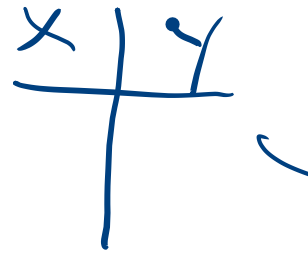
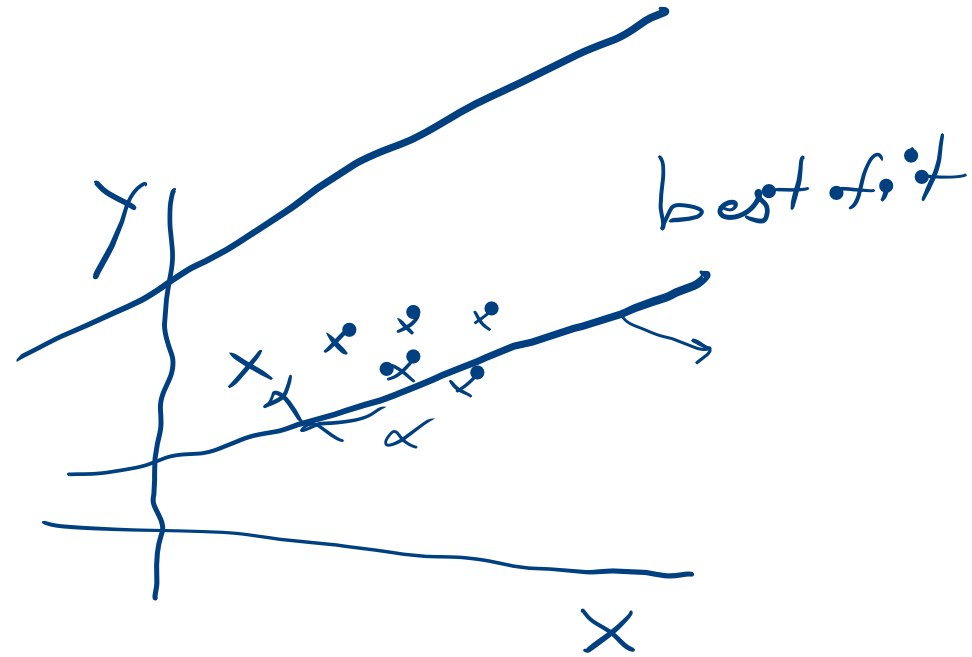
LR

SR MR polynomial

SR





Multiple Regression

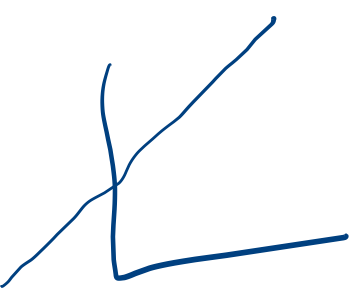
x_1	x_2	x_3	y

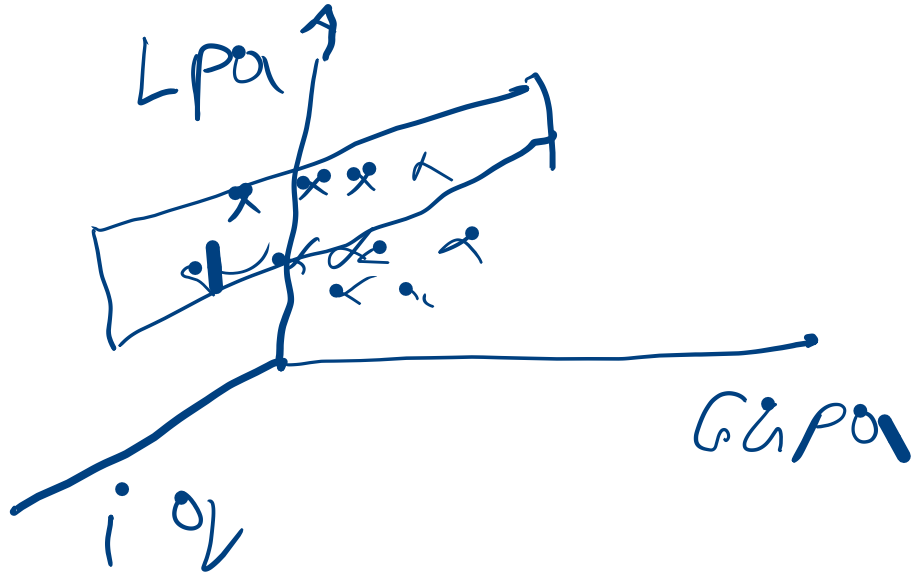
→ Extension of Simple Regression

cgpa	iq	gender	LPA

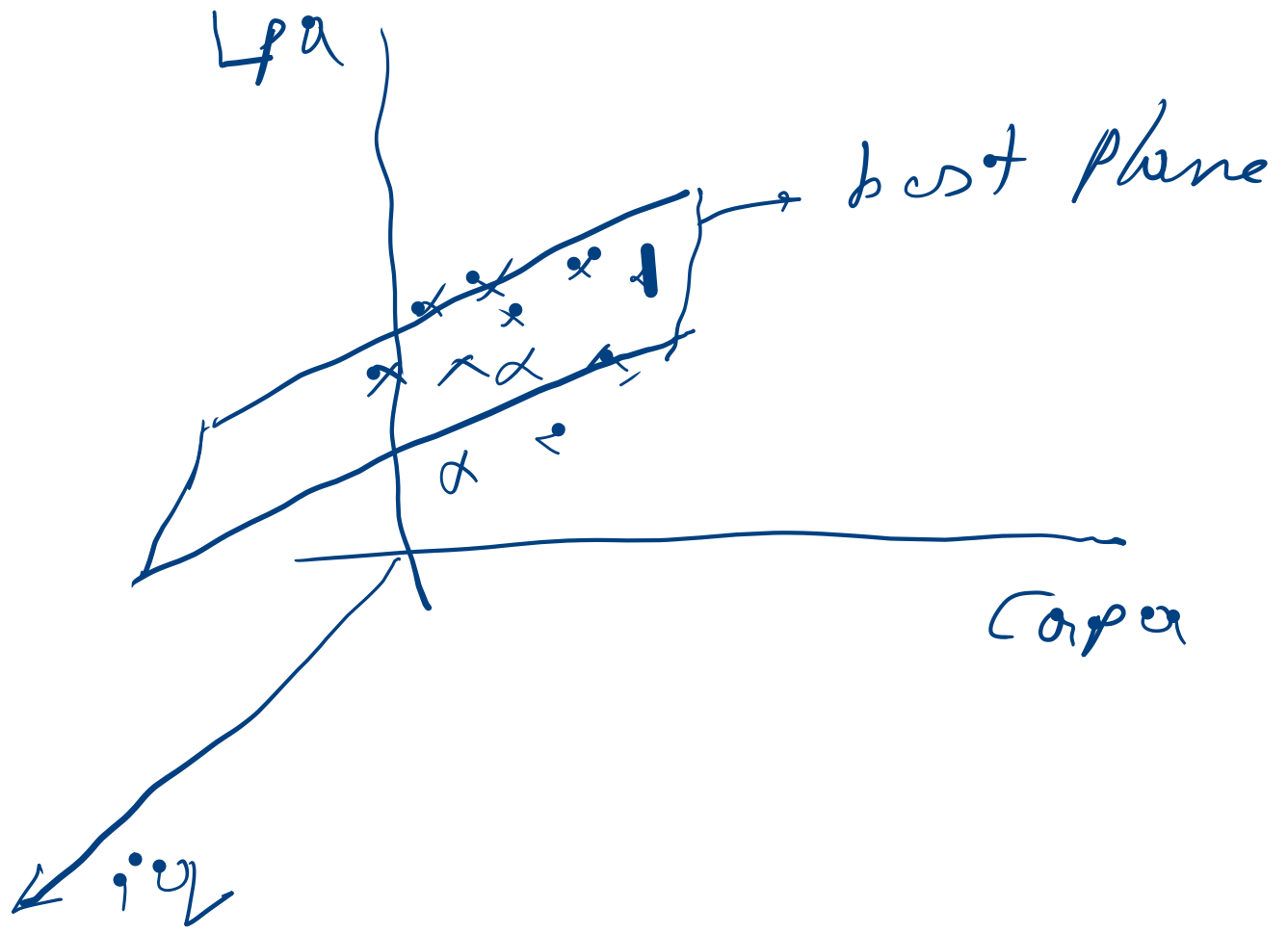
4D

3D-plane





3D



2D - Line
3D — Plane
4D — hyperplane

2D

$y = mx + b$ → offset / intercept

CGPA	LPA

x_1 CGPA	x_2 IQ	LPA

3D

$$y = m x_1 + m x_2 + b$$

$\beta_0, \beta_1, \beta_2 \leftarrow \hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

No. of coefficient → weights

4D → $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

nD = $y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i$

2D → $y = mx + b$

→ $y = \beta_0 + \beta_1 x_1$

3D — $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

— $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

4D — $y = \beta_0 + \beta_1 x_1 - \cdots \beta_n x_n$

nD — $y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i$

↳ No. of column ⇒ n

No. of feature. No. of coeffi. = n + 1

coefficient

CGPA | IQ | Y → IPA

No. of feature = 2 → 3

3D

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

$X_1 \rightarrow$ CGPA

$X_2 \rightarrow$ IQ

Y = IPA

$$IPA = \beta_0 + \beta_1 \cdot CGPA + \beta_2 \cdot IQ$$

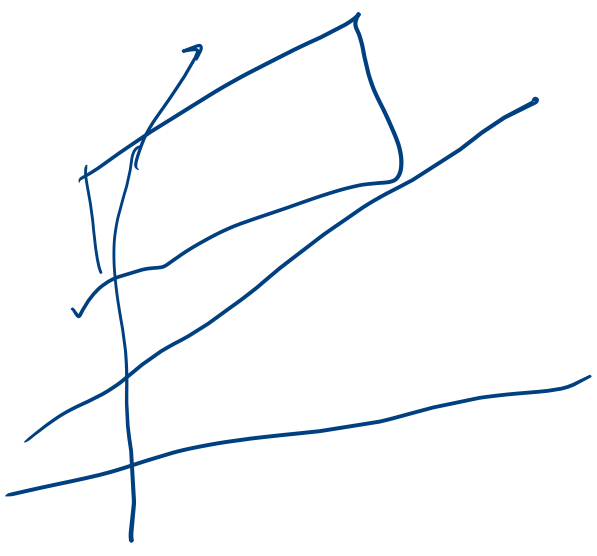
β_1 β_2 $\beta_1 \uparrow$ $\beta_2 \downarrow$

$\beta_0 \rightarrow$ offset (intercept)

nD

$$\hat{y} = \beta_0 + \sum_{i=1}^{n} \beta_i x_i$$

2D $\Rightarrow$ $y = \beta_0 + \beta_1 x_1$



3D $\Rightarrow$ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$

4D $\Rightarrow$ $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$

nD $\Rightarrow$ $y = \beta_0 + \beta_1 x_1 + \cdots + \beta_n x_n$

$$= \beta_0 + \sum_{i=1}^{n} \beta_i x_i$$

$$y = mx + b$$

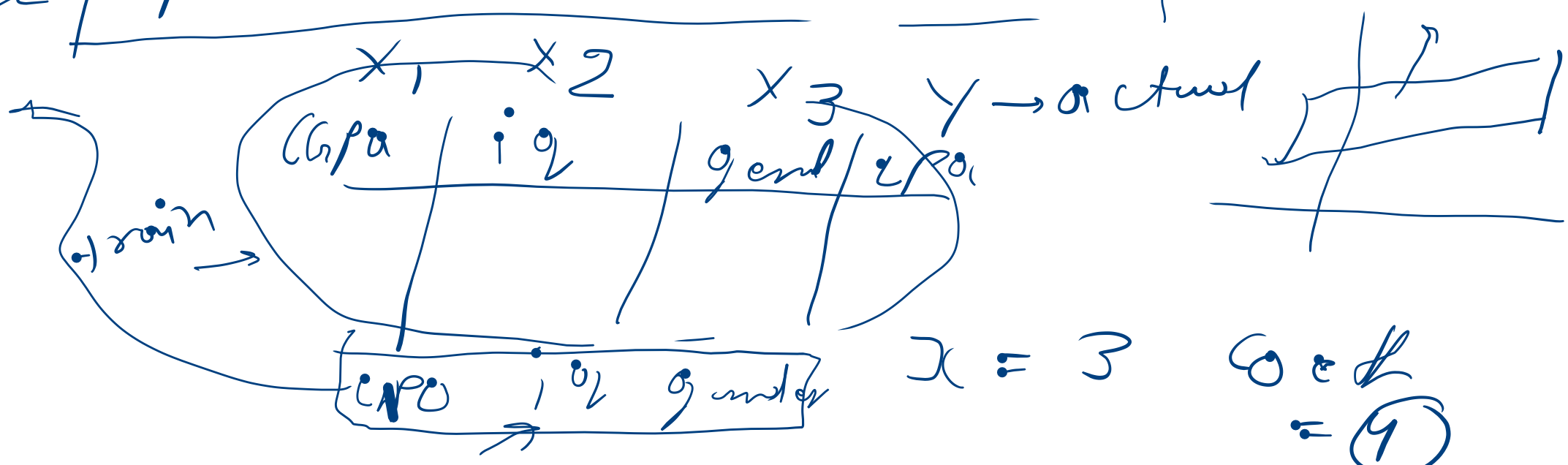
$$y = \beta_1 x + \beta_0 \quad \rightarrow 2D$$

y → GPA

100 students
shape (100, 4) → 3 feature, 1 term
↳ samples

y = actual

predicted $\hat{y} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$ $[\beta_0, \beta_1, \beta_2, \beta_3]$



$x = 3$ $\theta = ?$

$= 1$

$$\hat{Y}_{100} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_{100} \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 x_{11} & \beta_2 x_{12} & \beta_3 x_{13} \\ \beta_0 & \beta_1 x_{21} & \beta_2 x_{22} & \beta_3 x_{23} \\ \beta_0 & & & \\ \vdots & & & \\ \beta_{100} & \beta_1 x_{1001} & \beta_2 x_{1002} & \beta_3 x_{1003} \end{bmatrix}$$

n rows & m col

$$\begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} \beta_0 & \beta_1 x_{11} & \beta_2 x_{12} & \cdots & \beta_m x_{1m} \\ \beta_0 & & & & \\ \vdots & & & & \\ \beta_0 & \beta_n x_{n1} & \beta_n x_{n2} & \cdot & \beta_n x_{nm} \end{bmatrix}$$

$$\begin{bmatrix} 1 & x_{12} & x_{13} & \cdots & x_{1m} \\ 1 & x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & x_{n2} & x_{n3} & & x_{nm} \end{bmatrix} \cdot \begin{bmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \vdots \\ \beta_m \end{bmatrix}$$

Coefficient matrix

y = actual

$\hat{y}$ = predicted

$$\hat{y} = X\beta$$

x_1	x_2	x_3
x_{11}	x_{12}	x_{13}
x_{21}	x_{22}	x_{23}
x_{31}	x_{32}	x_{33}

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}$$ actual

$$\hat{y} = \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$ predict.

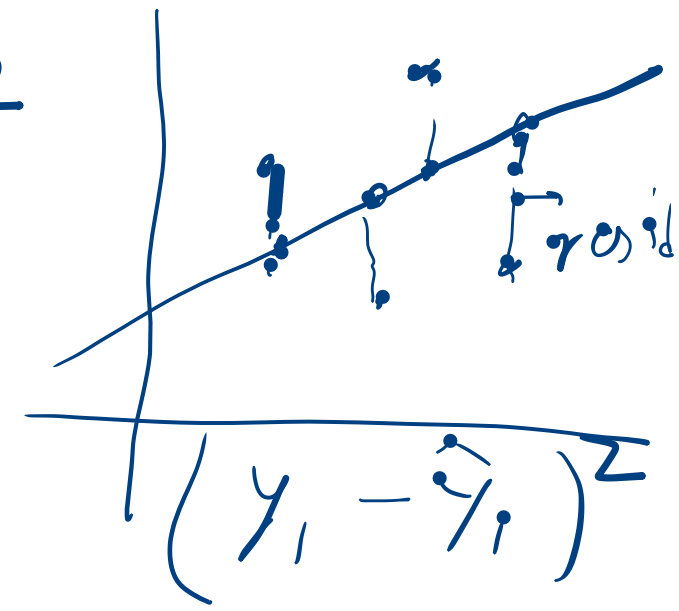
$$e = y - \hat{y}$$

$$e = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} - \begin{bmatrix} \hat{y}_1 \\ \hat{y}_2 \\ \vdots \\ \hat{y}_n \end{bmatrix}$$

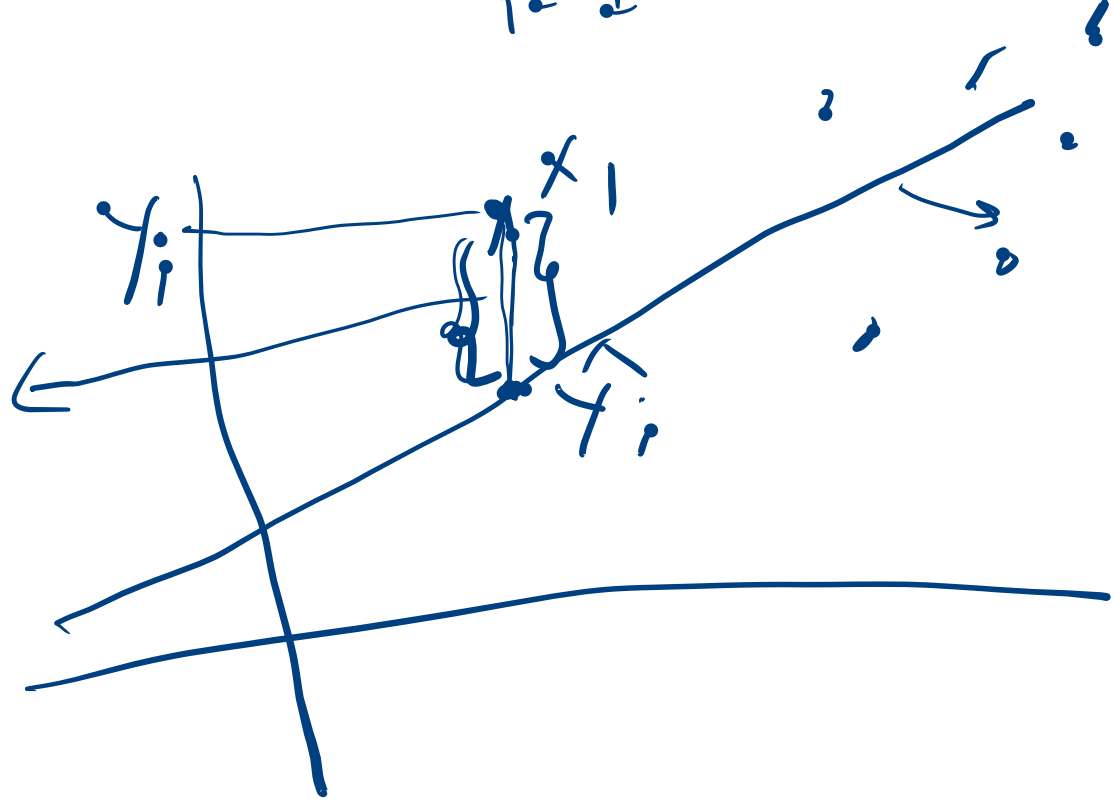
$$e = \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix}$$

Single linear regression

$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$



$\sum_{i=1}^{n} (y_1 - \hat{y}_i)^2$



$$E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$E = e^T e$$

$$\begin{bmatrix} y_1 - \hat{y}_1 & y_2 - \hat{y}_2 & \cdots & y_n - \hat{y}_n \end{bmatrix} \times \begin{bmatrix} y_1 - \hat{y}_1 \\ y_2 - \hat{y}_2 \\ \vdots \\ y_n - \hat{y}_n \end{bmatrix} =$$

$$(y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3) - \cdots (y_n - \hat{y}_n)^2$$

$$E = \quad E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2$$

$$E = e^T e$$

$$\hookrightarrow (y - \hat{y})^T (y - \hat{y})$$

$$= (y^T - \hat{y}^T)(y - \hat{y})$$

$$\hat{y} = X\beta$$

$$= \left[y^T - (X\beta)^T\right]\left[y - X\beta\right]$$

$$E = y^T y - y^T X\beta - (X\beta)^T y + (X\beta)^T X\beta$$