Statistical Methods in AI (CSE 471)
Ensemble Methods

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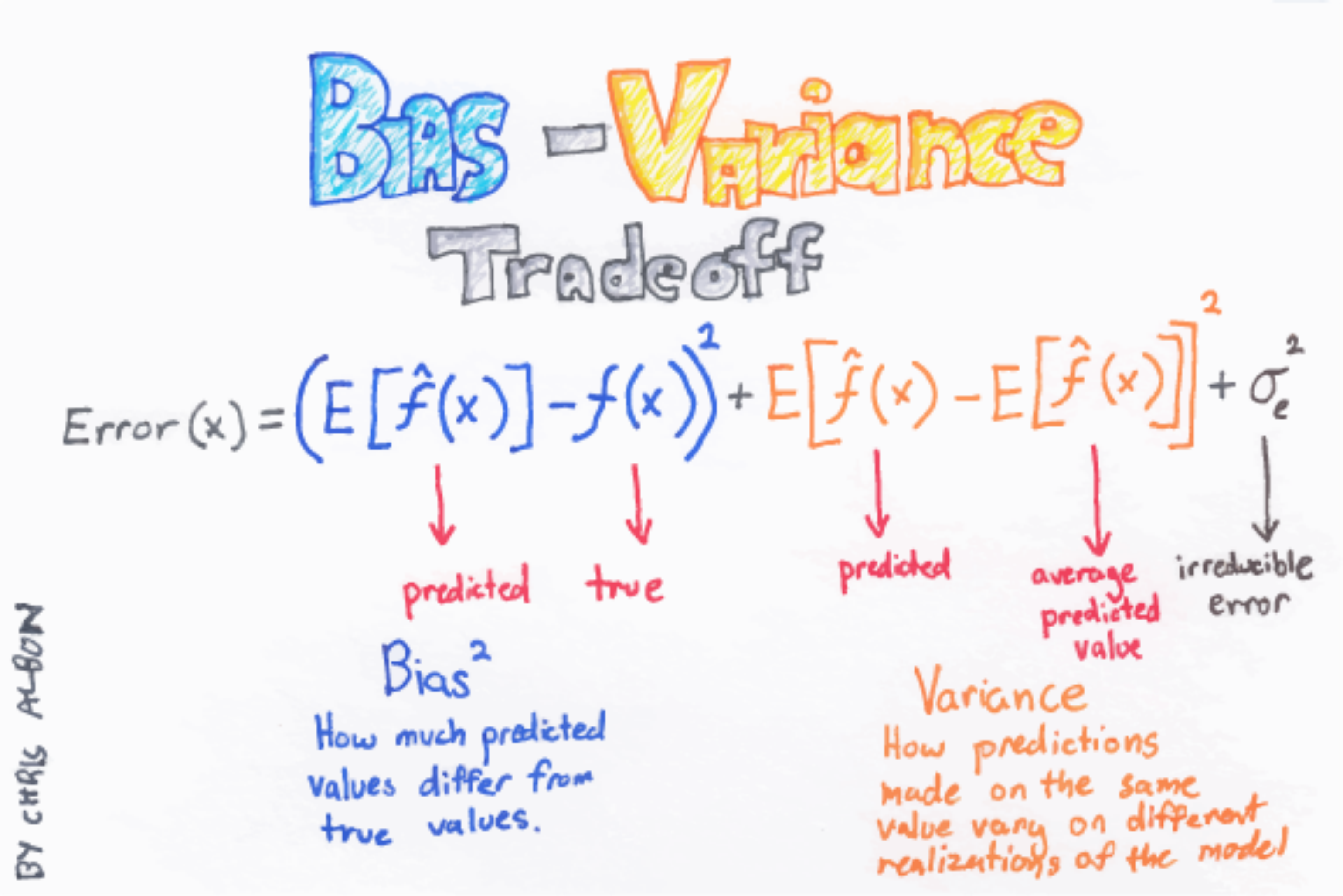
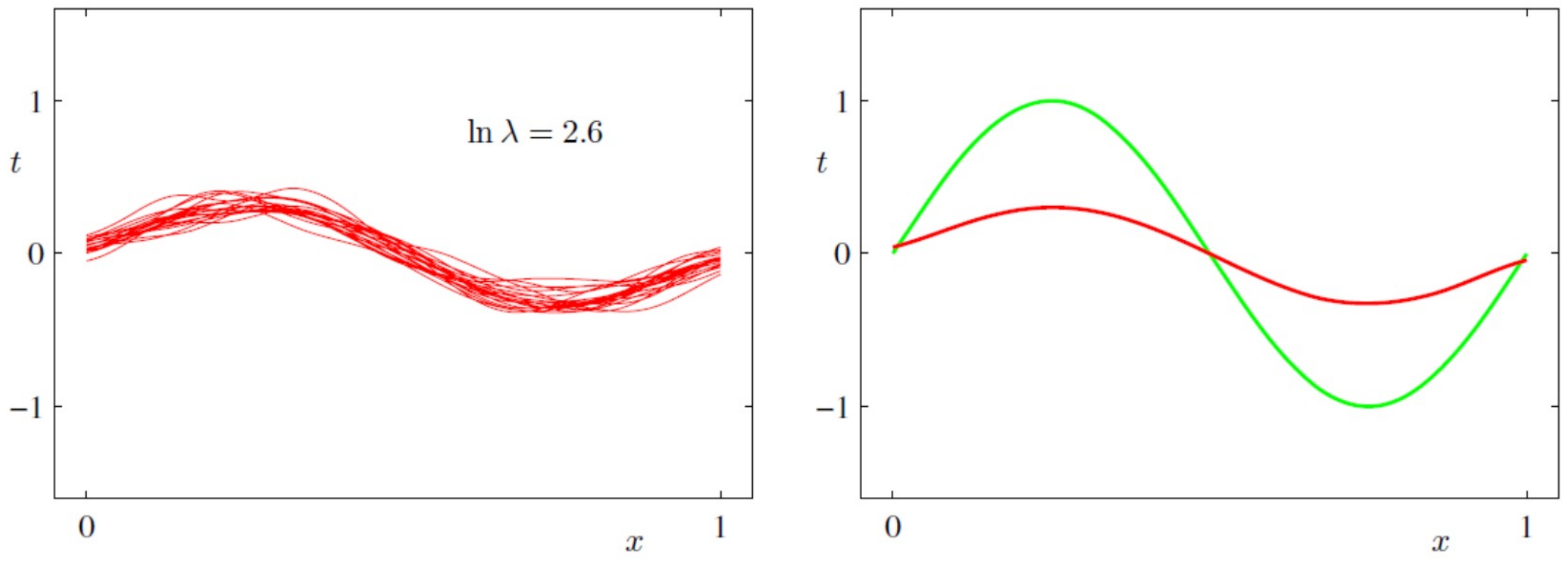
Many slides and figures from: Dr. Markus Kalisch ETH Zurich, Criminisi et al. Microsoft Research, StatQuest with Josh Starmer

Bias Variance trade-off

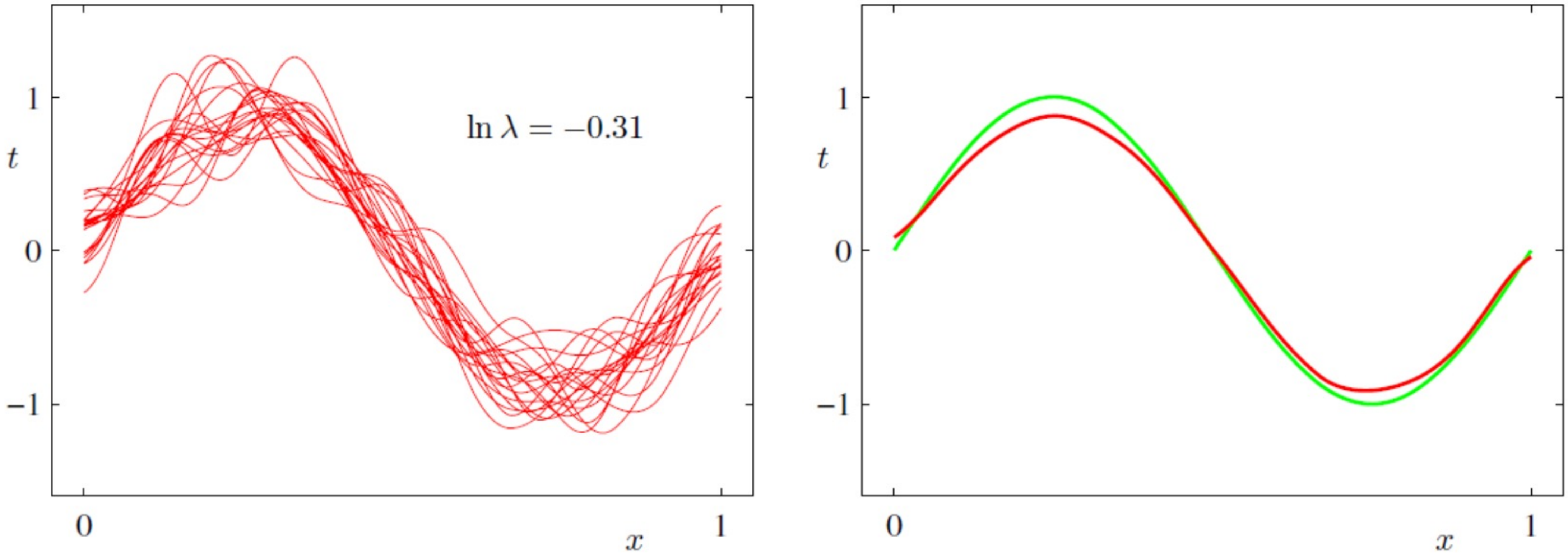


Courtesy — Chris Albon

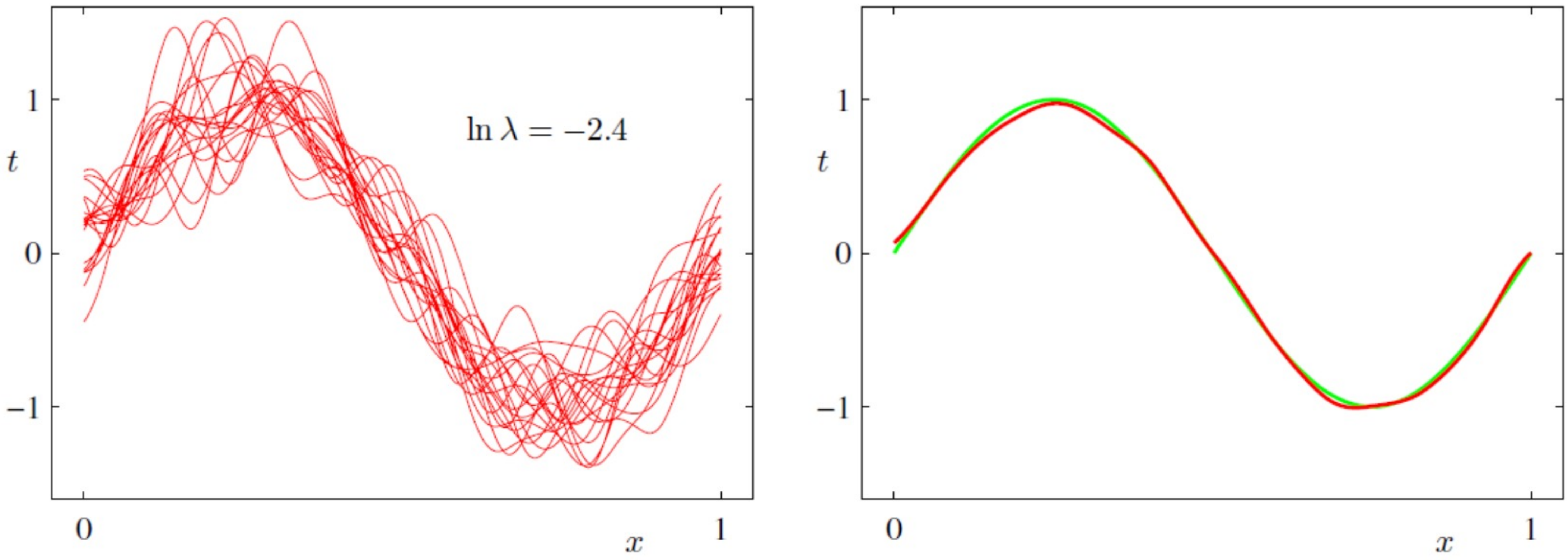
- Fitting a polynomial to 1d input (linear regression) for various regularization values.
- We can sample different datasets and see the variance in predictions and bias (loss of averaged prediction)



- Left: Predictions trained on various sampled datasets. Low variance
- Right: The mean prediction. High bias



- Left: Predictions trained on various sampled datasets. high variance
- Right: The mean prediction. Low bias



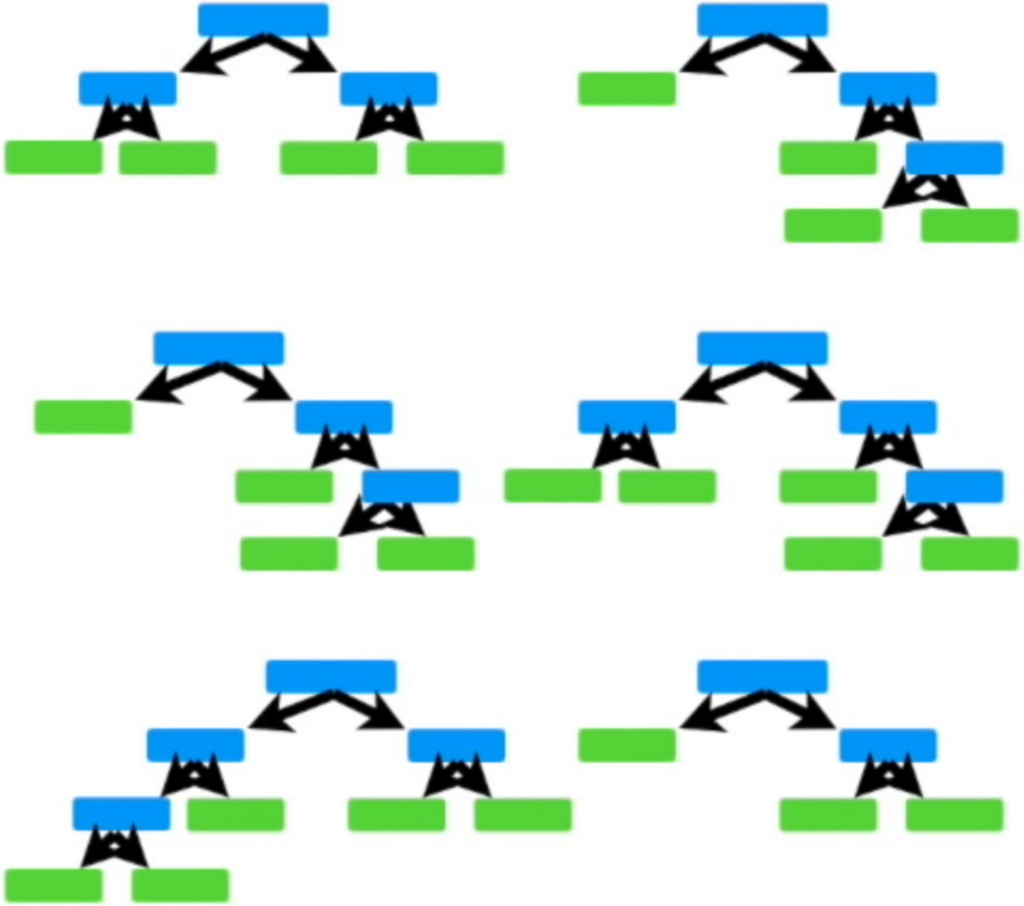
- Left: Predictions trained on various sampled datasets. Higher variance
- Right: The mean prediction. Lower bias

Boosting

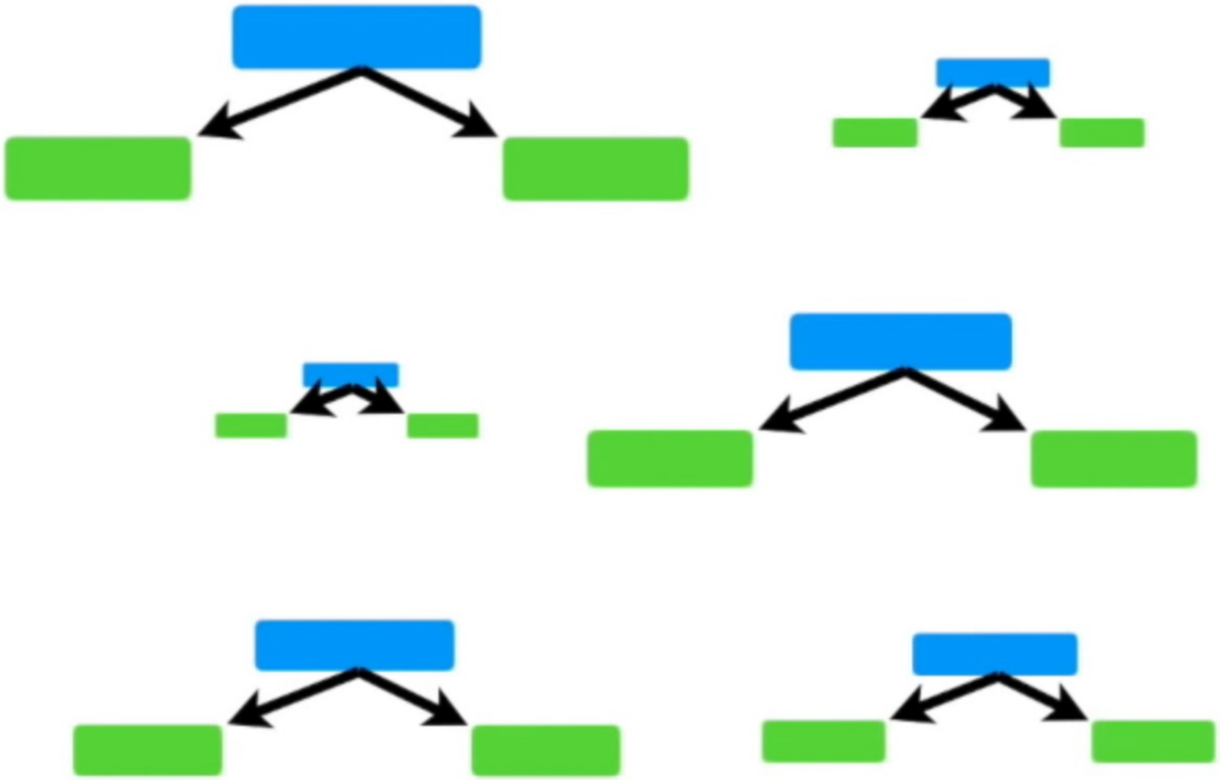
The idea of probabilistic sampling

Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
Yes	Yes	205	Yes	1/8
No	Yes	180	Yes	1/8
Yes	No	210	Yes	1/8
Yes	Yes	167	Yes	1/8
No	Yes	156	No	1/8
No	Yes	125	No	1/8
Yes	No	168	No	1/8
Yes	Yes	172	No	1/8

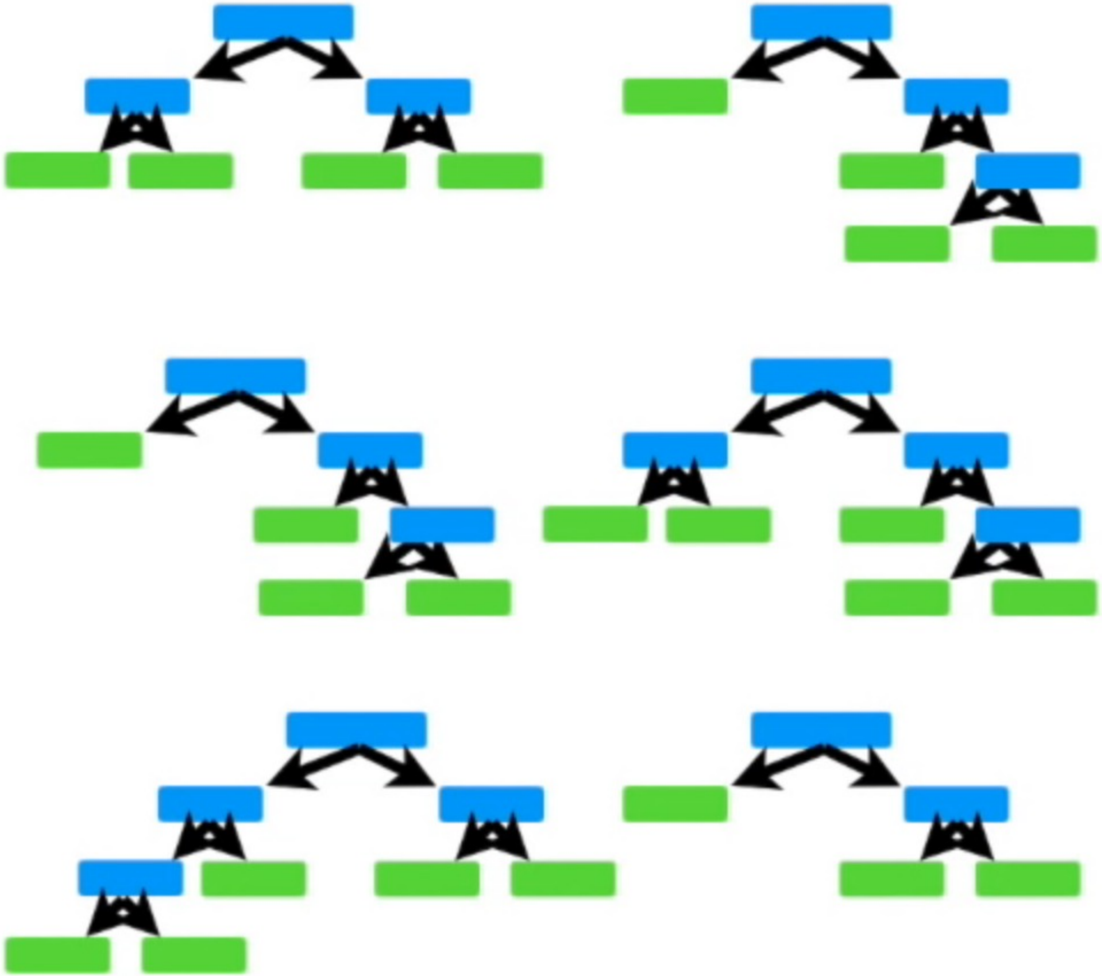
In a **Random Forest**, each tree has an equal vote on the final classification.



In contrast, in a **Forest of Stumps** made with **AdaBoost**, some stumps get more say in the final classification than others.

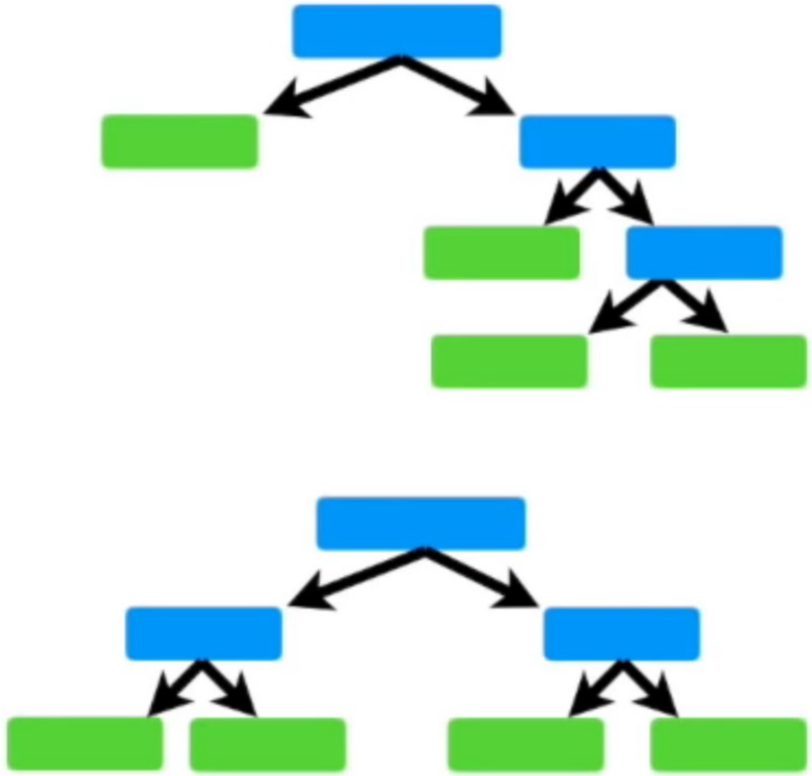


Lastly, in a **Random Forest**, each decision tree is made independently of the others.

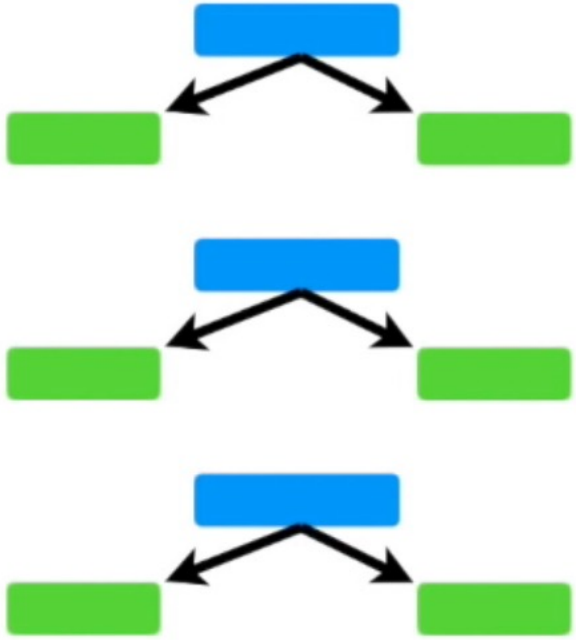


In contrast, in a **Forest of Stumps** made with **AdaBoost**, order is important.

In a **Random Forest**, each time you make a tree, you make a full sized tree.



In contrast, in a **Forest of Trees** made with **AdaBoost**, the trees are usually just a **node** and two **leaves**.



Adaboost

- In Adaboost we assign (non-negative) weights to points in the data set, which are then normalised so that they sum to one
- Iteratively learn new classifier
- In each iteration, we generate a training set by sampling from the data using the weights
- After learning the current classifier, we increase the (relative) weights of the data points which are misclassified by the current classifier
- The final classifier is the weighted majority voting by all classifiers

Adaboost

- Let $\{(X_1, y_1),\ldots,(X_n,y_n)\}$ be the data. We take y_i in $\{-1, +1\}$
- Let $w_i(k)$ denote the weight for the ith data point at kth iteration
- Let h_k be the classifier learnt at kth iteration, we take $h_k(X)$ in $\{-1,+1\}$
- We assume error rate of each classifier on its training data is less than 0.5

Given: $(x_1, y_1), \ldots, (x_m, y_m)$ where $x_i \in \mathscr{X}$, $y_i \in \{-1, +1\}$.
Initialize: $D_1(i) = 1/m$ for $i = 1, \ldots, m$.
For $t = 1, \ldots, T$:

- Train weak learner using distribution D_t.
- Get weak hypothesis $h_t : \mathscr{X} \to \{-1, +1\}$.
- Aim: select h_t with low weighted error:

$$\varepsilon_t = \Pr_{i \sim D_t}\left[h_t(x_i) \neq y_i\right].$$

- Choose $\alpha_t = \frac{1}{2} \ln\left(\frac{1-\varepsilon_t}{\varepsilon_t}\right)$.
- Update, for $i = 1, \ldots, m$:

$$D_{t+1}(i) = \frac{D_t(i) \exp(-\alpha_t y_i h_t(x_i))}{Z_t}$$

  where Z_t is a normalization factor (chosen so that D_{t+1} will be a distribution).

Output the final hypothesis:

$$H(x) = \operatorname{sign}\left(\sum_{t=1}^{T} \alpha_t h_t(x)\right).$$

Fig. 1 The boosting algorithm AdaBoost.

Lets take an example

Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight
Yes	Yes	205	Yes	1/8
No	Yes	180	Yes	1/8
Yes	No	210	Yes	1/8
Yes	Yes	167	Yes	1/8
No	Yes	156	No	1/8
No	Yes	125	No	1/8
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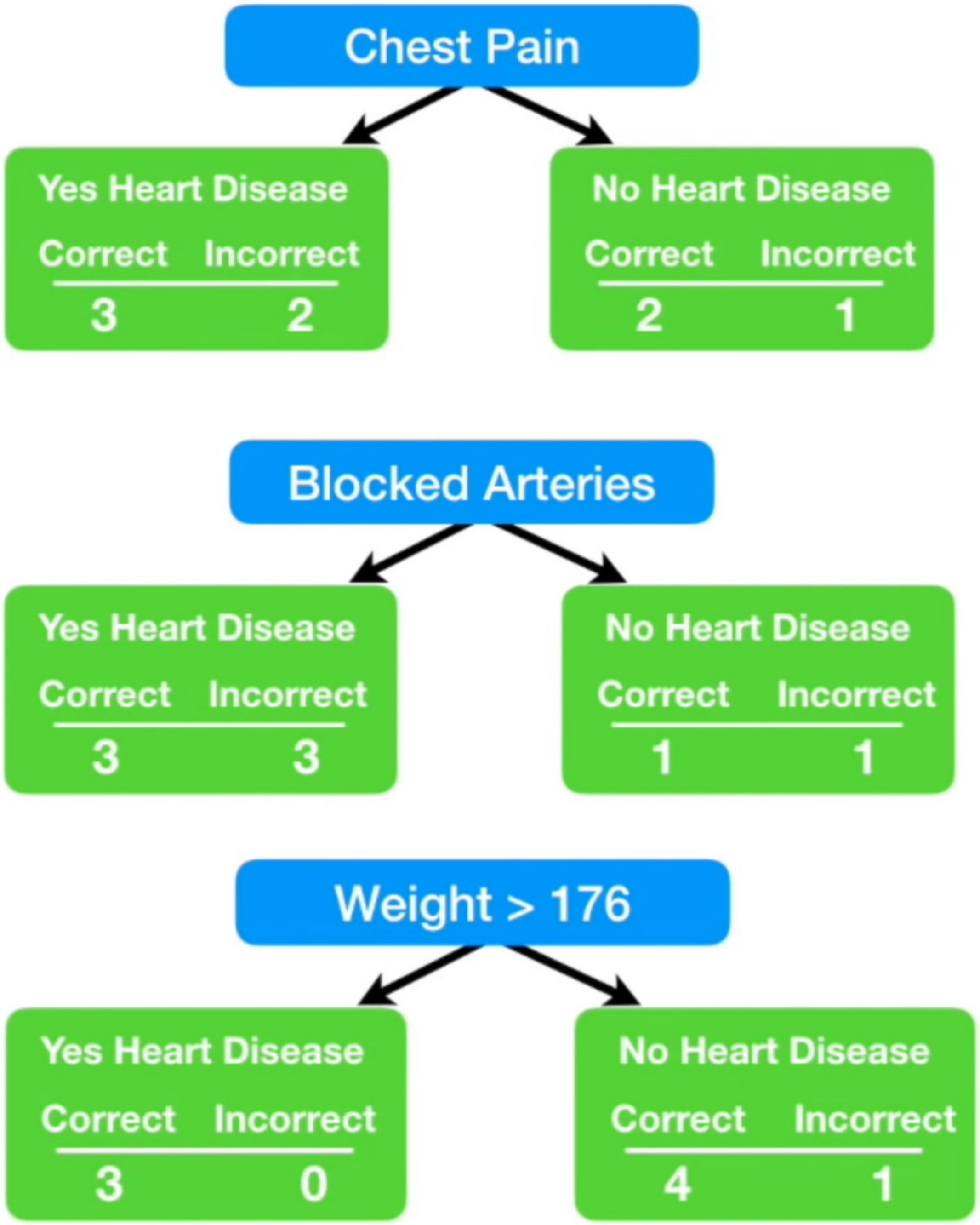

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Thus, in this case, the **Total Error** is **1/8**.

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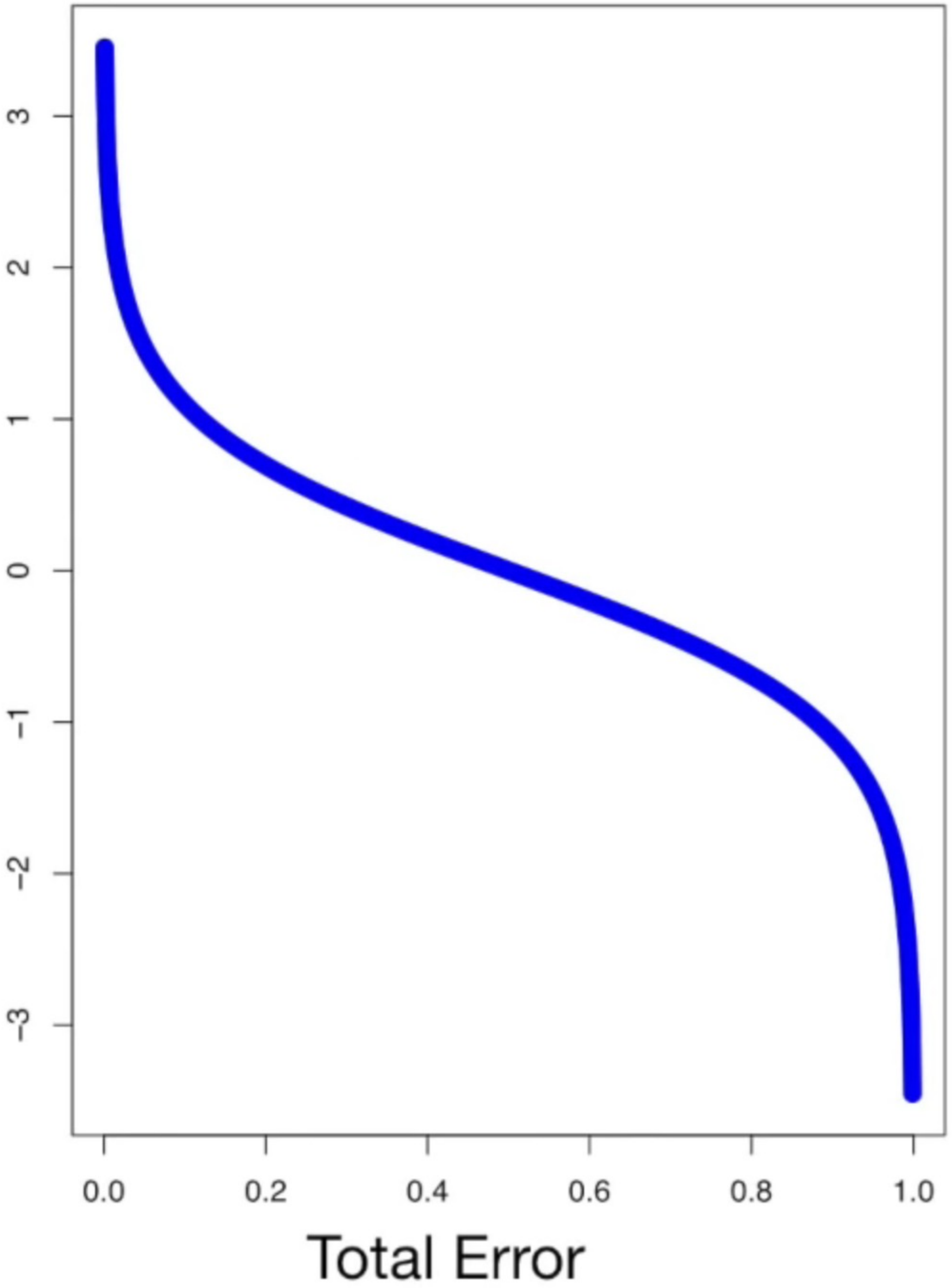
Thus, in this case, the **Total Error** is **1/8**.

$$\text{Amount of Say} = \frac{1}{2}\log\left(\frac{1 - \text{Total Error}}{\text{Total Error}}\right)$$



$$\text{Amount of Say} = \frac{1}{2}\log\left(\frac{1 - \text{Total Error}}{\text{Total Error}}\right)$$

$$\text{Amount of Say} = \frac{1}{2}\log(\ 7\ ) = 0.97$$

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$$\text{New Sample Weight} = \text{sample weight} \times e^{\text{amount of say}}$$

$$= \frac{1}{8} e^{0.97} = \frac{1}{8} \times 2.64 = 0.33$$

$$\text{New Sample Weight} = \text{sample weight} \times e^{-\text{amount of say}}$$

$$= \frac{1}{8} e^{-0.97} = \frac{1}{8} \times 0.38 \boxed{= 0.05}$$

Chest Pain	Blocked Arteries	Patient Weight	Heart Disease	Sample Weight	New Weight	Norm. Weight
Yes	Yes	205	Yes	1/8	0.05	0.07
No	Yes	180	Yes	1/8	0.05	0.07
Yes	No	210	Yes	1/8	0.05	0.07
Yes	Yes	167	Yes	1/8	0.33	0.49
No	Yes	156	No	1/8	0.05	0.07
No	Yes	125	No	1/8	0.05	0.07
Yes	No	168	No	1/8	0.05	0.07
Yes	Yes	172	No	1/8	0.05	0.07

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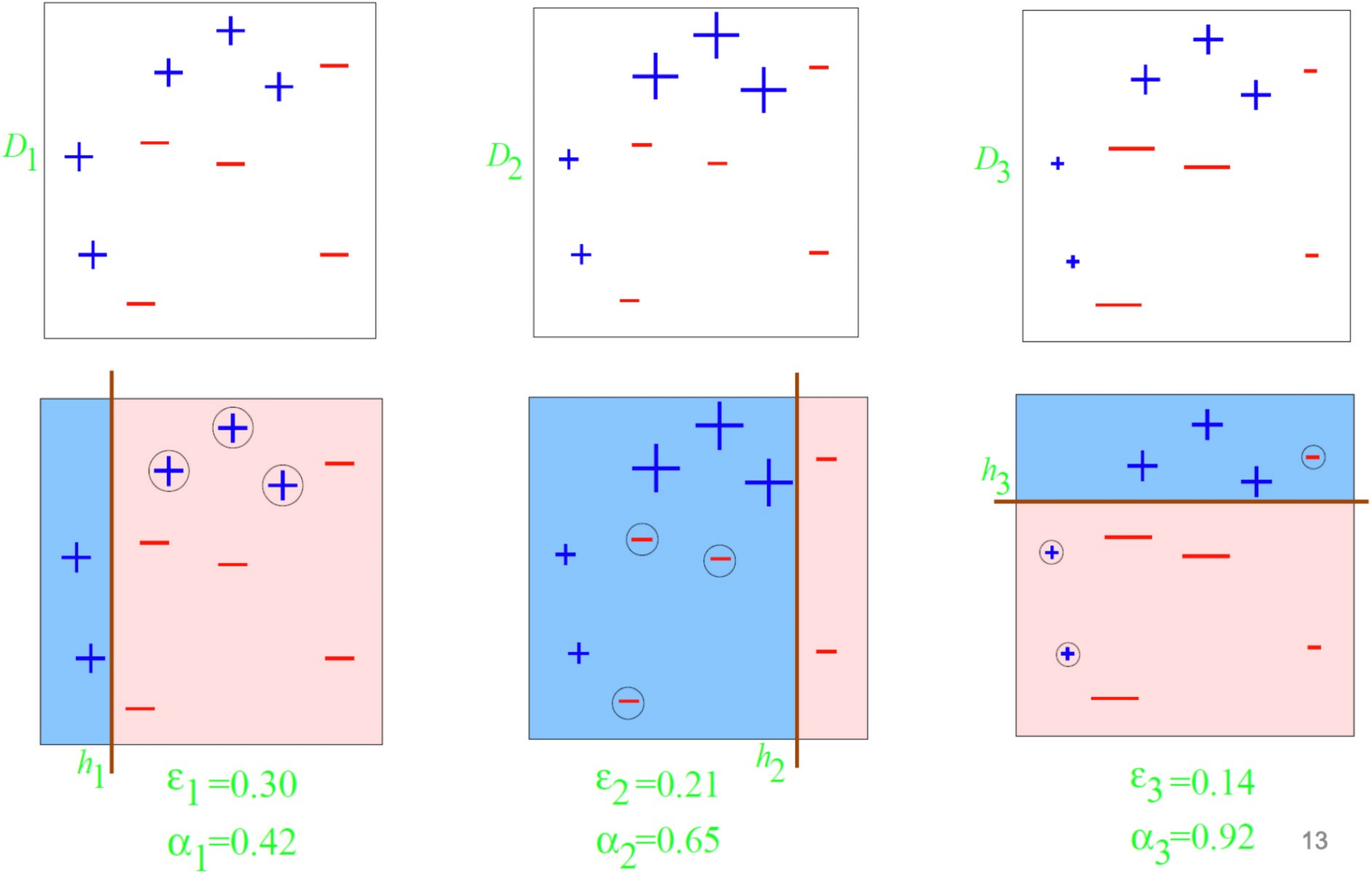
Yes	Yes	172	No	0.07

Chest Pain	Blocked Arteries	Patient Weight	Heart Disease
No	Yes	156	No
Yes	Yes	167	Yes
No	Yes	125	No
Yes	Yes	167	Yes
Yes	Yes	167	Yes
Yes	Yes	172	No
Yes	Yes	205	Yes
Yes	Yes	167	Yes

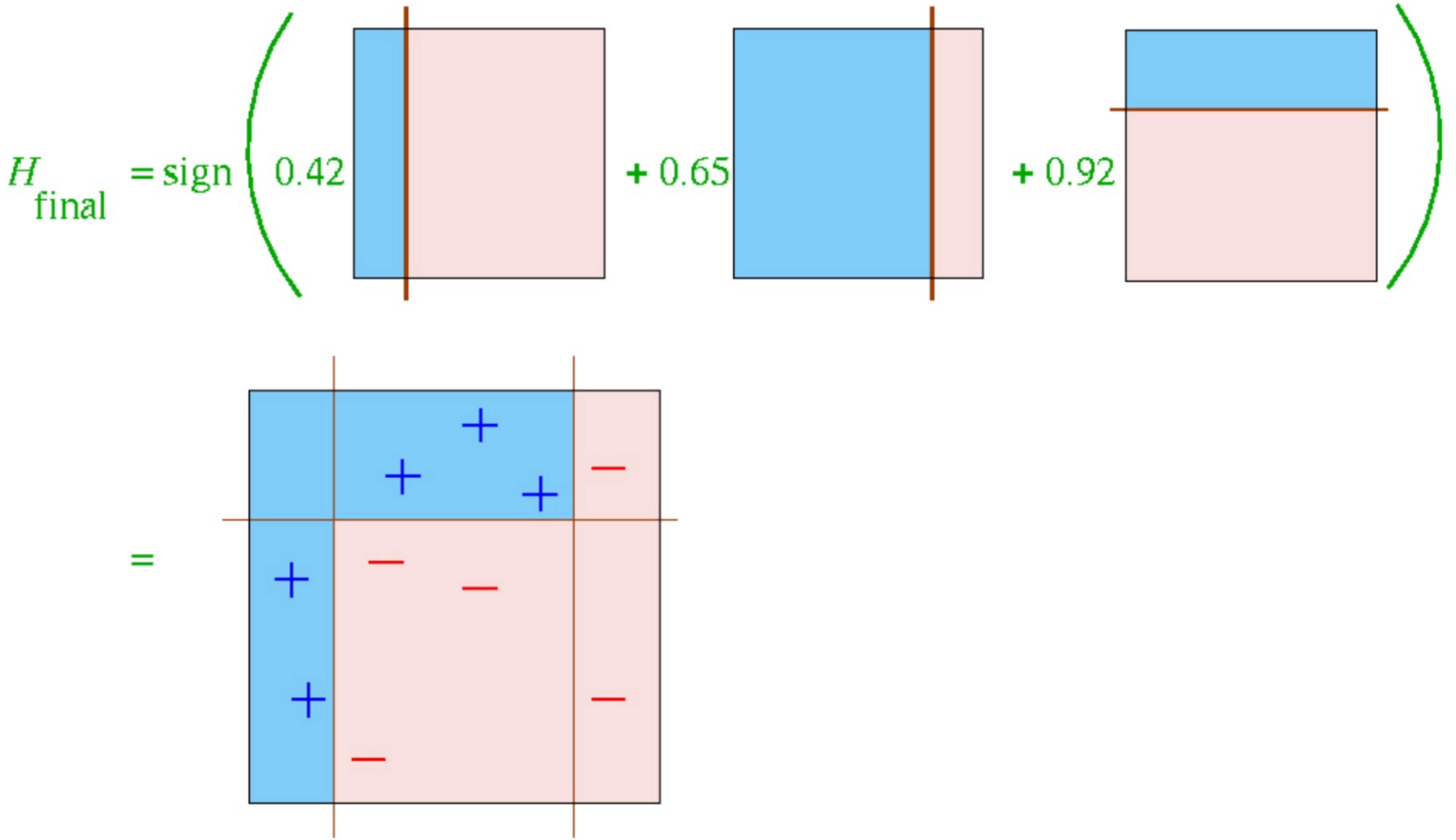
Ultimately, the patient is classified as **Has Heart Disease** because this is the larger sum.

Has Heart Disease **Total = 2.7**

Total = 1.23 Does Not Have Heart Disease



D_1
D_2
D_3
h_1
h_2
h_3
$\varepsilon_1=0.30$
$\alpha_1=0.42$
$\varepsilon_2=0.21$
$\alpha_2=0.65$
$\varepsilon_3=0.14$
$\alpha_3=0.92$


$H_{\text{final}} = \text{sign}\left(0.42 \; + 0.65 \; + 0.92\right)$
=
+
−