SHYAM STEEL

Theory of Computation

$\phi + \varepsilon = \varepsilon$

$\phi + \phi = \phi$

$a + \phi = a$

$a + \varepsilon = a + \varepsilon$

$\varepsilon + \varepsilon = \varepsilon$

$a \cdot \varepsilon = a, \varepsilon a, aE\varepsilon$ etc.

$a \cdot \phi = \phi$

$\phi \cdot \varepsilon = \phi$

$R \cdot \varepsilon = R$, $R \cdot \phi = \phi$

$R + \Sigma^* = \Sigma^*$

R means regular expression.

$\phi^* = \varepsilon + \phi = \varepsilon$

a^{Kn+L}

if $L > K$

no. of states = $L+1$

if $L < K$

no. of states = K.

$(a+b)^* a (a+b)^{n-1}$

nth element from end.

no. of states = 2^n

$(a+b)^{n-1} a (a+b)^*$

nth element from start

no. of states = $(n+1)+1$

Top goals	Things to do	Notes

Closure

Finite lang.

i. Complement → Not closed

ii. Kleene Closure *, + → Not closed.

Infinite Lang.

1. Union → closed.
2. Concatination → closed
3. Kleene closure → closed.

Regular lang.

Not closed → Subset, Infinite [∪, ∩, −, subset, substitution, ·]

Finite Subset is closed.

DCFL

closed → $\bar{L}$; prefix, h^{-1}, Finite Subset.

CFL

Not closed → Intersection, $\bar{L}$, $L_1 - L_2$, subset, Quotient, Finite Intersection & Difference

Infinite all.

Top goals	Things to do	Notes

Recursive lang.

Not closed :- Subset, ~~f~~ substitution, h(),
Infinite all.

★ ε free h() → closed.

Recursive Enumerable lang.

Not closed :→ $\bar{L}$, $L_1 - L_2$, Subset, finite diff.
Infinite all.

$\overline{REL}$ = May or may not REL

Top goals	Things to do	Notes

SHYAM STEEL

DCFL ∪ DCFL = CFL

Regular − DCFL = DCFL

Regular − CFL = Need not be CFL

≠ Set of all regular ∩ Set of all DCFL = Set of all reg.

CFL − DCFL ⇝ CFL ∩ $\overline{DCFL}$ ⇒ CFL ∩ DCFL → CSL
DCFL − CFL (need not be CFL)

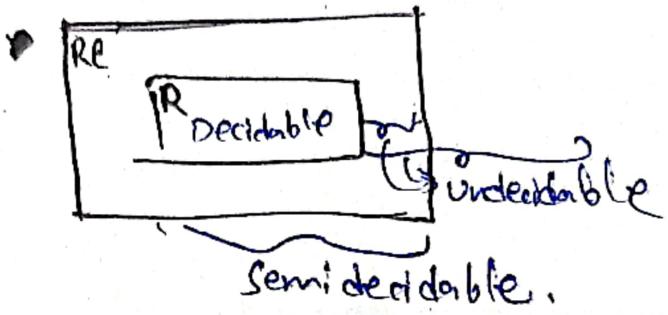
≠ DCFL₁ ∩ DCFL₂ ⇒ Always CSL

CFL₁ ∩ CFL₂ ⇒ Need not be CFL, DCFL, Regular.

Recursive — REL = Need not be RE
mean. Can be Recursive
Can be Not REL.

Top goals	Things to do	Notes

SHYAM STEEL



Set of : Finite, Regular ..... REL → SD and UD

: Infinite , Not REL → Not REL → UD

~~Undecidable and undecidable.~~

TM accepting

ε, ab, Something → SD and UD

Nothing, finite lang, everything → Not REL

only ab → Not REL

* SDUD → semidecidable and undecidable.
↳ RE but not Rec.

M is TM, Is L(M) regular → undecidable.

(i) TM, M is $L(M) = \phi$ → Undecidable

CFG, $L(G) = \phi$ → Decidable. but $= \Sigma^*$ is undecidable.

(ii) CFG, G_1 & G_2 is $L(G_1) = L(G_2)$ → Undecidable.

(iii) Class of reg. lang is not closed under infinite Union.

$U(ab) U(a^2b^2) U \ldots$ Infinite Union = $a^n b^n$ not regular.

(iv) P reducable/reduces to Q.

↳ $P \leq Q$

means machine used for Q, then can be used for P.

(v) Phase Structured Lang → TM

(vi) CFL → $O(n^3)$ ↳ polynomial.

(vii) Set of REL = closed under intersection

(viii) If NFA = n state than D.FA = 2^n state

Top goals	Things to do	Notes
If DFA has	(Let k) n states. then	NFA will
	have max.	2^n state.
	$k \leq 2^n$.	

SHYAM STEEL

Pumping Lemma (uses Pigeon hole)

Let L is reg. lang.

there exist pumping length P

such that $\forall w \in L$

$|w| \geq P$, $x y z = w$

$|y| > 0$

$|xy| \leq P$

$\forall i \geq 0$, $x y^i z \in L$

Moore

Inside State-C

More State

At-least k ~~#~~ state

↳ k no. of Symbol.

Mealy

On Transition.

lesser state

$S \to aS \mid Sa \mid \#$

↳ $L = \{a^* \# a^*\}$

↳ RL.

Not a regular grammer.

∴ Ambiguous

Top goals	Things to do	Notes

SHYAM STEEL

n = length.

Chomsky NF

2n−1 states

N.T → N.T N'T | Terminal

N N | T

Greibach NF

n. states

N.T → Terminal Non-Terminal*

T N*

Simplification of CFG.

1. Null rule $X \to \varepsilon$

$S \to aA \mid \varepsilon$

$A \to aSb \mid b$

→

$S \to aA$

$A \to aSb \mid b \mid ab$

2. Unid rule $X \to Y$

$S \to aA \mid \overset{\curvearrowright}{S}$

$A \to ab \mid B$

$B \to b \mid C \mid A$

→

$S \to aA$

$A \to ab \mid b \mid C$ or

$B \to b \mid ab \mid C$

3. Useless rule

* PDA accepts
  - ↳ using Final State
  - ↳ using Empty Stack

DPDA accepts
- ↳ using Final State

Top goals	Things to do	Notes
δ_{PDA} :	$Q \times \Sigma_\varepsilon \times \Gamma^*$ →	$2^{Q \times \Gamma^*}$
δ_{DPDA} :	$Q \times \Sigma \times \Gamma$ →	$Q \times \Gamma^*$ ↑ stack alp.

SHYAM STEEL

$\{ww\}$ → not CFL

→ Inverse of it is CFL not DCFL

$\{ww^R\}$ → CFL but not DCFL

$\{a^n b^n\}^{reverse}$ → DCFL, not reg.

Σ^* → Countable infinity 2^{Σ^*} is uncountable.

Ambiguity prob of CFG → Undecidable.

For CFGs $L(G_1) \cap L(G_2) = \phi$ Undecidable.

In case of membership prob.

Recursive → Undecidable

But if no. of steps are fixed → decidable

For every non-deterministic TM there exist an equivalent Deterministic TM.

no. of substrings $= \frac{n(n+1)}{2} + 1$ ← ϵ

Top goals	Things to do	Notes

SHYAM STEEL

1. RE - Recursive = RE ∩ $\overline{\text{Recursive}}$

= RE ∩ Recursive = RE.

2. Regular - CFL = Regular ∩ $\overline{\text{CFL}}$

= Regular ∩ CSL = CSL [CSL and $\overline{\text{CSL}}$]

3. CFL - RE = CFL ∩ $\overline{\text{RE}}$

= CFL ∩ Not RE = Not RE.

4. Recursive - RE = Recursive ∩ Not RE

Not RE

Recursive ∩ Recursive

= Recursive.

Undecidable

- CFG

Decidable

- Regular lang.
- DCFL
- Regular Grammer.

$\overline{\{a^n b^n c^n \mid n \geq 1\}}$ = → CFL.

$a^n b^n c^n$ = CSL.

and $\overline{a^n b^n c^n} = a^n b^m c^p$ where $n \neq m$ and $m \neq p$

Top goals	Things to do	Notes
		∴ CFL.

1. Max no. of Substring = $\frac{n(n+1)}{2} + 1(\epsilon)$
(None of the element is same)

2. $\phi^* = \epsilon$ 3. $\phi \cdot R = \phi$ 4. $R + \Sigma^* = \Sigma^*$
[Annihilation]

5. $\omega \times \omega^R$ $\omega, x \in (a+b)^+$ ⟶ Regular

6. $An + B$ → $A > B$ A states
→ $A < B$ B+1 states.

7. From last → k^{th} 2^K

From First → k^{th} $(K+1)+1$

8. $a^m b^n$, m = odd, n = odd ⟶ Regular.

9. $a^m b^n$ $m+n$ = even. ⟶ Regular.
$m+n$ = odd.

10. $a^m b^n$ $LCM(m,n)=1$ Regular
$GCD(m,n)=1$ Not Regular

11. a^{m^n} ⟶ Regular.

12. q_1/q_2 $abb/bb = a$ $abb/a = \phi$

Pumping Lemma.

13. $xyz = \omega$, $|\omega| \geq p$ [Depend on the no. of state (in DFA)]
i. $|y| > 0$
ii. $|xy| \leq p$
iii. $\forall i \geq 0$, $xy^i z \in L$

14. Every RG generates RL
If grammer is not Regular then it may generates RL.
$S \to aS \mid Sa \mid \#$ ⟶ $\{a^* \# a^*\}$ RL.
[Ambigious Gram.]

15. Chomsky NF $V \to VV \mid t$ $2n-1$ steps.
16. Greibach NF $V \to tV^*$ n steps.

17. $L = \{wcw^Ry\}$ → not CFL
$\bar{L}$ is CFL but not DCFL

18. $\{wcw^R\}$ L & $\bar{L}$ is CFL, not DCFL

19. $\{a^n b^n\}^{reverse}$ DCFL not regular.

20. $\{a^n b^n c^n y\}$ not CFL, $\bar{L}$ is CFL not DCFL

21. DCFL ∪ DCFL = CFL But DCFL ∩ DCFL cancel. = CSL.

22. Reg. − CFL = need not be CFL
DCFL − CFL = " / CSL
CFL − DCFL = " / CSL
Rec − REL = Need not be REL means either Recursive or not REL.

23. $\overline{REL}$ = may or may not be REL
= either recursive or not RELs.
= Never be "REL but not rec".

Equivalance Class → No. of state in minimal DFA.

Closure

Finite lang $\bar{L}$ ×
• Complement, Kleene closure *,+ [Not closed]

Infinite Lang
• Union, Concatination, Kleene closure [closed]

Regular Lang.
• Subset, Infinite All. [Not closed.]
Finite subset [closed]

DCFL $\bar{L}$ ✓
• $\bar{L}$, Prefix, h^{-1}, finite subset [closed]

CFL $\bar{L}$, $L_1 \cap L_2$, $L_1 - L_2$, L_1 / L_2 ×
• Intersection, Compliment, $L_1 - L_2$, subset
• Quotient, Finite Intersection & Difference
• Infinite all [Not closed]

Recursive Lang
• Subset, Substitution, $h()$, Infinite all [Not closed]
• ϵ free $h()$ [closed]

Recursive Enumerable Lang
• $\bar{L}$, $L_1 - L_2$, subset, finite diff.
Infinite all [Not closed]

Undecidable
• CFGs, $L(G_1) \cap L(G_2) = \phi$
• Ambiguity prob. of CFG
• $L(G_1) = L(G_2)$ for CFGs.
• $L(G) = \Sigma^*$
•
• TM, $L(M)$ = regular
• $L(M) = \phi$

$\overline{\Sigma^*}$ Countable infinity | 2^{Σ^*} Uncountable.