ALIAH UNIVERSITY
DEPARTMENT OF MATHEMATICS AND STATISTICS
COURSE: ENGINEERING MATHEMATICS-II
COURSE CODE: MATUGBS02 and MA 134
Total marks : $10 \times 8 = 80$
Time allotted : 3 hours

Unless otherwise stated, notations carry their usual meanings. Candidates should read the following instructions carefully before answering the questions:

Instructions: Answer any **four** questions from part **A** and any **four** questions from part **B**.

Part-A

1. (a) If $A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$, show that $A^2 - 4A - 5I_3 = O$. Hence obtain a matrix B such that $AB = I_3$.

   (b) Prove that the matrix $A = \frac{1}{3}\begin{pmatrix} 1 & -2 & 2 \\ 2 & -1 & -2 \\ 2 & 2 & 1 \end{pmatrix}$ is orthogonal. Utilise this to solve the following system of equations
   $x - 2y + 2z = 2$
   $2x - y - 2z = 1$
   $2x + 2y + z = 7$.

2. (a) Solve the system of equations by matrix inversion method
   $x + z = 0$
   $3x + 4y + 5z = 2$
   $2x + 3y + 4z = 1$.

   (b) Find the rank of the rectangular matrix $\begin{pmatrix} 0 & 0 & 5 & -3 \\ 2 & 4 & 3 & 5 \\ -1 & -2 & 6 & -7 \end{pmatrix}$.

3. (a) Use Caley-Hamilton theorem to find A^{100}, where $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$.

   (b) Without expanding prove that $\begin{vmatrix} 0 & -a & -b \\ a & 0 & c \\ b & -c & 0 \end{vmatrix} = 0$.

4. (a) Solve the system of equations by Cramer's rule
   $x + y + z = 6$
   $x + 2y + 3z = 14$
   $x - y + z = 2$.

   (b) Show that the set of vectors $\{(1,0,0), (1,1,0), (1,1,1)\}$ is a basis of $\mathbb{R}^3$.

5. (a) Find the cubic roots of 1.
   (b) Solve the equation $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1 = 0$.

6. (a) Solve the algebraic equation $x^4 + x^2 - 2x + 6 = 0$, it is given that $1 + i$ is a root of the above equation.
   (b) Apply Descartes' rule of sign to examine the nature of the roots of the equation $x^4 + 2x^2 + 3x - 1 = 0$.

Part-B

7. (a) Find the differential equation of all circles which pass through the origin and whose centres are on the y-axis.
   (b) Solve: $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$.

8. (a) Solve: $(x^3 + 3xy^2)dx + (y^3 + 3x^2y)dy = 0$.
   (b) Solve: $(y + \frac{y^3}{3} + \frac{x^2}{2})dx + \frac{(x+xy^2)}{4}dy = 0$.

9. (a) Solve by the method of variation of parameters: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = e^x \tan x$.
   (b) Solve : $(x^2D^2 - xD + 2)y = x \log x$, $(D = \frac{d}{dx})$

10. (a) Find the eigen values and eigen functions of the eigen value problem $\frac{d^2y}{dx^2} + \lambda y = 0$ satisfying with the conditions $y(0) + y'(0) = 0$ and $y(1) + y'(1) = 0$.
    (b) Transform the differential equation $yp^2 + x^3p = x^2y$ $(p = \frac{dy}{dx})$ to Clairaut's form by using the transformation $x^2 = u$ and $y^2 = v$ and hence solve it.

11. (a) Solve by the method of undetermined co-efficients : $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = x^3 - x + e^{-2x}$
    (b) Solve by reducing to normal form $\frac{d^2y}{dx^2} - 4x\frac{dy}{dx} + (4x^2 - 3)y = e^{x^2}$.

12. (a) Determine the general solution of the system of differential equations: $\frac{dx}{dt} + 5x + y = e^t$ $\frac{dy}{dt} + 3y - x = e^{-2t}$.
    (b) Solve by changing the independent variable $(1+x)^2\frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 4\cos(\log(1 + x))$.