3/02/21

Exponential Distribution

$f(x) = \lambda e^{-\lambda x}$, $0 \le x$ and $0 \le \lambda$
$= 0$, o.w

Q] The amount of time that a watch will run without having to be reset is a random variable having an exponential distribution with mean 120 days. Find the probability that such a watch will
i) have to be set in less than 24 days.
ii) not have the to reset in at least 180 days.

Soln:- $1/\lambda = 120$ i.e. $\boxed{\lambda = 1/120}$
x = amount of time that a watch will run without having to be reset.

$P(x < 24)$
$= \int_0^{24} \lambda e^{-\lambda x} dx$

$= \int_0^{24} (1/120) e^{-x(1/120)} dx$

$= 1 - e^{-0.2}$

$= 0.1812$

Q] Studies of a single-machine-tool system showed that the time, the machine operates before the breaking down is exponentially distributed with a mean 10 hrs.

i) Find the probability that the machine operates for

a) at least 12 hrs before the breaking down

b) at least 14 hrs but fail before 20 hrs

ii) If the machine has already been operating 8 hrs find the probability that it will last another 4 hrs

iii) If the machine has already been operating 6 hrs find the probability that it will another 4 hrs.

Sol^n:- Let x be life in hours of a machine

$$x \sim Exp(\lambda)$$

Mean $= 1/\lambda = 10$ i.e. $\lambda = 1/10$

$$x \sim Exp(\lambda = 0.1)$$

pdf is $f(x) = 0.1 e^{-0.1x}$, $x \geq 0$

$= 0$, o.w.

cdf is $F(X=x) = p(X \leq x) = 1 - e^{-0.1x}$, $0 \leq x$

so, $p(X \geq x) = e^{-0.1x}$

a) p (the machine operates for at least 12 hrs before the breaking down)

$= p(X \geq 12) = e^{-0.1(12)}$

$= \underline{\underline{0.301194}}$

Date
Page

4/02/21

b) P(the machine operates for atleast 14 hrs but fails before 20 hrs)

$$= p(14 \le x < 20) = F(20) - F(14)$$

$$= \{1 - e^{-0.1(20)}\} - \{1 - e^{-0.1(14)}\}$$

$$= \underline{\underline{0.1126}}$$

ii) ii) If the machine has already been operating 8 hrs the probability that it will last another 4 hrs

Using Memoryless property,
Required probability is $p(x > 12 / x > 8) = p(x > 4)$

$$= e^{-0.1(4)} = \underline{\underline{0.6703}}$$

iii) If the machine has already been operating 6 hrs, the probability that it will last another 4 hrs

Using Memoryless property,
Required probability is $p(x > 4 + 6 / x > 6) = p(x > 4)$

$$= e^{-0.1(4)} = \underline{\underline{0.6703}}$$

Date
Page

4/02/21

Normal Distribution

$$f(x) = \frac{1}{\sigma\sqrt{(2\pi)}} e^{-\frac{1}{2}((x-m)/\sigma)^2}, \quad -\infty < x < \infty$$

where, $-\infty < \mu < \infty$ and $0 < \sigma^2 < \infty$

Q.] Example: (Type I)

If $z \sim N(0,1)$

Find $P(0 < z < 0.95)$, $P(z > 0.95)$, $P(z < -0.95)$

$P(|z| \leq 0.95)$

Sol^n:-

$P(0 < z < 0.95) = 0.3289$

$P(z < 0.95) = 0.5 + \cancel{0.328}\ 0.3289$

$P(z > 0.95) = 0.5 - 0.3289$

$P(z < -0.95) = 0.5 - 0.3289$

$P(|z| \leq 0.95) = 2(0.3289)$

Q.] Type - II (Example)

what is the value of the constant c if

i) $p(0 < z < c) = 0.2291$

$\therefore c = 0.61$

ii) $p(z < c) = 0.7291 = 0.5 + 0.2291$

$\therefore c = 0.61$

iii) $p(z < c) = 0.2291$

$\therefore c = -0.74$

→ (continue - on 5/02/21)

iv) $p(z > c) = 0.2291$

$\therefore c = 0.74$

5/02/21

Q.] Type III : Example

If $X \sim N(3, 16)$, then what is $P(4 \le X \le 8)$?

Sol^n:- $P(4 \le X \le 8)$

$$= P\left(\frac{4-3}{4} \le \frac{x-3}{4} \le \frac{8-3}{4}\right)$$

$$= P(1/4 \le z \le 5/4)$$

$$= P(z \le 1.25) - P(z \le 0.25)$$

$$= 0.3944 - 0.0987$$

$$= 0.2957$$

Q.] Type IV : Example:

→ The marks obtained by students in a certain examination follow a normal distribution with a mean 70 and standard deviation 5. If 1000 students appeared at an examination. Calculate the number of students scoring more than 75 marks

Sol^n:- R.V. X = marks obtained by students

$\sigma = 5$, $m = 70$

Date
Page

p (student scoring more than 78 marks) = $p(x > 78)$

$= p\left(\frac{x-m}{\sigma} > \frac{78-70}{8}\right)$

$= p(z > 1)$

$= 0.5 - 0.3413$

$= 0.1587$

The number of students scoring more than 78 marks =

$= 1000(0.1587)$

$= ~~78~~\ 158.7$

$= \underline{\underline{(159)}}$

Date ______
Page ______

Q.] The monthly salary of a company XYZ were found to be normally distributed with mean Rs. 3000 and SD Rs 250, what should be the minimum salary of the worker in a company XYZ so that the probability that he belongs to top 5%.

Soln:- S.N.V. $z = \frac{x-m}{\sigma} = \frac{x-3000}{250} = \frac{x-3000}{250}$

We have to find value of z_1 for a given probability 0.05

$p(z > z_1) = 0.05$

$p(0 < z < z_1) = 0.5 - 0.05 = 0.45$

$z_1 = 1.64$

$\frac{x-3000}{250} = 1.64$

$x = 3000 + 250(1.64) = \text{Rs}.\ 3410$