

① Given a dynamic table that double in size when it needs more space. Find the amortized runtime for inserting n elements

② use the aggregate method
To insert n elements using the aggregate method with can be done in 2 ways.

Case 1 :-

If we don't take and need to allocate new memory then

$\begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ O(1) & O(1) & O(1) & O(1) & O(1) & O(1) & O(1) & O(1) \end{matrix}$

so the sequence of n inserts

$$O(n) + O(2n) = O(n)$$

so, replace $O(1)$ in above example

$$O(1) + O(2n) = O(1)$$

thus the amortized runtime is $O(n)$ for inserting n elements is $O(1)$

for case 2 :-

If we allocate new memory

$$i = 2^k + 1, \quad k = 1, 2, 3, \dots$$

to include the capacity & double the size of array

then we need to allocate new memory

for inserting the element n in new array

$$\text{runtime time} = 2^k + 1 \quad \text{if } i = 2^{k+1} \quad \text{case 1}$$

$$= 1 \quad \text{otherwise case 2.}$$

⑥ use the accounting method

using the accounting method. charge 2 units to each insertion.

when the table double in size from m to $2m$ units.

The credits exactly pay for the copy cost of $O(m)$

total credit is $m + 2m + 4m + \dots$

$$n/2 * m = O(n)$$

Pseudo code:-

initialize table with capacity = 1

for $i = 1$ to n

if table is new table with size

$2 * \text{current size}$

insert element i into table

initialize charges = 0

initialize credits = 0

for $i = 1$ to n

charges + = 2

credit + = m

total charges = $2 * n = O(n)$

total credit = $m + 2m + \dots n/2 + m = O(n)$

cost per insertion = $\text{total} / n = O(n) / n = O(1)$

\therefore routine per insertion = $O(1)$

total time for inserting n elements is $O(n)$.