

# K-Map → Minimization technique.

↳ Gray code. →

|   | Binary | Gray |
|---|--------|------|
| 0 | 0 0    | 0 0  |
| 1 | 0 1    | 0 1  |
| 2 | 1 0    | 1 1  |
| 3 | 1 1    | 1 0  |

Binary → Gray

B = 1 0 1 1 1  
 ↓ ↓ ↓ ↓ ↓  
 G = 1 1 1 0 0

Gray  
 Binary

1 1 1 0 0  
 ↓ ↓ ↓ ↓ ↓  
 1 0 1 1 1 → 10111 =

K-Map.

| A \ B       | $\bar{B}$ | B   |
|-------------|-----------|-----|
| 0 $\bar{A}$ | 0 0       | 0 1 |
| 1 A         | 1 0       | 1 1 |

| A \ BC      | $\bar{B}\bar{C}$ 0 | $\bar{B}C$ 1 | $B\bar{C}$ 3 | $BC$ 2 |
|-------------|--------------------|--------------|--------------|--------|
| 0 $\bar{A}$ |                    |              |              |        |
| 1 A         |                    |              |              |        |

| A \ B               | $\bar{B}$ | B  | $\bar{C}$ | C  |
|---------------------|-----------|----|-----------|----|
| 00 $\bar{A}\bar{B}$ | 0         | 1  | 3         | 2  |
| 01 $\bar{A}B$       | 4         | 5  | 7         | 6  |
| 11 $AB$             | 12        | 13 | 15        | 14 |
| 10 $A\bar{B}$       | 8         | 9  | 11        | 10 |

We have to find neighbor.  
 when only 1 bit is different

No diagonal relation if more than 1 bit diff.

$$AB + \bar{A}B$$

| A \ B       | $\bar{B}$ | B |
|-------------|-----------|---|
| 0 $\bar{A}$ |           | 1 |
| 1 A         |           | 1 |

→ common is B Ans

$$f(A,B,C) = \sum m(0,1,3,5,6,7)$$

$$f(A,B,C) = \sum m(0,1,3,5,6,7)$$

|             | 00 | 01 | 11 | 10 |
|-------------|----|----|----|----|
| 0 $\bar{A}$ | 1  | 1  | 1  | 0  |
| 1 $A$       | 0  | 1  | 1  | 1  |

Common

$$\bar{A}\bar{B} + AC + AB + \bar{A}C$$

$$\bar{A}\bar{B} + AB + (\bar{A} + A)C$$

$$\bar{A}\bar{B} + AB + C$$

Don't Care

→ Combination of inputs on which o/p may or may not depend

$$f(A,B,C) = \sum m(0,1,6,7) + \sum d(3)$$

|             | 00 | 01 | 11 | 10 |
|-------------|----|----|----|----|
| 0 $\bar{A}$ | 1  | 1  | X  | 0  |
| 1 $A$       | 0  | 0  | 1  | 1  |

$$\bar{A}\bar{B} + AB$$

$$\sum m(0,1,2) + \sum d(3)$$

|   |   |   |   |
|---|---|---|---|
| 1 | 1 | X | 1 |
|---|---|---|---|

$\bar{A}$

$$\sum m(0,1,7) + \sum d(3,5)$$

|   |   |   |   |
|---|---|---|---|
| 1 | 1 | X | 0 |
| 0 | X | 1 | 0 |

$$\bar{A}\bar{B} + AC$$

\*

|   |   |   |   |
|---|---|---|---|
| 1 | X | 1 | 0 |
| 0 | 0 | X | X |

✓

$$\bar{A}\bar{B} + \bar{A}C$$

|   |   |   |   |
|---|---|---|---|
| 1 | X | 1 | 0 |
| 0 | 0 | X | X |

$$\bar{A}\bar{B} + BC$$

X

|   |   |   |   |
|---|---|---|---|
| 1 | 1 | 0 | X |
| 0 | 1 | 1 | 0 |
| 1 | X | X | 1 |
| 0 | 0 | 0 | 1 |

$$\bar{A}\bar{B}\bar{C} + BD + A\bar{C}\bar{D}$$

|   |   |   |  |   |
|---|---|---|--|---|
| X |   |   |  | X |
|   |   | 1 |  | X |
|   | 1 | 1 |  | X |
| X |   |   |  | X |

1 quad  
1 pair.

|   |   |   |   |
|---|---|---|---|
| 1 | X | X | 1 |
| X | X |   |   |
|   | X | X |   |
| 1 | X | X | 1 |

$$f(A, B) = \bar{A}\bar{B} + \bar{A}B + AB = \sum m(0, 1, 3)$$

$$= \bar{A} \cdot \bar{B} \cdot 1 + \bar{A}B \cdot 1 + AB \cdot 0 + AB \cdot 1$$

|           |           |     |
|-----------|-----------|-----|
|           | $\bar{B}$ | $B$ |
| $\bar{A}$ | 1         | 1   |
| $A$       | 0         | 1   |

$$\rightarrow \bar{A} \cdot 1 + B \cdot 1$$

$$= \bar{A} + B$$

$$f(A, B, C) = \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}$$

↓ To standard Canonical form

$$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}(\bar{C} + C)$$

$$= \bar{A}\bar{B}C + \bar{A}BC + A\bar{B}\bar{C} + A\bar{B}C$$

|           |                  |            |      |            |
|-----------|------------------|------------|------|------------|
|           | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
| $\bar{A}$ |                  | 1          | 1    |            |
| $A$       | 1                | 1          |      |            |

$$\rightarrow \bar{A}\bar{B} + \bar{A}C$$

|           |           |     |
|-----------|-----------|-----|
|           | $\bar{B}$ | $B$ |
| $\bar{A}$ | 1         | 1   |
| $A$       | 1         | 1   |

$$\bar{A} \cdot C + A\bar{B} \text{ Ans}$$

|           |           |     |
|-----------|-----------|-----|
|           | $\bar{B}$ | $B$ |
| $\bar{A}$ | 1         |     |
| $A$       | 1         | 1   |

function is

$$= \bar{A}\bar{B}C + \bar{A}B + AB\bar{C}$$

$$\downarrow$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C} \rightarrow \bar{A}$$

|           |                  |            |      |            |
|-----------|------------------|------------|------|------------|
|           | $\bar{B}\bar{C}$ | $\bar{B}C$ | $BC$ | $B\bar{C}$ |
| $\bar{A}$ |                  | 1          |      |            |
| $A$       | 1                | 1          |      | 1          |

Boolean Alg

$$= \bar{A}\bar{B}C + \bar{A}B + A\bar{B}\bar{C}$$

$$= \bar{A}\bar{B}C + \bar{A}B\bar{C} + \bar{A}BC + A\bar{B}\bar{C}$$

$$= \bar{B}C + (\bar{A} + A) + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$= \bar{B}C + A\bar{C}(\bar{B} + B)$$

$$= \bar{B}C + A\bar{C}$$

$$\bar{B}C + A\bar{C}$$

or

$$\bar{A}\bar{B}C + \bar{A}B + \bar{A}B\bar{C} + A\bar{B}\bar{C}$$

$$= \bar{B}[\bar{A}C + A] + \bar{A}[\bar{B} + B\bar{C}]$$

$$= \bar{B}[\bar{A} + A] + \bar{A}[\bar{B} + B\bar{C}]$$

$$= \bar{B}[A + C] + \bar{A}[\bar{B} + B\bar{C}]$$

$$= \bar{B}[A + C] + \bar{A}[\bar{B} + B\bar{C}]$$

$$= \bar{B}[A + C] + \bar{A}[\bar{B} + B\bar{C}]$$

$$= \bar{B}C + A\bar{C}$$



|   |   |
|---|---|
| C | 1 |
| 1 | C |

Case-1  $C = 0$

|   |   |
|---|---|
| D | A |
| 1 | 1 |

$\Rightarrow A\bar{C}$

|   |   |
|---|---|
| C | 1 |
| 1 | 0 |

$\Rightarrow \bar{B}C$

$$= A\bar{C} + \bar{B}C$$

POS

$$F(A, B, C) = \prod M(0, 1, 3, 6, 7)$$

$$= (A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)(\bar{A}+\bar{B}+\bar{C})$$

|   |     |     |     |     |
|---|-----|-----|-----|-----|
|   | B+C | B+C | B+C | B+C |
|   | 00  | 01  | 11  | 10  |
| A | 0   | 0   | 0   |     |
| A | 1   |     | 0   | 0   |

$$(A+B+C)(A+B+\bar{C})(A+\bar{B}+C)(\bar{A}+\bar{B}+C)$$

$$(A+B)(A+\bar{C})(\bar{A}+\bar{B})$$

In SOP

|   |    |    |    |    |
|---|----|----|----|----|
|   | BC | BC | BC | BC |
|   | 00 | 01 | 11 | 10 |
| A | 1  | 1  | 1  |    |
| A |    |    | 1  | 1  |

$$F = \bar{A}\bar{B} + \bar{A}C + AB$$

$$= (A+B)(A+\bar{C})(\bar{A}+\bar{B})$$

Implicants - Prime Implicants - Essential Prime Implicants  
 $f(A, B, C) = \sum m(0, 1, 3, 6, 7)$   
 Terms in corner

|   |    |    |    |    |
|---|----|----|----|----|
|   | BC | BC | BC | BC |
|   | 00 | 01 | 11 | 10 |
| A | 1  | 1  | 1  |    |
| A |    |    | 1  | 1  |

3 pair  
 Total implicants = 5  $\rightarrow$  No. of terms  
 $\bar{A}\bar{B} + AB + AC \rightarrow$  reduced POS

|   |    |    |    |    |
|---|----|----|----|----|
|   | BC | BC | BC | BC |
|   | 00 | 01 | 11 | 10 |
| A | 1  | 1  | 1  |    |
| A |    |    | 1  | 1  |

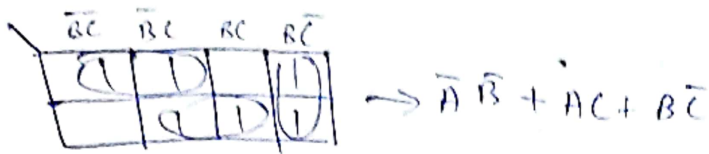
3 pair = 4 pair  
 $\bar{A}\bar{B} + AB + BC$   
 As  $\bar{A}BC$  & 1 can pair in two ways. And both are same

|   |    |    |    |    |
|---|----|----|----|----|
|   | BC | BC | BC | BC |
|   | 00 | 01 | 11 | 10 |
| A | 1  | 1  | 1  |    |
| A |    |    | 1  | 1  |

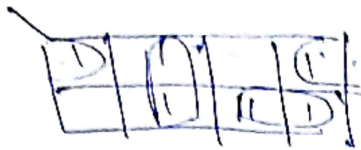
Implicants = 6  $\rightarrow$  EPI =  $\{\bar{A}\bar{B} + AB\}$

PI = 3  $\rightarrow$  EPI =  $\{\bar{A}\bar{B}, C, AB\}$

Total no. of possible ways to form pair is Prime I



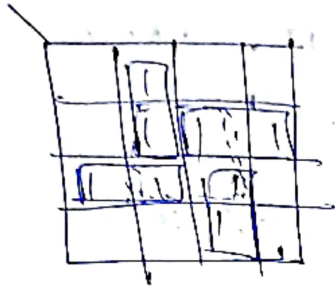
No common.



$$I = PI = 6$$

$$I = 6.$$

$EPJ = 0$ , No term in common



$I = 8$  If only one answer is possible all is ~~one~~  $EPJ$ .

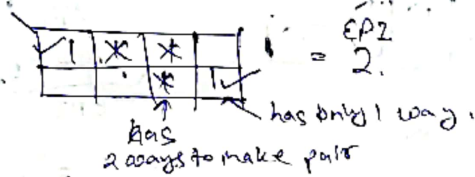
$$EPJ = 4$$

$$PI = 4 + 1 = 5$$

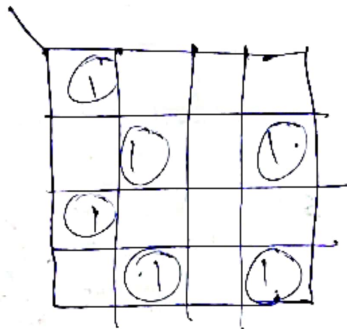
$$\bar{A}B\bar{C} + \bar{A}BC + A\bar{B}C + AB\bar{C}$$

But after making <sup>big</sup> group don't make small group inside

$EPJ$  can also be checked by the no. of 1's can have only 1 possibility to make pair.



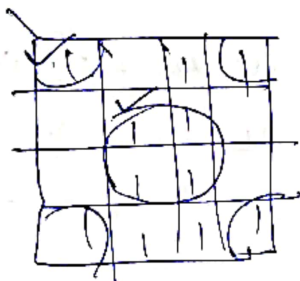
★



$$I = 6$$

$$PI = 6$$

$$EPJ = 6$$

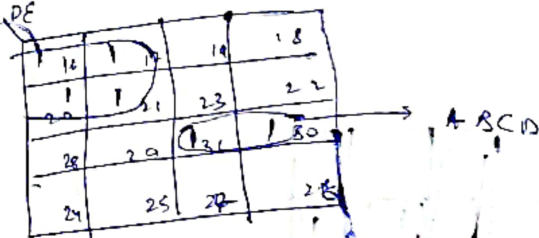
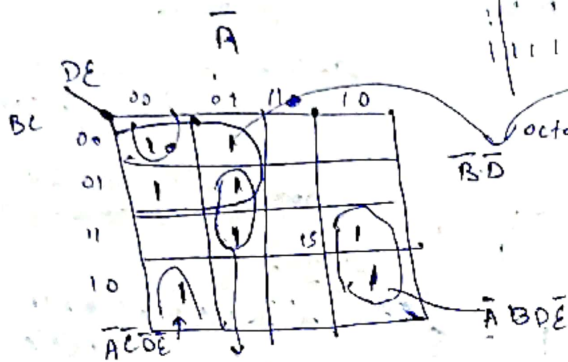


$$\bar{A}\bar{C} + \bar{B}C + AB$$



5 variable K-map.

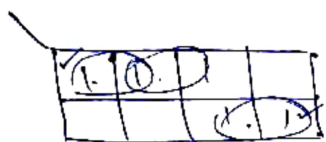
| A | B | C | D | E |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 1 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 |



In octa  
3 variable  
reduced

0, 1, 4, 5, 8, 10, 13, 14, 16, 17, 20, 21, 30, 31

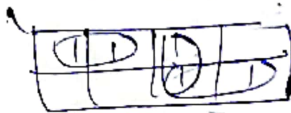
$$\bar{B}\bar{D} + \bar{A}\bar{C}\bar{D}\bar{E} + \bar{A}\bar{C}\bar{D}E + \bar{A}B\bar{D}\bar{E} + ABCD$$



I = 5 P1 = 4 EPI = 2

Selective P1 = 1

Reduced P1 = 1

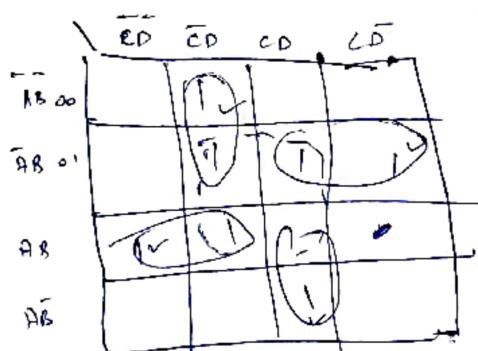


$$\begin{array}{l} \bar{A}\bar{B} + \bar{A}B + \bar{A}\bar{C} \\ \hline \text{EPI} \quad \text{SPI} \\ \bar{A}\bar{B} + \bar{A}B + \bar{A}\bar{C} \\ \hline \text{EPI} \quad \text{SPI} \end{array}$$

$$P1 = \{ \bar{A}\bar{B}, \bar{A}B, \bar{A}\bar{C}, \bar{A}C \}$$

Reduced P1 means  
not common.

SPI is Answer sharing  
2 part EPI & SPI  
common part  
In answer  
but not  
common.



I = 8

P1 = 5

EPI = 4

SPI = 0 No SPI.

RP1 = 1 { BD }

$$\{ \bar{A}\bar{C}\bar{D}, \bar{A}B\bar{C}, \bar{A}B\bar{C}, \bar{A}C\bar{D}, \bar{A}C\bar{D}, \bar{A}B\bar{C}, \bar{A}B\bar{C}, \bar{A}C\bar{D} \}$$

Answer  
also.

# Combinational Circuit

## Steps

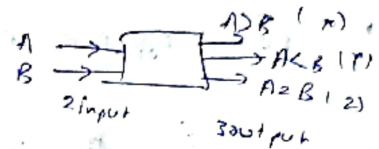
1. Find the no. of inputs & outputs.
2. Write the truth table.
3. Write the Logical Expression.
4. Minimization.
5. Hand write Implementation.

↳ Circuit without feedback or memory.  
↳ Static Circuit.

## 1. Design a one bit Comparator.

Step 2.

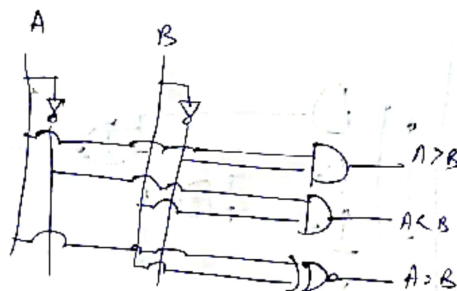
| A | B | $A > B$<br>X | $A < B$<br>Y | $A = B$<br>Z |
|---|---|--------------|--------------|--------------|
| 0 | 0 | 0            | 0            | 1            |
| 0 | 1 | 0            | 1            | 0            |
| 1 | 0 | 1            | 0            | 0            |
| 1 | 1 | 0            | 0            | 1            |



$$X = A \cdot \bar{B} \quad Y = \bar{A} \cdot B \quad Z = \bar{A} \cdot \bar{B} + A \cdot B$$

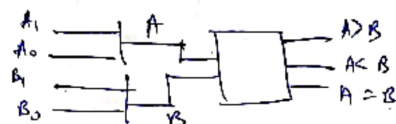
Step 3 No minimization is possible

Σ



## 2. Design a 2 bit Comparator.

| A <sub>1</sub> | A <sub>0</sub> | B <sub>1</sub> | B <sub>0</sub> | $A > B$<br>X | $A < B$<br>Y | $A = B$<br>Z |
|----------------|----------------|----------------|----------------|--------------|--------------|--------------|
| 0              | 0              | 0              | 0              | 0            | 0            | 1            |
| 1              | 0              | 0              | 1              | 0            | 1            | 0            |
| 2              | 0              | 1              | 0              | 0            | 1            | 0            |
| 3              | 0              | 1              | 1              | 0            | 1            | 0            |
| 4              | 1              | 0              | 0              | 1            | 0            | 0            |
| 5              | 1              | 0              | 1              | 0            | 0            | 1            |
| 6              | 1              | 1              | 0              | 0            | 1            | 0            |
| 7              | 1              | 1              | 1              | 0            | 1            | 0            |
| 8              | 1              | 0              | 0              | 1            | 0            | 0            |
| 9              | 1              | 0              | 1              | 1            | 0            | 0            |
| 10             | 1              | 1              | 0              | 0            | 0            | 1            |
| 11             | 1              | 1              | 1              | 0            | 0            | 1            |
| 12             | 1              | 1              | 0              | 1            | 0            | 0            |
| 13             | 1              | 1              | 1              | 1            | 0            | 0            |
| 14             | 1              | 1              | 0              | 1            | 0            | 0            |
| 15             | 1              | 1              | 1              | 0            | 0            | 1            |



$$X = \sum m(4, 8, 9, 12, 13, 14)$$

$$Y = \sum m(1, 2, 3, 6, 7, 11, 14)$$

$$Z = \sum m(0, 5, 10, 15)$$

Total 16 combinations.

Equal values = 4

$$X > Y = X < Y = 16 - 4 / 2 = 6$$

For 4 bits  
1+1+2+2=4

1 0 0 0  
1 0 1 0  
2 1 0 1  
2 1 1 1



Minimize action.

X

|                       | $\bar{A}_0 \bar{B}_0$ | $\bar{A}_0 B_0$ | $A_0 \bar{B}_0$ | $A_0 B_0$ |
|-----------------------|-----------------------|-----------------|-----------------|-----------|
| $\bar{A}_1 \bar{B}_1$ | 0                     | 0               | 0               | 0         |
| $\bar{A}_1 B_1$       | 1                     | 0               | 0               | 0         |
| $A_1 \bar{B}_1$       | 0                     | 1               | 0               | 0         |
| $A_1 B_1$             | 0                     | 1               | 1               | 1         |

$$A_1 \bar{B}_1 + A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0$$

$$X = A_1 \bar{B}_1 + A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 \bar{B}_0$$

Y

|   |   |   |   |
|---|---|---|---|
| 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 1 |
| 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 0 |

$$\bar{A}_1 B_1 + \bar{A}_1 \bar{A}_0 B_0 + \bar{A}_0 B_1 B_0$$

$$Y = \bar{A}_1 B_1 + B_0 \bar{A}_0 (\bar{A}_1 + B_1)$$

~~Z~~ =

Z

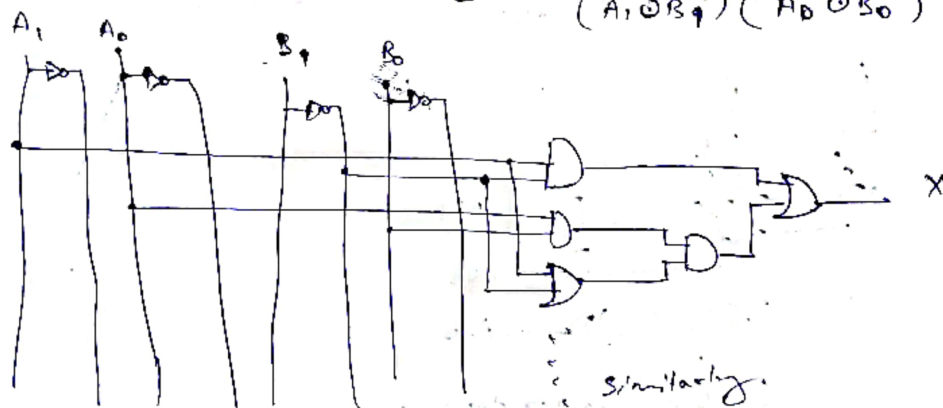
|   |   |   |   |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

$$\rightarrow \bar{A}_1 \bar{A}_0 \bar{B}_1 \bar{B}_0 + \bar{A}_1 \bar{A}_0 \bar{B}_1 B_0$$

$$+ \bar{A}_1 A_0 B_1 B_0 + A_1 \bar{A}_0 B_1 \bar{B}_0$$

$$\rightarrow \bar{A}_1 B_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0) + A_1 B_1 (\bar{A}_0 \bar{B}_0 + A_0 B_0)$$

$$Z = (A_1 \oplus B_1) (A_0 \oplus B_0)$$



Short cut 1

|   |   |   |   |
|---|---|---|---|
| 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 |
| 0 | 0 | 0 | 1 |

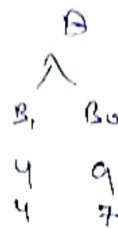
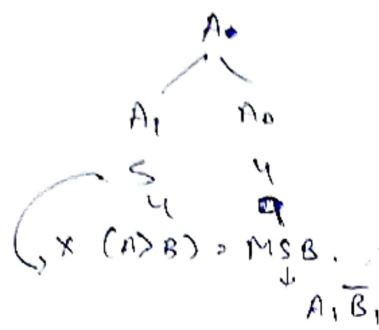
$$A_1 \bar{B}_1 + \bar{A}_1 A_0 \bar{B}_1 \bar{B}_0 + A_1 A_0 B_1 \bar{B}_0$$

$$\Rightarrow A_1 \bar{B}_1 + (A_1 \oplus B_1) A_0 \bar{B}_0 \rightarrow X \underline{A > B}$$

Similarly  $Y = \bar{A}_1 B_1 + (A_1 \oplus B_1) \bar{A}_0 B_0$



# Main Short-cut:



★ when smaller bit is given

$$4 > 2$$

$$A > B$$

$$n > 2$$

MSB = MSB than LSB's

$(A_1 > B_1) (A_0 > B_0)$  must be smaller than - must be smaller.

$$Y(A < B) = \overline{A_1} B_1 + (A_1 \odot B_1) (\overline{A_0} B_0)$$

$$52 < 69$$

$$A_1 > B_1$$

$$\therefore \overline{A_1} B_1$$

example

$$42 < 47$$

$$2 > 7$$

$$A_0 > B_0 \therefore \overline{A_0} B_0$$

$$Z(A = B) = 42 = 42$$

$$\therefore (A_1 \odot B_1) (A_0 \odot B_0)$$

For 3 bit.



$$A > B = A_2 \overline{B_2} + (A_2 \odot B_2) A_1 \overline{B_1} + (A_2 \odot B_2) (A_1 \odot B_1) A_0 \overline{B_0}$$

# 1 bit comparator.

Total combinations = 4

Equal condition = 2

Unequal = 2

Greater = Less condition = 1

2 bit

16

4

12

6

3 bit

64

8

56

28

n bit

$2^{2n}$

$2^n$

$2^{2n} - 2^n$

$(2^{2n} - 2^n) / 2$

★ A → 3 bit B → 2 bit

A > B?

| A <sub>2</sub> | A <sub>1</sub> | A <sub>0</sub> | B <sub>1</sub> | B <sub>0</sub> |
|----------------|----------------|----------------|----------------|----------------|
| 0              | 0              | 0              | 0              | 0              |
| 0              | 0              | 0              | 0              | 1              |
| 0              | 0              | 0              | 1              | 0              |
| 0              | 0              | 0              | 1              | 1              |
| 0              | 0              | 1              | 0              | 0              |
| 0              | 0              | 1              | 0              | 1              |
| 0              | 0              | 1              | 1              | 0              |
| 0              | 0              | 1              | 1              | 1              |
| 0              | 1              | 0              | 0              | 0              |
| 0              | 1              | 0              | 0              | 1              |
| 0              | 1              | 0              | 1              | 0              |
| 0              | 1              | 0              | 1              | 1              |
| 0              | 1              | 1              | 0              | 0              |
| 0              | 1              | 1              | 0              | 1              |
| 0              | 1              | 1              | 1              | 0              |
| 0              | 1              | 1              | 1              | 1              |
| 1              | 0              | 0              | 0              | 0              |
| 1              | 0              | 0              | 0              | 1              |
| 1              | 0              | 0              | 1              | 0              |
| 1              | 0              | 0              | 1              | 1              |
| 1              | 0              | 1              | 0              | 0              |
| 1              | 0              | 1              | 0              | 1              |
| 1              | 0              | 1              | 1              | 0              |
| 1              | 0              | 1              | 1              | 1              |
| 1              | 1              | 0              | 0              | 0              |
| 1              | 1              | 0              | 0              | 1              |
| 1              | 1              | 0              | 1              | 0              |
| 1              | 1              | 0              | 1              | 1              |
| 1              | 1              | 1              | 0              | 0              |
| 1              | 1              | 1              | 0              | 1              |
| 1              | 1              | 1              | 1              | 0              |
| 1              | 1              | 1              | 1              | 1              |

2<sup>5</sup> combination = 32

→ compare between A<sub>1</sub>, A<sub>0</sub> & B<sub>1</sub>, B<sub>0</sub>

Basically 2 bit comparator.

combination 2<sup>4</sup> = 16

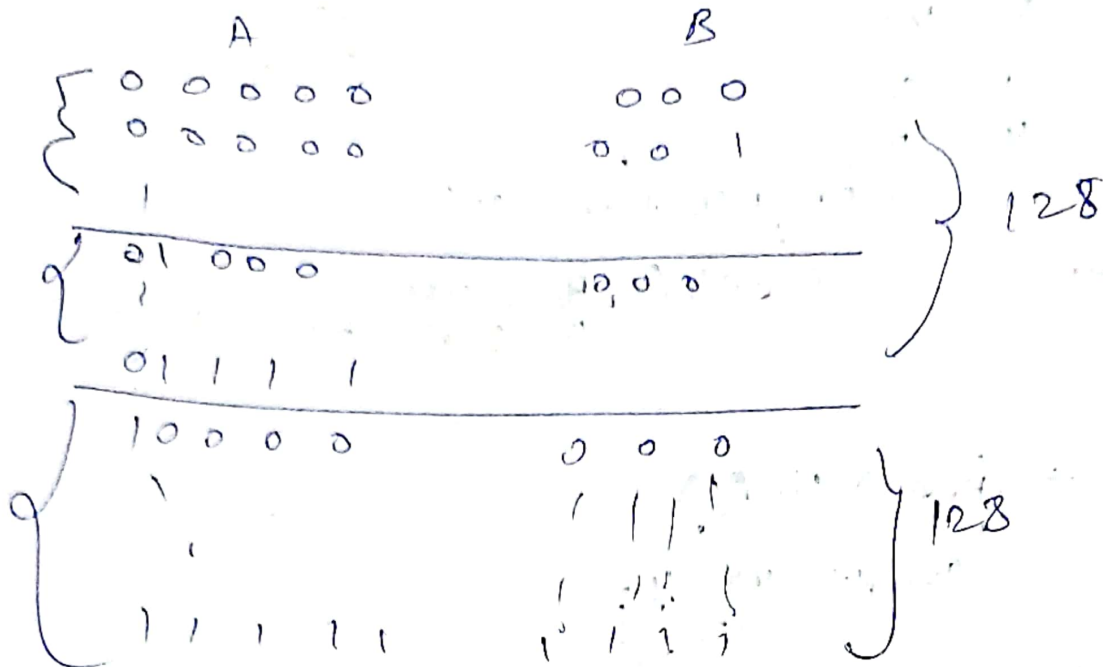
equal = 2<sup>2</sup> = 4

∴ A > B = (16 - 4) / 2 = 6

Always A > B

∴ A > B = 16 - 4 = 12

# A → 5 bit  
B → 3 bit



Total combination  $2 \times 2^{5+3} = 256$

$A_4 > B_4 \rightarrow 256/2 = 128$

$A_3 > B_3 \rightarrow 128/2 = 64$

Now  $A_2 A_1 A_0 > B_2 B_1 B_0$

Basically 3 bit comparison

$\therefore$  Total combo  $= 2^{3+3} = 2^6 = 64$

Equal  $2^3 = 8$

$A > B = (64 - 8) / 2 = 56 / 2 = 28$

$\therefore A > B$   $128 + 64 + 28 = 220$