Numerical methods

20/7/22

- The Problem which is defined mathematically convert it into Programmatically form by using the algorithms of methods, is done by numerical methods. The N.M. not Provides or may not Provides the exact Solution but it Provids approximate Solution, which are acceptable for the Solution.

- Hence there may be a Possibility of errors by the numerical methods but can be resolved using error optimisation methods / techniques. Hence this error handling methods are i) Exact error
ii) Relative error
iii) Percentage error

i) For, ex - $x_T = 3.4213$

Let $x_T = 3.4213$ is a true value of x

After solution we got $x_A = 3.422312$ is a approximate value of x.

This is called exact error.

$$\boxed{E_A = |x_T - x_A|}$$

[Exact error is the Subtraction of approximate ~~error~~ value of a sum from its true value]

$= |3.4213 - 3.422312|$

$= ~~0.00~~ ~~88~~ ~~1~~ ~~0.00001~~ 2$

$= -1.012 \times 10^{-3}$

[By using this methods we can calculate exact, Round not errors]

ii) $\boxed{E_R = \frac{E_A}{|x_T|} \qquad E_R = \frac{|x_T - x_A|}{x_T}}$ [It is the ratio of exact error and true value of a Soln.]

$= \frac{-1.012 \times 10^{-3}}{3.4213}$

$= 2.95 \times 10^{-4}$

iii) $\boxed{E_P = E_R \times 100 = \frac{|x_T - x_A|}{x_T} \times 100}$

[Percentage error is defined as Relative error × 100]

Conversion of a number into decimal Places

Rules – (with example)

1) $x_T = 11.437362$ convert it into 4 decimal places.

Ans $x_A = 11.4374$ (if the no. of the decimal Place+1 if higher than 5 then add 1 at the numeric value of that decimal Place)

2) $x_T = 12.437322$ (No changes)

$x_A = 12.4373$

3) a) $x_T = 12.437352$ [If odd then change]

$x_A = 12.4374$

b) $x_T = 12.437652$ [If even no change]

$x_A = 12.4376$

Significant figure

The total no. of digits including both integral Part and decimal Part is the no. of Significant figure.

• So, the no. with decimal places and it's Significant figure are different.

$x = 11.437362$

8 significant figure

(It's decimal place is 6, but it's sig. fig. is 8, they are not same)

Assignment - ① Find E_A, E_R, E_P of the following no. -

i) $\log 40$

ii) e^{10}

iii) $\tanh 70$.

iv) $\epsilon = 0.0085$

v) 12.712358

[Find the Sol^n for each no. upto 4 decimal places]

② Provided a general formula for the true & error when approximating and relatives the derivative of $f(x) = -x^2 + 3x$ at $x = 2$, using an arbitrary h. Determine the largest value of h such that the absolute / relative error will be not (E_R) more than 0.5%

[Polynomial = Find the function depending on the values it's called interpolation]

25/7/22

Interpolation (Here Points are known)

Polynomial = deg - 1

Interpolation means finding a polynomial function f based on initial information of independent and dependent variables.

$X = x_0, x_1, x_2, x_3 \ldots x_n \rightarrow$ independent variable

$Y = y_0, y_1, y_2, y_3 \ldots y_n \rightarrow$ Dependent variable

$Y = f(x)$ [Polynomial function]

- The nature of the p.f $f(x)$ is based on the nature of independent variables. If this set of independent variables will be either in equal interval or in unequal interval

$$\begin{bmatrix} 2 & 4 & 6 & 8 & 10 & \rightarrow \text{equal interval} \\ 11 & 0 & 12 & 9 & & \rightarrow \text{unequal} \ \text{"} \end{bmatrix}$$

The methods for finding the polynomial functions are -

For unequal independent variable:

① Langrang Interpolation formula

② Newton's divided difference I. F.

For equal independent variable:

③ Newton's forward I.F.

④ Newton's backward I.F.

Lagrange Interpolation formula is defined as:-

if $x_0, x_1, x_2 \ldots x_n$ be independent variables while the dependent variables are $y_0, y_1, y_2 \ldots y_n$

Π = Product
Σ = Summation

Then L.I.F is defined as

$$f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)} y_1$$

$$\ldots + \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})} \times y_n$$

$$\frac{\prod_{j=1}^{n}(x-x_j)}{\prod_{j=1, i=0}(x_i-x_j)} \times y_i + \frac{\prod_{j=0, j\neq 1}^{n}(x-x_j)}{\prod_{j=0, j\neq 1, i=1}^{n}(x_i-x_j)} \times y_i + \ldots \quad (1)$$

$$= \sum \frac{\prod_{j=0}^{n}(x-x_j)}{\prod_{j=0, i\neq j}(x_i-x_j)} \times y_i$$

= Lagrange multiplier

Ex-(1) Find the P.F by using the given data

	x	y	
x_0	2	-1	y_0
x_1	1	0	y_1
x_2	3	4	y_2
x_3	7	1	y_3

$$f(x) = \frac{(x-1)(x-3)(x-7)}{(2-1)(2-3)(2-7)} \times (-1) + \frac{(x-2)(x-3)(x-7)}{(1-2)(1-3)(1-7)} \times 0$$

$$+ \frac{(x-2)(x-1)(x-7)}{(3-2)(3-1)(3-7)} \times 4 + \frac{(x-2)(x-1)(x-3)}{(7-2)(7-1)(7-3)} \times 1$$

$$= \frac{(x-1)(x-3)(x-7)\times(-1)}{1\times(-1)\times(-5)} + \frac{(x-2)(x-1)(x-7)}{1\times 2\times(-4)} \times 4 + \frac{(x-2)(x-1)(x-3)}{5\times 6\times 4}$$

$$= \frac{(x-1)(x-3)(x-7)\times(-1)}{\cancel{+}5} + \frac{(x-2)(x-1)(x-7)}{-\cancel{8}\,2} + \frac{(x-2)(x-1)(x-3)}{120}$$

$$= (x^2-4x+3)(x-7) + \frac{(x^2-3x+2)(x-7)}{(-2)} + \frac{(x^2-3x+2)(x-3)}{120}$$

$$= x^3-4x^2+3x-7x^2+28x-21 + \frac{x^3-3x^2+2x-7x^2+21x-14}{(-2)} + \frac{x^3-3x^2+2x-3x^2+9x-6}{120}$$

$\boxed{\begin{array}{l} f(x) = 5x+3 \\ x=11 \\ f(11) = 58 \text{ at } 11 \end{array}}$

- If there are n initial given points $(x_0, x_1, \ldots x_n)$ then the polynomial of degree $n-1$.

Exercise – Find the polynomial $f(x)$ by using the following data

(1)

	x_0	x_1	x_2	x_3
x:	11	13	14	15
y:	-1	0	2	1
	y_0	y_1	y_2	y_3

find $f(0)$, $f(1)$, $f(2)$

(2)

	x_0	x_1	x_2	x_3	x_4
x:	-2	-3	-5	3	2
y:	10	20	30	40	50
	y_0	y_1	y_2	y_3	y_4

, find $f(100)$, $f(200)$

(3) Write a program in C to solve this two problems (?). [By array]

(1) $$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)} \times y_0 + \frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)} \times y_1 + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_2-x_0)(x_2-x_1)\cdots(x_2-x_n)} \times y_2 + \frac{(x-x_0)(x-x_1)\cdots(x-x_n)}{(x_3-x_0)(x_3-x_1)\cdots(x_3-x_n)} \times y_3$$

$$= \frac{(x-13)(x-14)(x-15)}{(11-13)(11-14)(11-15)} \times (-1) + \frac{(x-11)(x-14)(x-15)}{(13-11)(13-14)(13-15)} \times 0 + \frac{(x-11)(x-13)(x-15)}{(14-11)(14-13)(14-15)} \times 2 + \frac{(x-11)(x-13)(x-14)}{(15-11)(15-13)(15-14)} \times 1$$

$$= \frac{(n-13)(n-14)(n-15)}{(-2)\times(-3)\times(-4)}\times(+1) + \frac{(n-11)(n-13)(n-15)}{3\times1\times(-1)}\times 2 + \frac{(n-11)(n-13)(n-14)}{4\times2\times1}$$

$$= \frac{(n-13)(n-14)(n-15)}{24} - \frac{2(n-11)(n-13)(n-15)}{3} + \frac{(n-11)(n-13)(n-14)}{8}$$

$$= (n-13)\left[\frac{(n-14)(n-15)}{24} - \frac{2(n-11)(n-15)}{3} + \frac{(n-11)(n-14)}{8}\right]$$

$$= (n-13)\left[\frac{n^2-29n+210}{24} - \frac{2(n^2-26n+165)}{3} + \frac{n^2-25n+154}{8}\right]$$

$$= (n-13)\left[\frac{n^2-29n+210-16(n^2-26n+165)+3(n^2-25n+154)}{24}\right]$$

$$= \frac{(n-13)}{24}\left[n^2-29n+210-16n^2+416n-2640+3n^2-75n+462\right]$$

$$= (n-13)\left[-12n^2+312n-1968\right]\times\frac{1}{24}$$

$$= \frac{1}{24}\left[-12n^3+312n^2-1968n+156n^2-4056n+25584\right]$$

$$= \left[-12n^3+468n^2-6024n+25584\right]\times\frac{1}{24}$$

$$f(n) = \left[-12n^3+468n^2-6024n+25584\right]\times\frac{1}{24}$$

$$f(0) = \left[-12\times(0)^3+468\times(0)^2-6024\times(0)+25584\right]\times\frac{1}{24}$$

$$= 25584\times\frac{1}{24} = 1066$$

$$f(1) = \left[-12\times(1)^3+468\times(1)^2-6024\times(1)+25584\right]\times\frac{1}{24}$$

$$= 20016\times\frac{1}{24} = 834$$

$$f(2) = \left[-12\times(2)^3+468\times(2)^2-6024\times(2)+25584\right]\times\frac{1}{24}$$

$$= \left[-12\times8+462\times4-6024\times2+25584\right]\times\frac{1}{24}$$

$$= 15288\times\frac{1}{24}$$

$$= 637$$

(3)

$$f(x) = \frac{(x+3)(x+5)(x-3)(x-2)}{(-2+3)(-2+5)(-2-3)(-2-2)} \times 10$$

$$+ \frac{(x+2)(x+5)(x-3)(x-2)}{(-3+2)(-3+5)(-3-3)(-3-2)} \times 20$$

$$+ \frac{(x+2)(x+3)(x-3)(x-2)}{(-5+2)(-5+3)(-5-3)(-5-2)} \times 30$$

$$+ \frac{(x+2)(x+3)(x+5)(x-2)}{(3+2)(3+3)(3+5)(3-2)} \times 40$$

$$+ \frac{(x+2)(x+3)(x+5)(x-3)}{(2+2)(2+3)(2+5)(2-3)} \times 50$$

$$= \frac{(x^2+8x+15)(x^2-5x+6)}{1 \times 3 \times (-\cancel{5}) \times (-\cancel{4})_2} \times \cancel{10}^{2} + \frac{(x^2+7x+10)(x^2-5x+6)}{(-1)\times(\cancel{2})\times(-\cancel{6})_3\times(-\cancel{5})} \times \cancel{20}$$

$$+ \frac{(x^2+5x+6)(x^2-5x+6)}{(-\cancel{3})\times(-\cancel{2})\times(-8)\times(-7)} \times \cancel{30}\ \cancel{10}\ 5 + \frac{(x^2+5x+6)(x^2+3x-10)}{\cancel{5}\times 6 \times \cancel{8} \times 1} \times \cancel{40}$$

$$+ \frac{(x^2+5x+6)(x^2+2x-15)}{2\ 4 \times \cancel{5} \times 7 \times (-1)} \times \cancel{50}\ \cancel{10}\ 5$$

$$= \frac{x^4-5x^3+6x^2+8x^3-40x^2+48x+15x^2-75x+90}{6}$$

$$- \frac{x^4-5x^3+6x^2+7x^3-35x^2+42x+10x^2-50x+60}{3}$$

$$+ \frac{5(x^4-5x^3+6x^2+5x^3-25x^2+30x+6x^2-30x+36)}{56}$$

$$+ \frac{x^4+3x^3-10x^2+5x^3+15x^2-50x+6x^2+18x-60}{6}$$

$$- \frac{5(x^4+2x^3-15x^2+5x^3+10x^2-75x+6x^2+12x-90)}{14}$$

$$= \frac{x^4+3x^3-19x^2-27x+90}{6} - \frac{x^4+2x^3-19x^2-8x+60}{3}$$

$$+ \frac{5(x^4-13x^2+36)}{56} + \frac{x^4+8x^3+11x^2-32x-60}{6}$$

$$- \frac{5(x^4+7x^3+x^2-63x-90)}{14}$$

$$= \left[\frac{n^4+3n^3-19n^2-27n+90}{6} - \frac{n^4+2n^3-19n^2-8n+60}{3} + \frac{n^4+8n^3+11n^2-32n-60}{6}\right] + \left[\frac{5(n^4-13n^2+36)}{56} - \frac{5(n^4+7n^3+n^2-63n-90)}{14}\right]$$

$$= \frac{n^4+3n^3-19n^2-27n+90-2(n^4+2n^3-19n^2-8n+60)+n^4+8n^3+11n^2-32n-60}{6}$$

$$+5\left[\frac{n^4-13n^2+36-4(n^4+7n^3+n^2-63n-90)}{56}\right]$$

$$= \frac{\cancel{2n^4}+11n^3-8n^2-59n+30-\cancel{2n^4}-4n^3+38n^2+16n-120}{6}$$

$$+\frac{5}{56}\left[n^4-13n^2+36-4n^4-28n^3-4n^2+252n+360\right]$$

$$= \frac{7n^3+30n^2-43n-90}{6} + \frac{5}{56}\left[-3n^4-28n^3-17n^2+252n+396\right]$$

$$= \frac{28(7n^3+30n^2-43n-90)+15(-3n^4-28n^3-17n^2+252n+396)}{168}$$

$$= \frac{196n^3+840n^2-1204n-2520-45n^4-420n^3-255n^2+3780n+5940}{168}$$

$$= \frac{-45n^4-224n^3+585n^2+2576n+3420}{168}$$

$$f(n) = \frac{1}{168}\left[-45n^4-224n^3+585n^2+2576n+3420\right]$$

$$f(100) = \frac{1}{168}\left[-45\times(100)^4-224\times(100)^3+585\times(100)^2+2576\times100+3420\right]$$

$$= -28.08\times10^6$$

$$f(200) = \frac{1}{168}\left[-45\times(200)^4 - 224\times(200)^3 + 585\times(200)^2 + 3576\times 200 + 3420\right]$$

$= -439.1\times10^6$

(3)

i)

```
#include<stdio.h>
int main()
{
  int n, i, j;
  Printf("enter the value of n\n");
  Scanf("%d", &n);
  int a[n], b[n];
  Printf("enter the values for 1st array\n");
  Scanf("%d", &a[i]);
  Printf("enter the values for 2nd array\n");
  Scanf("%d", &b[j]);
```

ii)

```
#include <stdio.h>
int main()
{
  int x,n, i, j, chosen; k; k1=0;
  Printf("enter the value of n\n");
  Scanf("%d", &n);
  int a[n], b[n]; a[chosen];
  Printf("enter the values of 1st array\n");
  for (i=0; i<n; i++)
  {
    Scanf("%d", &a[i]);
  }
  Printf("enter the values of 2nd array\n");
  for (j=0; j<n; j++)
  {
    Scanf("%d", &b[j]);
  }
  Printf("take the chosen value for the operation\n");
  for (chosen=0; chosen<n; chosen++)
  {
    Scanf("%d", &a[chosen]);
  }
  for (i=0; i<n; i++)
  {
    for (j=0; j<n; j++)
    {
```

$$k = \prod_{\substack{i=0}}^{n-1} \frac{x - a[i]}{a[chosen] - a[i]} \times b[j]$$

$\prod^{n-1}_{i,j=0,\ chosen \neq i}$

$k1 = k1 + k;$

```
    }
```

```
}
~~for(i=0; i<n; i++)~~
Printf("%.d is the value of the problem\n", k);
return 0;
}
```

#

27/7/22

CW

② Newton's devided difference I.F.

Im If there are initial Points $[(x_0, y_0), (x_1, y_1) \dots (x_n, y_n)]$ then the Palynomial will be defined as -

$$f(x) = f(x_0) + (x - x_0) + \underbrace{f[x_0, x_1]}_{1st\ DD} + (x - x_0)(x - x_1) + \underbrace{f[x_0, x_1, x_2]}_{2nd\ DD} + (x - x_0)(x - x_1)(x - x_2) + f[x_0, x_1, x_2, x_3] \dots (x - x_0)(x - x_1) \dots (x - x_n) + \underbrace{f[x_0, x_1, x_2 \dots x_n]}_{nth\ DD}$$

[DD = devided diffence]

Ex - Find the Palynomial using the following initial Points -

	x_0	x_1	x_2	x_3	x_4	x_5
x :	1	3	4	5	7	10
y :	3	31	69	131	351	1011
	y_0	y_1	y_2	y_3	y_4	y_5

Compute the DD table -

	x	y	1st DD. $f[x_0, x_1]$	2nd DD
x_0	1	y_0 3	$\frac{31-3}{3-1} = 14$	$\frac{38-14}{4-1} = 8$
x_1	3	y_1 31	$\frac{69-31}{4-3} = 38$	$\frac{62-38}{5-3} = 12$
x_2	4	y_2 69	$\frac{131-69}{5-4} = 62$	$\frac{110-62}{7-4} = 16$
x_3	5	y_3 131	$\frac{351-131}{7-5} = 110$	:
x_4	7	y_4 351		
x_5	10	y_5 1011		

$\frac{1011-351}{10-7} = 220$ | $\frac{220-110}{10-5} = 22$

3DD	4th	5th
$\frac{12-8}{5-1} = 1$	$1-1 = 0$	$0-0 = 0$
$\frac{16-12}{7-3} = 1$	$1-1 = 0$	
$\frac{22-16}{10-4} = 1$		

• If there are n number of terms, then there will be $n-1$ divided difference.

$$f(x) = f(x_0) + (x-x_0)\times 14 + (x-x_0)(x-x_1)\times 8 + (x-x_0)(x-x_1)(x-x_2)\times 1$$

$$= 3 + (x-1)\times 14 + (x-1)(x-3)\times 8 + (x-1)(x-3)(x-4) \to$$

[As $(x-x_0)(x-x_1)(x-x_2)$...]

[So last value will never come]

at $f(100)$

$$f(100) = 3 + (100-1)\times 14 + (100-1)(100-3)\times 8 + (100-1)(100-3)(100-4)$$

$$= 3 + 99\times 14 + 99\times 97\times 8 + 99\times 97\times 96$$

$$= 1000101$$

(2) Use the following given data to find the polynomial using Newton D.D. I.F. (Imp)

x: 0 1 2 3 4

y: 0 1 8 27 64

DD table -

x	y	1st DD	2nd DD	3rd DD	4th DD
0	0	$\frac{1-0}{1-0} = 1$	$\frac{7-1}{2-0} = 3$	$\frac{6-3}{3-0} = 1$	$1-1 = 0$
1	1	$\frac{8-1}{2-1} = 7$	$\frac{19-7}{3-1} = 6$	$\frac{9-6}{4-1} = 1$	
2	8	$\frac{27-8}{3-2} = 19$	$\frac{37-19}{4-2} = 9$		
3	27	$\frac{64-27}{4-3} = 37$			
4	64				

$$f(x) = 0 + 1\times(x-x_0) + 3\times(x-x_0)(x-x_1) + 1\times(x-x_0)(x-x_1)(x-x_2)$$

$[\because f(x_0) = y_0 = 0]$

$$= 1\times(x-0) + 3(x-0)(x-1) + x(x-1)(x-2)$$

$$= x + 3[x(x-1)] + x[x^2-3x+2]$$

$$= x + 3[x^2-x] + x^3 - 3x^2 + 2x$$

$$= \cancel{x} + \cancel{3x^2} - \cancel{3x} + x^3 - \cancel{3x^2} + \cancel{2x}$$

$$= x^3$$

② Newton's forward I.F —

Sm If there are given $(x_0, y_0), (x_1, y_1) \ldots (x_n, y_n)$ then, the newton's forward I.F can be defined as -

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \cdots \frac{p(p-1)(p-2)\cdots(p-n+1)}{n!}\Delta^n y_0$$

$\left[p = \frac{x - x_0}{h}, \; h = \text{common difference} \right]$

Δ = forward difference

Steps -

① Find the common diff from the given set of points.

② Compute forward diff table.

③ Find the Polynomial

$\boxed{\begin{array}{l} \Delta f(x) = f(x+1) - f(x) \\ \nabla f(x) = f(x_n) - f(x_{n-1}) \end{array}}$

Ex - ①

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
0	0	1	6	6	0
1	1	7	12	6	
2	8	19	18		
3	27	37			
4	64				

$h = (n+1-n)$
$= 1$

$$f(n) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0$$

$$= 0 + p\times 1 + \frac{p(p-1)}{2}\times \cancel{3}\,6 + \frac{p(p-1)(p-2)}{6}\times 6$$

$$= 0 + \frac{(h-0)}{1}\times 1 + n(n-1)\times 3 + n(n-1)(n-2)$$

$$\boxed{p = \frac{n-0}{1} = n}$$

at $n=100$,

$$f(100) = 0 + 100 + 100\times 99\times 3 + 100\times 99\times 98 = 10^6$$

- $\boxed{\text{If there are } n \text{ number of terms, then there will be } n-1 \text{ no of forward difference}}$

(4) Newton backword I.F – If there is given $[(x_0, y_0), \ldots$

it is ~~defined as~~ the given initial points are $[(x_0,y_0), (x_1,y_1), \ldots (x_n,y_n)]$ then the polynomial will be defined as

$$f(x) = y_n + p\times\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots + \frac{p(p+1)(p+2)\cdots(p+n-1)}{n!}\nabla^n y_n$$

$$\left[p = \cancel{\frac{x-x_n}{h}} = \frac{x-x_0}{h}\right]$$
$h =$ common difference

Ex – 1

x	y	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$
0	0				
1	1	1			
2	8	7	6		
3	27	19	12	6	
4	64	37	18	6	0

$$f(n) = 64 + p\times 37 + \frac{p(p+1)}{2!}\times 18 + \frac{p(p+1)(p+2)}{3!}\times 6$$

$$= 64 + n\times 37 + \frac{n(n+1)}{2}\times 18 + \frac{n(n+1)(n+2)}{6}\times 6$$

$$\boxed{p = \frac{n-0}{1} = n}$$

at $n = 100$

$$f(100) = 64 + 100\times 37 + 100\times 101\times 9 + 100\times 101\times 102$$

$= 1124864$

- If there are n numbers of terms, then there will be $n-1$ no. of backward difference.
- N.F.I.F can be applied for both unequal and equal intervals but, N.B.I.F can be applied for only equal intervals
- $E\{f(x)\} = f(x+h)$

[E = Expectation]

$\Rightarrow E\{f(x)\} - f(x) = f(x+h) - f(x)$

$\Rightarrow (E-1) f(x) = \Delta f(x)$

$\Rightarrow E - 1 = \Delta$

$\Rightarrow E = \Delta + 1$

The relation between expectation and forward difference = forward diff + 1

Exercise

① ~~Take the initial points~~ take 5 initial points with common difference between their x values then apply both Newton's forward and backward I.F formula to find the polynomial. Check the correctness of polynomial at $x=100$ ii) If the forward and backward formulas give different outcomes then find exact, relative and percentage error. iii) How you can resolve the problem with four significant figures?

iii) write a C program for the solution of this one.

① Let, the common difference is = 1 (h)

the 5 initial values are below—

~~x: 0 1 2 3 4~~

~~y: 0 10 20 30 40~~

~~D.D table~~

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
x_0 0	y_0 0			
		1		
x_1 1	y_1 10		2	
		3		
x_2 2	y_2 20		1	
		2		
x_3 3	y_3 30		2	
		4		
x_4 4	y_4 40			

x: 0 1 2 3 4

y: 0 1 8 27 64

D.D table (for forward difference)

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$
x_0 0	y_0 0				
		1			
x_1 1	y_1 1		6		
		7		6	
x_2 2	y_2 8		12		0
		19		6	
x_3 3	y_3 27		18		
		37			
x_5 4	y_4 64				

Newton's forward interpolation formula

$$f(x) = y_0 + P\Delta y_0 + \frac{P(P-1)}{2!}\times\Delta^2 y_0 + \frac{P(P-1)(P-2)}{3!}\times\Delta^3 y_0 + \frac{P(P-1)(P-2)(P-3)}{4!}\times\Delta^4 y_0$$

$$= 0 + x\times 1 + \frac{x(x-1)}{2}\times 6 + \frac{x(x-1)(x-2)}{6}\times 6$$

$\left[\because P = \frac{x-x_0}{h} = \frac{x-0}{1}\right]$ $[\because h=1]$

at $x=100$,

$$f(100) = 100 + 100\times 99\times 3 + 100\times 99\times 98$$

$$= 10^6$$

$$= 1000000$$

D.D table (for backward difference)

x	y	∇y_n	$\nabla^2 y_n$	$\nabla^3 y_n$	$\nabla^4 y_n$
0	0				
		1			
1	1		6		
		7		6	
2	8		12		0
		19		6	
3	27		18		
		37			
4	64				

Newton's backward interpolation formula

$$f(x) = y_n + P\nabla y_n + \frac{P(P+1)}{2!}\times\nabla^2 y_n + \frac{P(P+1)(P+2)}{3!}\times\nabla^3 y_n + \frac{P(P+1)(P+2)(P+3)}{4!}\times\nabla^3 y_n$$

$$= 64 + x\times 37 + \frac{x(x+1)}{2}\times 18 + \frac{x(x+1)(x+2)}{6}\times 6$$

$\left[\because P = \frac{x-x_0}{h} = x\right]$ $[\because h=1]$

at $x=100$,

$$f(100) = 64 + 100\times 37 + 9\times 100\times 101 + 100\times 101\times 102$$

$$= 1124864$$

Now we can see forward and backward formulas give different outcomes.

(i) Now,

Exact error $(E_A) = |x_F - x_B|$

$= |10^6 - 1124864| = 124864$

[where x_F = forward formula's given outcome
x_B = Backward " " "]

Relative error $(E_R) = \frac{E_A}{x_F}$ [Let assume that x_F is the true value]

$= \frac{|x_F - x_B|}{x_F}$

$= \frac{124864}{10^6}$

$= 0.124864$

Percentage error $(E_P) = E_R \times 100$

$= 0.124864 \times 100$

$= 12.4864\%$

(ii) x_F in 4 significant figures $= 1000$
x_B " " " " $= 1124$

Now,

$E_A = |x_F - x_B| = |1000 - 1124|$

$= 124$

$E_R = \frac{E_A}{x_F} = \frac{|x_F - x_B|}{x_F} = \frac{124}{1000} = 0.124$

[Let assume that x_F is the true value]

$E_P = 0.124 \times 100 = 12.4\%$

$(E_R \times 100)$

1/8/22

Polynomial

Algebric — Transendental

- The polynomial $= 0$ is an eqn. hence the example of algebric eqn are —

$x^2 - 3x + 2 = 0$
$x^4 - 3x^2 + 5x + 6 = 0$
$x^5 - 6x^4 + 11x^3 + 10x^2 + 6x + 7 = 0$

→ Algebric eqn

roots

$e^x \tan x + x \log x + e^{(\cos x + \sin x)} = 0$ — Transendental

- Hence there are some numerical methods for the soln of both algebric and transendental eqns.

i) Bisection method
ii) Interpolation method
iii) Newton's Raphson's method
iv) Regular False method

(i)–(iii) → Iterative methods (programming)

step 1

Find the interval in which the root of eqn lies (for all the methods)

[The root will be lie if $f(a) + f(b) < 0$]
& lies between the interval (hearing

step-2

Test for convergence for Iteration (Is the interval clearing can be applied in this method or not)

method and Newton's Raphsons method.

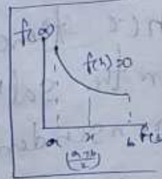
step-3

Compute iteration table for the soln of x.

(1) Bisection method

Ex-①

Find the roots of the eqn $x^2 - 3 = 0$



$f(x) = x^2 - 3$

$f(0) = -3 < 0$

$f(1) = 1 - 3 = -2 < 0$ Sign change

$f(2) = 2^2 - 3 = 1 > 0$ → stop

~~f(3)~~ $[1, 2]$

Compute it by using bisection method (Iteration table-1.1)

	a	b	f(a)	f(b)	$t = \frac{a+b}{2}$	f(t)
1)	1	2	-2	1	1.5	$-0.75 < 0$
2)	1.5	2	-0.75	1	1.75	$0.062 > 0$
3)	1.5	1.75	-0.75	0.062	1.625	$-0.359 < 0$
4)	1.625	1.75	-0.3594	0.062	1.6875	$-0.1523 < 0$
5)	1.6875	1.75	-0.1523	0.062	~~0.0625~~ 1.7188	$-0.0457 < 0$
6)	1.7188	1.75	~~-0.1523~~ -0.0457	0.062	1.7344	~~0.0092~~ < 0

Find the soln for 1 decimal place, $a = b = 1.7$

Find ... 2 ..., $b = 1.75$, $a = 1.7188$

[Updation → is called iteration]

Exercise

② Solve this by using bisection method –

① $x^4 - x - 10 = 0$

② $x = e^{-x}$

③ $e^{-x} = 3\log x$

④ $e^{-x} * (x^2 + 5x + 2) + 1 = 0$

① $x^4 - x - 1 = 0$

$f(x) = x^4 - x - 1$

$f(0) = -1 < 0$

$f(1) = 1 - 1 - 1$
$= -1 < 0$

$f(2) = 16 - 2 - 1$
$= 13$

$[1, 2]$

Compute it by using bisection method –

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1) 1	2	1.5	-1	13	$-0.25 < 0$
2) 1.5	2	1.75	-0.25	13	$6.629 > 0$
3) 1.5	1.75	1.625	-0.25	6.629	$4.348 > 0$
4) 1.5	1.625	1.5625	-0.25	4.348	$3.398 > 0$
5) 1.5	1.5625	1.53125	-0.25	3.398	$2.966 > 0$
6) 1.5	1.53125	1.5156	-0.25	2.966	$2.761 > 0$ *

(2) $x^4 - x - 10 = 0$

$f(x) = x^4 - x - 10$

$f(0) = -10 < 0$

$f(1) = \cancel{1} - \cancel{1} - 10$
$= -10 < 0$

$f(2) = 16 - 2 - 10$
$= 4$

[1,2]

compute it by using bisection method-

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1) 1	2	1.5	-10	4	$-0.25 < 0$
2) 1.5	~~1.5~~ 2	1.75	-0.25	4	$6.629 > 0$
3) 1.5	1.75	1.625	-0.25	6.629	$4.348 > 0$
4) 1.5	1.625	1.5625	-0.25	4.348	$3.398 > 0$
5) 1.5	1.5625	1.53125	-0.25	3.398	$2.966 > 0$
6) 1.5	1.53125	1.5156	-0.25	2.966	$2.761 > 0$

(3) $x = e^{-x}$ the eqn is $\Rightarrow x - e^{-x} = 0$

$f(x) = x - e^{-x}$

$f(0) = 0 - 1 = -1 < 0$

$f(1) = 1 - e^{-1}$
$= 1 - \frac{1}{e}$
$= 0.632 > 0$

$f(2) = 1 - e^{-2}$
$= 1 - \frac{1}{e^2}$
$= 0.864 > 0$

$f(3) = 1 - e^{-3}$
$= 1 - \frac{1}{e^3}$

[0,1]

compute it by using bisection method

a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1) 0	1	0.5	-1	0.632	$0.393 > 0$
2) 0	0.5	0.25	-1	0.393	$0.221 > 0$
3) 0	0.25	0.125	-1	0.221	$0.118 > 0$
4) 0	0.125	0.0625	-1	0.118	$0.06 > 0$
5) 0	0.0625	0.03125	-1	0.06	$0.031 > 0$
6) 0	0.03125	0.015625	-1	0.031	$0.016 > 0$

④ $e^{-x} = 3\log x$ the eqn is $e^{-x} - 3\log x = 0$

$f(x) = e^{-x} - 3\log x$

$f(1) = e^{-1} = 0.36 > 0$

$f(2) = e^{-2} - 3\log 2$

$= \frac{1}{e^2} - 3\log 2$

$= -0.76 < 0$

$\therefore [1, 2]$

compute i.t by using bisection method.

	a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1)	1	2	1.5	2.72	-0.76	-0.305
2)	1	1.5	1.25	2.72	-0.305	-4.22×10^{-3}
3)	1	1.25	1.125	2.72	-4.22×10^{-3}	0.171
4)	1.125	1.25	1.1875	0.171	-4.22×10^{-3}	0.081
5)	1.1875	1.25	1.21875	0.081	-4.22×10^{-3}	0.038
6)	1.21875	1.25	1.23434	0.038	-4.22×10^{-3}	0.017

⑤ $e^{-x} * (x^2 + 5x + 2) + 1 = 0$

$f(x) = e^{-x} * (x^2 + 5x + 2) + 1$

$f(0) = 1 \cdot (0 + 0 + 2) + 1$

$= 3$

$f(1) = \frac{1 + 5 + 2}{e} + 1$

$= 3.94$

$f(2) = \frac{4 + 10 + 2}{e^2} + 1$

$= 3.16$

$f(3) = \frac{9 + 15 + 2}{e^3} + 1$

$= 2.29$

$f(4) = \frac{16 + 20 + 2}{e^4} + 1$

$= 1.69$

$f(5) = \frac{25 + 25 + 2}{e^5} + 1$

$= 1.35$

$f(6) = \frac{36 + 30 + 2}{e^6} + 1$

$= 1.16$

$f(7) = \frac{49 + 35 + 2}{e^7} + 1$

$= 1.107$

$f(0) = 3 > 0$

$f(-1) = e^1 * (1 - 5 + 2) + 1$ $[0, -1]$

$= -4.4 < 0$

compute i.t by using bisection method -

	a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1)	0	-1	-0.5	3	-4.4	~~0.848~~ ~~1.061~~ 0.587
2)	-0.5	-1	-0.75	0.5887	-4.4	-1.514
3)	-0.5	-0.75	-0.625	0.587	-1.514	-0.372
4)	-0.5	-0.625	-0.5625	0.587	-0.372	0.129
5)	-0.5625	-0.625	-0.59375	0.129	-0.372	-0.116
6)	-0.5625	-0.59375	-0.5781	0.129	-0.116	8.312×10^{-3}

3/8/22

- fixed Point ~~in~~ iterative method -
If there may not be possible to find the interval then we will use fixed Point iterative method.

- To reduce the search space for finding the roots of the eqn we will use the fixed Point iterative method. The steps for this method are -

1) Given $f(x) = 0$
2) Solve $f(x)$ for x, i.e convert $f(x)$ in the form of $x = g(x)$
3) Start with initially at $x = x_0$, so the general form is $\boxed{x_{n+1} = g(x_n)}$ (general form)

Ex-① $x^2 - 2x + 1 = 0$

① $x = \frac{x^2+1}{2}$ ② $x = \sqrt{2x-1}$ ③ $x(x-2) = -1$ $\Rightarrow x = \frac{-1}{x-2}$

Just only one method have to be excuted. No need to execute all methods

$xe^x + \tan e^x + \cos x = 0$

④ $x = \frac{-(\tan e^x + \cos x)}{e^x}$

$\boxed{\text{Division of the function } f(x) \text{ [derivative]}}$

$\boxed{\text{By derivative we can find the track of the function}}$

x_0
$x_1 = g(x_0)$
$x_2 = g(x_1)$
$\vdots$
$x_{12} = g(x_{11})$
$x_{13} = g(x_{12})$

Test for ~~convergent~~ Convergence
$|x_{n+1} - x_n| < \epsilon$
Alternatively
$|g'(x)| < 1$

4) Test for convergence
5) Compute table and tabulation

① Find the soln of the given problem.

$x^2 - 2x - 8 = 0$

$x = \sqrt{2x+8}$ — ①

$x = \frac{x^2-8}{2}$ — ②

$f(x) = x^2 - 2x - 8$
$f(0) = -8 < 0$
$f(1) = 1 - 2 - 8$
$= -9 < 0$
$f(2) = 4 - 4 - 8$
$= -8 < 0$
$f(3) = 9 - 6 - 8$
$= -6 < 0$
$f(4) = 16 - 8 - 8 = 0$
$f(5) = 25 - 10 - 8 = 7 > 0$

interval $[3, 5]$
$a = 3$, $b = 5$

2.134 x_9 –
2.1314 2.1 (ans)

itr	n	x_n	$x_{n+1} = g(x_n) = \frac{x_n^2 - 8}{2}$ [n = 0]
1	0	3	$x_{n+1} = \frac{3^2-8}{2} = 0.5 = x_1$
2	1	0.5	$x_{n+1} = \frac{0.5^2-8}{2} = -3.875 = x_2$ [n=1]
3	2	-3.875	$x_{n+1} = \frac{(-3.875)^2-8}{2} = 3.5 = x_3$ [n=2]
4	3	3.5	$x_{n+1} = \frac{(3.5)^2-8}{2} = 2.125 = x_4$ [n=3]
5	4	2.125	$x_{n+1} = \frac{(2.125)^2-8}{2} = -1.74 = x_5$ [n=4]

(6 -1.74 $x_{n+1} = \frac{(-1.74)^2 - 8}{2} = -2.48$ once $(n=5)$

Ans, $x = -2$ (as, -1.74, -2.48 are almost converged)

② Example

$x^3 + 2x^2 + x - 1 = 0$

①

② $3x - \cos x - 1 = 0$

③ $\cos x - x e^x = 0$

• Newton's Raphson's method

According to this method the given eqn $f(x) = 0$ derives the root by using

$$\boxed{\begin{aligned} x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \\ x_2 &= x_1 - \frac{f(x_1)}{f'(x_1)} \\ &\vdots \\ x_{n+1} &= x_n - \frac{f(x_n)}{f'(x_n)} \end{aligned}}$$

, [x_0 = initial]

[where f' is the derivative of f]

Ex-①

$f(x) = x^5 - x - 1 = 0$ $f'(x) = 5x^4 - 1$

$f(0) = 0 - 0 - 1 = -1 < 0$

$f(1) = 1 - 1 - 1 = -1 < 0$

$f(2) = 2^5 - 2 - 1 = 61 (+ve) > 0$

[1, 2] [we can take any value for in between 1, 2, like $x_0 = 1.5$]

x_0

[one initial Point and it is fixed]

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ $= x_n - \frac{x^5 - x - 1}{5x^4 - 1}$
0	1.5	$f(1.5) = 8.89$	$f'(1.5) = 44.56$	$x_1 = 1.38$
1	1.38	$f(1.3) = 2.52$	$f'(1.3) = 21.37$	$x_2 = 1.18$
2	1.18	0.52	12.72	$x_3 = 1.14$
3	1.14	~~38.5~~ 0.065	~~132.6~~ 10.55	$x_4 = 1.13$
4	~~1.57~~ 1.13	~~12.41~~ -0.048	~~56.23~~ 10.05	$x_5 = 1.13$
5	1.13	~~3.7~~ -0.048	~~25.9~~ 10.05	$x_6 = 1.13$
~~6~~	~~1.21~~	~~0.93~~	~~14.56~~	$x_7 = 1.15$

Soln. 1.13 (converged val of x_{n+1})

- Regular False method

There ~~abbr~~ may be occurr abrupt situation in such a way that the soln will be at far iteration. Hence to control this Problem ~~of~~ over the soln of fixed point iteration and network's raph son's method the Regular False method employed.

Assume interval is $[x_0, x_1]$ [x_2 = root]

$$x_2 = x_1 - f(x_0) \cdot \frac{(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$x_2 = \cancel{x_1 f(x_1) - x_1 f(x_0) - x_1 f}$$

$$\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

general form -

$$\boxed{x_{n+1} = \frac{x_{n-1} * f(x_n) - x_n f(x_{n-1})}{f(x_n) - f(x_{n-1})}}$$

the conditions are - (i) If $f(x_2) = 0$ then root is x_2.

(ii) If $f(x_0) \times f(x_2) < 0$ then update $x_1 = x_2$

(iii) If $f(x_2) \times f(x_1) < 0$, then update $x_0 = x_2$

Ex-(1) Solve $x^2 - 4x - 7 = 0$

$f(x) = x^2 - 4x - 7$

$f(0) = -7 < 0$

$f(1) = 1 - 4 - 7$
$= -10 < 0$

$f(2) = 4 - 8 - 7$
$= -11 < 0$

$f(3) = 9 - 12 - 7 < 0$

$f(4) = 16 - 16 - 7 < 0$

$f(5) = 25 - 20 - 7 < 0$

$f(6) = 36 - 24 - 7 = +ve$

$[5, 6]$

$$x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}$$

itr	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	update
1	5	f(5)	6	f(6)	1.6667	-0.5 -.781	$x_0 = x_2$
~~1.6667~~ 2	1.6667	-.5787					

For interval [1, 2] $x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}$

itr	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	update
1	1	-1	2	5	1.6667	-0.5787	$x_0 = x_2$
2	1.6667	-.5787	2	5	1.2934	-0.12954	$x_0 = x_2$
3	1.2934	-0.12954	2	5			
4	1.2931						

Just example

$f(x) = x^6 - x - 1$, $x_2 = \frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}$

itr	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	update
1	1	-1	2	29	-1.03	1.22	$x_1 = x_2$
2	1	-1	-1.03	1.22	-0.08	-0.92	$x_0 = x_2$
3	-0.08	-0.92	-1.03	1.22	0.49	-1.48	$x_0 = x_2$
4	~~1.48~~ -0.49	-1.48	-1.03	1.22	0.786	-1.6	$x_0 = x_2$
5	0.786	-1.6	-1.03	1.22	0.24	-1.24	$x_0 = x_2$
6	0.24	-1.24	-1.03	1.22	0.4	-1.4	$x_0 = x_2$

for Problem (ex-1)

$f(x) = x^2 - 4x - 7$

interval = [5, 6]

itr	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	update
1	5	-2	6	5	5.3	-0.11	$x_0 = x_2$
2	5.3	-0.11	6	5	-5.32	42.58	$x_1 = x_2$
3	5.3	-0.11	-5.32	42.58	-5.27	41.85	$x_1 = x_2$
4	5.3	-0.11	-5.27	41.85	-5.27	41.85	$x_1 = x_2$

Not correct

final soln, x_2 = (converged val)

itr	x_0	$f(x_0)$	x_1	$f(x_1)$	x_2	$f(x_2)$	update
1	5	-2	6	5	5.2	-0.76	$x_0 = x_2$
2	5.2	-0.76	6	5	5.3	-0.11	$x_0 = x_2$
3	5.3	-0.11	6	5	5.3	-0.11	$x_0 = x_2$
4	5.3	-0.11	6	5	5.3	-0.11	$x_0 = x_2$

Converge

$\therefore x_2 = 5.3$ (root)

10/8/22

$\int f(x)\,dx$ — continuous

$\sum f(x)$ — discrete

Numerical integration - Here the methods for the sol^n of finding the numeric value of an integration or derive. If the given function is either algebric or trigonamatrical or transidental the same set of neumerical methods can be applied. (for an integration or derivation)

1) Trapizodal method

Step 1

1) f(x)
2) a, b (lower limit, upper limit)
3) No. of iterations n
4) steps h

• 0, 1 (LL, UL)
h = 0.2
0, 0.2, 0.4, 0.6, 0.8, 1

• a = 0
h = 8
h = .2

0, 0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4
a_0, a_1, a_2, ... a_7
y_0, y_1, ... y_7
$f(a_0)$, $f(a_1)$

Step-2

(1) Initial term (y_0)
(2) last term (y_n)
(3) Remaining term

term = $y_0 = f(x_0)$
$y_1 = f(x_1)$
⋮

Step-3

Apply the algorithms (method)

$$I = \int_a^b f(x)\,dx = \frac{h}{2}\left[\text{first term} + \text{last term} + 2\times\left(\text{sum of all remaining term}\right)\right]$$

$$I = \frac{h}{2}\left[y_0 + y_n + 2\times(y_1 + y_2 \cdots y_{n-1})\right]$$

Ex-(1)

Solve $\int_0^1 e^x\,dx$ with, $h = 0.4$.

$f(x) = e^x$, $a = 0$, $b = 1$, $h = 0.4$

$I = \frac{0.4}{2}\left[1 + e + 2\times(e^{0.4} + e^{0.8})\right]$

$= 0.2$

$a_0 = 0$, $a_1 = 0.4$, $a_2 = 0.8$, $a_3 = 1$

$b_0 = 1$, $b_1 = e^{0.4}$, $b_2 = e^{0.8}$, $a_3 = e$

Ex-(2)

Find the $\int_{-1}^{1} \tan(e^x)\,dx$, ~~h = 0.3~~ n = 12

$f(x) = \tan(e^x)$, $a = -1$, $b = 1$, $h = \frac{b-a}{n} = \frac{1+1}{12} = \frac{2}{12}$

$I = \frac{0.2}{2}\left[6.4\times10^{-3} + 0.03 + 2\times(7.8\times10^{-3} + 9.5\times10^{-3} + 0.01 + 0.01 + 0.02 + 0.02 + 0.03 + 0.03 + 0.04)\right]$

$a_0 = -1$	$b_0 = 6.4\times10^{-3}$
$a_1 = -0.8$	$b_1 = 7.8\times10^{-3}$
$a_2 = -0.6$	$b_2 = 9.5\times10^{-3}$
$a_3 = -0.4$	$b_3 = 0.01$
$a_4 = -0.2$	$b_4 = 0.014$
$a_5 = 0$	$b_5 = 0.017$
$a_6 = 0.2$	$b_6 = 0.02$
$a_7 = 0.4$	$b_7 = 0.03$

$a_8 = 0.6$ | $b_8 = 0.03$
$a_9 = 0.8$ | $b_9 = 0.04$
$a_{10} = 0.1$ | $b_{10} = 0.05$

$= 0.039$

$$I = \frac{0.2}{2}\left[6.4\times10^{-3} + 0.05 + 2\times(7.8\times10^{-3} + 5.5\times10^{-3} + 0.01 + 0.01 + 0.02 + 0.02 + 0.03 + 0.3 + 0.04)\right]$$

$= 0.0411$

2) Simpson's 1/3 rule

$$I = \int_a^b f(x)\,dx = \frac{h}{3}[\text{1st term} + \text{last term} + 4(\text{sum of all odd terms}) + 2(\text{sum of all even terms})]$$

$$I = \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + y_5 \dots) + 2(y_2 + y_4 + y_6 \dots)]$$

Exercise

1) Solve $\int_{-1}^{1} \tan(e^x)\,dx$, $h=2$ using Simpson 1/3 rule.

2) Write a C programme to solve the above integrations using both trapezoidal and Simpson 1/3 rule and compare the performance based on outcomes of this integrations.

16/8/22

3) Simpson's 3/8 rule

$$I = \int_a^b f(x)\,dx = \frac{3h}{8}[y_0 + y_n + 3(y_1 + y_2 + y_4 + y_5 \dots y_{n-1}) + 2(y_3 + y_6 + y_9 + y_{12} + \dots y_{n-3})]$$

$h = \frac{b-a}{n}$

n - no. of terms

4) Weddle's rule

$$I = \int_a^b f(x)\,dx = \frac{3h}{10}[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6 + 6y_7 + \dots + y_{n-3} + 5y_{n-2} + y_{n-1} + 6y_n]$$

5) Boole's Rule

$$I = \int_a^b f(x)\,dx = \frac{2h}{45}[7(y_0 + y_n) + 32(y_1 + y_3 + y_5 + \dots + y_{n-3} + y_{n-1}) + 12(y_2 + y_6 + y_{10} + \dots + y_{n-6} + y_{n-2}) + 14(y_4 + y_8 + y_{12} + y_{16} + \dots + y_{n-8} + y_{n-4})]$$

odd term

even terms with 4 interval from y_2

even terms with 4 interval from y_4

Ex-①

Solve $I = \int_{-1}^{1} e^{-\frac{\mu x}{m}} dx$, μ & m are constant

Solve using weddle's and Boole's Rule, and derive the difference between the two integers values.

$$I = \int_{-1}^{1} e^{-\frac{\mu x}{m}} dx$$

$b = 1$, $a = -1$

$h = \frac{1+1}{2}$ Divide the interval into the following points where $h = 1$

$\therefore f(x) = e^{-\frac{\mu x}{m}}$

$a_0 = -1$	$b_0 = e^{\frac{\mu}{m}}$
$a_1 = 0$	$b_1 = 1$
$a_2 = 1$	$b_2 = e^{-\frac{\mu}{m}}$

weddle's Rule,

$$I = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + \dots + y_{n-3} + 5y_{n-2} + y_{n-1} + 6y_n\right]$$

$$= \frac{3\times 1}{10}\left[e^{\frac{\mu}{m}} + 5\times 1 + 1\times e^{-\frac{\mu}{m}}\right]$$

$$= \frac{3}{10}\left[5 + e^{\frac{\mu}{m}} + e^{-\frac{\mu}{m}}\right]$$

$$= \frac{3}{10}\left[5 + e^{\frac{\mu}{m}}(1 + e^{-1})\right] = \frac{3}{10}\left[5 + e^{\frac{\mu}{m}} + \frac{1}{e^{\frac{\mu}{m}}}\right]$$

$$= \frac{3}{e^{\frac{\mu}{m}}\cdot 10}\left[5e^{\frac{\mu}{m}} + e^{(\frac{\mu}{m})^2} + 1\right]$$

$$= \frac{0.3}{e^{\frac{\mu}{m}}}\left[e^{(\frac{\mu}{m})^2} + 5e^{\frac{\mu}{m}} + 1\right]$$

Boole's Rule

$$\frac{2\times 1}{45}\left[7\left(e^{\frac{\mu}{m}} + e^{-\frac{\mu}{m}}\right) + 32\times(1)\right]$$

$$= \frac{2}{45}\left[7\left\{\frac{e^{(\frac{\mu}{m})^2} + 1}{e^{\frac{\mu}{m}}}\right\} + 32\right]$$

$$= \frac{0.04}{e^{\frac{\mu}{m}}}\left[7\left(1 + e^{(\frac{\mu}{m})^2}\right) + 32e^{\frac{\mu}{m}}\right]$$

$$= \frac{0.04}{e^{\frac{\mu}{m}}}\left[7e^{(\frac{\mu}{m})^2} + 32e^{\frac{\mu}{m}} + 7\right]$$

The diff between them, $\frac{1}{e^{\frac{\mu}{m}}}\left[0.3\left\{5e^{\frac{\mu}{m}} + e^{(\frac{\mu}{m})^2} + 1\right\} - 0.04\left\{7e^{(\frac{\mu}{m})^2} + 32e^{\frac{\mu}{m}} + 7\right\}\right]$

$= \frac{0.22}{e^{\frac{\mu}{m}}}\left[e^{(\frac{\mu}{m})^2} + e^{\frac{\mu}{m}} + 7\right]$

② $I = \int_{-1}^{1} e^{\frac{-(\cos x + \sin x)}{m}} dx$ — Same ans (Previous)

Numerical differentiation

Finding the function $f(x)$ from $f'(x)$, there are the methods of infinite series expunction by using tylor or euler series expantion. ~~Doing~~ During the expunction of this series most of the higher order expunctions are removed.

Errors ~~of the~~ optimization

Let Suppose x_T is the true value of x and x_A is the approximate value of x. then the error E will be

Boole's Rule,

$$I = \int_{-1}^{1} e^{-\frac{(\cos x + \sin x)}{m}}$$

$a = -1,\ b = 1$

$h = 1$, Divide the integers into the following parts where, $h = 1$

$f(x) = e^{-\frac{(\cos x + \sin x)}{m}}$

$a_0 = -1$	$b_0 = e^{-\frac{(\cos 1 - \sin 1)}{m}} = e^{-\frac{1}{m}}$
$a_1 = 0$	$b_1 = e^{-\frac{(\cos 0 - \sin 0)}{m}} = e^{-\frac{1}{m}}$
$a_2 = 1$	$b_2 = e^{-\frac{(\cos 1 + \sin 1)}{m}} = e^{-\frac{1}{m}}$

weddle's Rule,

$$I = \frac{3 \times 1}{10}\left[e^{-\frac{1}{m}} + 5e^{-\frac{1}{m}} + e^{-\frac{1}{m}}\right]$$

$$= \frac{3}{10}\left[8e^{-\frac{1}{m}}\right]$$

$$= 2.4e^{-\frac{1}{m}}$$

Boole's Rule,

$$I = \frac{2 \times 1}{45}\left[7\left(e^{-\frac{1}{m}} + e^{-\frac{1}{m}}\right) + 32 \times e^{-\frac{1}{m}}\right]$$

$$= \frac{2}{45}\left[14e^{-\frac{1}{m}} + 32e^{-\frac{1}{m}}\right]$$

$$= 2.04e^{-\frac{1}{m}}$$

The difference between them $= 2.4e^{-\frac{1}{m}} - 2.04e^{-\frac{1}{m}}$

$= 0.36e^{-\frac{1}{m}}$

$= |x_T - x_A|$

i) If $E = 0$, there is no error

ii) If $E \neq 0$, then apply error minimization

Steps of error minimization (for all methods)

1) Let $E_0 = E$, E_0 = initial error

2) Find the decimal place upto which error is to be minimized.

Here consider a small value $\epsilon = 0.1$ if there is one decimal place

$\epsilon = 0.01$ if there is two decimal place

$\epsilon = 0.001$

3) E_{new} = error at n decimal place

4) $|E_0 - E_{new}| < \epsilon$

5) $E_0 = E_{new}$ (if a condition is happen)

while ($|E_0 - E_{new}| > \epsilon$)

$E_0 = E = E_{new}$

• The error minimization problem is to find the generic soln of the given particular problem with optimal soln.
[Applicable for all]

22/8/22

Exercise

1) Solve this numerical integrations

i) $\int_0^{\pi/2} \frac{\cos x}{1+\cos^2 x}\, dx$ (exact value : 0.632)

ii) $\int_0^{\pi} \frac{dx}{5+4\cos x}$ (exact value 1.0471)

iii) $\int_0^1 e^{-x^2}\, dx$ (exact value : 0.74)

iv) $\int_0^{\pi} \sin^3 x \cos^4 x\, dx$ (exact value 0.11428)

v) $\int_0^1 (1+e^{-x}\sin(4x))\, dx$ (exact : 1.3082)

Numerical Differentiation

$\frac{dy}{dx} = \frac{df}{dx}$ y' or y'' or y''' ... (find)

$f(x) = x^3 - 4x + 5y + z$

$y' = \left[\frac{dy}{dx}\right]$ at $(x = \alpha)$

find the value of diff at some point

The methods for soln of numerical differentiations are - i) method based on the finite difference operators

(ii) method based on interpolation.
(iii) method based on undetermind coefficients.

$\Delta x = f(x+h) - f(x)$
finite difference.

method-(i)
Ex-① To find the value of the derivative of the function given by $f(x) = \sin x$ at $x = 1$ with $h = 0.003906$

Find $f'(x) \rightarrow f'(1)$.

For finding the soln of this we have to find 3 premitives (backward, center, forword)

1) Backward difference

$D = \frac{d}{dx}$ (differential operator)

$$D_h^- f(x) = \frac{f(x) - f(x-h)}{h} \quad [\text{backward}]$$

$$= \frac{f(1) - f(1 - 0.003906)}{0.003906}$$

$$= \frac{f(1) - f(0.996094)}{0.003906}$$

$= 0.5419$

2) Center difference -

$$D_h f(x) = \frac{f(x+h) - f(x-h)}{2h}$$

$$= \frac{f(1+0.003906) - f(1-0.003906)}{2\times 0.003906}$$

$$= \frac{f(1.006)}{2\times 0.003906}$$

$= 0.5403$

3) Forward difference -

$$D_h^+ f(x) = \frac{f(x+h) - f(x)}{h} = 0.5386$$

$\frac{0.5419 + 0.5403 + 0.5386}{3} = f'(1)$

$$\boxed{f'(x) = \frac{f(x+h) - f(x)}{h}}$$

$$f''(x) = \frac{d}{dx}[f'(x)] = \frac{d}{dx}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$= D\left[\frac{f(x+h)}{h}\right] - D\left[\frac{f(x)}{h}\right]$$

$$D[f(x+h)] = \frac{f(x+2h) - f(x+h)}{h^2}$$

$$D\, f(x) = \frac{f(x+h) - f(x)}{h^2}$$

$$\frac{f(x+2h) - f(x+h) - f(x+h) + f(x)}{h^2}$$

$$\boxed{f''(x) = \frac{f(x+2h) - 2f(x+h) + f(x)}{h^2}}$$

$$\boxed{f'''(x) = \frac{f(x+3h) + A\, f(x+2h) + B f(x+h) + C f(x)}{h^3}}$$

$$f''(x) = \frac{f(x+2h) + A\, f(x+h) + B f(x)}{h^2}$$

$A = -2$
$B = 1$

$$\boxed{f^{n}(x) = \left[f(x+nh) + a_1 f(x+(n-1)h) + \dots + a_{n-1} f(x+h) + a_n f(x) \right] \frac{1}{h^n}}$$

[n^{th} derivative]

2) method based on interpolation –

Here the Solⁿ of f(x) is based on the given initial points with some step h. Then find the set of initial points such as $(x_0, y_0) \dots (x_n, y_n)$. To find the interpolating function.

$f(x) = \sin x \qquad x_0 = 1, \quad h = 0.25$

$x_1 = 1.25$, $\boxed{x_{n+1} = x_n + h}$

$x_2 = 1.5$

$x_n = 2$

3) method of undetermined coefficients

Discussed already above finding the unknown coefficients $a_1, a_2, \dots a_n$ from the given n^{th} derivative we can find the Solⁿ for numerical differentiation.

Diff eqn = $\frac{d^2y}{dx^2} + 12x\frac{dy}{dx} + e^{xy} = 2\sin x$

we find $y = f(x)$

by using numerical methods

Soln of ~~num~~ differential eqn using numerical methods - (3 methods Euler's, Euler's modified, Runga kutta method)

1) Euler's method is derived using taylors series expansion.

taylors series = $f(x+h) = f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \dots \infty$

$O\left(\{h f(x)\}\right)^{n=h} = 0$, $n=2$

$f(x+h) = f(x) + h f'(x)$

$h f'(x) = f(x+h) - f(x)$

$f'(x) = \frac{f(x+h) - f(x)}{h}$

According to Euler's theorem, if there is (x_0, y_0) or $y_0(x_0)$ be the initial point with h the ~~dem~~ soln of the differential eqn can be find out by

using $\boxed{\begin{aligned} x_{n+1} &= x_n + h \\ y_{n+1} &= y_n + h f(x_n, y_n) \end{aligned}}$ Imp

(2)

24/8/22

Numerical differential eqn on soln

Euler's method

$x_{n+1} = x_n + h$ h = step

$y_{n+1} = y_n + h f(x_n, y_n)$ $\boxed{\frac{dy}{dx} = f(x,y)}$

Ex-1 solve $\frac{dy}{dx} = x + 2y$, given $y(0) = 0$, $h = 0.25$

Find $y = f(x, y) = ?$

$f(x, y) = x + 2y$, $h = g$, $x_0 = 0$, $y_0 = 0$, $h = 0.25$

$x_1 = x_0 + h = 0.25$, $y_1 = y_0 + h \underset{f(x_0, y_0)}{\cdot} = ~~0.25~~ \ 0 + 0.25 \times (x_0 + 2y_0)$

$= 0 + 0.25(0.25 + 2\times 0)$

$= 0$

$x_2 = x_1 + h = 0.25 + 0.25 = 0.5$, $y_2 = y_1 + h f(x_1, y_1)$

$= 0 + 0.25 \times (0.25 + 0)$

$= 0.0625$

$x_3 = 0.5 + 0.25 = 0.75$, $y_3 = y_2 + h f(x_2, y_2)$

$= 0.0625 + 0.25(0.5 + 2\times 0.0625)$

$= 0.21875$

$\therefore y = y_3 = 0.21875$ [initial points must be given, h also given]

[If h is not given then continue when till the values not converged]

n	x_n	y_n
0	0	1
1	0.25	0.0
2	.50	0.0625
3	0.75	0.2187
4	1	0.515
5	1.2	0.5175

converge

Till two decimal place the y is = 0.51

EX-2

Solve $5\frac{dy}{dx} - y^2 = -x^2$ with $y(0) = 1$, $h = \frac{1}{2}$, $N = 4$

$f(x,y) = \left(\frac{y^2 - x^2}{5}\right)$

$x_0 = 0$ $y_0 = 1$

$x_{n+1} = x_n + h$

~~$x_1 = 0 + 0.5$~~

$$y_{n+1} = y_n + h f(x_n, y_n)$$
$$= y_n + \frac{1}{2}\left(\frac{y_n^2 - x_n^2}{5}\right)$$
$$= y_n + \frac{1}{10}(y_n^2 - x_n^2)$$

$x_1 = 0.5$ $y_1 = 1 + \frac{1}{10}\cdot(1)$
$= 1.1$

$x_2 = 1$ $y_2 = 1.1 + \frac{1}{10}\{(1.1)^2 - (0.5)^2\}$
$= \frac{290}{250} = \frac{29}{25}$

$x_3 = 1.5$ $y_3 = \frac{29}{25} + \frac{1}{10}\left[\left(\frac{29}{25}\right)^2 - (1)^2\right]$
$= 1.2390$

$x_4 = 1.5 + .5 = 2$ $y_4 = 1.676$

$x_5 = 2.5$ $y_5 = 0.9046$

$x_6 = 3$ $y_6 = 0.3606$

There is a possibility of not geting convergence. Because, x is not dependent but y is dependent of previous x.

- Due to increase of test of convergence and also the final o/p desire optimal solution may not be converge with the given itaration. So, the Euler's method is modified which is modified Eular's theorem.

According to modified Euler's theorem–

$$\boxed{y_1^{n+1} = y_0 + \frac{h}{2}\left[f(x_0, y_0) + f(x_{n+1}, y_{n+1}^n)\right] \quad n = 0, 1, 2}$$

$y_{n+1} = y_n + h f(x_n, y_n)$ Im

$\frac{h}{2}[\ ] + \frac{h}{2}[\ ]$

$$\boxed{y_1^n = y_n + h f(x_n, y_n)}$$

Ex-① Solve $\frac{dy}{dx} = x+y$, $y=1$, $x=0$

Compute y for $x = 0.05$

$f(x,y) = x+y$, $h = 0.05 - 0 = 0.005$

$y_1^0 = y_0 + h f(x_0, y_0)$; $x_1 = x_0 + h = 0 + 0.05 = 0.05$

$= 1 + 0.05(0+1)$

$= 1.05$ (y_1 at 0 iteration)

$y_1^1 = y_0 + \frac{h}{2}[(x_0+y_0) + (x_1+y_1^0)]$

$= 1 + \frac{0.05}{2}[(0+1) + (0.05+1.05)]$

$= 1.0525$

$x_2 = x + h$

$= 0.05 + 0.05$

$y_1^2 = y_0 + \frac{h}{2}[(x_0+y_0) + f(x_2, y_1^{(1)})]$

$\boxed{y_1^2 = 1.0526}$

$x_3 = 0.1 + 0.05$

$= 0.15$

$y_1^3 = y_0 + \frac{h}{2}[(x_0+y_0) + f(x_3, y_1^{(2)})]$

$= 1.0526$

$\boxed{y_1^2 = y_1^3 = 1.0526}$

i.e. the solⁿ, $y = 1.0526$ at $x = 0.05$

Exercise

Solve - ① $y = 0.25e^{2x} - 0.5x - 0.25$ at $x=0$, $y=1$, $h=0.25$ using Euler's modified theorem.

② Do a programme on the previous question.

[If y is given then 1st find y', then do everything]

$y = 0.25e^{2x} - 0.5x - 0.25$

$\frac{dy}{dx} = 0.25 \times e^{2x} \times 2 - 0.5$

$= 0.5e^{2x} - 0.5$

$f(x) = 0.5e^{2x} - 0.5$, $h = 0.25$

$x_1 = x_0 + h$

$= 0 + 0.25$

$= 0.25$

y_1^0 (y_1 at 0 iteration) $= y_0 + h f(x_0)$

$= 1 + 0.25 \times 0$

$= 1$

$y_1^1 = y_0 + \frac{h}{2}[(x_0+y_0) + (x_1+y_1^0)]$

$= 1 + \frac{0.25}{2}[(0+1) + (0.25+1)]$

$= 1.28125$

$x_2 = x_1 + h$
$= 0.25 + 0.25$
$= 0.5$

$y_1^1 = 1.28125$

$y_1^2 = y_0 + \frac{h}{2}[(x_0+y_0) +$

$y_1^1 = y_0 + \frac{h}{2}[(0.5e^{2x_0} - 0.5) + (0.5e^{2x_1} - 0.5)]$

$= 1 + \frac{0.25}{2}[(0.5e^0 - 0.5) + (0.5e^{0.5} - 0.5)]$

$= 1.041$

$x_2 = x_1 + h = 0.25 + 0.25 = 0.5$

$y_1^1 = 1.041$

$y_1^2 = y_0 + \frac{h}{2}[(0.5e^{2x_0} - 0.5) + (0.5e^{2x_2} - 0.5)]$

$= 1 + \frac{0.25}{2}[(0.5 - 0.5) + (0.5e^1 - 0.5)]$

$= 1.107$

$x_3 = x_2 + h = 0.5 + 0.25 = 0.75$

$y_1^2 = 1.107$

$y_1^3 = 1 + \frac{0.25}{2}[(0.5 - 0.5) + (0.5e^{2\times 0.75} - 0.5)]$

$= 1.217$

1/9/22

Runge Kutta method for solution of ODE —

This method is heighted with the degree of implementation of the modified euler's method (RK method). This method has 4th degree of implementation.

① 1st order RK method

$y_1 = y_0 + hf(x_0, y_0)$

② 2nd order RK method

$y_1 = y_0 + \frac{1}{2}(k_1 + k_2)$

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + h, y_0 + k_1)$

③ 3rd order RK method -

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + k_3)$

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k_3 = hf(x_0 + h, y_0 + k')$

$k' = hf(x_0 + h, y_0 + k_1)$

④ 4th order RK method

$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

$k_1 = hf(x_0, y_0)$

$k_2 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$

$k_3 = hf(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$

$k_4 = hf(x_0 + h, y_0 + k_3)$

Ex - ①

From the given $\frac{dy}{dx} = x^2 + y^2$, $y(1) = 1.2$, find $y(1.05) = ?$ using 4th order RK method.

$x_0 = 1$, $y_0 = 1.2$, $h = 0.05$ $[\because 1.05 - 1]$

$f(x, y) = x^2 + y^2$

$k_1 = hf(x_0, y_0)$
$= 0.05\,[(x_0)^2 + (y_0)^2]$
$= 0.05\,[(1)^2 + (1.2)^2]$
$= 0.122$

$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$
$= h\left[\left(x_0 + \frac{h}{2}\right)^2 + \left(y_0 + \frac{k_1}{2}\right)^2\right]$
$= 0.05\left[\left(1 + \frac{0.05}{2}\right)^2 + \left(1.2 + \frac{0.122}{2}\right)^2\right]$
$= \cancel{0.05 f [1.025}$
$= 0.05\,[(1.025)^2 + (1.261)^2]$
$= 0.1320$

$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$
$= 0.05 f\left(1 + \frac{0.05}{2}, 1.2 + \frac{0.1320}{2}\right)$
$= 0.05 f(1.025, 1.266)$
$= 0.05\,[(1.025)^2 + (1.266)^2]$
$= 0.1326$

$k_4 = hf(x_0 + h, y_0 + k_3)$
$= 0.05 f(1 + 0.05, 1.2 + 0.1326)$
$= 0.05\,[(1.05)^2 + (1.3326)^2]$
$= 0.1439$

$k_1 = 0.122$, $k_2 = 0.1320$, $k_3 = 0.1326$
$k_4 = 0.1439$

$y_1 = 1.2 + \frac{1}{6}(0.122 + 2 \times 0.1320 + 2 \times 0.1326 + 0.1439)$
$= 1.3325$

$y(1.05) = 1.3325$

Exercise

① $\frac{dy}{dx} = 2x + 3y^2$, $y(0.1) = 1$, $h = 0.01$

② $\frac{dy}{dx} = \frac{y - x}{y + x}$, $y(0) = 1$, $h = 0.2$

Find the 4th order RK method by using

If h is not given we will find $x - x_0$, where x is the given value for which you have to find y and x_0 is the given initial val.

$\therefore \boxed{h = x - x_0}$, If h is given, x_0 is given and ... is find the val of x for which you have to find y then $\boxed{x = h + x_0}$

12/9/22

systems of solutions of linear equation

m = no. of linear equations
n = " " variables

Case 1-(i) If m = = n → unique solutions
(ii) m > n → more solutions
(iii) m < n → No solutions

systems of solutions of linear eqn has 2 ways → Non iterative method and iterative method

Non iterative method → Gauss Elimination
→ LU Factorization

Iterative method → Gauss Jacobian
→ Gauss (S.) seidel (Fastest)

① Gauss Elimination method-

① we've given a system of linear equations.
② convert the linear eqn into matrix from
③ Perform Row wise co-operations
④ Perform coloumn wise operations
⑤ find the solutions for unknown

Ex-① $x+y+z=1$
$2x-y+2z=2$
$3x-2y-2z=3$

step 2
$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -1 & 2 \\ 3 & -2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

next step → find ~~Argument~~ Augmented matrix

$$\therefore A_g = \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 2 & -1 & 2 & : & 2 \\ 3 & -2 & -2 & : & 3 \end{bmatrix}$$

(Performing Row wise operation)

$$= \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -3 & 0 & : & 0 \\ 0 & -5 & -5 & : & 0 \end{bmatrix} \quad [R_2 \to R_2 - 2R_1] \quad [R_3 \to R_3 - 3R_1]$$

$$= \begin{bmatrix} 1 & 1 & 1 & : & 1 \\ 0 & -3 & 0 & : & 0 \\ 0 & 0 & -15 & : & 0 \end{bmatrix} \quad [R_3 \to 3R_3 - 5R_2]$$

Now from the above we can rewrite →

$x+y+z=1$
$-3y=0$
$-15z=0$

$\therefore x=1, y=0, z=0$ (Ans)

Ex-② $3x - 4y - z + w = 12$

$x - y + z + 13w = 8$

$2x + y - z - w = 9$

$x - y - z - 2w = 2$

Argument matrix (A_g) =

$$\left[\begin{array}{cccc|c} 3 & -4 & -1 & 1 & 12 \\ 1 & -1 & 1 & 13 & 8 \\ 2 & 1 & -1 & -1 & 9 \\ 1 & -1 & -1 & -2 & 2 \end{array}\right]$$

Steps ① change R_2, R_3, R_4 using R_1

② change R_3, R_4 using R_2

③ change R_4 using R_3

$$\cancel{\left[\begin{array}{cccc|c} -3 & -4 & -1 & 1 & \\ -38 & 51 & 14 & 0 & \\ 5 & -3 & -2 & 0 & \\ 7 & -9 & -3 & 0 & \end{array}\right]} \quad \cancel{\begin{bmatrix} R_2 = R_2 - 13R_1 \\ R_3 = R_3 + R_1 \\ R_4 = R_4 + 2R_1 \end{bmatrix}}$$

$$= \left[\begin{array}{cccc|c} 3 & -4 & -1 & 1 & 12 \\ 4 & -5 & 0 & 14 & 20 \\ -1 & 3 & 0 & -2 & -3 \\ -2 & 3 & 0 & -3 & -10 \end{array}\right] \quad \begin{bmatrix} R_2 = R_2 + R_1 \\ R_3 = R_3 - R_1 \\ R_4 = R_4 - R_1 \end{bmatrix}$$

$$= \left[\begin{array}{cccc|c} 3 & -4 & -1 & 1 & 12 \\ 4 & -5 & 0 & 14 & 20 \\ 3 & 0 & 0 & 12 & 17 \\ 2 & 0 & 0 & 27 & 10 \end{array}\right] \quad \begin{bmatrix} R_3 = R_3 + R_2 \\ R_4 = 5R_4 + 3R_2 \end{bmatrix}$$

$$= \left[\begin{array}{cccc|c} 3 & -4 & -1 & 1 & 12 \\ 4 & -5 & 0 & 14 & 20 \\ 3 & 0 & 0 & 12 & 17 \\ 0 & 0 & 0 & 57 & -4 \end{array}\right] \quad [R_4 = 3R_4 - 2R_3]$$

now from the above we can rewrite

$3x - 4y - z + w = 12$

$4x - 5y + 14w = 20$

$3x + 12w = 17$

$57w = -4$

$\Rightarrow w = \frac{-4}{57} = -0.07$

$3x + 12 \times \left(\frac{-4}{57}\right) = 17$

$\Rightarrow 3x = 17 + \frac{48}{57}$

$= \frac{969 + 48}{57}$

$\Rightarrow x = \frac{1017}{171} = 6$

$x \times C - 5y + 14 \times 0.07 = 20$

$\Rightarrow 24 - 0.98 - 5y = 20$

$\Rightarrow 5y = 3.02$

$\Rightarrow y = 0.604$

$3 \times C - 4 \times 0.604 - 7 \times -0.07 = 12$

$\Rightarrow z = 3.514$

14/9/22

Systems of sol^ns of linear eq^ns

① $3x - 4y + 4z = 2$

$x + y - z = 11$

$x + 4y + 3z = 12$

② LU - factorization

Lower triangular, Upper triangular

(It tells that whatever the sol^n is, will be divided into 2 parts)

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$
$$\vdots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

$m = n$

$A_g = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} & : & b_1 \\ a_{21} & a_{22} & \cdots & a_{2n} & : & b_2 \\ \vdots & \vdots & & & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} & : & b_m \end{bmatrix}$ (Augmented matrix)

①

$A_g = \begin{bmatrix} 3 & -4 & 4 & : & 2 \\ 1 & 1 & -1 & : & 11 \\ 1 & 4 & 3 & : & 12 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 & 4 & : & 2 \\ 0 & 7 & -7 & : & 31 \\ 0 & 37 & 5 & : & 34 \end{bmatrix}$ $\begin{bmatrix} R_2 \to 3R_2 - R_1 \\ R_3 \to 3R_3 - R_1 \end{bmatrix}$

$= \begin{bmatrix} 3 & -4 & 4 & : & 2 \\ 0 & 7 & -7 & : & 31 \\ 0 & 0 & 294 & : & -909 \end{bmatrix}$ $\left[R_3 \to 7R_3 - 37R_2 \right]$

$3x - 4y + 4z = 2$ (Gauss elemination)

$7y - 7z = 31$

$294z = -909$

(2) $2x + y - z = 2$

$x - y + 3z = 1$

$2x + y - z = 3$

$Ax = b$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 2 & 1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$A = LU$ — (1)

$\cancel{Ax} = L\underbrace{Ux}_{y}$ $\quad Ux = y$ — (3)

$\cancel{Ux} = y$ $\quad Ly = b$ — (2)

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 2 & 1 & -1 \end{bmatrix}_{3\times 3} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

(Lower triangular matrix) (lower triangular matrix)

$$\begin{bmatrix} \checkmark & \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark & \checkmark \\ \checkmark & \checkmark & \checkmark & \checkmark \\ \vdots & & & \checkmark \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ l_{21} & 1 & 0 & 0 \\ l_{31} & l_{32} & 1 & 0 \\ l_{41} & l_{42} & l_{43} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} & u_{14} \\ 0 & u_{21} & u_{23} & u_{24} \\ 0 & 0 & u_{33} & u_{34} \\ 0 & 0 & 0 & u_{44} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} u_{11}+0+0 & u_{12}+0+0 & u_{13} \\ u_{11}\cdot l_{21}+0 & l_{21}\cdot u_{12}+u_{22} & l_{21}\cdot u_{13}+u_{23} \\ u_{11}\cdot l_{31} & u_{12}\cdot l_{31}+u_{22}\cdot l_{32} & u_{13}\cdot l_{31}+l_{32}\cdot u_{23}+u_{33} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 1 & -1 \\ 1 & -1 & 3 \\ 2 & 1 & -1 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{11}l_{21} & l_{21}u_{12}+u_{22} & l_{21}u_{13}+u_{23} \\ u_{11}l_{31} & u_{12}l_{31}+u_{22}l_{32} & u_{13}l_{31}+l_{32}u_{23}+u_{33} \end{bmatrix}$$

$u_{11} = 2$

$u_{12} = 1$

$u_{13} = -1$

$u_{11}l_{21} = 1$

$\Rightarrow 2\times l_{21} = 1$; $\Rightarrow l_{21} = \frac{1}{2}$

$l_{21}u_{12} + u_{22} = -1$

$\Rightarrow \frac{1}{2}\times 1 + u_{22} = -1$

$\Rightarrow u_{22} = -\frac{3}{2}$

$l_{21}u_{13} + u_{23} = 3$

$\Rightarrow \frac{1}{2}\times(-1) + u_{23} = 3$

$\Rightarrow u_{23} = 3 + \frac{1}{2}$

$= \frac{7}{2}$

$u_{11}l_{31} = 2$

$\Rightarrow l_{31} = 1$

$u_{12}l_{31} + u_{22}l_{32} = 1$

$\Rightarrow 1\times 1 + \left(-\frac{3}{2}\right)\cdot l_{32} = 1$

$\Rightarrow -\frac{3 l_{32}}{2} = 0$

$\Rightarrow l_{32} = 0$

$u_{13}l_{31} + l_{32}u_{23} + u_{33} = -1$

$\Rightarrow -1\times 1 + 0\times\frac{7}{2} + u_{33} = -1$

$\Rightarrow u_{33} = 0$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \quad U = \begin{bmatrix} 2 & 1 & -1 \\ 0 & -\frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$

$Ly = b$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} y_1 \\ \frac{y_1}{2} + y_2 \\ y_1 + y_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$y_1 = 2$

$\frac{y_1}{2} + y_2 = 1 ; \Rightarrow \frac{2}{2} + y_2 = 1$
$\Rightarrow y_2 = 0$

$y_1 + y_3 = 3 ; \Rightarrow 2 + y_3 = 3 ; \Rightarrow y_3 = 1$

$Ux = y$

$$\begin{bmatrix} 2 & 1 & -1 \\ 0 & -\frac{3}{2} & \frac{7}{2} \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x + y - z \\ 0 - \frac{3y}{2} + \frac{7z}{2} \\ 0 + 0 + 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

$2x + y - z = 2$ — (1)

$-\frac{3y}{2} + \frac{7z}{2} = 0$, let $z = 1$

$\Rightarrow \frac{3y}{2} = \frac{7z}{2}$

$\Rightarrow y = \frac{7z}{3}$ — (ii) $\Rightarrow y = \frac{7}{3}$

from (1), $2x + \frac{7z}{3} - z = 2$

$\Rightarrow 2x + \frac{7}{3} - 1 = 2 ; \Rightarrow x = \frac{1}{3}$

(3) $$\begin{bmatrix} 3 & -4 & 7 \\ 1 & 5 & -1 \\ 1 & -11 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \times \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$L = [\ ?\ ]$, $U = [\ ?\ ]$

(2) $Ly = b$

$[\ \ ] \times \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = [\ \vdots\ ]$

$Ux = y$

(3) $[\ \ ] \times \begin{bmatrix} x \\ y \\ z \end{bmatrix} = [\ \ ]$

(2) $2x + y - z = 2$
$x - y + 3z = 1$
$2x + y - z = 3$
ans by LU factorization

$x = \frac{1}{3}, y = \frac{7}{3}, z = 1$

The difference between term = 0.306

10/9/22

(3) Gauss Jacobian method
(Iterative methods)

$a_{11}x_1 + a_{12}x_2 \cdots + a_{1n}x_n = b_1$

$\vdots$

$a_{n1}x_1 + a_{n2}x_2 + \cdots a_{nn}x_n = b_2$

Ex-(1)

$$3x - 4y + 7z = 4 \rightarrow x = (4 + 4y - 7z)\frac{1}{3}$$

$$x - y + 3z = 2 \longrightarrow y = (2 - x - 3z)$$

$$3x + 4y + 11z = 3 \rightarrow z = \frac{1}{11}(3 - 3x - 4y)$$

Generalized form -

$$x^k = \frac{1}{3}(4 + 4y^{k-1} - 7z^{k-1})$$ [k, k-1 = iteration]

$$y^k = (2 - x^{k-1} - 3z^{k-1})$$ [In iteration we have some initial values]

$$z^k = \frac{1}{11}(3 - 3x^{k-1} - 4y^{k-1})$$ [k will be also given]

Solve that problem (1) by using Gauss Jacobian method where given $x^0 = 2, y^0 = 1, z^0 = 3$

(or) $x^0 = y^0 = z^0 = 0$

Iteration table

	K	x	y	z
(A)	0	$x^0=0$	$y^0=0$	$z^0=0$
	1	$x^1=\frac{4}{3}$	$2=y^1$	$\frac{3}{11}=z^1$
	2	$x^2=\frac{5}{12}$	$\frac{6}{17}=y^2$	$\frac{7}{13}=z^2$
	10	1.212	2.129	3.12
	11	1.233	2.11	3.121

→ We can stop [as the value of x, y, z are converging]

The difference between them = 2.46 ... = 0.36 ...

Gauss seidel methode (upgradation of Gauss Jacobian method)

$$x^K = \frac{1}{3}(47 + 4y^{K-1} - 7z^{K-1})$$

$$y^K = (2 - x^K - 3z^{K-1}) \quad [x \text{ is upgraded}]$$

$$z^K = \frac{1}{11}(3 - 3x^K - 4y^K)$$

K	x	y	z
0	$x^0=0$	$y^0=0$	$z^0=0$
1	$x^1=\frac{4}{3}$	$y^1=2-\frac{4}{3}-3\cdot 0=\frac{2}{3}$	$\frac{1}{11}(3-3\cdot\frac{4}{3}-4\cdot\frac{2}{3})=z^1$
2	$\frac{37}{33}=x^2$	$\frac{30}{33}=y^2$	z^2

(S. of. S. of Linear eqns from Gauss Jacobian)

*To solve the problem from Gauss Jacobian methode to Gauss Seidel method the following steps are needed —

i) Arrange the given set of eqns with the test of convergence $|a_{ii}| >$ $|a_{ij}| + |a_{ik}|$ $[i \neq j \neq k]$ [a = absolute]

ii) Solve the 1st eqn value for the 1st variable, 2nd eqn for the 2nd variable and so on.

iii) Use iteration table and start the computation from the given initial points, in seeded of that always use the upgreated computed value for the next value.

$|3| \ngtr |2| + |-7|$ $\quad |7| > |3| + |2|$

Ex (2)

$3x + 2y - 7z = 1 \rightarrow$ (i)

$x + 12y + 10z = 2 \rightarrow$ (ii)

$13x + y + 2z = 3 \rightarrow$ (iii)

Step — (1)

$13x + y + 2z = 3 \longrightarrow x^K = \frac{1}{13}(3 - y^{K-1} - 2z^{K-1})$

$x + 12y + 10z = 2 \longrightarrow y^K = \frac{1}{12}(2 - x^K - 10z^{K-1})$

$3x + 2y - 7z = 1 \longrightarrow z^K = \frac{1}{7}(3x^K + 2y^K - 1)$

Ex of conversion of Gauss Jacobian method to Gauss seidal method and then soln.

The difference between them $= 2.42\,m^{-1} - 2.04\,e\,m^{-1}$
$= 0.36\,e\,m^{-1}$

The matrix form of Gauss Jacobian and Gauss seidel method

$Ax = B$ [If merge A and B = Augmented matrix]

$A = LU$

$\boxed{A = D - L - U}$ [D = Diagonal matrix, L = Lower triangular matrix, U = upper triangular matrix]

$Ax = b$

$\Rightarrow (D - L - U)x = b$

$\Rightarrow Dx - Lx - Ux = b$

$\Rightarrow Dx = (L+U)x + b$

$\Rightarrow x = D^{-1}(L+U)x + D^{-1}b$

$\Rightarrow x = D^{-1}(L+U)x + D^{-1}b$

$\Rightarrow x^k = T_{jacobi}\, x^{k-1} + C$ $[T_{jacobi} = D^{-1}(L+U)]$ $[D^{-1}b = C]$

$$\boxed{x^k = T_{jacobi}\, x^{k-1} + C}$$

matrix form of Gauss Jacobi method

$Ax = b$

$(D - L - U)x = b$

$\Rightarrow (D - L)x = Ux + b$

$\Rightarrow x = (D-L)^{-1}Ux + (D-L)^{-1}b$

$$\Rightarrow \boxed{x^k = T_{seidel}\, x^{k-1} + E}$$ $[E = (D-L)^{-1}b]$ $[T_{seidel} = (D-L)^{-1}U]$

matrix form of Gauss seidel method

(3)

NM ~ Previous year Qn

13/12/22

1) What are iterative and non-iterative methods for the soln of system of linear equations. Where have they been used and why? Solve the below questions using Gauss Elimination and Gauss Seidel methods and compare the results.

$$4x + y + 2z = 4$$
$$3x + 8y - 2z = 5$$
$$7x + y + 11z = 7$$

a) Iterative & non iterative methods for the soln of system of linear eqn –

Non-iterave method –

i) Gauss Elimination

ii) LU Factorization

Iterative method –

i) Gauss Jacobian

ii) Gauss Seidel method

b) They are used in iterative design and prototyping as ~~it has~~ they have following benefits -

1) Greater efficiency and faster time to market.

2) Lower product development costs.

3) Roundoff errors are not given a chance to "accumulate".

The difference between them = $2.4e^{-\frac{1}{m}} - 2.04e^{-\frac{1}{m}}$
$= 0.36e^{-\frac{1}{m}}$

Gaussi elimination method — $\begin{bmatrix} 4 & 1 & 2 \\ 5 & 8 & -2 \\ 7 & 1 & 11 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 7 \end{bmatrix}$

$$Ag = \left[\begin{array}{ccc:c} 4 & 1 & 2 & 4 \\ 5 & 8 & -2 & 5 \\ 7 & 1 & 11 & 7 \end{array}\right]$$

$$= \left[\begin{array}{ccc:c} 4 & 1 & 2 & 4 \\ 0 & 27 & -18 & 0 \\ 0 & -3 & 30 & 0 \end{array}\right] \quad \begin{matrix} [R_2' = 4R_2 - 5R_1] \\ [R_3' = 4R_3 - 7R_1] \end{matrix}$$

$$= \left[\begin{array}{ccc:c} 4 & 1 & 2 & 4 \\ 0 & 27 & -18 & 0 \\ 0 & 0 & 756 & 0 \end{array}\right] \quad [R_3' = 27R_3 + 3R_2]$$

now from the above we can write

$4x + y + 2z = 4 ;\Rightarrow 4x + 0 + 0 = 4 ;\Rightarrow x = 1$

$27y - 18z = 0 ;\Rightarrow 27y - 0 = 0 ;\Rightarrow y = 0$

$756z = 0$

$\Rightarrow z = 0$

$x = 1, y = 0, z = 0$

The difference between them = $2.4e^{-1} - 2.04e^{-1}$ = $0.36e^{-1}$

Gauss Seidel method —

$4x + y + 2z = 4$

$\Rightarrow 4x = 4 - y - 2z$

$\Rightarrow x = \frac{1}{4}(4 - y - 2z)$ — (i)

$5x + 8y - 2z = 5$

$\Rightarrow 8y = 5 - 5x + 2z$

$\Rightarrow y = \frac{1}{8}(5 - 5x + 2z)$ — (ii)

$7x + y + 11z = 7$

$\Rightarrow 11z = 7 - 7x - y$

$\Rightarrow z = \frac{1}{11}(7 - 7x - y)$ — (iii)

generalized form

$x^k = \frac{1}{4}(4 - y^{k-1} - 2z^{k-1})$ — (i)

$y^k = \frac{1}{8}(5 - 5x^k + 2z^{k-1})$ — (ii)

$z^k = \frac{1}{11}(7 - 7x^k - y^k)$ — (iii)

let $x_0, y_0, z_0 = 0$

k	x	y	z
0	$x^0 = 0$	$y^0 = 0$	$z^0 = 0$

2) State the general Newton's-Raphson method with the criteria for convergence. Find the roots of equation: $f(x) = 5x^2 - 3\tan(x) - 3$ using Newton's Raphson methods upto 3 decimal places. Can we apply iteration method to find the root of the eqn?

a) done

the criteria for convergence -

it converges if $|f(x) \cdot f''(x)| < |f'(x)|^2$.

Also, this method fails if $f'(x) = 0$

b) $f(x) = 5x^2 - 3\tan x - 3$

$f'(x) = 10x - 3\sec^2 x$

$10x - \frac{3}{\cos^2 x}$

$f(0) = -3 < 0$

$f(1) = 5 - 3$
$= 2 > 0.$

interval $= [0, 1]$

n (iter)	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
0	0	-3	7	$x_1 = 0 + \frac{3}{7}$ = 0.42
1	0.42	-2.14	7	$x_2 = 0.42 + \frac{2.14}{7}$ = 0.72
2	0.72	-0.22	7	$x_3 = 0.72 + \frac{0.22}{7}$
3	0.75			

The difference between them $= 2.4e^{-\frac{1}{m}} - 2.04e^{-\frac{1}{m}}$
$= 0.36e^{-\frac{1}{m}}$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
0	0.5	1.776	2	$x_1 = 0.5 - \frac{1.776}{2} = -0.388$
1	-0.388	-2.227	-6.88	$x_2 = -0.388 - \frac{6.88}{2.227} = -3.477$
2	-3.477	57.63	-37.781	$x_3 = -3.477 + \frac{57.63}{37.781} = -1.951$

d) yes as,
: the fixed point iteration method is an iterative method to find the roots of algebraic and transcendental equations by converting them into a fixed point function.

3) State the ~~General~~ newton's forward and backward interpolation formula. Where these methods are different from Langrange and newton's divided difference interpolation formula. Find the function $f(x)$ using any these methods, where interpolating points are given by—

x	3	6	2	11	5
$f(x)$	10	20	30	40	50

The difference between them $= 2.4e^{-\frac{1}{3}} - 2.04e^{-\frac{1}{3}}$
$= 0.36e^{-\frac{1}{3}}$

2.04e 3

a) done

b) Langrange and newton's divided difference interpolation formula are used when the initial points given are in unequal interval. N.F.I.F and N.B.I.F are used when the given initial points are in equal interval. [+ But we can use 1st 2 methods for equal interval also]

c)

x	$f(x)$ [y_0]	1st DD	2nd DD	3rd DD	4th DD
x_0 3	10 y_0	$\frac{20-10}{6-3} = 3.33$			
x_1 6	20 y_1		$\frac{-2.5-3.33}{2-3} = 5.83$		
		$\frac{30-20}{2-6} = -2.5$		$\frac{0.722-5.83}{11-3} = -0.63$	
x_2 2	30 y_2		$\frac{1.11+2.5}{11-6} = 0.722$		$\frac{1.64+0.63}{5-3} = 1.135$
		$\frac{40-30}{11-2} = 1.11$		$\frac{-0.92-0.722}{5-6} = 1.64$	
x_3 11	40 y_3		$\frac{-1.67-1.11}{5-2} = -0.92$		
		$\frac{50-40}{5-11} = -1.67$			
x_4 5	50 y_4				

$f(x) = 10 + (x-3) \times 3.33 + (x-3)(x-6) \times 5.83$
$+ (x-3)(x-6)(x-2) \times (-0.63) +$
$(x-3)(x-6)(x-2)$
$(x-11) \times 1.135$

4) Discuss some numerical methods. Where these methods are applied. Using Simpson 3/8 and Weddle's rules solve the following integration problem with step value = 1·5 and compare results.

$$\int_1^5 (x + \log x)\, dx$$

a) done b) done

c) $h = 1·5$

$a_0 = 1$	$b_0 = (1 + \log 1) = 1$
$a_1 = 1 + 1·5 = 2·5$	$b_1 = (2·5 + \log 2·5) = 3$
$a_2 = 2·5 + 1·5 = 4$	$b_2 = (4 + \log 4) = 4·6$
$a_3 = 4 + 1·5 = 5$	$b_3 = (5 + \log 5) = 5·6$

$f(x) = (x + \log x)$

Simpson 3/8 rule,

$$\int_1^5 (x + \log x)\, dx = \frac{3 \times 1·5}{8}\left[1 + 5·6 + 2\underbrace{(0)}_{\text{3's multiple}} + 3 \times (3 + 4·6)\right]$$

$= 16·5375$

Weddle's rule,

$$I = \frac{3 \times 1·5}{10}\left[1 + 3 \times 5 + 4·6 \times 1 + 5·6 \times 6\right]$$

$= 24·39$

Comparison $= 24·39 - 16·5375$

$= 7·8525$

The val. from W. R. is 7·8525 greater than S. 3/8 R.

2·04 e m

The difference between them $= 2·4e^{-\frac{1}{m}} - 2·04e^{-\frac{1}{m}}$

$= 0·36e^{-\frac{1}{m}}$

5) What are transcendental eqn? Explain LU factorization for system of sol's of linear eqns. What is the order of the truncation error of the trapezoidal rule as function of n, the numbers of trapezoids?

a) An equation over the real (or complex) numbers that isn't algebric i.e it is an eqn containing polynomials, logarithmic functions, trigonometric functions, and exponential functions.

b) LU decomposition of a matrix is the factorization of a given square matrix into two triangular matrices, one upper triangular matrix and one lower triangular matrix, such that the product of these two matrices gives the original matrix. It includes 3 steps —

i) $A = LU$, ii) $LY = b$, iii) $UX = Y$ [A = Augmented matrix, L = Lower triangular matrix, U = Upper " "]

c) The error in the trapezoidal rule is of the order h^2.

There are 3 stages of selection process - Prelims, mains and interview.

c) If $\Delta f(x) = f(x+h) - f(x)$, then a constant k, Δk equals? Show that Gaussian quadrature using $n+1$ points is exact for polynomial of degree $k \leq 2n+1$. What is significant digits for 0.0216×10^4,

given: $\Delta f(x) = f(x+h) - f(x)$

To find: for constant k, Δk =

c) Soln, $\Delta f(x) = f(x+h) - f(x)$

Let $f(x) = k$ where k is a constant

Then $f(x+h) = k$ [as if we add some thing or subtract something from a constant it will remain constant]

$\therefore \Delta k = k - k = 0$

The difference between them $= 2.42^{-1}_{3} - 2.042^{-1}_{3}$

$= 0.38^{-1}_{3}$

b) $\int S(x) P_{n+1}(x)\, dx = 0$. The first equality is because the degree of $R(x)$ is less than the number of points used in the quadrature formula. The 2nd equality is because the points x_i were chosen to be the zeros of P_{n+1}. So, we have shown that the quadrature formula is extract for polynomials of degree $\leq 2n+1$.

c) 3 significant figure

7) For the given real numbers x_0, x_1 and x_2, define the divided difference $f[x_0, x_1, x_2]$ of a real valued function $f(x)$. Find the value of $f[1, 0.5, 1]$ when $f(x) = \tan(x)$. Estimate the effect of data inaccuracy on results computed by trapezoidal and Simpson's rule. Find the approximate soln of the eqn $x\tan x - 1 = 0$ (since calculated in radians) in the interval $[0,2]$ using Bisection method. Obtain the number of iterations to be performed to obtain a soln whose absolute error is less than 10^{-3}.

a) $f(x) = f(x_0) + (x - x_0) f[x_0, x_1] + (x - x_0)(x - x_1) f[x_0, x_1, x_2]$

$\Rightarrow f[x_0, x_1, x_2] = \dfrac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0}$

b) $x_0 = 1$, $y_0 = \tan 1$ [as $f(x) = y = \tan x$]

$= 0.01$

$x_1 = 0.5$, $y_1 = \tan 0.5$

$= 8.7 \times 10^{-3}$

$x_2 = 1$, $y_2 = 0.01$

b) $f[1, 0.5, 1] = \dfrac{f(0.5, 1) - f(1, 0.5)}{1 - 1}$

$= \infty$

	h	y	1st DD $[f(x_0, x_1)]$	2nd DD $[f(x_0, x_1, x_2)]$
x_0	1	0.01 (y_0)	$\frac{8.7\times10^{-3} - 0.01}{0.5 - 1} = 2.6\times10^{-3}$	$\frac{2.6\times10^{-3} - 2.6\times10^{-3}}{1 - 1} = 0$
x_1	0.5	8.7×10^{-3} (y_1)		
			$\frac{0.01 - 8.7\times10^{-3}}{1 - 0.5} = 2.6\times10^{-3}$	
x_2	1	0.01 (y_2)		

$f(x) = 0.01 + (x-1)\times 2.6\times10^{-3} + (x-1)(x-0.5)\times 0$

$f(x) = 0.01 + (x-1)\times 2.6\times10^{-3}$

The trapezoidal rule with n = 6 partitions. (The "error" is the difference between the actual "true" value and the approximation. Errors in T and S rule can be calculated with a couple of straightforward formulas, these are useful when we want to increase the accuracy of an approximation.)

The difference between them $= 2.4e^{-\frac{1}{3}} - 2.04e^{-\frac{1}{3}}$
$= 0.36e^{-\frac{1}{3}}$

d) $f(x) = x\tan x - 1 = 0$
interval $[0, 2]$

Sol^n by Bisection method —

	a	b	$t = \frac{a+b}{2}$	$f(a)$	$f(b)$	$f(t)$
1)	0	2	1	−1	−0.9	−0.9
2)	1	1	~~−0.9~~ 1	−0.9	−0.9	−0.9
3)	1	1	1	−0.9	−0.9	−0.9

The approximate value is $a = b = 1$

e) ??
Not done by sir

a	b	$t = \frac{a+b}{2}$	$f(t) = x\tan x - 1$
0	2	1	0.556

$\frac{22}{7}$ rad → 180°

1 → $\frac{180\times7}{22}$ = 57.27

$\boxed{1 \text{ rad} = 57.27}$

$x = 1\times 57.27$

update a by t
if $f(a) = +ve$, and $f(a) = f(t)$
and b by t
if $f(b) = +ve$, and $f(b) = f(t)$

8) What are constrained and unconstrained optimizations? How can you solve the systems of non-linear eqn using numerical methods? How differential eqn's are solved using numerical methods. What are ways for the soln of complex optimization problems?

a) Optimization problems. Unconstrained simply means that the choice variable can take on any value — there are no restrictions. Constrained means that the choice variable can only take on certain values within a larger range.

b) We can resolve the systems of nonlinear eqn using the following n.m —
i) Newton's method, ii) Broyden's method
iii) the finite difference method.

c) Differential eqn's are solved by the use of the following n.m —
i) Euler's method, ii) Euler's modified method
iii) Runge-Kutta method.
(majorly used)

d) We've seen that we can solve optimization problems by following a five-step process. It is: visualize the problem, define the problem, write an eqn for it, find the minimum or maximum for the problem and answer the qn.

$2.04e^{-\frac{1}{3}}$

The difference between them $= 2.4e^{-\frac{1}{3}} - 2.04e^{-\frac{1}{3}}$
$= 0.36e^{-\frac{1}{3}}$

NM Prv Yr Qn Soln (2020)

1) State the General Newton's – Raphson method with the criteria for convergence. Find the roots of the eqn: $f(x) = 4x^2 - 2x - 3$ using Newton's-Raphson methods upto 3 decimal places. Can we apply iteration method to find the root of the equation?

a) ~~Dod~~ done.
~~b) done~~

b) $f(x) = 4x^2 - 2x - 3$

$f'(x) = 8x - 2$

$f(0) = -3$

$f(1) = 4 - 2 - 3$
$= -1$

$f(2) = 4 \times 4 - 4 - 3$
$= 9$

interval $= [1, 2]$

n	x_n	$f(x_n)$	$f'(x_n)$	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
0	$x_0 = 1$	-1	6	$x_1 = 1.167$
1	$x_1 = 1.167$	0.1134	7.336	$x_2 = 1.151$
2	$x_2 = 1.151$	-2.79×10^{-3}	7.208	$x_3 = 1.151$

root $= 1.151$

c) done

$= 2.04\, e^{-\frac{1}{m}}$

The difference between them $= 2.4e^{-\frac{1}{m}} - 2.04e^{-\frac{1}{m}}$
$= 0.36\, e^{-\frac{1}{m}}$

2) What are iterative and non-iterative methods for the solution of system of linear equation. Where have they been used and why? Solve the below question using Gauss Elimination method and Gauss Seidel method and compare their results.

$3x - y + 7z = 1$
$5x - 2y + z = 3$
$x - 4y + 2z = 2$

a) done
b) done

c) Gauss elimination method

$$\begin{bmatrix} 3 & -1 & 7 \\ 5 & -2 & 1 \\ 1 & -4 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$$

$$A_g = \left[\begin{array}{ccc:c} 3 & -1 & 7 & 1 \\ 5 & -2 & 1 & 3 \\ 1 & -4 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 3 & -1 & 7 & 1 \\ 0 & -1 & -32 & 4 \\ 0 & -11 & -1 & 5 \end{array}\right] \quad [R_2' = 3R_2 - 5R_1] \quad [R_3' = 3R_3 - R_1]$$

$$= \left[\begin{array}{ccc:c} 3 & -1 & 7 & 1 \\ 0 & -1 & -32 & 4 \\ 0 & 0 & 351 & -39 \end{array}\right] \quad [R_3' = R_3 - 11R_2]$$

$= 2.04e^{-\frac{1}{3}}$

The difference between them $= 2.4e^{-\frac{1}{3}} - 2.04e^{-\frac{1}{3}}$
$= 0.36e^{-\frac{1}{3}}$

Now from the above we can write,

$351z = -39$
$\Rightarrow z = \frac{-39}{351}$
$= -0.111$ — (i)

$-y - 32z = 4$
$\Rightarrow -y + 32 \times 0.111 = 4$
$\Rightarrow y = -0.448$

$3x - y + 7z = 1$
$\Rightarrow 3x + 0.448 - 7 \times 0.111 = 1$
$\Rightarrow 3x = 1.329$
$\Rightarrow x = 0.443$

$x = 0.443,\ y = -0.448,\ z = -0.111$

Gauss Seidel method,

$3x - y + 7z = 1$
$\Rightarrow 3x = 1 + y - 7z$
$\Rightarrow x = \frac{1}{3}(1 + y - 7z)$ — (i)

$5x - 2y + z = 3$
$\Rightarrow 2y = 5x + z - 3$
$\Rightarrow y = \frac{1}{2}(5x + z - 3)$ — (ii)

$x - 4y + 2z = 2$
$\Rightarrow 2z = 2 - x + 4y$
$\Rightarrow z = \frac{1}{2}(2 - x + 4y)$ — (iii)

generalized form –

$$x^{k} = \frac{1}{3}(1 + y^{k-1} - 7z^{k-1}) \quad \text{— (i)}$$

$$y^{k} = \frac{1}{2}(5x^{k} + z^{k-1} - 3) \quad \text{— (ii)}$$

$$z^{k} = \frac{1}{2}(2 - x^{k} + 4y^{k}) \quad \text{— (iii)}$$

assume, $x^0 = y^0 = z^0 = 0$

$x_0, y_0, z_0 = ?$

k	x	y	z
0	$x^0 = 0$	$y^0 = 0$	$z^0 = 0$
1	$x^1 = 0.6$	$y^1 = -0.33$	$z^1 = -0.469$
2	$x^2 = 0.49$	$y^2 = -0.45$	$z^2 = -0.1333$
3	$x^3 = 0.443$	$y^3 = -0.456$	$z^3 = -0.1122$
4	$x^4 = 0.44$	$y^4 = -0.446$	$z^4 = -0.1094$
5	$x^5 = 0.44$	$y^5 = -0.44$	$z^5 = -0.11$
6	$x^6 = 0.44$	$y^6 = -0.44$	$z^6 = -0.11$

$x = 0.44$

$y = -0.44$

$z = -0.11$

$= 2.04\, e^{-\frac{1}{3}}$

The difference between them $= 2.4e^{-\frac{1}{3}} - 2.04e^{-\frac{1}{3}}$

$= 0.36e^{-\frac{1}{3}}$

3) State Newton's forward and backward interpolation formula. Where these methods are different from Langrange and Newton divide difference interpolation formula. Find the function f(x) using any these methods, where interpolating points are given by –

x	1.5	3	4.5	6	7.5	9	10.5	12
y	1	2	3	4	5	6	7	8

a) done

b) done

c) the common diff of x is equal (1.5). So we will apply N.F.I.F.

x	y	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$	$\Delta^7 y_0$
1.5	1	1	0	0	0	0	0	0
3	2	1	0	0	0	0	0	
4.5	3	1	0	0	0	0		
6	4	1	0	0	0			
7.5	5	1	0	0				
9	6	1	0					
10.5	7	1						
12	8							

$p = \frac{x - x_0}{h}$

$= \frac{x - 1.5}{1.5}$

$f(x) = 1 + 1 \times \frac{x-1.5}{1.5} + 0 \times \frac{(x-1.5)(x-1.5-1)}{2!}$

$+ 0 \times \frac{(x-1.5)(x-1.5-2)}{3!} + 0 \times \frac{(x-1.5)(x-1.5-1)(x-1.5-2)(x-1.5-3)}{4!}$

$+ 0 \times \frac{(x-1.5)(x-1.5-1)(x-1.5-2)(x-1.5-3)(x-1.5-4)}{5!}$

$+ 0 \times \frac{(x-1.5)(x-1.5-1)(x-1.5-2)(x-1.5-3)(x-1.5-4)(x-1.5-5)}{6!}$

$+ 0 \times \frac{(x-1.5)(x-1.5-1) \cdots (x-1.5-6)}{7!}$

$= 1 + 1 \times \frac{x-1.5}{1.5}$

$= \frac{1.5 + x - 1.5}{1.5}$

$= \frac{x}{1.5}$

$f(x) = 0.67x$

$= 2.04 e^{-\frac{1}{n}}$

The difference between them $= 2.4 e^{-\frac{1}{n}} - 2.04 e^{-\frac{1}{n}}$

4) Discuss some numerical integration methods. where these methods are applied. using simpson's 3/8 and weddl's rule is, solve the following integration problem with step value = 1.5 and compare the results.

$\int_1^5 (1 + x \cdot \sin x)\,dx$

a) done

b) done

c) $h = 1.5$ $\qquad f(x) = (1 + x \sin x)$

$a_0 = 1$	$b_0 = (1 + 1 \cdot \sin 1) = 1.01$
$a_1 = 1 + 1.5 = 2.5$	$b_1 = (1 + 2.5 \sin 2.5) = 1.11$
$a_2 = 2.5 + 1.5 = 4$	$b_2 = (1 + 4 \sin 4) = 1.27$
$a_3 = 4 + 1.5 = 5$	$b_3 = (1 + 5 \sin 5) = 1.43$

simpson 3/8 rule,

$I = \int_1^5 \underset{(1 + x \sin x)}{\cancel{(x + \log x)}}\,dx = \frac{3 \times 1.5}{8} [1.01 + 1.43 + \cancel{2(1.43)} + 2 \times (0) + 3(1.11 + 1.27)]$

$= 5.38875$

weddle's rule,

$I = \int_1^5 (1 + x \sin x)\,dx = \frac{3 \times 1.5}{10} [1 \times 1.01 + 5 \times 1.11 + 1 \times 1.27 + 6 \times 1.43]$

$= 7.3845$

Comparison = 7.3845 − 5.38875
= 2

value of from the weddles rule ~~y~~ the value from simpson 3/8 rule and it is = 2

(ans) The val. of w.r is '2' greater than the val. of S. 3/8 r.

5) Discuss numerical soln of differential eqns. How Euler's method is derived taylor's series expansion method. Solve $\frac{dy}{dx} = 6 - \frac{2y}{x}$, $y(3) = 1$

1st, done

b) done.

c) $\frac{dy}{dx} = 6 - \frac{2y}{x}$

$x_0 = 3$, $y_0 = 1$ $f(x,y) = 6 - \frac{2y}{x}$

$h = ??$

$x_1 = x_0 + h$

$\Rightarrow \frac{1}{-3}\log\left(6 - \frac{3y}{x}\right) = \log x + -c$

$\Rightarrow -\frac{1}{3}\log\left(\frac{6x-3y}{x}\right) - \log x = -c$ [$-c$ = constant]

$\Rightarrow \frac{1}{3}\log\left(\frac{6x-3y}{x}\right) + \log x = c$

$\Rightarrow \log\left(\frac{6x-3y}{x}\right) + 3\log x = 3c = \log k$ (let $3c = \log k$ = constant)

$\Rightarrow \log\left(\frac{6x-3y}{x} \times x^3\right) = \log k$

$\Rightarrow x^2(6x - 3y) = k$ — (iv) — final soln

$y(3) = 1$, $x = 3$, $y = 1$

$\Rightarrow 9(18-3) = k$

$\Rightarrow k = 135$

∴ final soln $\boxed{x^2(6x - 3y) = 135}$ — (v)

6) State Runge-Kutta 4th order method. Where it has been used? What are transcedental eqns? Explain LU factorization method for system of soln of linear eqns.

a) done

b) They are used in –

i) to solve the differential eqns. i.e they are one of n. ms to solve differential eqns.

Runge-Kutta 4th order method is used to solve the differential equation $dy/dx = (y - x)$

c) done d) done

7) If $\Delta f(x) = f(x+h) - f(x)$, then a constant k, Δk equals ? what is the order of the truncation error of the the trapizoidal rule as function of n, the no. of trapizoid? Show Gaussian quadrature using $n+1$ points is exact for polynomials of degree $k \le 2n+1$

a) done c) done
b) done

8) two no. A and B are approximated as C and D, respectively. Find the relative error of $C \times D$. what is significant digits for 0.0185×10^4.
Let $x_A = 3.14$ and $y_A = 2.651$ be correctly rounded from x_T and y_T, to the no. of decimal digits shown. Find the smallest interval containing (i) x_T, (ii) y_T, (iii) $x_T - y_T$ (v) $x_T \times y_T$ and (vi) x_T / y_T.

a) ~~Let C be the true value and D be the a1~~

b) 3 significant digits.

a), c), d) - Not done by sir

9) For the given real no. x_0, x_1, x_2 define the divided difference $f[x_0, x_1, x_2]$ of a real valued function $f(x)$.
Find the value of $f[1, 0.5, 1]$ when $f(x) = \sin x$. Estimate the effect of data inaccuracy on results computed by Trapezoidal and Simpson's rule. Find the approximate soln of the eqn $x \sin x - 1 = 0$ ($\sin x$ is calculated in rad) in the interval $[0, 2]$ using Bisection method. obtain the no. of iterations to performed to obtain a soln whose ~~er~~ absalute error is less than 10^{-3}.

a) done, c) done

b) $x_0 = 1$, $y_0 = \sin 1 = 0.01$ [$f(x) = \sin x$]
$x_1 = 1.5$; $y_1 = \sin 1.5 = 0.02$
$x_2 = 1$, $y_2 = \sin 1 = 0.01$

	x	y	1st DD	2nd DD
x_0	1	0.01	$\frac{0.02 - 0.01}{0.5 - 1} = -0.02$	
x_1	0.5	0.02	$\frac{0.01 - 0.02}{1 - 0.5} = -0.02$	$\frac{-0.02 + 0.02}{1 - 1} = 0$
x_2	1	0.01		

$$f(x) = 0.01 + (x-1) \times (-0.02) + (x-1)(x-0.5) \times 0$$
$$= 0.01 - 0.02(x-1)$$
$$= -0.02x + 0.01 + 0.02$$
$$= 0.03 - 0.02x$$

d) $f(x) = x \sin x - 1 = 0$
interval $[0, 2]$

Soln by Bisection method -

	a	b	$t = \frac{a+b}{2}$	f(a)	f(b)	f(t)
1)	0	2	1	-1	-0.9	-0.9
2)	1	1	1	-0.9	-0.9	-0.9
3)	1	1	1	-0.9	-0.9	-0.9

the approximate val is $a = b = 1$

e) done