Module-1
Asymptotic Functions

1. **Properties**
2. **Examples**

Properties

As we have gone through the definition of this three notations let's now discuss some important properties of those notations.

General Properties :

If f(n) is O(g(n)) then a*f(n) is also O(g(n)) ; where a is a constant.

Example:

$f(n) = 2n^2+5$ is $O(n^2)$, then $7*f(n) = 7(2n^2+5) = 14n^2+35$

$f(n) = O(n^2)$

Similarly this property satisfies for both Θ and Ω notation.

We can say

If f(n) is Θ(g(n)) then a*f(n) is also Θ(g(n)) ; where a is a constant.

If f(n) is Ω (g(n)) then a*f(n) is also Ω (g(n)) ; where a is a constant.

Reflexive

- If f(n) is given then f(n) is O(f(n)).
- Example: $f(n) = n^2$; $O(n^2)$ i.e O(f(n))
- Similarly this property satisfies for both Θ and Ω notation.
- We can say
- If f(n) is given then f(n) is Θ(f(n)).
- If f(n) is given then f(n) is Ω (f(n)).

Transitive

If f(n) is O(g(n)) and g(n) is O(h(n)) then

f(n) = O(h(n)) .

Example: if f(n) = n , g(n) = n^2 and h(n)=n^3

n is O(n^2) and n^2 is O(n^3)

then n is O(n^3)

Similarly this property satisfies for both Θ and Ω notation.

We can say

If f(n) is Θ(g(n)) and g(n) is Θ(h(n)) then f(n) = Θ(h(n)) .

If f(n) is Ω (g(n)) and g(n) is Ω (h(n)) then f(n) = Ω (h(n))

Symmetric Properties

- If $f(n)$ is $\Theta(g(n))$ then $g(n)$ is $\Theta(f(n))$.
- Example: $f(n) = n^2$ and $g(n) = n^2$
- then $f(n) = \Theta(n^2)$ and $g(n) = \Theta(n^2)$
- This property only satisfies for Θ notation.

Transpose Symmetric Properties

- If f(n) is O(g(n)) then g(n) is Ω (f(n)).
- Example: $f(n) = n$, $g(n) = n^2$
- then n is $O(n^2)$ and n^2 is Ω (n)
- This property only satisfies for O and Ω notations.

Some More Properties

- If $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$ then $f(n) = \Theta(g(n))$
- If $f(n) = O(g(n))$ and $d(n)=O(e(n))$
  then $f(n) + d(n) = O(\max(g(n), e(n)))$
- Example:
  - $f(n) = n$ i.e $O(n)$
  - $d(n) = n^2$ i.e $O(n^2)$
  - then $f(n) + d(n) = n + n^2$ i.e $O(n^2)$
- If $f(n)=O(g(n))$ and $d(n)=O(e(n))$
- then $f(n) * d(n) = O(g(n) * e(n))$
- Example:
  - $f(n) = n$ i.e $O(n)$
  - $d(n) = n^2$ i.e $O(n^2)$
  - then $f(n) * d(n) = n * n^2 = n^3$ i.e $O(n^3)$