CW

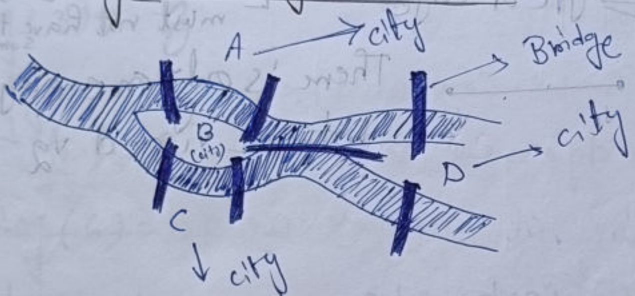
Discrit mathematics

20/7/22

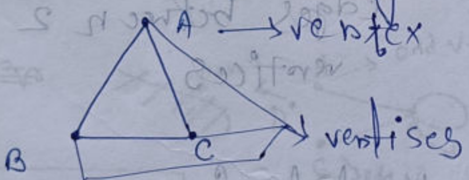
• Graph Theory

Pictorial form of any real life cinario.
The graph is a cinarico which we draw on the paper to understand the real life cinareo.

• Koinsberg 7 bridge Problem



Graph is denoted by → $G = (V, E)$



$V = \{A, B, C\}$

$E = \{\{A,B\}, \{B,C\}, \{C,A\}\}$

'V' is set of vertices

'E' is edgs (edge is something connect 2 vertics)

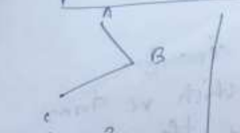
[$\{A, B\} \Longrightarrow$ one edge]

[There is an edge between A and B]

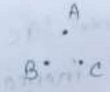
• No. of vertics of any graph is called order of a graph

$\boxed{|V| = \text{order of a graph}}$

- $\boxed{\text{No. of edges} = |E| = \text{size of graph}}$



order = 3
size = 2

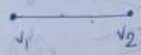


order = 3
size = 0

[It is not necessarily that if the order of a graph is 3, it's size must have to be 3]
[i.e → order and size must not have to be same]

*** Type of edge

Direct edge



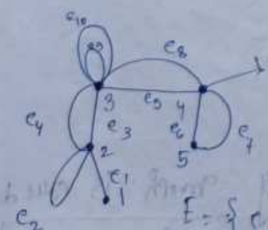
- There is only one edge between v_1 and v_2

→ Parallel edges

more than one edges between 2 vertices

→ self loop

edge that starts from one point(v) and ends in the same point(v)



vertex

order = 5
$V = \{1, 2, 3, 4, 5\}$
size = 10
$E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$

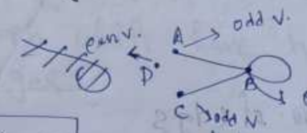
- Degree of any vertex

Representation [$deg(v_i)$ [not fixed, we can use our own choice]]

$deg(v_i)$ = No. of edges incidenting on v_i.

$deg(1) = 1$
$deg(2) = 5$ [deg of 1 self loop = 2]
$deg(3) = 8$
$deg(4) = 4$
$deg(5) = 2$
$deg(6) =$

vertices's degree type → odd degree vertices (odd vertex)
→ even " " " (even vertex)



$\boxed{deg(v_i) = 0}$ → Isolated vertex

$\boxed{deg(v_i) = 1}$ → Pendent "

- Theorem

▷ For any graph $G = (V, E)$ summation of degrees of all the vertices are even.

Proof. We know deg of any vertex is no. of edges that is incidenting on the vertex

Now,

[optional-Example]

Type	Contribution in total deg
1 single edge	2
1 parallel edge	2
1 self loop	2

So, from this we can understand that for every one edge we have total 2 degrees. In summation of degrees we get 2 for every single edge.

$\therefore$ Sum of degrees of all the vertices $= 2 *$ No. of edges

$$\therefore \sum_{i=1}^{n} \deg(v_i) = 2 * \text{No. of edges}$$

From this we conclude that summation of all degrees of all vertices are even.

2] For any graph $G = (V, E)$ no. of odd vertices will be always even.

Proof: We know $\sum_{i=1}^{n} \deg(v_i) = \text{even}$

$$\Rightarrow \sum_{i=1 (\text{even})}^{n} \deg(v_i) + \sum_{i=n (\text{odd})}^{n} \deg(v_i) = \text{even}$$

$$\Rightarrow \text{even} + \sum_{i=1 (\text{odd})}^{n} \deg(v_i) = \text{even}$$

$$\Rightarrow \boxed{\sum_{i=1 (\text{odd})}^{n} \deg(v_i) = \text{even}}$$

~~the degree of~~ The summation of all odd deg vertices is even, when the no. of all odd deg vertices are even.

So, we can say that no. of odd deg vertices will be always even. [As we can get even value by the summation of even no. of odd values]

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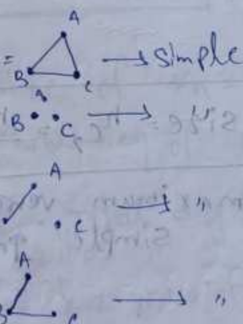
Types of graphs – i) simple graph (No self loop, No parallel edges)

ii) A graph $G = (V, E)$ is called simple graph which have no self loop, no parallel edges.

order 3, G_3 =



→ simple graph

G_n = n = order

for normal graph

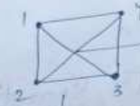
in one graph (simple) both 0 and $n-1$ deg is not possible

for $n = 3$
min edge $= 0$
max " $= 3$

Deg. of every vertex for min $= 0$ to
Deg. of every vertex for max $= n-1$

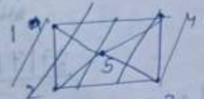
G_4 =  → simple

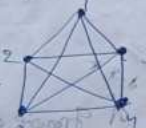
for n = 4
min edge = 0
max = 6

 this is not vertex

max edge = 6

• what is the max and min size possible for a simple graph of order n.

G_5 = 

max edge 10 ← 

① ** simple graph

for order n

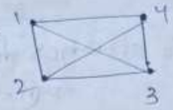
min size = 0

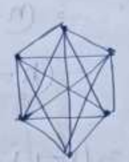
$\boxed{\text{max size} = {}^nC_2 = \frac{n(n-1)}{2}}$

② Complete graph (maximum version of simple graph)

Denoted by K_n

Ex = K_3 = 

K_4 = , K_5 = 

K_6 : 

$\boxed{\text{Deg of every vertex in } K_n = n-1}$

G = (V, E)

A graph where there is no self loop, no Parallel edges and there are every possible edges. This is called complete graph. [Every vertex is connected with all the remaining vertices by single edge]

• $\boxed{\text{Size of } K_n = {}^nC_2}$

$\frac{5 \times 4}{2}$

Theorem.3} Size of $K_n = \frac{n*(n-1)}{2}$

Proof: we know deg of each and every vertex for K_n

1st type ⇒ is $n-1$ ∴ Total deg = $n \times (n-1)$

⇒ $2 \times$ No of edges = $n \times (n-1)$

∴ No of edges = $\frac{n \times (n-1)}{2}$

2nd type → For 1st vertex no of edges drawn → $(n-1)$

For 2nd ... → $(n-2)$

For 3rd ... → $(n-3)$

...

For nth ... $= (n-n) = 0$

$0 + 1 + 2 + \dots + (n-1)$
$= \frac{n(n-1)}{2}$

③ Regular graph – If degree of all the vertices in that graph $G = (V, E)$ are same.

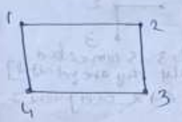
- Every complete graph is regular graph
- All regular graphs are simple graph — False

Ex–

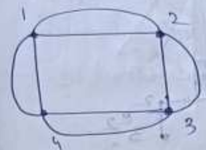
→ Regular but not simple

[Regular graph can contain any type of loop, but simple g. contains only direct edge]

As we know that regular is graph is that which degree of all the vertices are same. But there in this graph ~~must have~~ should not be present any self loop or parallel edges. If they are present they the graph is not simple graph.



→ ① Simple graph
② Regular graph [∵ All deg are same]
③ but not complete graph



→ ① Regular graph
② But not simple and not complete also

- All regular graph is not complete.

i) A graph of order 4 might have degrees 0, 2, 3, 4 — False [∵ 1 odd deg vertices is not possible]

ii) A simple graph of order 4 might have degrees like (0, 1, 2, 3) — False [∵ In 1 graph both max and min degree present can not be possible]

iii) A simple graph of order 6 might have degrees like (0, 0, 0, 1, 2, 5) — False

[∵ As both max and min deg can not be present in the same graph]

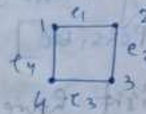
• ~~Bipartite graph~~

[1,3 is connected but they are not adjacent]

vertex adjacency:- v_1 and v_2 are adjacent when they share atleast one common edge.

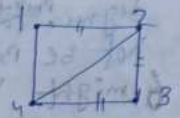
• edge adjacency:-

e_1 and e_2 are adjacent when both the edges / they meet at common point.



e_4 and e_2 are not adjacent but $\{e_1,e_2;\ e_2,e_3;\ e_3,e_4;\ e_4,e_1\}$ are adjacent

(4) Bi-Partite graph



X ⇓

this is not bipartite graph

$V = \{1,2,3,4\}$

$V_1 = \{1,3\}$

$V_2 = \{2,4\}$

they are adjacent. we can not place them both in V_2

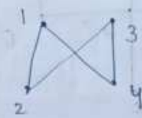
⇓

This is a bi-partite graph

$\{1,3\}$

$\{2,4\}$

Same graph

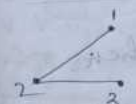


1, 3 - no adjacency / no edge

1, 2 - sharing edge

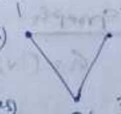
Same set - adjacency ⇓ Not possible

different set adjacency ⇓ Possible

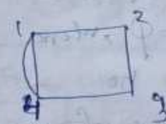


$= \{1,3\}$ (2 vertices)

$= \{2\}$ (1 vertices)

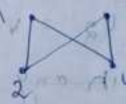


⇒ Bipartite

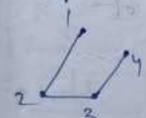


$= \{1,3\}$

$= \{2,4\}$

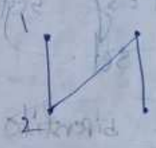


⇒ Bipartite [Parallel or self loop will be included]

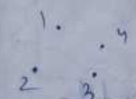


$= \{1,3\}$

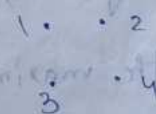
$= \{2,4\}$



⇒ Bipartite



⇒ Bipartite



• There is no need that there must have an adjacency between the vertices of different sets.

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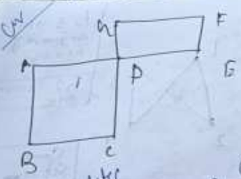
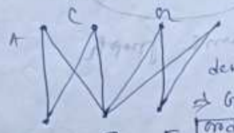


$V_1 = \{A, C, G, E\}$

$V_2 = \{B, D, F\}$

m = the no of vertices in 1st set
n = " " " 2nd "

Bipartite



denoted by $G_{m,n}$

$\boxed{\text{order} = m+n}$

$\Rightarrow G_{4,3}$

inseem Bipartite

Bipartite graph

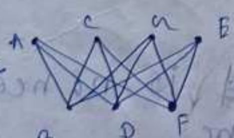
A Graph $G = (V, E)$ is called bipartite when we can divide V in V_1 and V_2 such that $V_1 \cup V_2 = V$ and $V_1 \cap V_2 = \phi$, where ~~vertices~~ in any set there is no adjacency between the vertices of same set.

⑤ • Complete bipartite graph

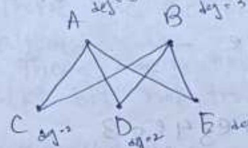
Denoted by $K_{m,n}$

This is the maximum version of bipartite graph

$K_{4,3}$ (↑ diagram) $\Rightarrow$ Complete Bipartite



$K_{2,3}$ =



$\boxed{\text{Orders of } K_{m,n} = m+n}$

$\boxed{\text{Size of } K_{m,n} = m \times n}$

$\boxed{\text{Deg of vertices in } K_{m,n} \text{ either } m \text{ or } n}$

$\boxed{\text{max deg of any vertex in } K_{m,n} = \max(m,n)}$

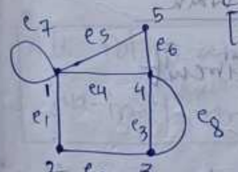
$\boxed{\text{There are no parallel edges or self loop in B.G}}$

$K_5 \Rightarrow$ edges / size $= {}^5C_2 = 10$

[There should be all possible edges between the V_i of different sets]

• **Walk** [The move in graph is walk]
[It is a unit] [starts with a vertex ends with a vertex]

[Representation - vevevev]



adjacent

$v_1 \; e \; v_2 e v_3 e \; v_4$

① v_1 and v_2 should be adjacent

② v_2 " v_3 " " "

③ v_3 " v_4 " " "

Points

i) It starts with v and end with v

ii) Every 2 consecutive vertex are adjacent.

iii) Every 2 consecutive edges are adjacent.

iv) It is a combination of vertex, edge – vertex, edge →

Ex – ① $2 e_2 3 e_3 4 e_8 3$
this is correct walk.

parallel edges have 2 adjacent points.

② $1 e_7 1 e_4 4 e_2 2$
This is not a correct walk.

• walk
→ open walk [This will starts at one vertex and ends with the other vertex]
(Ex – $2 e_1 1 e_4 4 e_3 3$)
→ closed walk [This will starts at one v and ends with the same v]
(Ex – $1 e_4 4 e_6 5 e_5 1$)

• open walk
→ Trail
→ Path

• closed walk
→ Circuit
→ cycle

• The closed version of trail is called circuit.
• The closed version of path is called cycle.

• Condition to be trail :– ① There are no edge repetation allowed.
② There ~~are~~ is no restriction in v repetation in trail.

Ex :– i) open walk but not trail
$1 e_4 4 e_6 5 e_5 1 e_4 4$
ii) $1 e_7 1 e_4 4 e_3 3 e_8 4$ → open walk and trail

• Condtition to be Path
① There is no vertex repetation allowed.
② self loop, Parallel edge will not come in the path.

Ex – (i) not a trail not a path
– $1 e_4 4 e_6 5 e_5 1 e_4 4$
(ii) Not a path → $1 e_7 1 e_4 e_3 3 e_8 4$

All paths are trail but all trails are not path

Where there is no v. repetation there will be also no. e repetation

(iii) ~~$3 e_3 4 e_4 1 e_1 2 e_3 3$~~
$3 e_3 4 e_8 3 e_2 2$
This is trail but not path

• the condition to be cycle—

① Same v in start and end but no same repetition in between start and end vertex.

② As well as no edge repetition between start and end vertex (not compulsory)

③ Self loop can not come.

④ Set Parallel path can not come

• the condition to be circuit—

① Self loop can come, as if there is any v repetition ~~between~~ then this will not make a problem.

② No edge repetition allowed.

Ex — i) $3\,e_3\,4\,e_8\,3\,e_3\,4\,e_8\,3$ → not cycle

ii) $1\,e_7\,1$ → cycle

including $1e_71$ write a cycle—

iii) $3e_3\,4\,e_4\,1\,e_7\,1\,e_1\,2\,e_2\,3$ → Circuit, Not a cycle

• where there are no self loop and parallel edge then we can write the walk as

2 3 4 1 2

• All cycles are circuit but all circuits are not cycle

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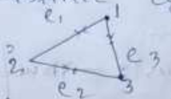
⑥ • Cyclic graph (C_n)

The graph $G=(V,E)$ which contains one distinct cycle is called cyclic graph.

diff $K_{4,3}$	$G_{4,3}$
size = 2	size = v − 1 2
1 possible	more than 1 possible

C_3



→ this is cyclic graph

[n = order]

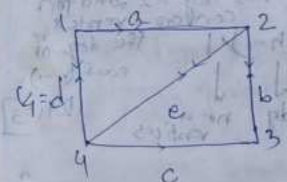
$1e_1\,2\,e_2\,3e_3\,1$ → one cycle

$1e_3\,3e_2\,2\,e_1\,1$ → one cycle

$2e_1\,1e_3\,3\,e_2\,2$ → one cycle

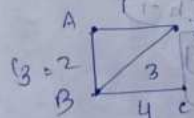
$2\,e_2\,3\,e_3\,1\,e_1\,2$ → one cycle

Total 6 cycles



more than one distinct cycle → not a cyclic graph

Here 3 distinct cycles [▷, △, □]. So it is not a cyclic graph



A 2 B 4 C → A

v repetition not allowed

[G_4 subgraph → C_3]

[Pendent vertices are not cyclic graph]

C_5 , C_2 , C_1

$\boxed{\text{deg of every vertex in } C_n \text{ equals } 2 \text{ — True}}$

each and

$\boxed{\text{Size of } C_n = 2 \times n}$

Q. Wheel graph (W_n):-

$W_n \Rightarrow C_{n-1}$ + adding all vertices from centre vertex,

$W_4 \Rightarrow C_{4-1} \Rightarrow C_3 =$  wheel graph

$W_5 \Rightarrow C_4 =$ 

[As C_n has deg = 2, and for center vertex the deg of each $(n-1) = 2+1 = 3$]

total degrees of W_n = $\underbrace{3}_{\text{deg}} \times \underbrace{(n-1)}_{\text{no. of vertices}} + \underbrace{(n-1)}_{\text{no. of vertices}} \times \underbrace{1}_{\text{deg}}$

$\boxed{\text{deg of } W_n = 4(n-1)}$

Size of W_n = total deg of vertices of W_n = $4(n-1)$

$\Rightarrow 2 \times$ Size of ~~the~~ $W_n = 4(n-1)$

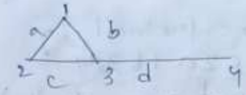
$\Rightarrow \boxed{\text{Size of } W_n = 2(n-1)}$

(8). Euler graph

[The circuit that contains all the edges of this graph]

If there is a cyclic graph it must be circuit

euler circuit including all edges



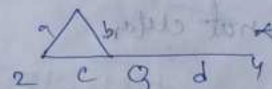
2 c 3 b 1 a 2 ⟶ circuit

- When a graph is called euler graph, if there is atleast one euler circuit of the graph.
- The euler circuit is that which includes all the edges.

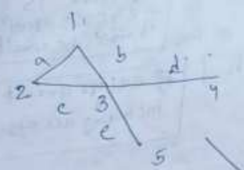
 ⟶ This is not euler graph (as this is not connected graph)

A connected graph in which there is atleast one euler circuit, that graph is called euler graph — Euler graph

- Semieuler graph

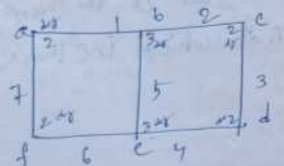
 ⟵ 4 d 3 c 2 a 1 b 3 (trail, connected euler trail)

↘ As this graph contains only euler trail, no eulerian circuit is called semi euler graph.



(trail, euler trail),
5e 3c 2a 1b 3d4
(but not euler circuit)

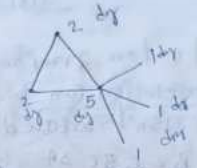
→ semieular graph



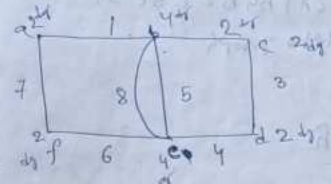
Semieular graph
. not euler

How to identify that this graph is euler graph or it is a normal graph —
(short cut)

no. of odd deg vertices	graph type
> 2	not semieular, not euler
= 2	semieular, not euler
0	euler

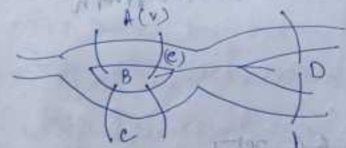


→ no euler trail

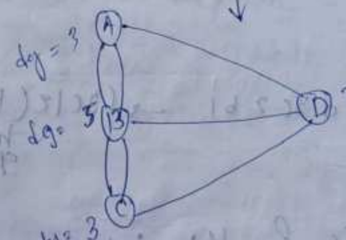


→ euler graph

~~b 1 a 7 f 6 e 4~~   e 5 b 1 a 7 f 6 e 8 b 2 c 3 d 4 e



graphical representation

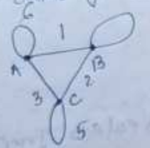


⟹ not a euler or semieular graph

We can not cover all the bridges by at a ~~only~~ one time ⟹ there must be bridge repitation.

4|8|22

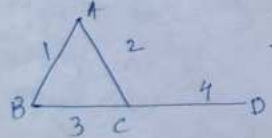
~~All regular graphs are not eulor graph.~~
~~All regular graphs are~~ edge repetition can be possible.
As regular g can be edge repetetion isn't allowed.
But in eulor g. edge repetetion isn't allowed.



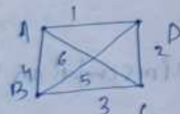
A 6 A 1 B 4 B 2 C 5 C 3 A
⊢ eulor graph

• ~~Hamiltonial graph~~

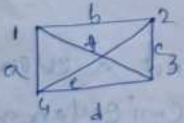
(Hamiltonial cycle) ⟶ Including all vertex
↓
no veptex repetation



⟶ not hamiltonial graph

K_4 = 

~~B 4 A 6 C 3 B 5~~ 1

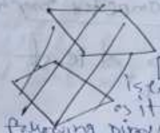


1 a 4 d 3 c 2 b 1 ⟶ cycle (Hamiltonial cycle)

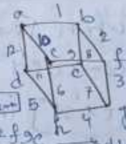
• Every cyclic graph is hamiltonial graph
• cyclic g. don't follow dirac's theorem though it is hamiltonial graph

1) All complete graphs are eulor graph — False
2) All complet graphs are hamiltonial graph — True (As it is cycle complete g. so we can get atleast one cycle from here)
hamiltonial cycle

• K_4 = by applying dirac's theorem ⟶ this is hamiltonial graph;  C_{42} ⟶ hamiltonial but not satifies dirac's theorem as it is cyclic graph



[But it is exceptional as it is not following Dirac's, ore's theorem]



⟶ this is not eulor graph

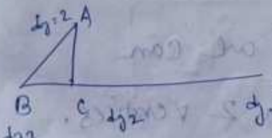
e 6 h 4 g 7 c 11 d 12 a 1 b 2 f 9 e ⟶ Hamiltonial cycle (Hamiltonial graph)

• 1) Dirac's Theorem –
G_n would be hamiltonial if deg of each and every vertex will be $\frac{n}{2}$, where n is the order of the graph.
atleast

G_n would be hamiltonial when deg of each and every vertex will be atleast $(n/2)$

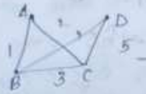
If all the vertices are connected to one another by only one edge – complete g.

• 2] ore's theorem –
$$dg(v_i) + dg(v_j) \geq n$$
where v_i and v_j are not adjacent.



$dg(B) = 2$
$dg(D) = 1$
$2 + 1 \leq 4$
not hamiltonial

Applying dirac's theo



→ Hamiltonial graph

All v deg are 2/3 · [$\geq n/2$, $n=4$]

- TSP – requirement → shortest hamiltonian cycle.

(10) • ~~Parallel~~ graph connected and disconnected graph

Connected – If when there is at least one path [∃ – there exist] in between every pair of vertices chosen from the graph.

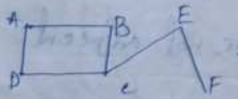
A g. is connected graph

→ Not connected

∴ → disconnected graph

A 3 C
A 1 B 2 C

[Disconnected graph is a collection of some ~~connective graph~~ components]



→ Connected

If graph is cyclic then we can draw 2 paths between 2 vertices.

[∴ Every vertex has 2 direction]

→ component / connected graph

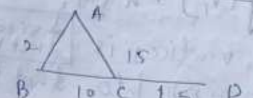
component

component

[Component it self is a connected graph]

[Hamiltonil, euler graph must be connected graph]

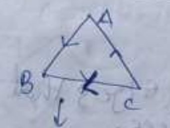
- (11) weighted graph



If we assign values over edges then weighted, if not not weighted.

(12) [A directed g. also called a digraph, is a graph in which the edges have a direction. This is usually indicated by an arrow on the edge.]

- Di- graph ⊂ Directed graph



not cyclic

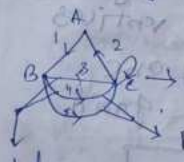
V = {A, B, C}

E = {{A, B}, {C, B}, {C, A}}

[A is incidenting on B]

[A is adjacent to B]

[not B is adjacent to A]



→ cyclic graph

Parallel

not parallel

E = {1, 2, 3, 4, 5}

Degree (Here) In deg – no. of edges that is coming into that v.

~~In deg~~ Out deg – " " " out " .

In deg (A) = 1   outdeg (A) = 1   [Here contribution of self loop = 2 (1 for in, 1 for out) i.e. deg of 1 self loop here = 2]

In deg (B) = 2   outdeg (B) = 2

In deg (C) = 3   Out deg (C) = 3

- $\sum_{i=1}^{n} \text{indeg}(v_i) = \sum_{i=1}^{n} \text{out deg}(v_i) = n$ (?) [size atleast n/2]

we know that the in deg vertices is that vertices the no of edges that comming into.

	In deg contribution	out deg contribution
1 single edge →	1	1
1 parallel edge	1	1
1 self loop	1	1

out deg vertices is the no. of edges that comming out from that vertices.
we know that the summation of in deg vertices and the summaction of out deg vertices are both equal, and they both are equal to the total no. of edges of the graph.

$\therefore$ ~~Sum of indegs of all v. / Sum of out degs of all v.~~ = ~~n~~/2×

we know also that, sumetion of degs of all v = 2 × no of edgs.

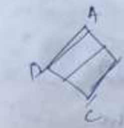
$\therefore \sum_{i=1}^{n} \text{in deg}(v_i) / \sum_{i=1}^{n} \text{out deg}(v_i) = 2 \times$ no of edgs

$\Rightarrow n = 2 \times$ no. of edgs [n = total no. of edgs]

$\Rightarrow$ no. of edges = n/2 [$\because$ Total no. of edges must be atleast n/2]

-  → simple digraph

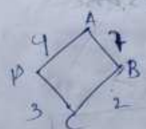
If K_n is a digraph, then size = n(n−1)
[As 2 directions are possible]



- Bipartite graph is not digraph

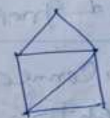
$K_{2,3}$ = 

- the cycle that existing in $K_{m,n}$ have the length atleast 4 or more than 4, no odd length cycle will exist here.



A 1 B 2 C 3 D 4 A

length = 4



→ min length = 3
→ max length = 5

[K_n must have atleast 1 cycle]

- $K_{m,n}$ must contain cycle, but there is no odd length cycles.

10/8/22

No need

1) There are 3 v and there are 2 components the min edge/size, max edge/size?

$n=3$, $K=2$

if $n=3$
$K=2$
min size = 1
max size = infinite

[for drawing 2 components we need a edge]

$n=4$
$K=2$
min size = 1
max size = $\frac{n-k}{2}$ ∞ [if there are n no of v and K components are here]

$\frac{(n-k)(n-k+1)}{2}$

for simple disconnected graph

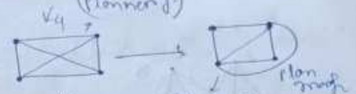
simple disconned graph of orders n and k components - max size = $\frac{(n-k)(n-k+1)}{2}$

Same.ps
① consider a simple g. G with K components. If each component has $n_1, n_2 \ldots n_k$ vertices, then max num # edges in G is —?

⑬ Planar graph

~~When~~ A graph is planner when a plan g is exist there, i.e. when that that can be converted into planers graph.

• Plan graph - A graph where there are no edges crossing, but obey all the properties of planners graph, from which it is converted.



V_4 (planner g.) → plan graph (isomorphic graph)

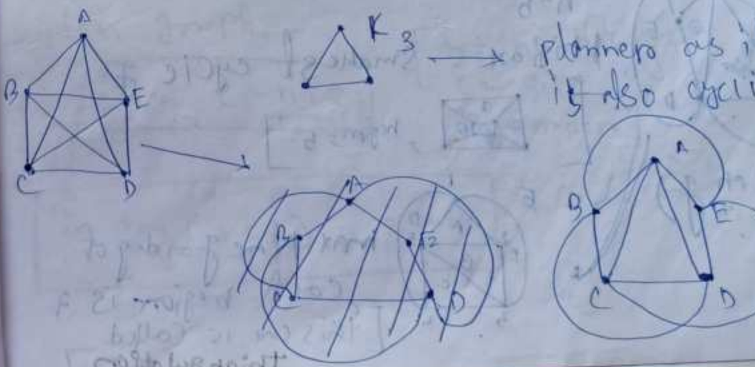
• Not planner g → isnt plan graph
Those g. which are not planner, it will not have any plan graph also.

• Planner graph — that can be convertable into plane graph.

(planners graph) (plan g.)

$n = 5 - 4 + 2 = 3$

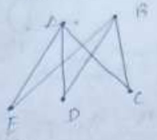
• Cyclic g. are plan as well as planners.
• wheel g. are also always plane graph.

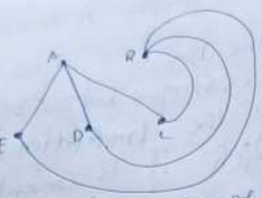


K_3 → planners as it is also cyclic graph

K_5 is not a planner graph

$K_{2,3}$ =

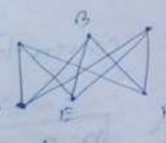


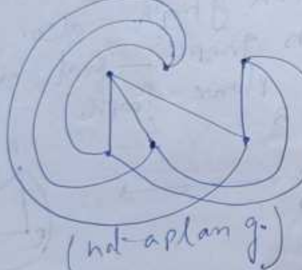


(plan graph)

∴ planner graph

$K_{3,3}$ =

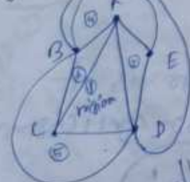




(not a plan g.)

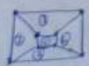
not a planner g

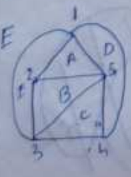
n = 1, size = 0 ⟶ trivaial graph
where there exist 1 vertices and, no edge exist in this graph.



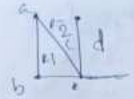
(plan g)

n = 5
1 region = smallest cycle g

[ , region = 5]



max plane g = deg of each region is 3
[this one is called triangulation]

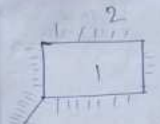


⟶ region = 2 (outer area is also called region)

~~deg of any region~~

deg (reg 1) = 3 [How many edges that forming that region]

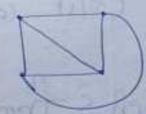
deg (reg 2) = 5 [outer edge will contribute 2 for degree]



⟶ deg(2) = 6
deg(1) = 4

for K_4 max plane g = plan g

n = 4,



⟶ max plane g ⟶ [no edges are left ~~form~~]
⟶ triangulation [deg of each and every reg = 3]

$p = {}^4C_2 - 4 + 2$
$= 4$ no of regions

• Eular's formula for no. of region in any plan graph.

$$\boxed{r = e - n + 2}$$

[r = no. of regions
e = size
n = order]

$$\boxed{\sum_{i=1}^{n} \deg(\text{reg}\, i) = 2 * e}$$

Pendent edges if the Pendent edge is in inner region –

$p = 4 - 4 + 2$

$dg(2) = 3$

$dg(1) = 5$

For triangulation – $\sum_{i=1}^{n} dg(reg\ i) = 2 \times e$

$\Rightarrow 3 \times p = 2e$

$\Rightarrow 3(e - n + 2) = 2e$

$\Rightarrow e = 3n - 6$

For max plan $g = \boxed{e = 3n - 6}$ (condition)

[Plane graph which exist continue max plane graph]

For the smaller graphs from max g – plan

$\boxed{e \leq 3n - 6} \rightarrow$ For planner g. (condition)

$C_3 = 3 \leq 3$, $e \leq 3$

$3 \leq 3 \rightarrow$ this is triangulation also

$5 \leq 6$ but it is not triangulation

(below g case) Smaller than max

K_5, $e = \binom{5}{2} \leq 3 \cdot 5 - 6$

$10 \leq 9 \rightarrow$ not a Planner g.

$K_{2,3}$, $6 \leq 5 \times 3 - 6 = 9 \rightarrow$ Planner g.

$K_{3,3}$, $9 \leq 6 \times 3 - 6 = 12 \rightarrow$ not Planner

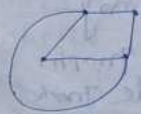
- K_n is planner for $n \leq 4$
- $K_{m,n}$ is not planner if both $m, n \geq 3$ [or $m \neq n \geq 3$]

$K_{2,200}$ = planner

$K_{1,200}$ = planner

$K_{3,2}$ = planner g

- There is no triangulation in bipartite g.



Rectangulation

no C_3, can have C_4

- $\boxed{\sum_{i=1}^{n} dg(reg\ i) = 2 \times e}$ → For bipartite graph (only applicable for those graph where there is no odd length cycle)

$\Rightarrow 4 \times p = 2e$

$\Rightarrow 2[e - n + 2] = e$

$\Rightarrow \boxed{e = 2n - 4}$

$K_{2,3}$, $6 \leq 2 \times 5 - 4$

$6 \leq 6 \rightarrow$ Planner as well as rectriangulation steps for recognizing a planner graph

1) find C_3 / C_4 in that graph

2) i) If you can find C_3 in this random graph then apply formula $e \leq 3n-6$ [n = order, e = size]; $e \leq 3n-6$ is the case of traingulation, i.e. if $e = 3n-6$ it is max plane graph (~~that has~~ the deg of the all regions of max plane graph ≥ 3). If $e < 3n-6$ it is the case for below traingulation (i.e they are planner, but not have ~~all~~ deg of all regions ≥ 3)

ii) If you can find C_4 in this random graph then apply formula $e \leq 2n-4$ [n = order, e = size], $e = 2n-4$ is the case of ~~traingulatio~~ rectraingulation i.e if $e = 2n-4$ it is max plane graph (the deg of all regions of max plane graph ≥ 4). If $e \leq 2n-4$ it is the case of below rectraingulation (i.e they are planner but not have deg of all regions ≥ 4)

Point to be noted—

① $e \le 2n-4$ is mainly applicable for bipartite graph, where there are no odd length cycles and the length of the cycles are must be ≥ 4. But this formula is applicable for all random graph ($e \le 3n-6$ is not applicable for all random graphs, ex- $K_{3,3}$)

② K_n is not planner for $n \nleq 4$

③ $K_{m,n}$ is not planner of both $m,n \geq 3$.

- if you can see a complete graph then you can mind this formula ② or can go through the random graph. planner or not recognising steps (← previous).

- If you can see a complete bipartite graph then you can mind this formula ③ or can go through the recognization of random graph planner or not steps (← any previous)

Theorem
For any nontrivial connected simple graph degree of atleast 2 vertices will be same.

nontrivial, $n \neq 1$, $n > 1$

we can not more no. of different deg of atleast 2 will same

Ex. $n = 3$

We know that the graph that contains the order 1 and size = 0 is called trivial graph. So, those graphs that has the order more than 1 and size is more than 0 is called non trivial graph. A simple graph is that graph which have no self loop no parallel edges. ~~i.e only has the direct edges.~~ And the connected graph is when there exist atleast one path in between every pair of vertices chosen from the graph; graph is called.
We know that the degrees of a simple graph can be 0 to n-1. But a simple graph can not contain both 0 and deg n-1. So one degree must be repeated.

• Region:- A region is defined to be an area of the plane that is bounded by edges and cant be further subdivided. The region can be both finite & infinite.

$K_{3,3} \Rightarrow 9 \le 12 - 4$
$9 \le 8$ not planner.

~~Qubi~~ (cubic g)

applying the formule — $e \le 2n - 4$
[$12 \le 2 \cdot 8 - 4 = 12$, planner]

max plane graph – ~~A graph~~ that has been drawn ~~till~~ ~~the maximum~~ edges If we can draw a ~~complete~~ graphs planner version keeping that maximum edges till which it will be a plan graph, it will called max plane graph.

• Degree of region – no of edges that creates that region / Total no of edges required to form that boundary / region.

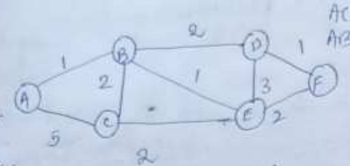
(or) • If we can draw a graph, keeping the maximum edges containing which it remains a plan graph, than this graph is called max plan graph.

Maximal planner g:- A simple g is called maximal planner if it is planner but adding any edge would destroy that property.

17/8/22

Q. Ibek

ABE ⇒ 1+1 = 2 (length)
ACE ⇒ 5+2 = 7 (")
ABCE ⇒ 1+2+2 = 5 (")



• Dijkstra's Algorithm (To find out the shortest path between two vertices)
Trying to find out the shortest path from A – F

Dijkstra's algorithm –

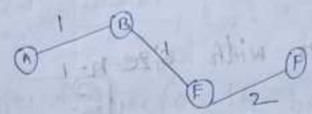
	A	B	C	D	E	F	
1st row	$\boxed{0}$ *	∞	∞	∞	∞	∞	(initialize statement)
selected A = 0	$\boxed{0}$	min$(\infty, 1+0) = \boxed{1}$ *	min$(\infty, 5+0) = 5$	min$(\infty, \infty+0) = \infty$	min$(\infty, \infty+0) = \infty$	∞	[selected values remain same in next row]
selected B = 1	$\boxed{0}$	$\boxed{1}$	min$(5, 2+1) = 3$	min$(\infty, 2+1) = 3$	min$(\infty, 1+1) = \boxed{2}$ *	min$(\infty, \infty+1) = \infty$	[no edge = previous value in next row.]
selected E = 2	$\boxed{0}$	$\boxed{1}$	min$(3, 2+1) = \boxed{3}$ *	min$(3, 3+\infty) = 3$	$\boxed{2}$ *	min$(\infty, 2+2) = 4$	
selected C = 3	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{3}$ *	$\boxed{2}$	min$(4, \infty+3) = 4$	
selected D = 3	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$	$\boxed{3}$	$\boxed{2}$	min$(4, 1+3) = \boxed{4}$ *	
selected F = 4	$\boxed{0}$	$\boxed{1}$	$\boxed{3}$				

(starting vertex given in question (in 1st) (A) (in last → F) [last in question])

The length of the shortest path between A – F is 4.

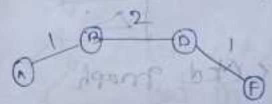
Back tracking

A → B → E → F (ABEF)



[Here considering the new value & more prioritize the values (no need of old consideration)]

ABDF (considering old value, new value)



[Here considering new value]

[numericals (try at h/w)]

①

TREE

Denoted by – T [All bipartite graphs are tree – F; All trees are bipartite graph – T]

It is a connected graph where there is no cycle.

∴ It is a connected acyclic graph.

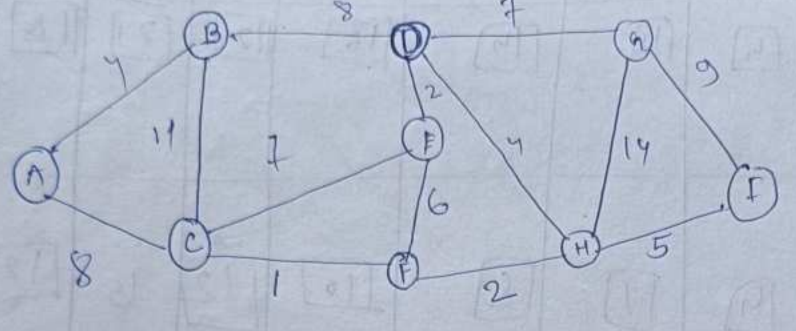
K_2 = tree, C_3 ⇒ not a tree

• • → not a tree, as it is disconnected.


→ tree, there is no cycle, no self loop, no parallel edges, connected graph

Dijkstra's Algorithm

(1) Find the shortest path from B to I.



from A to I → Direction

	B	A	C	D	E	F	G	H	I
Initial	$\boxed{0}$ *	∞	∞	∞	∞	∞	∞	∞	∞
Selected B=0	$\boxed{0}$	min (∞,4+0) ≃ $\boxed{4}$ *	min (∞,11+0) ≃ 11	min (∞, 8+0) = 8	∞	∞	∞	∞	∞
Selected A = 4	$\boxed{0}$	$\boxed{4}$	min (11, 8+4) = $\boxed{11}$ *	8	∞	∞	∞	∞	∞
Selected C=11	$\boxed{0}$	$\boxed{4}$	$\boxed{11}$	∞	min (∞,7+11) ≃ 18	min (∞,1+11) ≃ $\boxed{12}$ *	∞	∞	∞
Selected F=12	$\boxed{0}$	$\boxed{4}$	$\boxed{11}$	min (∞, 2+12) ≃ $\boxed{14}$ *	min (18, 6+12) ≃ 18	$\boxed{12}$	∞	∞	∞
Selected D=14	$\boxed{0}$	$\boxed{4}$	$\boxed{11}$	$\boxed{14}$	min (18, 2+14) ≃ $\boxed{16}$ *	$\boxed{12}$	min (∞, 7+14) ≃ 21	min (∞, 4+14) ≃ 18	∞
Selected E=16	$\boxed{0}$	$\boxed{4}$	$\boxed{11}$	$\boxed{14}$	$\boxed{16}$	$\boxed{12}$	~~min~~ 21	$\boxed{18}$ *	∞

Self loop, no parallel edges, connected graph

selected H = 18	0	4	11	14	16	12	min (21, 14+(8)) = 21	18	min (∞, 5+18) = 23
selected G = 21	0	4	11	14	16	12	21	18	min (23, 21+9) = 23
selected H = 12	0	4	11	8	10	12	15	12	17
selected F = 12	0	4	11	8	10	12	15	12	17
selected G = 15	0	4	11	8	10	12	15	12	~~17~~ 17

B - I shortest distance = 17

shortest path -

B — D — H — I

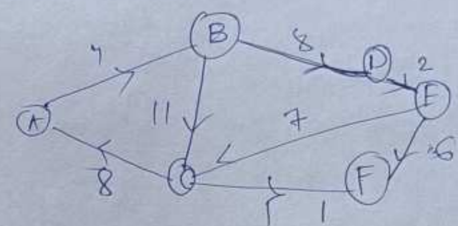
B to H shortest distance = 12

B to G " " " = 15

B — D — G

	shortest distance	path
B-A	4	B A
B-C	11	B C
B-D	8	B D
B-E	10	B D E
B-F	12	B C F
B-G	15	B D G
B-H	12	B D H
B-I	17	B D H I

selected D = 2	0	1	3	3	2



A - F

	A	B	C	D	E	F
Initial	0	∞	∞	∞	∞	∞
selected A = 0	0	4	∞	∞	∞	∞
selected B = 4	0	4	15	12	∞	∞
S D = 12	0	4	15	12	14	∞
S, E = 14	0	4	min (15, 14+7) = 15	12	14	20
S, D = 15	0	4	15	12	14	16

Shortest distance = 16 , Path = A B C F

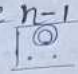
Tree, There is no cycle, no self loop, no parallel ...

Bipartite g → any cycle → F
$K_{m,n}$ ⟶ cycle ⟶ T

If a tree is of order n then size will be - $\boxed{n-1}$

→ 1 edge X - disconnected ⎤ not
1 edge ↓ - cyclic ⎦ tree

i) Any g of order n with size n-1 is a tree - F

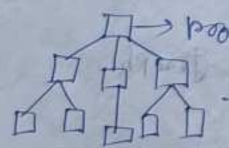


ii) Any simple graph order n with size n-1 is a tree - F

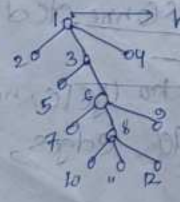
- Tree is minimally connected graph
- If you selected any pair of v.v there will only one path in the g.

↓ same statement

- There is exactly one path between every pair of vertices in a tree.



→ root node
[V ⟶ node in tree]
→ rooted tree



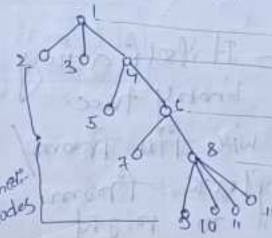
→ root node

Height

level = 0, 1, 2, 3, 4

| Level (2) = level (1) + 1
| 2 is the child of 1
| High level = Parent
| Level low = child

The node which has no child = leaf node
Ex = 2, 3, 5, 7, 9, 10, 11, 12

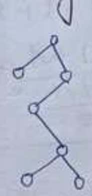


level = 0 — height = 1
" = 1 — " 2
" = 2 — " 3
" = 3 — " 4
" = 4 — " 5

Intermediate nodes

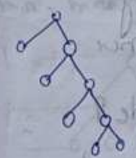
Types of tree (on the basis of how many children of a Parent node)

1) Binary tree - No. of childe of node max = 2



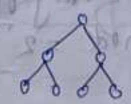
(0, 1, 2) childe nodes possible,
But in the binary tree the node childe greater than 2 → not possible.

2) strictly Fully binary tree - (2-tree)
(0 or 2 childe)



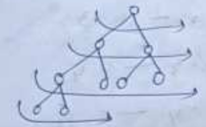
In this type of tree the no. of child node should be 2 or 0

3) Fully binary tree -



(All levels should be full by possible child nodes)

4) Complete binary tree — It itself a fully binary tree.



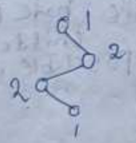
It will fill from/complete from left to right for all the levels.

18/6/22

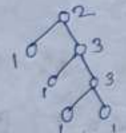
i) c/w

Any node is called leaf node when it's degree is 1.

In tree deg = 0 not possible.



max deg = 3
min deg = 1] → binary tree



max deg = 3
min deg = 1] → strictly binary tree

1) order of 2-tree is n, no. of leaf node?
fully binary

[l = no of leaf nodes]

[The deg of root node = 2]

$\sum_{i=1}^{n} deg(v_i) = 2 \times e = 2 \times (n-1)$

$\Rightarrow 2 + l \times 1 + 3(n-l-1)$

$\Rightarrow 2 + l - 3l - 3 = 2n - 2$ $+3n$

$\Rightarrow 3n - 2l - 1 = 2n - 2$

$\Rightarrow n + 1 = 2l$

$\Rightarrow \boxed{l = \frac{n+1}{2}}$

ii) order of 2-tree is n, no. of leaf node?

$\sum_{i=1}^{n} deg(v_i) = 2 \times e = 2 \times (n-1)$

$\Rightarrow 2 + l \times 1 + 3(n-l-1) = 2n - 2$

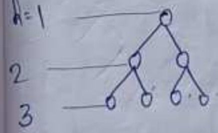
$\Rightarrow 2 + l + 3n - 3l - 3 = 2n - 2$

$\Rightarrow n + 1 = 2l$

$\Rightarrow \boxed{l = \frac{n+1}{2}}$

root node = deg 2
leaf node = deg 1
remaining nodes = deg 3

iii) There is a complete binary tree of height h, then min nodes and max nodes (order min, max order)?



h = 3, min = 4 (order)
max = 7 (order)

h = 4, min = 8 (order)
max = 15 (order)

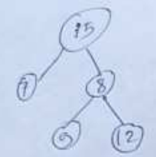
any level - order

$\boxed{\text{min order} = 2^{h-1}}$

$\boxed{\text{max order} = 2^{h} - 1}$

~~Traversal in a tree~~

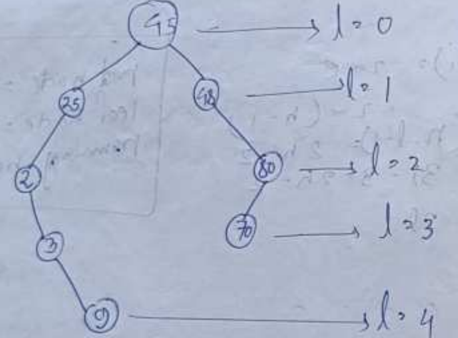
BST (Binary search tree)



BST don't store same element (redundant element)

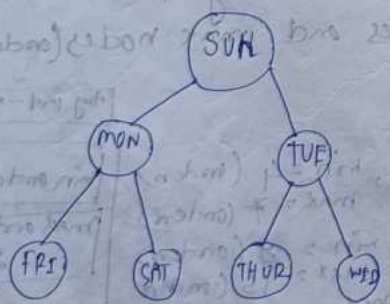
(1) sequence of no. I want to ~~for~~ entr in BST

45, 25, 48, 80, 2, 3, 9, 70



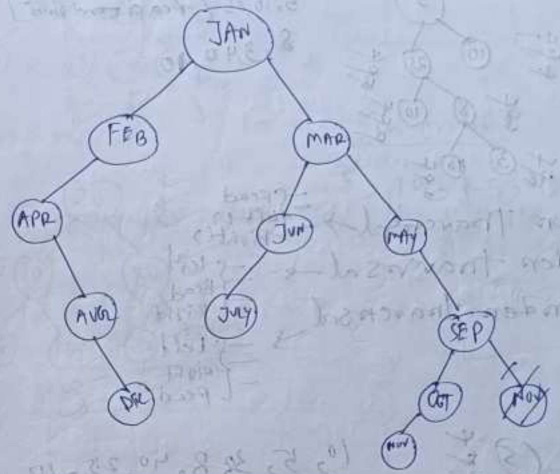
h = 5

Sun, MON, TUE, WED, THUR, FRI, SAT

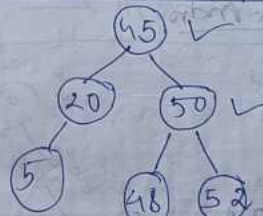


alphabetically cheque

JAN, FEB, MAR, APR, MAY, JUN, JULY, AUG, SEP, OCT, NOV, DEC



45	20	50	52	48	5



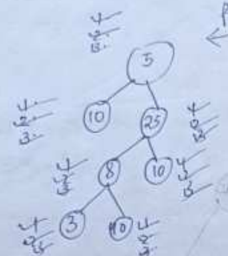
5 checking

3 checking

$\boxed{\text{checking} = \log_2 n}$

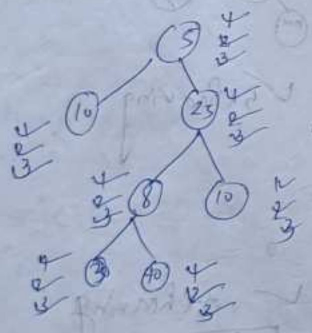
No. of comparisons

Traversal of a tree



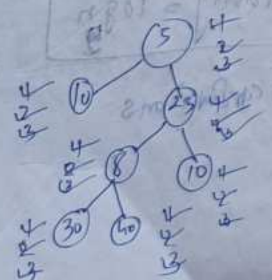
Pre-order
5, 10, 25 [Read and Print]
8, 3, 40, 10

1) Pre-order traversal → Read, left(), Right()
2) In order traversal → left, Read, Right
3) Post order traversal → Left, Right, Read



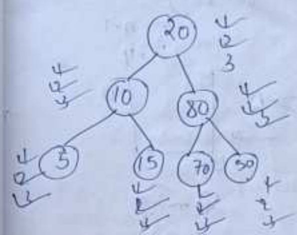
10, 5, 3, 8, 40, 25, 10

In-order

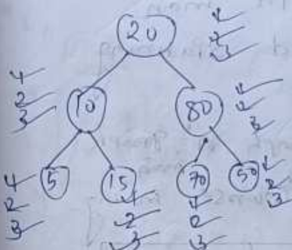


10, 30, 40, 8, 10, 25, 5

Post-order

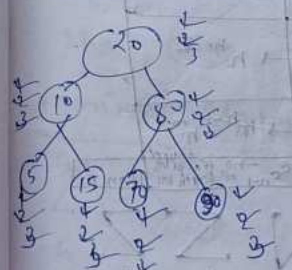


Pre-order
20, 10, 5, 15, 80, 70, 90



in-order
5, 10, 15, 20, 70, 80, 90

BST → in order traversal → get sorted list

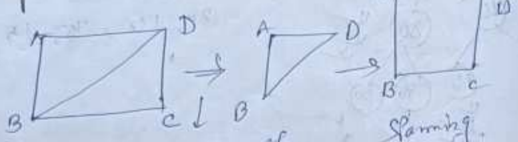


5, 15, 10, 70, 90, 80, 20

Post order

Spanning tree (subgraph of a graph)

$e = n-1$
$n = 4$



$n = 4$ subgraph Spanning tree

Adding all vertices of the main graph, then it is called spanning tree and it is a subgraph.

Spanning tree is a subgraph of main graph including all the v and ofcourse making a tree.

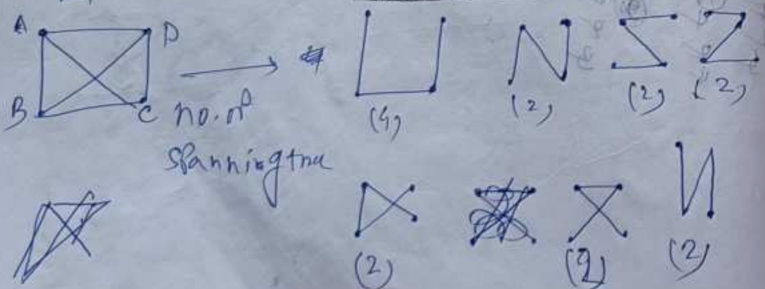
$C_3 \longrightarrow 3$

not spanning tree

C_4 no. of spanning tree $\longrightarrow 4$

Type of graph	No. of spanning tree
C_n	$\longrightarrow n$
K_n	$\longrightarrow n^{n-2}$
$K_{m,n}$	$\longrightarrow m^{n-1} \times n^{m-1}$
for any graph of order n	$e_{C_{n-1}}$ — no. of distinct cycles of length less than n

K_4



no. of spanning tree

$K_{2,3} \longrightarrow 2^{2} \times 3^{1} = 12$ (no. of spanning trees)

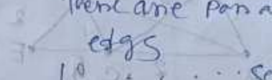
$e_{C_{n-1}}$ — no. of distinct cycles of length less than n

no. of vertices $= n$

${}^{6}C_{3} - (4+0+0)$

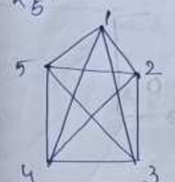
[no 2 length, no 1 length cycles]

[2 length cycles possible when there are parallel edges



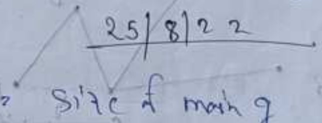
1 2 . . . self loop]

- K_5 —



${}^{10}C_{4} - (+)$

not possible humanly (so, formula previous)



25/8/22

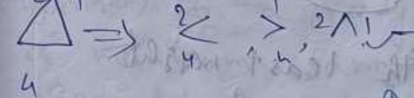
CW

chord $= e - (n-1)$ [e = size of main g, $(n-1)$ = The size of tree]

$\Downarrow$

That edges, that not coming in tree

minimal spanning tree



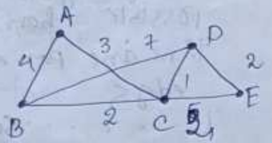
$\Rightarrow$ minimal spanning tree

A spanning tree formed from a weighted graph and sum of weight values will ~~values~~ be minimum — minimal spanning tree

1) Kruskal's Algorithm] → to find MST (minimal spanning tree)
2) Prim's Alogrithm

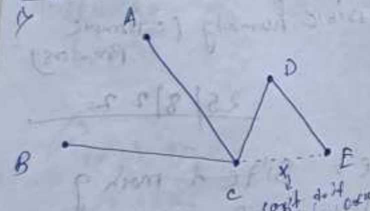
Kruskal's algorithm finds a minimum spanning forest of an undirected-weighted g. if the g. is connected it finds a minimum spanning tree. ~~It is a~~ (greedy algorithm)

1) Kruskal's Algorithm



[• Initial size
size of tree = 0
size = ~~0 1 2~~ 3]

Step 1



[Add every v in the same way like they are in the main graph]

can't do it cause it form cycle

Step-2

2) set [arrange the edges in the ascending order]

edge	{C,D}	{B,C}	{D,E}	{A,C}	{A,B}	{B,D}
weight	1	2	2	3	4	7

Step-3

3) Select the edge with least weight value. if selected edge not forms cycle

3.1) if (size<n), then draw that edge

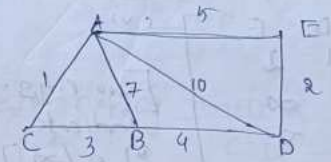
3.1) size++

3.2) go to step 3.1

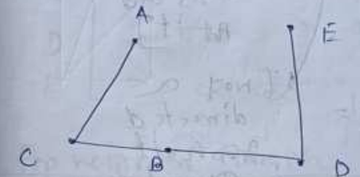
Step-4

4) stop.

2) Prim's Algorithm



Step-1

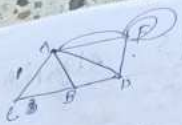


Step 2

edges	AC	ED	CB	BD	AE	AB	AD
weight	1	2	3	4	5	7	10

Prim's algorithm (also known as Jarnik's algorithm) is a greedy algorithm that finds a minimum spanning tree for a weighted undirected graph. this means it finds a subset of the edges that forms a tree that includes every v., where the total weight of all edges in the tree is minimized.

Adjuncency matrix



$$
\begin{array}{c|c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\left[\begin{array}{ccccc}
- & 1 & 1 & 1 & 2 \\
1 & - & 1 & 1 & \infty \\
1 & 1 & - & \infty & \infty \\
1 & 1 & \infty & - & 1 \\
2 & \infty & \infty & 1 & 4
\end{array}\right]
\end{array}
$$

(v) ↓ (v)

[No edges = indicate by 0/∞]

[If edge = then count the no of edges and put it].

[Not a directed graph, it is symmetric graph]

Row wise / col wise sum = deg

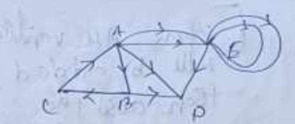
deg(A) = 5

deg(C) = 2

deg(E) = 7

Sum of any row / sum of any colourn indicates the deg of the ~~edge~~ vertices present in that graph.

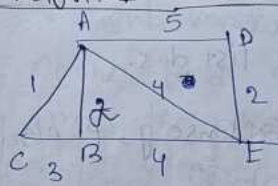
In adjacency matrix the summation of any row under a v. or summation of any ~~vertex~~ col. under a v. ~~is~~ indicates the deg of that v.



Adjancency matrix

$$
\begin{array}{c|c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} &
\left[\begin{array}{ccccc}
- & 1 & \infty & 1 & 2 \\
\infty & - & 1 & 1 & \infty \\
1 & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & \infty & \infty \\
\infty & \infty & \infty & 1 & 4
\end{array}\right]
\end{array}
$$

For weighted graph



Step - No. 1

Step-1

On the satisfaction of this con. site ch, all the v. will included automatically

Step-1 (As live Kruskal's Algo)

Step-1

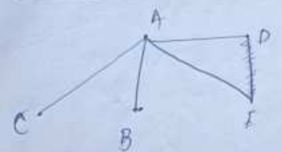
$$
\begin{array}{c|c}
 & \begin{array}{ccccc} A & B & C & D & E \end{array} \\
\begin{array}{c} \rightarrow A \\ \rightarrow B \\ \rightarrow C \\ \rightarrow D \\ \rightarrow E \end{array} &
\left[\begin{array}{ccccc}
- & 2 & 1 & 5 & 4 \\
2 & - & 3 & \infty & 4 \\
1 & 3 & - & \infty & \infty \\
5 & \infty & \infty & - & 2 \\
4 & 4 & \infty & 2 & -
\end{array}\right]
\end{array}
$$

selected previously

[Choose the least value between A and C, 1 is selected previously]

Initial site = 0

Step-2



[When All vertices will be added then also the size (n-1) will covered automatically]

K Algo	P Algo
1st element $(n^2) \rightarrow 100$	1) $n \rightarrow 10$ [no of comparisons]
2nd $(n^2-1) \rightarrow 99$	2) $2n \rightarrow 20$
3) Here, T.C = ~~O(V2)~~ O(E log V)	3) Here, T.C = O(V2)
4) K-Algo may have disconnected g parts	4) In P algo, all the g. elements must be connected
5) It prefers heap d.s	5) It prefers list d.s.
6) In case of ~~dense g.~~ sparse g., K Algo runs faster	6) In case of dense g., P Algo runs faster

* BFS, DFS [DSA], B-Tree, ~~ABL~~ AVL Tree, B+ tree

- Travelling salesman Problem (TSP) = TSP is an algorithmic problem tasked with finding the shortest route between a set of points and locations that must be visited.

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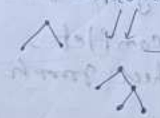
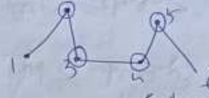
Disconnected tree



→ Forest - Collection of disconnected ~~tree~~ graphs or (trees)

Cut vertex If the v is removed the adjacent edge is also removed automatically.



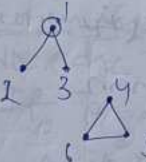
(Not a cut vertex).

It is a single v whose removal makes a connected graph disconnected. i.e v is cut vertex if v's removal makes a connected graph disconnected.

~~Bridge~~

Ex - from the above graph - 3, 4, 2, 5.

Pendant v could not be cut vertices.



Condition to be cut v in disconnected graph

$C(G-V) > C(G)$

v is a cut vertex if

$C(G-V) > C(G)$

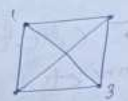
1 is cut vertex, as we remove 1 then no of components are - 3.

[C(G) - no of components in G

C(G-V) = no of components in (G-V) graph]

There are no cut v in cyclic graph.
There are no cut v in complete, wheel graph

K_4 -



3 → cut v

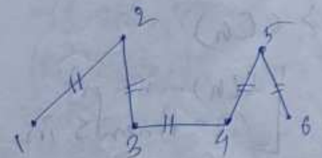
Bridge / (cut edge)

Condition to be cut edge

$c(G-e) > c(G)$

e is a bridge if the above condition is correct.

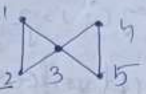
[$c(G)$ = No. of comp. in main graph
$c(G-e)$ = " in $(G-e)$ graph]



All of the edges are bridge.

C_4 =

There are no bridge in cyclic graph, complete graph.



→ There are no bridge in this graph.

Pendent edges always be bridge.

→ This graph is a graph where there is cut vertex but no bridges.

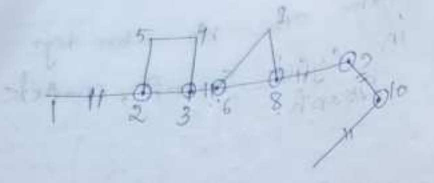
K_2 =

This graph is a graph where there is no cut vertex but has bridge

Theorem:

* No edge of a cycle could be bridge.
* If $e = \{u, v\}$ is a bridge, deg of u or v [$\deg(u \text{ or } v)$] ≥ 2 → u or v is a cut vertex.

bridge — 1 two n edge {1, 2} the deg(1) = 1, deg(2) = 2



cut vertex

* After removing bridge the no of comp increase = 1

But after removing cut v the no of comp increases = more than 1

*So, ~~if~~ $C(G-e) > C(G)$ can be ~~cut~~ written $C(G-e) = C(G) + 1$

$C(G-v) > C(G)$ can be written

so $C(G-v) = C(G) + n$.

5 — Same example.

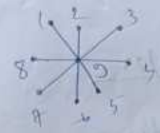
Vertex covers (set of V that covers all of the edges of that graph)

= {1, 2}, {1, 3}, {1, 2, 3}, {1} → X this is not a vertex cover.

minimum vertex covers — {1, 2}, {1, 3}

{1, 2, 3} → vertex cover (but it is not minimum)

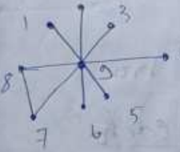
[All the edges are incidenting on any of vertices in the ...]



{9} → A minimum vertex covers

{1, 2, 3, 4, 5, 6, 7, 8, 9} → maximum vertex covers

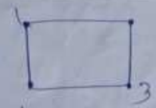
{7, 9} → vertex covers.



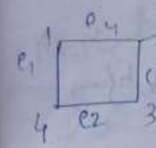
{9, 8} or {9, 7} → minimum vertex covers

•edge covers.

cut set vertices



C_4 remove → {1, 3} cut set V, {2, 4}



→ {2}
{2, 4} → X as {2} is cut set v

cut - set edges

There should not be any subcut - set edges

~~we can~~ there should not and not sub cut set
v, but {2} is cut set being vertices

{e1, e3} → ✓

{e1, e3, e4} → × selected edges

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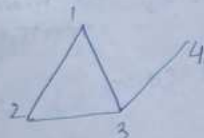
def

Independent set — no adjacency between the vertices of a set
(set of vertices)



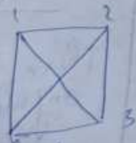
= {1, 4}, {2, 4}

{1, 2, 4} → ×

This is a set of v where no 2 vertices are adjacent to each other.

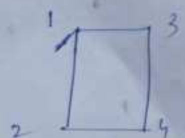
min independent set · {1}, {2}, {3}, {4}

For K_4:

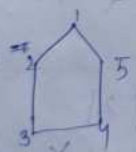


max independent set = {1}, {2}, {3}, {4}

C_4 =



C_5 =



= {1, 3, 4} → ×

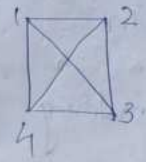
max independent set · {1, 4}, {2, 3}

C_6 =

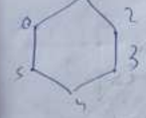
= {1, 3, 5} ✓

All bipartite graph's two different set is one independent set.

Clique (set of vertices) — All the vertices are adjacent to one another.



{1, 2, 3, 4}

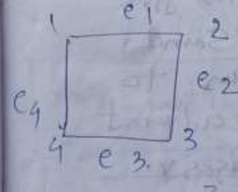


max 2 vertex click.

{1, 2} → ✓

matching (Independent set of edges)

no two edges are adjacent to each other.



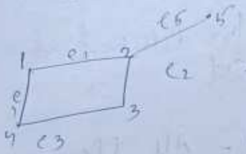
= {e1}, {e2} ... ✓

= {e1, e4} → ×

{e1, e3}, {e2, e4} → ✓

Perfect matching — All the vertices of the graph will be covered by the selected edges, then selected edges are perfect matching.

$\{e_1, e_3\}$ → Perfect matching



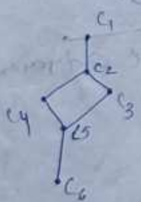
$= \{\{e_2, e_5\}, \{e_3, e_5\}\}$

But not a perfect matching.

maximal/maximum matching – In maximal matching there is a high possibility to get perfect matching.

Graph colouring

1) vertex colouring
2) Edge "
3) Region/maps

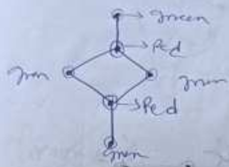


If you are giving 1 colour to ~~a~~ cannot give that same colour to the adjacent ~~edges~~ v.

vertex colouring = giving colour to those v. those who are not adjacent.

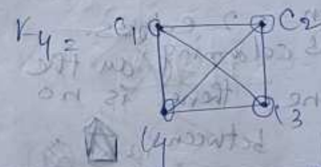
vertex colouring = colouring all the v of the graph by maintaining the condition that we can't give the same colour to those v which are adjacent.

chromatic no – the least no. of colours required to colour all the v. of that graph. / the proper colouring of the g.



chromatic no = 2

Proper colouring – colouring all the vertices [colouring all the v. of the graph = Proper colouring]. Proper colouring is applicable for ~~Real~~ chromatic no.



chromatic no. for $K_n = n$

C_n . n is ~~given~~ even, chromatic no = 2
" . odd, " " = 3

The chromatic no. for any tree = 2

" " " . bipartite graph = 2

* A graph whose ch.no is 2 is bipartite graph

. . 2 → tree

acyclic

$\boxed{W_n,\ n \text{ is even } c.no = 4 \quad n \text{ odd } = 3}$

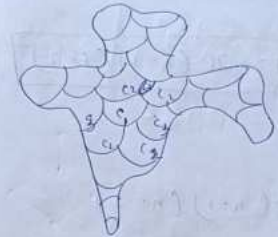
edge colouring

Colouring all the edges of the g., where there is no common vertex between 2 edges.
that means colouring all the edges where there is no adjacency between 2 edges

C.I = 3

Colouring all the e. of the gr. by maintaining the condition that we can't give the same colour to those e. which are adjacent

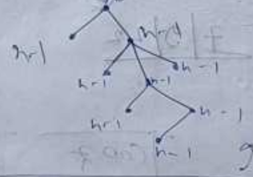
Chromatic ~~No~~ Index - Colouring all the edges of the graph



Chromatic Polynomial (this is a function of x)

$\boxed{P(x) = n(n-1)(n-2)}$ ← for ... [x = no. of colours given to colour your graph]

Chromatic polynomial for that graph



$\boxed{P(x) = n(n-1)^{n-1}}$ ← For tree of order n

(as if n-1, n-1 are same)

$\boxed{P(n) = n(n-1)^3}$

$\boxed{P(n) = n(n-1)^{n-1}}$

for cyclic g of order n

$P(x) = x(x-1)(x-2)$

$P(n) = n(n-1)(n-2)$

$n=0$, $P(n)=0$

$n=1$, $P(n)=0$

$n=2$, $P(n)=0$

$n=3$, $P(n)=6$

i.e 3 is ch. no.

If you have 3 colours

you can colour the

graph in 6 way.

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set theory

~~Relation~~

$|Z|$ = set of integer no.

$Z = \{-\infty \ldots -1, 0, 1, \ldots \infty\}$

$|Z|$ = cardinality

$S_1 = \{1\}$ → single-ton set

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

mutually exclusive sets

$A = \{1, 2, 3\}$

$B = \{2, 3, 4\}$

operations on set

1. union of 2 sets (OR)

$A \cup B$

$A = \{1, 2, 3\}$

$B = \{3, 4, 5, 6\}$

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

2. Intersection of 2 sets (AND)

$A \cap B$

$A = \{1, 2, 3\}$

$B = \{3, 4, 5, 6\}$

$A \cap B = \{3\}$

4. Complement of any set

$A^c / A' = U - A$

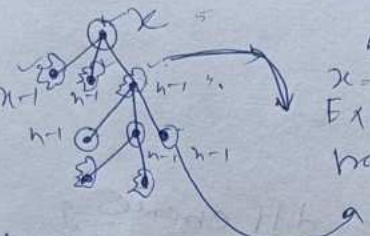
$[U \cap A^c]$

3. Substraction between 2 sets

in A but not in $B = A - B = \{1, 2\} = A \cap B^c$

i.e the element present in only A not in B.

Chromatic polynomial



5
x=5
Except that 1 Colour
rest of Colours (5-1 = 4
n-1)

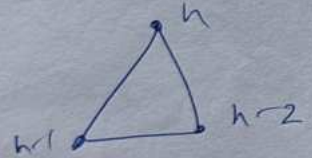
T_n, $P(n) = x(n-1)^{n-1}$ → For n nodes

$P(0) = 0$

$P(1) = 0$

$P(2) = 2(1)^{n-1} = 2$

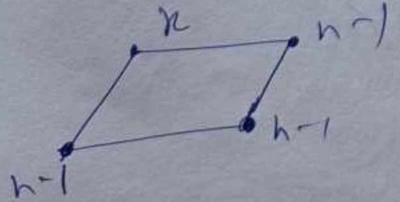
$P(3) = 3(2)^{10} \simeq 3000$ → 11 nodes



$P(n) = n(n-1)(n-2)$

Chromatic no = 3 (As for 0, 1, 2 $P(n)=0$)

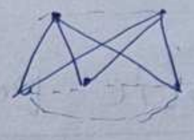
$P(3) = 3 \times 2 \times 1$
$= 6$, so we can Colour the graph by 6 ways



operations on set

$K_{2,3}$

order = 5



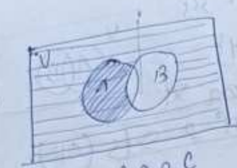
K_n, K_{n_1, n_2}

what is the size difference ?

$n_1 + n_2 = n$ $\quad \boxed{{}^{n}C_2 - n_1 \times n_2}$

${}^{2}C_2 + {}^{3}C_2 = 1 + 3 = 4 = ?$

↳ the no. edges in complete bipartite graph which ~~are~~ will not be present in the graph!



$A \cap B^c$

Commutative property

$A \cup B = B \cup A$

$A \cap B = B \cap A$

Union and intersection of set theory are commutative.

Subtraction on 2 sets are not commutative as $A - B \neq B - A$ [$A \cap B^c \neq B \cap A^c$]

Demorgans theorem

① $\overline{(A \cup B)} = \bar{A} \cap \bar{B}$

② $\overline{(A \cap B)} = \bar{A} \cup \bar{B}$

Set theory holds the De morgan's theorem.

①

(Proved)

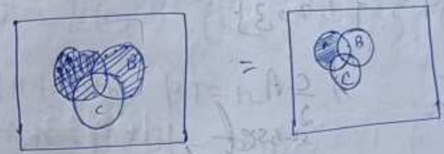
Associative property

$A \cup (B \cup C) = (A \cup B) \cup C$

$A \cap (B \cap C) = (A \cap B) \cap C$

Set theory are associative property. Intersection and union of

*. $A - (B - C) = (A - B) - C$



$B \cap C'$
$A \cap \bar{C}$
$A \cap B \cap C$
$\cap C'$

$A - (B - C) = A - (B \cap C') = A \cap \overline{(B \cap C)}$
$= A \cap (\bar{B} \cup C)$
$= (A \cap \bar{B}) \cup (A \cap C)$ (LHS)

$(A - B) - C = (A \cap B^c) - C = (A \cap B^c) \cap C' = (A \cap B^c) \cap \bar{C}$

Set theory holds associativity. $A - B = A \cap B'$

subtraction of

$A \cup (A \cap B)$
$= A$

$A + (A \cdot B)$
$= A \cdot 1 + A \cdot B$
$= A (1 + B)$
$= A$

$A - 1 = A$

Power set

set of subsets of any set

$A = \{1, 2, 3\}$

$\{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{1, 3\}$
$\{1, 2, 3\}$

$\{1\}, \{2, 3\} \longrightarrow$ proper subset

let $S = \{2, 3\}$ $\quad S \subsetneq A$ proper sub set

let $S_1 = \{1, 2, 3\}$ $\quad S_1 \subseteq A$ subset

1 set contains n no of elements.

total no. of subsets - $\boxed{2^n}$

If $|A| = n$, then no. of subsets possible is $= 2^n$

If $|A| = n$, then $|P(A)| = 2^n$

~~power set no of~~ subsets

cardinality of A power set = no. of subsets possible

If $|A| = n$ then,

$|P(P(A))| = 2^{2^n}$

$2 \cdot P = 2 \cdot 2$

* $A^c = 1 - A$

6) Cartesian Product

| | = cardinality means count

$A = \{1, 2, 3\}$ $\quad A \times B \neq B \times A$

$B = \{a, b\}$ but * $|A \times B| = |B \times A|$

$= |A| \times |B|$

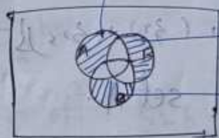
$A \times B = \{(1, a), (1, b), (2, a), (2, b),$
$(3, a), (3, b)\}$

doesn't hold Commutative property.

$B \times A = \{(a, 1), (b, 1), (a, 2), (a, 3), (b, 2), (b, 3)\}$

* If $|A| = m$, $|B| = n$

then, $|A \times B| = mn$
$= |B \times A|$



$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Relation

$R_1 \subset \underset{m}{A} \times \underset{n}{B}$

No of relations possible $= 2^{mn}$

$R_1 = \{(1,a), (2,3)\}$

R_1 is a relation of A with B

$R_2 = \{\varphi\}$

$R_3 = \{(1,a), (1,3), (1,c), (2,a), (2,3), (2,c)\}$

Qu

$|A| = n$ $A = \{1,2,3\}$

$|A \times A| = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

This is a relation on set A.

Types of relation

1) Reflexive Relation $\forall (a,a) \in R$ where a is an element of set A $[a \in A]$.

$A = \{1,2,3\}$

$R_1 = \{(1,2)\}$ × not reflexive

$R_2 = \{(1,1)\}$ × not reflexive

$R_3 = \{(1,1), (2,2), (3,3)\}$ ✓ Reflexive

[All (1,1), (2,2), (3,3) should have to present in the R]

$R_4 = \{(1,1), (1,2), (2,2), (3,3), (2,1)\}$ ✓ Reflexive

$R_5 = \{\varphi\}$ × Not Reflexive

2) Ir-Reflexive

$\forall a, (a,a) \notin R$ where $a \in A$.

$A = \{1,2,3\}$

$R_2 = \{(1,1)\} \to$ ×

$R_1 = \{(1,2)\} \to$ ✓

$R_3 = \{(1,1), (2,2), (3,3)\} \to$ ×

$R_4 = \{(1,1), (1,2), (2,2), (3,3), (2,1)\}$ ×

$R_5 = \{\varphi\}$ ✓

[Any of (1,1), (2,2), (3,3) should not be present there in that R]

- If $|A| = n$

$|A \times A| = n^2$

The no. of subsets possible $= \boxed{2^{n^2}}$

The no. of relations possible on set $A = \boxed{2^{n^2}}$

- $\{(1,1), (1,2) \cdots (1,n)$

$(2,1), (2,2) \cdots (2,n)$

$\vdots$

$(n,1), (n,2) \cdots (n,n)\}$

If $|A| = n$
then no. of reflexive relations possible on set $A = \boxed{2^{n^2-n}}$

- If $|A| = n$
Then no. of ir-reflexive relations possible on set $A = \boxed{2^{n^2-n}}$

3) Symmetric

If $(a,b) \in R$ then $(b,a) \in R$

$A = \{1, 2, 3\}$

$R_1 = \{\phi\} \longrightarrow$ ✓

$R_2 = \{(1,1)\} \longrightarrow$ ✓

$R_3 = \{(1,2), (2,1)\} \longrightarrow$ ✓

$R_4 = \{(1,1), (1,2)\} \longrightarrow \times$

$R_5 = \{(1,2), (2,1), (1,1), (2,2)\} \longrightarrow$ ✓

If $|A| = n$
Then no. of symmetric relations possible on set $A = \boxed{2^{n + \left(\frac{n^2-n}{2}\right)}}$

$= \boxed{2^{\frac{n^2+n}{2}}}$

automatically   1 choice on 2 elements

$$\left\{\begin{matrix} (1,1), & (1,2) & \cdots & (1,n) \\ (2,1), & (2,2) & \cdots & (2,n) \\ \vdots & & & \\ (n,1), & (n,2) & \cdots & (n,n) \end{matrix}\right\}$$

4) A symmetric

If $(a,b) \in R$ then $(b,a) \notin R$

$A = \{1,2,3\}$

$R_1 = \{\phi\} \longrightarrow$ ✓

$R_2 = \{(1,1)\} \longrightarrow \times$

$R_3 = \{(1,2), (2,1)\} \longrightarrow \times$

$R_4 = \{(1,1), (1,2)\} \longrightarrow \times$

$R_5 = \{(1,2), (1,3), (2,3)\} \longrightarrow$ ✓

$R_6 = \{(1,2), (2,1), (1,1), (2,2)\} \longrightarrow \times$

- If $|A| = n$

$\therefore$ Then no. of Asymmetric relations possible on set $A = \boxed{3^{\frac{n^2-n}{2}}}$

$\{(2,1), (1,2)\}$ ✓ or ✓

$\{(3,1), (1,3)\}$

$\{(3,2), (2,3)\}$ or ×, ×

$$\left\{\begin{matrix} (1,1), & (1,2) & \cdots & (1,n) \\ (2,1) & (2,2) & \cdots & (2,n) \\ \vdots & & & \\ (n,1) & (n,2) & \cdots & (n,n) \end{matrix}\right\}$$

3 (3 choices)

How 3 comp

5) Transitive

If $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

$A = \{1,2,3\}$

$R_1 = \{\phi\} \rightarrow ✓$

$R_2 = \{(1,1)\} \rightarrow ✓$

$R_3 = \{(1,2),(2,1)\} \rightarrow ✗$

$R_4 = \{(1,2),(1,3),(2,4)\} \rightarrow ✗$

$R_5 = \{(1,2),(1,3),(2,4),(1,4)\} \rightarrow ✓$

$R_7 = \{(1,2),(3,2)\} \rightarrow ✓$ [if $a,b \in R$ and also $(b,c) \in R$]

$R_8 = \{(1,2),(2,1),(1,1),(2,2)\} \rightarrow ✓$

- If any relation reflexive, symmetric, transitive $\rightarrow$ that is called equivalent relation.

$A = \{1,2,3\}$ $R_1 = \{(1,1),(2,2),(3,3)\} \rightarrow ✓$

$R_2 = \{(1,1),(2,2),(3,3),(1,2),(2,1)\} \rightarrow ✓$

6) Anti symmetric

If $(a,b) \in R$ and $(b,a) \in R$, then $a=b$

$A = \{1,2,3\}$

$R_1 = \{(1,2),(2,1)\} \rightarrow ✗$

$R_2 = \{(1,1),(2,2),(3,3)\} \rightarrow ✓$

Relation having only diagonal elements

$R_3 = \{(1,1)\} \rightarrow ✓$, $R_4 = \{(1,1),(1,2)\} \rightarrow ✓$

- Any relation reflexive antisymmetric, transitive that relation will be called Partially ordered relation.

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$(1,1)$, $(1,2)$, —, $(1,3)$
$(2,1)$, $(2,2)$, —, $(2,3)$
$(3,1)$, $(3,2)$, $(3,3)$

The no. of antisymmetric relations possible on set $A = \boxed{2^n \times 3^{\frac{n^2-n}{2}}}$

[Dependent $= *$
Independent $= +$]

$(1,1)$, $(1,2)$, $(2,1)$

1) $R = \{(a,b) \mid a-b \% 5,\ a,b \in Z\}$

(1) For reflexive,

If $(a,a) \in R$, then it must satisfy that $a-a$ is divisable by 5

$\forall a, (a,a) \in R$

$\therefore$ This is a reflexive relation.

2) For transitive,

If $(a,b) \in R$ and $(b,c) \in R$ then $(a,c) \in R$

$(20,15) \in R$ $(15,5) \in R$

$(20,5) \in R$

∴ If (a,b) is divisiable by 5 and R

$a-b=5e$ and (b,c) is also divisiable by 5 $b-c=5*e$. so, $(a-c)$ is also divisable by 5 $a-c=5C$.

∴ This is transitive relative.

3) For symmetric,

If (a,b) is divisiable by 5

$a-b=5e$

then (b,a) is also divisiable by 5

$b-a=5C$

$\Rightarrow -(a-b)=5C$

$\Rightarrow a-b=5K$

So, this is equivalent relation.

2) equivalent class, $[a]=\{(a,x) \mid x,$ for x Relation satisfied$\} = \{x \mid (a,x) \in R\}$

$[0]=\{0,5,10,15,20,-5,-10 \ldots\}$

$[1]=\{+1,+6,+11,\ldots\}$

$[2]=\{+2,+7,+12,\ldots\}$ [based on Previous m]

3) $R_2 \{(a,b) \mid a \leq b,\ a,b \in Z\}$

1) For Reflexive,

~~If (a,a)~~ It is Reflexive, as $a \leq a$ ∴ $\forall a, (a,a) \in R$

2) For ~~symmetric~~ transitive,

as, $a \leq b$, $b \leq c$

$(a,b) \in R$ $(b,c) \in R$

$a \leq c$

$(a,c) \in R$

∴ It is transitive Relation.

3) For anti-symmetric,

It is ~~as~~ anti symmetric, as.

$a \leq b$, $\Rightarrow (a,b) \in R$, ~~then~~ and

$b \nleq a \Rightarrow (b,a) \notin R$

∴ a must equal to b $\Rightarrow (a=b)$

4) For symmetric,

if $a \leq b$ then $(a,b) \in R$ but ~~$b \leq a$~~ is not possible ∴ $(b,a) \notin R$

This one is partially ordered relation.

~~Po~~ Poset (Partially ordered set) → $(Z, \preceq)$

[Integers are partially ordered with $\leq$]

(4) $(D_{12}, /)$
$D_{12} = \{1, 2, 3, 4, 6, 12\}$ [/ = divisible by]
$(D_{12}, /)$ is indicating
$R = \{(a,b) \mid a/b,\ a,b \in D_{12}\}$,
3) Is this a poset or not?

For Reflexive,
It is reflexive as a/a
ie $(a,a) \in R$

For transitive,
It is ~~not~~ transitive;
If $a/b \Rightarrow (a,b) \in R$
and $b/c \Rightarrow (b,c) \in R$
then must a/c $(a,c) \in R$

For ~~sy~~ symmetric,
this is not symmetric
as $a/b \Rightarrow (a,b) \in R$
but b/a can not possible
$\therefore (b,a) \notin R$

$\therefore$ The set is poset

Inverse of a Relation

$A = \{1, 2, 3\}$
$R = \{(a,b)\}$, $R^{-1} = \{(b,a)\}$
$R_1 = \{(1,2), (1,3)\}$
$R_1^{-1} = \{(2,1), (3,1)\}$

Complement of any Relation

$\bar{R} = R^C = \{(a,b) \in R^C \mid (a,b) \notin R\}$
$A = \{1, 2, 3\}$
$R_1 = \{(1,2), (1,3)\}$
$R_1^C = \{(1,1), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

How to take closure

Reflexive Closure → [closure is a pairs of elements that are is required to make a relation that which we want (like r, t, s etc)] [including R_1 we have to write closure R_1^+]

$A = \{1, 2\}$
$R_1 = \{(1,2)\}$
the elements that make it reflexive closure
$R^+ = \{(1,1), (2,2), (1,2)\}$

$R^s = \{(1,2), (2,1)\}$ " symmetric

what makes the R_1 set transitive closer

$R^* = \{(1,2),(2,1),(1,1),(2,2)\}$

↓ graph

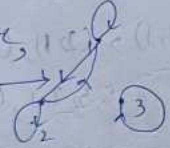
$A = \{1, 2, 3\}$ graph

$R = \{\phi\}$ ⟶ 1 2 3

R^*

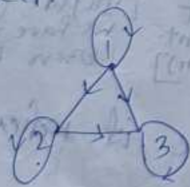
Reflexive relation ⟶

Symetric, transitive, Equivalent relation ⟶



also

The Relation's graph s are always directed.



Partially ordered relation

CW

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① $(N, /)$ — is it a poset?

$N = \{1, 2, 3, 4, \ldots\}$

$R = \{(a,b) \mid a/b, a, b \in N\}$

a/a,

$\forall a, (a, a) \in R$ ∴ Reflexive

$(a, b) \in R$ ~~$\not\Rightarrow (b,a) \in R$~~ ∴ a/b

but $(b, a) \notin R$ ∵ b is not divisible by b, where a is divisiable by b.

∴ Not symmetric,

$(a, b) \in R$, $(b, c) \in R$

↓

$(a, c) \in R$

∴ This is Transitive.

antisymmetric as it is symmetric, as well as, it is ~~transitive~~ reflexive

∴ It is Poset

Cover of any ~~der~~ element

① $(D_{12}, /)$ $D_{12} = \{1, 2, 3, 4, 6, 12\}$

$4/2 = 2$

2 is cover of 1 $2/1$

3 ...... $3/1$

4 ...... 2 $4/1$ as $4/2$, $2/1$

2/4 ∴ will not come 2 which come =2 is similar divisors

As between 4 and 1, there is an element 2 which is coverable. So 4 cannot be a cover of 1 nal symmetric,

(b,c) ← n

A cycle has symmetricity;
ie starting point & 2 directions.
If ~~a cy~~ the order of a
cyclic graph is n (C_n) then the
no. of Starting points are = n
and each Starting points has
2 directions ∴ then no of cycles = 2n
possible

$D_{12} = \{1, 2, 3, 4, \ldots\}$

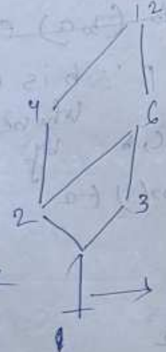
2, 1)

2/1

6 ---- 2
6 ---- 3
12 ---- 6, 4

12/4   4/2   2/1   Immidiate dividers
12/6, 6/3, 3/1   will be there,
12/6, 6/2, 3/1   the between element
can't come

12 → maximal



If it is not poset the hash diagram of it can't be drawn

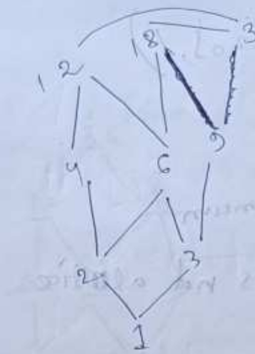
⟹ Hash diagram

1 → minimal

② $(D_{36}, /)$

D_{36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}

2, 3 Covers of 1
4, 6 ... 2
6, 9 ... 3
12 ... 4
12, 18 ... 6
18 ... 9



36 ---- 12
36 ---- 18
~~36~~

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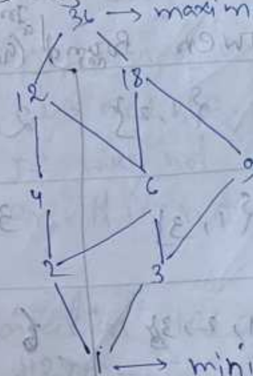
Q ①

D_{36} = {1, 2, 3, 4, 6, 9, 12, 18, 36}

36 → maximal

2, 3 is covers of 1
6, 4 ... 2
6, 9 ... 3
12 ... 4
12, 18 ... 6
18 ... 9
36 ... 12
36 ... 18



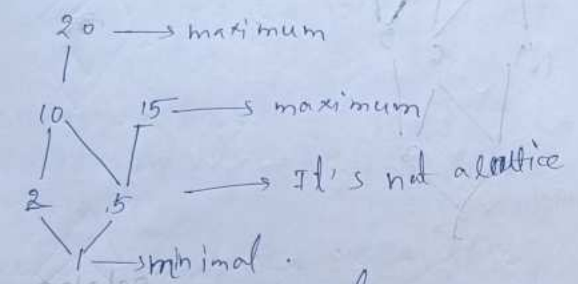
1 → minimal

It is a poset. [As the if it is poset then only the hash diag. can be drawn]

The element which is not covers of others element — minimal.

The element which has no cover element — maximal

① poset ({1, 2, 5, 10, 15, 20}, /)

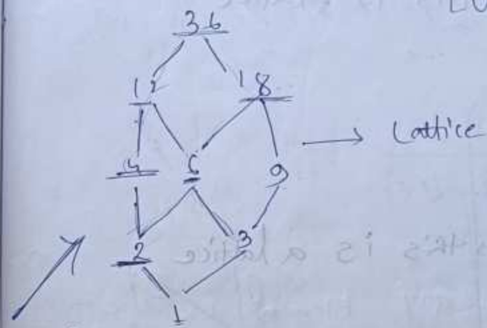


- minimal, maximal elements can be more than one.

	Lower Bound (LB)	greatest of LB (GLB)	Upper Bound (UB)	Least of UB (LUB)
{4,6}	{1, 2}	2	{36}	36
{6,9}	{1, 3}	3	{18, 36}	18
{6,12}	{1, 2, 3}	6	{12, 36}	~~36~~ 12
2 {18,3}	{3, 1}	3	{18, 36}	18
{2,5}	{1}	1	NULL (NO elements)	NULL (NO elements)

GLB → meet

LUB → join



LATTICE Any hasse diagram will called a lattice, if ~~any~~ randomly chosen any pair of elements will have LUB and GLB, then it is called lattice. If will not have any LUB or GLB then it is not lattice.

(When there is a direct connection then in LB → LB itself like {12, 6} → LB → {1, 3, 36}
and in UB → UB itself like {12, 6} → UB = {12, 36})

[Direct edge, then → ex. → {18, 3}, then 3 will be GLB, 18 will be LUB, between element will not come]

[|| → connected directly
Horizontally → not connected]

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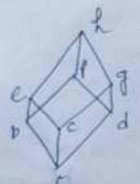
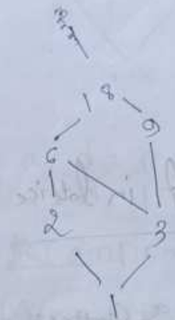
→ This is a lattice

① (D18, |)

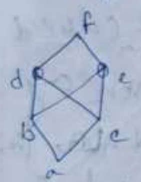
D18 = {1, 2, 3, 6, 9, 18}



→ this is a lattice



	LB	GLB	UB	LUB
{2, 9}	{1}	{1}	{18}	{18}

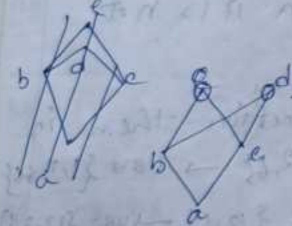


→ This is not lattice

{b, c}	GLB	UB	LUB
	a	{d, e, f}	×

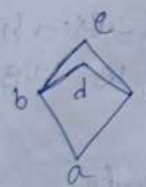
Semilattice – only GLB is there or only LUB is there.

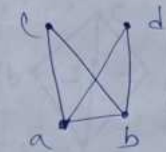
lattice – Both GLB and LUB is Present



→ same level $\boxed{\text{which one is greater we don't know}}$

Same figure



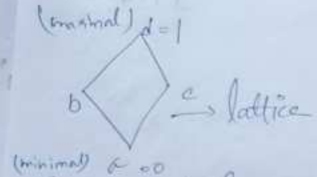


→ Not Lattice, not semilattice also

{c, d}	LB	UB	GLB	LUB
	×	{c, d}	×	×

Notation of GLB – ∧ (meet)

" " LUB – ∨ (join)



(bounded shape)
[there there will be maximal element and minimal element]

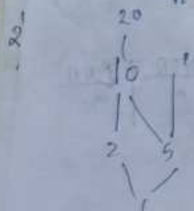
Complement of any element in lattice

$a \vee b = 1$
$a \wedge b = 0$ } then a is a complement of b

It always defined on finite no. of sets (only on bounded figure)

$\angle \to |$
G G'

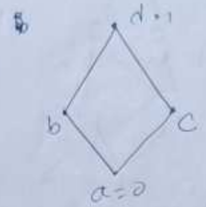
a' a simple graph G is G'
$G_n' = K_n - G_n$



→ Not a complemented lattice

Not bounded figure = no complement

All bounded lattice = complemented lattice → False

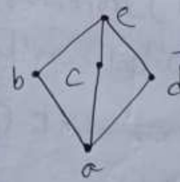


$b \wedge c = 0$
$b \vee c = 1$ } b & c are complemented to one another

$a \wedge d = 0$
$a \vee d = 1$

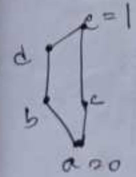
So, it's complemented lattice.

Complemented lattice – If there exist atleast one complement (the pair of elements must have GLB and LUB) for every element, then that lattice is called complemented lattice.



→ Complemented lattice

table



element	compl
a	e
e	a
b	c
c	b
c	d
d	c

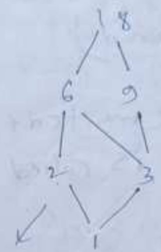
b	d
d	b

⇓
modified table

element	Compl
a	e
b	c, d
c	b, d
d	b, c
e	a

$b \wedge d = b \rightarrow$ (not be comp) ×
$b \vee d = d \rightarrow$ ×

element	Comp l
a	e
b	c
c	b, d
d	c
e	a



Non complemented lattice

element	Compl
2	9
6	×
3	×

6 has no complement !!

Distributive Lattice

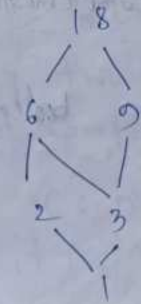
1) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

2) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

If there will be any formation of this condition (upper), then it's called distributive lattice.

H.W

① Is ~~is~~ this lattic is D.L? $\boxed{\text{Ans = yes}}$



yes this is distributive Lattice.

let $a = 2$, $b = 9$, $c = 3$

$a \vee (b \wedge c) = 2 \vee (9 \wedge 3)$
$= 2 \vee 3$
$= 6$ LHS

$(a \vee b) \wedge (a \vee c) = (2 \vee 9) \wedge (2 \vee 3)$

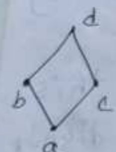
= 18 ∧ ~~18~~ 6
= ~~18~~ 6 RHS ∴ LHS = RHS (Proved)

Boolean lattice

A lattice which is complemented + distributive, that lattice is called boolean lattice.

The Previous one —

It is not a boolean lattice as it is distributive but it isn't complemented lattice.



⇒ $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
$= a \vee a$ | $= b \wedge c$
$= a$ (LHS) | $= a$ (RHS)

LHS = RHS

∴ Distributive lattice.

It is also complemented lattice.

∴ This is a boolean lattice.

Totally ordered ~~set~~ sets or tosets.

1 poset will be called toset when each elements are comperable with one another by the operators

~~One Poset~~ $(Z, \leq) \rightarrow$ poset

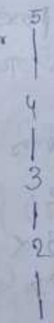
2, 3
$3 \leq 2$

$D_{18} = \{1, 2, 3, 6, 9, 18\}$

No relation between 6, 9 ∴ that's why 6, 9 are not comperable.

toset will be like that —

Poset = A poset is called toset when every two elements of that set are comparable with one another by operators.



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Q) $(D_{18}, |) = \{1, 2, 3, 6, 9, 18\}$

Every two elements of this set are not comperable.

Hence $(6, 9) \notin R$ and $(9, 6) \notin R$

So it is not a toset.

(2) $(\mathbb{Z}, 1)$ B = {all integers}

$(10, 5) \in R$

but $(9,5) \notin R$

(3) $(D_{18}, \leq)$

This is a poset

This is also a toset, as every two elements of this set are comparable to each other.

$(\{1, 2, 3, 6, 9, 18\}, \leq) \longrightarrow$ poset $\longrightarrow$ toset

$1 \leq 2,\ 1 \leq 9$ ......

18 = 1 ← {1, 3} → GLB LUB (cons)

LUB (9)

6

GLB (3) → Hash diagram of toset

2

1 = 1

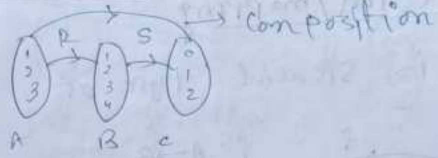
$1 < 2$

$1 < 3$ ✗ as $2 < 3$

Hash diagram of every tosets are straight line.

one toset which's sets has 2 entries then it is Complemented lattice - only → [If one toset has 2 entries then it is complemented lattice]

Composition of Relations -



$R \subseteq A \times B$, $S \subseteq B \times C$

the happening, 1st happening $f \circ (S(x))$

$S \circ R \subseteq A \times C$

composite sign

Notation meaning = R is happening 1st, then S is happening.

$R = \{(1,1), (1,2), (3,4), (2,3)\}$ A×B

$S = \{(1,0), (1,2), (3,1), (4,2)\}$ B×C Composition A×C

So, $S \circ R = \{(1,0), (1,2), (3,2), (2,1)\}$

$R \circ S = \{(1,3), (3,1), (3,2), (4,3)\}$? ...

$S \circ R \neq R \circ S$

This does not follow commutative property (Composition between 2 relations)

$R = \{(1,0), (1,2), (3,4), (2,3)\}$

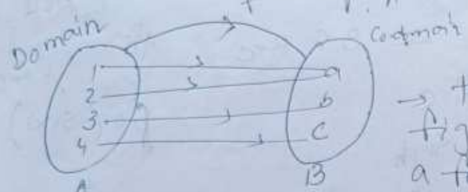
$R = \{(0,1), (0,2), (3,4), (2,3)\}$

$R \circ R = \{(1,1), (1,2), (1,3), (2,4)\}$

(3)

Function/mapping

Function is special type of relation.



→ this fig. is a function

R = {(1,a), (3,b), (4,c) (2,a)}

From point is domain, from set = domain set

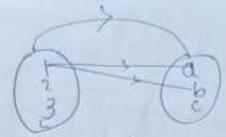
to " " co-domain, to " = co domain set

Image of 3 is b

'b' is preimage is '3'

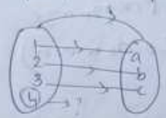
The each and every element of the domain should have exacty one image.

Every ele. of a domain will have exactly 1 image.



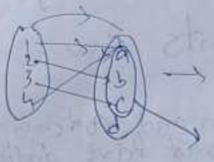
R = {(1,a), (1,b)} → X

Not a function



this is not a function

* R = {(1,a), (2,b), (3,c)} → X



this is a function

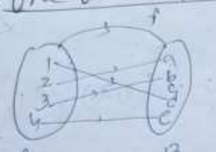
→ Range

The elements of codomain whose have pre images that are called range. (Not necessary each and every elements of codomain should have pre-image)

calsification of mapping

(1) one - one
(2) many one
(3) Into
(4) onto

1) one-one mapping



A B

x f(x)

(No more than one preimage in codomain)

If $x_1 \neq x_2$

then $f(x_1) \neq f(x_2)$

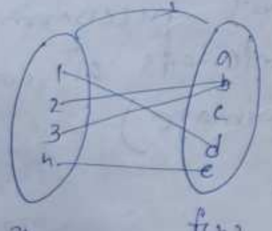
Then it's called one-one mapping

Each and every elements have distinct image.

[Each and every elements of domain and codomain set should have distinct image and pre image. If ~~f~~ it is, then its one-one mapping] ~~X~~

2] many-one mapping

more than 1 elements have same image in c.d set. f (in domain set)

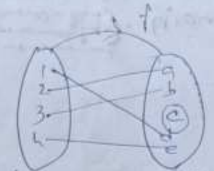


A B

x f(x)

3) Into mapping

Range will be less than codomain.



A B

x f(x)

Range = {a, b, d, e}

Codomain = {a, b, c, d, e}

∴ Range < Codomain

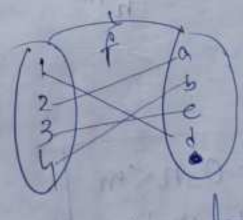
One-one + Into mapping = Injective function

many one + Into mapping = many-into

one-one + onto = Bijective function/mapping

4) Onto mapping

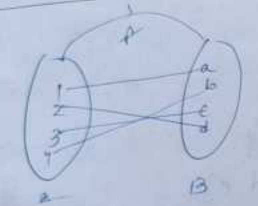
When Range = Codomain



Every elements of the codomain should have pre images.

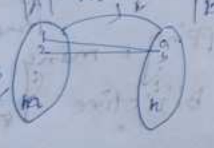
If a function is bijection then no. of elements in domain = no. of elements in codomain. (~~no. of codomain elements in domain have distinct pre-image~~)

Bijective mapping fig. —



How many functions are possible —

$|A| = m$, f, $|B| = n$



$f : A \to B$

image possibilities

$1 \longrightarrow n$

$2 \longrightarrow n$

$3 \longrightarrow n$

$m \longrightarrow n$

$\therefore n^m$

$\boxed{\text{total no. of function Possible} = n^m}$

No. of one-one functions $= 0$, $n < m$

$= n(n-1)(n-2) \ldots (n-m+1)$, $n \geq m$

$= {}^nP_m$

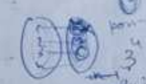
For one-one mapping if the no. of elements in domain > no. of elements in codomain then, it's not possible to draw one-one mapping.

No. of onto functions Possible —

$$\Rightarrow \sum_{i=0}^{n-1} {}^nC_i \cdot (n-i)^m$$

Total no. of functions possible $= n^m$ $\left[\begin{matrix} n(A) = m \\ n(B) = n \end{matrix}\right]$

No of Onto function possible =



3^4

a no pre-image

$- {}^3C_1 (2)^4 +$

${}^3C_2 (1)^4$

Case 1

n^m [All cases are included here, a ~~empty, b empty~~ no preimage, b no preimage ... has no preimage]

Case 2

$n^m - {}^nC_1 (n-1)^m$ [As during case 1 we included all cases include 1 element of codomain has no p.i. so we have to exclude it]

Case 3

$n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m$ [As during case 2 we exclude 1 element have no preimage, if was only 1 element of 3 and by doing this we excluded - rest of 2 elements have no preimage, as we don't know between 3 which one we going to be delete as, there are 3 possibilities]

Like this ~~this~~ that process will be going on ~~due~~ till $(n-1)$th element, as $(n-n)^m = 0$.

$\therefore$ the formula is =

$n^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - \ldots$

$= {}^nC_0 (n-0)^m - {}^nC_1 (n-1)^m + {}^nC_2 (n-2)^m - \ldots$

(1) The no. of images possible for 1 = n

2 = n

3 = n

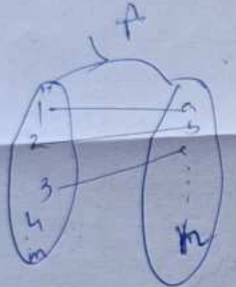
$m = n^m$ ($n \cdot n \cdot n \cdots$ m times)

$\therefore$ the total no. of func. possible $= n^m$

(2) No. of one-one func possible $= 0$, if $n < m$

$= n(n-1)\cdots(n-m+1)$, if $n \geq m$

$= {}^nP_m$



The no. of image possible for 1 = n

2 = $(n-1)$

3 = $(n-2)$

$m = (n-m+1)$

[as $2 = n-(1)$, $3 = n-(2)$, $4 = n-(3)$, $m = n-(m-1)$]

$\therefore$ the total no. of one one relations possible $= n(n-1)\cdots(n-m+1) = {}^nP_m$

$= \sum_{i=0}^{n-1} {}^nC_i (n-i)^m$ (proved) [R = real no.]

- $f : R \to R_0,\ f(x) = 2x+1$
  $g : R \to R,\ g(x) = x^2$

Injective, but not bijective, one-one.

Not onto

$gof = g(f(x))$
$= g(2x+1)$ $gof \neq fog$
$= (2x+1)^2$

$fog = f\{g(x)\}$
$= f(x^2)$
$= 2x^2+1$

Imp If
(1) $f : A \to B \Rightarrow$ one-one
$g : B \to C \Rightarrow$ one-one   //are correct
then $gof = A \to C$; one-one   //is correct

2) If
$f : A \to B$
$g : B \to C$
and $gof : A \to C$; one-one

⇓

f is also one-one.

3) If
$f : A \to B$, onto
$g : B \to C$, onto
then $gof : A \to C$; onto

4) If $f : A \to B$
$g : B \to C$
and $gof : A \to C$, onto

⇓

g is onto function

4) If $f : A \to B$, bijective
$g : B \to C$, bijective
then $gof : A \to C$, bijective

5) ~~If $f : A \to B$~~
~~$g : B \to C$~~
~~$gof : A \to C$~~

We can calculate inverse of ~~only~~ bijective function only

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mathematical logic

~~Compositional logic.~~

1) Proportional logic

2) First order logic/Predicate logic.
→ quantifiers

3) Theorem argument

Proposition - The Statement/argument in which the logic is applied.

1) If rains today, then there is no cricket match — Proposition statement.

2) If you study 10 hr/day (then) you can score 80% +

$\underbrace{\text{you study 10 hr/day}}_{\text{Proposition}}$ then (connected) $\underbrace{\text{you can score 80\% +}}_{\text{Proposition}}$

S: Studying 10 hr/day
m: getting 80% + marks

$S \longrightarrow m$

1) If it rains today, then there is no cricket match.

p: Rains today
m: match is taking place

$p \longrightarrow \neg m$ → if p, then not m
↓
negation (not)

3) Ram studies 10 h/day (Compositional Statement)
&
Sita runs 10 km/h

r: Ram - - - 10 h/day
S: S - - 10 km/h
↓
$r \wedge S$
↓
conjuction
(this one called connectives also)

[$p \to q$
If p then q]
[≡ equivalent sign]

$\boxed{\neg p \vee q}$ // same as $\boxed{p \to q \;\; \downarrow \text{same} \;\; p \equiv q}$

p	q	$\neg p$ (negation)	$p \wedge q$ (conjuction) (like intersection)	$p \vee q$ (disjuction) (like union)	$p \to q$ (implies) ($p \equiv q$)	$p \leftrightarrow q$ bidirectional (p iff q)	$q \to p$
F	F	T	F	F	T	T	T
F	T	T	F	T	T (As p is false then Ram can be conceived q (scalar f))	F	F
T	F	F	F	T	F	F	T
T	T	F	T	T	T	T	T

$\bar{p} + q$
T
T
F
T

$$\boxed{p \leftrightarrow q = (p \to q) \wedge (q \to p)}$$

↓
$\boxed{\text{xnor gate}}$

Calculating conjunction and takes the negative (like NOT)

$\rightarrow$ calculating disjunction and takes the neg

p	q	p ↑ q	p ↓ q
F	F	T	T
F	T	T	F
T	F	T	F
T	T	F	F

1) Tautology = Always true results if a statement

$p \vee \neg p \Rightarrow T$

p	¬p	p ∨ ¬p
T	F	T
F	T	T

2) Contradiction - Always false results if a statement

$p \wedge \neg p \Rightarrow F$ if a statement

3) Contingency: Not always T results, not always F results

Ex - $p \wedge q$, $p \vee q$ etc. all

4) Valid - when it results always True.

5) Satisfiable - For any combination of inputs if it results T atleast once.

Ex - $p \vee q$

All valid statements are satisfiable.

If insatisfiable $\rightarrow$ then it have to be contradiction

If valid $\rightarrow$ then it have to be tautology.

(1) $(p \rightarrow q) \rightarrow (q \rightarrow p)$ it is tautology

$\overline{q}+p$	
T	T
T	F
F	T
T	T

T
T
T
T

It follows demorgan's law

$$\boxed{\begin{aligned}\neg(p \vee q) &= \neg p \wedge \neg q \\ \neg(p \wedge q) &= \neg p \vee \neg q\end{aligned}}$$

$(\overline{p}+q) \rightarrow (\overline{q}+p)$

$= \overline{(\overline{p}+q)} + (\overline{q}+p)$ $[\neg(\neg p) = p]$

$= (p \cdot \overline{q}) + (\overline{q}+p)$

$= (p\bar{q} + \bar{q}) \cdot (p\bar{q} + p)$

$= \bar{q}(p+1) \cdot p(\bar{q}+1)$ } one way

$= \bar{q} \cdot 1 \quad p \cdot 1$

$= p\bar{q} + p + \bar{q}$ — or you can solve it by t.t

$= p + \bar{q}$

$= p \vee \neg q \Rightarrow T$

(this statement is tautology)

(ii) $\boxed{p \to q \equiv \neg q \to \neg p}$

$p \to q$

$= \bar{p} + q$

$\neg q \to \neg p$

$= \bar{\bar{q}} + \bar{p}$

$= p + \bar{q}$

$p \to q$ — T T F T

(a) (b)

$\neg q$	$\neg p$	$\neg q \to \neg p$	$p \to q$	$a \equiv b$
F	T	T	T	T
F	F	T	T	T
T	F	F	F	T
F	[illegible]	T	T	T

$\therefore$ this upper statement is valid, so it's a tautology also.

1) Identity ~~element~~ laws

~~Identity laws~~ $p \wedge T = p$

$p \vee F = p$

2) Domination laws

$p \vee T = T$

$p \wedge F = F$

3) Idempotent law

$p \vee p = p$

$p \wedge p = p$

4) Double negative law

$\neg(\neg p) = p$

5) Commutative law

$a \vee b \equiv b \vee a$

$a \wedge b \equiv b \wedge a$

6) Associative law

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

7) Distribution law

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \vee r)$

8) Demorgan's law

$\neg(p \vee q) = \neg p \wedge \neg q$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

9) Absorption law

$p \vee (p \wedge q) \equiv p$

$p \wedge (p \vee q) \equiv p$

10) Negation law

$p \vee (\neg p) = T$

$p \wedge (\neg p) = F$

1) $p \to q \equiv \neg p \vee q$ **

2) $p \to q \equiv \neg q \to \neg p$ *

3) $p \wedge q \equiv \neg(q \to \neg p)$

4) $p \vee q \equiv \neg p \to q$ *

5) $(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

6) $(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$

7) $(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$ **

8) $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$ **

[If want to prove, prove tautology, then this eqn are obviously valid]

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i) $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$ **

ii) $p \leftrightarrow q \equiv (\neg p \leftrightarrow \neg q)$

iii) $p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

Rules of inferences

Ex (1) It is raining today, then no match is there

r: raining today

m: no match

(2) It is raining today

(1) $r \to m$ (T)

(2) r (T)

→ Premises

All premises will add by AND ($\wedge$)

$\therefore m$ (T) → Conclusion

or, $\boxed{(r \to m) \wedge r \to m}$

tautology

$(\bar{r} + m)\, r \to m$

$= mr \to m$

$= \overline{mr} + m$

$= \bar{m} + \bar{r} + m$

$= 1$

$(r \to m)\, r \to m$

$= \overline{(\bar{r} + m)\, r} + m$

$= m\bar{r} + m$

$= m(\bar{r} + 1)$

$= m$

1) modus ponens

$p \to q$ (or) $(p \to q) \wedge p \to q$

p

$\therefore q$

2) modus tollens

$p \to q$ $\quad (p \to q) \wedge \neg q \longrightarrow \neg p$

$\neg q$

$\therefore \sim p$

3) Hypothetical syllogism

$p \to q$ $\quad (p \to q) \wedge (q \to r) \longrightarrow (p \to r)$

$q \to r$

$\therefore p \to r$

not equivalent
it is simplies to
? its not transitive

4) Disjunctive syllogism

$p \vee q$ (T)

$\sim p$ (T)

$\therefore q$ (T)

5) Addition

p

$\therefore p \vee q$

[p is true if you add something with p it will be p always]

6) simplification

$\dfrac{p \wedge q}{\therefore p}$ and $\dfrac{p \wedge q}{\therefore q}$

7) Conjuction

p

q

$\therefore p \wedge q$

8) Resolution

$p \vee q$

$\sim p \vee r$

$\therefore q \vee r$

$(p+q)(\bar{p}+r) \longrightarrow (q+r)$

$= \overline{(p+q)(\bar{p}+r)} + (q+r)$

$= \overline{(pr + \bar{p}q + qr)} + (q+r)$

$= \overline{r(p+q)} + q + r$

$= \bar{r} + \overline{(p+q)} + q + r$

$= (r+\bar{r}) + \overline{(p+q)} + q$

$= 1 + \overline{(p+q)} + q$

$= 1$

$(p+q)(\bar{p}+r)$

$(q+r)$

$= pr + \bar{p}q + qr$

$(q+r)$

$= (\bar{p}\bar{r} \cdot p\bar{q} \cdot \bar{q}\bar{r}) + r + q$

$= (\bar{p}+r)(\bar{q}+r)$

$(p+r)(\bar{q}+r)$

$(\bar{p}+r)(p+r)(\bar{q}+r)$

$= \overline{(p+q)} + \overline{(\bar{p}+r)} + q + r$

$= p\bar{q} + p\bar{r} + q + r$

$= p\bar{q} + p\bar{r} + q + r$

$p\bar{q} + p\bar{r} + qp + q\bar{r} + pr + \bar{p}r$

$= p + p\bar{p} + p\bar{q} + q$

$= (p+p)(r+\bar{p}) + (\bar{p}+q)$

$1 + (r+p)(p+q)$

2/11/22

$(p \to q) \wedge p \to q$

$\Rightarrow (\bar{p} + q) \wedge p \to q$

$\Rightarrow pq \to q$

$\Rightarrow \overline{pq} + q$

$\Rightarrow \bar{p} + \bar{q} + q$

$\Rightarrow 1 + \bar{p}$

$= 1$

1 means tontology.

(1) $p \to q$
(2) p

$\therefore q$

As (1) and (2) → Conclusion

So then it is tontology. So this statement is correct or valid.

(1) Which one is tontology?

$((a \to b) \wedge (b \to c)) \longrightarrow (a \to c)$

in exam it should be $(\neg a \vee b) \wedge (\neg b \vee c)$

$\Rightarrow (\bar{a} + b) \cdot (\bar{b} + c) \longrightarrow (a \to c)$

$\Rightarrow (\bar{a}\bar{b} + \bar{a}c + bc) \longrightarrow (a \to c)$

$\Rightarrow (\bar{a}\bar{b} + \bar{a}c + bc) \longrightarrow (\bar{a} + c)$

$\Rightarrow \overline{(\bar{a}\bar{b} + \bar{a}c + bc)} + (\bar{a} + c)$

$\Rightarrow (a + b) \cdot (a + \bar{c}) \cdot (\bar{b} + \bar{c}) + (\bar{a} + c)$

$\Rightarrow (a + a\bar{c} + ab + b\bar{c}) \cdot (\bar{b} + \bar{c}) + \bar{a} + c$

$\Rightarrow a\bar{b} + a\bar{c} + a\bar{c}\bar{b} + ab\bar{c} + b\bar{c} + \bar{a} + c$

$= a\bar{c} + a\bar{b} + \cancel{b\bar{c}} + \bar{a} + c$

$(\bar{b} + \bar{c} + \bar{a} + c)$

$(a + \bar{b} + \bar{a} + c)$

$(a + b + \bar{a} + c)$

$= 1 \cdot 1 \cdot 1$

$= 1$

a	b	c	$a \to b$	$b \to c$	and of this 2	$a \to c$	
0	0	0	T	T	T	T	T
0	0	1	T	T	T	T	T
0	1	0	T	F	F	F	T
0	1	1	T	T	T	T	T
1	0	0	F	F	F	F	T
1	0	1	F	F	F	F	T
1	1	0	F	F	F	F	T
1	1	1	T	F	F	T	T

Propositional logics don't follow from duality.

$\underbrace{(a \to b) \wedge (b \to c)}_{\bar{a} + c} \to (a \to c)$

$\bar{a} + c$

$a \to b$
$b \to c$

$\therefore a \to c$

or $\overline{(\bar{a} + b)(\bar{b} + c)} + \bar{a} + c$

$= \overline{(\bar{a} + b)} + \overline{(\bar{b} + c)} + \bar{a} + c$

$= a\bar{b} + b\bar{c} + \bar{a} + c$

$= a\bar{b} + \bar{a} + b\bar{c} + c$

$= \bar{a} + \bar{b} + b + c = 1$

Q ?? There ~~in~~ are 2 types of people in island. 1 type is zombie ~~that~~ who always says truth and 2nd types is evil ~~that~~ who always says false. We have 2 random people A, B. We have to tell what type of people they are by given answer?

A says "B is a zombie (good one)"
B says "we 2 are of opposite type"

~~Hence A = zombie (Good one), B = evil~~

p: A is zombie, ¬p: A is evil
q: B is zombie, ¬q: B is evil

$\boxed{p, q = \text{propositions}}$

(i) Case 1

P = T, A is zombie
↓
B is zombie

The wordings of B

i q = T, T

$(p \wedge \neg q) \vee (\neg p \wedge q)$

T ← T∧T ∨ F

Case :- ¬p = T — A is evil ↓ B is evil — As we consider that p = T

(ii) Case :- 2 ¬q = T, B is zombie
↓
A is evil

As we consider that p = T so that statement is not correct.

3/11/22

First order logic / Predicate logic

a: 2+1, b: 3+1, c: 4+1 → proposition ⇒ p(n) = n+1 → Predecate logic

Propositional logic: only 1 statement / logic belongs here.

Predecate logic: collection of propositionals logics.

All the students of the class has appeared

S: Students has appeared

But we can't able to add 'all'. So where those statements — All, some, few, none are present, there pro- logic is not efficient, pre. logic should be

(i) Case 1, we consider that $p = T$

$\therefore$ A is zombie

i.e ~~A~~ always says truth.
that means B is a zombie.
that means B always says truth.
But B says that "we 2 are of opposite types". But from here we can see that A and B both are zombie,
So, it is not satisfy the assumption $p=T$.
So ~~the~~ ~~Same~~ our consideration isn't ~~correct~~.

(ii) Case 2: we consider $q = T$, i.e B is zombie who always says truth.
That means, A and B are of opposite type, ~~and A says that B is zombie~~.
and B says that they are opposite.
$\therefore$ A is evil. and A says that B is zombie, i.e. B is saying truth, that is "A is evil".
so it satisfies our assumption. $q=T$

So we can give a conclusion that B is zombie who always says truth and A is evil.

d

n

ho

B

a	b	c	$a \to b$ (p)	$b \to c$ (q)	$p \wedge q$	$a \to c$ (r)	$(p \wedge q) \to r$
0 F	0 F	0 F	T	T	T	T	T
F 0	0 F	ϕ T	T	T	T	T	T
F 0	1 T	0 F	T	F	F	T	T
F 0	1 T	1 T	T	T	T	T	T
T 1	0 F	0 F	F	T	F	F	T
T 1	0 F	1 T	F	T	F	T	T
T 1	1 T	0 F	T	F	F	F	T
T 1	1	1 T	T	T	T	T	T

(:

applied here.

Quantifiers
↓
(numbers)

$\forall$ → for all

$\exists$ → there exists, for a few

So, for that prev statement, we have to write

$\forall x \; S(x)$

appeared

• Some of the students have appeared the class.

$S(x) = x$ has appeared the class

$\exists x \; S(x)$ or $\neg \forall x \; S(x)$

There exists none, append the class

$= \neg \exists x \; S(x)$ or $\forall x \neg S(x) =$

4) none of the students have appeared in the class.

$\boxed{\text{Demorgan's theorem} - \quad \forall x \neg S(x) = \neg \exists x \; S(x)}$

5)

$f(x): x+1 > x, \; \forall x$ [$\forall$ → as for all x the $f(x)$ will be always true]

$P(x): x > 3, \; \exists x$ [$\exists$ → as not for all x the $P(x)$ is true, so we have to give $\exists$]

• All the students of class have studied DS.

$S(x)$: x is a student of the class

$D(x)$: x has studied DS

$\forall x (S(x) \longrightarrow D(x))$

• Some of the students of the class have studied DS.

$S(x)$: x is a student of the class

$D(x)$: x has studied DS

$\exists x (S(x) \wedge D(x))$

$\forall x (S(x) \to D(x))$

$\equiv \forall x (\neg S(x) \vee D(x))$

How to break

- Everyone in the Class is perfect.
  $S(x)$: x is a student of the Class.
  $P(x)$: x is perfect

1 $\exists x\,(S(x) \land P(x))$ ~~$\forall x(S(x) \rightarrow P(x))$~~ in the Class

- one of the Students has studied DS.

2 $\exists x\,(S(x) \land D(x))$

- $\forall x\,(S(x) \longrightarrow \neg D(x)) =$
  Nobody of the ~~Cal~~ Class has studied DS.

- None of this Class has studied DS. $\equiv$ There doesn't exist anyone in the Class who has studied DS.

4 ~~$\neg\forall x\,(S(x) \rightarrow D(x))$~~

- $= \neg\exists x\,(S(x) \land D(x))$

$\forall x\,(S(x) \longrightarrow \neg D(x))$

5 $\equiv \forall x(\overline{S(x)} + \overline{D(x)})$

$\equiv \forall x\,(\overline{S(x) \cdot D(x)})$

$=$

$\equiv \forall x(\neg S(x) \lor \neg D(x))$

$\equiv \forall x(\neg(S(x) \land D(x)))$

$\equiv \neg\exists x\,(S(x) \land D(x))$

this two Statements are Same.

- $\exists x\,(S(x) \land \neg D(x))$ $\equiv$ Some of the Students of the class have not studied DS.

- $\exists x(\neg S(x) \land D(x))$ $=$ There exist few people outside of the Class has studied DS.
  ~~$\equiv \neg\forall x\,(S(x) \land D(x))$~~

- $\exists x\,(\neg S(x) \land \neg D(x))$ $\Rightarrow$ There exist few people outside of the Class has not studied DS
  $= \neg\forall x\,(S(x) \lor D(x))$
  There exist noone outside of the Class has not studied DS.

$\boxed{\forall D(x) = \neg D(x)}$

- None of my friends are perfect.
  $F(x)$: x is a ~~fr~~ friend of mine
  $P(x)$: x is perfect

$= \neg\exists x\,(F(x) \land P(x))$

or $\forall x\,(F(x) \longrightarrow \neg P(x))$

$\boxed{\text{note: } \exists \rightarrow \text{in clude } \land,\ \forall \text{ includes } \rightarrow}$

<u>Properties</u>

1) 1) $\forall x\,(A(x) \land B(x)) \equiv \forall x\, A(x) \land \forall x\, B(x)$ *

2) $\exists x\,(A(x) \lor B(x)) \equiv \exists x\, A(x) \lor \exists x\, B(x)$ *

3) $\exists x\,(A(x) \land B(x)) \longrightarrow \exists x\, A(x) \land \exists x\, B(x)$ *

~~4)~~

$\cancel{\exists x\,(A(x) \land \exists x\, B(x))}$

2) $\cancel{\forall x\,(A(x) \lor B(x)) \rightarrow}$

4) $\forall x\, A(x) \land \forall x\, B(x) \longrightarrow \forall x\,(A(x) \lor B(x))$ *

5) $\neg \forall x\, A(x) \equiv \exists x\, \neg A(x)$

6) $\neg \exists x\, A(x) \equiv \forall x\, \neg A(x)$

3 7) $\exists x\,(P(x) \rightarrow Q(x)) \equiv \exists x\, P(x) \rightarrow \exists x\, Q(x)$ *

8) $\forall x\, P(x) \rightarrow \forall x\, Q(x) \equiv \forall x\,(P(x) \rightarrow Q(x))$ *

<u>Nested Predicate</u>

$P(x, y) : x + y = 7$

4) 1) $\forall x \forall y\, P(x,y) \equiv \forall y \forall x\, P(x,y)$

~~2) $\forall x \forall y\, P(x,y) \equiv \exists y\, \exists$~~

2) $\exists x \exists y\, P(x,y) \equiv \exists y \exists x\, P(x,y)$

~~3)~~ $\forall x \forall y\, P(x,y)$

5) 4) $\exists y \forall x\, P(x,y) \longrightarrow \exists y \forall x\, P(x,y)$

$\exists y \forall x\, P(x,y) \longrightarrow \forall x \exists y\, P(x,y)$

~~5) $(\beta \rightarrow \exists x$~~

$\{a, b\}$

$(\beta \longrightarrow \exists x\, \alpha(x) \longrightarrow \forall x\,(\beta \rightarrow \alpha(x))$

is it valid (tautology) or not?

Considering

$\alpha(a) = T$

$\alpha(b) = F$

$\beta = T$

$\beta \longrightarrow \alpha(a) \lor \alpha(b)$: $\beta = T$, $\alpha(a) = T$, $\alpha(b) = F$, $\alpha(a) \lor \alpha(b) = T$, $\therefore T$

$(\beta \rightarrow \alpha(a)) \land (\beta \rightarrow \alpha(b))$: $T \rightarrow T = T$, $T \rightarrow F = F$, $T \land F = F$

$T \longrightarrow F \quad \therefore F$

Not a valid statement.

$\boxed{\begin{array}{l}\text{mine} \\ \beta \rightarrow \exists x\, \alpha(x) = \\ \beta \rightarrow \alpha(a) \lor \alpha(b)\end{array}}$

2/11/22

Recursive mathematical function / Recurrence Relation

- Basic step
- recursive step

fact

$a_n = n * a_{n-1}$ (Recursive)

$= 1, \; n = 0, 1$ (Basic)

fibo

$a_n = 0, \; n = 1$ (the first element of fibonacci series)

$= 1 \; n = 2$

$= a_{n-1} + a_{n-2}$

mathematical functions =

$[a_n - a_{n-1} - a_{n-2} = 0]$

1) $a_n + a_{n-1} + a_{n-2} = 0$
2) $a_n - 2a_{n-1} + a_{n-2} = 0$ } Homogenous
3) $a_n + 2a_{n-1} = n + 3 \rightarrow$ Non homogenous

Here no recursive term is present so it is in that side.

4)

5)

Degree of eqn → (1) deg = n-n+2 eqn = 2 (2 roots)

(3) n-n+1 = 1 (1 root)

Degree

1) $a_n = c \cdot r^n, \; c \neq 0$

$c r^n + c \cdot r^{n-1} + c \cdot r^{n-2} = 0$ (Auxiliary eqn)

$\Rightarrow r^n + r^{n-1} + r^{n-2} = 0$

$\Rightarrow r^n \left(1 + \frac{1}{r} + \frac{1}{r^2}\right) = 0$

$\Rightarrow r^2 + r + 1 = 0$ (from there we can calculate the root)

$r = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot 1}}{2 \cdot 1}$

$= \frac{-1 \pm \sqrt{-3}}{2}$

$= \frac{-1 \pm \sqrt{3}i}{2}$ (α - root)

$= -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$\alpha = -\frac{1}{2}, \; \beta = \frac{\sqrt{3}}{2}, \; \rho = \sqrt{\frac{1}{4} + \frac{3}{4}} = 1, \; \theta = \tan^{-1}\left(\frac{\sqrt{3}}{2} / -\frac{1}{2}\right) = -\frac{\pi}{3}$

Comparing soln $a_n = \rho^n (A \cos n\theta + B \sin n\theta)$

2) $a_n = c \cdot r^n \quad c \neq 0$, given $a_1 = 2, a_2 = 3$

$c r^n - 2 c r^{n-1} + c r^{n-2} = 0$

$\Rightarrow r^n - 2 r^{n-1} + r^{n-2} = 0$

$\Rightarrow r^n \left(1 - \frac{2}{r} + \frac{1}{r^2}\right) = 0$

Left page

$a_1 = c_1 + c_2,\ n=1$

$\Rightarrow c_1 + c_2 = 2$ — (i)

$c_1 + 2c_2 = 3$, $n=2$ — (ii)

1) $\Rightarrow p^2 - 2p + 1 = 0$

$\Rightarrow (p-1)^2 = 0$

$\Rightarrow \alpha = 1, 1$ (α = root)

$\therefore a_n = (c_1 + c_2 n) 1^n = \boxed{c_1 + c_2 n}$ (temporary solution)

$c_1 + c_2 = 2$, $c_1 + 2c_2 = 3$ $\Rightarrow -c_2 = -1$ $\Rightarrow c_2 = 1$, $c_1 = 1$

$\boxed{a_n = 1 + n}$ (particular solution)

For one root (α), $a_n = A(\alpha)^n$

2) For many distinct roots $= A\alpha^n + A\beta^n + \cdots$

For n / more than one same roots $= (c_1 + c_2 n)\alpha^n$ (for $n=2$)

(for $n=3$) $[(c_1 + c_2 n + c_3 n^2)\alpha^n]$

3)

16/11/22

4) $a_n - 2a_{n-1} + a_{n-2} = 3$, $a_0 =$, $a_1 =$ → Previous class

$C \cdot p^n$, $C \neq 0$

$C \cdot p^n - 2 \cdot C \cdot p^{n-1} + C \cdot p^{n-2} = 0$

$p^2 - 2p + 1 = 0$

$(p-1)^2 = 0$

$\alpha = 1, 1$

5)

For 2 same roots and 1 distinct root — $C(c_1 + c_2 n)\alpha^n + C \cdot \beta^n$

Right page

If we are getting $\alpha + i\beta$, $\alpha - i\beta$

$\therefore a_n = \rho^n (A\cos n\theta + B\sin n\theta)$ (temporary soln)

where, $\rho = \sqrt{\alpha^2 + \beta^2}$, $\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$

$a_0 = 2$, $a_1 = 3$ (given)

$a_0 = A = 2$

$a_n = 1^n(A\cos n\theta + B\sin n\theta) = 1^n\left(A\cos n\left(-\frac{\pi}{3}\right) + B\sin n\left(-\frac{\pi}{3}\right)\right)$

$= 1^n\left(A\cos\frac{n\pi}{3} - B\sin\frac{n\pi}{3}\right)$

$a_1 = 1\left(A\cos\frac{\pi}{3} - B\sin\frac{\pi}{3}\right) = 3$

$\Rightarrow 2 \cdot \frac{1}{2} - B \cdot \frac{\sqrt{3}}{2} = 3$; $\Rightarrow \frac{\sqrt{3}}{2}B = 1 - 3$; $\Rightarrow \frac{\sqrt{3}B}{2} = -2$ $\Rightarrow B = -\frac{4}{\sqrt{3}}$

$\therefore$ Particular soln — $a_n = (2\cos n\theta - \frac{4}{\sqrt{3}}\sin n\theta)$

1) $a_n - a_{n-1} + a_{n-2} = 0$

$\Rightarrow C \cdot p^n - C \cdot p^{n-1} + C \cdot p^{n-2} = 0$

$\Rightarrow p^n - \frac{p^n}{p} + \frac{p^n}{p^2} = 0$

$\Rightarrow p^2 - p + 1 = 0$

$\frac{1 \pm \sqrt{1-4}}{2}$

$A = \frac{1 \pm \sqrt{3}i}{2}$

$= \frac{1}{2} \pm \frac{\sqrt{3}}{2}i$

$a_n = r^n(A\cos n\theta + B\sin n\theta)$, $\alpha = \frac{1}{2}, \beta = \frac{\sqrt{3}}{2}$

$r = 1$, $\theta = \tan^{-1}\left(\frac{\sqrt{3}/2}{1/2}\right) = \frac{\pi}{3}$

$a_n = 1^n\left(A\cos\frac{n\pi}{3} + B\sin\frac{n\pi}{3}\right) \rightarrow$ temporary sol^n

$a_n - a_{n-1} - a_{n-2} = 0$

(2) $\Rightarrow r^2 - r - 1 = 0$

$r = \frac{1 \pm \sqrt{1+4}}{2}$

$= \frac{1 \pm \sqrt{5}}{2}$

$\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}$

$a_n = A\left(\frac{1+\sqrt{5}}{2}\right)^n + B\left(\frac{1-\sqrt{5}}{2}\right)^n \rightarrow$ temporary sol^n

Non-homogeneous eq^n

(3) $a_n + 2a_{n-1} = n+3$ [If in $n+3$ we had any constant (let constant $= A$), then $a_n^{p} = A$]

$a_n + 2a_{n-1}$

say $a_n^{(h)}$ / say $a_n = c \cdot r^n$

$a_n^{(h)} = c \cdot r^n, c \neq 0$

temporary sol^n for homogeneous part

$a_n^{(h)} = c \cdot r^n + 2 \cdot c r^{n-1} = 0$

$\Rightarrow r^n\left(1 + \frac{2}{r}\right) = 0$

$\Rightarrow r + 2 = 0$

$\Rightarrow r = -2$

temporary sol^n $a_n^{(h)} = A(-2)^n$

particular sol^n for non-homogeneous part -

$a_n^{(p)} = An + B$,

$(An+B) + 2[A(n-1) + B] = n+3$

$\Rightarrow An + B + 2[An - A + B] = n+3$

$\Rightarrow An + B + 2An - 2A + 2B = n+3$

$\Rightarrow 3An + 3B - 2A = n+3$

$\Rightarrow 3A = 1, 3B - 2A = 3$

$\Rightarrow A = \frac{1}{3}$, $\Rightarrow 3B - 2 \times \frac{1}{3} = 3$

$\Rightarrow 3B = 3 + \frac{2}{3} = \frac{11}{3}$; $B = \frac{11}{9}$

Non homogeneous eq^n

particular sol^n (How to write)

$a_n^{(p)} = A$ for $a_n + 2a_{n-1} - 2 = 0$

$a_n^{(p)} = (An^2 + Bn + C)$, for $a_{n-1} + 2a_{n-1} = n^2 + 3$

$a_n^{p} = (An+B)$, for, $a_{n-1} + 2a_{n-1} = n+3$

temp sol -

$a_n = A(-2)^n + \frac{n}{3} + \frac{11}{9}$

given $a_0 = 1$

$a_0 = 1$

$\Rightarrow A + \frac{11}{9} = 1$

$A = 1 - \frac{11}{9} = -\frac{2}{9}$

$\therefore a_n = -\frac{2}{9}(-2)^n + \frac{n}{3} + \frac{11}{9}$

Numericals Do by yourself

DM Previous years qn

1) Gm-A

a) All complete graphs are regular.

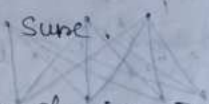
b) Every poset is a Lattice.
(also a complete semilattice)

c) A relation which is not reflexive is called as irreflexive relation. T

d) Degree of any vertex in a cyclic graph is 2.

e) Consider a simple graph G with k components. If each component has $n_1, n_2, \ldots, n_k$ vertices, then maximum number of edges in G is $\frac{(n-k+1)(n-k)}{2}$.

f) A graph with chromatic number 2 is a bipartite graph for sure.



g) Every regular graph is Euler graph. F.

h) Let ϕ be an empty set and $P(\phi)$ be a power set of ϕ then the cardinality of $P(P(P(\phi)))$ is $2^{2^{2^1}} = 16$.

• i) If a simple graph G with n vertices is isomorphic to its Complement G', then value of n or (n-1) must be multiple of ____.

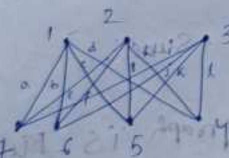
ii) Consider the binary relations R and S on non-empty set A. Both R and S are reflexive. Then R∪S is also reflexive. T

Gr-B

2) Find number of spanning trees in $K_{3,4}$. Find the vertex cover and edge cover for the same graph. Is it a planner graph.

the no. of spanning trees in $K_{3,4}$ - $3^{4-1} \times 4^{3-1} = 3^3 \times 4^2 = 27 \times 16 = 432$.

$K_{3,4}$ =



V. connections from $K_{m,n}$ = 2 diff sets are 2 v. conn always

vertex cover = {1, 2, 3}, {7, 6, 5, 4},
edge cover = {a, b, g, l}, {a, f, g, l}, {a, j, g, l}
{b, f, g, l} and so on.

may be $\frac{n}{2}+1=$ $\frac{n}{2}+1=2$

A graph is called planner g. when it can be converted into a plan graph. A plan graph is that where there are no edges crossings but obey all the properties of the graph from which it is converted (often main g.)
But we know, that if $K_{m,n}$ ~~is a complete~~ where $K_{m,n}$ is a complete bipartite graph if m, n both are ⩾ 3 then it isn't possible to draw a plan graph of this graph. So, clearly it isn't planner graph.

3) Find the sum of degrees of all vertices in any wheel graph of order 'n' (W_n)?

Wheel graph (W_n) of order n is a type of graph, which is the sumation of C_{n-1} i.e a cyclic graph of order n-1 and all the vertices are connected to the center vertex, i.e

$W_n = C_{n-1}$ + adding all the vertices from the center vertex

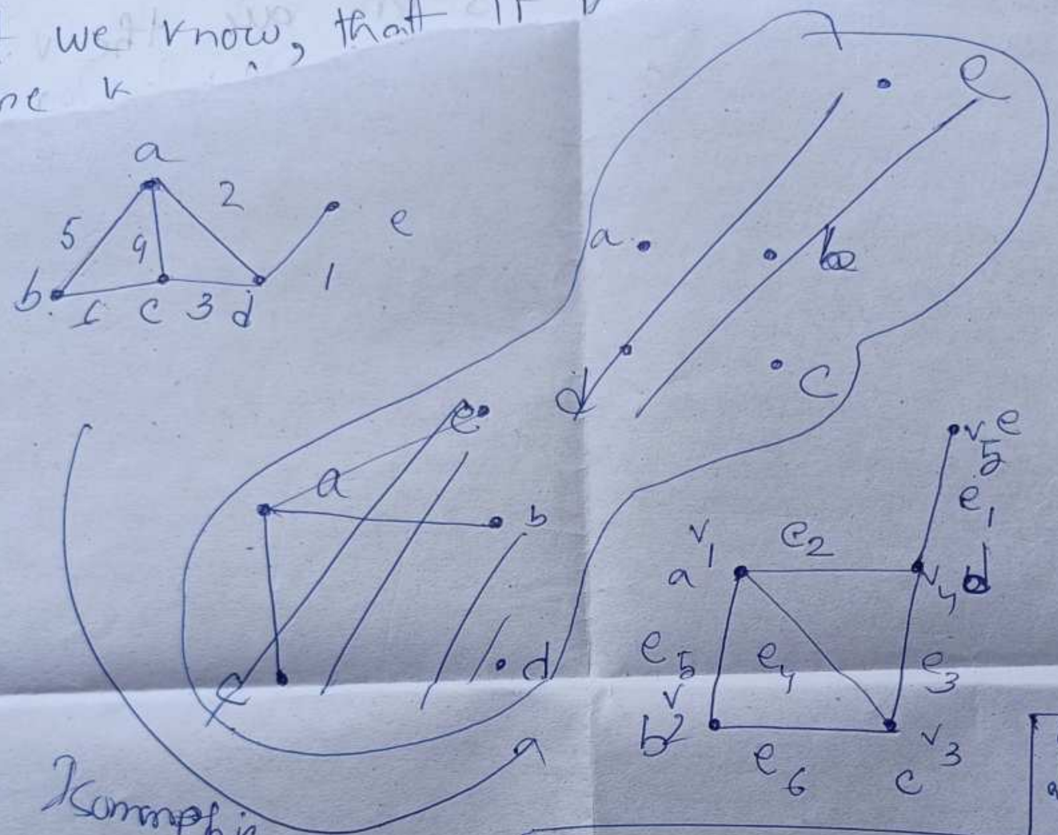
Example - The diagram of a wheel graph -

$W_4 = C_3$ + adding all the v. from the center v.

It is converted (cFAan main g.)

But we know, that if v

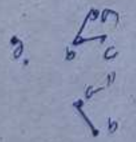
where k



Complement g.

$G_n' = K_n - G_n$

Isomorphic graph of this graph (maintain all properties of main graph) deg, edges, vertices



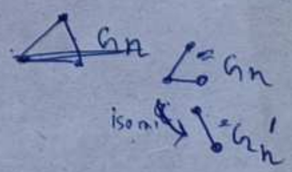
Q.n) i) G_n = simple g.

G_n' = complement g. of the simple g.

and G_n is isomorphic to G_n'

G_n, no of v. = n

G_n' = no. of v. n-1



$K_{3,4}$.

er for

ners graph

3, 4

4 32.

(own) = for $K_{m,n}$, 2 diff sets and covers always

5, 4}

, j, g,}

so on.

K_4 =

the sum of degrees of all the v. i.e. total degree $= (n-1)\cdot 3 + 1\times(n-1)$

$= 3n-3+n-1$

$= 4n-4$

$= 4(n-1)$

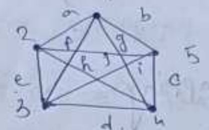
$\therefore$ Total degree $= 4(n-1)$

4) How many "Hamiltonian-cycles" possible for complete graph of order 5?

Firstly, a graph is hamiltonial-cycle is that which includes all the edges of that graph, (but there should not be any vertex repetition) vertices / without any v. repetition. The complete graph is the maximum version of simple graph. there should not be any self-loop, parallel edges and there should be all possible edges [between {a pair of vertices}], i.e all the v. should be connected

with the remaining v. by a single edge. It is denoted by K_n, where n is the order of the graph.

$\therefore K_5$ =



no. of hamiltonial-cycles possible for K_5 -

1 a 2 e 3 d 4 c 5 b 1

1 b 5 c 4 d 3 e 2 a 1

$\frac{(n-1)!}{2}$ where n is the order of the g.

$= \frac{(5-1)!}{2}$ or $\frac{n!}{n\times 2} = \frac{5!}{5\times 2} = \frac{120}{10} = 12$

$= \frac{24}{2}$

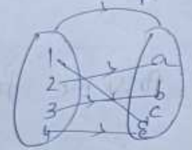
$= 12$

logic

K_5 has 12 distinct hamiltonial cycles, since every permutation of the 5 v. determines a hamiltonial cycles, but each cycle is counted 10 times due to symmetry (5 possible starting points * 2 directions)

5) How many one-to-one and onto functions are possible for functions from set A to set B, where $|A|=m$ and $|B|=n$?

one-one function / mapping :



A B

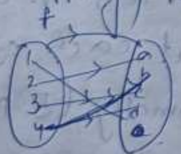
x $f(x)$

If $x_1 \neq x_2$

then $f(x_1) \neq f(x_2)$

then it's called one-one mapping. Each and every elements be of domain should have distinct image. Not more than one pre-image in codomain. There should

onto mapping / function :



When the range (the ~~no.~~ set of those elements in co-domain which have ~~a distinct~~ pre-image in domain) is equal to no of elements in co-domain and every elements of co-domain have pre-images, then this type of mapping is called onto mapping / function.

The functions possible from a set A to B, where $|A|=m$, $|B|=n$ is—

for one-one mapping,

no. of one-one functions possible,

$= 0, \quad n<m$

$= n(n-1)(n-2) \dots (n-m+1), \quad n \geq m$

$= {}^nP_m$

for onto mapping,

no. of onto functions possible $= \sum_{i=0}^{n-1} {}^nC_i (n-i)^m$

[derivation - done]

6) For a relation defined as $R: A \to A$, where $|A|=11$. How many reflexive, not-reflexive, symmetric and anti-symmetric relations possible?

Reflexive R = $\forall a\ (a,a) \in R$ where a is an element of set A $[a \in A]$, this type of relation is called reflexive R.

The no. of reflexive relations possible ~~for R:A→A~~ on set A where $|A| = 11$,

$2^{11^2-11} = 1.3 \times 10^{-33}$ or 2^{110}

[Ex-optional]

Non- reflexive / irreflexive - for all $a \in A$, if $(a,a) \notin R$ where $a \in A$, then this type of relation is called irreflexive relation. The no. of i. r. possible on set A

$\boxed{\text{Ex- optional}}$

$= 2^{11^2 - 11} = 2^{110}$

Symmetric r = If $(a,b) \in R$, then (b,a) must have belongs to R i.e. $(b,a) \in R$ where $a,b \in A$. If this happen then this type of relation is called symmetric r. The no. of s. r. possible on set A

$= 2^{\frac{11^2+11}{2}}$ $\boxed{\text{Ex- optional}}$

$= 2^{66}$

$= 7.3 \times 10^{19}$

anti-symmetric r - If $(a,b) \in R$ and $(b,a) \in R$, where $a,b \in A$ then a must have equal to b i.e. $a=b$, if it happens then this type of relation is called a. s. r. The no. of a. s. r. possible on set A

$= 2^{11} \times 3^{\frac{11^2-11}{2}}$

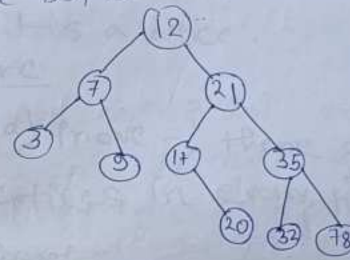
$= 2^{11} \times 2^{55}$

$= 2^{66}$

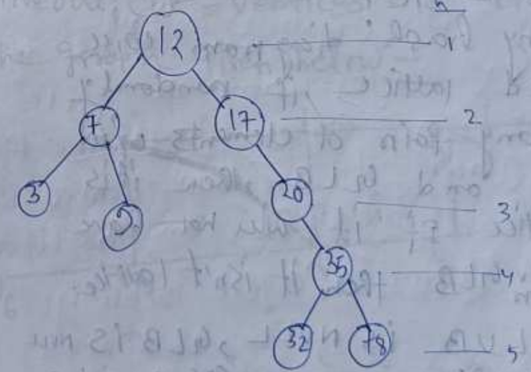
$= 7.3 \times 10^{19}$

7) Insert values given below (in the order) and form the binary search tree (BST) - values are (in order): 12, 21, 17, 7, 20, 3, 35, 32, 9, 78.
Delete 21 then mention the height of new BST.

The BST is-



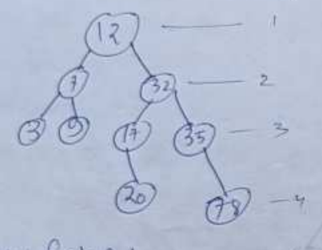
After deleting 21 - the values are 12, 17, 7, 20, 3, 35, 32, 9, 78



New height = 5

the inorder traversal of this BST is—
3 7 9 12 17 20 21 32 35 78
we have to delete 21, it has 2
child nodes, so we will replace
21 with its immidiate successor. (inorder)

∴ Now BST is,



new height = 4

8) Which of these Hasse diagram is/are not Lattice?

Lattice - Any hash diagram will be called lattice, if randomly choosen any pair of elements will have both LUB and GLB, then it is called lattice. If it will not have ~~and~~ LUB ~~and~~ or GLB then it isn't lattice.

1) for 1st case, LUB is NULL, GLB is NULL
(as f's covering, but d's covering) (as a and c are in same level)

∴ it isn't a lattice. for {b, f}

2) for 2nd case,
LUB = NULL (as f, g are in same level) for {f, g}
GLB = a
∴ it isn't lattice

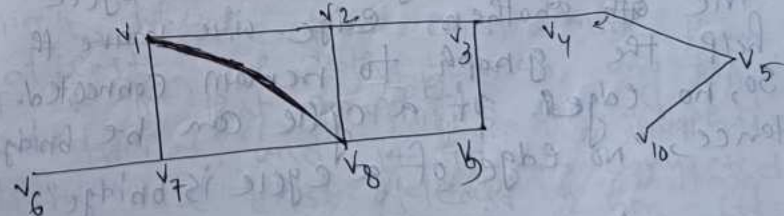
3) for case 3,
for {d, e}
LUB = f, GLB = a
∴ it is a lattice.

GrC

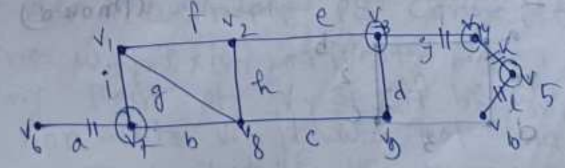
9) a) Prove — "there are at-least 2 non-cut vertices in every non-trivial connected graph".

b) Prove — "No edge of a cycle is bridge".

c) Findout cut-vertices and bridges in the graph given below —



c)



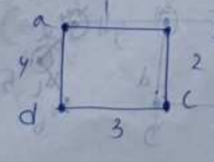
Bridges = a [{v_6, v_7}], j [{v_3, v_4}],
k [{v_4, v_5}], l [{v_5, v_{10}}]

cut-vertices = v_7, v_4, v_5 [By using the theorem,
if, e = {u, v} is a bridge and, deg [u or v] > 2, then u or v = cut - v]

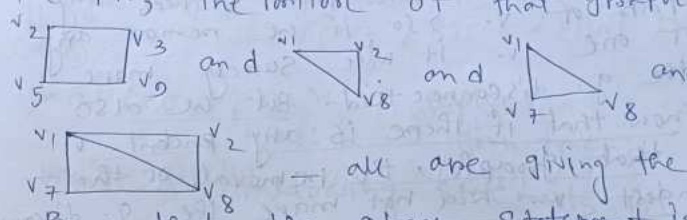
we know that the
2). cycle contains two paths or two ways, i.e. we can represent a cycle by two directions, that means a cycle have 2 ways. ~~If we~~ we know that if ~~we remove~~ a edge the removal of ~~one~~ a edge makes a connected g. disconnected then it, is called (the edge) bridge or cut-edge. So, if we ~~del~~ remove one edge from a cycle the ~~oth~~ another edge will ~~make~~ help the graph to remain connected.
So, no edges of a cycle can be bridge.
Hence, "No edge of a cycle is bridge".
(Proved)

By example -

C_4 =



If we, will remove any of edge from that graph it will remain connected still.
on the other hand the graph given in the qn, the portion of that graph -



— all are giving the
Prove that the above statement is correct, as no edges of ~~that~~ any of that graphs are bridge and all of the graphs are cycle.

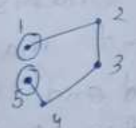
a) we know that the non-cut v. means the set of v. ~~the~~ which's removal don't makes a connected g. disconnected and the non-trivial graph is which's order is more than 1 and ~~edo~~ size is more than 0 and the connected g. is that graph where there exist ~~more than~~ atleast one path between any pair of v. in that graph. As the graph is non-trivial and it is ~~dis~~ connected there will exist atleast 2 non-cut v, as the graph is connected graph there will exist atleast one path between every pair of v, so if we will remove one non-cut v. it will not make the g. disconnected, but if we remove any 2 non-cut

v. it will surely make the g. disconnected [we cut-set vertex]]X.

(*) As the g. is connected g., there exist atleast one path between any pair of v., so if we remove any of one v. it will surely make the g. disconnected. But we also know that if there is any pendent v. in that graph, the removal of that pendent v. will not make the g. disconnected.

[And by this we can get at-least 2 vertices ~~wh~~ or can say two non-cut vertices as path has two directions, we can see this prove by an example also.]

a conn g with vertex may



[As a connected g. with pendant v. may has 2 possible directions we will get atleast 2 such v. whose removal will not make the g. disconnected or we can say we will get 2 non-cut v.]

Here 1 and 5 are 2 non-cut vertex, and the graph is non-trivial connected graph. Hence the above statement is proved.

10) When a graph will be called as a "planar" graph? Generate the below mentioned equations (to decide planarity of a graph) with the conditions and give one example of each (here e - size and n - order of graph.)

a) $e <= 3n-6$

b) $e <= 2n-4$

done

(Added, definition of max planner g., region, deg of region, triangulation, rectangulation)

(done)

11) a) Let $P(x)$: x is perfect

$F(x)$: x is your friend

write the following statements in symbolic form (use quantifiers)

I) No-one is perfect -

$\neg\exists x(\neg F(x) \land \neg P(x))$ or $\neg\forall x(F(x) \lor \neg P(x))$

II) All of your friends are perfect.

$\forall x(F(x) \rightarrow P(x))$

III) None of your friends are perfect -

$\neg\exists x(F(x) \land P(x))$ or $\forall x(F(x) \rightarrow \neg P(x))$

b) mention these equations are tautology or not - (Don't use truth table).

I) $((P \rightarrow R) \land (Q \rightarrow R)) \rightarrow ((P \land Q) \rightarrow R)$

II) $((P \rightarrow Q) \land (P \rightarrow R)) \rightarrow (P \rightarrow (Q \land R))$

1) $((P \rightarrow R) \land (Q \rightarrow R)) \rightarrow ((P \land Q) \rightarrow R)$

$\neg\big((\neg P \vee R) \wedge (\neg Q \vee R)\big) \vee (\neg(P \wedge Q) \vee R)$

$\equiv \neg(\neg P \vee R) \vee \neg(\neg Q \vee R) \vee \neg(P \wedge Q) \vee R$

$\equiv (P \wedge \neg R \vee Q \wedge \neg R) \vee P \wedge \neg Q \vee R$

$\equiv (P \wedge \neg R \vee \neg(P \wedge Q)) \vee Q \wedge \neg R \vee R$

$\equiv (\neg R \vee \neg(P \wedge Q)) \wedge (P \vee \neg(P \wedge Q)) \vee (\neg R \vee R) \wedge (Q \vee R)$

$\equiv (\neg R \vee \neg(P \wedge Q)) \wedge (P \vee \neg P \vee \neg Q) \vee (Q \vee R) \wedge T$

$\equiv \neg R \vee \neg P \vee Q \vee R \vee \neg Q$

$\equiv T + T + \neg P$

$\equiv T$

$\therefore$ It is tautology (md)

ii) $((P \leftrightarrow Q) \wedge (P \rightarrow R)) \rightarrow (P \rightarrow (Q \wedge R))$

$\equiv \neg\big((\neg P \vee Q) \wedge (\neg P \vee R)\big) \vee (\neg P \vee (Q \wedge R))$

$\equiv \neg(\neg P \vee Q) \vee \neg(\neg P \vee R) \vee \neg P \vee (Q \wedge R)$

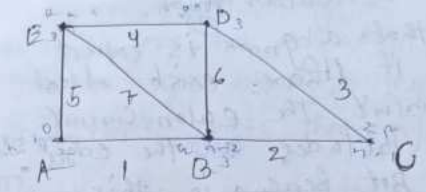
$\equiv (P \wedge \neg Q) \vee \neg P \vee (P \wedge \neg R) \vee (Q \wedge R)$

$\equiv (\neg Q \vee \neg P) \wedge (P \vee \neg P) \vee ((P \wedge \neg R) \vee Q) \wedge ((P \wedge \neg R) \vee R)$

$\equiv T \cdot (\neg Q \vee \neg P) \vee (P \vee Q) \wedge (\neg R \vee Q) \wedge (P \vee R) \wedge (\neg R \vee R)$

(1) PTO

1Q) Consider the graph given to answer the Qns below—



a) Find chromatic polynomial and mention chromatic number (x number of colors are given for colouring).

b) Define Euler graph. IS is Euler graph? Give the reason for your answer.

c) Define Hamiltonial graph. IS it Hamiltonial graph? Give reasone for your answer.

a) $P(x) = x(x-1)^3 (x-2)$

if $x = 3$, then $P(3) = 3 \times (3-1)^3 \times (3-2)$

$= 3 \times 8 \times 1$

$= 24$

$\therefore$ the chromatic no is 3, i.e. we need 3 colours to colours that above graph and we can colours it in 24 possible ways.

$\neg[(\neg P \vee R) \wedge (\neg Q \vee R)] \vee (\neg(P \wedge Q) \vee R)$

$= \neg(\neg P \vee R) \vee \neg(\neg Q \vee R) \vee \neg(P \wedge Q) \vee R$

$= (P \wedge \neg R \vee Q \wedge \neg R) \vee P \wedge \neg Q \vee R$

$= ($

11) $((P \to Q) \wedge (P \to R)) \to (P \to (Q \wedge R))$

$= \overline{(\bar{P} + Q) \cdot (\bar{P} + R)} + (\bar{P} + QR)$

$= \overline{(\bar{P} + Q)} + \overline{(\bar{P} + R)} + \bar{P} + QR$

$= (P\bar{Q} + \bar{P}) + (P\bar{R} + QR)$

$= (\bar{Q} + \bar{P})(P + \bar{P}) + (P\bar{R} + Q) \cdot (P\bar{R} + R)$

$= (\bar{Q} + \bar{P}) + (P + Q)(\bar{R} + Q) \cdot (P + R)(\bar{R} + R)$

$= (\bar{Q} + \bar{P}) + (P + Q)(\bar{R} + Q)(P + R)$

$= (\bar{Q} + \bar{P} + P + Q) \cdot (\bar{Q} + \bar{P} + \bar{R} + Q) \cdot (\bar{Q} + \bar{P} + P + R)$

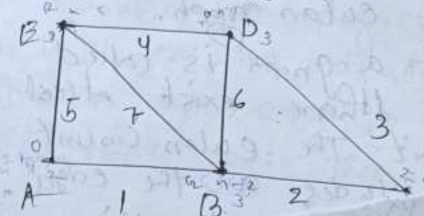
$= 1 \cdot 1 \cdot 1$

$= 1$

$= T \cdot (\neg Q \vee \neg P) \vee (P \vee Q) \wedge (\neg R \vee Q) \wedge (P \vee R) \wedge (\neg R \vee R)$

① PTO

12) Consider the graph given to answer the qns below —



$\overline{(\bar{P} + R) \cdot (\bar{Q} + R)} + (\overline{PQ} + R)$

$= \overline{(\bar{P} + R)} + \overline{(\bar{Q} + R)} + \overline{PQ} + R$

$= (P\bar{R} + Q\bar{R}) + \overline{PQ} + R$

$= (P\bar{R} + \overline{PQ}) + Q\bar{R} + R$

$= (\bar{R} + \overline{PQ})(P + \overline{PQ}) + (\bar{R} + R) \cdot (Q + R)$

$= (\bar{R} + \overline{PQ}) \cdot (P + \bar{P} + \bar{Q}) + 1 \cdot (Q + R)$

$= (\bar{R} + \bar{P} + \bar{Q}) + Q + R$

$= \bar{R} + \bar{P} + Q + R + \bar{Q}$

$= 1 + 1 + \bar{P}$

$= 1$

$= T$

and we can colour it in 24 possible ways.

b) done.
No it isn't a eulan graph.

As we know that a graph is called eulan g. if there exist atleast one eulan circuit. The eulan circuit is that which includes all the edges without any repetition of that graph. But here, in this graph no eulan circuits exists there in this graph.

c) done.
yes, it is hamiltonial graph.

As by Dirac's theorem the deg of each and every v. is at least $\frac{5}{2}$ (5 is the order of the graph.) here. [Dirac's theorem: A graph will be called H-g if deg of each and every v. will at least $\frac{n}{2}$, where n is the order of the g.]

on the other hand by ore's theorem

deg(A) + deg(D)

= 2 + 3

= 5 which is equal to the order of the graph (i.e = 5)

[Ore's theorem, $deg(v_i) + deg(v_j) \geq n$ where v_i and v_j are 2 vertices of that graph which ~~have~~ are not adjacent and n is the order of that graph]

[The summation of degs of any pair of v. which are not adjacent is always greater than equal to the order of the graph - ore's Theorem].

(4) (11)

$\equiv (\neg Q \vee \neg P) \vee (P \vee Q) \wedge (\neg R \vee Q) \wedge (P \vee R)$

$\equiv (\neg Q \vee \neg P \vee P \vee Q) \wedge (\neg Q \vee \neg P \vee \neg R \vee Q)$

$\wedge (\neg Q \vee \neg P \vee P \vee R)$

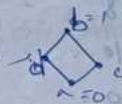
$\equiv T \cdot T \cdot T$

$\equiv T$

I-I is a tautology (proved)

• [As every elements are interconnected, there must have GLB & LUB for every pair of elements (i.e. complement)]

(13) (a) Complement of any element means



if $a \vee b = 1$ [join] [can be any no.]

$a \wedge b = 0$ [meet] [can be any no.] then a is a complement of b. [a and b are the element of the lattice]

So, here all the elements of this lattice has complement. So it is a complemented lattice. [As every elements are connected to rest of the elements of this lattice, and if the elements are connected it must have GLB and LUB].

13) Consider the set $D_{40} = \{1, 2, 4, 5, 8, 10, 20, 40\}$ and the relation divides (/) make a poset $(D_{40}, /)$.

a) Draw the Hasse Diagram of the poset.

b) Find upper Bound and Lower bound of {8, 10} and mention GLB and LUB.

c) Find join and meet for {4, 5}

d) Is it a Lattice? If yes then is it Complementary Lattice?

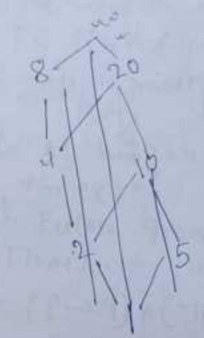
$D_{40} = \{1, 2, 4, 5, 8, 10, 20, 40\}$

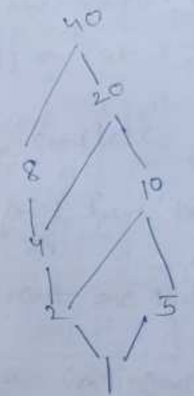
Po-set is = $(D_{40}, /)$

2	is the covers of	1
4	. . .	2
5	. . .	1
8	. . .	4
10	. . .	2, 5
20	. . .	4, 10
40	. . .	8, 20

2/1, 4/2, 8/1, 8/2, 8/4 = 2, 4/2 = 2, 4 divide division, 20/2 = 10, 20/4 = 5, 20/10 = 2, 10/5 = 2, 40/2 = 20, 40/4 = 10, 40/5 = 8, 40/8 = 5, 40/10 = 4, 40/20 = 2, 40/10 = 4

∴ a) hasse diagram is —





b)

	UB	LUB (join) (∨)	LB	GLB (meet) (∧)
{8, 10}	{40}	{40}	{2, 1}	{2}
c) {4, 5}	{20, 40}	{20}	{1}	{1}

[GLB = meet = ∧
LUB = join = ∨]

∴ c) join (LUB) = {20}
meet (GLB) = {1}

d) yes it is lattice.
we know that if there exist atleast one complement for every elements in the lattice then this lattice is called complemented lattice.

← P.T. before

Dm Previous yr Qn (2020)

Gp-A

1) a) All Regular graphs are simple. F

b) The relation $R = \{(a,b), (b,a)\}$ on set $X = \{a,b\}$ is irreflexive. T

c) All Strictly B.T. are complete B.T. F

d) A bridgeless graph can't have cut vertex. F

e) Euler graph has at least one Euler Trail. F (semi-euler g.)

f) $(p \to q) \wedge (\neg p \leftrightarrow q)$ is a Contingency. F (Contradiction)

g) Any graph with chromatic number 2 is a tree. F

h) If $|A| = n$ then $|P(A)|$ = Cardinality of Power set of $A = 2^n$ T

i) Vertex Connectivity is always less than or equal to Edge Connectivity for any graph. T $[K(G) \le \lambda(G)]$

j) Every TO-set is a Lattice. T

$(\bar{p}+q) \cdot [(\neg p \to q) \wedge (q \to \neg p)]$

$= (\bar{p}+q)\ [(p+q) \cdot (\bar{q}+\bar{p})]$

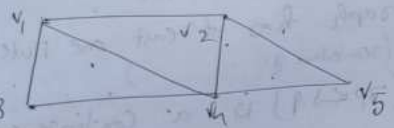
$= (p+q)(\bar{p}+q)$

$= pq + \bar{p}q + q = q$

G-C

9a) What is spanning Tree? mention some imp characteristics of spanning T. Done

b) calculate number of spanning trees possible for - complete graph - K_7, cyclic gr. - C_9 and the g given below:



no. of S.T = $K_7 = 7^{7-2} = 7^5 = 16807$

" " " $C_9 = 9$

$e = 7$ $n = 5$

" " " $= {}^7C_4 + (3+1)$

$= 35 + 4$

$= 39$

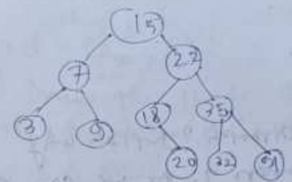
10a) Define B. S. T. - Done

b) Insert values given below (in order) and form the b. S. T.

c) Delete 22 and show BST

Values are in order: 15, 22, 18, 7, 20, 3, 35, 32, 9, 91

a)

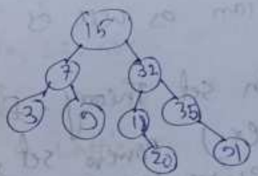


b) Inorder traversal -

3 9 7 15 18 20 22 32 35 91

After del 22 -

BST -



11) Look at the g. below and answer -

a) It is a Distributive Lattice? Explain why?

1) a b, c, d

$b \vee (c \wedge d) = b \vee d = c$ ∴ $b \vee (c \wedge d) = (b \vee c) \wedge (b \vee d)$

$(b \vee c) \wedge (b \vee d) = c \wedge c = c$ Satisfied

so it is a Distributive Lattice
the condi- $b \wedge (c \vee d) = (b \wedge c) \vee (b \wedge d)$ is
also satisfied.

so, it is a D.L.

b) Is it a planner graph? Explain why

yes, it is a Plannar g. As we can convert it into plane g. . Plane g. is that where there are no edge crossings but maintain all the properties of the ~~main g~~ g. from which it is converted. This given g. itself a plan as well as plannar g.

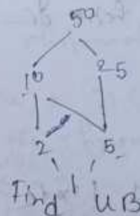
12. Define set in set Theory. Explain concepts (with ex.) Power set, subtraction of sets, compliments of any set and cartesian product of 2 sets.

Done.

Set - set theory is the branch of mathematical logic that studies sets, which can be informally described as collection of objects.

13) Consider the set $D_{50} = \{1, 2, 5, 10, 25, 50\}$ and relation divides(|) make a the Poset $= (D_{50}, |)$

a) Draw the Hash diagram of D50 with relation divides(|).



b) Find UB and LB of $\{5, 10\}$

UB = $\{10, 25, 50\}$

LB = $\{5, 2, 1\}$

d) Is it a Lattice?

It is ~~att~~ lattice as every element of this hash diagram has GLB and LUB.

c) Find GLB and LUB $\{5, 10\}$

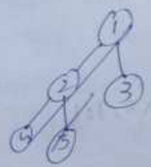
GLB = $\{\cancel{50}\ 5\}$

LUB = $\{10\}$

5-B

2) Prove that a non-empty and non-trivial simple g. has at least 2 v. of equal deg - done

3) A complete B. T. is having 62 nodes. Then calculate the height of that tree -



height = 1 · max 1 node
" = 2 " 2 "
" = 3 " 4 "
" = 4 = 8 "

~~62~~ =

the formula is $\log(n+1)$ n is the number of nodes.

∴ the minimum height is: 6

4) What is the chromatic polynomial of a complete Bipartite g. $K_{m,n}$ when 'λ' is the no. of colours given.

The chromatic ... = 2

$P(x) = x(x-1)$

The chromatic ...

$P(n) = n(n-)$

As the chromatic no. of the B. g. $K_{m,n}$ is = 2.

5) If there exists functions from set A to set B, where $|A| = m$ and $|B| = n$ then how many one to one and onto func are possible?

done

7) Show the statement given below is a tautology (without using T.T).

$(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$

$= (P+Q) \cdot (\overline{\bar{P} \cdot (\bar{Q}+\bar{R})}) \cdot (\bar{P} \cdot \bar{Q}) \cdot (\bar{P} \cdot \bar{R})$

$= (P+Q)(P+QR) \cdot (\bar{P} \cdot \bar{Q} \cdot \bar{R})$

$= (P+Q)(P+Q)(P+R)(\bar{P} \cdot \bar{Q} \cdot \bar{R})$

$= (P+Q)(P+R)\ \overline{P}\cdot\overline{Q}\cdot\overline{R}$

$= \cancel{(\overline{P}Q)(P+R)\overline{Q}\,\overline{P}\,\overline{R}}$

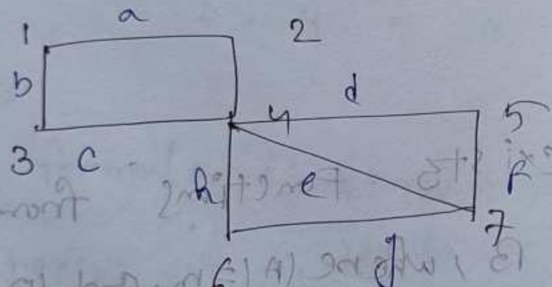
$= (Q\overline{P})(P+R)\,\overline{Q}\,\overline{R}$

$= (Q\overline{P})(P\overline{R})\,\overline{Q}$

$= 1$

$\therefore$ It is a tautology

8) Find Hamiltonial path, Hamiltonial cycle (if exists) and state whether the g. given below is Hamiltonial g. or not?



Here Hamiltonial path, Hamiltonial cycle ~~dont~~ doesn't exist as it is not a hamiltonial g. It don't follows Dirac's and Ore's theorem also.

Dm internal Qn salve

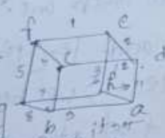
1) A graph is bipartite g. Prove that if and only if it contains no odd length cycle.

Image a bipartite g. there are many edges from one partition to other one. Assume that there is an odd-length cycle. To construct a cycle, we need to follow some edges & at the end finish at the same node we started. However, since the g. is bipartite, having an additional edge in the cycle means that we need to change the partition. we end up at the other partition if we have odd number of edges and the same partition if we have even no of edges. Also this condition is "if and only if" which means that if we have a cycle in a b.g., then we have even no. of edges in the cycle.

2) Define Eular & Hamiltonial g. Draw graph for i) Hamiltonial & Non Eularian ii) Eular but not Hamiltonial

Done

i) Ex-1



→ Hamiltonial g.

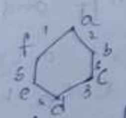
as it satisfies Dirac's theorem [the deg of each and every v of this g. is = 3 which is not $\geq \frac{n}{2} = \frac{8}{2} = 4$]

But there exist atleast one hamiltonial cycle - c 3 d 11 a 9 b 8 g 5 f 1 e 3 h 6 7 c

This doesn't follow Ore's theorem also.

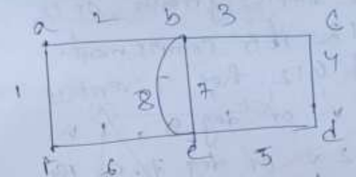
Ex-2

C_6 =



This doesn't satisfies Dirac's and Ore's theorem [Dirac's theorem, the deg of every of this g. C_6 should be ≥ 3, but the deg of each v is = 2, Ore's theorem $deg(v_a) + deg(v_d)$ should be ≥ 6, but $deg(v_a) + deg(v_d) = 4 \ngeq 6$]. But it contains a hamiltonial cycle - a 2 b 6 c 3 d 4 e 5 f 1 a

(ii)

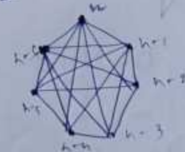


It contains a Eular circuit - e 8 b 2 a 1 f 6 e 7 3 c 4 d e. So, it is eular graph. But there doesn't exist any ~~eular~~ Hamiltonial cycle.

3) A complete g. of order '7', find chromatic polynomial for that. Determine wether it is possible to construct a graph with '12' edges such that two of the v. have degree '3' & remaing v. have degree '4'.

~~P(n) =~~

K_7 =



for K_7

$P(x) = n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6)$

the chromatic no. for $K_7 = 7$

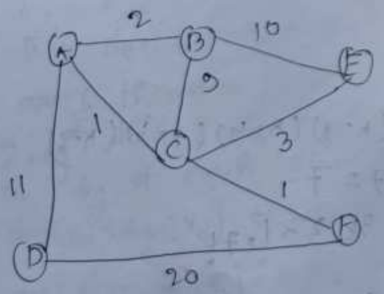
$P(7) = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 7!$

$= 5040$

∴ we can colour the g. K_7 by 7 colours and in 7! possible ways.

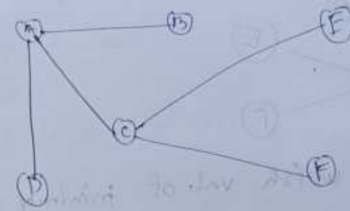
Let G12 be a simple graph of 12 vertices, and H12 its Complement. It is known that G12 has 7 vertices of deg 10, 2 v. of deg 9, 1 v. of deg. 8 and 2 v. of deg 7. So, the above qn is not possible.

5) Find minimal Spanning tree for a graph-



By Kruskal's Algo,

S-1



S-2

edges	AC	CF	AB	CE	BC	BE	AD	DF
weight	1	1	2	3	9	10	11	20

Initial Site = 0

Step-3

1) Arrange the edges by acending order.

2) 2.1 – Find the edge with least value

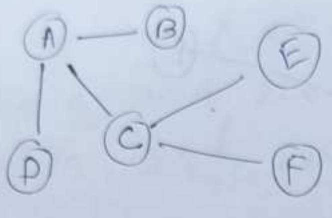
2.2. – If ~~that~~ e (size < n) Condition is Satisfied and the edge is not forming a cycle then draw the edge

2.3. site++

3) continue step 2.1 to 2.3 until site < n

4) STOP

∴ The ~~sp.~~ minimal spanning tree is—

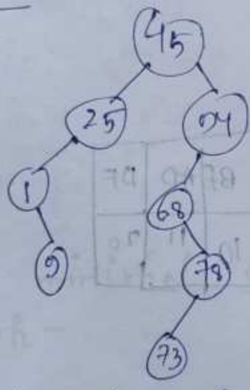


the minimum weigh val. of minimal spanning tree = 11 + 1 + 2 + 3 + 1
= 18

c) Form a BST for the entries - 45, 25, 1, 9, 94, 68, 78, 73. Delete 1 & 68.

BST

T :



After deletion of 1 & 68 -

[Inorder traversal of BST = 1 - 9 - 25 - 45 - 68 - 73 78 94]

T :