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1. Introduction

It is observed that majority of the real world problems generally involve optimizing multiple conflicting objectives. As these optimization formulations involve multiple objectives, the objective function is formulated as a vector and it is treated as a vector optimization or a multi-objective optimization problem (MOOP). A multi-objective problem involving multiple, conflicting objectives may be combined into a single-objective scalar function. This approach is called the weighted sum method.

1.1. Background

The weighted sum method, a commonly used technique in optimization problems, has distinctive advantages of greater search efficiency and computational capabilities. This method, as the name suggests, scalarizes a set of conflicting objective functions, by pre-multiplying each of the objective function by predefined weights. However, it is often criticized for its inability to predict the logic behind the weight selection. It was suggested that the weights scientifically determine the decision maker's preference for a particular objective. Various approaches have been suggested for the weight selection process as well.

1.2. Objectives

Our primary objective here is to use a probabilistic technique to obtain weights to be used for the weighted sum algorithm. Comparisons are then made with existing and traditional weight-determination techniques.

General form of weighted sum problems

By definition, the weighted-sum method reduces a positively weighted convex sum of the objectives as follows:

$$min \sum_{i=1}^{n} w_i \, . \, f_i(x)$$

$$\sum_{i=1}^{n} w_i = 1;\ w_i > 0, i = 1, \dots, n$$

Minimization of this single-objective function will give an efficient solution for the multi objective problem. The process involves scalarizing the conflicting objectives into a single objective function.

1.3. Purpose and Scope

The purpose here is to decide on a scientific technique to improve the weights obtained deterministically in solving optimization problems using weighted sum technique. The approach here is probabilistic as compared to the usual deterministic approach to weight selection.

The method works best when one has to work with a small data set.

2. Survey of Technologies

PARETO OPTIMAL SOLUTION

Given a set of solutions, the non-dominated solution set is a set of all the solutions that are not dominated by any member of the solution set. The non-dominated set of the entire feasible decision space is called the **Pareto-optimal set**. The boundary defined by the set of all point mapped from the Pareto optimal set is called the **Pareto optimal front.**

MULTINOMIAL DISTRIBUTION

It is a multivariate generalization of Binomial Distribution.

Suppose a multinomial experiment consists of *n* trials, and each trial can result in any of *k* possible outcomes: $E_1, E_2, \ldots, E_k$. Suppose, further, that each possible outcome can occur with probabilities $p_1, p_2, \ldots, p_k$. Then, the probability (P) that E_1 occurs n_1 times, E_2 occurs n_2 times, . . . , and E_k occurs n_k times is:

$P = [\, n! / (n_1! * n_2! * \ldots n_k!) \,] * (p_1 n_1 * p_2 n_2 * \ldots * p_k n_k)$

where $n = n_1 + n_2 + \ldots + n_k$.

DIRICHLET DISTRIBUTION

It is a multivariate generalization of Beta distribution. Dirichlet distribution of order k(≥2) with parameters $a_1, a_2, \ldots, a_k > 0$ has the following probability density function:

$$g(x_1, x_2, \ldots, x_k) = \frac{\Gamma\left(\sum_{i=1}^{k} a_i\right)}{\prod_{i=1}^{k} \Gamma(a_i)} \prod_{i=1}^{k} {x_i}^{a_i - 1}$$

3. Requirements and Analysis

PROBLEM DEFINITION

This paper focuses on solving optimization problems using weighted sum technique. More specifically, it focuses on a scientific technique to improve the weights obtained deterministically in solving optimization problems using weighted sum technique. It probabilistically generates the weights for conflicting objective functions using a model based on multinomial distribution and Dirichlet prior.

PLANNING AND SCHEDULING

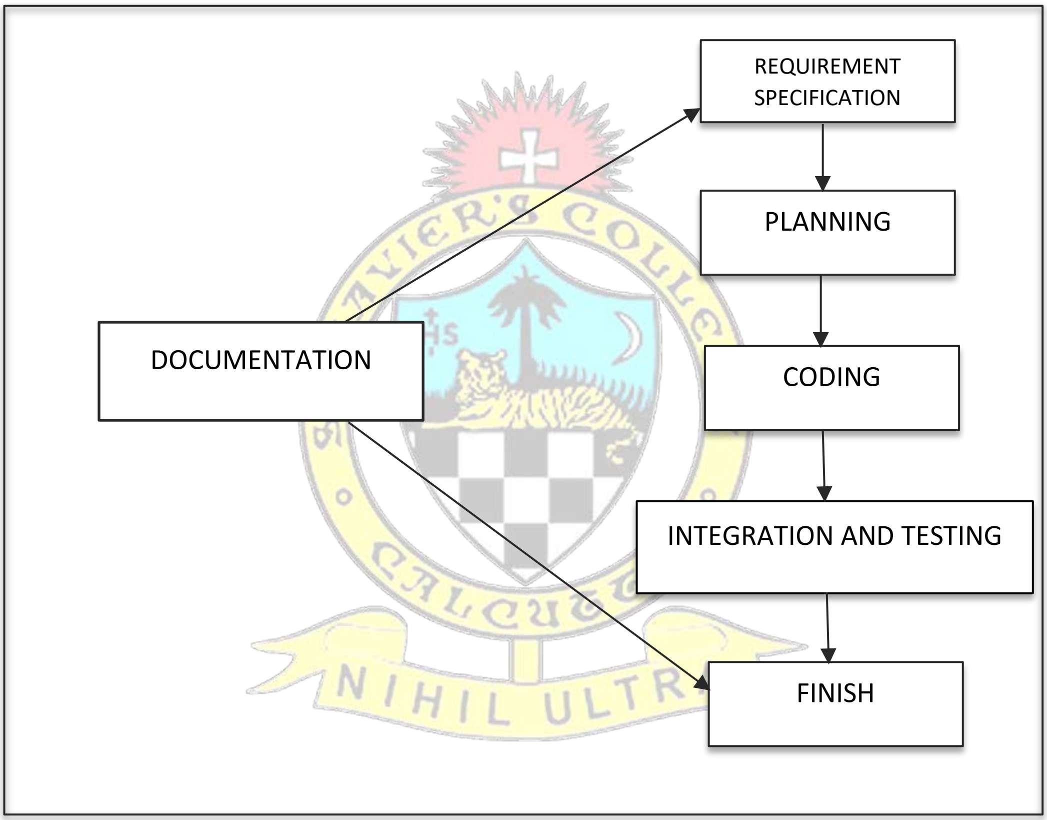

Fig. PERT CHART

SOFTWARE AND HARDWARE REQUIREMENTS

Hardware Requirements:

- Operating Systems: Windows 10/ Windows 7 Service pack1/ Windows Server 2016/ Windows Server 2012 R2/ Windows Server 2012
- RAM: 4 GB
- Disk Space:2 GB of HDD space for Matlab only, 4-6 GB for a typical installation. A full installation of all MathWorks products may take upto 23 GB of disk space.
- Processor: Any Intel or AMD x86-64 processor
- Browser: Internet Explorer 9 and above, Firefox, Google Chrome

Software Requirements:

- MatLab(any standard version)
- BlueJ

PRELIMINARY PRODUCT DESCRIPTION

The proposed method aims at improving the existing methods of weight determination as data driven choice of weights through appropriate probabilistic modeling ensures, on an average, much reliable results than non-probabilistic techniques. This technique is ideally suited for applications where one has to depend on a small data set.

CONCEPTUAL MODELS

To demonstrate the use of the proposed technique, a modeled situation has been devised. In a simulated graph, one has to reach from point A to point B via a number of intermediate nodes. The nodes in between may vary and all nodes in between need not be visited in the process. The best path is to be found out. However, there are three parameters on which the final path depends unlike the usual one (which is, in the majority of cases, distance between the points). Thus, the final result depends on the users' preference between distance between points, average time required to reach one point from another (which may depend of factors other than distance, such as condition of the roads) and average availability of parking space.

4. References

- Multi-Objective Optimization using evolutionary algorithms by Kalyanmoy Deb, Published by John Wiley & Sons, LTD.

- Genetic Algorithms for Multi-objective Optimization: Formulation, Discussion and Generalization - Carlos M. Fonseca and Peter J. Fleming

- Stochastic weight determination to solve multi-objective optimization problem using the weighted sum approach – Romit S. Beed