OR is the scientific approach to decision-making that uses advanced analytical methods like mathematics, statistics, & algorithms to solve complex real world problems like minimizing costs, maximizing profits.

Characteristics of OR

1. Inter-disciplinary team approach: The optimum sol^n is found by a team of scientists selected from various disciplines.

2. Imperfectness of solutions: Improved the quality of sol^n.

3. Use of scientific research: Uses scientific research to reach optimum sol.

4. To optimize total O/P: It tries to optimize by maximizing profit & minimizing loss.

Applications of OR

1. Purchasing, Procurement & Exploration.
2. Production Management
3. Marketing
4. Personnel Management
5. Research & development

Phases of OR

1. Defining the problem & gathering data.
2. Formulating a mathematical model.
3. Deriving sol^n from the model.
4. Testing the model & its solns.
5. Preparing to apply the model.
6. Implementation.

Explanation.

1. The 1st task is to study the relevant system & develop a well defined statement of problem.
2. This phase is to reformulate the problem in terms of mathematical symbols & expressions.
3. This phase is to develop a procedure for deriving sol^n to the problem.
4. After deriving the sol^n it is tested as a whole for errors if any.

5. The process of testing and improving a model to increase its validity is commonly referred as Model Validation

6. The last phase of an OR study is to implement the system as prescribed by the management

Linear Programming Problem

LP is a method of optimising operations with some constraint. The main objective of LP is to maximize or minimize the profit or cost. It consist of linear function which are subjected to constraint in the form of linear eqn. LP is considered as an important technique that is used to find the optimum resource utilisation

$$\text{Maximize or minimize, } Z = C_1x_1 + C_2x_2 + \cdots + C_nx_n$$

Transportation problem:

It is one of the subclass of LPP. The objective is to transport various quantities of a single homogeneous commodity that are initially shared at various origins to diff. destination in such a way that the transportation cost is minimized.

To achieve this we must know the amount & location of available supplies & quantities demanded.

Mathematical formulation

Consider a Transportation problem with m origin (rows) & n destinations (col). Let C_{ij} be the cost of transportating one unit of product from ith origin to jth destination. Let a be the quantity of commodity available at origin i & b be the quantity of commodity needed at destination j. x_{ij} is the quantity

		Destination				
O R I G I N		1	2	--- ---	n	Capacity
	1	x_{11}	x_{12}		x_{1n}	a_1
	2	⋮	⋮		⋮	a_2
	m	x_{m1}	x_{m2}		x_{mn}	⋮
	Demand	b_1	b_2	b_3	b_n	a_n

The LP model representing the transportation problem is given by

$$\text{Minimize, } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}\, x_{ij}$$

The LP model representing the transportation problem is said to be balanced if $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ i.e. total supply = total demand

Feasible solⁿ: Any set of non-neg. allocation $(x_{ij} \geq 0)$ which satisfies row & col sum (rim requirement) is called feasible solⁿ.

A feasible solⁿ is called basic feasible solⁿ if the no of non-neg. allocation is equal to $m+n-1$

Optimal solⁿ: It is a feasible solⁿ which minimizes the total cost. The solⁿ of a transportation problem can be obtained in 2 stages • initial & • optimum. Initial solⁿ can be obtained using any of 3 methods.

i) North West Corner Rule.

ii) Least Cost Method or Matrix Minima Method

iii) Vogel's Approximation Method

obtain initial basic solⁿ of a transportation prblm where cost & rim req table is given below.

Destination Origin	D1	D2	D3	supply
O1	2	7	4	5
O2	3	3	1	8
O3	5	4	7	7
O4	1	6	2	14
Demand	7	9	18	

Here, $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = 34$

	D1	D2	D3	supply
O1	⑤ $\boxed{2}$	$\boxed{7}$	$\boxed{4}$	~~5~~ 0
O2	② $\boxed{3}$	⑥ $\boxed{3}$	$\boxed{4}$	~~8~~ ~~6~~ 0
O3	$\boxed{5}$	③ $\boxed{4}$	④ $\boxed{7}$	~~7~~ ~~4~~ 0
O4	$\boxed{1}$	$\boxed{6}$	⑭ $\boxed{2}$	~~14~~ 0
Dem.	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	

Here we get the feasible solⁿ to given table which is given by: $x_{11} = 5$, $x_{21} = 2$, $x_{22} = 6$, $x_{32} = 3$, $x_{33} = 4$, $x_{43} = 14$

Total cost $= (5 \times 2) + (2 \times 3) + (6 \times 3) + (3 \times 4) + (4 \times 7) + (14 \times 2)$

$= ₹ 102$

Least Cost or Matrix Minima Method.

5. This method is used to find an initial basic feasible solⁿ

S1 Setup the transportation table.

6. S2 Find the cell with lowest cost

S3 Allocate as much as possible to that cell.

S4 Adjust the supply & demand

S5 Repeat S2 – S4

solve the TP using LC Method.

	D1	D2	D3	D4	Sup.
O1	1	2	3	4	6
O2	4	3	2	0	8
O3	0	2	2	1	10
Dem.	4	6	8	6	24

	D1	D2	D3	D4	Sup
O1	1	2 ⑥	3	4	~~6~~0
O2	4	3	2 ②	0 ⑥	~~8~~ ~~2~~
O3	0 ④	2	2 ⑥	1	~~10~~ ~~8~~
Dem.	~~4~~0	~~6~~0	~~8~~ ~~6~~0	~~6~~0	

Now, the solⁿ is given by: $x_{31}=4$, $x_{12}=6$, $x_{23}=2$, $x_{33}=6$, $x_{24}=6$.

$4\times0 + 6\times2 + 2\times2 + 6\times2 + 6\times0$ = ₹ 28.

Vogel's Approximation Method.

This is another way to find an initial feasible solⁿ for a TP

S1 Setup the transportation table.

S2 For each row & column compute penalty

S3 Find row or col with highest penalty.

S4 In that row or col, find the cell with lowest cost.

S5 Allocate as much as possible to that cell.

S6 Adjust supply & demand

S7 Recompute penalties for remaining table

S8 Repeat S3 – S7 until all supplies & demands are fulfilled.

Find the initial basic feasible solⁿ for the TP by VAM.

	D1	D2	D3	D4	Sup.
O1	11	13	17	14	250
O2	16	18	14	10	300
O3	21	24	13	10	400
Dem.	200	225	275	250	950

	D1	D2	D3	D4	Sup.	P1	P2	P3	P4	P5	P6
O1	$\boxed{11}$ 200	$\boxed{13}$ 50	$\boxed{17}$	$\boxed{14}$	~~250~~ ~~50~~ 0	2	1	—	—		
O2	$\boxed{16}$	$\boxed{18}$ 175	$\boxed{14}$	$\boxed{10}$ 125	~~300~~ ~~125~~ 0	4	4	4	← 4		
O3	$\boxed{21}$	$\boxed{24}$	$\boxed{13}$ 275	$\boxed{10}$ 125	~~400~~ ~~125~~ 0	3	3	3	3	3	10
Dem	~~200~~ 0	~~225~~ ~~175~~ 0	~~275~~ 0	~~250~~ ~~125~~ 0							
P1	5↑	5	1	0							
P2		5↑	1	0							
P3		6↑	1	0							
			1	0							
P5			13↑	10							
P6				10							

Here, No of allocated cell = 6 = m+n-1

There are 6 +ve independent allocations

Basic feasible sol^n is:

$11\times200 + 50\times13 + 175\times18 + 275\times13 + 125\times10 + 125\times10$

$= ₹\ 12075.$

North West Corner Rule

S1 start $\overset{at}{\wedge}$ the top-left cell of cost matrix.

S2 Allocate as much as possible to that cell.

S3 Subtract the allocation from the corresponding supply & demand.

S4 If supple becomes 0, move down to next source. If demand becomes 0, move right to the next destination.

S5 Repeat S2-S4 until all supply & demand are satisfied

obtain the initial basic feasible sol^n of TP whose cost & rim req^n $\overset{is}{\wedge}$ given.

Des / ori	D1	D2	D3	Supply
O1	2	7	4	5
O2	3	3	1	8
O3	5	4	7	7
O4 ~~Demand~~	1	6	2	14
Dem.	7	9	18	34

	D1	D2	D3	Sup
O1	$\boxed{2}$ ⑤	$\boxed{7}$	$\boxed{4}$	~~5~~ 0
O2	$\boxed{3}$ ②	$\boxed{3}$ ⑥	$\boxed{1}$	~~8~~ ~~6~~ 0
O3	$\boxed{5}$	$\boxed{4}$ ③	$\boxed{7}$ ④	~~7~~ ~~4~~ 0
O4	$\boxed{1}$	$\boxed{6}$	$\boxed{2}$ ⑭	~~14~~ 0
Dem.	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~14~~ ~~18~~ 0	

The given TP is balanced i.e.,

total demand = total supply.

$x_{11} = 5$, $x_{21} = 2$, $x_{22} = 6$, $x_{32} = 3$, $x_{33} = 4$, $x_{43} = 14$.

Min^m Trans. cost $= (5\times2) + (2\times3) + (6\times3) + (3\times4) + (4\times7) + (14\times2)$

$= ₹\ 102.$

Optimality Test: Once the initial basic feasible solⁿ has been computed, the next step is to determine whether the solⁿ is optimum or not.

Optimality test can be conducted on any initial basic feasible solⁿ of a TP provided such an allocation has exactly m+n-1, non-neg. allocations.

↳ To perform optimality test we use MODI method.

MODI method

An optimization technique used in transportation problems to find optimal solⁿ from an initial basic feasible solⁿ.

S1 Find initial basic feasible solⁿ of TP

S2 Compute u & v for each row i and col j

$C_{ij} = u_i + v_j$ (for all allocated cells)

Start by assigning $u_1 = 0$

Use the above eqⁿ to compute other u & v values

S3 Calculate opportunity cost (Δ) for unallocated cells.

$\Delta_{ij} = C_{ij} - (u_i + v_j)$

S4 Perform optimality check.

- if all Δ values > 0 → optimal solⁿ
- if Δ values < 0 → not optimal.
- if Δ values ≥ 0 → solⁿ is optimum, but an alternate solⁿ exist

S5 Improve the solⁿ

Solve the following TP & obtained the optimum solⁿ

O \ D	D1	D2	D3	D4	Supply
O1	2	2	2	1	3
O2	10	8	5	4	7
O3	7	6	6	8	5
Demand	4	3	4	4	

↳ The given TP is balanced.

	D1	D2	D3	D4	Supp.	P1	P2	P3	P4	P5
O1	2 ③	2	2	1	~~3~~ 0	1	—	—	—	
O2	10	8	5 ③	4 ④	~~7~~ ~~3~~ 0	1	1	← 3	—	
O3	7 ①	6 ③	6 ①	~~6~~ 8	~~8~~ ~~5~~ ~~4~~ ~~1~~ 0	0	0	0	0	0
Dem	~~4~~ ~~1~~ 0	~~3~~ 0	~~4~~ ~~1~~ 0	~~4~~ 0						
P1	5↑	4	3	3						
P2	3	2	1	4↑						
P3	5↑	2	1	—						
P4	7↑	6	6	—						
P5	—	6↑	6	—						

No. of occupied sol $= m+n-1 = 6$
& the occupied cells are independent
∴ solⁿ is non-degenerate

$x_{11} = 3$, $x_{31} = 1$, $x_{32} = 3$, $x_{23} = 3$, $x_{33} = 1$, $x_{24} = 4$

Initial basic feasible solⁿ $= (3 \times 2) + (1 \times 7) + (3 \times 6) + (3 \times 5) + (1 \times 6) + (4 \times 4)$

or Trans. Cost $=$ ₹ 68.

→ Now, optimal solⁿ (Applying MODI method).

O \ D	D1	D2	D3	D4	Supp.	
O1	2 ③	2	2	1	3	u_1
O2	10	8	5 ③	4 ④	7	u_2
O3	7 ①	6 ③	6 ①	8	5	u_3
Dem.	4	3	4	4		
	v_1	v_2	v_3	v_4		

Max^m allocation is in u_3 ∴ assume $u_3 = 0$.

$C_{31} = u_3 + v_1$
$7 = 0 + v_1$
$\boxed{v_1 = 7}$

$C_{32} = u_3 + v_2$
$6 = 0 + v_2$
$\boxed{v_2 = 6}$

$C_{33} = u_3 + v_3$
$6 = 0 + v_3$
$\boxed{v_3 = 6}$

$C_{11} = u_1 + v_1$
$2 = u_1 + 7$
$\boxed{u_1 = -5}$

$C_{23} = u_2 + v_3$
$5 = u_2 + 6$
$\boxed{u_2 = -1}$

$C_{24} = u_2 + v_4$
$4 = -1 + v_4$
$\boxed{v_4 = 5}$

→ Now, opportunity cost for all unallocated cells $(\Delta_{ij} = c_{ij} - (u_i + v_j))$.

$\Delta_{12} = C_{12} - (u_1 + v_2) = 2 - (-5+6) = 1$

$\Delta_{13} = C_{13} - (u_1 + v_3) = 2 - (-5+6) = 1$

$\Delta_{14} = C_{14} - (u_1 + v_4) = 1 - (-5+5) = 1$

$\Delta_{21} = C_{21} - (u_2 + v_1) = 10 - (-1+7) = 4$

$\Delta_{22} = C_{22} - (u_2 + v_2) = 8 - (-1+6) = 3$

$\Delta_{34} = C_{34} - (u_3 + v_4) = 8 - (0+5) = 3$

In this problem all Δ_{ij} values > 0 $\therefore$ solⁿ is unique & optimum.

The solⁿ is given by: $x_{11} = 3$, $x_{23} = 3$, $x_{24} = 4$, $x_{31} = 1$, $x_{32} = 3$, $x_{33} = 1$

Trans. cost = ₹68

Degeneracy in TP

In a TP, if the no of non-neg independent allocations is less than $m+n-1$, [m = no. of rows, n = no. of cols], there exists a degeneracy. This may occur at the initial stage or at subsequent iteration.

solve the TP & obtain the optimal solⁿ.

O \ D	D1	D2	D3	Sup.
O1	2	2	3	10
O2	4	1	2	15
O3	1	3	1	40
Dem	20	15	30	

Given table is balanced.

→ Initial basic feasible solⁿ using NWCR

	D1	D2	D3	Sup
O1	⑩ 2	2	3	~~10~~ 0
O2	⑩ 4	⑤ 1	2	~~15~~ ~~5~~ 0
O3	1	⑩ 3	㉚ 1	~~40~~ ~~30~~ 0
D	~~20~~ ~~10~~ 0	~~15~~ ~~10~~ 0	~~30~~ 0	

$x_{11} = 10$, $x_{21} = 10$

$x_{22} = 5$, $x_{32} = 10$

$x_{33} = 30$.

$m+n-1 = 5$

$\because$ no. of occupied cells = 5 and all cell allocations are independent we get non-degenerate solⁿ

Trans. cost = $2\times10 + 4\times10 + 1\times5 + 3\times10 + 1\times30$ = ₹125

→ Now optimal solⁿ.

	D1	D2	D3	sup	
O1	⑩ 2	2	3	10	u_1
O2	⑩ 4	⑤ 1	2	15	u_2
O3	1	⑩ 3	㉚ 1	40	u_3
	20	15	30		
	v_1	v_2	v_3		

Assume, $v_1 = 0$ (check for max allocation col / row)

$C_{11} = u_1 + v_1 \Rightarrow 2 = u_1 + 0 \Rightarrow u_1 = 2.$

$C_{21} = u_2 + v_1 \Rightarrow 4 = u_2 + 0 \Rightarrow u_2 = 4$

$C_{22} = u_2 + v_2 \Rightarrow 1 = 4 + v_2 \Rightarrow v_2 = -3$

$C_{32} = u_3 + v_2 \Rightarrow 3 = u_3 - 3 \Rightarrow u_3 = 6$

$C_{33} = u_3 + v_3 \Rightarrow 1 = 6 + v_3 \Rightarrow v_3 = -5$

→ calculate Δ_{ij} for all unoccupied cell $\Delta_{ij} = c_{ij} - (u_i + v_j)$

$\Delta_{12} = c_{12} - (u_1 + v_2)$ ~~⇒~~ $\Delta_{13} = c_{13} - (u_1 + v_3)$

$= 2 - (2 + (-3))$ $= 3 - (2 + (-5)) = 6.$

$= 3$

$\Delta_{31} = c_{31} - (u_3 + v_1) = -5$

$\Delta_{23} = c_{23} - (u_2 + v_3) = 3$

$\because \Delta_{31} < 0$ ⇒ Sol^n is not optimum

So, to make sol^n optimum → follow these steps.

	D1	D2	D3	Sup
O1	2 ⑩	2	3	
O2	⑩ −x 4	⑤ 1+x	2	
O3	$\Delta<0$ 1+x	⑩ 3−x	㉚ 1	
Dem				

calculate x.

$x = \min(10, 10) = 10$

Now, update table

* we take allocated val. of −x then find min^m.

	D1	D2	D3	Sup
O1	2 ⑩	2	3	
O2	4 (10−10)	1 (5+10)	2	
O3	1 ⑩	3 (10−10)	1 ㉚	
Dem				

⇒

Iteration table.

	D1	D2	D3	Sup
O1	2 ⑩	2	3	
O2	4	1 ⑮	2	
O3	1 ⑩	3	1 ㉚	
Dem				

Now, in updated table, allocated cells $= 4 < m+n-1$

Hence, problem of degeneracy arises here.

Steps for degeneracy.

Among the empty cell, we choose an empty cell having the least cost, which is an independent position. If such cell is more than one then choose any one.

Allocate the small +ve quantity $\varepsilon > 0$ in the cell.

	D1	D2	D3	Sup
O1	2 ⑩	2 (ε)	3	
O2	4	1 ⑮	2	
O3	1 ⑩	3	1 ㉚	
Dem				

» This 2 is at independent posi (SE)

So, to resolve degeneracy we need to add an empty cell (1, 2) and allocate $\varepsilon > 0$

Now, $m+n-1 = 5$

Hence, sol^n is non-degenerable.

Again perform both steps to check & solⁿ is optimal or not.

	D1	D2	D3	Sup	
O1	2 ⑩	2 (ε)	3		$u_1 = 0$ (let)
O2	4	1 ⑮	2		u_2
O3	1 ⑩	3	1 ㉚		u_3
Dem.					
	v_1	v_2	v_3		

$c_{ij} = u_i + v_j$ (for all occupied cell)

On solving,

$u_2 = -1$, $u_3 = -1$, $v_1 = 2$, $v_2 = 2$

$v_3 = 2$

Now, $\Delta_{ij} = c_{ij} - (u_i + v_j)$ [for all unoccupied cell].

$\Delta_{13} = 1$, $\Delta_{21} = 3$, $\Delta_{23} = 1$, $\Delta_{32} = 2$ [on solving]

Here all Δ values ≥ 0 ∴ solⁿ is optimal

Solⁿ is: $x_{11} = 10$, $x_{31} = 10$, $x_{12} = \varepsilon$, $x_{22} = 15$, $x_{33} = 30$

$TC = (2\times10) + (1\times10) + (1\times15) + (1\times30) + (2\times\varepsilon) = (75 + 2\varepsilon) \approx ₹75.$

Assignment Problem

It is a fundamental concept in OR that deals with the optional assignment of a no of resources to an equal no of tasks in such a way that the total cost is minimized or total profit is maximized.

Suppose there are n jobs to be performed by n person. Assume that each person can do each job at a time. The problem is to determine that which job should be assigned to which person so that total cost is minimized or total profit is maximized.

let c_{ij} be cost of doing job (each).

Minimize total cost $(Z) = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$

x_{ij} denotes that jth job is to be assigned to ith person.

[illegible]

$d_{11} = 4$, $d_{21} = 5$, $d_{23} = 1$, $d_{32} = 2$

[illegible] optimal

[illegible]

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person can do each job at a time. The problem is to determine
that which job should be assigned to which person so that total cost
is minimized or total profit is maximized.

let C_{ij} be cost of doing job (i,j).

Minimize total cost $(Z) = \sum_{i=1}^{n} \sum_{j=1}^{n} C_{ij} x_{ij}$

x_{ij} denotes that i^{th} job is to be assigned to j^{th} person.

Hungarian Method Procedure

sol^n of assignment problem can be assived at by using Hungarian method.

S1 Prepare a cost matrix. If cost matrix is not square then add dummy row/col with zero cost element.

S2. Subtract the min^m element in each row from all elements of respective rows.

S3 Modify resulting matrix by subtracting min^m ele of each col from all elements of respective cols. Thus obtained modified matrix

~~Thus obtain &~~

S4 Then draw min^m no. of horizontal & vertical lines to cover all zeros in resulting matrix.

let the min^m no of lines = N. Now 2 possible cases.

Case I: if N = n (n = order of matrix) then an optional assignment can be made.

Case II: if N < n then proceed to S5

S5 Determine the smallest uncovered element in matrix. Subtract this min^m element from all uncovered elements & add same element at intersection of horizontal & vertical lines. Thus obtained 2nd modified matrix

S6. Repeat S3 & S4 until we get case (i) of S4.

S7 Examine rows successively until a row-wise exactly single 0 is found. Circle (o) this zero to make assignment. Then mark a cross (x) over all zeros if lying in col of circled zero, showing that they can't be considered for future assignment. Continue in this manner until all zeros have been examined. Repeat same process for columns.

S8 Repeat S6 until one of the following situation arises.

If no unmarked 0 left, then process ends

If there lie more than one unmarked 0 in any col or row, circle one of the unmarked 0 & mark cross in cells of remaining zeros in its row or col. Repeat this process until no unmarked 0 is left in matrix.

S9 Thus exactly one marked circle zero in each row & col of matrix is obtained. The assignment corresponding to these marked circled zeros will give the optimal assignment

Types of Assignment problem
- Balanced (n×n)
- Unbalanced (n×m)

Solve the assignment problem using Hungarian method

Machines.

Jobs	A	B	C	D	E
1	15	8•	16	18	19
2	9•	15	24	9	12
3	12	9	4•	4	4
4	6•	12	10	8	13
5	15	17	18	12•	20·

• — min^m elem.

S1+S2

	A	B	C	D	E
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

(Rowwise detection)

After performing colwise detection, we will get same table because 0 is min in each column. So, im not redrawing same table.

S3.

	A	B	C	D	E
1	5	$\boxed{0}$	8	10	11
2	0	6	15	0	3
3	8	5	⓪	0	0
4	$\boxed{0}$	6	4	2	7
5	3	5	6	$\boxed{0}$	8

Row → 1 zero mark & del. col.

Row → 1> zero ~~skip.~~

Now perform colscan and apply same rule

Here no of lines = 4 = N and n = 5

∴ N < n.

Now find smallest uncovered element

min(8, 11, 15, 3, 4, 7, 6, 2) = 3.

	A	B	C	D	E
1	5	0	5	10	8
2	0	6	12	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5.

and intersection values are (8, 5, 0). so, add this 8, 5, 0 and 3 and rewrite in table and write remaining values as it is.

Now go to S3. and repeat S3.

	A	B	C	D	E
1	5	$\boxed{0}$	5	10	8
2	0	6	12	0	$\boxed{0}$
3	11	8	$\boxed{0}$	3	0
4	$\boxed{0}$	6	1	2	4
5	3	5	3	$\boxed{0}$	5

∴ this is the only 0 remaing ∴ mark & del.

This 0 already deleted

here, N = 5, n = 5 and N = n, sol^n is optimal.

So, now we can form assignment.

Jobs	Machine
1	B
2	E
3	C
4	A
5	D

Min^m total cost = (1,B) = 8 ; (2,E) = 12 ; (3,C) = 4 ; (4,A) = 6

& (5,D) = 12.

= 8+12+4+6+12 = ₹ 42

Solve the AP. Find optimal solution of jobs to machines to minimize the total processing time and also find for which machine no job is assigned.

Jobs	Machines				
	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
~~0~~	~~0~~	~~0~~	~~0~~	~~0~~	~~0~~

Since the matrix in not balanced. ∴ we add a dummy Job 5 with corresponding entries 0.

Jobs	Machines.				
	A	B	C	D	E
1	4	3	6	2 min^n	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
5	0	0	0	0	0

Jobs	Machines				
	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

modified mat

here N = 4 and n = 5 → N < n → sol^n in not optimal.

So, from uncovered value, we get min^m = 0.

Jobs	Machines				
	A	B	C	D	E
1	1	0	3	0	4
2	0	2	1	5	6
3	2	1	0	0	3
4	2	1	0	4	0
5	0	0	0	1	0

Now, n=5 and N=5 → N=n

→ sol^n is now optimal we can get :

Jobs	M/C				
	A	B	C	D	E
1	1	0	3	0	4
2	$\boxed{0}$	2	1	5	6
3	2	1	0	0	3
4	2	1	0	4	0
5	~~0~~	0	0	1	0

- row wise check → only one zero → mark → & cross all particular column zero
- more than 1 zero → skip

→ Same Rule for colwise

Now we will randomly choose any one zero and and then cross all zero of this zero's row & col.

randomly sel

	A	B	C	D	E
1	1	$\boxed{0}$	3	~~0~~	4
2	$\boxed{0}$	2	1	5	6
3	2	1	~~0~~	$\boxed{0}$	3
4	2	1	$\boxed{0}$	4	~~0~~
5	~~0~~	~~0~~	~~0~~	1	$\boxed{0}$

After that D contains single zero so mark it & delete its resp. rows & col zero

Again choose randomly any zero. → col (4, C) → choosen.

∴ cross its resp. zero in row & col

→ done in upper table

→ now remaing only one zero (5, E)

∴ → optimal assignment

~~Mach.~~ Job.	Machin.
1	B
2	A
3	D
4	C

(1, B) = 3, (2, A) = 10, (3, D) = 1, (4, C) = 6, (5, E) = 0.

So, optimal solⁿ is = 3 + 10 + 1 + 6 + 0 = 20 hr.

Maximization in Assignment Problem.

Q. Find assignment of mechanics to job that will result in max^m profit.

Job

mechanic	A	B	C	D	E
1	62	78	50	111	82
2	71	84	61	73	59
3	87	92	111	71	81
4	48	64	87	77	80
5	0	0	0	0	0

max^m ele

Given matrix is not a sq. matrix. Hence we introduce a dummy mechanic 5 with all elements 0

Now we convert this profit matrix into loss matrix by subtracting all elements from highest element: (111)

49	33	61	0 min^m	29
40	27	50	38	52
24	19	0	40	30
63	47	24	34	31
111	111	111	111	111

Here, we r selecting min^m element row wise.

Row minima or row reduction matrix:

49	33	61	0	29
13	0	23	11	25
24	19	0	40	30
39	23	0	10	7
0	0	0	0	0

here we r selecting min^m ele. colwise –

↳ in each col min val = 0

→ Redraw same table for col minima or col reduction matrix

49	33	61	0	29
13	0	23	11	25
24	19	0	40	30
39	23	0	10	7
0	0	0	0	0

Now, N = 4 & n = 5 → N < n.

∵ N < 4 ∴ sol^n is not optimal.

— Update matrix (2nd modified matrix).

— min (uncovered values) = 7.

For uncovered values : uncovered val = uncovered val − 7

For values that lies on line intersection — cume +7 then write

49	40	68	0	29
6	0	23	4	18
17	19	0	33	23
32	23	0	3	0
0	7	7	0	0

Now, here N = 5 = n. — sol^n we will get will be optimal.

∴ we make the assignment.

49	40	68	$\boxed{0}$	29
6	$\boxed{0}$	23	4	18
17	19	$\boxed{0}$	33	23
32	23	$\cancel{0}$	3	$\boxed{0}$
$\boxed{0}$	7	7	$\cancel{0}$	$\cancel{0}$

— The optimum assignment is.

mechanics	Job
1	D
2	B
3	C
4	E
5	A

— The max^m profit is given by : 111 + 84 + 111 + 80 + 0

= ₹ 386