minimum of the function

Goal $\Rightarrow$ Find m and b

Reduce the error

$$\text{Cost function} = \frac{\sum (y - \hat{y})^2}{n}$$

SSE

MSE = Mean Square error

Goal Reach 20 Km destination

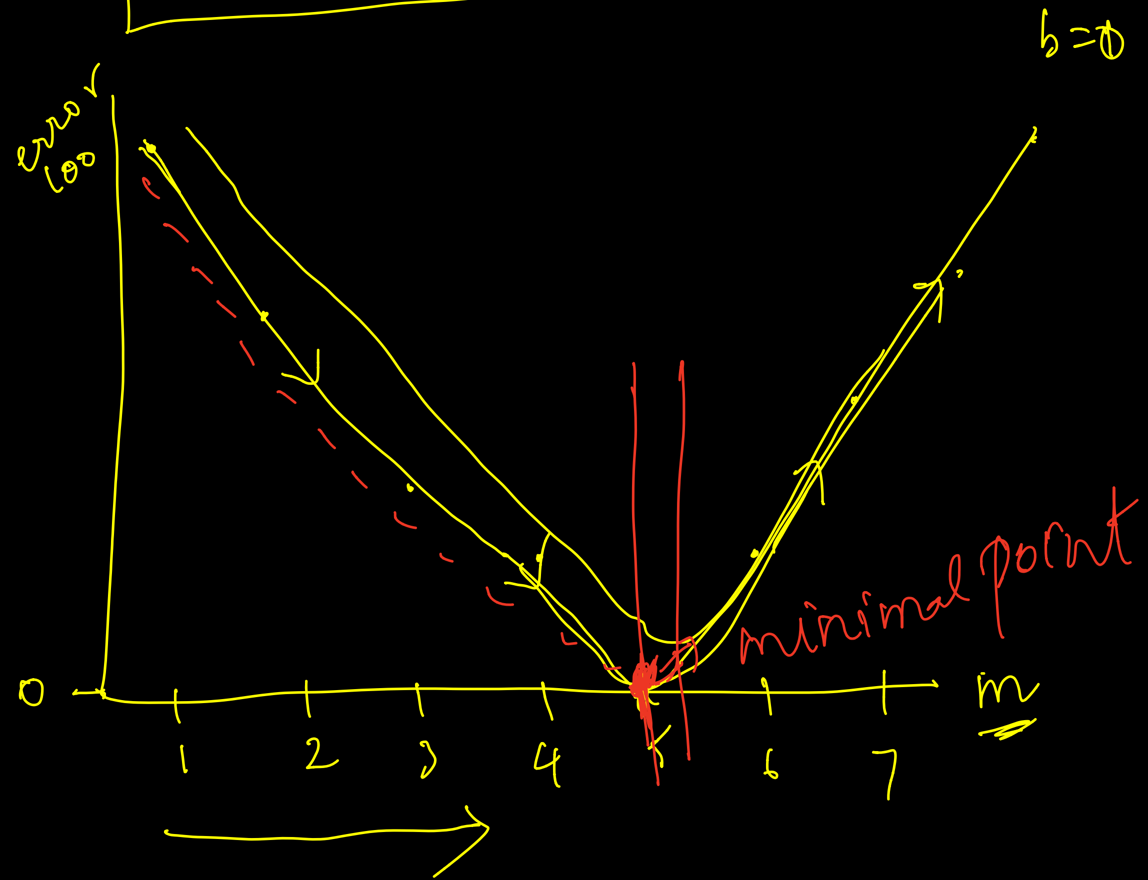
Cost

20 Rupees

1. Cab =
   Uber = 300/-
   Ola = 280/-
2. two wheeler cab = 120/-
3. Metro = 60/-
4. Electric = 30/-

Train

$\boxed{5.\ \text{Bus} = 18}$

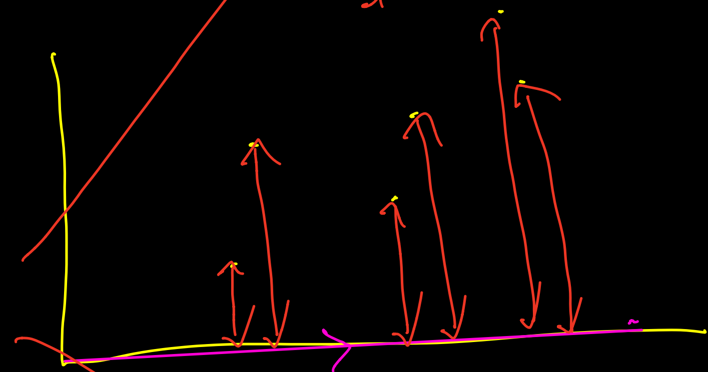


New_m = Old_m - (Step function)

Non Techinal

1. Substitute value m and b = 0

$y = mx + b = 0$



error (MSE) = 100

2. Change m and b based on previous value

m_new = m_old − (step function)

1. direction ✓ (Partial derivative)
2. How much (learning rate)

$m = 0 \rightarrow$ (+) (−)

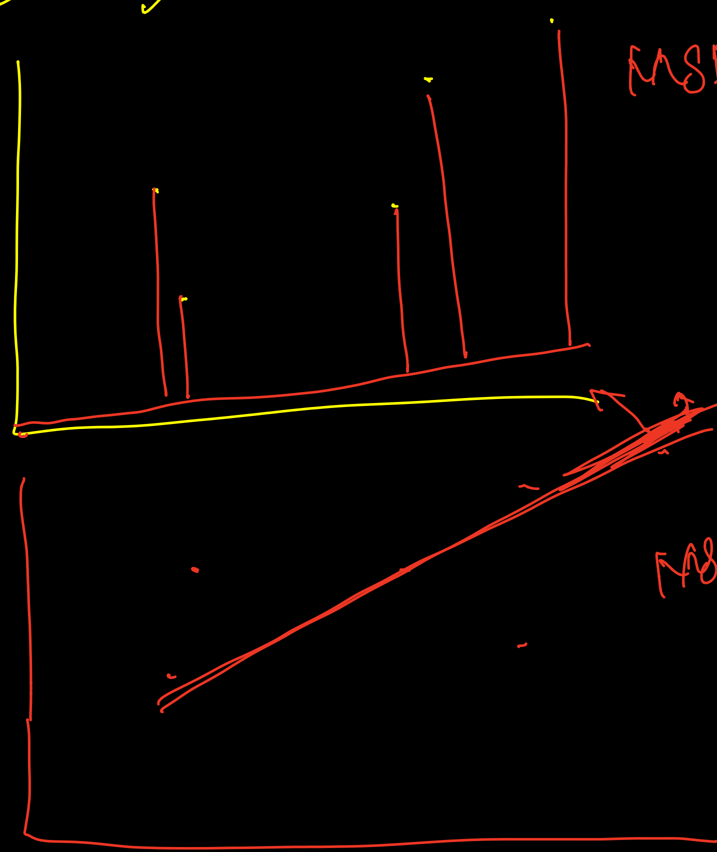
$m = \rightarrow$ 0.01, 0.001, 0.2

b_new = b_old − Step function

1. direction
2. How much

m_new = 0.01   1000

b_new = 0.01



MSE = 98.9

MSE = 20.2

Step 4 = Update m and b

Step 5 = 2 to 4