MCS2 Linear Algebra: End Semester Exam

November 2021

Instructions:

- Question paper consists of four sections:
- Total Marks 30
- A, B, C, D with x questions in each section
- Section A is common to all and must be attempted
- Section B/C/D is to be attempted based on your roll-no:
  - Let r be your roll no.
  - Take r mod 3 = k
  - If k = 0, attempt section B
  - If k = 1, attempt section C
  - If k = 2, attempt section D

Timings:

- 9:45 - 10:00 am - TA review questions.
- 10:00 - 10:10 am - Read instructions and clarify any doubts related to the question.
- 10:10 - 11:10 am - Attempt questions.
- 11:10 -11:15 am - Time to upload the answers.

1 SECTION A: Answer any two questions

1. (a) Using Gaussian elimination, solve for x, y and z in $3x+2y+z=-1$, $2x-y+4z=3$

(b) Using Gauss-Jordan elimination, solve for x, y and z in
$a+b+c+d=4$,
$a+2b+3c+4d=10$,
$a+3b+6c+10d=20$,
$a+4b+10c+20d=35$

2. (a) Use Cramer's rule to solve the given linear system
$x+y-z=1, x+y+z=2, x-y=3$.

(b) Use Gauss Jordan method to find the inverse of the matrix.
$$A=\begin{pmatrix}1&0&0&0\\0&1&0&0\\0&1&0&0\\a&b&c&d\end{pmatrix}$$

3. For each of the following matrices, either find a matrix P such that $P^{-1}AP$ is a diagonal matrix. If such a matrix does not exist, give clear reasons for the same.

(a) $A=\begin{pmatrix}-1&4&2\\-3&4&0\\-3&1&3\end{pmatrix}$

(b) $A=\begin{pmatrix}5&0&0\\1&5&0\\0&1&5\end{pmatrix}$

4. Consider R^3 with two bases: the standard basis $E=\{e_1,e_2,e_3\}$ and the basis $B=\{f_1,f_2,f_3\}$, where $f_1=\frac{1}{\sqrt{3}}(1,1,1), f_2=\frac{1}{\sqrt{6}}(1,-2,1), f_3=\frac{1}{\sqrt{2}}(1,0,1)$

(a) Find the change of basis matrix from E to B, i.e., $[P]_B^E$.

(b) Find the matrix representation $[T]_E$ of the linear operator T on R^3 having eigen vectors $\{f_1,f_2,f_3\}$ and corresponding eigen values 1,1/2,-1/2 respectively.

2 SECTION B: Answer any two questions :

1. If $A=\begin{pmatrix}0&1\\2&1\end{pmatrix}$, show A is diagonalizable and hence compute A^{10}.

2. Let Q is an orthogonal matrix, then show that 1) Q^{-1} is orthogonal 2) $det(Q)=\pm 1$

3. Orthogonally diagonalize the matrix $A=\begin{pmatrix}2&1&1\\1&2&1\\1&1&2\end{pmatrix}$

3 SECTION C: Answer any two questions :

1. If $A = \begin{pmatrix} 1 & k & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, find the value of k for which A is diagonalizable.

2. Let A and B are two similar matrices of size $n \times n$, then show that 1) A and B have the same eigenvalues 2) A^m and B^m are similar matrices.

3. If Q is an orthogonal matrix, prove that any matrix obtained by rearranging the rows of Q is also orthogonal.

4 SECTION D: Answer any two questions :

1. If $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, find the condition under which A is diagonalizable and not diagonalizable.

2. Let $\lambda_1, \lambda_2, ., , , , \lambda_n$ be the complete set of eigen values of the $n \times n$ matrix A. Prove that 1) $det(A) = \lambda_1 \lambda_2 .... \lambda_n$ and $tr(A) = \lambda_1 + \lambda_2 + ... + \lambda_n$.

3. If A and B are orthogonally diagonalizable show that 1) $A + B$ is orthogonally diagonalizable and 2) AB is orthogonally diagonalizable if $AB = BA$.