Instructions for Finding The Convex Hull of a Set of Points on Paper

Introduction

The convex hull of a set of points is a shape used to get a sense of what the points look like. It is defined as the polygon which has the entire set of points inside it but also has the smallest perimeter. There are many algorithms which computers use to find the convex hull of a set of points, but most are not approachable for people. This tutorial adapts the Jarvis march method to provide an easy way to find the convex hull of points on a piece of paper. After reading this tutorial, you will be able to show off a neat math trick to your friends and learn a little about computational geometry along the way.

Materials Needed:

- Scrap Paper
- Two markers of different colors

Experience Needed:

- Knowledge of basic geometry terminology (points, lines)
- (Optional but helpful) Basic origami

Warnings:

- Do not use important documents, as you will be drawing on and folding the paper.
- You will be folding paper, so be cautious of papercuts.

The Procedure

Setup

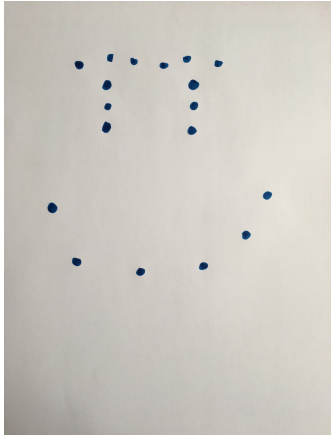1) Draw a set of points on a blank piece of paper using one of the colors (blue). Figure 1 provides an example.

Figure 1

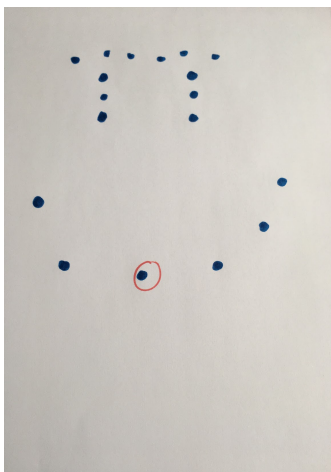2) Find the lowest point on the paper among those you've drawn. Circle it with the other color (red), as in Figure 2.

Figure 2

3)

Fold the paper horizontally through that lowest point, folding the empty side part underneath so that you can still see all the points, as in Figure 3.
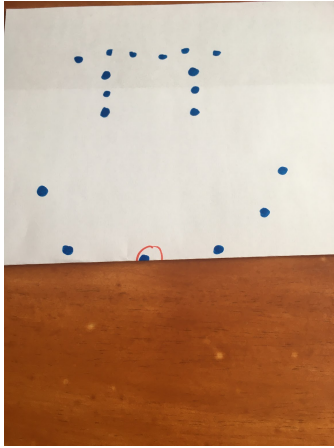

Figure 3

4) Find the point to the right of that lowest point on the paper that forms the smallest angle with the new bottom of the paper, as in Figure 4.
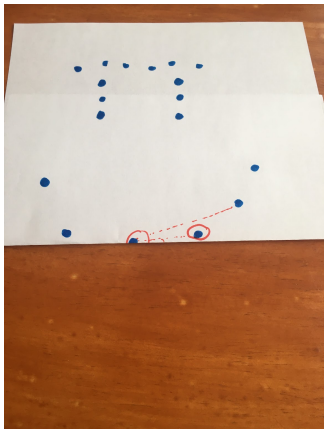

Figure 4

5) Fold the paper along the line between the lowest point and that new point, and tuck it underneath as before. See Figure 5 for an example.
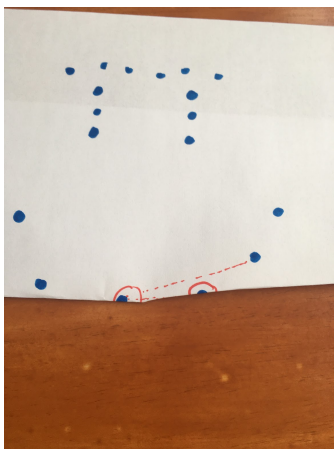

Figure 5

6) Rotate the paper such that the new point is now the lowest point. Repeat steps 3 through 5 until you reach back to the original point, as in Figure 6.
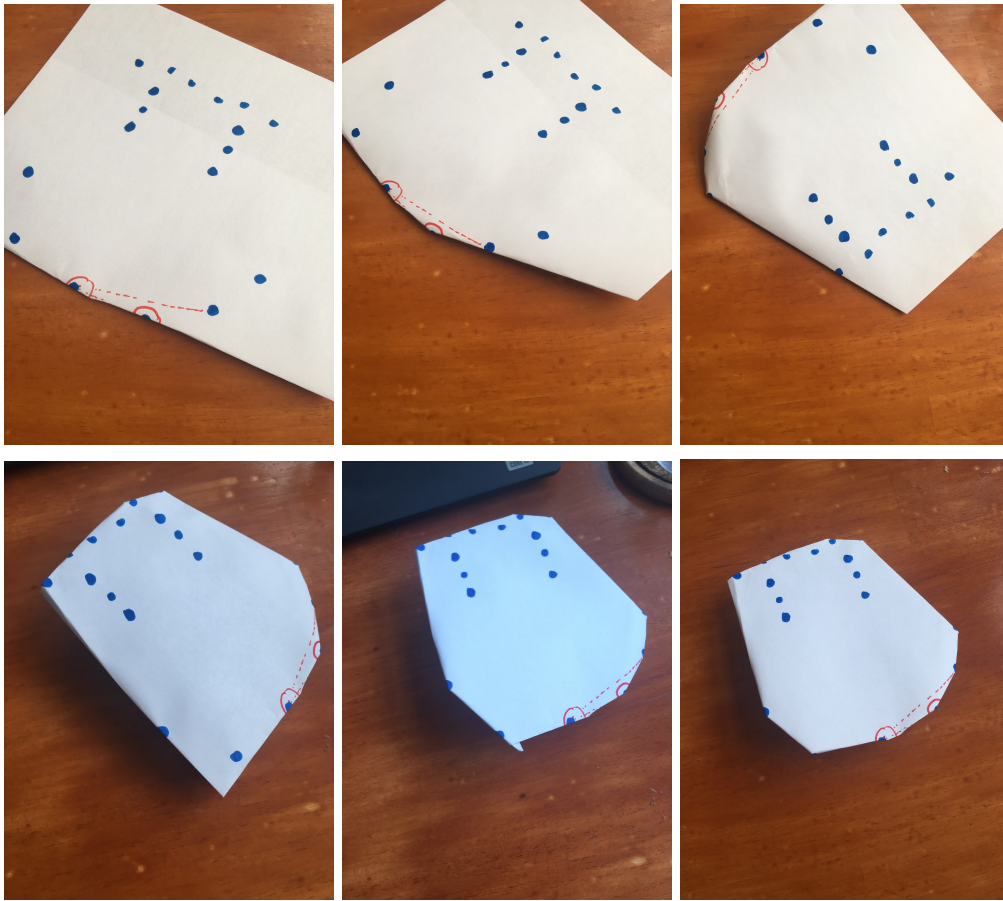
Figure 6

Drawing the Hull

7) Run along the side of the folded paper with your red marker. Keep going until the entire rim is covered. See Figure 7 for an example
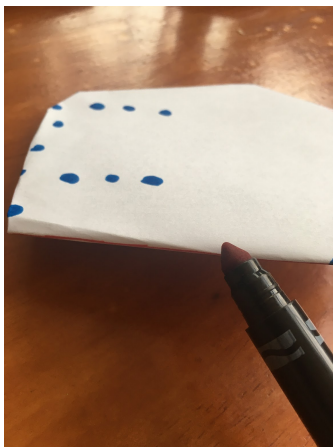
Figure 7

8) Unfold your paper and the result will be the convex hull of the points! See Figure 8 for an example end product.
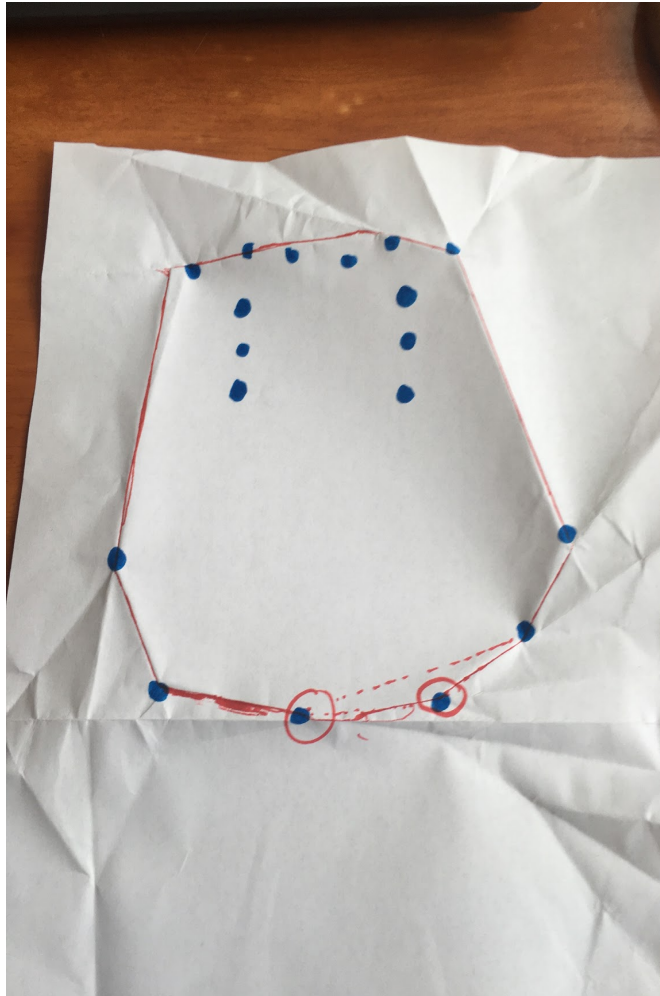

Figure 8

Conclusion

You should now be able to find the convex hull of any set of points by hand. For a more theoretical understanding, you can look at R.A. Jarvis' paper in the Works Cited. Now you have a new math trick to show all your friends and maybe impress a few interviewers.

Works Cited:

Jarvis, R. A. (1973). "On the identification of the convex hull of a finite set of points in the plane". Information Processing Letters. 2: 18–21. doi:10.1016/0020-0190(73)90020-3