Group-A

1) What are the various types of models?

Ans. (i) Iconic or physical
(ii) Analogue or schematic models.
(iii) Symbolic or Mathematical models

2) Define a feasible region?

Ans A region in which all the constraints are satisfied simultaneously is called feasible region

3) State the necessary and sufficient condition for the existence of a feasible solution to a transportation problem

Ans The necessary and sufficient condition for the existance solution is a solution that satisfies all the conditions of supply and demand.

4) What are the main phases of a project

Ans, The three phases of a project are planning, scheduling and control.

5) What are the different types of inventories?

Ans Raw materials, work in progress, Materials, Safety stock, Eco-economic decoupling, cycle inventory.

Group B

Q1. What are the advantages of linear programming techniques?

Ans. Linear programming (LP) techniques offer numerous advantages across various fields, making them invaluable tools for optimization problems. Here are some key benefits:

i.Optimal Resource Allocation:LP helps in the efficient allocation of limited resources such as materials, labor, and capital to achieve the best possible outcome, whether it's maximizing profit or minimizing cost.

ii.Simplified Decision-Making:LP provides a clear framework for decision-making by modeling complex relationships and constraints in a straightforward mathematical form. This helps in making informed and objective decisions.

iii.Versatility and Applicability:LP can be applied to a wide range of problems across different industries, including manufacturing, transportation, finance, telecommunications, agriculture, and military operations. It can handle various types of problems like production scheduling, transportation logistics, and portfolio optimization.

iv.Objective Analysis:LP allows for objective analysis based on quantitative data. It eliminates biases and ensures that decisions are made based on mathematical logic and empirical data rather than intuition or subjective judgment.

v.Ability to Handle Multiple Constraints:LP can manage multiple constraints simultaneously, ensuring that all relevant limitations and requirements are considered in the solution. This is particularly useful in real-world scenarios where multiple constraints often coexist.

vi.Efficiency in Finding Solutions:LP techniques, especially with the help of modern computer software, can solve large-scale problems efficiently. Algorithms such as the Simplex method and interior-point methods are designed to find optimal solutions quickly even for complex problems.

vii.Cost Reduction:By optimizing the use of resources and minimizing waste, LP can lead to significant cost reductions. This is particularly beneficial in industries where cost control is critical for maintaining competitiveness.

viii.Improved Planning and Control:LP aids in better planning and control by providing a systematic approach to problem-solving. It helps in forecasting, planning production schedules, managing inventories, and controlling operations more effectively.

Q2. State the characteristics of standard form and write the standard form of linear programming problem in matrix form?

Ans. Characteristics of Standard Form in Linear Programming:

1.Objective Function of Maximization Type:

(The standard form requires the objective function to be a maximization problem. If it is a minimization problem, it can be converted to a maximization problem by multiplying the objective function by -1.)Example: Maximize $Z=c1x1+c2x2+...+cnxn$

2.All Constraints Expressed as Equations:All constraints must be in the form of equations (i.e., with equality signs). Inequalities (≤, ≥) in the constraints must be converted to equalities by adding slack, surplus, or artificial variables.

Example: $a11x1+a12x2+...+a1nxn=b1$

3.RHS of Each Constraint is Non-Negative:The right-hand side (RHS) of each constraint equation must be non-negative. If any RHS value is negative, it can be made non-negative by multiplying the entire equation by -1.

Example: $a11x1+a12x2+...+a1nxn=b1$ where b1>=0

4.All Variables are Non-Negative:

All decision variables must be non-negative, meaning xi≥0x i≥0 for all i.

Example: $x1,x2,...,xn$≥0x

Matrix Form of Standard Linear Programming Problem:

The standard form of a linear programming problem can be written in matrix notation as follows:

Maximize Z=CX Subject to:$AX=b$ and X≥0

Where:

i.C is a 1 × n row vector of coefficients in the objective function.

ii.X is an n × 1 column vector of decision variables.

iii.A is an m × n matrix of coefficients for the constraints.

iv.b is an m × 1 column vector of the RHS values of the constraints.

v.The constraints X≥0 ensure that all decision variables are non-negative.

Example of a Linear Programming Problem in Standard Form:

Given the following linear programming problem:

Maximize

$Z=3x1+2x2$

Subject to:

2x1+2x2=4

x1+3x2 = 5

x1,x2>=0

In matrix form, this can be expressed as:

Maximize

Z=[3 2][$x1$

$x2$]

Subject to:

[2 1 x1 4

=

1 3] x2   5

x1   0

>=

x2   0

This example illustrates the characteristics and matrix representation of a linear programming problem in standard form.

Q3. Write the Vogel's approximation method for solving transportation problems?

Ans.Vogel's Approximation Method (VAM) is a heuristic for finding a good initial feasible solution to a transportation problem. This method attempts to reduce the transportation cost by considering the penalty costs, which represent the difference between the smallest and the next smallest cost in each row and column.

Here's a step-by-step guide to applying VAM:

Step 1: Calculate Penalty Costs

For each row and each column of the cost matrix, calculate the penalty cost. The penalty cost is the difference between the smallest and the second smallest cost in that row or column.

Step 2: Identify the Highest Penalty

Identify the row or column with the highest penalty cost. If there is a tie, you can choose arbitrarily among the tied penalties.

Step 3: Allocate as Much as Possible

In the selected row or column, find the cell with the smallest cost. Allocate as much as possible to this cell, which is the minimum of the supply and demand for that cell.

Adjust the supply and demand for the row and column by subtracting the allocated amount.

If a row or column supply or demand is met (i.e., becomes zero), cross out that row or column.

Step 4: Recalculate Penalties

Recalculate the penalties for the remaining rows and columns (ignoring any that have been crossed out).

Step 5: Repeat

Repeat steps 2 to 4 until all supplies and demands are satisfied.

Example

Consider a transportation problem with the following cost matrix, supplies, and demands:

D1	D2	D3	Supply	
S1	19	30	50	7
S2	70	30	40	9
S3	40	8	70	18
Demand		5	8	21

Step-by-Step Solution using VAM

Calculate Penalties:

Row penalties:

Row S1: 30−19=11

Row S2: 40−30=10

Row S3:40−8=32

Column penalties:

Column D1:40−19=21

Column D2:30−8=22

Column D3:50−40=10

Identify Highest Penalty:

The highest penalty is 32 for Row S3.

Allocate to Smallest Cost Cell in S3:

In Row S3, the smallest cost is 8 (at column D2).

Allocate min(18, 8) = 8 to cell (S3, D2).

Adjust supplies and demands: S3's supply becomes 10, D2's demand becomes 0.

Cross out column D2.

Updated table:

D1	D2	D3	Supply	
S1	19	30	50	7
S2	70	30	40	9
S3	40	X	70	10
Demand		5	X	21

Recalculate Penalties:

Row penalties:

Row S1:50−19=31

Row S2: $70-40=30$

Row S3: $70-40=30$

Column penalties (ignore crossed out columns):

Column D1: $40-19=21$

Column D3: $50-40=10$

Identify Highest Penalty:

The highest penalty is now 31 for Row S1.

Allocate to Smallest Cost Cell in S1:

In Row S1, the smallest cost is 19 (at column D1).

Allocate min(7, 5) = 5 to cell (S1, D1).

Adjust supplies and demands: S1's supply becomes 2, D1's demand becomes 0.

Cross out column D1.

Updated table:

D1	D2	D3	Supply	
S1	X	30	50	2
S2	70	30	40	9
S3	40	X	70	10
Demand		X	X	21

Recalculate Penalties:

Row penalties:

Row S1: $50-30=20$

Row S2: $70-40=30$

Row S3: $70-40=30$

Column penalties (only for column D3):

Column D3: $70-40=30$

Identify Highest Penalty:

The highest penalty is now 30 for Row S2 and Row S3. Choose one arbitrarily (let’s choose S2).

Allocate to Smallest Cost Cell in S2:

In Row S2, the smallest cost is 40 (at column D3).

Allocate min(9, 21) = 9 to cell (S2, D3).

Adjust supplies and demands: S2's supply becomes 0, D3's demand becomes 12.

Cross out row S2.

Updated table:

D1	D2	D3	Supply	
S1	X	30	50	2
S2	X	X	X	X
S3	40	X	70	10
Demand		X	X	12

Recalculate Penalties:

Row penalties:

Row S1:50−30=20

Row S3:70−40=30

Column penalties (only for column D3):

Column D3:70−50=20

Identify Highest Penalty:

The highest penalty is 30 for Row S3.

Allocate to Smallest Cost Cell in S3:

In Row S3, the smallest cost is 40 (at column D3).

Allocate min(10, 12) = 10 to cell (S3, D3).

Adjust supplies and demands: S3's supply becomes 0, D3's demand becomes 2.

Cross out row S3.

Updated table:

D1	D2	D3	Supply	
S1	X	30	50	2
S2	X	X	X	X
S3	X	X	X	X
Demand		X	X	2

Allocate Remaining Demand:

Allocate remaining 2 to cell (S1, D3).

Adjust supplies and demands: S1's supply becomes 0, D3's demand becomes 0.

Cross out row S1 and column D3.

Final allocation:

	D1	D2	D3
S1	5	0	2
S2	0	0	9
S3	0	8	10

Final Allocations:

x 11=5

x12=0

x13=2

x21=0

x22=0

x23=9

x31=0

x32=8

x33=10

Total Minimum Cost Calculation:

Total Cost=5×19+2×50+9×40+8×8+10×70

Total Cost=95+100+360+64+700=1319

Vogel's Approximation Method provides an initial feasible solution with a total transportation cost of 1319. This solution can be further refined using optimization methods like the MODI method (Modified Distribution Method) to find the optimal solution if needed.

Q4. Explain Unbalanced Assignment Problem?

Ans. If the number of rows is not equal to the number of columns in the cost matrix of the assignment problem or if the cost matrix of the given assignment problem is not a square matrix, then the given assignment problem is said to be unbalanced.

Unbalanced Assignment Problem

An assignment problem involves assigning a set of tasks to a set of agents in such a way that the total cost or time is minimized, or the total profit is maximized. Typically, this problem is represented in the form of a cost matrix where the rows represent the tasks and the columns represent the agents.

Characteristics:

Unequal Number of Rows and Columns: When there are more tasks than agents or more agents than tasks, the cost matrix is rectangular rather than square.

Incomplete Assignment:Not every task can be assigned to an agent directly, or some agents may not be assigned any task.

Example of Unbalanced Cost Matrix:Suppose we have 4 tasks and 3 agents with the following cost matrix:

	Agent 1	Agent 2	Agent 3
Task 1	9	2	7
Task 2	6	4	3
Task 3	5	8	1
Task 4	7	6	2

Here, there are 4 rows (tasks) and 3 columns (agents), making the matrix unbalanced.

Steps to Solve Unbalanced Assignment Problems:

To convert an unbalanced assignment problem into a balanced one, we follow these steps:

Identify the Imbalance:

Determine whether there are more tasks than agents or more agents than tasks.

Add Dummy Rows/Columns:

If there are more tasks than agents, add dummy columns (agents) with costs of zero.

If there are more agents than tasks, add dummy rows (tasks) with costs of zero.

In the example above, we have more tasks than agents. Therefore, we add a dummy column with zero costs:

	Agent 1	Agent 2	Agent 3	Dummy Agent
Task 1	9	2	7	0
Task 2	6	4	3	0
Task 3	5	8	1	0
Task 4	7	6	2	0

Now, the cost matrix is balanced with 4 rows and 4 columns.

Apply the Assignment Algorithm:

Use the Hungarian algorithm or any other suitable method to solve the balanced assignment problem. This algorithm will ensure an optimal assignment of tasks to agents, including the dummy agents.

Q5. Distinguish between PERT and CMP?

Ans. For 2 marks

PERT	CPM
(i) Event oriented.	(i) Activity oriented.
(ii) Probabilistic.	(ii) Deterministic.
(ii) Three time estimates	(iii) Time is fixed.

namely optimistic,

pessimistic, most likely are

given.

(iv) Resources such as labour, (iv) No limitation of resources.

equipment, materials are

limited.

Aspect	PERT	CPM
Abbreviation	PERT stands for Project Evaluation and Review Technique.	CPM stands for Critical Path Method
Definition	PERT is a technique of project management which is used to manage uncertain (i.e., time is not known) activities of any project.	CPM is a technique of project management which is used to manage only certain (i.e., time is known) activities of any project.
Orientation	It is event oriented technique which means that network is constructed on the basis of event.	It is activity oriented technique which means that network is constructed on the basis of activities.
Model Type	It is a probability model.	It is a deterministic model.
Focus	It majorly focuses on time as meeting time target or estimation of percent completion is more important.	It majorly focuses on Time-cost trade off as minimizing cost is more important.
Precision	It is appropriate for high precision time estimation.	It is appropriate for reasonable time estimation.
Nature of Job	It has Non-repetitive nature of job.	It has repetitive nature of job.
Crashing	There is no chance of crashing as there is no certainty of time.	There may be crashing because of certain time bound.
Dummy Activities	It doesn't use any dummy activities.	It uses dummy activities for representing sequence of activities.

Aspect	PERT	CPM
Sustainability	**It is suitable for projects which required research and development.**	**It is suitable for construction projects.**

Q6. What are the main Objectives of an inventory Model?

Ans. The main objectives of an inventory model are to optimize the management of inventory within an organization. Effective inventory management is crucial for ensuring that the right amount of inventory is available at the right time, minimizing costs, and meeting customer demand. Here are the key objectives:

i.Minimize Total Costs:

Ordering Costs: Reduce the costs associated with placing orders, including administrative expenses, shipping, and handling.

Holding Costs: Minimize the costs of storing inventory, such as warehousing, insurance, and spoilage.

Shortage Costs: Avoid the costs related to stockouts, which can include lost sales, customer dissatisfaction, and expedited shipping fees.

ii.Ensure Product Availability:

Ensure that sufficient inventory is available to meet customer demand without interruption. This helps in maintaining high service levels and customer satisfaction.

iii.Optimize Inventory Levels:

Maintain optimal inventory levels to balance the costs of holding too much inventory against the risks and costs of having too little inventory. This includes determining appropriate reorder points and order quantities.

iv.Improve Cash Flow:

Efficient inventory management can free up capital that is otherwise tied up in excess inventory, improving the organization's cash flow and allowing for better financial management.

v.Increase Efficiency and Productivity:

Streamline inventory processes to improve overall efficiency and productivity within the supply chain, reducing lead times and enhancing operational performance.

vi.Support Production Planning:Align inventory levels with production schedules to ensure a smooth and continuous production process, avoiding delays caused by material shortages.

vii.Enhance Decision Making:Provide accurate data and analysis to support strategic decision-making regarding purchasing, production, and sales. This includes forecasting demand and planning for future inventory needs.

viii. Reduce Waste:Minimize waste by reducing overstock and obsolete inventory, thus contributing to sustainability efforts and cost savings.

ix.Improve Customer Service:By ensuring that products are available when customers need them, inventory management contributes to higher customer satisfaction and loyalty.

x.Adapt to Market Changes:Enable the organization to respond swiftly to changes in market demand, seasonal variations, and other external factors by maintaining flexible and responsive inventory systems.

Q7. Write down short note on Queue Discipline?

Ans. Queue discipline refers to the set of rules and methods used to determine the order in which customers or items in a queue are served. It is a crucial aspect of queuing theory in operations research, impacting the efficiency and effectiveness of service systems. Different types of queue disciplines are applied based on the specific requirements and objectives of the service system. Here are some common types of queue disciplines:

First-Come, First-Served (FCFS):

Advantages: Simple and fair, ensuring that no customer is favored over another.

Last-Come, First-Served (LCFS):

Advantages: Useful in specific scenarios where recent items need immediate attention.

Shortest Processing Time First (SPT):

Advantages: Can significantly reduce the average waiting time for all customers.

Longest Processing Time First (LPT):

Advantages: Helps to clear out large tasks that might otherwise cause delays.

Priority Queuing:

Advantages: Ensures that critical or high-priority customers are served promptly.

Round Robin (RR):

Advantages: Ensures a fair allocation of resources among customers, preventing any single customer from monopolizing the service.

Random Selection for Service (RSS):

Advantages: May be useful in certain experimental or theoretical scenarios.

Shortest Remaining Processing Time (SRPT):

Advantages: Minimizes the average completion time for tasks.

Importance of Queue Discipline

Efficiency: Proper queue discipline can significantly enhance the efficiency of a service system by reducing waiting times and ensuring optimal utilization of resources.

Customer Satisfaction: Different disciplines can be tailored to improve customer satisfaction, such as prioritizing urgent cases in healthcare or ensuring fairness in service delivery.

Operational Control: Helps in managing and controlling the flow of customers or items, making it easier to predict and handle peak times and service demands.

Strategic Planning: Informs decisions about resource allocation, staffing, and system design to meet service goals and customer expectations.

Q8. State different Types of games?

Ans. The different types of games are:

1. Two-Person Games vs. n-Person Games:

Two-Person Games: In these games, there are only two players involved. Each player has multiple choices or strategies to select from, but the number of players is strictly limited to two.

n-Person Games: These games involve more than two players. The term "n-person" signifies that the number of players can be any number greater than two.

2. Zero-Sum Games:

Definition: A zero-sum game is one in which the sum of the gains and losses of all players is zero for every possible outcome. If one player gains a certain amount, the other players collectively lose the same amount.

Example: Classic board games like poker, where one player's winnings are exactly balanced by the other players' losses.

3. Two-Person Zero-Sum Games:

Definition: These are a specific type of zero-sum game where there are exactly two players. The gain of one player is equal to the loss of the other. These games are often represented using a pay-off matrix, which can be rectangular in shape.

Characteristics:

(a) Only Two Players: The game involves only two participants.

(b) Finite Number of Strategies: Each player has a limited set of strategies to choose from.

(c) Specific Pay-Offs: Each combination of strategies chosen by the players results in a specific pay-off.

(d) Total Pay-Off is Zero: The sum of the pay-offs for both players is zero at the end of each game.

Group - C

1. LPT Graphically

Minimize $Z = x+y$ subjected to $5x+9y \le 45$

$x+y \ge 2$

$y \le 4$

$x, y \ge 0$

Solution,

Replace all the inequalities of the constraints by formula equation.

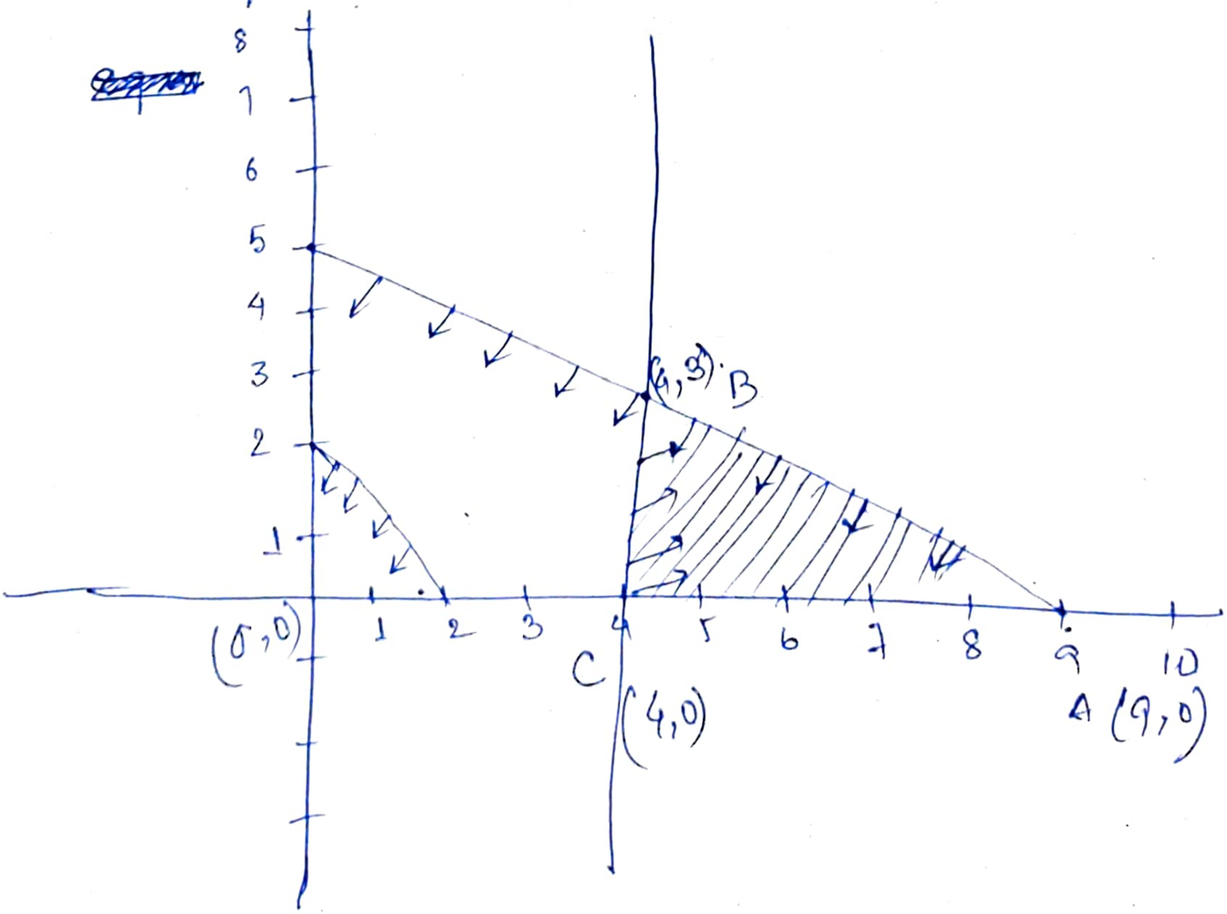
$$5x+9y = 45 \quad \text{--- (1)}$$
$$x+y = 2 \quad \text{--- (2)}$$
$$y = 4 \quad \text{--- (3)}$$

eq(1) $5x+9y=45$

$\frac{x}{9}+\frac{y}{5}=1$

eq(2) $x+y=2$

$\frac{x}{2}+\frac{y}{2}=1$



$A(9,0)$ $B(4,3)$ $C(4,0)$

Multiplying equation 2 with 5

$$\begin{array}{r} \cancel{5x} + 9y = 45 \\ \cancel{5x} + 5y = 10 \\ \hline 4y = 35 \end{array}$$

$$y = 35/4 = 8.8.$$

$$5x + 9y = 45$$

$$5x = 45 - \frac{35}{4} \cdot 9$$

$$5x = 45 - 78.75$$

$$= 33.75$$

$$\boxed{x = 6.75}$$

The feasible region ~~is~~ ABC

B is the point of intersection of line B intersect $y \leq 4$ and $5x + 9y \leq 45$

On solving we get B(~~4~~ 6.75, 8.8)

Now eq (4) and eq (3) $\times$ (9)

$$5x + \cancel{9y} = 45$$
$$\cancel{9y} = 36$$

$$5x = 9 \qquad y = \frac{45-9}{9}$$

$$x = 9/5 \qquad y = \frac{36}{9}$$

point

A (9, 0)

B (9/5, 36/9)

C (4, 0)

value of $z = x + y$

9

4

Maximum value

The maximum value of occurs at point ___ and optional solution is

$x =$ ___ $y =$ ___

2. Solve the following Linear Programming using simplex method.

Max $\Rightarrow Z = 3x + 2y$ subj. to $x + y <= 4$

$x - y <= 2$

$x, y >= 0$

Solution.

By introducing slack variables S_1, S_2 converting the problem in standard form

Max : $Z = 3x + 2y + 0 \cdot S_1 + 0 \cdot S_2$

$$\begin{bmatrix} x + y + S_1 = 4 \\ x - y + S_2 = 2 \end{bmatrix}$$

~~[illegible]~~ where, $x, y, S_1, S_2 \geq 0$

if $x, y = 0$.

$\Rightarrow$ An initial basic feasible solution is given by

$x = y = 0$ $\quad S_1 = 4 \quad S_2 = 2$

writing in matrix form $AX = B$

$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad X = \begin{bmatrix} X \\ Y \\ S_1 \\ S_2 \end{bmatrix} \quad B = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

An initial simplex table.

		c_j	3	2	0	0	Min Ratio (X_B/x)
C_B	X_B	B	x	y	S_1	S_2	
0	S_1	4	1	1	1	0	
0	S_2	2	1	1	0	1	
$(Z_j - C_j)$							

Calculation for initial simplex Table.

$Z_j - C_j = C_B X_j - C_j$ where,

$Z_1 - C_1 = C_{B_1} X_1 - C_1$

$Z_2 - C_2 = C_{B_2} X_2 - C2$

and so on

Short trick

$Z_j = \sum_{i=1}^{2} (C_{Bi})(a_{ij})$

$0 \times 1 + 0 \times 1 = 0$

leaving variable. Entering variable. Key element

C_{Bi}	C_j	3	2	0	0	Solu.	Ratio
	Basic variable	x	y	s_1	s_2		
0	s_1	1	1	1	0	4	$4/1$ = 4
0	s_2	1	1	0	1	2	$2/1$ = 2
Z_j	Key row	0	0	0	0	0	

$C_j - Z_j$ → 3 2 0 0

Key column

For optimality condition

Max ~~MAX~~ all $C_j - Z_j \leq 0$

Min all $C_j - Z_j \geq 0$

Select the maximum value that will be Key column

Now check in Ratio least value (2)
So (S2) row is key row

First Iteration → entering

C_{Bi}	C_j / BV	3	2	0	0	Soln	Ratio
		x	y	s_1	s_2		
2	y	1	1	1	0	4	4
0	row leaving S_2	key col 0	1	-1	1	-2	→ key row
	Z_j	1	2	2	0		
	$C_j - Z_j$	2	0	-2	0		

pivot

(selecting maximum post. val) & Replace with Entering variable and leaving variable.

Now key element ta ber korar pore key element theke sob ta ke ~~subtract~~ dividide korte hobe

S2 row (find new value) Formula

$$\text{New value} = \text{old value} - \frac{\text{corr. key col. value} \times \text{corr. key row. value}}{\text{Key Element}}$$

$$\Rightarrow 1 - \frac{1\times1}{1} \Rightarrow 1-1=0$$

$$\Rightarrow 1 - \frac{0\times1}{1} \quad 1-0 \Rightarrow 1$$

$$\Rightarrow 0 - \frac{1\times1}{1} = -1$$

$$\Rightarrow 1 - \frac{1\times0}{1} = 1-0=1$$

$$\Rightarrow 2 - \frac{1\times4}{1} \Rightarrow 2-4=-2$$

After this Z_j and $C_j - Z_j$

For iteration - II

C_{Bi}	C_j / BV	3 (X)	2 (Y)	0 (S_1)	0 (S_2)	Solution
2	y	0	0	0	0	0
3	x	0	0	0	0	0

$C_j - Z_j =$

Apply formula,

$$1 - \frac{1 \times 1}{0} \qquad 1 - \frac{1 \times 1}{0} = 0$$

$Z_j = C_{Bij} + a_{ij}$

$C_j - Z_j =$ find

Check optimality → reached.

Now, $x =$ ✓ $y =$ ✓ $Z =$ ✓

Since $Z_j - C_j \geq 0$ for all j
the solution is optimum
and is given by
$y =$ __
$x =$ __
$Z =$ __

The optimal solution is given by

Max :

$$Z = CB \times g$$

$$= 3 \times x_1 + 2 \times y$$

$$= \_\_ + \_\_ = \checkmark$$

Ans

Long Q3 Transportation problem (NWCM)

origin/Desc.	D1	D2	D3	Supply
O1	2	7	4	5
O2	3	3	1	8
O3	5	4	7	7
O4	1	6	2	14
Demand	7	9	18	34

(i) first check wheather the equation is balanced or not.

(ii) Total supply = 34
Total demand = 34

The problem is balanced TP as the Total supply is equal to the total demand we obtain the initial feasible soln as follows.

Or/Desc	D1	D2	D3	Supp
O1	2	7	4	5
O2	3	3	1	8
O3	5	4	7	7
O4	1	6	2	14
Demand	7	9	18	

origin/Desc	D1	D2	D3	supply
O1	2 $\boxed{5}$	7	4	~~5~~=0
O2	3 $\boxed{2}$	3 $\boxed{6}$	1	~~8~~=~~6~~=0
O3	5	4 $\boxed{3}$	7 $\boxed{4}$	~~7~~ ~~4~~ 0
O4	1	6	2 $\boxed{14}$	~~14~~ 0
Demand	~~7~~ 2 0	~~9~~ 3 0	18 ~~14~~ 0	

Now min cost

↳ cost

2

3 Quantity

~~minimum~~

ori/Desc	D1	D2	D3	supply
O1	2 $\boxed{5}$	7	4	5
O2	3 $\boxed{2}$	3 $\boxed{6}$	1	8
O3	5	4 $\boxed{3}$	7 $\boxed{4}$	7
O4	1	6	2 $\boxed{14}$	14
Demand	7	9	18	34

Minimum cost

$5\times2 + 3\times2 + 6\times3 + 4\times3 + 7\times4$
$+ 14\times2 = 102$

If unbanded

O.	D_1	D_2	D_3	S.	dumy
S_1	4	8	8	76	0
S_2	16	24	16	82	0
S_3	8	16	24	77	0
D	72	102	41		20

TS = 235

TD = 215

235-215=20

The problem is unbalanced TP as the total supply is greater than total demand.

Minimum cost NWCR = ₹ —

6 allocated cell = 6 (m+n-1)

$3+4-1=6$

Source
S_1
S_2
S_3

desi
4

unbalanced TP

$D > S$

$\sum b_j > \sum a_i$

add a dumy row with cost zero. and supply equals to $(d-s)$

$D < S$

$\sum b_j < \sum a_i$

add d. col.

So the solution is non-degenerate

$m+n-1$

The Maximin - Minimax Principle

This principle is used for the selection of optimal strategies by two players. Considered two players A and B. A is a player who wishes to maximize his gains, while player B wishes to minimize his losses. Since A would like to maximize his minimum gain, we obtain for player A, the value called maximin value and the corresponding strategy is called the maximin strategy.

On the other hand, since player B wishes to minimize his losses, a value called the minimax value, which is the minimum of maximum losses is found. The corresponding strategy is called minimax strategy. when these two are equal (maximin value = minimax value), the corresponding strategies are called optimal strategies and the game is said to have a saddle point. The value of the game is given by the saddle point.

The selection of maximin and minimax strategies by A and B is based upon the so-called maximin - minimax principle, which guarantees the best of the worst results.

Saddle point: A saddle point is a position in the pay-off matrix where, the maximum of row minima coincides with the minimum of column maxima.

The pay-off at the saddle point is called value of the game.

We shall denote the maximin value by $\underline{\gamma}$ the minimax value of the game by $\bar{\gamma}$ and the value of the game by γ

Notes: (i) The game is said to be fair if,

maximin value = minimax value = 0, i.e., $\bar{\gamma} = \underline{\gamma} = 0$

(ii) The game is said to be strictly determinable if,

maximin value = minimax value $\neq 0$. $\underline{\gamma} = \gamma = \bar{\gamma}$.

@Start Practicing

Q: Solve the game whose pay-off matrix is given below.

Player B

$$
\text{Player A} \quad
\begin{array}{c}
 \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ \text{Column maxima}
\end{array}
\begin{array}{c}
\begin{array}{ccccc} B_1 & B_2 & B_3 & B_4 & B_5 \end{array} \\
\begin{bmatrix} -2 & 0 & 0 & 5 & 3 \\ 3 & 2 & 1 & 2 & 2 \\ -4 & -3 & 0 & -2 & 6 \\ 5 & 3 & -4 & 2 & -6 \end{bmatrix} \\
\begin{array}{ccccc} 5 & 3 & 1 & 5 & 6 \end{array}
\end{array}
\begin{array}{c}
\text{Row minima} \\ -2 \\ 1 \\ -4 \\ -6 \\ \
\end{array}
$$

$\text{Maxi(minimum)} = \underline{\gamma} = \text{Max}(-2, 1, -4, -6) = 1$

$\text{Min (Maximum)} = \bar{\gamma} = \text{Min}(5, 3, 1, 5, 6) = 1$

Since, $\underline{\gamma} = \bar{\gamma} = 1$, there exists a saddle point.

Value of the game = 1

The position of saddle point is the optimal strategy and is given by (A_2, B_3)

Q: Determine which of the following two-person zero-sum games are strictly determinable and fair. Given the optimum strategy for each player in the case of strictly determinable games.

(i) Player B

$$\begin{array}{c|cc|c} & B_1 & B_2 & \\ \text{Player A} \quad A_1 & -5 & 2 & -5 \\ A_2 & -7 & -4 & -7 \\ \hline & -5 & 2 & \end{array}$$

$\text{Min (Maximum)} = \text{Min}(-5, 2) = -5$

$\text{Max min} = \text{Max}(-5, -7) = -5$

$\bar{\gamma} = \underline{\gamma} = -5 \neq 0$

∴ The game is strictly determinable.

Value of the game $= -5$

Optimal strategy is the position (A_1, B_1).

(ii) Player B

$$\begin{array}{c|cc|c} & B_1 & B_2 & \text{Row min.} \\ \text{Player A} \quad A_1 & 1 & 1 & 1 \\ A_2 & 4 & -3 & -3 \\ \hline \text{Column. max.} & 4 & 1 & \end{array}$$

$\text{Max (minimum)} = \text{Max}(1, -3) = 1$

$\text{Min (Maximum)} = \text{Min}(4, 1) = 1$

$\bar{\gamma} = \underline{\gamma} = 1 \neq 0$

∴ The game is strictly determinable

value of the game $= 1$

Optimal strategy is the position is (A_1, B_2).

Q4. Assignment Problem → Balanced
→ unblanced

n – job   n – person

→ each person can do each job at a time through with varying degree of efficiency

Minimum cost.

Hungarian Method.

Balanced

n=5

Jobs	Machine A	B	C	D	E
1	13	8	16	18	19
2	9	15	29	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

Solution Check Balanced or not n×n

STEP I :- Sub. the minimum element from all the elemts in Resp. row, we get row reduction matrix as.

Machine

Jobs	A	B	C	D	E
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

ST.2 Sub. the minimum element from all the elements in Resp. colum we are doing colum. Reduction matrix as.

Machine

Jobs	A	B	C	D	E
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

STEP 3.

Draw minimum no. of horizontal and vertical line to cover all zero.

If $N = n$ n = order of matrix
then an optimal solution can be made.

If $N < n$, then go to next step.
$4 < 5$

Machine

Jobs	A	B	C	D	E	
1	5	$\boxed{0}$	8	10	11	Row 1 zero → mark.
2	0	6	15	0	3	Row → Max 0
3	8	5	$\boxed{0}$	0	0	skip
4	$\boxed{0}$	6	4	2	7	
5	3	5	6	$\boxed{0}$	8	

1st wise. row select col. deleted

then col. wise app.

STEP 4 This solution is not optimal

So,

Determine the smallest uncovered element (x).

a) write uncovered value = uncovered value $- x$

b) Intersection value = Intersection value $+ x$

c) Line value (other values) as same

$x = 3$

$8-3=5$	$11-3=8$
$15-3=12$	$3-3=0$
$4-3=1$	$7-3=4$
$6-3=3$	$8-3=5$

Jobs	Machine A	B	C	D	E
1	5	0	5	10	8
2	0	6	12	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

STEP 5: Go to step 3.

	A	B	C	D	E
1	5	$\boxed{0}$	5	10	8
2	0	6	12	0	$\boxed{0}$
3	11	8	$\boxed{0}$	3	0
4	$\boxed{0}$	6	1	2	4
5	3	5	3	$\boxed{0}$	5

no. of line = 5
order = 5

So, we can form an Assignment

Jobs	Machine
1	B
2	E
3	C
4	A
5	D

Minimum Total cost

$$8 + 12 + 4 + 6 + 12 = 42$$

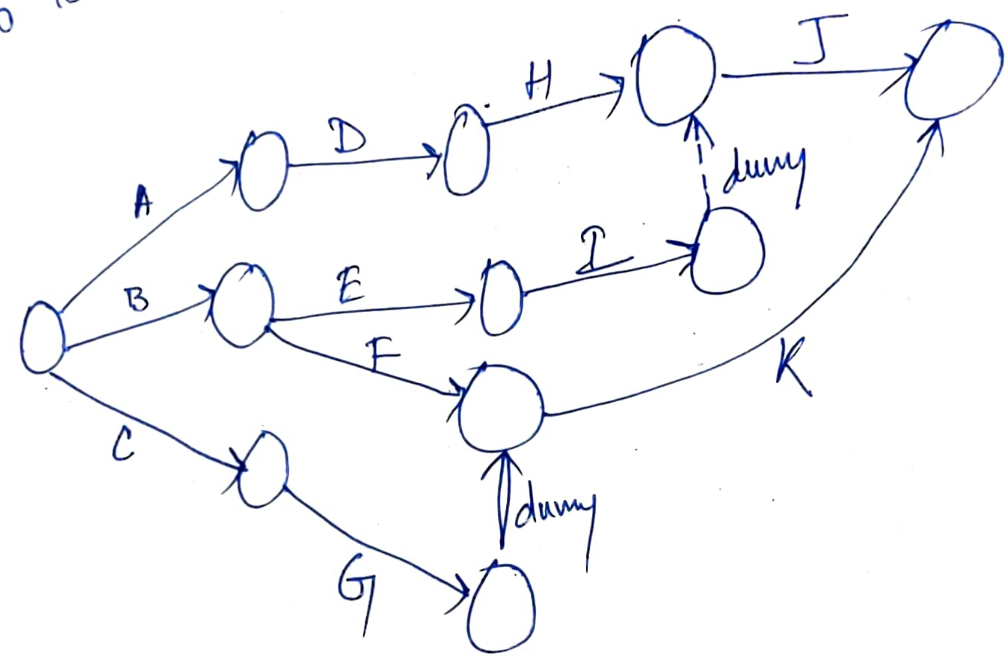
Q.6 ← Network scheduling problem →

It is a technique used for planning and Scheduling large projects in the field of Construction, maintaince, fabrication and purchasing of Computer System etc.

Construction of Network

Activity		Predecessor
A	→	—
B	→	—
C	→	—
D	→	A
E	→	B
F	→	B
G	→	C
H	→	D
I	→	E
J	→	H, I
K	→	F, G

Sample how do it



Final diagram