COL333 Assignment 3

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1 Part 1

- There are two variables for each vertex. Let N be the number of vertices in the graph. For vertex x_i (1 $\leq$ i $\leq$ N), variable g_1, i is represented as i for G_1 and similarly variable $g_{2,i}$ is represented as i+N for G_2. If x_i is included in G_1 then $g_{1,i}$ is 1 else 0. Similarly, if x_i is included in G_2 then $g_{2,i}$ is 1 else 0.
- For graph G_1, $s_{1,i,j}$ is represented as (2*N+1)+i*(k_1+2)+j (0$\leq$i$\leq$N and 0$\leq$j$\leq$$k_1$+1) and for G_2, $s_{2,i,j}$ is represented as (2*N+1)+(N+1)*(k_1+1)+i*(k_2+2)+j (0$\leq$i$\leq$N and 0$\leq$j$\leq$$k_2$+1). $s_{1,i,j}$ is 1 if j vertices are chosen from the first i vertices for G_1, similarly for G_2.
- For G_1, as we want a clique of size k_1. So $g_{1,i} = 0$ if degree(x_i) $\leq$ k_1 - 1, similarly for G_2.
- Since a vertex can't be in both G_1 and G_2, so we generated the clause

$$-g_{1,i} \text{ or } -g_{2,i} \text{ for } 1 \leq i \leq N.$$

- If edge is not present between x_i and x_j, we generated clauses -$g_{1,i}$ or -$g_{1,j}$ for G_1, similarly for G_2, -$g_{2,i}$ or -$g_{2,j}$.
- Initializing the base case:

$$s_{1,i,0} = 1 \text{ for } 0 \leq i \leq N$$

$$s_{1,0,j} = 0 \text{ for } 1 \leq j \leq k_1 + 1$$

- Array $s_{1,i}$ can be seen as a unary number which represents how many vertices are selected from the first i vertices.

$s_{1,i,j} = s_{1,i-1,j}$ or ($s_{1,i-1,j-1}$ and $g_{1,i}$) for $1 \leq i \leq N$ and for $1 \leq j \leq k_1$+1

- To force the size to be exactly equal to k_1:

$$s_{N,k_1} = 1 \text{ and}$$

$$s_{N,k_1+1} = 0.$$

- Similar clauses are generated for G_2.

2 Part 2

We have used binary search on the size of clique k.

We have defined two types of literals g_i and $s_{,i,j}$, where g_i represents nodes of the graph, and $s_{i,j}$ represents at least j of the first i nodes being true.

1. Degree Constraint Clause: If the degree of node g_i is less than $k-1$, then set g_i to 0.

$$\text{Clause: if degree}(g_i) < k-1, \text{ then } \neg g_i$$

2. Edge Constraint Clause: If there is no edge between nodes g_i and g_j, then both g_i and g_j should be 0.

$$\text{Clause: if no edge between}(g_i, g_j), \text{ then } \neg g_i \text{ and } \neg g_j$$

3. Base Size Constraint Clauses:

$$s_{0,j} = 0 \text{ for } 1 \leq j \leq k+1$$

$$s_{i,0} = 1 \text{ for } 0 \leq i \leq N$$

4. Size Constraint Clauses: For $1 \leq i \leq N$ and $1 \leq j \leq k+1$:

$$\text{Clause: } s_{i,j} \Leftrightarrow (s_{i-1,j} \text{ or } (g_i \text{ and } s_{i-1,j-1}))$$

5. Final Constraint to Fix Size to be k:

$$s_{N,k} = 1$$

$$s_{N,(k+1)} = 0$$