Test Name : M21_EndEval_H1_MCS1 Probability and Statistics_9th October 2021_3:30 PM
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Test Start Time	Marks Scored	Total Questions
09/10/2021, 15:30:45	18.0 / 40.0	4

Attempted Questions	Correct Questions	Incorrect Questions
4	4	0

Skipped Questions	Pending Evaluation
0	0

List of Sections

Question 1 Marks per question : 10.0 **Marks Scored : 2.0**

Q No.	Q. Type	Status	Marks	
1	File Upload	✔	2.0	Hide Answer

Alex lives in the Big Island of Hawaii where there is an active volcano and has a home security system. One day he comes home and finds the alarm to be on (A =1). Either it could have gone off because of a robbery (B = 1), or an earthquake might have made it on (E = 1)? He listens to the radio and the news could give an earthquake alert (R = 1). However, the alarm is not directly influenced by any report on the Radio. Also the Radio alerts are independent of his house getting robbed. Similarly the earthquake also does not depend upon his house getting burgled. A,B,E,R are binary valued random variables which are 1 when alarm sounded, house burgled, earthquake happened, radio alerted for earthquake respectively.

The following probabilities are given to you:

$\Pr[B=1] = 0.01, \Pr[E=1] = 0.000001$

The conditional probabilities of A given values of B, E ($\Pr[A=1 \mid B, E]$) and the conditional probabilities of R given E ($\Pr[R=1 \mid E]$) are given by tables bellow:

$\text{Alarm}=1$	Burglar	Earthquake
0.9999	1	1
0.99	1	0
0.99	0	1
0.0001	0	0

$\text{Radio}=1$	Earthquake
1	1
0	0

1. What is the probability that house was robbed given the alarm rang?
2. What is the probability that house was robbed given that both the alarm rang and radio issued an earthquake alert?

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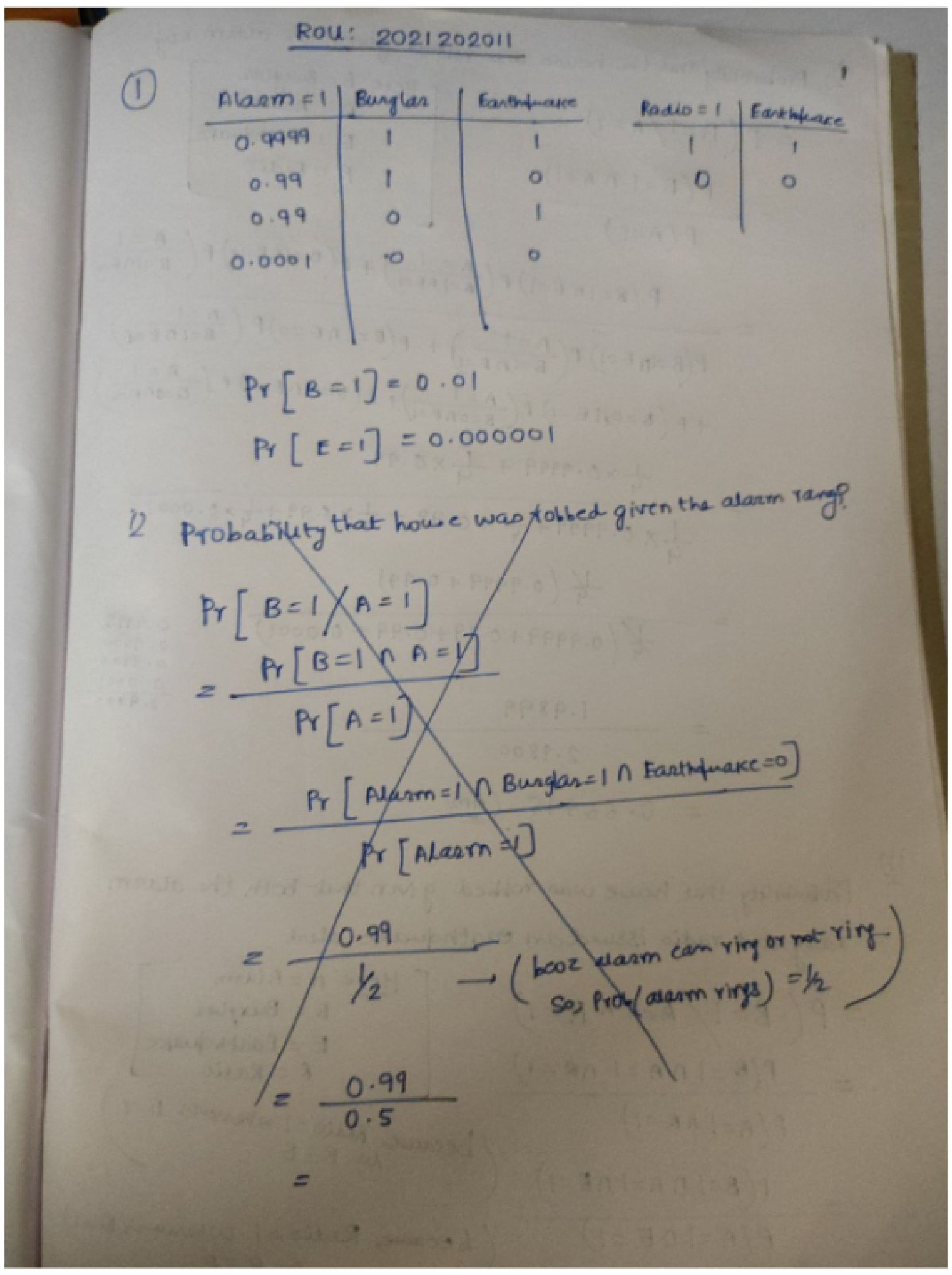

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(1)

Alarm = 1	Burglar	Earthquake
0.9999	1	1
0.99	1	0
0.99	0	1
0.0001	0	0

Radio = 1	Earthquake
1	1
0	0

$Pr[B=1] = 0.01$

$Pr[E=1] = 0.000001$

2) Probability that house was robbed given the alarm rang?

$Pr[B=1 / A=1]$

$= \frac{Pr[B=1 \cap A=1]}{Pr[A=1]}$

$= \frac{Pr[Alarm=1 \cap Burglar=1 \cap Earthquake=0]}{Pr[Alarm=1]}$

$= \frac{0.99}{1/2}$ → (booz alarm can ring or not ring. So, Prob(alarm rings) = 1/2)

$= \frac{0.99}{0.5}$

=

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Evaluator Comments

incorrect

Question 2 Marks per question : 10.0 Marks Scored : 2.0

Q No.	Q. Type	Status	Marks

1	File Upload	✔	2.0	Hide Answer

A box has n balls numbered from 1 to n. Suppose you keep picking a ball randomly each time and put it back in the box before the next pick.

1. Let X be the random variable denoting the first time at which you have seen a ball twice. Find the PMF of X.
2. Let T_i be the random variable corresponding to the time taken for seeing a new ball, after you have seen i different balls. Find the PMF of T_i.
3. Let T be the random variable corresponding to the first time at which you have encountered all the n balls. Find $\mathbb{E}T$.

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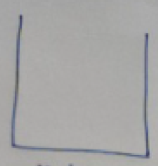

n balls
$(1 \dots n)$

1) X = r.v. denoting 1st time at which you have seen a ball twice

Let, r be the number of trials in between seeing the ball ~~twice~~ first and then 2nd time

$$\therefore P(X=n) = \frac{((n-1)!)^n}{(n!)^n} = \left(\frac{(n-1)!}{n!}\right)^n = \left(\frac{1}{n}\right)^n = \frac{1}{n^n}$$

Arrangement of all other balls except the ball which we have already seen

Arrangement of all balls

2) T_i = r.v. denoting time taken for seeing a new ball after seeing i diff balls.

3) The best case when we can encounter n balls at first chance is the nth trial.

Then, if not $(n+1)$ th trial (In this case, 2 trial would have same balls, rest all diff balls)

$(n+2)$ ——

⋮

so on

number of extra trials

number of extra trials than n

$$\therefore E(T) = 0 \times \frac{1}{n!} + 1 \times \frac{1}{\frac{(n+1)!}{2!}} + 2 \times \frac{1}{\frac{(n+2)!}{3!}} + 3 \times \frac{1}{\frac{(n+3)!}{4!}} + \dots \infty$$

$$= \sum_{K=1}^{\infty} K \cdot \frac{1}{\frac{(n+K)!}{K!}}$$

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Question 3 Marks per question : 10.0 **Marks Scored : 7.0**

Q No.	Q. Type	Status	Marks	
1	File Upload	✔	7.0	Hide Answer

Suppose you are taking an MCQ test with 100 questions which has 5 options and 1 marks each and negative marking (-1) for wrong answer. Suppose a student gives a random answer among the 5 for each of the problems. Give an upperbound on the probability that the student has a score ≥ 0 using

1. Markov's Inequality
2. Chebyshev's Inequality

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3) 100 Questions

$\therefore n = 100$

Each Question has 5 options. If correct (+1) if wrong (−1)

Student randomly answers among the 5 for each problem.

Now. probability of success = probability of giving correct answer

$= 1/5$

reward for success $= +1$

probability for failure = probability of giving wrong ans

$= 4/5$

reward for failure $= -1$

$$\therefore E(X_i) = 1/5 \times 1 + (4/5)(-1)$$
$$= 1/5 - 4/5$$
$$= -3/5.$$

$$var(X_i) = E(X_i^2) - E^2(X_i)$$
$$\cancel{= ((1/5)^2 \times 1) + (-4/5)}$$
$$= ((1)^2 \times 1/5 + (-1)^2 \times 4/5) - (-3/5)^2$$
$$= (1/5 + 4/5) - \frac{9}{25}$$
$$= \frac{5}{5} - 9/25$$
$$= 1 - 9/25 = 16/25$$

Now. $X = X_1 + X_2 + \dots + X_{100}$ [$\because$ total 100 Q Each Q is identical]

$$\therefore E(X) = E(X_1) + E(X_2) + \dots + E(X_{100})$$
$$= 100 \times E(X_i)$$
$$= 100 \times (-3/5) = -60$$

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Section - 4 Marks per question : 10.0 **Marks Scored : 7.0**

Q No.	Q. Type	Status	Marks	
1	File Upload	✔	7.0	Hide Answer

Consider a random (undirected) graph $G(V, E)$ on the vertex set $V = \{1, \cdots, n\}$. That is for every $\{i, j\}$ $(i \neq j)$ is independently chosen to be an edge in E with probability $\frac{1}{2}$.

1. A triangle in the graph is a set of edges $\{\{i, j\}, \{j, k\}, \{k, i\}\}$ where i, j, k are distinct. Find the expected number of triangles in G.
2. A k-clique in a graph is a set of k vertices $S \subseteq V$, $|S| = k$, such that there is a edge in G between every pair of vertices in S. Show that:

$$\Pr[\text{there is no } k\text{-clique in } G] \geq 1 - \binom{n}{k} \frac{1}{2^{\binom{k}{2}}}$$

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4.

1) Set $t \stackrel{def}{=} \binom{n}{3}$, fix any ordering of the t possible sets of 3 nodes, and consider accordingly $X_1, X_2, \ldots, X_t$ the indicator random variables

where $X_j = \begin{cases} 1 & ; \text{ iff } j\text{th set defines a triangle} \\ 0 & ; \text{ else} \end{cases}$

We are interested in $E \sum_{j=1}^{t} X_j$

where the expectation is taken over the $\binom{n}{2}$ i.i.d draws defining the edges.

$\therefore$ By linearity of expectation

$$E \sum_{j=1}^{t} X_j = \sum_{j=1}^{t} E(X_j) = t\, E X_1 = t p^3$$

Here given $p = 1/2$ (independently chosen edge probability $= 1/2$)

$$\therefore E \sum_{j=1}^{t} X_j = t \cdot (1/2)^3 = t/8$$

2) The probability ~~the~~ of any particular set of k vertices forming a k clique is $p^{\binom{k}{2}}$. The prob of there being at least 1 k-clique is less than sum of all these probabilities,

and thus $< \binom{n}{k} \cdot p^{\binom{k}{2}}$

Here, $p = 1/2$ $\therefore$ $< \binom{n}{k} \left(\frac{1}{2}\right)^{\binom{k}{2}}$

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