

Figure 4.10 shows the schematic of a cocurrent contacting apparatus. The notations used for the phase flow rates and concentrations have the same significances as in Figure 4.7. Following the procedure used in the previous section, the steady-state material balance equation (with the concentrations expressed in the mole ratio unit) over *envelope 1* in Figure 4.10 may be written as

$$G_s(Y_1 - Y) = L_s(X - X_1) \quad (4.29)$$

The above equation represents a straight line having a slope $-L_s/G_s$ and passing through the point (X_1, Y_1) . The equation relates the bulk concentrations of the phases (X, Y) at any section inside the apparatus with the terminal concentrations (X_1, Y_1) at one end of it. Equation (4.29) is the equation of the ‘operating line’ for cocurrent contact.

A material balance over *envelope 2* that encloses the entire apparatus is given below.

$$G_s(Y_1 - Y_2) = L_s(X_2 - X_1) \quad (4.30)$$

The relation (4.30) shows that the operating line equation (4.29) is satisfied by the point (X_2, Y_2) . So in the case of cocurrent contact also, the operating line joins the points representing the terminal concentrations of the two phases.

Figure 4.11 shows the equilibrium curve on the $X-Y$ plane together with the operating line, PQ . If the bulk concentrations of the phases at any section are (X, Y) , represented by the point M on the operating line, then MN' gives the overall gas-phase driving force and MN'' gives the overall liquid-phase driving force at this section. If a line of slope $-k_x/k_y$ is drawn through M and meets the equilibrium curve at the point M' , it gives the interfacial concentrations at the particular section. If the phases are allowed to be in contact for a sufficiently long time, the outgoing streams will reach equilibrium, and the equilibrium concentrations will be given by the point F . Thus the equilibrium curve, in effect, is also the locus of the point representing the interfacial concentration. If the point representing the concentrations of the phases at the inlet to the apparatus lies on the other side of the equilibrium curve, the operating line is represented by the line RS . As the sign of the driving force changes, transfer of the solute now occurs from phase L to phase G .

The material balance can also be written using the mole fraction unit instead of the mole ratio unit. By invoking arguments as in the case of countercurrent contact, we can say that the operating line will be a curve in this case as well.

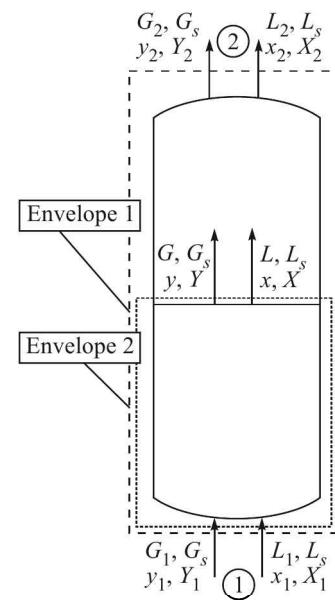


Figure 4.10 Mass balance in a steady state cocurrent process (notations as in Figure 4.7).

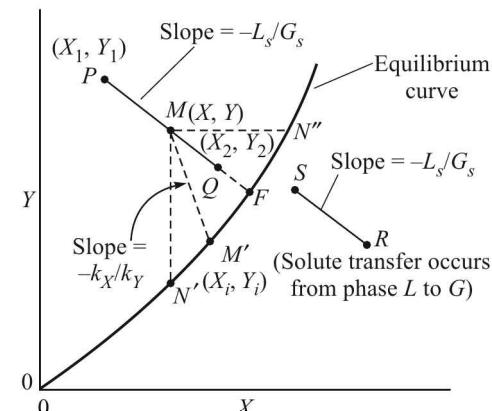


Figure 4.11 Operating lines for cocurrent contact (concentration in the mole ratio unit).

4.6 MASS TRANSFER IN STAGE-WISE CONTACT OF TWO PHASES

The schematic of a countercurrent contacting apparatus presented in Figure 4.7 does not say anything about its internal construction. While we leave the details of the construction and operation of such a device for the next chapter, it will be useful to mention here how the phases are brought into intimate contact in order to achieve efficient transfer of the solute from one phase to the other. For example, in *continuous countercurrent contact* of a gas-phase and a liquid-phase, the liquid may trickle down a packed bed of inert solid while the gas flows up through it (we call it a ‘packed tower’). A large area of contact of the phases is thus provided and the capacity of mass transfer of the apparatus is enhanced. In the *stage-wise contact* of the two phases (Figure 4.12), the liquid is fed at the top of a tower or column and it flows down the column from one ‘tray’ (or ‘plate’) to the next below (it is called a *tray tower* or *plate tower*). The gas flows up bubbling through the liquid on a tray to the one above.

4.6.1 Stage

Any device or combination of devices in which two immiscible phases are brought into intimate contact in order to achieve mass transfer (of one or more solutes) from one phase to the other is called a *stage*. An efficient contact of the phases in a stage *tends* to bring them to equilibrium. If the contacting is so efficient that the phases reach equilibrium when they leave, the stage is called an *ideal stage*. This leads to the definition of ‘stage efficiency’. The stage efficiency gives a measure of how close to the equilibrium the phases may reach in a stage. An ideal stage has 100% efficiency. A group of interconnected stages in which the phases flow from one stage to other, in sequence, is called a *cascade*.

4.6.2 Determination of the Number of Stages in Countercurrent Contact

In general, a graphical construction is the most convenient method for the purpose when a single solute is transferred. Consider an N -tray column (Figure 4.12) in which the concentrations of the gas and the liquid phases entering or leaving the different stages are shown. In the case of stage-wise contact, it is a convention that the subscript of a concentration term or a flow rate term denotes the stage from which the stream concerned comes out. For example, Y_n denotes the concentration (in mole ratio unit) of the gas phase leaving the n th plate or tray of a tower. The top tray of the column is the tray or plate number ‘1’, and the bottom tray is the tray number ‘ N ’. Accordingly, in Figure 4.12, the concentration of phase G entering the column is denoted by Y_{N+1} (the stream, as if, comes from the *hypothetical* $(N + 1)$ th stage preceding stage N). Similarly the phase L entering plate 1 at the top is imagined as a stream coming from the

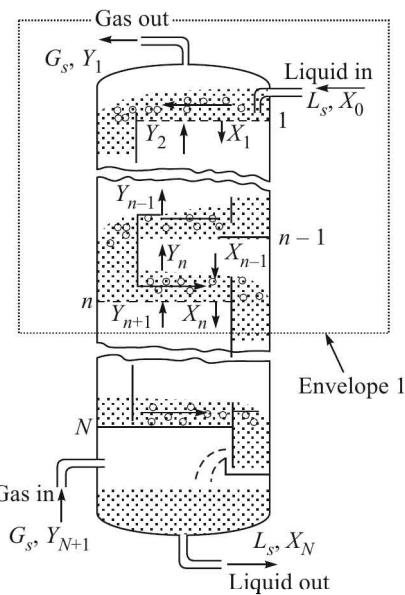


Figure 4.12 Countercurrent contact in a tray tower.

hypothetical zeroth plate. The concentration of this stream is called X_0^+ . The flow rates of the phases on solute-free basis, G_s and L_s , remain constant along the column although the total flow rates, G and L , do vary.

The material balance equation over *envelope 1* in Figure 4.12 enclosing the top n trays gives

$$G_s(Y_{n+1} - Y_1) = L_s(X_n - X_0) \quad (4.31)$$

Equation (4.31) is identical to Eq. (4.26) except that the subscripts of the concentration terms are different. It is the equation of the operating line, and (X_n, Y_{n+1}) is any point on this line that represents the concentrations of the two phases, one leaving the n th tray and the other entering into it. The overall material balance for the entire column is

$$G_s(Y_{N+1} - Y_1) = L_s(X_N - X_0) \quad (4.32)$$

So the operating line is a straight line joining the points (X_0, Y_1) and (X_N, Y_{N+1}) .

The concentration of phase G leaving the n th plate is Y_n , and that of phase L leaving the same plate is X_n . If the plate is *ideal*, the phases reach equilibrium as they leave. Therefore, the concentration pair (X_n, Y_n) should lie on the equilibrium curve, $Y^* = \varphi(X)$.

The above results can be directly utilized for the graphical determination of the number of ideal stages. The equilibrium curve and the operating line (PQ) are drawn on the X - Y plane in Figure 4.13(a). Let us start from the point $Q(X_N, Y_{N+1}$; bottom of the column). A vertical line through Q meets the equilibrium line at point $D(X_N, Y_N)$. A horizontal line through D meets the operating line at the point E which must have an abscissa X_{N-1} . This is because (X_n, Y_{n+1}) is a point on the operating line, and putting $n = N - 1$, we see that (X_{N-1}, Y_N) should lie on this line. The region QDE stands for the N th plate. By drawing successive vertical and horizontal line segments or steps between the operating line and the equilibrium curve, we do the graphical construction for the number of plates. This continues till the point $P(X_0, Y_1)$ is reached. The full construction is further explained under Example 4.6.

There is an obvious question in this connection. What should we do if the construction does not end up exactly at the point P ? We get *fraction of a plate* in the last step of construction. This is explained in Figure 4.13(b). The idea behind expressing the number of ideal plates as a mixed fraction is that if we divide this number by the *overall plate efficiency*, we get a better estimate of the number of real trays required for the separation^{††}.

In the graphical construction described in this section we start at the point Q on the operating line. The construction can also start at the other terminal of the column represented by the point P on the operating line. This alternative construction gives the same number of ideal trays. Conversely, if the number of ideal trays in a column is given, the degree of removal of the solute from a phase that can be achieved in it can be determined graphically; but this will involve a *trial-and-error procedure*. A trial operating line is drawn and it is checked if the desired concentration is reached at the other terminal after the given number of trays are fitted between the equilibrium line and the operating line.

[†] In the case of a continuous contact equipment, the subscripts 1 and 2 are generally used to denote the terminal concentrations as stated in Section 4.5.1.

^{††} Let the number of ideal trays be $5\frac{1}{2}$, and the overall plate efficiency be 0.5. The number of real trays = $(5\frac{1}{2})/(0.5) = 11$. But, if instead, we take the next whole number (i.e. 6) as the number of ideal trays, the number of real trays would have been $6/(0.5) = 12$, which is one more than that required.

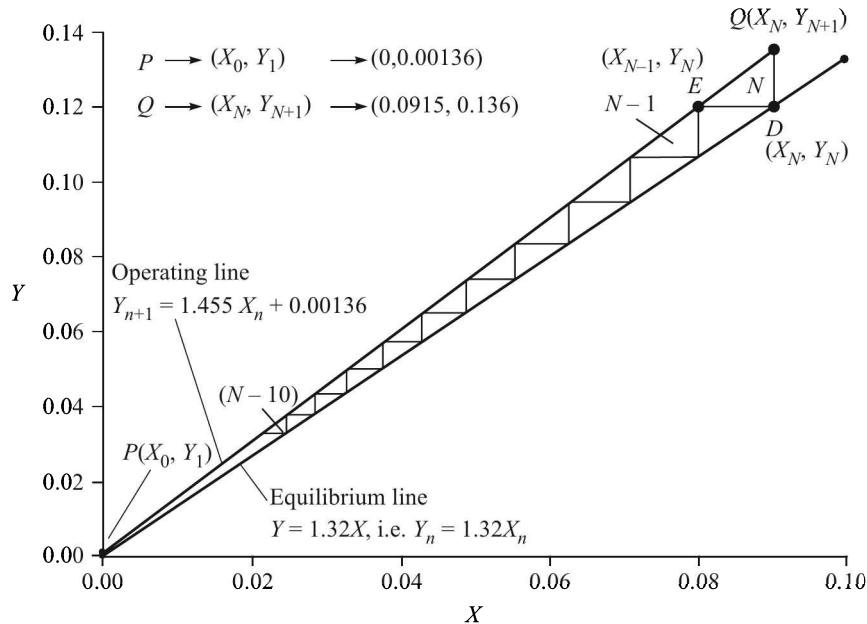


Figure 4.13(a) Graphical construction of stages for countercurrent mass transfer (data from Example 4.5).

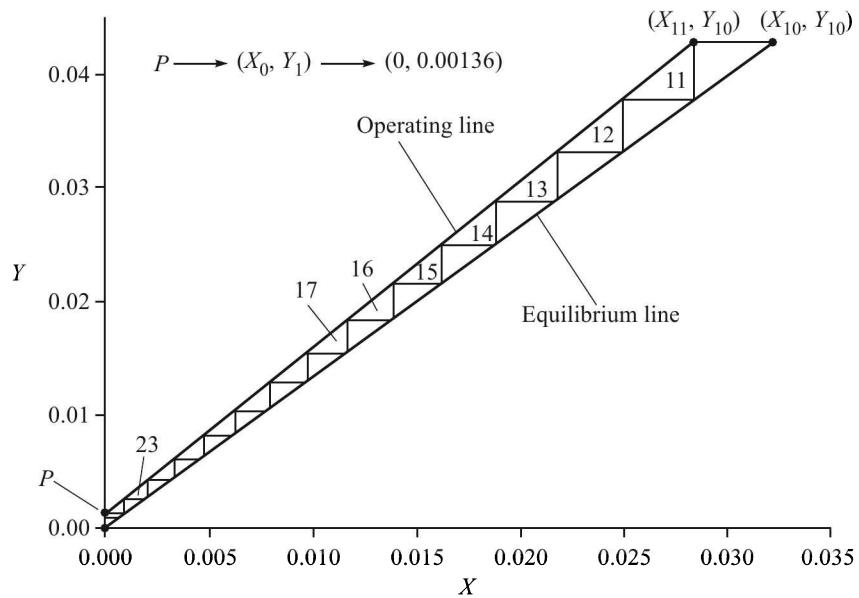


Figure 4.13(b) Graphical construction of stages in the low concentration.

4.6.3 Algebraic Determination of the Number of Ideal Stages

The number of ideal stages can be calculated algebraically if both operating and equilibrium lines are straight. We first consider the case of transfer of the solute from phase G to L (call it

absorption). As pointed out earlier, if the concentrations are expressed in the mole ratio unit and the phase flow rates are on solute-free basis, the operating line is always linear. If the equilibrium line is also straight, say, given by $Y = \alpha X$, the point (X_n, Y_n) lies on this line, i.e.

$$Y_n = \alpha X_n \quad (4.33)$$

Substituting for X_n in the operating line Eq. (4.31) and rearranging,

$$\begin{aligned} Y_{n+1} - Y_1 &= \frac{L_s}{G_s}(X_n - X_0) = \bar{A}Y_n - \bar{A}\alpha X_0; \quad \bar{A} = \frac{L_s}{\alpha G_s} \\ \Rightarrow Y_{n+1} - \bar{A}Y_n &= Y_1 - \bar{A}\alpha X_0 \end{aligned} \quad (4.34)$$

In the case of gas absorption, the quantity $\bar{A} = L_s/\alpha G_s$ is called the *absorption factor*. This quantity has a direct mathematical significance—it is the ratio of the slope of the operating line to that of the equilibrium line, $(L_s/G_s)/\alpha$. Equation (4.34) is a first-order non-homogeneous linear ‘difference equation’ that can be solved by using the standard technique. The corresponding homogeneous equation is

$$Y_{n+1} - \bar{A}Y_n = 0 \quad (4.35)$$

Assuming a solution of the form: $Y_n = K_1 Z^n$, and substituting in Eq. (4.35),

$$K_1 Z^{n+1} - \bar{A} K_1 Z^n = 0 \quad \Rightarrow \quad Z = \bar{A} \quad (4.36)$$

Since the difference Eq. (4.34) is non-homogeneous, it will have a ‘particular solution’ which is a constant in this case (see Wylie, 1975). Assuming $Y_n = K_2$ (= constant) as the particular solution and substituting in Eq. (4.34),

$$K_2 - \bar{A}K_2 = Y_1 - \bar{A}\alpha X_0 \quad \Rightarrow \quad K_2 = \frac{Y_1 - \bar{A}\alpha X_0}{1 - \bar{A}} \quad (4.37)$$

The general solution of Eq. (4.34) is now obtained as

$$Y_n = K_1 Z^n + K_2 = K_1 (\bar{A})^n + \frac{Y_1 - \bar{A}\alpha X_0}{1 - \bar{A}} \quad (4.38)$$

The constant K_1 can be determined by using the known terminal condition: for $n = 0$, $Y_0 = \alpha X_0$.

$$K_1 = \alpha X_0 - \frac{Y_1 - \bar{A}\alpha X_0}{1 - \bar{A}} = \frac{\alpha X_0 - Y_1}{1 - \bar{A}} \quad (4.39)$$

Putting the value of K_1 in Eq. (4.38), we get the complete solution of Eq. (4.34) as

$$Y_n = \left(\frac{\alpha X_0 - Y_1}{1 - \bar{A}} \right) (\bar{A})^n + \frac{Y_1 - \bar{A}\alpha X_0}{1 - \bar{A}} \quad (4.40)$$

Equation (4.40) gives the concentration of the solute (Y_n) in phase G leaving the n th plate of the column. The concentration of the phase L leaving the same tray is readily obtainable using Eq. (4.33). To determine the total number of ideal plates, we put $n = N + 1$ and $Y_n = Y_{N+1}$ in Eq. (4.40).

$$Y_{N+1} = \left(\frac{\alpha X_0 - Y_1}{1 - \bar{A}} \right) (\bar{A})^{N+1} + \frac{Y_1 - \bar{A}\alpha X_0}{1 - \bar{A}}$$

A rearrangement of the equation gives

$$\bar{A}^N = \frac{Y_{N+1}[(1/\bar{A}) - 1] - [Y_1/\bar{A} - \alpha X_0]}{\alpha X_0 - Y_1}$$

Rearranging further and taking logarithm of both sides,

$$N = \frac{\log \left[\left(\frac{Y_{N+1} - \alpha X_0}{Y_1 - \alpha X_0} \right) \left(1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right]}{\log \bar{A}} \quad (4.41)$$

The above equation is the well-known *Kremser Equation* (also called the Kremser–Brown–Souders equation) originally developed in connection with gas absorption in a plate column. However, it is applicable to other mass transfer operations as well (for example, counter-current adsorption). Charts based on Eq. (4.41) are available for ready use (see Ludwig, 1997). A typical chart is given in Figure 4.14.

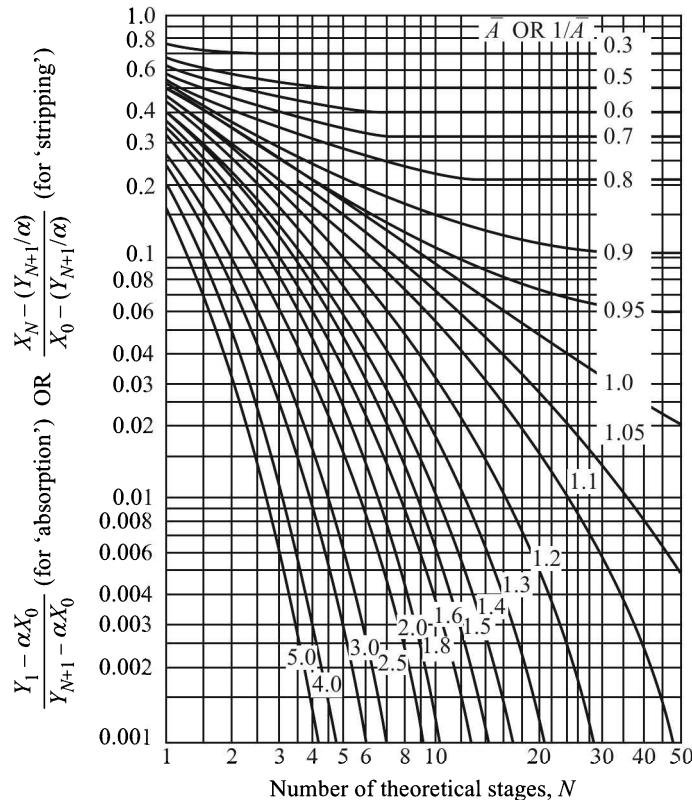


Figure 4.14 Kremser Chart [Eqs. (4.41) and (4.44)]; parameters of the chart: \bar{A} for 'absorption', $1/\bar{A}$ for 'stripping'.

The solution given by Eq. (4.40), does not apply when $\bar{A} = 1$. In this case Eq. (4.34) becomes

$$Y_{n+1} - Y_n = Y_1 - \bar{A} \alpha X_0 \quad (4.42)$$

If we write this equation for $n = N, N - 1, N - 2, \dots, 1$, and add, we get

$$\begin{aligned} Y_{N+1} - Y_1 &= N(Y_1 - \bar{A}\alpha X_0) \\ \Rightarrow N &= \frac{Y_{N+1} - Y_1}{Y_1 - \bar{A}\alpha X_0} \end{aligned} \quad (4.43)$$

Equation (4.43) can be given a physical interpretation. When the absorption factor $\bar{A} = L_s/(\alpha G_s) = 1$, the slope of the operating line becomes $L_s/G_s = \alpha$, which is the same as the slope of the equilibrium curve, Eq. (4.33). So the operating and the equilibrium lines become parallel. The change in the concentration of a phase at each stage (which is equivalent to the distance between the lines) is equal. Therefore, the number of plates will be equal to the ratio of the total concentration change over the column to the concentration change over a single plate or stage.

In the case of transfer of the solute from phase L to G (if the phase G is a gas and the phase L is a liquid, transfer of the solute from L to G is called *stripping*), and the number of theoretical plates or stages is given by

$$N = \frac{\log \left[\left(\frac{X_0 - (Y_{N+1}/\alpha)}{X_N - (Y_{N+1}/\alpha)} \right) (1 - \bar{A}) + \bar{A} \right]}{\log(1/\bar{A})} \quad (4.44)$$

The inverse of the absorption factor, $\bar{S} = 1/\bar{A} = \alpha G_s/L_s$, is called the stripping factor.

The following points are to be noted in connection with the use of Kremser equation.

- (a) In practical situations, the absorption factor \bar{A} may vary along the column to some extent if the equilibrium relation is not strictly linear. If it is so, it is suggested that the geometric mean of the absorption factors at the top and the bottom trays, i.e. $\bar{A} = \sqrt{\bar{A}_1 \bar{A}_N}$ be used.
- (b) In Eqs. (4.41) and (4.44), the concentrations are taken in mole ratio (or mass ratio) unit. If the solute concentrations are low (say, within about 7%), the same equations with concentrations expressed in the mole fraction unit (x and y) may be used to determine the number of ideal plates with reasonable accuracy. But the absorption factor will vary along the column because of changes in the total flow rates and because of any deviation of the equilibrium relation from linearity. Again, the geometric mean value of the absorption factors at the top ($\bar{A}_1 = L_1/\alpha G_1$) and at the bottom ($\bar{A}_N = L_N/\alpha G_N$) is to be used.

EXAMPLE 4.6 (Number of trays for countercurrent contact) It is required to remove 99% of the solute C from a solution of C in G by using a ‘pure’ solvent L in a counter-current cascade. The feed containing 12% C in the mixture enters the column at the bottom at the rate of 6000 kg/h. The solvent enters at the top at a rate of 7685 kg/h. Write down the equation of the operating line. Determine the number of trays required to perform the separation if the overall tray efficiency is 40%. The equilibrium relation is linear, $Y = 1.32X$, where Y = kg C per kg C -free G , and X = kg C per kg C -free L .

Solution

Feed concentration = 12%; feed rate on solute-free basis, $G_s = (6000)(1 - 0.12) = 5280 \text{ kg}$.

Mass of solute entering = $(6000)(0.12) = 720 \text{ kg}$; fraction of the solute removed = 0.99.

Using the *mass ratio unit*, feed concentration, $Y_{N+1} = 0.12/0.88 = 0.136$; exit concentration, $Y_1 = 0.00136$ (see Figure 4.15].

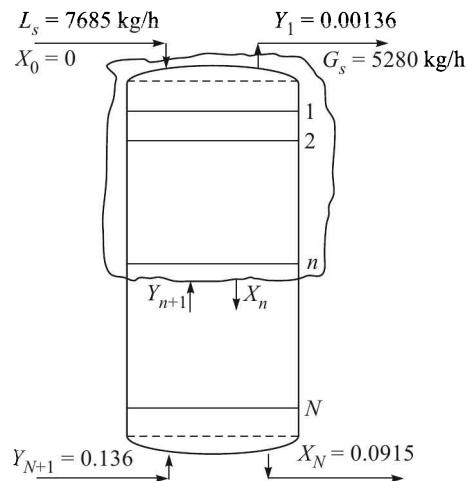


Figure 4.15 The input and output streams.

The entering solvent does not have any C in it; i.e. $X_0 = 0$. Solvent input rate, $L_s = 7685 \text{ kg/h}$.

Material balance over the column: $G_s(Y_{N+1} - Y_1) = L_s(X_N - X_0)$,

$$\text{i.e. } 5280(0.136 - 0.00136) = 7685(X_1 - 0) \quad \text{or} \quad X_1 = 0.0915$$

Equation of the operating line:

By a material balance over the envelope, Figure 4.15,

$$\begin{aligned} G_s(Y_{n+1} - Y_1) &= L_s(X_n - X_0); \text{ but } X_0 = 0 \\ \Rightarrow 5280(Y_{n+1} - 0.00136) &= 7685X_n \quad \text{i.e.} \quad Y_{n+1} = 1.455X_n + 0.00136 \end{aligned}$$

This is the operating line which passes through the terminal points: top (0, 0.00136), and bottom (0.0915, 0.136).

Determination of the number of ideal trays:

The equilibrium line, $Y = \alpha X = 1.32X$ (slope, $\alpha = 1.32$), is drawn on the $X-Y$ plane. The operating line, PQ , is also drawn through the terminal points as above. The number of ideal stages is obtained by the construction of steps on the diagram.

In the lower part of the diagram, the driving force is pretty small. In order to ensure accuracy, the construction of steps in this region is done separately using enlarged scales. The construction is shown in Figure 4.13(a) for the top eleven trays, and Figure 4.13(b) for the rest. The number of ideal trays is 23.8. The number of real trays is obtained by dividing this number by the fractional tray efficiency.

$$\text{The number of real trays} = 23.8/0.4 = 59.5, \quad \boxed{\text{say 60}}.$$

In the given problem, both the equilibrium and operating lines are linear. So the number of ideal trays can also be calculated from the Kremser equation, Eq. (4.41) as well.

The absorption factor, $\bar{A} = L_s/\alpha G_s = 7685/(1.32)(5280) = 1.103$

$$\Rightarrow N = \frac{\log \left[\left(\frac{Y_{N+1} - \alpha X_0}{Y_1 - \alpha X_0} \right) \left(1 - \frac{1}{\bar{A}} \right) + \frac{1}{\bar{A}} \right]}{\log \bar{A}}$$

$$= \frac{\log \left[\left(\frac{0.136 - 0}{0.00136 - 0} \right) \left(1 - \frac{1}{1.103} \right) + \frac{1}{1.103} \right]}{\log 1.103} = \boxed{23.7}$$

Number of real trays = $23.7/0.4 = 59$ real trays.

The number of ideal trays can also be determined using the Kremser chart, Figure 4.14. We have

$\bar{A} = 1.103$ and $\frac{Y_1 - \alpha X_0}{Y_{N+1} - \alpha X_0} = \frac{0.00136 - 0}{0.136} = 0.01$. From the chart (Figure 4.14), the number of ideal stage, $N = \boxed{22.5}$.

4.6.4 The Number of Ideal Stages for Steady State Crosscurrent Contact

The arrangement of flow of the phases in this kind of contact is shown in Figure 4.16. The phase G flows from one stage to the next sequentially. An amount of fresh L phase is added to each stage and the phases are brought into intimate contact. The phases leaving a stage are then separated. The phase G flows to the next stage, and the phase L , along with the solute transferred into it, is taken out of the apparatus. The amount of phase G on solute-free basis (denoted by G_s) flowing from one stage to the next remains constant (because the carrier and the solvent phases, G and L , are assumed to be mutually immiscible). But the amounts of phase L (and also its concentrations) fed to the different stages may not be the same.

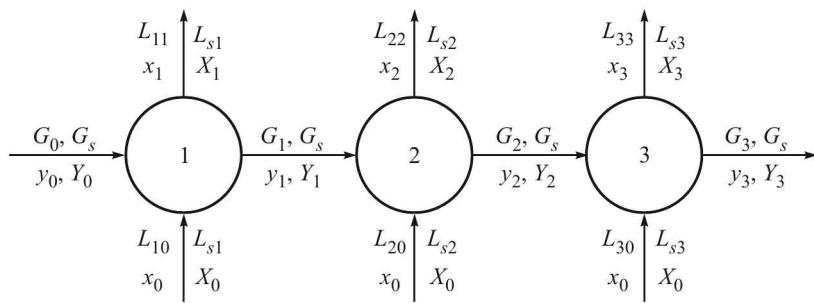


Figure 4.16 Flow arrangement in a cross-flow cascade.

It is obvious that in an individual stage of the cascade, the contact is cocurrent. So we can use the results of Section 4.6.2 in order to determine the number of ideal contact stages required to achieve a desired degree of separation. The graphical method of determination is illustrated in

Figure 4.17. The equilibrium curve is drawn on the X - Y plane. The operating line for stage 1 is drawn through the point $P_1(X_0, Y_0)$ with a slope $-L_{s1}/G_s$. The operating line is extended to meet the equilibrium curve at the point Q_1 if the stage is ideal. The operating line for stage 2 starts from P_2 (which has an ordinate $Y = Y_0$, and abscissa corresponding to the point Q_1) and will have a slope $-L_{s2}/G_s$. The operating lines for the three stages are P_1Q_1 (slope $= -L_{s1}/G_s$), P_2Q_2 (slope $= -L_{s2}/G_s$), and P_3Q_3 (slope $= -L_{s3}/G_s$). If equal amounts of phase L are supplied to the stages, the operating lines will be parallel. If all the stages are *ideal*, the construction procedure can be continued to find out the number of stages required to reduce the concentration of G phase to a specified level. Alternatively, if the number of stages in a cross-flow cascade is given, any other unknown quantity (for example, the flow rate of a phase, the degree of separation attainable, etc.) of the system can be determined. A trial-and-error procedure may be required. The procedure of calculation is the same if the mass transfer occurs from phase L to G .

For a system having a linear equilibrium relation (i.e. if $Y = \alpha X$), the cross-flow cascade problem can be readily solved algebraically. The operating and the equilibrium line equations in terms of the input and output concentrations for each stage are written. The resulting set of equations can be solved for any unknown or unknowns as required. An algebraic approach is also possible if the equilibrium relation, even if nonlinear, is available in the form of an equation (rather than tabulated data). The methods are illustrated in Example 4.9 for mass transfer from phase L to phase G .

EXAMPLE 4.7 (Batch adsorption from a solution) An adsorbent, which is a modified clay, is used to separate an organic compound A from an aqueous solution. One kilogram of the solution containing 10% of the organic was treated with varying amounts of the clay in a number of laboratory tests. The following data were collected.

Gram clay used	15	40	60	100	135	210	325	450
% A in the solution	9.1	7.81	6.93	5.66	4.76	3.38	2.44	1.48
<i>in equilibrium</i>								

- (a) Do the test data fit the Freundlich adsorption isotherm in the form $Y = aX^\beta$, where Y = gram solute per gram clay, and X = gram solute per gram solute-free solvent?
- (b) How much of the adsorbent is required to recover 90% of the solute from 1000 kg of 10% solution?
- (c) If 678 kg of the adsorbent is used per 1000 kg of the solution, how much of the solute is recovered if the treatment is done in two stages, using half of the clay in each stage?

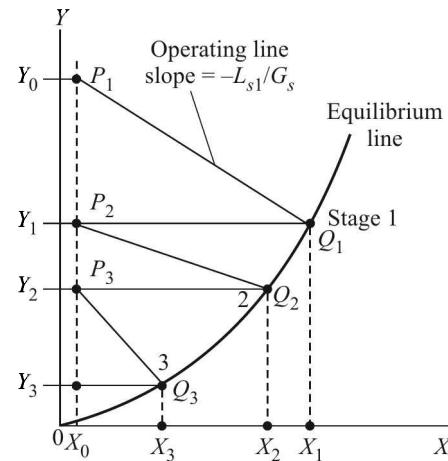


Figure 4.17 Graphical construction of a cross-current cascade (Mass transfer from phase G to phase L).

Hints: The given data are to be converted to

$$X = \frac{\text{kg solute}}{\text{kg solute-free clay}} \quad \text{and} \quad Y = \frac{\text{kg solute}}{\text{kg solute-free solvent (water)}}$$

A sample calculation is shown for the third data point.

Mass of clay taken = 60 g; mass of solution taken initially = 1000 g (100 g solute + 900 g water); % solute (A) in the solution after equilibration = 6.93%.

$$\text{Mass of } A \text{ in the solution in equilibrium} = [(900)/(1 - 0.0693)] - 900 = 67 \text{ g}$$

$$\text{Mass of } A \text{ absorbed} = 100 - 67 = 33 \text{ g}$$

$$X = \frac{33}{60} = 0.55; \quad Y = \frac{67}{900} = 0.0744$$

Calculated data: $X = 0 \quad 0.667 \quad 0.592 \quad 0.550 \quad 0.460 \quad 0.407 \quad 0.326 \quad 0.238 \quad 0.189$
 $Y = 0 \quad 0.1 \quad 0.0847 \quad 0.0744 \quad 0.060 \quad 0.050 \quad 0.035 \quad 0.025 \quad 0.015$

(a) The calculated data are plotted on log-log scale from which we get by fitting a straight line

$$Y = 0.173X^{1.36} \quad \text{i.e.} \quad \alpha = 0.173; \quad \beta = 1.36$$

(b) Mass of solution treated = 1000 kg (900 kg water + 100 kg solute); 90 kg solute to be removed. If M kg adsorbent is used, at equilibrium, $X = 90/M$; $Y = 10/900 = 0.0111$

Putting these equilibrium values of X and Y in the Freundlich's isotherm, we have $M = \boxed{678 \text{ kg}}$.

(c) Two stage contact—use $678/2 = 339$ kg of clay in each stage.

Mass of the aqueous phase (on *solute-free basis*), $G_s = 900$ kg; mass of clay, $L_s = 339$ kg.

Slope of the operating line = $-L_s/G_s = -339/900 = -0.376$; feed concentration, $Y_0 = 100/900 = 0.111$; the feed clay is solute-free, i.e. $X_0 = 0$.

Draw the equilibrium line ($Y = 0.173X^{1.36}$). Locate the point $P_1(0, 0.111)$ on the Y -axis; draw a line of slope -0.376 through P_1 to meet the equilibrium line at $Q_1(X_1, Y_1)$. Draw a horizontal line through Q_1 to meet the Y -axis at $P_2(0, Y_1)$. Draw the operating line for the second cross-current stage through P_2 and parallel to P_1Q_1 to meet the equilibrium line at Q_2 . Then P_2Q_2 is the operating line for the second stage. At the point Q_2 , the concentration of the solute (in the aqueous phase) is $Y_2 = 0.006$. Mass of solute leaving the second stage = $(900)(0.006) = 5.4$ kg. The graphical construction is left as an exercise.

EXAMPLE 4.8 (Batch adsorption from a gas) The equilibrium distribution of moisture between air and a newly developed adsorbent at 25°C is given by

$$Y = 0.0337X$$

where Y = kg moisture per kg dry air, and X = kg moisture per kg dry adsorbent.

- (a) In an experiment on the adsorption of moisture by this material, a cylindrical vessel, provided with a frictionless piston, is filled with 0.2 m³ of air containing moisture at a partial pressure of 20 mm Hg and then 50 g of the adsorbent is added into the vessel. The total pressure within the cylinder remains at 1 atm and the temperature is 25°C. Calculate the partial pressure of moisture in the vessel, the amount of water vapour adsorbed, and the final volume of air after equilibrium is established.

- (b) How much of the adsorbent is required to remove 99% of the moisture from 100 m³ of air at 25°C, 1 atm total pressure and 80% relative humidity by (i) a single-stage batch contact, (ii) two-stage batch contact such that 60% of the total solid required is used in the first stage and the remaining 40% in the second stage?

Given: vapour pressure of water at 25°C = 23.6 mm Hg.

Hints: (a) Mass of dry air in the vessel = 229.2 g; mass of moisture in the vessel at the start = 3.87 g.

Let the final (i.e. after the equilibrium is attained) partial pressure of moisture in the vessel be p mm Hg, and the mass of moisture be m g. Then, at equilibrium,

$$Y = m/229.2; \quad X = Y/0.0337 = 0.1295m$$

Also,

$$\begin{aligned} X &= (\text{mass of moisture removed})/(\text{mass of adsorbent}) \\ &= (3.87 - m)/50 = 0.1295m \quad \text{i.e.} \quad m = 0.518 \text{ g} \end{aligned}$$

If V is the final volume of air in the vessel (1 atm, 25°C), mass of air in it is 229.2 g as before, i.e.

$$\text{Mass of air, } V \left(\frac{760 - p}{760} \right) \left(\frac{273}{298} \right) \left(\frac{28.8}{22.414} \right) = 229.2$$

$$\text{Mass of moisture, } V \left(\frac{p}{760} \right) \left(\frac{273}{298} \right) \left(\frac{18}{22.414} \right) = 0.518$$

Solving these two equations,

$$V = \boxed{0.1954 \text{ m}^3}$$

$$p = \boxed{2.74 \text{ mm Hg}}$$

and the mass of moisture absorbed = 3.032 g.

(b)(i) Mass of dry air, $G_s = 114.8 \text{ kg}$; mass of moisture = 1.83 kg ; $Y_0 = 1.83/114.8 = 0.01594 \text{ kg}$ moisture per kg dry air. Final mass of moisture in the vessel = 1% of the initial = 0.0183 kg ; $Y_1 = 0.0183/114.8 = 1.594 \times 10^{-4} \text{ kg/kg}$ dry air.

$$X_0 = 0; \quad X_1 = ? \quad \text{Material balance: } G_s(Y_0 - Y_1) = L_s(X_1 - X_0)$$

Apply equilibrium relation: $Y_1 = 0.0337X_1$ to get $L_s = \boxed{383 \text{ kg}}$.

(b)(ii) Total mass of adsorbent used = $L_{s1} + L_{s2}$; $L_{s1}/L_{s2} = 60/40 = 1.5$; $X_0 = 0$.

Material balance for Stage 1: $L_{s1}(X_1 - X_0) = G_s(Y_0 - Y_1)$, i.e. $L_{s1}X_1 = 1.83 - 114.8Y_1$

Stage 2: $L_{s2}(X_2 - X_0) = G_s(Y_1 - Y_2)$, i.e. $L_{s2}(X_2 - 0) = 114.8(Y_1 - Y_2)$

Apply the equilibrium relation: $Y_1 = 0.0337X_1$; and $Y_2 = 0.0337X_2$; $Y_2 = 1.594 \times 10^{-4}$, given

Solving the above equations,

$$L_{s1} = 42.5 \text{ kg}$$

$$L_{s2} = 28.4 \text{ kg}$$

Total mass of the adsorbent = 70.9 kg

EXAMPLE 4.9 (Multistage crosscurrent contact) The equilibrium distribution of a solute *C* between solvents *A* and *B* (up to 30% of *C* in solution in *A*) is given by

$$Y = 3.75X$$

where *X* and *Y* are the concentrations of *C* in *A* and *B* respectively, both in the mass ratio unit (i.e. mass of the solute per unit mass of solute-free solvent). The solvents *A* and *B* are practically immiscible.

It is required to calculate the amount of the solvent *B* required to separate 95% of *C* from 1000 kg of a 15% (by mass) solution of *C* in *A* for the following separation schemes: (a) an ideal single-stage contact; (b) an ideal three-stage crosscurrent contact, the amount of the solvent used in each stage being equal; (c) a very large number of crosscurrent contacts using an infinitesimal amount of the solvent in each stage.

Solution

Basis: 1000 kg of the feed solution containing 15% *C* (solute) and 85% *A* (carrier). Let *L* denote the feed solution (*C* in *A*) and *G* denote the extracting solvent (*B*) phase.

$$\text{Amount of the carrier phase in the feed, } L_s = 1000(1 - 0.15) = 850 \text{ kg}$$

Feed concentration (on solute-free basis), $X_0 = (15 \text{ kg } C)/(85 \text{ kg carrier}) = 0.1765 \text{ kg solute per kg carrier, } A$.

At the outlet, 95% of the solute is removed; exit concentration of the phase *L* is

$$(0.15)(0.05)/850 = 0.00882 \text{ kg } C \text{ per kg } A$$

(a) Single-stage contact [see Figure 4.18(a)]

$$\text{Mass balance over the stage: } L_s(X_0 - X_1) = G_s(Y_1 - Y_0)$$

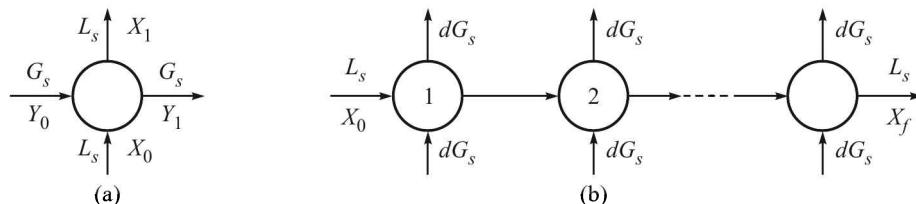


Figure 4.18 (a) Single-stage ideal contact. (b) An infinitesimally small quantity of phase *G* is added to each of the infinite number of crosscurrent stages.

The phases are in equilibrium as they leave the *ideal stage*. Therefore, $Y_1 = 3.75X_1$. Putting the values of various quantities, i.e.

$$X_0 = 0.1765; Y_1 = 3.75X_1 = (3.75)(0.00882) = 0.0331 \text{ kg } C \text{ per kg } B; Y_0 = 0$$

in the mass balance equation, we have

$$(850)(0.1765 - 0.00882) = G_s(0.0331 - 0)$$

or
$$G_s = \frac{(850)(0.1765)}{0.0331} = \boxed{4306 \text{ kg}}$$

(b) An ideal three-stage crosscurrent cascade (see Figure 4.16)

The mass balance equations over the three stages are given below (equal amount of B is used for each stage).

$$\text{Stage 1: } L_s(X_0 - X_1) = G_s(Y_1 - Y_0)$$

$$\text{Stage 2: } L_s(X_1 - X_2) = G_s(Y_2 - Y_0)$$

$$\text{Stage 3: } L_s(X_2 - X_3) = G_s(Y_3 - Y_0)$$

Given: $L_s = 850 \text{ kg}$; $X_0 = 0.1765$; $X_3 = 0.00882$; $Y_0 = 0$; $Y_1 = 3.75X_1$; $Y_2 = 3.75X_2$ and $Y_3 = 3.75X_3$; $G_s = ?$

Putting the above values in the mass balance equations,

$$\text{Stage 1: } 850(0.1765 - X_1) = G_s(3.75X_1)$$

$$\text{Stage 2: } 850(X_1 - X_2) = G_s(3.75X_2)$$

$$\text{Stage 3: } 850(X_2 - 0.00882) = G_s(3.75)(0.00882)$$

The above three mass balance equations can be solved for the three unknowns, X_1 , X_2 , and G_s . But the equations are nonlinear. A graphical method of solution is therefore more convenient. The procedure of graphical solution of such a problem is given below (see also Section 4.6.4).

Step 1: Draw the equilibrium line on the X - Y plane (in the given case, the line is straight).

Step 2: Locate the concentrations of the input streams to the first stage, (X_0, Y_0) .

Step 3: Draw three operating lines of equal slopes (because the same amount of solvent, G_s is used in each stage). One end of such an operating line lies on the horizontal line $Y = Y_0 (= 0)$ and the other end on the equilibrium line. A trial-and-error construction method may have to be adopted so that the last operating line (that corresponds to the third stage) ends up at the point (X_3, Y_3) on the equilibrium line where $X_3 = 3.75Y_3$.

The construction is shown in Figure 4.19. The equal slope of the lines $= L_s/G_s = -2.187 \Rightarrow G_s = 388.7$. The total amount of extracting solvent required $= 3 \times 388.7 = \boxed{1166 \text{ kg}}$.

This is substantially less than that required if the separation is done in a single stage.

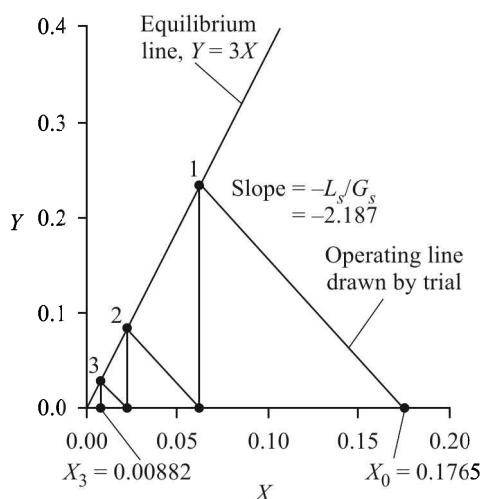


Figure 4.19 Graphical construction of three crosscurrent stages by trial.

(c) A cascade having an infinite number of crosscurrent contact stages [see Figure 4.18(b)]

While the feed stream moves from one stage to the next with gradually reducing concentration of the solute in it, a small amount, dG_s , of the solvent B is added to each stage.

$$\text{The material balance for the } n\text{th stage: } -L_s dX = (Y - Y_0) dG_s = Y dG_s$$

Here dX is the change in the concentration of the stream L as it moves through a stage. Since dX is negative, a minus sign is used in the above equation to maintain consistency of sign. Since the stages are ideal, the phases leaving a stage are in equilibrium, i.e. $Y = 3.75X$.

$$\Rightarrow -L_s dX = (3.75X) dG_s$$

On integration from $X = X_0$ (the feed concentration) to $X = X_f = 0.00882$, which is the terminal concentration,

$$\begin{aligned} -L_s \int_{X_0}^{X_f} \frac{dX}{X} &= \int_0^{G_s} dG_s \quad \Rightarrow \quad L_s \ln \frac{X_0}{X_f} = 3.75G_s \\ \Rightarrow 850 \ln \left(\frac{0.1765}{0.00882} \right) &= 3.75G_s \quad \text{or} \quad G_s = 679.2 \text{ kg} \end{aligned}$$

which is the required amount of solvent.

EXAMPLE 4.10 (A crosscurrent extraction battery) A feed having X_0 kg solute per kg carrier A is treated with a solvent B in an N -stage crosscurrent cascade. The feed rate is R_s kg/h (solute-free basis), and an equal amount of pure solvent, E_s kg/h, is supplied to each of the stages. If the equilibrium relation is linear ($Y = \alpha X$), determine the number of ideal stages required to reduce the solute concentration in phase A to X_N (this phase is called the ‘raffinate’; see Chapter 8).

Solution

Refer to Figure 4.20. For $Y_{1i} = Y_{2i} = \dots = Y_{ni} = 0$, the material balance equation for the stage-1 becomes:

$$\begin{aligned} R_s(X_0 - X_1) &= E_s Y_1 = E_s \cdot \alpha X_1 \Rightarrow X_0 - X_1 = (\alpha E_s / R_s) X_1 = A' X_1 \Rightarrow X_0 = (A' + 1) X_1 \\ &\text{For stage-2} \quad X_1 = (A' + 1) X_2 \\ &\text{For stage-3} \quad X_2 = (A' + 1) X_3 \\ &\vdots \quad \vdots \\ &\text{For stage-}N \quad X_{N-1} = (A' + 1) X_N \end{aligned}$$

Multiplying the above equations,

$$X_0 = (A' + 1)^N X_N \quad \Rightarrow \quad \text{Number of ideal stages,} \quad N = \frac{\log(X_0/X_N)}{\log(A' + 1)}$$

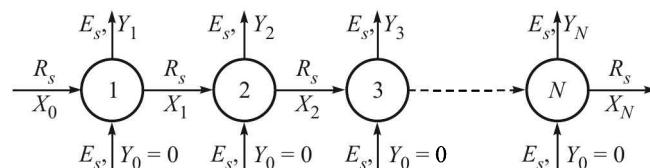


Figure 4.20 Crosscurrent extraction battery.