

## NUMBER SYSTEM

Numbers are collection of certain symbols or figures called digits.

The common number system in use is Decimal number system. In this system we use ten symbols each representing a digit. These are 0,1,2, 3,4,5,6,7,8 and 9. A combination of these figures representing a number is called a numeral. We also have Binary number system. It uses only 0 and 1. There are other number systems too. We would confine our discussion to Decimal number system in this topic.

Every digit has a face value which equals the value of the digit itself, irrespective of its place in the numeral. Each digit in a number or numeral has a place value besides its face value. For a given number / numeral we begin from the extreme right as unit's place, ten's place, hundred's place, thousand's place and so on.

Example : The number 795698 is represented as shown below :

Lacs $10^5$	Ten thousands $10^4$	Thousands $10^3$	Hundreds $10^2$	Tens $10^1$	Units $10^0$
7	9	5	6	9	8

We read it as :

Seven lac ninety five thousand six hundred and ninety eight.

In the above number,

The place value of 8 is  $(8 \times 10^0) = 8$

The place value of 9 is  $(9 \times 10^1) = 90$

The place value of 6 is  $(6 \times 10^2) = 600$  and so on.

The face value of a digit in a number is the value of digit itself wherever it may be .

Thus, the face value of 7 in the above numeral is 7, the face value of 9 is 9 and so on.

### Types and Nature of Numbers.

**Natural numbers :** These are also called counting numbers as these numbers are the ones which we use for counting purpose. It is represented by N.

eg :  $N = \{1, 2, 3, \dots, \infty\}$

**Whole numbers :** It includes all natural numbers with zero. We can denote it by W .

Eg :  $W = \{0, 1, 2, \dots, \infty\}$

**Integers :** It includes all whole numbers along with negative numbers. It is represented by I

eg :  $I = \{-\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty\}$

Thus we see that whole numbers are nothing but positive integers and zero. Similarly, natural numbers consists of positive integers.

**Even number :** A number which is completely divisible by 2 is called an even number. In other words such numbers has 2 as a factor when they are written as a product of different numbers.

Eg : 2, 4, 6, 8,.....

A number is said to be a factor or submultiple of another when it divides the other exactly. For example 5 and 3 are factors of 15.

**Odd number :** These numbers are not completely divisible by 2. In other words, a number which is not even is an odd number.

Eg : 1, 3, 5, 7,.....

It may be noted that zero is an exception to this even-odd classification.

**Real number** : Any measurement carried out in the physical world leads to some meaningful figure or value or number. This number is called real number. It consists of two groups :

- (i) **Rational number** : A rational number can always be represented by a fraction of the form  $\frac{p}{q}$ , where  $p$  and  $q$  are integers and  $q$  is not equal to zero. All integers and fractions are rational numbers.

- (ii) **Irrational number** : An irrational number can't be expressed in the form of  $\frac{p}{q}$ , where  $q \neq 0$

Eg :  $\sqrt{3}$ ,  $\sqrt{2}$ ,  $\sqrt{5}$

It gives only an approximate answer in the form of a fractional or decimal number. The digits after the decimal point are non-ending. The same holds true for  $\pi = 3.14....$  which is again irrational. Alternatively we can say that an infinite non-recurring decimal number is an irrational number.

**Prime number** : A prime number is a number which has no factors besides itself and unity, *ie*, it is divisible only by itself and 1 and by no other numbers.

Eg : 2, 3, 5, 7, 11, 13, 17, 19, 23,-----

The largest prime number known so far is  $2^{2281} - 1$  which is a number of about 700 digits.

**Note :**

- (1) 2 is the only even number which is prime.  
(2) All prime numbers other than 2 are odd numbers but all odd numbers are not prime numbers, for example, 9 is an odd

number but it is not a prime number as it is divisible by 3.

**Composite number** : A composite number is one which has other factors besides itself and unity. Thus it is a non-prime number.

Eg : 4, 6, 9, 14, 15, .....

**Note** : (1) 1 is neither prime nor composite.

(2) A composite number may be even or odd.

The number of ways in which a number  $N$  can be expressed as the product of two factors which are relatively prime to each other is  $2^{M-1}$ , where  $M$  is the number of different prime factors of  $N$ .

Eg :  $540 = 2^2 \times 3^3 \times 5$

Prime factors of 540 are 2,3 and 5

$\therefore M = 3$

$\therefore$  Number of ways =  $2^{3-1} = 4$

ie  $20 \times 27$ ,  $4 \times 135$ ,  $108 \times 5$ ,  $540 \times 1$ .

**Consecutive Integers** : These are series of numbers differing by 1 in ascending order.

Eg : 5, 6, 7, 8, .....

**Fractions** : A fraction is a number which represents a ratio or divisions of two integers.

It is expressed in the form  $\frac{a}{b}$ ,

where 'a' and 'b' are integers. 'a' is called the numerator and 'b' is called the denominator. A fraction cannot have zero ( $b \neq 0$ ) as a denominator since division by zero is not defined.

Zero divided by any integer is always zero. A fraction with 1 as the denominator is the same as its numerator.

Two fractions are equivalent if they represent the same ratio or number. So if we

multiply or divide the numerator and denominator of a fraction by the same non-zero integer, the result obtained will be equivalent to the original fraction.

**Decimals :** A collection of digits (0, 1, 2, .....9) after a period ( called the decimal point) is called a decimal fraction.

Eg : 0.629, 0.013, 10.652, etc.

Every decimal number represents a fraction. These fractions have denominators with power of 10.

$$\text{eg: } 0.5 = \frac{5}{10}, 0.45 = \frac{4}{10} + \frac{5}{100}$$

A number containing a decimal point is called a decimal number.

**Mixed number :** A mixed number consists of a whole number and a fraction.

Eg :  $4 \frac{3}{5}$  is a mixed fraction.

This is equivalent to  $4 + \frac{3}{5}$  and hence

can be written as  $4 \frac{3}{5} = 4 + \frac{3}{5}$

Here 4 is the whole number and  $\frac{3}{5}$  is the fraction.

## TEST OF DIVISIBILITY

1. **Divisibility by 2 :** A given number is divisible by 2, if its unit digit is any one of 0, 2, 4, 6 and 8.
2. **Divisibility by 3 :** A given number is divisible by 3, if the sum of its digits is divisible by 3.
3. **Divisibility by 4 :** A given number is divisible by 4, if the number formed by its last two digits is divisible by 4 or the last two digits are 00.

4. **Divisibility by 5 :** A given number is divisible by 5, if its unit digit is either 0 or 5.
5. **Divisibility by 6 :** A given number is divisible by 6, if it is divisible by both 2 and 3.
6. **Divisibility by 8 :** A given number is divisible by 8, if the number formed by its last three digits is divisible by 8 or the last three digits are 000.
7. **Divisibility by 9 :** A given number is divisible by 9, if the sum of its digits is divisible by 9.
8. **Divisibility by 10 :** A given number is divisible by 10, if its unit digit is 0.
9. **Divisibility by 11 :** A given number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.
10. **Divisibility by 12 :** A given number is divisible by 12, if it is divisible by both 3 and 4.
11. **Divisibility by 14 :** A given number is divisible by 14, if it is divisible by both 2 and 7.
12. **Divisibility by 15 :** A given number is divisible by 15, if it is divisible by both 3 and 5.
13. **Divisibility by 16 :** A given number is divisible by 16, if the number formed by the last 4 digits of the given number is divisible by 16 or the last four digits are 0000.
14. **Divisibility by 18 :** A given number is divisible by 18, if it is divisible by both 2 and 9.
15. **Divisibility by 20 :** A given number is divisible by 20, if it is divisible by both 4 and 5.
16. **Divisibility by 7, 13, 17 and 19 :**

There is no direct method to test the divisibility by 7, 13, 17 and 19. In all the

above cases, the operator for checking is the number itself or its prime factors. But here the operator is not the number itself or its factors.

In this case each number has a particular operator. For example, the operator for 7 is 2, 13 is 4, 17 is 5 and 19 is 2.

To find the operator for 7; consider the multiples of 7, such that the multiple is very near to the multiple of 10 (*ie*  $21 = 3 \times 7$  is near to  $20 = 2 \times 10$ ). Here 2 (the number multiplied by 10 to get 20) is taken as the operator. Similarly the multiple of 13 (*ie*  $3 \times 13 = 39$ ) near to the multiple of 10 (*ie*  $40 = 4 \times 10$ ) is 39. Here 4 (the number multiplied by 10 to get 40) is taken as the operator. Through the same way we can determine the operator for 17 as 5 and that for 19 as 2.

The multiple of 7 and 17 (*ie*  $3 \times 7 = 21$  and  $3 \times 17 = 51$ ) are one more than the multiple of 10 (*ie*  $20 = 2 \times 10$  and  $50 = 5 \times 10$ ), therefore the operator for 7 and 17 are called one more operator. Also the multiple of 13 and 19 (*ie*  $3 \times 13 = 39$  and  $1 \times 19 = 19$ ) are one less than the multiple of 10 (*ie*  $40 = 4 \times 10$ , and  $20 = 2 \times 10$ ), therefore the operator for 13 and 19 are called negative operators.

#### **Divisibility by 7 :**

To test the divisibility by 7, multiply the last digit of the given number by the operator of 7 (*ie* 2) and subtract the product from the number obtained by removing the last digit of the given number. Again take the last digit of the difference and multiply by the operator, the product is subtracted from the number obtained by removing the last digit of the just previous difference. The above process is repeated. If the difference obtained is zero or a multiple of 7, then we say that the number is divisible by 7.

Eg : To test the number 3563 is divisible by 7.

The operator for 7 is 2 and the last digit of the given number is 3. Then its product is  $3 \times 2 = 6$ .

The number obtained by removing the last digit is 356.

Then  $356 - 6 = 350$

Here the difference is 350 and its last digit is 0. The product with operator is  $2 \times 0 = 0$ .

The number obtained by removing the last digit of the difference is 35

Then  $35 - 0 = 35$ .

Here 35 is a multiple of 7. Therefore the number 3563 is divisible by 7.

#### **Divisibility by 13**

Multiply the last digit of the given number by the operator for 13 (*ie* 4) and add the product to the number obtained by removing the last digit of the given number. Again take the last digit of the sum (previously obtained) and multiply by the operator. Add the product to the number obtained by removing the last digit of the sum (just previously obtained). The above process is repeated. If at last the sum obtained is the multiple of 13, then we say that the given number can be divisible by 13.

Eg : To test the number '6539' is divisible by 13.

The operator for 13 is 4, and the last digit of the given number is 9; then its product is  $4 \times 9 = 36$ .

The number obtained by removing the last digit of the given number is 653.

Then, sum =  $653 + 36 = 689$

Here the sum is 689, its last digit is 9 and its product with the operator is  $9 \times 4 = 36$ .

The number obtained by removing the last digit of the sum is 68.

Then sum,  $68 + 36 = 104$

Again  $4 \times 4 = 16$

Then sum  $= 10 + 16 = 26$

Here 26 is a multiple of 13. Therefore the number 6539 is divisible by 13.

**Divisibility by 17 :** Same as 7

**Divisibility by 19 :** Same as 13.

### SOLVED EXAMPLES.

1. What is the difference in the place value and the face value of 8 in 78975?

Ans :

The place value of 8 is 8000

The face value of 8 is 8

$\therefore$  the required difference  $= 8000 - 8 = 7992$

2. The numbers  $p, p + 2, p + 4$  are all primes. Find the value of  $p$ .

Ans : When  $p = 3$ , the given numbers are 3, 5, 7; which are prime.

3. If  $42k8$  is a multiple of 9, find the value of  $k$

Ans : Sum of digits  $= (4 + 2 + k + 8)$   
 $= (14 + k)$ .

Now, least value of  $k$  for which  $(14 + k)$  is divisible by 9 is  $k = 4$ .

4. Find the least number of five digits that is exactly divisible by 456.

Ans :

The least five digit number  $= 10000$ .  
 when 10000 is divided by 456, the remainder is 424.

$\therefore$  the required number

$$= 10000 + (456 - 424) = 10032$$

5. When a certain number is multiplied by 13, the product consists entirely of fives.

Which is the smallest of such numbers.?

Ans : By hit and trial we find that 555555 is divisible by 13.

$$\frac{555555}{13} = 42735$$

$\therefore$  the required number  $= 42735$

6. A number when divided by 779 gives 47 as remainder. On dividing the same number by 19, what would be the remainder ?

Ans :

When the number is divided by 779, let the quotient be  $k$ .

$$\therefore \text{number} = 779k + 47$$

$$= 19 \times 41k + 19 \times 2 + 9$$

$$= 19(41k + 2) + 9$$

So, when the number is divided by 19, the quotient is  $(41k + 2)$  and the remainder is 9

7. A positive number when decreased by 4 is equal to 21 times the reciprocal of the number. Find the number.

Ans : Let the number be  $x$

$$\text{Then } x - 4 = \frac{21}{x}$$

$$\text{ie } x^2 - 4x - 21 = 0$$

$$(x - 7)(x + 3) = 0 \Rightarrow x = 7$$

8. A certain number consists of two digits whose sum is 9. If the order of the digits is reversed, the new number is 9 less than the original number. The original number is :

Ans :

Let ten's digit be  $x$  and unit's digit be  $y$ .

$$\text{Then } x + y = 9$$

$$(10x + y) - (10y + x) = 9 \Rightarrow x - y = 1$$

$$\text{Solving } x + y = 9 \text{ \& } x - y = 1,$$

$$\text{we get } x = 5 \text{ \& } y = 4$$

$$\therefore \text{ the required number} = 54$$

9. A boy was asked to multiply a certain number by 25. He multiplied it by 52 and got his answer more than the correct one by 324. The number to be multiplied was :  
 Ans : Let the required number be  $x$   
 Then  $52x - 25x = 324$   
 $\Rightarrow x = \frac{324}{27} = 12$
10. A number whose fifth part increased by 5 is equal to its fourth part diminished by 5. Find the number.  
 Ans : Let the required number be  $x$   
 Then  $\frac{x}{5} + 5 = \frac{x}{4} - 5$   
 ie  $\frac{x}{4} - \frac{x}{5} = 10 \Rightarrow x = 200$
4. In an examination, a student was asked to find  $\frac{4}{5}$  th of a number. By mistake, he found  $\frac{5}{4}$  th of it and his answer was 180 more than the correct answer. Find the given number.  
 1) 81                      2) 890  
 3) 400                     4) 500
5. A sum is divided between Vishnu and Ananthan such that  $\frac{3}{5}$  th of the amount of Vishnu is equal to  $\frac{5}{7}$  th of the amount of Ananthan. If Vishnu gets Rs. 750, how much does Ananthan get?  
 1) Rs. 450                2) Rs. 600  
 3) Rs. 630                4) Rs. 730
6.  $\frac{4}{5}$  th of a number exceeds its  $\frac{2}{3}$  rd of  $\frac{9}{10}$  th by 120. Find the number.  
 1) 600                      2) 140  
 3) 800                      4) 660

### PRACTICE TEST

1.  $\frac{3}{4}$  th of  $\frac{1}{3}$  rd of  $\frac{4}{5}$  th of a number is 80, find that number  
 1) 300                      2) 80  
 3) 14                        4) 400
2.  $\frac{4}{5}$  of a number exceeds its  $\frac{2}{3}$  rd by 20. Find the number  
 1) 150                      2)  $\frac{3}{2}$   
 3) 160                      4) 300
3. Sum of two numbers is  $\frac{1}{3}$  rd of  $\frac{1}{5}$  th of 195 and their product is  $\frac{1}{6}$  th of  $\frac{1}{4}$  th of 960. Find  $\frac{1}{3}$  rd of the difference between them.  
 1) 1                         2) 9  
 3) 3                         4) 27
7. At an election, a candidate who gets  $\frac{3}{4}$  th of the total votes, is elected by a majority of 2000 votes. The total number of votes polled and the number of votes secured by the candidate who was elected, are respectively,  
 1) 4000, 3000            2) 8000, 6000  
 3) 4500, 2500            4) 5000, 3000
8. One-third of the total marks are required to pass an examination. A candidate who gets 178 marks, fails by 22 marks. The total marks in the examination are  
 1) 550                      2) 600  
 3) 535                      4) 660
9. A sum is divided between Raju and Biju such that  $\frac{5}{6}$  th of the amount of Raju is

- equal to  $\frac{4}{5}$  th the amount of Biju. If Raju gets Rs. 240, find the sum.
- 1) Rs. 500                      2) Rs. 250  
3) Rs. 490                      4) Rs. 400
10. The number of students in a school are 1125 more than  $\frac{1}{4}$  th of it. Find the total strength
- 1) 175                              2) 1500  
3) 2000                            4) 1225
11. A water tank having 1300 litres of capacity was filled by adding 75 litres of water and as many buckets of water as each bucket had a capacity. What was the capacity of each bucket in litres ?
- 1) 32                                2) 35  
3) 48                                4) 45
12. The amount that Satheesh had was Rs. 675 more than  $\frac{3}{5}$  th of  $\frac{1}{9}$  th of Rs. 450. Find the amount.
- 1) Rs. 700                      2) Rs. 705  
3) Rs. 625                      4) Rs. 605
13.  $\frac{5}{8}$  th of 120% of a number is 235 more than 245. Find that number.
- 1) 523                                2) 684  
3) 576                                4) 640
14. A basket contains 44 more than  $\frac{1}{5}$  th of total number of apples. How many apples are there in the basket ?
- 1) 66                                2) 55  
3) 65                                4) 33
15. In a library,  $\frac{1}{2}$  of the books are story books,  $\frac{2}{3}$  rd of the remaining books are reference books and the remaining 250 books are encyclopaedias. Find the total number of books in the library.
- 1) 1,200                            2) 1,500  
3) 2,000                            3) 750
16. In a village,  $\frac{5}{9}$  th of the population are adults.  $\frac{1}{2}$  of the adults are male,  $\frac{3}{5}$  th of the adult females are illiterate. If 800 adult females are illiterate, then the population of the village is
- 1) 4,000                            2) 4,800  
3) 9,000                            4) 5,600
17. In an examination, a student was asked to find  $\frac{5}{7}$  of a number. By mistake, he found  $\frac{7}{5}$  of it. His answer was 96 more than the correct answer. The given number is
- 1) 196                                2) 128  
3) 156                                4) 140
18.  $\frac{7}{8}$  th of a number exceeds its  $\frac{7}{16}$  th by 84. Find  $\frac{3}{4}$  th of the number.
- 1) 144                                2) 192  
3) 138                                4) 204
19. The sum of three consecutive even integers is 132. Find the difference between 3 times the least and  $\frac{1}{2}$  of the greatest.
- 1) 44                                2) 89  
3) 144                                4) 103
20. The sum of four consecutive numbers is 90. Find the difference between five times the third number and 4 times the fourth number.
- 1) 21                                2) 29  
3) 24                                4) 19

21. 24 is divided into two parts such that 6 times the first part exceeds 4 times the second part by 4. Find the first part.  
 1) 14                      2) 16  
 3) 10                      4) 12
22. The sum of 11 results is 55. The sum of the first 6 results is 30 and that of the last 6 is 48. Find the sixth result.  
 1) 20                      2) 24  
 3) 13                      4) 23
23. The total age of 9 boys is 128. The total age of the first 4 boys is 62 and that of the next 4 boys is 48. Find the age of the 9th boy.  
 1) 16                      2) 18  
 3) 12                      4) 17
24. 340 soldiers are arranged in a parade such that the number of soldiers in each column is the same as the number of columns. Find how many soldiers are excluded from the group to make such an arrangement possible.  
 1) 16                      2) 21  
 3) 14                      4) 24
25. The smallest number which must be subtracted from 1300 to make it a perfect square is  
 1) 2                      2) 3  
 3) 4                      4) 6
26. Three books and a pen cost Rs. 70 and 2 pens and a book cost Rs. 40. How much will a book and a pen cost ?  
 1) Rs. 10                      2) Rs. 20  
 3) Rs. 30                      4) Rs. 40
27. Each student in a class contributed as many paise as the number of students in the class. The teacher contributed Rs. 13 to make the total collection of Rs. 49. How many students are there in the class?  
 1) 70                      2) 50  
 3) 60                      4) 65
28. The total marks secured by A, B and C are 540. A's marks were three times that of B's and B's marks were twice that of C. A's marks are how much more than those of C ?  
 1) 154                      2) 300  
 3) 265                      4) 286
29. When 6 is added to a number and the sum is multiplied by 8, the result is same as when 26 is multiplied by 10 and 12 is added to the product. The number is  
 1) 28                      2) 23  
 3) 26                      4) 25
30. Two pens and a pencil cost Rs. 18 and two pencils and a pen cost Rs. 12. Find the cost of a pen  
 1) 8                      2) 2  
 3) 16                      4) 6
31. If the square of a number of two digits is subtracted from the square of the number formed by interchanging the digits, the largest number by which the result is always divisible is  
 1) 9                      2) 10  
 3) 11                      4) 99
32. The smallest number, which when subtracted from the sum of squares of 11 and 13 gives a perfect square, is  
 1) 1                      2) 4  
 3) 5                      4) 9

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### ANSWERS TO PRACTICE TEST

- |         |         |         |         |         |         |         |
|---------|---------|---------|---------|---------|---------|---------|
| 1. (4)  | 2. (1)  | 3. (1)  | 4. (3)  | 5. (3)  | 6. (1)  | 7. (1)  |
| 8. (2)  | 9. (3)  | 10. (2) | 11. (2) | 12. (2) | 13. (4) | 14. (2) |
| 15. (2) | 16. (2) | 17. (4) | 18. (1) | 19. (4) | 20. (4) | 21. (3) |
| 22. (4) | 23. (2) | 24. (1) | 25. (3) | 26. (3) | 27. (3) | 28. (2) |
| 29. (1) | 30. (1) | 31. (4) | 32. (1) |         |         |         |



BANK PROBATIONARY OFFICER  
QUANTITATIVE APTITUDE  
BASIC ARITHMETIC OPERATIONS

**ADDITION**

- (i). Positive no. + Positive no. = Positive no.  
Eg :  $+2 + +5 = +7$
- (ii) Negative no. + Negative no = Negative no.  
Eg :  $-4 + -6 = -10$
- (iii) Negative no. + Positive no. = Difference of nos. and sign of greater no.  
Eg :  $+9 + -11 = -2$ ,  $-10 + +4 = -6$

**QUICKER & SHORT CUT METHODS FOR ADDITION**

In the Bank PO examination, there will be a lot of situations in which you will have to add various numbers. But the most important thing is to add numbers in lightning speed. You should develop the habit of seeing the numbers and adding them instantly. You will not have the time to write down the numbers with a pen on a piece of paper and calculate in the usual manner.

1. The moment you see  
 $9 + 5$ , the number 14 should flash in your mind.
2. As soon as you see  
 $7 + 4 + 9$ , the number 20 should come.  
  
Remember, you should not even read the numbers as seven plus four plus nine. Reading is time consuming. You just see and calculate. Your eye can recognise these numbers as  $7 + 4 + 9$  and instantly the mind can come out with the answer 20.
3. When you see a number, understand and represent it with shorter possible words.

(a) 748 should be understood as seven forty eight, not as seven hundred and forty eight.

(b) 1098 should be understood as ten ninety eight, not as one thousand and ninety eight.

(c) 89876 should be eighty nine eight seven six.

4. Double column addition will enable you to add numbers quicker and faster.

(a) Take the example of  $78 + 65$ . The moment you see the numbers visualise in your mind that they are  $78 + 60 + 5$ . This way you can straight away get the answer 143.

(b)  $84 + 43 + 16$  should be visualised as  $(84 + 40 + 10 + 9) = 143$ .

(c)  $6328 + 4233 + 2495$ . Here try the double column addition

63	28
42	33
<u>24</u>	<u>95</u>
130	56

1st double column is  $28 + 33 + 95 = 28 + 30 + 90 + 8 = 156$ .

Write 56, and 1 is carried.

2nd double column is  $63 + 42 + 24 + 1 = 63 + 40 + 20 + 7 = 130$

Once you master double column addition, you can easily visualise numbers in the addable form and add numbers horizon-