10) A Project schedule has the following characteristics.

Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time (days)	4	1	1	1	6	5	4	8	1	2	5	7

a) Construct a n/w diagram.
b) Compute the earliest time & latest even time.
c) Determine the critical path & total project duration.

Not in syllabus

OR pyq soln 23

22/5/25

S-A

1. What are the various types of models?

1) Linear programming model – Optimize with linear constraints.

2) Integer Programming model – Like ~~LPP~~ LP with integers.

3) Non-linear programming m. – Involves non-linear functions.

4) Dynamic Prog. m. – Solves in stages.

5) Queuing m. – Analyzes waiting lines.

6) Simulation m. – Imitates real systems

7) Inventory m. – manages stock levels.

8) Game theory m. – Competitive decisions.

9) Network m. – Graph-based problems.

10) Decision theory – Choices under uncertainty.

2) Define a feasible region.
A feasible region is the set of all possible Solⁿs that satisfy all the constraints of a given problem. In short, the area containing all points that meet the problem's constraints.

3) State the necessary & sufficient condition for the existance of a feasible soln to a transportation problem. Done

4) What are the main phases of a product? Done

5) What are the diff. types of inventories?
~~i) Raw m~~ i) Cycle inventory [not in syllabus]
ii) Safety (Buffer) "
iii) Pipeline (transit) "
iv) Decoupling "
v) Anticipation "
vi) Dead "

G-B
1) What are the advantages of linear programming techniques?
i) Optimal resource use - maximize profit/minimize cost efficiently.
ii) Clear decision making - Provides a structured, mathematical approach.
iii) Handles constraints - Considers multiple restrictions simultaneously.
iv) Flexibility - Can be applied to various industries and problems.
v) Improves productivity - Helps in better planning & resource allocation.
vi) Supports sensitivity analysis - shows how changes affect outcomes.

2) State the characteristics of standard form & write the standard form of linear programming problems to matrix form?
Characteristics of standard form (LPP)
i) Objective func is to be maximized.
ii) All constraints are equalities (=).
iii) All variables are non-negative ($x_i \geq 0$)
iv) RHS of constraints is non-negative.
Matrix form of standard LPP -
maximize $\boxed{Z = C^T x}$, Subject to: $\boxed{Ax = b,\ x \geq 0}$
[C = objective coefficients
x = decision variables
A = constraint coefficients
b = RHS vals]

3) Write Vogel's approximation method for solving T.P.? [Done.]

4) Explain unbalanced assignment problem. [Done]

5) Distinguish b/w PERT & CMP.

PERT	CMP
i) It stands for program evaluation & review technique.	i) It stands for critical path method.
ii) ~~Here time is estimated & unce~~ Focuses on time estimation & uncertainty.	ii) Focuses on time & cost optimization.
iii) Activity time is probabilistic.	iii) Activity time is ~~determinis~~ deterministic.
iv) Cost analysis is not emphasized.	iv) Here cost analysis is strongly focused.
v) Application – Research & development.	v) ~~Appl~~ Application – construction, maintenance.

6) What are the main objectives of an inventory model? [N. I. S] → (Not in syllabus)

i) minimize total cost, ii) Avoid stockouts, iii) optimize order size & timing, iv) Ensure smooth operations, v) maintain service level.

7) Write down short note on queue discipline.

8) State diff. types of games. [NIS]

i) Cooperative – players from coalitions.

ii) Non-cooperative – Players act independently.

iii) Zero-sum – one's gain = another's loss.

iv) Non-zero sum – All can gain or loss.

v) Symmetric – Same strategies for all.

vi) Asymmetric – Different strategies / payoffs.

vii) Perfect information – All moves are known.

viii) Sequential – Players move in turns.

ix) Imperfect info – Some moves are ~~kn~~ hidden.

x) Simultaneous – Players move at once.

Solution to Group C Question 1

Problem Statement:

Minimize $z = x + y$

Subject to:

1. $5x + 9y \leq 45$
2. $x + y \geq 2$
3. $y \leq 4$
4. $x, y \geq 0$

Step 1: Identify the Constraints and Plot the Feasible Region

1. Constraint 1: $5x + 9y \leq 45$
   - When $x = 0, y = 5$
   - When $y = 0, x = 9$
   - Plot the line joining (0, 5) and (9, 0). The feasible region is below this line.
2. Constraint 2: $x + y \geq 2$
   - When $x = 0, y = 2$
   - When $y = 0, x = 2$
   - Plot the line joining (0, 2) and (2, 0). The feasible region is above this line.
3. Constraint 3: $y \leq 4$
   - A horizontal line at $y = 4$. The feasible region is below this line.
4. Non-negativity Constraints: $x, y \geq 0$
   - The feasible region is in the first quadrant.

Step 2: Find the Corner Points of the Feasible Region

The feasible region is bounded by the intersection points of the constraints. Let's find these points:

1. Intersection of $5x + 9y = 45$ and $x + y = 2$:

Step 2: Find the Corner Points of the Feasible Region

The feasible region is bounded by the intersection points of the constraints. Let's find these points:

1. Intersection of $5x + 9y = 45$ and $x + y = 2$:
   - Solve the system:
     $x + y = 2 \Rightarrow x = 2 - y$
     Substitute into $5x + 9y = 45$:
     $5(2 - y) + 9y = 45$
     $10 - 5y + 9y = 45$
     $4y = 35 \Rightarrow y = 8.75$
     $x = 2 - 8.75 = -6.75$
     Not feasible (since $x < 0$).
2. Intersection of $5x + 9y = 45$ and $y = 4$:
   - Substitute $y = 4$:
     $5x + 36 = 45$
     $5x = 9 \Rightarrow x = 1.8$
     Point: (1.8, 4)
3. Intersection of $x + y = 2$ and $y = 4$:
   - Substitute $y = 4$:
     $x + 4 = 2 \Rightarrow x = -2$
     Not feasible (since $x < 0$).
4. Intersection of $x + y = 2$ and $y = 0$:
   - Substitute $y = 0$:
     $x = 2$
     Point: (2, 0)
5. Intersection of $5x + 9y = 45$ and $y = 0$:
   - Substitute $y = 0$:
     $5x = 45 \Rightarrow x = 9$
     Point: (9, 0)
6. Intersection of $x = 0$ and $x + y = 2$:
   - Substitute $x = 0$:
     $y = 2$
     Point: (0, 2)
7. Intersection of $x = 0$ and $5x + 9y = 45$:
   - Substitute $x = 0$:
     $y = 5$
     But $y \leq 4$, so not feasible.
8. Intersection of $x = 0$ and $y = 4$:
   - Point: (0, 4)

Step 3: Evaluate the Objective Functio
Each Corner Point

Step 3: Evaluate the Objective Function at Each Corner Point

The feasible corner points are:

1. (1.8, 4)
2. (2, 0)
3. (9, 0)
4. (0, 2)
5. (0, 4)

Now, evaluate $z = x + y$ at these points:

1. $z = 1.8 + 4 = 5.8$
2. $z = 2 + 0 = 2$
3. $z = 9 + 0 = 9$
4. $z = 0 + 2 = 2$
5. $z = 0 + 4 = 4$

Step 4: Identify the Minimum Value of z

The minimum value of z is 2, achieved at the points:

- (2, 0)
- (0, 2)

Final Answer:

The optimal solution is $z = 2$ at the points $(2, 0)$ and $(0, 2)$. This means there are multiple optimal solutions along the line segment joining these two points. The minimum value of z is 2.

Qn 2

Solution to Group C Question 2

Problem Statement:

Maximize $z = 3x + 2y$

Subject to:

1. $x + y \leq 4$
2. $x - y \leq 2$
3. $x, y \geq 0$

Step 1: Convert Inequalities to Equations and Introduce Slack Variables

To solve using the Simplex method, we first convert the inequalities into equations by introducing slack variables s_1 and s_2:

1. $x + y + s_1 = 4$
2. $x - y + s_2 = 2$

The objective function becomes:

$z = 3x + 2y + 0s_1 + 0s_2$

Step 2: Construct the Initial Simplex Tableau

We set up the initial tableau with the coefficients of the constraints and the objective function:

Basis	x	y
s_1	1	1
s_2	1	-1
z	-3	-2

- Basis: The initial basic variables are s_1 and s_2.
- Objective Row: The coefficients of x

Step 2: Construct the Initial Simplex Tableau

We set up the initial tableau with the coefficients of the constraints and the objective function:

s_1	s_2	RHS
1	0	4
0	1	2
0	0	0

- Basis: The initial basic variables are s_1 and s_2.
- Objective Row: The coefficients of x and y are negative, indicating th[...]e current solution is not optimal.

Step 3: Determine the Entering and Leaving Variables

1. Entering Variable: Choose the most negative coefficient in the objective row. Here, x has a coefficient of -3, so x enters the basis.
2. Leaving Variable: Calculate the minimum ratio $\frac{\text{RHS}}{\text{Pivot Column}}$ for the positive entries in the x-column:
   - For s_1: $\frac{4}{1} = 4$
   - For s_2: $\frac{2}{1} = 2$

   The minimum ratio is 2, so s_2 leaves the basis.

Step 4: Perform the Pivot Operation

Step 4: Perform the Pivot Operation

The pivot element is at the intersection of the x-column and the s_2-row (value = 1).

1. Update the Pivot Row (s_2):
   Divide the entire row by the pivot element (1):
   $$x - y + s_2 = 2 \rightarrow x - y + s_2 = 2$$
2. Update the Other Rows:
   - For s_1: Subtract the pivot row from s_1:
     $$(1-1)x + (1-(-1))y + (1-0)s_1 + (0-1)s_2 = 4-2$$
     $$0x + 2y + s_1 - s_2 = 2$$
   - For z: Add 3 times the pivot r[...]o the objective row:

the objective row:

$$(-3 + 3 \cdot 1)x + (-2 + 3 \cdot (-1))y + (0 + 3 \cdot 0)s_1 + (0 + 3 \cdot 1)s_2 = 0 + 3 \cdot 2$$

$$0x - 5y + 0s_1 + 3s_2 = 6$$

The updated tableau is:

Basis	x	y
s_1	0	2
x	1	-1
z	0	-5

Step 5: Second Iteration

1. Entering Variable: The most negative

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the objective row:

$$(-3+3\cdot 1)x+(-2+3\cdot(-1))y+(0+3\cdot 0)s_1+(0+3\cdot 1)s_2=0+3\cdot 2$$

$$0x-5y+0s_1+3s_2=6$$

The updated tableau is:

s_1	s_2	RHS
1	-1	2
0	1	2
0	3	6

Step 5: Second Iteration

1. Entering Variable: The most negative

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Step 5: Second Iteration

1. Entering Variable: The most negative coefficient in the objective row is -5 for y, so y enters the basis.
2. Leaving Variable: Calculate the minimum ratio:
   - For s_1: $\frac{2}{2} = 1$
   - For x: $\frac{2}{-1}$ (ignore, since the denominator is negative)

   The minimum ratio is 1, so s_1 leaves the basis.

Step 6: Perform the Pivot Operation

The pivot element is at the intersection of the y-column and the s_1-row (value = 2).

The pivot element is at the intersection of the y-column and the s_1-row (value = 2).

1. Update the Pivot Row (s_1):

   Divide the entire row by 2:

   $$0x + y + 0.5s_1 - 0.5s_2 = 1$$

2. Update the Other Rows:

   - For x: Add the pivot row to the x-row:

     $$1x + (-1+1)y + (0+0.5)s_1 + (1-0.5)s_2 = 2+1$$

     $$x + 0y + 0.5s_1 + 0.5s_2 = 3$$

   - For z: Add 5 times the pivot row to the objective row:

     $$0x + (-5+5\cdot 1)y + (0+5\cdot 0.5)s_1 + (3+5\cdot(-0.5))s_2 = 6+5\cdot 1$$

$$(-0.5))s_2 = 6 + 5 \cdot 1$$

$$0x + 0y + 2.5s_1 + 0.5s_2 = 11$$

The updated tableau is:

Basis	x	y
y	0	1
x	1	0
z	0	0

Step 7: Check for Optimality

The objective row has no negative coefficients, so the current solution is optimal.

$$(-0.5))s_2 = 6 + 5 \cdot 1$$

$$0x + 0y + 2.5s_1 + 0.5s_2 = 11$$

The updated tableau is:

s_1	s_2	RHS
0.5	-0.5	1
0.5	0.5	3
2.5	0.5	11

Step 7: Check for Optimality

The objective row has no negative coefficients, so the current solution is optimal.

coefficients, so the current solution is optimal.

- Basic Variables:
  $x = 3, y = 1$
- Non-Basic Variables:
  $s_1 = 0, s_2 = 0$
- Objective Value:
  $z = 3(3) + 2(1) = 11$

Final Answer:

The optimal solution is $x = 3, y = 1$, achieving a maximum value of $z = 11$.

is, (A_1, B_2).

Example 19.4 Solve the game whose pay-off matrix is given below.

$$\begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix}$$

Solution

$$\begin{array}{cc} & \begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \quad \text{Row minima} \\ \text{Player } A \begin{array}{c} A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} & \begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix} \begin{array}{c} -2 \\ 1 \\ -4 \\ -6 \end{array} \\ \text{Column maxima} & \begin{array}{ccccc} 5 & 3 & 1 & 5 & 6 \end{array} \end{array}$$

Maxi (minimum) $= \underline{\gamma} = \text{Max}(-2, 1, -4, -6) = 1$.

Mini (maximum) $= \overline{\gamma} = \text{Min}(5, 3, 1, 5, 6) = 1$.

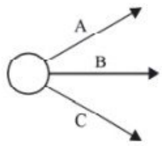
Since, $\underline{\gamma} = \overline{\gamma} = 1$, there exists a saddle point. Value of the game is 1. The position of the saddle point is the optimal strategy and is given by, $[A_2, B_3]$.

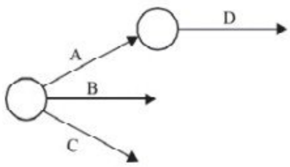
EXERCISES

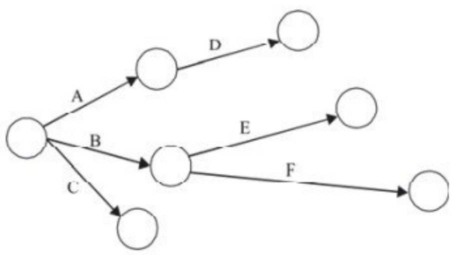
Example 15.2 Construct a network for each of the projects whose activities and their precedence relationships are given below.

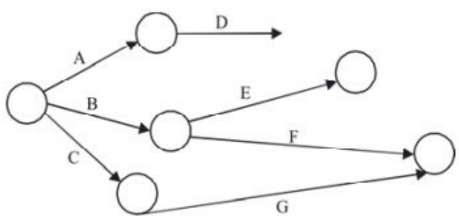
Activity	***A***	***B***	***C***	***D***	***E***	***F***	***G***	***H***	***I***	***J***	***K***
Predecessor	–	–	–	*A*	*B*	*B*	*C*	*D*	*E*	*H, I*	*F, G*

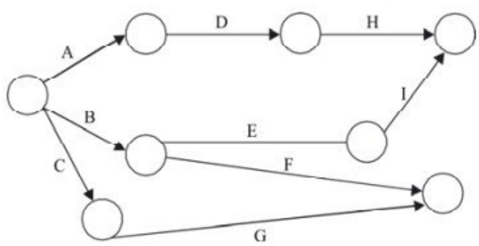
Solution *A*, *B* and *C* are the concurrent activities as they start simultaneously. *B* becomes the predecessor of activities *E* and *F*. Since the activities *J* and *K* have two preceding activities, a dummy may be introduced (if possible).

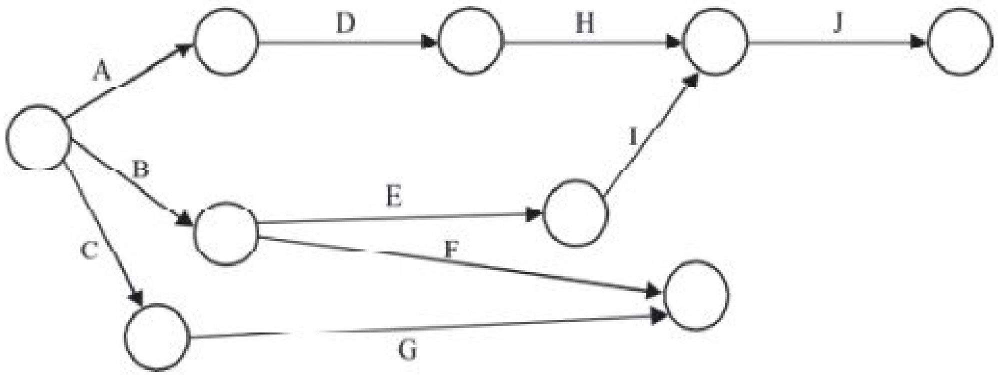












Finally we have,