

Numbers

Objective

This projectlet will use recreational number theory as a means to understand performance considerations. As the “problem size” grows the solution cost - time and/or resources grow as well but a judicious choice of algorithms and data structures will determine the viability of the solution.

In addition, this projectlet will encourage a layered approach to the solution. A core layer with a clean interface specification will support other higher layers with an eventual top layer to solve the specific problem.

REFERENCE

There will be many terms used in this projectlet that would require precise definitions. Following sites could be used as references.

<https://www.wikipedia.org/>

<https://www.wolframalpha.com/>

https://en.wikipedia.org/wiki/Luhn_algorithm

https://en.wikipedia.org/wiki/International_Standard_Book_Number

User needs and requirements

Library

Id	Need/Requirement
1	A library to support explorations in recreational number theory is needed with the following core routines.
2	Given a number a routine to return a list of decimal digits
3	Given a list of decimal digits, a routine to return the value
4	Given a number a routine to return a list of all divisors
5	Given a number a routine to return a list of its prime factors
6	Given a list of numbers, a routine to return the sum of all the numbers
7	Given a list of numbers, the product of all of the numbers
8	Given a list of numbers, to return a list of all squares, cubes, ... other powers

Id	Need/Requirement
9	Given 2 numbers, to return the greatest common divisor
10	Generate a series of Fibonacci numbers
11	Given an array of numbers generate pairwise gcd's
12	Generate a series of taxicab numbers of a specified order
13	Enumerate numbers which are squares but can be expressed as sums of pairs of squares. What is the maximum number of such ways. Expand to cubes. ie cubes which can be exposed as sums of pairs of cubes.
14	Sum of squares of n different integers. Product of any 2 also is a sum of squares of n different integers

Applications

Id	Need/Requirement
1	Given a number - report if it is prime
2	Given a number - report if it is perfect
3	Given a number - report if it is a Kaprekar number
4	Given a number - report if it is a Harshad number
5	For a list of Fibonacci series, report the gcd of all pairs of numbers
6	Enumerate taxicab numbers (Ramanujan-Hardy numbers) of the order 3, 4. Description: A number expressible as sum of 2 different cubes ie $a^3 + b^3$; 2 different powers of 4 and so on.
7	Given a number - validate whether it could be a Visa/Mastercard/American Express credit card number. This requires implementation of the Luhn algorithm.
8	Given an ISBN-10 identification of a book, convert it into an ISBN-13 number and vice-versa

User wants

1	A density graph of primes (e.g. no in each range of 1000)
2	A density graph of fibonacci numbers

Example usage

SIMPLE TESTS ON NUMBERS

```
No 4096-----
DigitsOf
[4 0 9 6 ]
Value is 4096
DivisorsOf
[1 2 4 8 16 32 64 64 128 256 512 1024 2048 4096 ]
PrimeFactorsOf
[1 2 2 2 2 2 2 2 2 2 2 2 ]
Product of all those 4096
IsPrime False
IsPerfect False
IsHarshad False
IsHappy True
Iskaprekar False
```

PAIRWISE GCD OF FIBONACCI NUMBER SERIES

1043008345	527452805	: 5
1043008345	132996290	: 5045
1043008345	1640061041	: 29
1043008345	1511089245	: 5
1043008345	2001647585	: 5
1043008345	1479474520	: 5
1043008345	66908715	: 5
1043008345	1669358405	: 5
1043008345	602368435	: 5
1043008345	1528872610	: 5
1043008345	532750075	: 5
1043008345	318507655	: 5
1043008345	872122360	: 5
1043008345	196167680	: 5
1043008345	848533765	: 145
1043008345	2111895522	: 1009
1043008345	1675338205	: 5
1043008345	1739456245	: 5
1043008345	1597594320	: 5

TIMING COMPARISONS FOR PAIRWISE GCD COMPUTATION

1024 ELEMENTS

```
real 0m0.352s
user 0m0.149s
sys 0m0.060s
```

4096 ELEMENNTS

real. 0m4.146s
user 0m1.754s
sys 0m0.562s

TAXICAB NUMBERS OF ORDER 3

7872536

184	118
196	70

7957504

184	120
198	58

8077888

186	118
192	100

TAXICAB NUMBERS OF ORDER 4

635318657

134	133
158	59