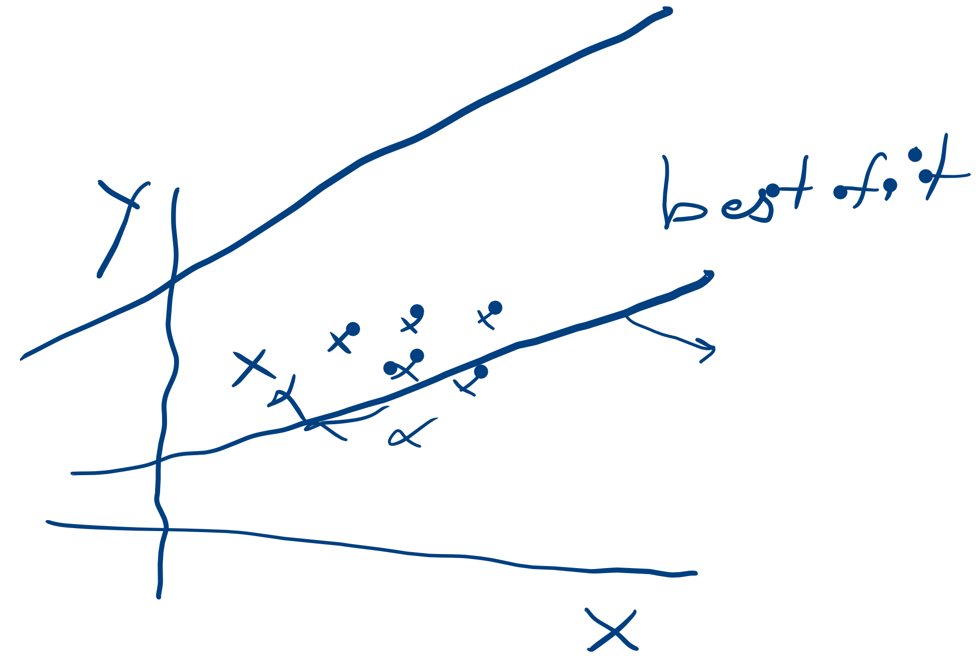
LR

SR | MR | polynomial

SR

x	y



Multiple Regression

Extension of Simple Regression

x_1	x_2	x_3	y

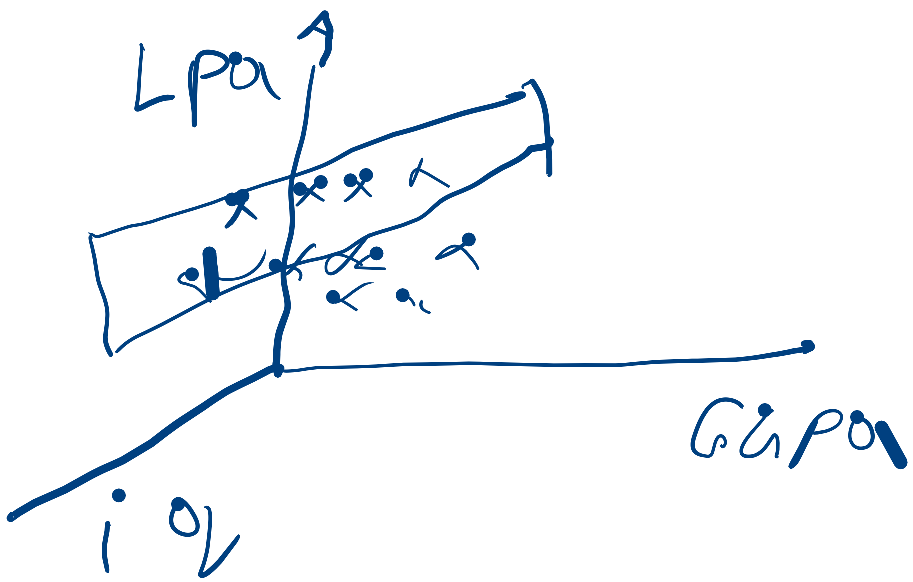
4D

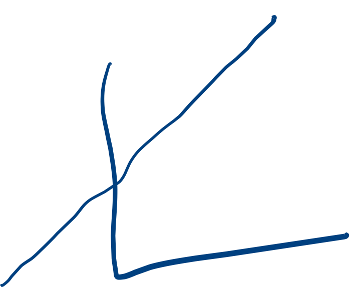
cgpa	iq	gender	Lpa

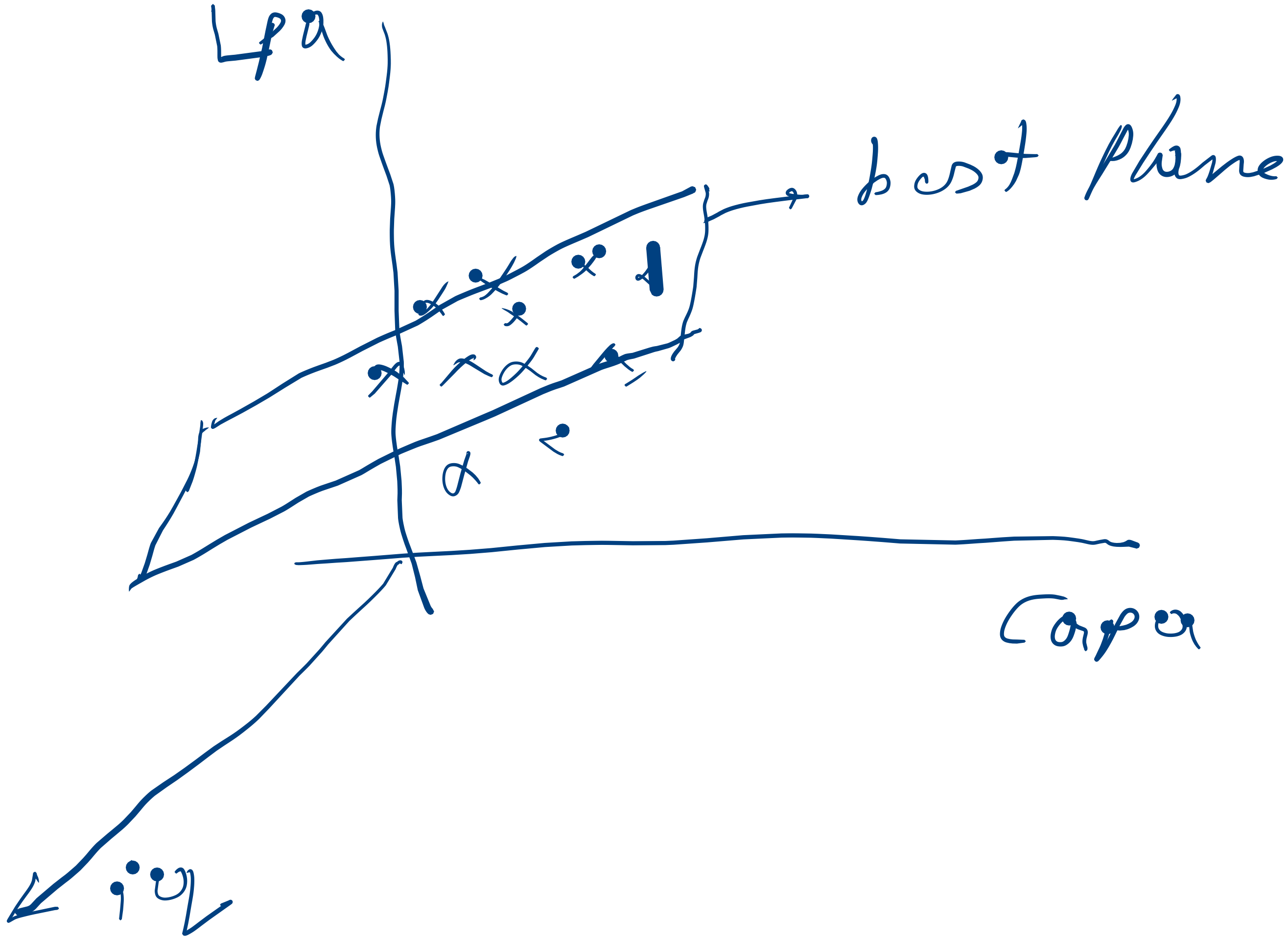
3D

Cgpa	iq	Lpa

3D - plane







2D - Line
3D - Plane
4D - hyperplane

2 D

$\hat{y} = mx + b$ → offset intercept

Capa	Lpa

x_1 Capa	x_2 iq	Lpa

3D

$$\boxed{y = m x_1 + m x_2 + b}$$

$$\{\beta_0, \beta_1, \beta_2\} \leftarrow \hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

No. of cofficient

weights

4D → $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

$$nD = \boxed{y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i}$$

2D $\longrightarrow$ $y = mx + b$

$\longrightarrow$ $y = \beta_0 + \beta_1 x_1$

$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

3D —— $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

4D — $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots \beta_n x_n$

nD —— $y = \beta_0 + \beta_1 x_1$ —— . . .

$$\boxed{y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i}$$

No. of column $\Rightarrow$ n

No. of feature. no. of ~~of~~ coeffi.. = n + 1 coefficient

CGPA, iq | $Y \to$ lpa

no. of feature $= 2$

3

3D

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

$x_1 \to$ CGPA

$x_2 \to$ iq

$Y =$ lpa

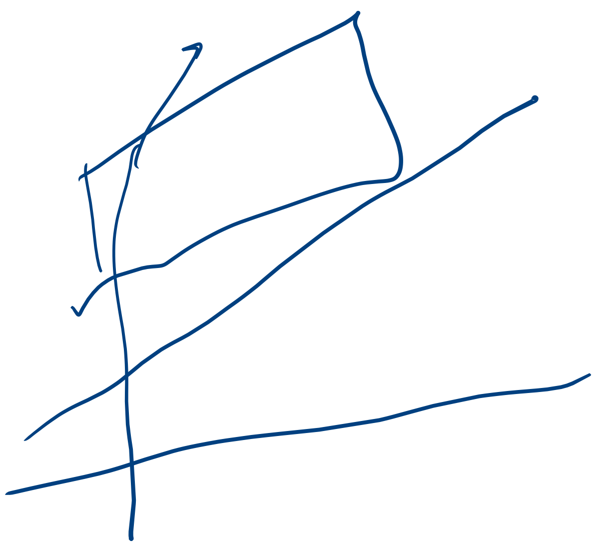
$$\text{lpa} = \beta_0 + \beta_1 \times \text{CGPA} + \beta_2 \times \text{iq}$$

β_1 β_2 $\beta_1 \uparrow$ $\beta_2 \downarrow$

$\beta_0 \to$ offset (intercept)

nD

$$\boxed{\hat{y} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i}$$



2D $\Rightarrow$ $y = \beta_0 + \beta_1 x_1$

3D $\rightarrow$ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

4D $\Rightarrow$ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

nD $= y = \beta_0 + \beta_1 x_1 \ldots \beta_n x_n$

$$= \beta_0 + \sum_{i=1}^{n} \beta_i x_i$$

$y = mx + b$

$y = \beta_1 x + \beta_0 \rightarrow 2D$

Gupa

100 students

shape (100, 4) → 3 feature, 1 target

sample

$Y \Rightarrow$ actual

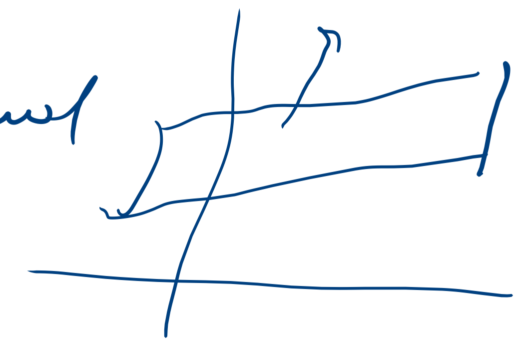
predicted ← $\boxed{\hat{Y} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3}$

$[\beta_0, \beta_1, \beta_2, \beta_3]$

X_1	X_2	X_3	Y → actual
CGPA	iq	gender	LPA

train →

CGPA	iq	gender



$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_{100} \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 X_{11} & \beta_2 X_{12} & \beta_3 X_{13} \\ \beta_0 & \beta_1 X_{21} & \beta_2 X_{22} & \beta_3 X_{23} \\ \beta_0 & & & \\ \vdots & & & \\ \beta_{100} & \beta_1 X_{1001} & \beta_2 X_{1002} & \beta_3 X_{1003} \end{bmatrix}$$

n rows and m col

$$\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 X_{11} & \beta_2 X_{12} & \cdots & \beta_m X_{1m} \\ \vdots & \vdots & \vdots & & \vdots \\ \beta_0 & \beta_n X_{n1} & \beta_n X_{n2} & \cdot & \beta_n X_{nm} \end{bmatrix}$$

$$
\begin{bmatrix}
1 & x_{12} & x_{13} & \cdots & x_{1m} \\
1 & x_{21} & x_{22} & \cdots & x_{2m} \\
\vdots & \vdots & \vdots & & \vdots \\
n & x_{n2} & x_{n3} & & x_{nm}
\end{bmatrix},
\quad
\begin{bmatrix}
\beta_0 \\
\beta_1 \\
\beta_2 \\
\vdots \\
\beta_m
\end{bmatrix}
$$

$\beta \rightarrow$ Coefficient matrix

β

Y = actual

$\hat{Y}$ = predict.

X

$$\boxed{\hat{Y} = X\beta}$$

x_1	x_2	x_3
x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	y_{23}
x_{31}	y_{32}	y_{33}

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} \qquad \hat{Y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$

$$e = Y - \hat{Y}$$

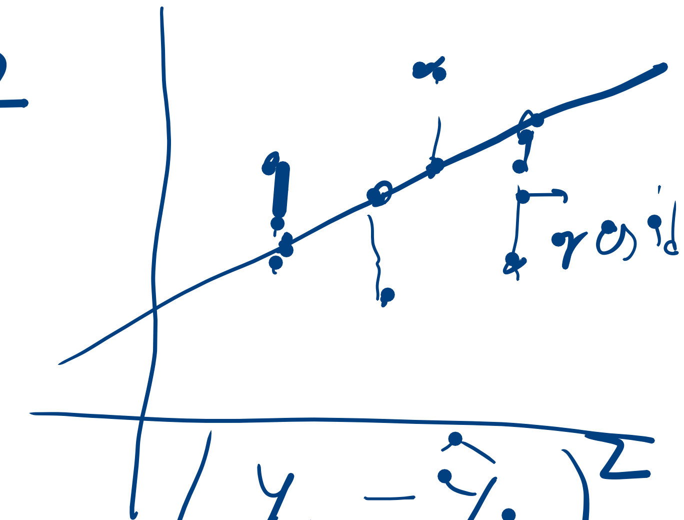
$$e = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}$$

$$e = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$

Single linear regression

$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$



$$(y_i - \hat{y}_i)^2$$

$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$\boxed{E = e^T e}$$

$$[(y_i - \hat{y}_i)\ (y_2 - \hat{y}_2) \cdots y_n - \hat{y}_n] \times \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix} =$$

$$(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3) - \cdots (y_n - \hat{y}_n)^2$$

$$E = \quad E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

multiple regression:

$$E = e^T e = (y - \hat{y})^T \cdot (y - \hat{y})$$