

## Queue

→ CPU Scheduling, Ready Queue, Device Resource Sharing.

Empty

→  $front = -1$  &  $rear = -1$

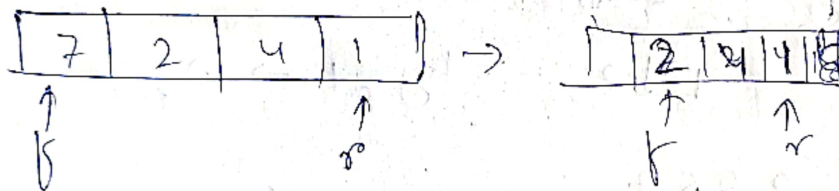
full

→  $rear = Max - 1$

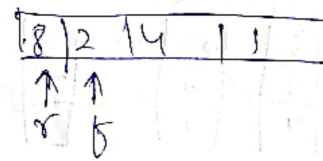
## In Circular Queue

★  $rear = (rear + 1) \% Max$

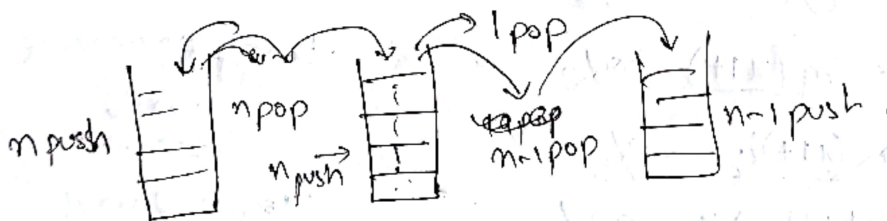
( $front = (rear + 1) \% Max$ ) || ( $front = 0$  &  $rear = Max - 1$ )



Implementing Queue using 2 stacks



n - enqueue & 1 Dequeue



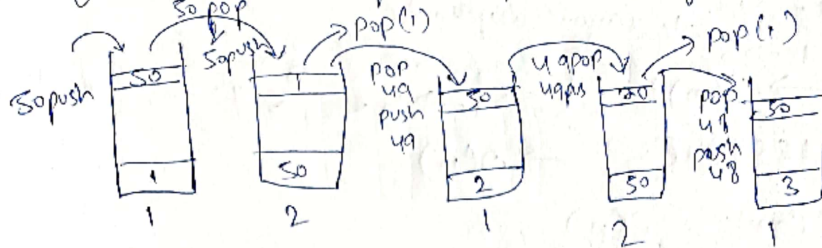
$$\therefore \text{Enqueue} = n + n + n - 1 = 3n - 1 \text{ push}$$

$$\text{Dequeue} = n + 1 + n - 1 = 2n \text{ pop.}$$

## Rule

1. push  $\rightarrow$  S1
2. pop & push to S2
3. <sup>only</sup> 1. pop ~~here~~ ~~for~~ for Dequeue
- ★ 4. Pop all from S2 & push to S1  $\rightarrow$  for stability

∴ for ~~data~~ So enqueue & 2 Dequeue.



∴ Enqueue =  $50 + 50 + 49 + 49 + 48 = 246$

Dequeue =  $50 + 1 + 49 + 49 + 1 + 48 = 198$

444.

## Deque

Double ended Queue.

Insert rear  
Delete rear

Delete front  
Insert front

## Priority Queue :

Inserted in any order but deletion will be based on priority.

Heap Max  
Heap Min

## Linked List

Array is fixed length.

for insertion.

→ ordered Array → shift & insert

Unordered " → insert at end.

for deletion

→ Ordered Array → Delete & shift

Unordered Array → Delete & shift

last element to that position

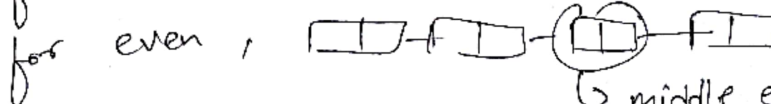
## Null pointer dereferencing

The head pointer is null.

and we are try to access the data & next results error.

return middle element

for odd, as usual



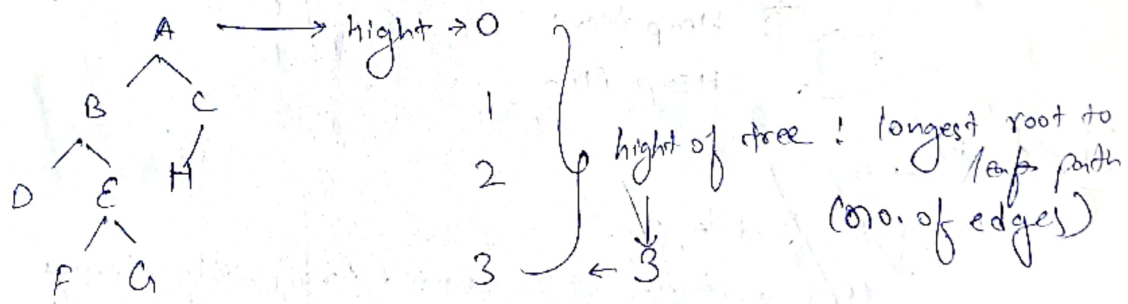
- Suppose 2 set
- ① Union  $\rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 3 \rightarrow 1 \rightarrow 5$  + remove duplicate  $\rightarrow O(n)$
  - ② Intersection  $\rightarrow O(n)^2$   $\frac{1 \rightarrow 2 \rightarrow 4}{3 \rightarrow 1 \rightarrow 5} \rightarrow$  Finalised duplicate  $\rightarrow O(n^2)$
  - ③ membership  $\rightarrow 1 \in S? 4 \in S? \rightarrow O(n)$
  - ④ Cardinality  $\rightarrow$  Count  $O(n)$
  - ⑤ etc

## Tree

- Tree is set of nodes & edges
- Recursion Linear (factorial)
- Fibonacci series (Non-linear)
- Acyclic graph
- Tree having  $n$  node always has  $n-1$  edges.
- Tree is a connected graph.

Rooted Tree one node is selected as root

Except leaf node, All are Internal node including Root node.



E is at Depth 2

G is at Depth 3

- \* Ancestor! E's ancestor is B A (parent & above in path)
- \* Descendant! B's descendant is E F G (child & below in path)

## General Tree

$\rightarrow$  In which each node is having 0 or more children.

Forest  
 $\rightarrow$  It is collection of 1 or more general Tree.

& each general tree called component

★ No. of edges in forest having  $P$  components &  $n$  nodes  
Total =  $(n - P)$  edges

K-ary tree  $\rightarrow$  0 or K children

\* If L is no. of Leaves & I is no. of Internal nodes

$$L = I(K-1) + 1$$

Degree of Node: no. of edges its connected to.

Handshaking  $\rightarrow$  Sum of degree =  $2 \times$  no. of edges (simple graph)

I is internal node, degree of Internal =  $K+1$  (with exception (root)).

$$\text{Total nodes} = L + I$$

$$\text{Total no. of edges} = L + I - 1$$

Handshaking Theorem.

$$\underbrace{L \times 1}_{\text{leaf degree}} + \underbrace{I(K+1)}_{\text{Internal node degree}} - 1 = 2(L + I - 1)$$

2 | Edges |

# Complete n-ary tree. Each node has n children.

$$L = 41, I = 10, n = ?$$

A. 3

B. 4

D. 6

$$L = I(n-1) + 1$$

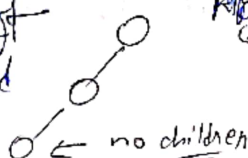
$$41 = 10(n-1) + 1$$

$$41 = n \Rightarrow n = 5$$

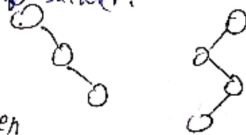
Binary Tree: 0, 1 or 2 children

Skewed Binary Tree: in which each node is having 1 child except 1 (leaf node).

Left skewed



Right skewed



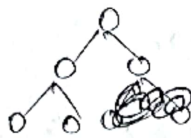
How many distinct Skewed Binary Tree possible

$$\star \frac{1}{n+1} 2^n C_n$$

n = no. of node.



Full Binary tree.  
(Comp2)

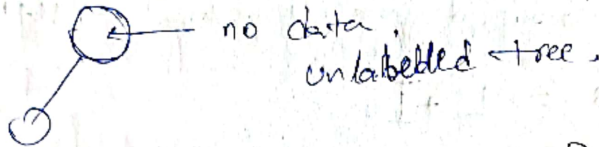


~~All leaf are at same height~~

Minimum Height of Binary Tree with  $n$  node.

$$\lceil \log_2 n \rceil$$

Counting Unlabelled Tree

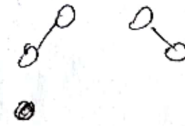


no. of unlabelled trees possible?

1 2 5 14 42

1.  $n=1$   $T(1) = 1$

2.  $n=2$   $T(2) = 2$



3.  $n=3$   $T(3) = 5$

L	R	
2	0	$\rightarrow 2$
0	2	$\rightarrow 2$
1	1	$\rightarrow 1$

$\rightarrow 5$

4.  $n=4$   $T(4) = 14$

L	R	
3	1	$\rightarrow 5$
1	3	$\rightarrow 5$
2	2	$\rightarrow 2 \times 2$

$\rightarrow 14$

5.  $n=5$   $T(5) = 42$

L	R	
4	1	$\rightarrow 14$
1	4	$\rightarrow 14$
3	2	$\rightarrow 10$
2	3	$\rightarrow 10$

$\frac{1}{n+1} 2^n C_n$  Catalan No.

Binary Tree

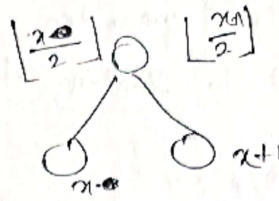
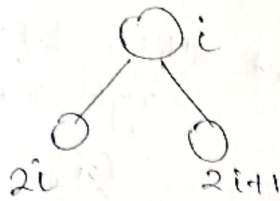
In B<sup>ary</sup> tree if  $N_0$  is no. of leaf node,  $N_1$  is no. of node with 1 children &  $N_2$  is no. of node with 2 children,

$$N_0 = N_2 + 1$$

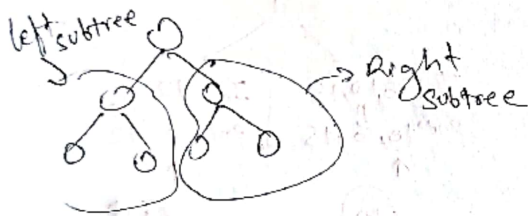
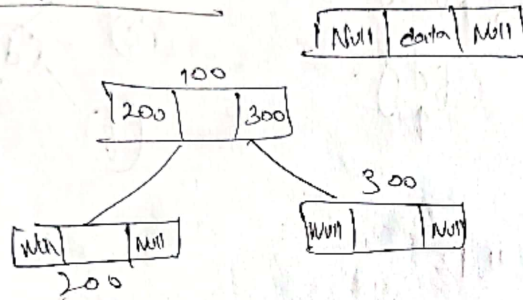
## Binary Tree Representation

Heap: Sequential Representation

→ An array is used to store Binary Tree



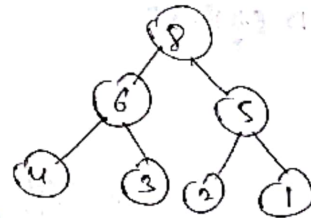
node Representation



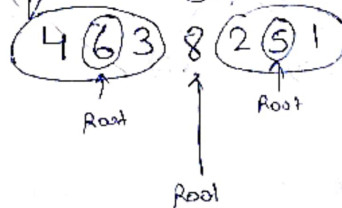
Tree structure is Recursive

## Binary Tree Traversal

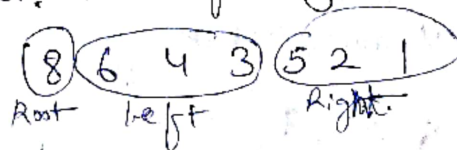
1. Inorder
2. Preorder
3. Postorder



1. Inorder: Left, Root, Right



2. Preorder: Root Left Right



a b c d f g e ← Preorder.

a c b f d g a e ← Inorder.

1. Find root in preorder.

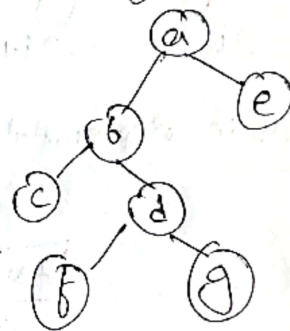
2. Take the root in Inorder traversal and split in Left and right subtree

Preorder

b c d f g → d f g

Inorder.

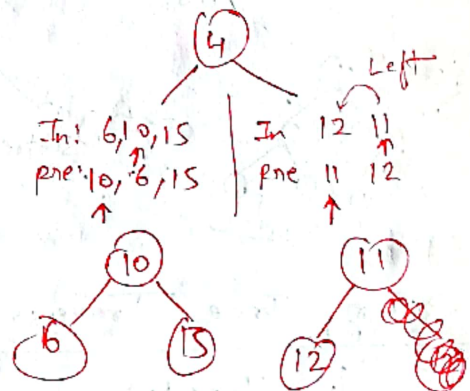
a b f d g



11

Preorder: 4, 10, 6, 15, 11, 12

Inorder: 6, 10, 13, 4, 12, 11



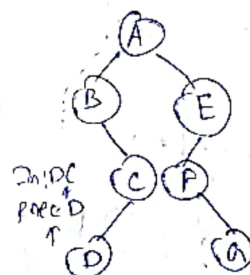
# Pre: A B C D E F G

In: B D C A F G E

Post: ?

In: B D C  
Pre: B C D A

In: D C  
Pre: C D

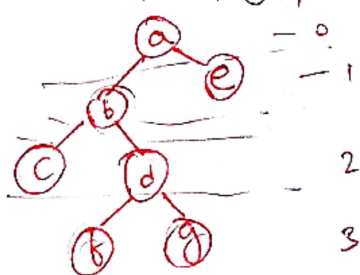


In: F G E  
Pre: E F G

∴ Post: D C B G F E A

# Post: a f g d b e a

Inorder: c b f d g a e



\* height = ? 3

y no. of nodes in Left subtree

z no. of nodes in Right subtree

$$10 \times 12^4 + 32^2$$

$$10 \times 3 + 2^5 + 3 \times 1$$

$$30 + 32 + 3$$

$$65 \text{ Ans.}$$

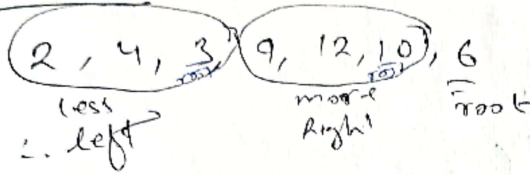


BST  $\rightarrow$  Inorder is Increasing Order Sequence (Unique)

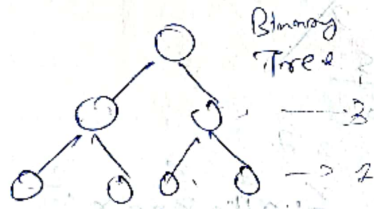
Preorder.



Post order



# a d b e f g h is post order traversal of full binary tree  
Can we construct the tree?



a d b e f g h  
 Left right root.  
 $\therefore$  can be made

3  
 7  
 15  
 31  
 63  
 127

$2^{h+1} - 1$

★ BST : Inorder ! Can't be constructed  
 2-arr Tree : Preorder & post order traversal we can  
 Construct Tree Uniquely

Time Complexity of Search operation in BST

$O(h)$ 

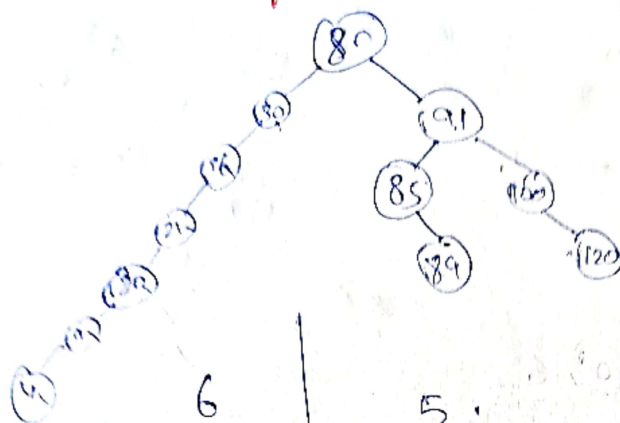
- skewed  $O(n)$
- balanced  $O(\log_2 n)$

(h) Left most Node : Min element (BST)  
 Right " " : Max " (BST)  $\rightarrow$  Not always Leaf



T. 251

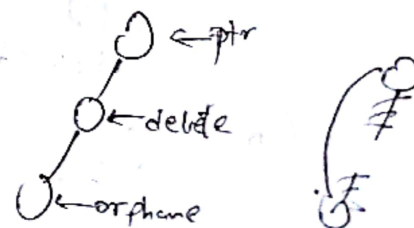
with 80 as Root of BST. The no. of BST possible is         .



$\frac{1}{6 \times 1} \quad 6 \times 2 \quad 6$        $\frac{1}{5 \times 1} \quad 5 \times 2 \quad 5$   
 $\frac{1}{7} \quad 12 \quad 6$        $\frac{1}{6} \quad 10 \quad 5$   
 $\frac{1}{7} \times \frac{12 \times 10}{2} = \frac{60}{7}$        $\frac{1}{6} \times \frac{10 \times 9}{2} = \frac{45}{6}$   
 $\frac{1}{7} \times \frac{2 \times 11 \times 10 \times 4 \times 8 \times 7}{6 \times 5 \times 4 \times 3 \times 2} = 132$        $\frac{1}{6} \times \frac{2 \times 10 \times 9 \times 8 \times 7 \times 6}{3 \times 4 \times 3 \times 2} = 42$   
 $132 \times 42 = 5544$   
Combination

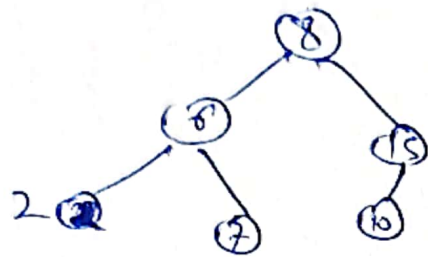
- ① Deleting Leaf node! Direct Delete.

- (2) Deleting node with 1 child: hold the parent of the node to be deleted and connect the orphaned child of the deleted node to the parent of deleted node.



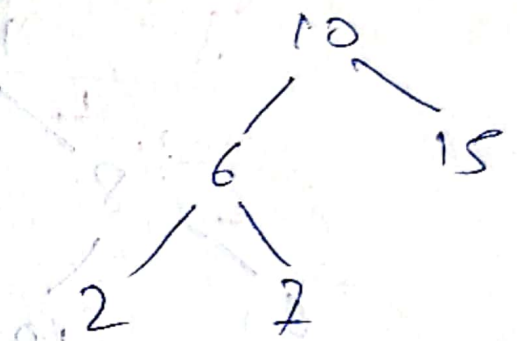
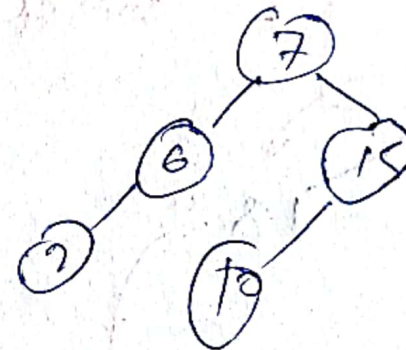
③ Deleting node with 2 children:

The node is replaced with Inorder predecessor  
or Inorder Successor.



In order = 2 6 7 8 10 15

↑                      ↑  
 predec.              Successor.



In order.

min don't have predecessor  
max " " successor.