Recursion Tree Method

1. A tree where each node represents the cost of a certain recursive sub-problem
2. Sum up the values in each node to get the cost of entire algorithm.

I: Draw a recursion tree based on given recurrence relation.

II: Cost of each level
Total no. of levels in recursion tree
No. of nodes in last level
Cost of last level.

III: Add cost of all levels of recursion tree and simplify the expression so obtained in terms of asymptotic notation.

Problem-1: Solve the following recurrence relation using recursion tree method:

$$T(n) = 2T(\tfrac{n}{2}) + n$$

1: $T(n)$ Level 0

$T(\frac{n}{2})$ $T(\frac{n}{2})$ Level 1

$T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(\frac{n}{4})$ $T(n/4)$

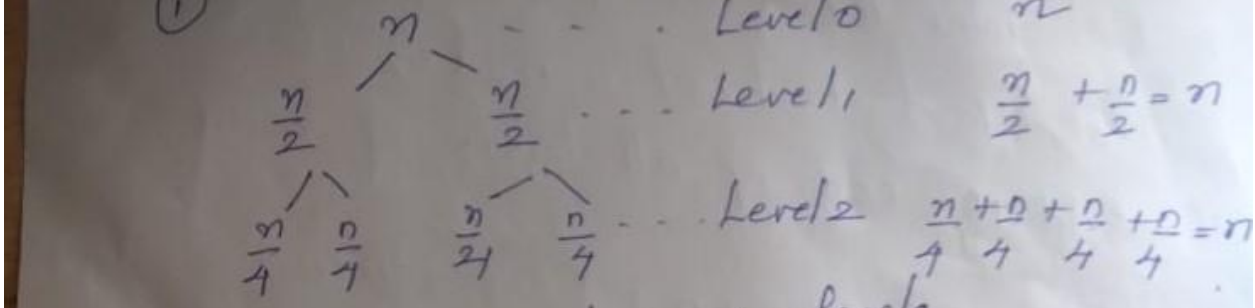
⋮

* A problem of size n will get divided in 2 subproblem of size n/2.
* Then, each sub-problem of size $\frac{n}{2}$ will get divided into 2 sub-problems of size $n/4$
* At the bottom layer, the size of subproblem will reduce to 1.

The given recurren relation shows:

* The cost of dividing a problem of size n into 2 sub-problems and combining its solution is n.
* Cost of dividing a problem of size n/2 into 2 sub problems and then combining its solution is $\frac{n}{2}$ . . . . .

(1) (2)



(3) Determine the total no. of levels

Level-0 = $\frac{n}{2^0}$

Level-1 = $\frac{n}{2^1}$

Level-2 = $\frac{n}{2^2}$

Continuining we have size of subproblem at level i = $\frac{n}{2^i}$

the last level, size of sub-problem becomes 1.

$$\frac{n}{2^x} = 1$$

$$= 2^x = n$$

Logarithm on both sides $x = \log_2 n$

∴ Total no. of levels in recursion tree

Determine the no. of nodes in last level

Level 0 has 2^0 ie., 1 node
Level 1 has 2^1 ie., 2 nodes
Level 2 has 2^2 ie., 4 nodes
:
Level $\log_2 n$ has $2^{\log_2 n}$ nodes ie., n nodes

To Determine the cost of last level,

Cost of last level $= nT(1) = \theta(n)$

6) Add the cost of all levels of recursion tree

$$T(n) = \underbrace{\{n + n + n + \cdots\}}_{\log_2 n \text{ Levels}} + \theta(n)$$

$$n \log_2 n + \theta(n)$$

$$= n \log_2 n + \theta(n)$$

$$= \theta(n \log_2 n)$$