MCS2 Linear Algebra: Mid Semester Quiz

October 2021

1 Questions: Each carries 5 marks

1. Let M be the set of all complex 2×2 matrices with determinant equal to 0. Find out whether M is a subspace of $C^{2\times 2}$ (the set of all 2×2 complex matrices).

2. Show that the set W of all polynomials of the form $a + bx - bx^2 + ax^3$ $(a, b \in R)$ is a subspace of P_3 (set of all real polynomials upto degree 3).

3. Let $T : M_{2\times 2} \to M_{2\times 2}($ $M_{2\times 2}$ be the set of all 2×2 matrices) be the linear transformation defined by $T(A) = A^T$. Find the Kernel and Range of the linear transformation T

4. Find the Kernel and Range of the differential operator(linear transformation) $D : P_3 \to P_2$ (P_i set of all real polynomials upto degree i) defined by $D(p(x)) = \frac{d(p(x))}{dx}$.

5. Let u, v and w be distinct vectors of a vector space V. Show that if $[u, v, w]$ is a basis for V then $[u + v + w, v + w, w]$ is also a basis for V.

6. Show that a basis of a subspace W is (a) a maximal independent subset of W (b) a minimal spanning set of W.

7. Find out whether the following transformations are linear or not, (a) $T : R \to R$ defined by $T(x) = x + 1$ (b) $T : R \to R$ defined by $T(x) = 2^x$.

8. Find out whether the following transformations are linear or not, (a)$T : M_{m\times n} \to R$ defined by $T(A) = tr(A)$ (b) $T : M_{m\times n} \to R$ defined by $T(A) = rank(A)$

9. Let $T : V \to W$ be a linear transformation and let $B = [v_1, v_2, \cdot v_n]$ be a spanning set for V. Then $T(B) = [T(v_1), T(v_2), \cdot T(v_n)]$ spans the range of T.

10. Let S be a subspace of R^n. Then show that any two basis for S has same number of vectors.