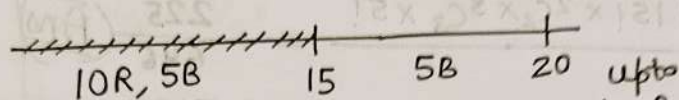


1.2

10 Red cards, 10 blue cards.

Shuffled and numbered from 1 to 20.

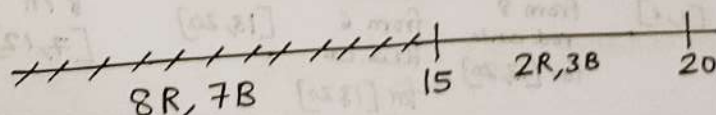
 \therefore Sample space = $20!$ (a) All red cards are assigned numbers ≤ 15 We have to place all 10 red cards ~~before~~ ^{upto} 15. \therefore we need to place $(15-10) = 5$ blue cards ~~before~~ ^{upto} 15 \therefore Favourable events *

$$= \left({}^{10}C_{10} \times {}^{10}C_5 \times 15! \times {}^5C_5 \times 5! \right)$$

\swarrow \swarrow \swarrow \swarrow \swarrow
 Choosing 10 red cards from 10 red cards for $[1, 15]$ choose 5 blue cards from 10 blue cards for $[1, 15]$ Arrange first 15 cards $[1, 15]$ Choosing 5 blue cards from 5 blue cards for $[16, 20]$ Arranging last 5 cards $[16, 20]$

 \therefore Probability (All red cards are assigned numbers ≤ 15)

$$= \frac{\text{Favourable events}}{\text{Sample space}} = \frac{{}^{10}C_{10} \times {}^{10}C_5 \times 15! \times {}^5C_5 \times 5!}{20!} = \frac{21}{1292} \quad (\text{Ans}) = 0.0162$$

(b) Exactly 8 red cards are assigned numbers ≤ 15 We have to place 8 red cards ~~upto~~ 15 \therefore We need to place $(15-8) = 7$ blue cards upto 15.As a result, we need to place 2 red cards and 3 blue cards in from $[16, 20]$

∴ Favourable events

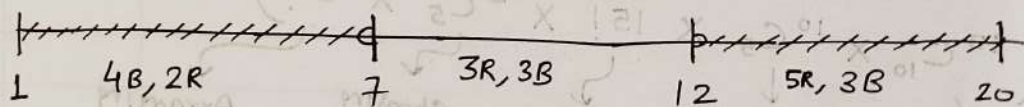
$$= ({}^{10}C_8 \times {}^{10}C_7 \times 15! \times {}^2C_2 \times {}^3C_3 \times 5!)$$

choosing 8 red cards from 10 red cards for [1, 15]
 choose 7 blue cards from 10 blue cards for [1, 15]
 Arranging first 15 cards [1, 15]
 choosing 2 red cards from 2 red cards for [16, 20]
 choosing 3 blue cards from 3 blue cards for [16, 20]
 Arranging last 5 cards [16, 20]

∴ Probability (Exactly 8 red cards ≤ 15)

$$= \frac{{}^{10}C_8 \times {}^{10}C_7 \times 15! \times {}^2C_2 \times {}^3C_3 \times 5!}{20!} = \frac{225}{646} \text{ (Ans)} = 0.3482$$

- (c) Exactly 5 red cards are assigned numbers > 12 and exactly 4 blue cards " " " < 7



We have place 4 blue cards and 2 red cards in [1, 6]

" " " 3 blue cards and 3 red " " [7, 12]

" " " 3 blue " and 5 red " " [13, 20]

∴ Favourable events

$$= ({}^{10}C_2 \times {}^{10}C_4 \times 6! \times {}^8C_5 \times {}^6C_3 \times 8! \times {}^3C_3 \times {}^3C_3 \times 6!)$$

choose 2 red from 10 red for [1, 6]
 choose 4 blue from 10 blue for [1, 6]
 Arranging first 6 cards [1, 6]
 choose 5 red cards from 8 red cards for [13, 20]
 choose 3 blue cards from 6 blue cards for [13, 20]
 Arranging cards in [13, 20]
 Arranging cards in [7, 12]

∴ Probability (Exactly 5 ^{red} cards > 12 and Exactly 4 blue cards < 7)

$$= \frac{{}^{10}C_2 \times {}^{10}C_4 \times 6! \times {}^8C_5 \times {}^6C_3 \times 8! \times {}^3C_3 \times {}^3C_3 \times 6!}{20!}$$

$$= \frac{4200}{46189} \text{ (Ans)} = 0.09093$$

1.1

Case 1

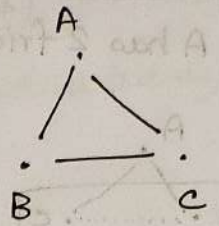
Let's say, there are 3 persons A, B, C.

Now, Probability (A, B, C are mutual friend)

 \equiv Probability (AB, BC, AC all are friends) $= P(A \text{ and } B \text{ frnd}) \times P(B \text{ and } C \text{ are frnd}) \times P(A, C \text{ are frnd})$

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8} = \frac{1}{2^3}$$

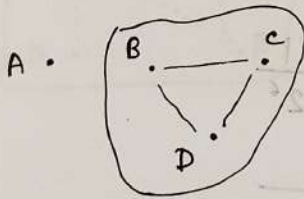


Case 2

Now, Let's say, there are 4 persons A, B, C, D.

Case 2.1

A has 0 friend. Probability (at least 3 mutual friend)

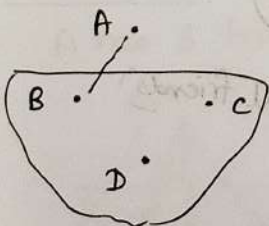
 ~~\equiv Probability (B, C, D are mutual friend)~~ $= {}^3C_0 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \text{Probability (B, C, D are mutual friend)}$

\downarrow Selecting 0 edges from $\{AB, AC, AD\}$
 \downarrow Probability of $\{AB, AC, AD\}$ edge not present

$$= 1 \times \frac{1}{2^3} \times \frac{1}{2^3} = \frac{1}{2^6}$$

Case 2.2

A has 1 friend.



Probability (at least 3 mutual friend)

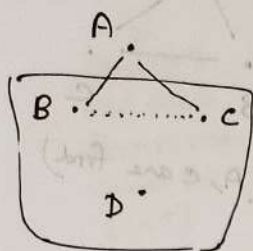
 $= {}^3C_1 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \text{Probability (B, C, D are mutual friend)}$

\downarrow Selecting any 1 edge from $\{AB, AC, AD\}$
 \downarrow Probability of $\{AB \text{ present, } AC \text{ not }, AD \text{ not }\}$

$$= \frac{3}{2^3} \times \frac{1}{2^3} = \frac{3}{2^6}$$

Case 2.3

A has 2 friend Probability (at least 3 mutual friend)

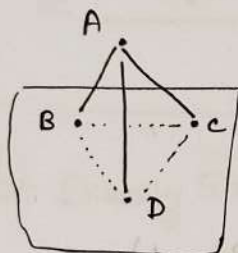


$$= {}^3C_2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \text{Prob}(B, C \text{ are friend})$$

$$= \frac{3}{2^3} \times \frac{1}{2} = \frac{3}{2^4} = \frac{12}{2^6}$$

Case 2.4

A has 3 friend Probability (at least 3 mutual friend)



$$= {}^3C_3 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \text{Prob}(\text{Either } B, C \text{ or } C, D \text{ or } B, D \text{ are mutual friend})$$

$$= \frac{1}{2^3} \times (1 - \text{Prob}(\text{none of } B, C, D \text{ are mutual friend}))$$

$$= \frac{1}{2^3} \times (1 - \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2})$$

$$= \frac{1}{2^3} \times (1 - \frac{1}{2^3}) = \frac{1}{2^3} - \frac{1}{2^6}$$

$$= \frac{7}{2^6}$$

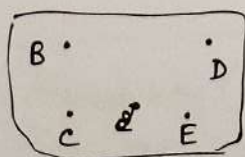
\therefore For four persons, Prob(at least 3 mutual friend)

$$= \frac{1}{2^6} + \frac{3}{2^6} + \frac{12}{2^6} + \frac{7}{2^6} = \frac{23}{2^6}$$

Case 3 (The final answer)

Case 3.1

A has 0 friend Probability (at least 3 mutual friend)



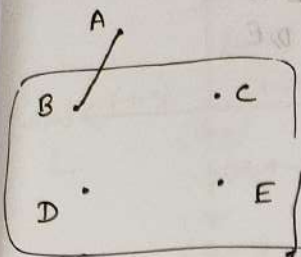
$$= {}^4C_0 \times \frac{1}{2^4} \times \text{Prob}(B, C, D, E \text{ are mutual friends})$$

$$= \frac{1}{2^4} \times \frac{23}{2^6} = \frac{23}{2^{10}}$$

①

Case 3.2

A has 1 friend



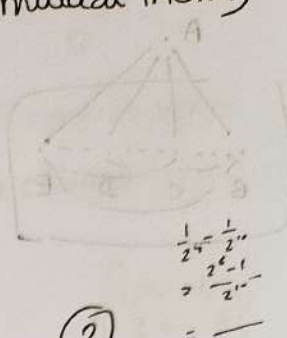
Probability (at least 3 mutual friend)

$$= 4C_1 \times \frac{1}{2^4} \times \text{Prob}(B, C, D, E \text{ are mutual friends})$$

Choose any edge from $\{AB, AC, AD, AE\}$

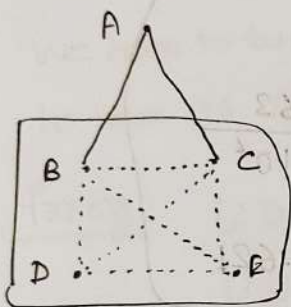
Prob of $\begin{cases} AB \text{ present,} \\ AC \text{ not } \\ AD \text{ not } \\ AE \text{ not } \end{cases}$

$$= \frac{4}{2^4} \times \frac{2^3}{2^6} = \frac{92}{2^{10}}$$



Case 3.3

A has 2 friends



Probability (at least 3 mutual friend)

$$= 4C_2 \times \frac{1}{2^4} \times \left[\text{Prob}(B, C \text{ mutual friend}) + \frac{1}{2} \text{Prob}(B, D, E \text{ or } C, D, E \text{ mutual friend}) \right]$$

$$= \frac{6}{2^4} \left[\frac{1}{2} + \frac{1}{2} P(\Delta BDE \cup \Delta CDE) \right]$$

$$= \frac{6}{2^4} \left[\frac{1}{2} + \frac{1}{2} P(\Delta BDE) + \frac{1}{2} P(\Delta CDE) - \frac{1}{2} P(\Delta BDE \cap \Delta CDE) \right]$$

$$= \frac{6}{2^4} \left[\frac{1}{2} + \frac{1}{2 \times 2^3} + \frac{1}{2 \times 2^3} - \frac{1}{2 \times 2^5} \right]$$

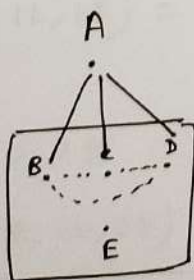
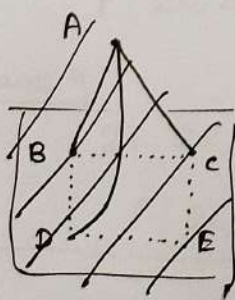
$$= \frac{6}{2^5} + \frac{6}{2^8} + \frac{6}{2^8} - \frac{6}{2^{10}} = \frac{6 \times 2^5 + 6 \times 2^2 + 6 \times 2^2 - 6}{2^{10}} = \frac{6 \times 2^5 + 6 \times 2^3 - 6}{2^{10}}$$

$$= \frac{234}{2^{10}}$$

$$\begin{aligned} & \frac{6 \times 2^5 + 6 \times 2^3 - 6}{2^{10}} \\ &= \frac{6 \times 32 + 6 \times 8 - 6}{2^{10}} \\ &= \frac{240 + 48 - 6}{2^{10}} \\ &= \frac{282}{2^{10}} \end{aligned}$$

Case 3.4

A has 3 friends



Probability (at least 3 mutual friends)

$$= 4C_3 \times \frac{1}{2^4} \times \text{Prob}(B, C, D \text{ or } B, C, E \text{ or } B, D, E \text{ or } C, D, E \text{ are mutual friends})$$

Choose 3 edges from $\{AB, AC, AD, AE\}$

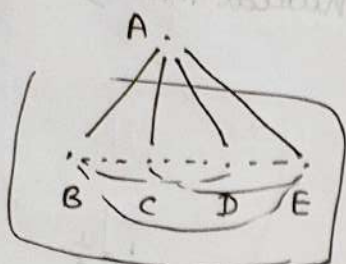
$$= \frac{4}{2^4} \times \left[1 - \text{Prob}(B, C, D \text{ or } B, C, E \text{ or } B, D, E \text{ or } C, D, E \text{ are not mutual friends}) \right]$$

$$= \frac{4}{2^4} - \frac{4}{2^7} = \frac{8 \times 4 - 4}{2^7} = \frac{28}{2^7} = \frac{224}{2^{10}}$$

$$P \times 2^3 = 224$$

case 3.5

A has 4 friends.



Probability (at least 3 mutual friends)
 $= {}^4C_4 \times \frac{1}{2^4} \times \text{Prob}(B,C \text{ or } B,D \text{ or } B,E \text{ or } C,D \text{ or } C,E \text{ or } D,E \text{ m.f.})$

$$= \frac{1}{2^4} \times \left(1 - \frac{1}{2^6}\right)$$

$$= \frac{1}{2^4} - \frac{1}{2^{10}} = \frac{2^6 - 1}{2^{10}} = \frac{63}{2^{10}}$$

⑤

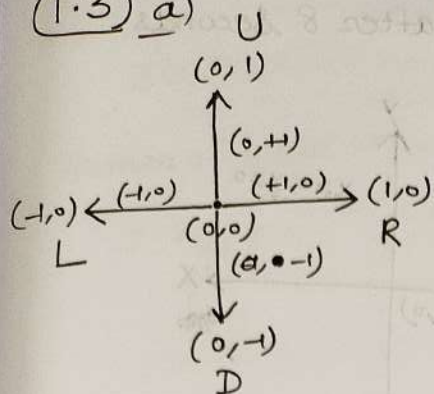
\therefore Probability (at least 3 mutual friends among 5 persons)

$$= ① + ② + ③ + ④ + ⑤$$

$$= \frac{23}{2^{10}} + \frac{92}{2^{10}} + \frac{234}{2^{10}} + \frac{224}{2^{10}} + \frac{63}{2^{10}}$$

$$= \frac{636}{2^{10}} = \frac{636}{1024} \quad (\text{Ans}) = 0.621$$

(1.3) a)



Four moves are possible

(0,1) (1,0) (-1,0) (0,-1)
U R L D

In 6 sec, we can take 6 moves and
for each move we have 4 choices

$$\therefore \text{Sample Space} = 4^6$$

$$\left\{ \begin{array}{l} U \leftarrow D \\ L \leftarrow R \end{array} \right\}$$

~~In order to take 6 steps and what~~

In order to get back to origin after 6 seconds,

We have to take 6 steps and whatever step we take, we have
to take its opposite step same number of times.

foreg UDUDLR

UUUDDD

case 1

3U, 3D

$$P(3U, 3D) = \frac{\frac{6!}{3! \times 3!}}{4^6} = \frac{20}{4^6}$$

case 2

3L, 3R

$$P(3L, 3R) = \frac{\frac{6!}{3! \times 3!}}{4^6} = \frac{20}{4^6}$$

case 3

2U, 2D, 1L, 1R

$$P(2U, 2D, 1L, 1R) = \frac{\frac{6!}{2! \times 2!}}{4^6} = \frac{180}{4^6}$$

case 4

2L, 2R, 1U, 1D

$$P(2L, 2R, 1U, 1D) = \frac{\frac{6!}{2! \times 2!}}{4^6} = \frac{180}{4^6}$$

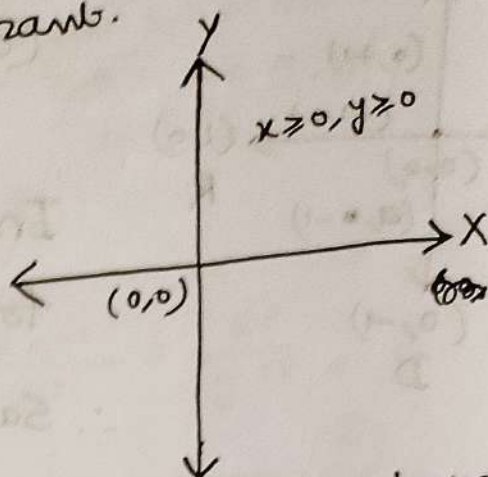
$$\begin{aligned} \therefore P(\text{at } t=6 \text{ } x=0, y=0) &= P(3U, 3D) + P(3L, 3R) + P(2U, 2D, 1L, 1R) \\ &\quad + P(2L, 2R, 1U, 1D) \\ &= \frac{20}{4^6} + \frac{20}{4^6} + \frac{180}{4^6} + \frac{180}{4^6} = \frac{400}{4^6} \\ &= 0.0976 = \frac{400}{4096} \text{ (Ans)} \end{aligned}$$

1.3b

The particle comes back to the origin after 8 seconds by remaining only in first quadrant.

Here also, there are 4 moves possible.

$(0,1)$, $(0,-1)$, $(1,0)$, $(-1,0)$
U D R L.



To move back in origin after 8 seconds, we have to take 8 steps and whatever step we take, we have to take opposite step same number of times.

But, here the constraint is → The particle ~~can't~~ can remain only in 1st quadrant.

We will take the help of catalan numbers to solve this problem.

$$\text{Total Sample Space} = 4^8 \quad \Bigg| \quad \text{Catalan number}(C_n) = \frac{1}{n+1} \binom{2n}{n}$$

Here, C_n = String of length $2n$ consisting of n X's and n Y's such that no initial segment of the string has more Y's than X's.

$$= \frac{1}{n+1} \binom{2n}{n}$$

Case 1:

4U, 4D — — — — —

Number of ways in which particle comes back to $(x=0, y=0)$ at $t=8$ being in $x \geq 0, y \geq 0$

$$= {}^8C_8 \times C_4 = \frac{1}{5} \binom{8}{4} \quad \text{--- (1)}$$

Choosing (4U, 4D)
Total 8 positions from 8 positions

String of length 8 consisting of 4U and 4D such that no initial segment of the string has more D's than U's

Case 2:

4L, 4R — — — — —

$$\text{Number of ways} = {}^8C_8 \times C_4 = \frac{1}{5} \binom{8}{4} \quad \text{--- (2)}$$

Choosing 4L, 4R
Total 8 positions from 8 positions

String of length 8 consisting of 4L and 4R such that no initial segment of the string has more L's than R's

Case 3:

3U, 3D, 1L, 1R

Number of ways = ${}^8C_6 \times {}^6C_3 \times {}^2C_2 \times {}^1C_1$

\swarrow Choosing 6 positions from 8 positions for (3U, 3D)
 \swarrow String of length 6 consisting of 3U and 3D such that no initial segment of the string has more D's than U's
 \swarrow Choosing 2 positions from 2 pos. for (1L, 1R)
 \swarrow String of length 2 consisting of 1L and 1R s.t. no initial segment of the string has more L's than R's

$$= {}^8C_6 \times \frac{1}{4} ({}^6C_3) \times 1 \times \frac{1}{2} ({}^2C_1)$$

$$= {}^8C_6 \times {}^6C_3 \times \frac{1}{4} \quad \text{--- (2)}$$

Case 4:

3L, 3R, 1U, 1D

Number of ways = ${}^8C_6 \times {}^6C_3 \times {}^2C_2 \times {}^1C_1$

\swarrow Choosing 6 positions from 8 positions for (3L, 3R)
 \swarrow String of length 6 consisting of 3L and 3R such that no initial segment of the string has more L's than R's
 \swarrow Choosing 2 positions from 2 pos. for (1U, 1D)
 \swarrow String of length 2 consisting of 1U, 1D s.t. no initial segment of the string has more D's than U's

$$= {}^8C_6 \times \frac{1}{4} ({}^6C_3) \times 1 \times \frac{1}{2} ({}^2C_1)$$

$$= {}^8C_6 \times {}^6C_3 \times \frac{1}{4} \quad \text{--- (4)}$$

Case 5:

2U, 2D, 2L, 2R

$$\text{Number of ways} = {}^8C_4 \times C_2 \times {}^4C_4 \times C_2$$

Choosing 4 positions
from 8 positions
for (2U, 2D)

String of
length 4
consisting
of 2U and
2D such
that no
initial
segment of
string has
more D's
than U's

choosing 4
positions
from 4
positions
for (2L, 2R)

String of length 4
consisting of 2L and
2R such that no
initial segment
of string has more
L's than R's

$$= {}^8C_4 \times \frac{1}{3} ({}^4C_2) \times 1 \times \frac{1}{3} ({}^4C_2)$$

$$= {}^8C_4 \times {}^4C_2 \times {}^4C_2 \times \frac{1}{9} \quad \text{--- (5)}$$

\therefore Total Number of favourable cases =

$$\textcircled{1} + \textcircled{2} + \textcircled{3} + \textcircled{4} + \textcircled{5}$$

$$= \frac{1}{5} \times ({}^8C_4) + \frac{1}{5} \times ({}^8C_4) + \frac{1}{4} \times {}^8C_6 \times {}^6C_3 + \frac{1}{4} \times {}^8C_6 \times {}^6C_3 + {}^8C_4 \times {}^4C_2 \times {}^4C_2 \times \frac{1}{9}$$

$$= 14 + 14 + 140 + 140 + 280$$

$$= 588$$

\therefore Probability (Number of ways in which particle comes back to $(x=0, y=0)$ at $t=8$ remaining in $x \geq 0, y \geq 0$)

$$= \frac{588}{4^8}$$

$$= \frac{588}{65536} = 0.0089 (\text{Ans})$$