Modelling ocean–Level Rise

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1 Introduction

If we change the surface temperature of the earth, oceans take millennia to respond. However, it is not a bad assumption that the top layer of the ocean equilibrates quickly and below the top layer, the temperature is constant in a decadal timescale.

Thus, given a surface temperature, we know the temperature profile in equilibrium in this top layer. We will try to write a model for the height of this top layer as a function of the surface temperature.

2 Equations

On a global scale, we will assume that the top layer of the ocean is in hydrostatic balance i.e.

$$\frac{dp}{dz} = -\rho g \ . \tag{1}$$

3 The Base state

In real life, the base state has a non-zero temperature gradient and a non-uniform density profile. Also, the height of the top layer will depend on the melting polar ice. However, we shall assume that the density of the base state is uniform (ρ_0) and the temperature at the top is the same as at the bottom. The height of the base state is assumed to be D.

4 Boundary Conditions

The boundary conditions are imposed on pressure. The pressure difference between the top and the bottom of the layer will be equal to the total mass per unit area in it i.e

$$p(0) - p(h) = \rho_0 \, g \, D \ . \tag{2}$$

If the melting of polar ice is included, the RHS of the equation will have an additional term.

5 Getting the height

If T_s and T_{ocean} are the temperature at the top and the bottom of the ocean layer respectively, the equilibrium temperature profile will be

$$T(z) = T_s + (T_s - T_{\text{ocean}}) \, \frac{z-h}{h} \ . \tag{3}$$

Thus, the difference in temperature will be

$$\Delta T(z) = (T_s - T_{\text{ocean}}) \, \frac{z}{h} \ . \tag{4}$$

The change in density for small changes in absolute temperature can be written as

$$\rho(z) = \rho_0 \left(1 - \alpha \Delta T(z)\right) \ , \tag{5}$$

where α is the coefficient of thermal expansion for ocean water. Thus, from the boundary conditions of pressure, we get

$$p(0) - p(h) = \rho_0 \, g \, D = \int_0^h dz \ \rho(z) g \ . \tag{6}$$

Substituting $\rho(z)$, $\Delta T(z)$ and integrating we arrive at

$$D = h - \alpha \left(T_s - T_{\text{ocean}}\right) \frac{h}{2} \ . \tag{7}$$

Thus,

$$h = \frac{D}{1 - \frac{\alpha}{2} \left(T_s - T_{\text{ocean}}\right)} \ . \tag{8}$$

For small α and small temperature difference between the top and the bottom layer of the ocean, we can expand the denominator and write

$$h = D \left[1 + \frac{\alpha}{2} \left(T_s - T_{\text{ocean}}\right)\right] \ . \tag{9}$$

If for two different surface temperatures T_{s_1} and T_{s_2}, the height of the mixed layer is h_1 and h_2 respectively, then

$$h_2 - h_1 = D \frac{\alpha}{2} \left(T_{s_2} - T_{s_1}\right) \ . \tag{10}$$

6 Needed numerical values

1. Thermal expansion coefficient α and density ρ_0 of the ocean
   $\alpha = 1.65 \times 10^{-4}$ K^{-1} , $\rho_0 = 1027$ kg m^{-3}.
2. The depth of the top layer $D \sim 50 - 200m$.
3. The default density of the ocean.
4. The deep ocean temperature T_{ocean}.

7 Possible Corrections

1. Add a base density profile.
2. Add melting of ice.