Filter

ML models
- Linear regression (70 min)
- Logistic regression (35 min)
  - Introduction (5 min)
  - Calculating a probability (10 min)
  - Loss and regularization (10 min)
  - Test your knowledge (10 min)
  - What's next
- Classification (70 min)

Data
- Working with numerical data (85 min)
- Working with categorical data (50 min)
- Datasets, generalization, and overfitting (105 min)

Info | Chat | API

Home > Products > Machine Learning > ML Concepts > Crash Course

Was this helpful?

Logistic regression: Calculating a probability with the sigmoid function

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On this page
Sigmoid function
Transforming linear output using the sigmoid function
Exercise: Check your understanding

Many problems require a probability estimate as output. Logistic regression is an extremely efficient mechanism for calculating probabilities. Practically speaking, you can use the returned probability in either of the following two ways:

- Applied "as is." For example, if a spam-prediction model takes an email as input and outputs a value of `0.932`, this implies a `93.2%` probability that the email is spam.
- Converted to a binary category such as `True` or `False`, `Spam` or `Not Spam`.

This module focuses on using logistic regression model output as-is. In the Classification module, you'll learn how to convert this output into a binary category.

Sigmoid function

You might be wondering how a logistic regression model can ensure its output represents a probability, always outputting a value between 0 and 1. As it happens, there's a family of functions called **logistic functions** whose output has those same characteristics. The standard logistic function, also known as the sigmoid function (*sigmoid* means "s-shaped"), has the formula:

$$f(x) = \frac{1}{1 + e^{-x}}$$

Figure 1 shows the corresponding graph of the sigmoid function.

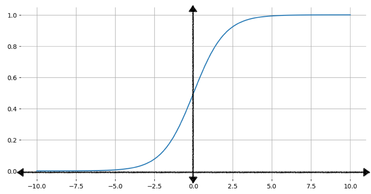

Figure 1. *Graph of the sigmoid function. The curve approaches 0 as x values decrease to negative infinity, and 1 as x values increase toward infinity.*

As the input, `x`, increases, the output of the sigmoid function approaches but never reaches `1`. Similarly, as the input decreases, the sigmoid function's output approaches but never reaches `0`.

Click here for a deeper dive into the math behind the sigmoid function

Transforming linear output using the sigmoid function

The following equation represents the linear component of a logistic regression model:

$$z = b + w_1x_1 + w_2x_2 + \ldots + w_Nx_N$$

where:

- z is the output of the linear equation, also called the log odds.
- b is the bias.
- The w values are the model's learned weights.
- The x values are the feature values for a particular example.

To obtain the logistic regression prediction, the z value is then passed to the sigmoid function, yielding a value (a probability) between 0 and 1:

$$y' = \frac{1}{1 + e^{-z}}$$

where:

- y' is the output of the logistic regression model.
- z is the linear output (as calculated in the preceding equation).

Click here to learn more about log-odds

Figure 2 illustrates how linear output is transformed to logistic regression output using these calculations.

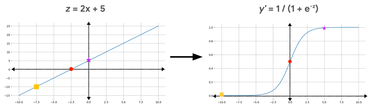

Figure 2. *Left: graph of the linear function z = 2x + 5, with three points highlighted. Right: Sigmoid curve with the same three points highlighted after being transformed by the sigmoid function.*

In Figure 2, a linear equation becomes input to the sigmoid function, which bends the straight line into an s-shape. Notice that the linear equation can output very big or very small values of z, but the output of the sigmoid function, y′, is always between 0 and 1, exclusive. For example, the yellow square on the left graph has a z value of −10, but the sigmoid function in the right graph maps that −10 into a y′ value of 0.00004.

Exercise: Check your understanding

A logistic regression model with three features has the following bias and weights:

$$b = 1$$
$$w_1 = 2$$
$$w_2 = -1$$
$$w_3 = 5$$

Given the following input values:

$$x_1 = 0$$
$$x_2 = 10$$
$$x_3 = 2$$

Answer the following two questions.

1. What is the value of *z* for these input values?

- −1 ☐
- 0 ☐
- 0.731 ☐
- 1 ✓

Correct! The linear equation defined by the weights and bias is $z = 1 + 2x_1 - x_2 + 5x_3$. Plugging the input values into the equation produces $z = 1 + (2)(0) - (10) + (5)(2) = 1$

Correct answer.

2. What is the logistic regression prediction for these input values?

- 0.268 ☐
- 0.5 ☐
- 0.731 ✓

As calculated in #1 above, the log-odds for the input values is 1. Plugging that value for *z* into the sigmoid function:

$$y = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-1}} = \frac{1}{1+0.367} = \frac{1}{1.367} = 0.731$$

Correct answer.

- 1 ☐

Key terms:

- Binary classification
- Log odds
- Logistic regression
- Sigmoid function

Help Center

Previous: Introduction (5 min)

Next: Loss and regularization (10 min)

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