Home
optimal = optimum

Extra topics

(1) Game theory — mixed strategy (without saddle point)

theory
Eg 2×2 Games without saddle point. Consider a 2×2 two-person zero-sum game without saddle point, having the payoff matrix for player A as

$$\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & a_{11} & a_{12} \\ A_2 & a_{21} & a_{22} \end{array}$$

The optimum mixed strategies,

$$S_A = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix} \quad \& \quad S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$$

$$\boxed{P_1 = \frac{a_{22} - a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} ; \quad P_1 + P_2 = 1; \quad P_2 = 1 - P_1}$$

$$\boxed{q_1 = \frac{a_{22} - a_{12}}{(a_{11}+a_{22}) - (a_{12}+a_{21})} ; \quad q_1 + q_2 = 1; \quad q_2 = 1 - q_1}$$

The ~~sat~~ val. of the game (V) =

$$\boxed{\frac{a_{11}a_{22} - a_{12}a_{21}}{(a_{11}+a_{22}) - (a_{12}+a_{21})}}$$

Eg (1) Solve the following payoff matrix. Also determine the optimal strategies & value of the game.

$$\begin{array}{c|c} & B \\ \hline A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & B \\ \hline A & \begin{bmatrix} 5 & 1 \\ 3 & 4 \end{bmatrix} \end{array}$$

It's clear that the payoff mat. doesn't possess any saddle point.

Let this be $\begin{array}{c|cc} & B_1 & B_2 \\ \hline A_1 & a_{11} & a_{12} \\ A_2 & a_{21} & a_{22} \end{array}$. The players will use mixed strategies.

The optimal mixed strategies,

$$S_A = \begin{bmatrix} A_1 & A_2 \\ P_1 & P_2 \end{bmatrix} \quad \& \quad S_B = \begin{bmatrix} B_1 & B_2 \\ q_1 & q_2 \end{bmatrix}$$

$$\left.\begin{aligned} P_1 &= \frac{4-3}{(5+4)-(3+1)} = \frac{1}{5} \\ P_2 &= 1 - \frac{1}{5} = \frac{4}{5} \end{aligned}\right\} S_A$$

$$\left.\begin{aligned} q_1 &= \frac{4-1}{5} = \frac{3}{5} \\ q_2 &= 1 - \frac{3}{5} = \frac{2}{5} \end{aligned}\right\} S_B$$

∴ The value of the game $= \dfrac{5\times 4 - 3\times 1}{5}$

$$= \frac{20-3}{5} = \frac{17}{5} (V)$$

∴ The optimum mixed strategies, $S_A = \left(\frac{1}{5}, \frac{4}{5}\right)$

$S_B = \left(\frac{3}{5}, \frac{2}{5}\right)$, value of the game $(v) = \frac{17}{5}$

② pyq 24 N/W scheduling (PERT/CPM)

a-c ⑮ A project schedule has the following characteristics:

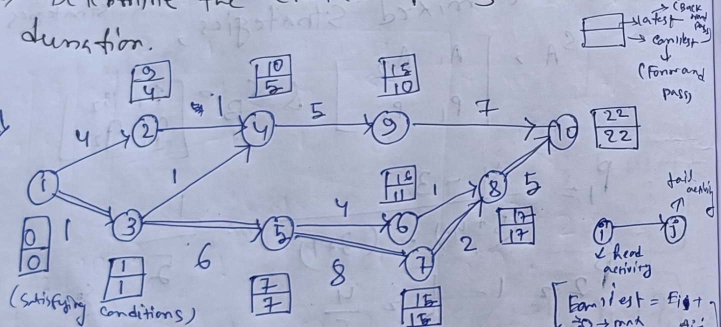
Activity	1-2	1-3	2-4	3-4	3-5	4-9	5-6	5-7	6-8	7-8	8-10	9-10
Time (days)	4	1	1	1	6	5	4	8	1	2	5	7

a) construct a n/w diagram.

b) compute the earliest event time & latest even time.

c) Determine the critical path & total project duration.

a) & b)



c) 1-3-5-7-8-10 → Critical path

Activity	time (t_{ij})	Earliest Start (ES) E_i	Earliest Finish (EF) $E_i + t_{ij}$	Latest Start (LS) $E_j - t_{ij}$	Latest Finish (LF) E_j	Total float TF = LS − ES
1-2	4	0	4	5	9	5
1-3	1	0	1	0	1	$\boxed{0}$
2-4	1	4	5	9	10	5
3-4	1	1	2	9	10	8
3-5	6	1	7	1	7	$\boxed{0}$
4-9	5	5	10	10	15	5
5-6	4	7	11	12	16	5
5-7	8	7	15	7	15	$\boxed{0}$
6-8	1	11	12	16	17	5
7-8	2	15	17	15	17	$\boxed{0}$
8-10	5	17	22	17	22	$\boxed{0}$
9-10	7	10	17	15	22	5

∴ Critical path = 1-3-5-7-8-10

∴ total project duration = 1 + 6 + 8 + 2 + 5 = 22 days

[Extra
Free float (FF) = $E_j - E_i - t_{ij}$
$= E_j - (E_i + t_{ij})$]

E_j (Earliest, but for tail node)	FF
4	0
1	0
5	0
5	3
7	0
10	0
11	0
15	0
17	0
17	0
22	0
22	5

(Not mentioned - But extra)

③ Simplex method (Not gonna attempt)

pyq 24 a-c ⑮ Solve the following linear programming problem using simplex method.

maximize $z = 2x + y$

subject to $x - y \leq 10$

$2x - y \leq 40$ $x, y \geq 0$

$\leq$ → slack (+)

$\geq$ → surplus (−)

max $z = 2x + y + 0s_1 + 0s_2$

$x - y + s_1 = 10$

$2x - y + s_2 = 40$

$x, y, s_1, s_2 \geq 0$

~~An initial~~ An initial basic feasible soln is given by,

$x = y = 0 \quad \therefore S_1 = 10,\ S_2 = 40$

writing in mat form. $AX = B$

$$\begin{array}{cccc} x & y & s_1 & s_2 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 40 \end{bmatrix}$$

An initial simplex table

incoming vector

		C_j	2	1	0	0	
C_B	B	x_B	x	y	s_1	s_2	min ratio $= x_B / \text{key row (x)}$
0	S_1	10	$\boxed{1}$	-1	1	0	$10/1 = 10$ (min)
0	S_2	40	2	-1	0	1	$40/2 = 20$
$Z_j - C_j$			$-2\uparrow$ (most neg)	-1	0	0	

key row

key ele

key column

out going vector

$$\boxed{Z_j - C_j = C_B x_j - C_j}$$

$Z_1 - C_1 = C_B \cdot x_1 - C_1$

$= (0 \cdot 1 + 0 \cdot 2) - 2$

$= -2$

min ratio $= x_B / \text{key row} = x_B / x$

1st it

incoming

		C_j	2	1	0	0	
C_B	B	x_B	x	y	s_1	s_2	min ratio $= x_B / y$
2	x	10 $10/1 = 10$	$1/1 = 1$	$\boxed{-1}$	1	0	-10
0	S_2	10	0	$\frac{1}{2}$	-1	1	20
$Z_j - C_j$			0	$-3\uparrow$	2	0	

key ele

out going

$x_B(S_2)$,

$$\frac{40 \times 1 - 10 \times 2}{2} = \frac{20}{2} = 10$$

y,

$$\frac{-1 + 2}{2} = \frac{1}{2}$$

S_1,

$$\frac{0 - 2}{2} = -1$$

$Z_1 - C_1 = C_B \cdot x_1 - C_1$
$= (2 + 0) - 2$
$= 0$

$Z_2 - C_2 = C_B \cdot x_2 - C_2$
$= (-2 + 0) - 1$
$= -3$

S_1,
$(2 + 0) - 0$
$= 2$

S_2,
$(0 + 0) - 0 = 0$

2nd it

		C_j	2	1	0	0
C_B	B	x_B	x	y	s_1	s_2
1	y	-10	-1	1	-1	0
0	S_2	7.5	$-\frac{1}{4}$	0	$\frac{1}{4}$	1
$Z_j - C_j$			-3	0	-1	0

s_2,

$$\frac{\frac{5}{2}+10}{2} = \frac{15}{2} = 7.5$$

x_1,

$$\frac{0-\frac{1}{2}}{2} = -\frac{1}{4}$$

s_1,

$$\frac{1-\frac{1}{2}}{2} = \frac{1}{4}$$

$z_1 - c_1 = C_B \cdot x_1 - c_1$
$= (-1+0) - 2$
$= -3$

$z_2 - c_2 = (1+0) - 1$
$= 0$

s_1, $(-1+0) - 0$
$= -1$

— — this is how continue

when $z_j - c_j \geq 0$ (for all), then, optimal.

~~Optimal~~.

After that. $\boxed{\max z = C_B \cdot x_B}$

final res.

(eg) $1 \times (-1) + 0.75$

= x ans