Kalyani Government Engineering College

Department of Information Technology

Name: - RAJARSHI ROY

Roll Number : - 10200221027

Email :- royrajarshi0123@gmail.com

Year :- 2nd

Semester : - 3rd

Subject : - BS-C 301(Mathematics-III)

Assignment :- CA-2

Keywords

Exact Equations

Integrating Factors

Bernoulli's Differential Equations

Introduction

Because they may be used to simulate a wide range of real-world scenarios, differential equations are significant in both mathematics and the sciences. Differential equations, for instance, can be used to simulate the motion of particles in a fluid or the trajectory of a projectile in physics. Differential equations can be used in biology to simulate population increase and the transmission of diseases. Differential equations are a useful tool for scientists and engineers because they can be used to simulate complex situations. Many physical phenomena are described by differential equations. They assist us in taking an observation from the real world and putting it into a mathematical form.

Exact Equations:

The first order differential equation of the form

$$M\,dx + N\,dy = 0 \text{ - - - - - - - - - - -(i)}$$

where both M and N are functions of x , y , is said to be exact, if there exist a function u(x,y) such that

$$M\,dx + N\,dy = du$$

Then equation (i) becomes du = 0 ,which on integration gives

$$U(x,y) = c \text{ , c being a constant}$$

Therefore u(x,y) = c is a solution of (i)

For example: $\log x\, dy + \frac{y}{x}\, dx = 0$ is an exact differential, since

$$\log x\, dy + \frac{y}{x}\, dx = d(y \log x)$$

Hence, $y \log x = c$, c being a constant, is a general solution of the equation.

Integrating Factors:

Differential equations which are not exact can sometimes be made exact after multiplying by a suitable factor(a function of x and/ or y) called Integrating Factor.

For example: the equation, xdy – ydx = 0 is not exact.

But after multiplying the equation by $1/x^2$, the equation becomes,

$$\frac{xdy - ydx}{x^2} = 0$$

Or, $d(\frac{y}{x}) = 0$, which is an exact and solution is $\frac{y}{x} = c$, c being a constant.

Rules for finding integrating factors:

Here we consider a differential equation of the form

$$M\,dx + N\,dy = 0$$

Which is not exact i.e, $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$

Rule 1. If $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}$ = f(x) , a function of x only , then $e^{\int f(x)dx}$ is an integrating factor of the differential equation.

Rule 2. If $\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{M}$ = g(y) , a function of y only , then $e^{\int g(y)dy}$ is an integrating factor of the differential equation.

Rule 3. If M and N both are homogeneous functions in x , y of same degree and $Mx + Ny \neq 0$, then $\frac{1}{Mx + Ny}$ is an integrating factor.

Rule 4. If the equation is of the form yf(xy)dx + xg(xy)dy =0 and Mx - Ny $\neq$ 0, then $\frac{1}{Mx - Ny}$ is an integrating factor of the equation.

Rule 5. If the equation if of the form

$$x^a y^b(mydx + nxdy) + x^{a\prime} y^{b\prime}(m'ydx + n'xdy) = 0$$

Where a, b, a', b', m, n, m', n' are all constant, then $x^h y^k$ is an integrating factor of the equation where

$$\frac{a+h+1}{m} = \frac{b+k+1}{n} \quad , \quad \frac{a\prime+h+1}{m\prime} = \frac{b\prime+k+1}{n\prime}$$

Bernoulli's Differential Equations:

The equation of the type

$$\frac{dy}{dx} + Py = Q\, y^n \text{ - - - - - - - - - (ii)}$$

Where P , Q are functions of x only or constant is known as Bernoulli's equation, This equation is reducible to linear equation.

Dividing both sides by y^n , we have

$$y^{-n}\frac{dy}{dx} + Py^{1-n} = Q \text{ - - - - - - - - - -(iii)}$$

Putting , $y^{1-n} = z$ i.e (1-n) $y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$

i.e $y^{-n}\frac{dy}{dx} = \frac{1}{(1-n)}\frac{dz}{dx}$

From (iii) we get,

$$\frac{1}{(1-n)}\frac{dz}{dx} + Pz = Q$$

$$\frac{dz}{dx} + (1\text{-}n)\ Pz = (1\text{-}n)Q$$

Which is a linear equation in z and can be solved.

Remark: putting, $z = \int y^{-n} dy$, (iii) can be reduced to a linear form

$\frac{dz}{dx} + (1\text{-}n)\ Pz = Q$ also.

Another type of Bernoulli's equation of the form

$$\frac{dx}{dy} + Px = Q\, x^n$$

Where P , Q are functions of y only or constants, can be reduced to linear equation by putting

$$x^{1-n} = z$$

Solved Examples:

2. Solve : $(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$

Solution:

Here, $M = x^2y - 2xy^2$ and $N = 3x^2y - x^3$ both are homogeneous function in x , y of same degree 3

Now, $Mx + Ny = (x^2y - 2xy^2)x + (3x^2y - x^3)y$

$= x^2y^2$

$\therefore$ I.F = $\frac{1}{x^2y^2}$

Multiplying the given differential equation by $\frac{1}{x^2y^2}$, we get,

$$\frac{1}{x^2y^2}(x^2y - 2xy^2)dx + \frac{1}{x^2y^2}(3x^2y - x^3)dy = 0$$

or, $(\frac{1}{y} - \frac{2}{x})dx + (\frac{3}{y} - \frac{x}{y^2})dy = 0$ which an exact equation

Hence, the solution of the equation is,

$$\int\left(\frac{1}{y} - \frac{2}{x}\right)dx + \int\frac{3}{y}dy = c$$

or, $\frac{x}{y}$ - $2\log x + 3\log y = c$

or, $\frac{x}{y} + \log\frac{y^3}{x^2} = c$, c being a constant

4. Solve : $x\frac{dy}{dx} + y = y^2 + \log x$

Solution:

The given equation can be written as

$\frac{dy}{dx} + \frac{y}{x} = y^2\frac{\log x}{x}$ - - - - - - - - - - - - - - - - - - - (i)

It is of Bernoulli's

Dividing both sides of (i) by y^2 we get,

$y^{-2}\frac{dy}{dx} + \frac{y^{-1}}{x} = \frac{\log x}{x}$ - - - - - - - - - - - - - - - (ii)

Put, $y^{-1} = z$

So that, $-y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$

$\therefore\ y^{-2}\frac{dy}{dx} = -\frac{dz}{dx}$

Then (i) takes the term,

$-\frac{dz}{dx} + \frac{z}{x} = \frac{\log x}{x}$

$\therefore\ \frac{dz}{dx} - \frac{z}{x} = -\frac{\log x}{x}$ - - - - - - - - - - - - - -(iii)

Which is a linear equation in z,

I.F $= e^{\int -\frac{1}{x}dx}$

$= e^{-\log x}$

$= x^{-1}$

Multiplying both sides of (iii) by x^{-1} we get,

$$x^{-1}\frac{dz}{dx} \text{ -z}x^{-2} = \text{-}x^{-2} \log x$$

or, $\frac{d}{dx}(\text{z}x^{-1}) = \text{-}x^{-2} \log x$

or, $d(\text{z}x^{-1}) = \text{-}x^{-2} \log x \text{ dx}$

Which on integration gives,

$$\text{z}x^{-1} = \text{-} \int x^{-2} \log x \text{ dx}$$

or, $y^{-1}x^{-1} = \text{-}\{ \log x \ \frac{x^{-1}}{1} \text{ - } \int \frac{1}{x} \frac{x^{-1}}{1} \text{ dx}$

or, $\frac{1}{xy} = \frac{\log x}{x} + \frac{x^{-1}}{1} + \text{c}$

or, , $\frac{1}{xy} = \frac{\log x + 1}{x} + \text{c}$

Which is the general required equation.

Concluding Remarks:

Differential Equations are used to describe a lot of physical phenomenon. They help us to observe something happening in real life and put it in a mathematical form. And Exact equations , Integrating factors and Bernoulli's has been discussed in this report which hold much potential in many real life situations as previously mentioned.

References:

1.Integral Calculus, Das and Mukherjee, U.N.Dhar Pvt. Ltd.

2.Engineering Mathematics(Volume – IIIA), Pal and Das, U.N.Dhur and Sons Pvt. Ltd.