Operation Research

For all if we write like this i.e., explain before doing math, will get more marks

Note (1) Imp • Graphical method 27/2/23

Ex 3.1 By graphical method solve the following LLP.

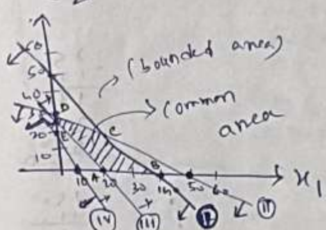
$\max z = 3x_1 + 4x_2$ (objective function)

Subject to
$5x_1 + 4x_2 \le 200$ — (i)
$3x_1 + 5x_2 \le 150$ — (ii)
$5x_1 + 4x_2 \ge 100$ — (iii)
$8x_1 + 4x_2 \ge 80$ — (iv)
$x_1, x_2 \ge 0$ — (v)

(decision variable)

$\boxed{\text{(i), (ii), (iii), (iv) — write the type of eq}^n \text{ in exam}}$

[$\le \rightarrow \swarrow$, $\ge \rightarrow \nearrow$]



Sol^n Replacing all the inequalities by equality.

[$\frac{x}{a} + \frac{y}{b} = 1$, (0,b), (a,0)]

from eq (i)

$5x_1 + 4x_2 \le 200$

$\Rightarrow \frac{x_1}{40} + \frac{x_2}{50} \le 1$ (40,0) & (0,50)

from (ii), $\frac{x_1}{50} + \frac{x_2}{30} \le 1$ (50,0) & (0,30)

from (iii), $\frac{x_1}{20} + \frac{x_2}{25} \ge 1$ (20,0) & (0,25)

from (iv) $\frac{x_1}{(\frac{9}{4})\ 10} + \frac{x_2}{20} \ge 1$ (10,0) & (0,20)

Plot each eq^n on the graph.

So, Feasible region is given by ABCDE

we know, $\boxed{A = (20,0)}$
$\boxed{B = (40,0)}$

for C,
we need to solve eqn ① & ⑪

$5x_1 + 4x_2 = 200$ — ① $\times 3$
$3x_1 + 5x_2 = 150$ — ⑪ $\times 5$

$15x_1 + 12x_2 = 600$
$\underline{-15x_1 - 25x_2 = -750}$

$\Rightarrow 13x_2 = 150$
$\Rightarrow x_2 = \frac{150}{13} = 11.54$

$5x_1 + 4 \times \frac{150}{13} = 200$
$\Rightarrow 5x_1 = 153.84$
$\Rightarrow x_1 = 30.77$

$\therefore \boxed{C = (30.77, 11.54)}$

~~for, D we need to solve ⑪ & ⑪⑪~~

~~$3x_1 + 5x_2 = 150$~~

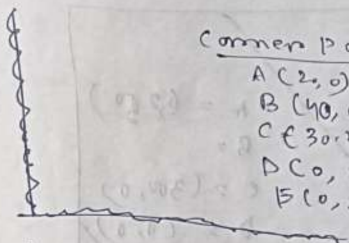
for D, $x_1 = 0$

putting in ⑪, ~~$3x_1 + 0 = 150$~~ ~~$\Rightarrow x_1 = 50$~~ $3 \times 0 + 5x_2 = 150$ $\Rightarrow x_2 = 30$

$\therefore \boxed{D = (0, 30)}$

for E, $x_1 = 0$
$\therefore$ from ⑪⑪,
$5 \times 0 + 4x_2 = 100$
$\Rightarrow x_2 = 25$

$\therefore \boxed{E = (0, 25)}$



Corner Points	val. of $z = 3x_1 + 4x_2$
A (20, 0)	$z = 60$
B (40, 0)	$z = 120$
C (30.77, 11.54)	$z = 138.47$ (max val. of z)
D (0, 30)	$z = 120$
E (0, 25)	$z = 100$

$\therefore$ the max val of z is obtained at point C(30.77, 11.54) & the val. is 138.47.

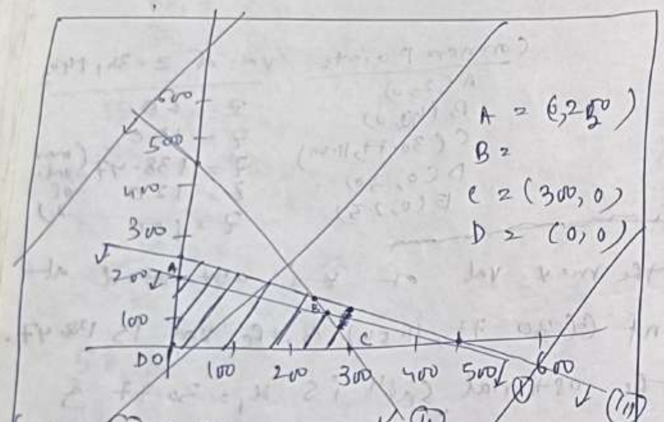
$\therefore$ the optimal unique soln is $x_1 = 30.77$ & $x_2 = 11.54$.

Case 1: A unique optimal solution. 4/3/25
Case 2: An infinite no. of optimal soln.

②
Eg 37
Solve the LPP by graphical method
maximize $z = 100x_1 + 40x_2$
subject to
$5x_1 + 2x_2 \le 1000$
$3x_1 + 2x_2 \le 900$
$x_1 + 2x_2 \le 500$
& $x_1, x_2 \ge 0$

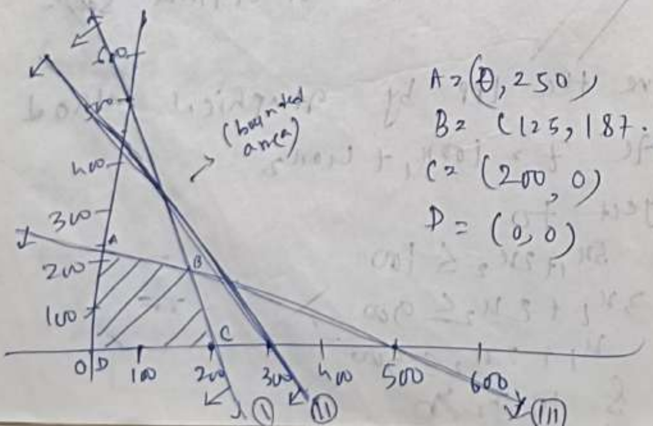
Solⁿ
Replace all the eqns of the constraints by eqn we get,

$5x_1 + 2x_2 = 1000$ — (I) → (200, 0) (0, 500)
$3x_1 + 2x_2 = 900$ — (II) → (300, 0) (0, 450)
$x_1 + 2x_2 = 500$ — (III) → (500, 0) (0, 250)



from (II) & (III) we get B,

$3x_1 + 2x_2 = 1000$



from (I) & (III) we get B,

$5x_1 + 2x_2 = 1000$
$x_1 + 2x_2 = 500$
(−) (−) (−)

$\Rightarrow 4x_1 = 500$
$\Rightarrow x_1 = 125$
$125 + 2x_2 = 500$
$\Rightarrow x_2 = 187.5$

Common point	value $Z = 100x_1 + 40x_2$
A (0, 250)	10000
B (125, 187.5)	20000 (max)
C (200, 0)	20000 (max)
D (0, 0)	0

∴ So the max val. of Z occurs at 2 vertices B & C. Since there are infinite no. of points on the line joining B & C, it gives the same max val. of Z. Thus there are infinite no. of optimal solⁿs for the LPP.

Case 3: Unbounded Solⁿ

Eg 38 Solve the following LPP.

max $Z = 3x_1 + 2x_2$
subject to
$x_1 - x_2 \geq 1$ $x_1, x_2 \geq 0$
$x_1 + x_2 \geq 3$

$x_1 - x_2 = 1$ — (i) (1,0) (0,-1)

$x_1 + x_2 = 3$ — (ii) (3,0) (0,3)

from (i) & (ii) we get,

$x_1 + x_2 = 3$

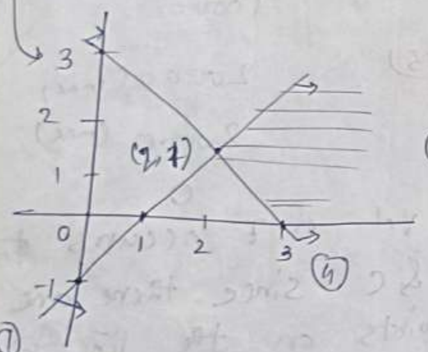
$x_1 - x_2 = 1$

$2x_1 = 4$

$\Rightarrow \boxed{x_1 = 2}$

$2 - x_2 = 1$

$\Rightarrow \boxed{x_2 = 1}$



(unbounded area)

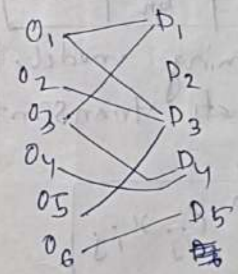
the solⁿ space is unbounded. Infact, the maximum value of z occurs at infinite. Hence, the problem has an unbounded ~~solqation~~ solution.

22/4/25

Transportation problem

The transportation problem is one of the subclasses of LPPs.

The objective is to transport various quantities of a single homogeneous commodity that are initially stored at various origins, to diff. destinations in such a way that the transportation cost is minimized. To achieve this we must know the amount & location of available supplies & the quantities demanded. Additionally we must know the cost of that results from transporting one unit of commodity from various origins to various destinations.



Mathematical formulation

Consider a transportation problem with m origin (rows) & n destination ~~to~~ (cols).

Let c_{ij} be the cost of transporting one unit of the product from the i-th origin to j-th destination. a_i be the quantity of commodity available at origin i. b_j be the quantity of commodity needed at destination j. x_{ij} is the quantity transported from i-th origin to jth destination.

The above transportation problem can be stated in the following tabular form —

	1	2	3	---	n	capacity/supply
1	x_{11} (quantity) c_{11}	x_{12} c_{12}	x_{13} c_{13}	—	x_{1n} c_{1n}	a_1
2	x_{21} c_{21}	x_{22} c_{22}	x_{23} c_{23}	—	x_{2n} c_{2n}	a_2
3	x_{31} c_{31}	x_{32} c_{32}	x_{33} c_{33}	—	x_{3n} c_{3n}	a_3
---	---	---	---	---	---	---
m	x_{m1} c_{m1}	x_{m2} c_{m2}	x_{m3} c_{m3}	—	x_{mn} c_{mn}	a_m
Demand	b_1	b_2	b_3	~~b_3~~ ---	b_n	

The linear programming model representing the ~~transformati~~ transportation problem is given by,

$$\boxed{\text{minimize } \underset{(\text{cost})}{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}}$$

subject to constraints —

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, 3 \ldots m \quad (\text{row sum})$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, 3 \ldots n \quad (\text{col sum})$$

$x_{ij} \geq 0$, [all the x_{ij} should be positive value for all i & j]

~~give~~ Given transportation problem is said to be $\boxed{\text{balanced if } \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$.

i.e, If the total supply = the total demand, then it is said to be balanced.

- **Feasible Solution**

Any set of ~~non~~ non-negative allocations ($x_{ij} > 0$) which satisfies the row & col sum (rim requirement) is called feasible solution.

- **Basic feasible solution —**

A feasible solⁿ is called basic feasible solⁿ if ~~its~~ the no. of non-negative allocations equals to $\boxed{m+n-1}$, where m is the no. of rows & n is the no. of cols in a transportation table.

- **Optimal Solution —**

Optimal solⁿ is a feasible solⁿ (not necessarily basic), which minimizes the total cost.

Transportation Problem

Unbalanced transportation Problem

Demand > Supply

$\Sigma b_j > \Sigma a_i$

Add a dummy row with Cost zero &

$\boxed{\text{supply equals to (demand} - \text{supply)}}$

Demand < Supply

$\Sigma b_j < \Sigma a_i$

add a dummy col with cost zero &

$\boxed{\text{demand} = (\text{supply} - \text{demand})}$

Balanced transportation Problem

$\boxed{\text{Demand} = \text{Supply}}$

$\Sigma a_i = \Sigma b_j$

(row sum)

The calculation of a transportation problem can be obtained in 2 stages, namely, initial & optimum solⁿ.

Initial solⁿ can be obtained using any of the 3 methods —

i) North-west - corner rule (NWCR)

ii) Least cost method or matrix minima method

iii) Vogel's approximation method (VAM) [(algo + eg) (H/W)]

Eg-① ✓ pyq

Obtain the initial basic solⁿ of a transportation problem where cost & min requirement table is given below —

Origin \ Destination	D_1	D_2	D_3	Supply
O_1	2	7	4	5 ← a_1
O_2	3	3	1	8 ← a_2
O_3	5	4	7	7 ← a_3
O_4	1	6	2	14 ← a_4
Demand	7	9	18	34
	b_1	b_2	b_3	balanced

→ (see to understand if there exists a feasible solⁿ)

• NWCR

Since $\sum_{i=1}^{4} a_i = 34 = \sum_{j=1}^{3} b_j$, there exists a feasible solⁿ to the transportation problem. (the T-P is balanced)

[if solⁿ is non-degenerate then it's also basic feasible]

We obtain the initial feasible solⁿs as follows:

The given transportation problem is stated in the following table.

(Start solving from N-W)

Origin \ Destination	D_1	D_2	D_3	Supply
O_1	[2] 5	[7]	[4]	~~5~~ 0
O_2	[3] 2	[3] 6	[1]	~~8~~ ~~6~~ 0
O_3	[5]	[4] 3	[7] 4	~~7~~ ~~4~~ 0
O_4	[1]	[6]	[2] 14	~~14~~ 0
Demand	~~7~~ ~~2~~ 0	~~9~~ ~~3~~ 0	~~18~~ ~~14~~ 0	

Hence we get the initial basic feasible solution to the given transportation problem which is given by,

$x_{11} = 5$, $x_{21} = 2$, $x_{22} = 6$, $x_{32} = 3$, $x_{33} = 4$, $x_{43} = 14$

∴ Total cost —
$= (5\times2) + (2\times3) + (3\times6) + (4\times3) + (7\times4) + (2\times14)$
$= 10 + 6 + 18 + 12 + 28 + 28$
$= 102$

Allocated cells = 6

$m+n-1 = 4+3-1 = 6$ & also no close paths i.e. cells are independent

∴ the solⁿ is $\boxed{\text{non-degenerate}}$

25/4/25

Chr √ policy

2nd • Least cost or matrix minima method

Algo –

Eg – Obtain an intial feasible Solⁿ to the following transprotation problem using the least cost/matrix minima method

Destination / Origin	D_1	D_2	D_3	D_4	Supply	
O_1	1	2	3	4	6	a_1
O_2	4	3	2	0	8	a_2
O_3	0	2	2	1	10	a_3
Demand	4	6	8	6	24 (balanced)	
	b_1	b_2	b_3	b_4		

Σa_i: 6, 8, 10 = 24

Σb_i: 4, 6, 8, 6 = 24

Solⁿ Since $\Sigma a_i = \Sigma b_j = 24$, there exists a feasible solⁿ to the transportation problem

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	1	2 (6)	3	4	~~6~~ 0
O_2	4	3	2 (2)	0 (6)	~~8~~ ~~2~~ 0
O_3	0 (4) min	2	2 (6)	1	~~10~~ ~~6~~ 0
Demand	~~4~~ 0	~~6~~ 0	~~8~~ ~~6~~ 0	~~6~~ 0	

(Start solving from least cost 0 (any))

In exam use pencil to ☐ to like this

1st Steps

Step 1: – [not this]

The first allocation is made in the cell (3,1), the magnitude is being $x_{31} = 4$. It satisfies the demand at the destination D_1 & we delete this column from the table as it is exhausted.

Step 2: –

The 2nd allocation is made in the cell (2,4) with magnitude $x_{24} = \min(6, 8) = 6$. Since it satisfies the demand at the dest. it's deleted from the table.

Step 3: –

The 3rd allocation is made in the cell (1,2), the magnitude being $x_{12} = \min(6, 6) = 6$, since it satisfies both the supply at the origin O_1, ~~D_2, the corresponding row~~ & the demand at the destination D_2, the corresponding row & column are deleted.

Step 4: – The fourth allocation is made in the cell (2,3), the magnitude being $x_{23} = \min(2, 8) = 2$

Step 5: – Finally the fifth allocation is made in the cell (3,3). The magnitude being $x_{33} = \min(6, 6) = 6$

Step 6: – Now all the rim requirement

have been satisfied & hence initial feasible sol^n is being obtained.

Here no. of allocations = 5, but $m+n-1 = 6$; it's basic sol^n also it's non-degenerate

The sol^n is given by

$x_{12} = 6,\ x_{23} = 3,\ x_{24} = 5,\ x_{31} = 4$

$x_{33} = 6$

Total cost $= (6 \times 2) + (2 \times 2) + (6 \times 0)$

to get it: moor method (min)

$+ (4 \times 0) + (6 \times 2)$

$= 12 + 4 + 0 + 0 + 12$

$= 28$

~~here m+n-1 = 6~~

• **Vogel's approximation method** (optimal)

Eg - find the initial basic feasible sol^n for the following transportation.

Sol^n Since $\Sigma a = \Sigma b = 950$ the given problem is balanced & there exists a feasible sol^n to the problem

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	11	13	17	14	250
O_2	16	18	14	10	300
O_3	21	24	13	10	400
Demand	200	225	275	250	950

→ Balanced

250 + 300 + 400 = 950
200 + 225 + 275 + 250 = 950
S = 950, D = 950

Sol^n

O \ D	D_1	D_2	D_3	D_4	Supply	P
O_1	11	13	17	14	250	2
O_2	16	18	14	10	300	4
O_3	21	24	13	10	400	3
Demand	200	225	275	250		
P	5	5	1	0		

The problem is balanced T.P. as the total supply = total demand. We find the initial basic feasible sol^n as given in the following table—

(We prefer this method the most, as the result is either optimal sol^n or nearest to the optimal sol^n.)

	D_1 (min)	D_2 (2nd min)	D_3	D_4	Supply	P_1	P_2	P_3	P_4	P_5
O_1	11 (200)	13 (50)	17	14	~~250~~ ~~50~~ 0	2	1	–	–	–
O_2	16	18 (175)	14	10 (125)	~~300~~ ~~125~~ 0	4	4	4	4	–
O_3	21	24	13 (275)	10 (125)	~~400~~ ~~125~~ 0	3	3	3	3	3
Demand	~~200~~ 0	~~225~~ ~~175~~ 0	~~275~~ 0	~~250~~ ~~125~~ 0						
P_1	5↑ (max P in P_1 value)	5	1	0						
P_2	–	5↑	1	0						
P_3	–	6↑	1	0						
P_4	–	–	1	0						

row wise penalty; col wise penalty

col delete → row change, col same
row delete → col change, row same

	D_1	D_2	D_3	D_4	Supply
O_1	11 (200)	13 (50)	17	14	250 / 50 / 0
O_2	16	18 (175)	14	10 (125)	300 / 125 / 0
O_3	21	24	13 (275)	16 (125)	400 / 125 / 0
Demand	200 / 0	225 / 50 / 0	275 / 0	250 / 125 / 0	500

	P_1	P_2	P_3	P_4	P_5
	2	1	1	1	1
	4	4	5	5.m	1
	3	3	3		3

	P_1	P_2	P_3	P_4	P_5
	5↑	—	—	—	—
	5	5↑	6↑	—	—
	1	1	1	1	1
	0	0	0	0	13↑ 10

P_5 — 13↑ 10

fresh

No. of allocated & cell $= m+n-1 = 6$

There are 6 positive independent allocations (forming no close path)

This ensures that the Sol^n is non-degenerate. So, basic feasible Sol^n.

The transportation cost (min) = [for this MODI method]

$= 11 \times 200 + 13 \times 50 + 18 \times 175 + 10 \times 125 + 13 \times 275 + 10 \times 125$

$= 12075$ (this is min cost)

(if we want vals, then)

the Sol^n is given by –

$x_{11} = 200$; $x_{12} = 50$; $x_{22} = 175$; $x_{24} = 125$;

$x_{33} = 275$; $x_{34} = 125$ [Here $m+n-1 > 6$ = no. of allocated cells → nondegenerate problem]

~~Step 1:~~ Steps [not this] The Vogel's approximate method takes into account not only the least cost c_{ij} but also the costs that just exceed c_{ij}.] Various steps of the method are given below –

Step 1:– For each row of the transportation table identify the smallest & the next to smallest cost. Determine the diff. b/w them for each row. These are called Penalties (opportunity cost). Put them along side of the transportation table by enclosing them in parenthesis against the respective rows. similarly compute these penalties for each col.

Step II – Identify the row/col with the largest penalty among all the rows & cols. If a tie occurs use any arbitrary tie breaking choice. Let the greatest penalty correspond to ith row & let C_{ij} be the smallest cost in the ith row. Allocate the largest possible amount $x_{ij} = \min(a_i, b_j)$ in the (i, j) of the cell & cross off the ith row or jth col in the usual manner.

Step III – Recompute the col & row penalty for the reduced transportation table & go to step II. Repeat the procedure until all the requirements are satisfied.

29/4/25

Internal assignment handwritten, need to submit sir & upload too

(S. Kalavant) Book eg 8.12 (pg-129)

(Book egs) eg 8.1 (NWCR, least cost) (Do the unbalanced problem) 8.3, 8.7 ...

Imp

Eg-8.12 Solve the transportation problem when the unit transportation costs, demands & supplies are given below

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	70
Demand	85	35	50	45	

Since the total demand $\Sigma b_j = 215$ is greater than the total supply $\Sigma a_i = 195$, the problem is an unbalanced T.P.

We convert this into a balanced T.P by introducing a dummy origin O_4 with cost 0 & giving supply = 215 − 195 = 20 units. (demand − supply)

O \ D	D_1	D_2	D_3	D_4	S
O_1	6	1	9	3	70
O_2	11	5	2	8	55
O_3	10	12	4	7	70
O_4	0	0	0	0	20
D	85	35	50	45	

Applying VAM –

P.T.O

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
	2	2	2	–	–	–	–
	3	3	3	3	6	–	–
	3	3	3	3	3	3	4
	0	–	–	–	–	–	–

P_1	6↑	1	2	3
P_2	4↑	4	2	4
P_3	–	4↑	2	4
P_4	–	7↑	2	1
P_5	–	–	2	1
P_6	–	–	4	7↑
P_7	–	–	↑4	–

The soln is given by,

$x_{11} = 65$, $x_{12} = 35$, $x_{22} = 30$, $x_{23} = 25$, $x_{33} = 25$, $x_{34} = 45$,

$x_{41} = 20$

the T.P. cost (min) $= 65 \times 6 + 35 \times 1 + 30 \times 5 + 25 \times 2 + 25 \times 4 + 45 \times 7 + 20 \times 0$

$= 1010$

$m+n-1 = 8-1 = 7$ = no. of allocated cells & the allocation is independent

∴ the soln is non-degenerate.

MODI

• Modi method (short note) → To find the optimal soln we need to apply this (the upper soln is basic feasible soln)

For eg (Pg 117)

Eg—

O \ D	D_1	D_2	D_3	D_4	Supply
O_1	$\boxed{2}$ (3)	$\boxed{2}$ Δ=1	$\boxed{2}$ Δ=1	$\boxed{1}$ Δ=1	3
O_2	$\boxed{10}$ Δ=4	$\boxed{8}$ Δ=3	$\boxed{5}$ (3)	$\boxed{4}$ (4)	7
O_3	$\boxed{7}$ (1)	$\boxed{6}$ (3)	$\boxed{6}$ (1)	$\boxed{8}$ Δ=3	5
Demand	4	3	4	4	15 → Balanced

$v_1 = 7$, $v_2 = 6$, $v_3 = 5$, $v_4 = 5$

$u_1 = -5$ (after solving through VAM method)

$u_2 = -1$

$u_3 = 0$

$m+n-1 = 6$ = no. of allocated cells. → non-degenerate soln. & also the allocation is independent

T.P. cost (min) $= 2 \times 3 + 5 \times 3 + 4 \times 4 + 7 \times 1 + 6 \times 3 + 6 \times 1$ $= 68$ (basic feasible soln)

(to get this we are calculating MODI method)

To find the optimal soln.- Applying MODI method

We determine a set of no. u_i & v_j for each row & col, such that $u_i + v_j = C_{ij}$ - For each occupied cell.

u_3 (3 allocations)

$C_{31} = u_3 + v_1$ ~~= 7 +~~ ; $\Rightarrow 7 = 0 + v_1$ $\Rightarrow v_1 = 7$

$u_{32} = u_3 + v_2$; $\Rightarrow 6 = 0 + v_2$; $\Rightarrow v_2 = 6$

$u_{33} = u_3 + v_3$; $\Rightarrow 6 = 0 + v_3$; $\Rightarrow v_3 = 6$

} any soln for occupied cells.

v_1 (2 allocations)

$C_{11} = u_1 + v_1$; $\Rightarrow 2 = u_1 + 7$; $\Rightarrow u_1 = -5$

C_{31} = done

v_2 (1 allocation)

C_{32} = done

v_3 (2 allocations)

$C_{23} = u_2 + v_3$; $\Rightarrow 5 = u_2 + 6$; $\Rightarrow u_2 = -1$

v_4 (1 allocation)

$C_{24} = u_2 + v_4$; $\Rightarrow 4 = -1 + v_4$; $\Rightarrow v_4 = 5$

∵ we solved the problem by VAM, we got optimal soln at very 1st if we didn't i.e soln by NWCR or least cost then we need to do next steps

We find the sum of u_i & v_j for each unoccupied cell & then find $\Delta_{ij} = C_{ij} - (u_i + v_j)$

$\Delta_{12} = C_{12} - (u_1 + v_2) = 2 - (-5 + 6) = 1$

$\Delta_{13} = C_{13} - (u_1 + v_3) = 2 - (-5 + 6) = 1$

$\Delta_{14} = C_{14} - (u_1 + v_4) = 1 - (-5 + 5) = 1$

$\Delta_{21} = C_{21} - (u_2 + v_1) = 10 - (-1 + 7) = 4$

$\Delta_{22} = C_{22} - (u_2 + v_2) = 8 - (-1 + 6) = 3$

$\Delta_{24} = C_{24} - (u_2 + v_4) = 8 - (-1 + 5) = 4$

∴ All the $\Delta_{ij} > 0$, the soln is already optimum & unique. [The sol. is given by, (= basic optimum feasible soln)

$x_{14} = 3$, $x_{23} = 3$, $x_{24} = 4$, $x_{31} = 1$, $x_{32} = 3$, $x_{33} = 1$]||

∴ T.P (B+ (min) = 68

Steps - 3rd

[not this]

Step 1 - Find the initial basic feasible soln of a T.P by using any one of the 3 methods.

Step 2 - Find out a set of no.s u_i & v_j for each row & col satisfying $u_i + v_j = C_{ij}$ for each occupied cell. To start with, we assign a no. '0' to any row or col having max no of allocations. If this max no. of allocations is more than 1, choose any 1 arbitrarily.

Step 3 - For each empty (unoccupied) cells we find the sum u_i & v_j written in the bottom left corner of that cell.

Step 4 - Find out the net evaluation value $\Delta_{ij} = C_{ij} - (u_i + v_j)$ for each empty cell, which is written at the bottom right corner of that cell. This step gives the opportunity optimality conclusion,

i) If all $\Delta_{ij} > 0$ (i.e, all the net evaluation values), the soln is optimum & an unique soln exists.

ii) If $\Delta_{ij} \geq 0$, then the solns is optimal, but an alternate soln exists.

iii) If at least one $\Delta_{ij} < 0$, the soln is not optimum. In this case, we go to the next step to improve the total T.P cost.

⊛ Eg of this PTO →

Step 5 - select the empty cell having the most negative value of Δ_{ij}. From this cell we draw a closed path by drawing horizontal & vertical lines with the corner cells occupied. Assign sign + & - alternately & find the min. allocation from the cell having neg. sign. This allocation should be added to the allocation having pos. sign & subtracted from the allocation having neg. sign.

Step 6 - The above step yields (gives) a better soln by making one (or more) occupied cell as empty & one empty cell as occupied. For this new set of basic feasible allocations, repeat from step(2) onwards, till an optimum basic feasible soln is obtained.

Eg

O \ D	D_1	D_2	D_3	D_4	S
O_1	200 [3]	50 [1]	[7]	[4]	230
O_2	[2]	250 [6]	100 [5]	[9]	350
O_3	[8]	[3]	250 [3]	150 [2]	400
D	200	300	350	150	1000

balanced

$\therefore$ m+n-1 = 6 = no. of allocated cells &

also the allocation is independent $\Rightarrow$ Non-degenerate soln (basic feasible)

$\therefore$ T.P cost (from the that method) $= 200 \times 3 + 1 \times 50 + 6 \times 250 + 5 \times 100 + 3 \times 250 + 2 \times 150 = 3700$

To find optimal Soln

We apply MODI method & find out a set of nos. u_i & v_j for which $u_i + v_j = c_{ij}$ (only for occupied cell)

O \ D	D_1	D_2	D_3	D_4	S	
O_1	−x 200 [3]	+x 50 [1]	[7]	[4]	250	$u_1 = 0$
O_2	+x [2]	250 [6]	−x 100 [5]	[9]	350	$u_2 = 5$
O_3	[8]	[3]	250 [3]	150 [2]	400	$u_3 = 3$
D	200	300	350	150	1000	
	$v_1 = 3$	$v_2 = 1$	$v_3 = 0$	$v_4 = -1$		

Got value, solve allocation for which (v_1, v_2, u_1, u_2 —) you got the value

$C_{11} = u_1 + v_1 \Rightarrow 3 = 0 + v_1$; $v_1 = 3$

$C_{12} = u_1 + v_2 \Rightarrow 0 + v_2 = 1$; $v_2 = 1$

$C_{22} = u_2 + v_2 \Rightarrow 6 = u_2 + 1$; $\Rightarrow u_2 = 5$

$C_{23} = u_2 + v_3 \Rightarrow 5 + v_3 = 5$; $\Rightarrow v_3 = 0$

$C_{33} = u_3 + v_3 \Rightarrow u_3 + 0 = 3$; $\Rightarrow u_3 = 3$

$C_{34} = u_3 + v_4 \Rightarrow 3 + v_4 = 2$; $\Rightarrow v_4 = -1$

for finding Δ_{ij}, we have $\Delta_{ij} = C_{ij} - (u_i + v_j)$ (for unoccupied cell)

$\Delta_{13} = C_{13} - (u_1 + v_3) = 7 - (0 + 0) = 7$

$\Delta_{14} = C_{14} - (u_1 + v_4) = 4 - (0 + -1) = 5$

$\Delta_{21} = C_{21} - (u_2 + v_1) = 2 - (5 + 3) = -6 < 0$

$\Delta_{24} = C_{24} - (u_2 + v_4) = 9 - (5 + (-1)) = 5$

$\Delta_{31} = 2$, $\Delta_{32} = -1 \Rightarrow < 0$

$\therefore$ Our soln is not optimal (as atleast $\Delta_{ij} < 0$)
[3700 is not an optimum soln]

the jth job)

Assignment Problem

(13.1.18)

Most neg. val b/w -6 & -1 = -6, ∴ we are choosing Δ_{21}

stepper motor is classified into GND

$x = \min(200, 250)$ [we ll check the one with neg x (i.e $-x$)]
$= 200$

[And also in that stucone path one $+x$ has val. but other $+x$ has no allocation, so we can't check it]

The modified sol^n is given in the following table —

	D_1	D_2	D_3	D_4	
O_1	[3]	[1] (250)	[7]	[4]	$u_1 = -5$
O_2	[2] (200)	[6] (50) $-x$	[5] (100) $+x$	[9]	$u_2 = 0$
O_3	[8]	[3] $+x$	[3] (250) $-x$	[2] (150)	$u_3 = -2$

allocating 200

250-200

[0 = no allocations]

$\begin{pmatrix} 2-0 & 1-6 & 3-5=-2; & 2-(-2) \\ 6-0, & 6-0 & , & \\ 5-0, & & & \end{pmatrix}$ Calculated $u_1, u_3, v_1, v_2, v_3, v_4$ (short cut)

$v_1 = 2 \quad v_2 = 6 \quad v_3 = 5 \quad v_4 = 4$

$\Delta_{11} = 6$; $\Delta_{13} = 7$; $\Delta_{14} = 5$; $\Delta_{24} = 5$; $\Delta_{31} = 8$; $\Delta_{32} = -1 < 0$ (again the sol^n is not optimal)

Most neg val = -1; ∴ choosing Δ_{32}

$x = \min(50, 250) = 50$

∴ The modified sol^n is given in the following table —

	D_1	D_2	D_3	D_4	
O_1	[3]	[1] (250)	[7]	[4]	$u_1 = -2$
O_2	[2] (200)	[6]	[5] (150)	[9]	$u_2 = 2$
O_3	[8]	[3] (50)	[3] (200)	[2] (150)	$u_3 = 0$
	$v_1 = 0$	$v_2 = 3$	$v_3 = 3$	$v_4 = 2$	

∴ T.P cost (min)
$= 1 \times 250 + 2 \times 200 + 5 \times 150 + 3 \times 50 + 3 \times 200 + 2 \times 150$
$= 2450$

$\Delta_{11} = 5$; $\Delta_{13} = 6$; $\Delta_{14} = 4$; $\Delta_{22} = 1$; $\Delta_{24} = 5$; $\Delta_{31} = 8$;

∴ all $\Delta_{ij} > 0$ ∴ the sol^n is optimum & unique.

Assignment Problem (Pg-148)

Suppose there are n jobs to be performed & n persons are available for doing these jobs. Assume that each person can do each job at a time, (though with varying degrees of efficiency). Let c_{ij} be the cost if the ith person is assigned to the jth job. The problem is to find an assignment (which job should be assigned to which person, on a one-to-one basis) so that the total costs of performing all the jobs is min.

It can be stated in the form of $n \times n$ cost matrix $[c_{ij}]$ of real no-s as given below -

(*)

Mathematical formulation (eg-(Pg-149))

This can be stated as,

$$\boxed{\text{minimize } Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} \cdot x_{ij}}$$

, where, $i = 1, 2, \ldots, n$ & $j = 1, 2, \ldots, n$.

Subject to the restrictions / constraints

$$x_{ij} = \begin{cases} 1, & \text{if the ith person is assigned jth job} \\ 0, & \text{if not} \end{cases}$$

$\sum_{j=1}^{n} x_{ij} = 1$ (one job is done by the ith person)

& $\sum_{j=1}^{n} x_{ij} = 1$ (only one person should be assigned the jth job) where, x_{ij} denotes that the jth job is to be assigned to the ith person.

$$
\begin{array}{c|c}
 & \text{Jobs} \\
 & \begin{array}{ccccccc} 1 & 2 & 3 & \cdots & j & \cdots & n \end{array} \\
\text{Persons} \begin{array}{c} 1 \\ 2 \\ 3 \\ \vdots \\ i \\ \vdots \\ n \end{array} &
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & \cdots & c_{1j} & \cdots & c_{1n} \\
c_{21} & c_{22} & c_{23} & \cdots & c_{2j} & \cdots & c_{2n} \\
c_{31} & c_{32} & c_{33} & \cdots & c_{3j} & \cdots & c_{3n} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
c_{i1} & c_{i2} & c_{i3} & \cdots & c_{ij} & \cdots & c_{in} \\
\vdots & \vdots & \vdots & & \vdots & & \vdots \\
c_{n1} & c_{n2} & c_{n3} & \cdots & c_{nj} & \cdots & c_{nn}
\end{bmatrix}
\end{array}
$$

-

Transportation Problem	Assignment Problem
i) Supply at any source may be any positive quantity a_i.	i) Supply at any source (machine) will be 1 (i.e) $a_i = 1$ (1 task → 1 person/machine/job)
ii) Demand at any destination may be any positive quantity b_j.	ii) Demand at any destination (job) will be 1 (i.e) $b_j = 1$
iii) One or more source to any no. of destination.	iv) One source to only one destination (job). (1 job → 1 person)

- ~~Ungarian~~ Hungarian method $\boxed{\text{not this}}$

Soln of an assignment problem can be arrived at, by using the Hungarian method. The steps involved in this method are as follows.

Step 1 – Prepare a cost matrix. If the cost matrix is not a square matrix then add a dummy row (or col) with zero cost element.

Step 2 – Subtract the min element in each row from all the eles of the respective rows.

Step 3 – Further modify the resulting matrix by subtracting the min ele of each col from all the eles of the respective cols. Thus, obtain the modified matrix.

Step 4 – Then, draw the min no. of horizontal & vertical lines to cover all zeros in the resulting matrix. Let the min no. of lines be N. Now there are 2 possible cases.

i) If $N = n$, where n is the order of the matrix, then an optimal assignment can be made. So make the assignment to get the required soln.

ii) If $N < n$, then proceed to step 5.

Step 5 – Determine the smallest uncovered element (element not covered by N lines).

Subtract this min ele. from all uncovered elems & add the same ele. at the intersection of horizontal & vertical lines. Thus, the 2nd modified matrix is obtained.

Step 6 – Repeat Steps 3 & 4 until we get the case (i) of Step 4.

Step 7 – (To make zero assignment) Examine the rows successively until a row-wise ~~exactly~~ exactly single zero is found. Circle (0) this zero to make the assignment. Then mark a cross (x) over all zeros if lying in the col of the circled zero, showing that they can't be considered for future assignment. Continue in this manner until all the zeros have been examined. Repeat the same procedure for ~~cols~~ col. also.

Step 8 – Repeat Step 6 successively until one of the following situation arises—

(i) If no unmarked zero is left, then the process ends or

(ii) If there lies more ~~than~~ than one unmarked zero ~~is left, then the processe~~ in any col / row, circle one of the unmarked zeros arbitrarily & mark a cross in the cells of remaining zeros in its row / col. Repeat the process until no unmarked zero is left in the matrix.

Step 9 – Thus, exactly one marked circled zero in each row & each col of the matrix is obtained. The assignment corresponding to these marked circled zeros will give optimal assignment.

~~Assignment~~ Hungarian method Problem eg (Assignment Problem) =

eg 1·13
eg 2·2

Eg = Using the following cost matrix, determine (a) optimal job assignment (b) the cost of assignments.

(while solving transpose it)

$$\text{mechanic (i)}\quad \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array}\begin{array}{c} \text{Jobs (j)} \\ \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 10 & 3 & 3 & 2 & 8 \\ 9 & 7 & 8 & 2 & 7 \\ 7 & 5 & 6 & 2 & 4 \\ 3 & 5 & 8 & 2 & 4 \\ 9 & 10 & 9 & 6 & 10 \end{bmatrix}\end{array}$$

n = 5 ~~jobs~~ / machine

Jobs (j)	A	B	C	D	E (i)
1	8	1	1	0	6
2	7	5	6	0	5
3	5	3	4	0	2
4	1	3	6	0	2
5	3	4	3	0	4

(Row reduction)

Jobs	A	B	C	D	E	
1	7	$\boxed{0}$	0	0	4	(s)
2	6	4	5	$\boxed{0}$	3	
3	4	2	3	0	$\boxed{0}$	(s)
4	$\boxed{0}$	2	5	0	0	(s)
5	2	2	2	0	2	

n = 5, N = 4 (No. of 0's that are marked & so need (more 0 to be marked)

∴ Here, (b) N < n, 4 < 5

Jobs	A	B	C	D	E	
1	7	0	0	0	4	(s)
2	6	4	5	$\boxed{0}$	3	
3	6	4	5	2	$\boxed{0}$	
4	$\boxed{0}$	2	5	0	0	
5	0	0	0	0	0	(s)
		(s)	(s)			

n = 2

Jobs	A	B	C	D	E
1	10	7	7	3	7
2	3	7	5	5	10
3	3	8	6	8	2
4	2	2	2	2	2
5	8	7	9	4	10

~~(Row~~ ~~reduction)~~

Jobs	A	B	C	D	E
1	7	6	4	0	6
2	0	4	2	2	7
3	0	5	3	5	6
4	0	0	0	0	9
5	4	3	0	0	6

(Row reduction)

(Here all cols don't have min val as '0', so we need col. reduction)

Jobs	A	B	C	D	E
1	7	6	4	$\boxed{0}$	2
2	$\boxed{0}$	4	2	2	3
3	0	5	3	5	2
4	0	$\boxed{0}$	0	0	0 (3)
5	4	3	$\boxed{0}$	0	2

(col reduction)

N = 4, n = 5, (no. of marked 0 = 4, that means i.e. 1 more 0 is needed to be marked)

Here N < n, 4 < 5

k = 2

modified mat.-

Jobs	A	B	C	D	E
1	7	4	4	$\boxed{0}$	0 (3)
2	$\boxed{0}$	2	2	2	1
3	0	3	5	7	$\boxed{0}$ (5)
4	0	$\boxed{0}$	0	0	0 (5)
5	4	1	$\boxed{0}$	0	0 (1)

(if there is 1 '0' in row ~~that~~ (for intersection, that will be marked if it wasn't cut, then consider it marked & leave the space, keep ~~make~~ it same)

N = 5, n = 5, also 5 ~~no~~ 0s have been marked.

$\therefore$ N = n

$\therefore$ we can form an assignment.

Jobs	machine
1	D
2	A
3	E
4	B
5	C

$\therefore$ min cost = 3 + 3 + 9 + 2 + 4
= 21

3rd ~~A~~ ~~AQ23~~

Eg- A Company has 5 jobs to be done on five machines. Any job can be done on any machine. The cost of doing the jobs on diff. machines are given below. Assign the jobs for diff. machines so as to minimize the total cost. (Eg 7.13) (Pg - 151)

Jobs	A	B	C	D	E
1	13	8	16	18	19
2	9	15	24	9	12
3	12	9	4	4	4
4	6	12	10	8	13
5	15	17	18	12	20

Sol^n

$n \times n = 5 \times 5$ $\therefore$ balanced

(1) subtract the min ele from all the elems in respective row, we get row reduce

matrix as-

jobs	A	B	C	D	E
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

(rowwise reduction)

(2) Subtract the min ele from all the ele in respective cols, we get cols reduction matrix as-

jobs	A	B	C	D	E
1	5	0	8	10	11
2	0	6	15	0	3
3	8	5	0	0	0
4	0	6	4	2	7
5	3	5	6	0	8

(moved)

(will perform col. reduction from the prev matrix (row reduced) not the main matrix)

(∵ we have 0 in all cols, some don't need this col red.)

(Every row will have a '0' for sure, so the res. of col. reduction will be same as row reduction) [Not necessarily for other qns]

(3) Draw min no. of horizontal & vertical lines (N) to cover all zero:

a) If $N = n$, n = order of mat., then an optimal soln can be made.

b) If $N < n$, then go to next step

Jobs	A	B	C	D	E	
1	5	$\boxed{0}$	8	10	11	
2	0	6	15	0	3	(skip)
3	8	5	$\boxed{0}$	0	0	(skip)
4	$\boxed{0}$	6	4	2	7	
5	3	5	6	$\boxed{0}$	8	

Row scan = Col delete
Col scan = Row delete

intersection val.

not covered 2 col scan

(Row scan, if all 0's are covered then ok. If not, then col. scan)

If Row → 1 zero ⇒ mark it

If Row → more zero ⇒ skip (more than 1 zero)

(Row scan → Col delete
Col scan → Row delete)

$N = 4$, $n = 5$

Here, (b) $N < n$; $4 < 5$

(If $n = 5$, the there should be 5 zeros, that will be marked, here we have 4 0s, now we need 1 more 0 to be marked, so we need to proceed to the next step.)

(4) Determine the smallest uncovered elem (x).

a) write uncovered value = uncovered val − x

b) Intersection val. = intersection val + x

c) line val. (other val) as same

$x = 3$

modified matrix is as

	A	B	C	D	E
1	5	0	5	10	8
2	0	6	12	0	0
3	11	8	0	3	0
4	0	6	1	2	4
5	3	5	3	0	5

(actual logic just add with the intersection val, i.e. ...)

(just filling other vals, not marked 0s)

⑤ ~~go to~~ Go to step 3.

Jobs \ machine	A	B	C	D	E
1	5	$\boxed{0}$	5	10	8
2	0	6	12	0	~~0~~ (s)
3	11	8	$\boxed{0}$	3	0 (s)
4	$\boxed{0}$	6	1	2	4
5	3	5	3	$\boxed{0}$	5

(Here getting which job is assigned to which machine directly)

N = 5 n = 5 (& also we have marked 5 0_s)

∴ N = n means

∴ We can form an assignment.

Jobs	machine
1	B
2	E
3	C
4	A
5	D

min total cost = 8 + 12 + 4 + 6 + 12

val. at 1-B from main matrix

> 42

②

6/5/25

In balanced assignment Problem, given cost matrix is a square matrix. for eg, $\boxed{\text{no. of Jobs = no. of machines}}$

• Unbalanced assignment Problem ✓ MP 23

Any assignment Problem is said to be unbalanced, if the cost matrix is not a square matrix, i.e. the no. of rows & cols are not equal ~~to the no. of~~ to make it balance, a dummy row/col with all the entries as zero is added.

J \ m	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

row = 4
col = 5
As row < col, so unbalanced assignment Problem

⇓

J \ m	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
5	0	0	0	0	0

→ Dummy row

After adding a dummy row with all entries as '0', the given assignment problem become balanced.

Eg-(5x6)

There are four jobs to be assigned to five machines. Only one job can be assigned to one machine. The amount of time in hours required for the jobs per machine are given the following matrix—

Jobs	Machines A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6

Find an optimum assignment of jobs to the machine to minimize the total processing time & also findout for which machine no job assigned. What is the total processing time to complete all the jobs.

Soln Step 1—

Since the cost matrix is not a square matrix, the given assignment problem is unbalanced. To make it balanced, a dummy job 5 with corresponding entries zero is added.

Here the modified matrix is given below—

	A	B	C	D	E
1					
2					
3					
4					
5					

	A	B	C	D	E
1	4	3	6	2	7
2	10	12	11	14	16
3	4	3	2	1	5
4	8	7	6	9	6
5	0	0	0	0	0

Step 2

We subtract the smallest element from all the elements in the respective rows. Hence the modified matrix is given below—

	A	B	C	D	E
1	2	1	4	0	5
2	0	2	1	4	6
3	3	2	1	0	4
4	2	1	0	3	0
5	0	0	0	0	0

(row reduction)

(all cols have 0, so no col reduction)

Since each col has min ele. as zeros, we draw minimum no. of lines to covers all zeros.

	A	B	C	D	E
1	2	1	4	$\boxed{0}$	5
2	$\boxed{0}$	2	1	4	6
3	3	2	1	0	4
4	2	1	$\boxed{0}$	3	0
5	0	$\boxed{0}$	0	0	0

(Here we will not get which job is assigned to which m directly)

(delete the row/col which has max 0s, if row & col both have max 0s, then delete any 1) (or) (can do as you were doing)

The no. of lines to covers all zeros = 4 < 5.
We form the 2nd modified matrix by subtracting the smallest uncovered element from the remaining uncovered ele & adding it to the ele. at the point of intersection of lines. Hence the 3rd modified matrix is given below—

$n = 1$ (smallest uncovered val.)

	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Again we draw minimum of lines to cover all zeros -

	A	B	C	D	E	
1	2	0	3	0	4	(s) ← row reductions
2	$\boxed{0}$	1	0	4	5	(s)
3	3	1	0	$\boxed{0}$	3	(s)
4	3	1	0	4	$\boxed{0}$	(s)
5	1	0	0	1	0	(s)
		(s)	(s)			

(col reductions)

~~Inst~~ (Just getting 1 '0' here, ~~every next rows & cols have more than 1 '0'~~) So, see which row/col has max '0s' & delete it. If both rows & cols have max '0s', delete any 1.

	A	B	C	D	E
1	2	0	3	0	4
2	0	1	0	4	5
3	3	1	0	0	3
4	3	1	0	4	0
5	1	0	0	1	0

Here the no. of lines drawn to covers all zeros = 5 = orders of the matrix. Hence, we can make the assignment. (Here we need to make the assignment, as we're not getting it directly.)

	A	B	C	D	E	
1	2	$\boxed{0}$	3	$\cancel{0}$	4	(s)
2	$\boxed{0}$	1	$\cancel{0}$	4	5	(s)
3	3	1	$\cancel{0}$	$\boxed{0}$	3	(s)
4	3	1	$\boxed{0}$	4	$\cancel{0}$	(s)
5	1	$\cancel{0}$	$\cancel{0}$	1	$\boxed{0}$	(s)
		(s)	(s)	(s)	(s)	

(If single 0 in row mark it & '$\cancel{0}$' all 0s of it's row & col Similarly for col)

→ ~~when~~ Just for this ~~row~~ col, we get this. For others rows & cols - we'll select any 1 0 arbitrarily & do the same. It will be for both row & col [(But while doing this case, after finishing this case, see if case (1) - upper one is happening or not (i.e if 1 0) → If yes do it's process)]

Optimal assignment-

[Here we are chosing 0 arbitrarily, so we could choose another 0s as well, so we will get multiple sols (multiple problem), but the min cost will be same]

Job	machine	cost (min)
1	B	3
2	A	10
3	D	1
4	C	6
5	E	0

∴ Since job 5 is dummy machine E, so no job is assigned. Hence optimum (min) ~~cost~~ time = 3+10+1+6 = 20 hrs

(multiple soln)

	A	B	C	D	E
1	2	$\cancel{0}$	3	$\boxed{0}$	4
2	$\boxed{0}$	1	$\cancel{0}$	4	5
3	3	1	$\boxed{0}$	$\cancel{0}$	3
4	3	1	$\cancel{0}$	4	$\boxed{0}$
5	1	$\boxed{0}$	$\cancel{0}$	1	$\cancel{0}$

Job	m	cost (min)
1	D	2
2	A	10
3	C	2
4	E	6
5	B	0

∴ optimum (min) time = 2+10+2+6 = 20 hrs

(more soln exists but the total hrs will be same)

[Unbalanced & multiple solns]

Chr (But for unbalanced also we can have the assignments like prev sums] [For balanced also we can get like this multiple solns]

8/5/23

- N/W Scheduling by PERT | CPM

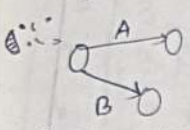
Eg 153) A < C, D, I ; B < G, F ; D < G, F ; F < H, K ; G, H < J ; I, J, K < E. Construct a n/w. (< is the predecessor of)

Soln A < C, which C can not be started until A is completed. That is, A is the preceeding activity to C.

The above constraints can be given in the following table-

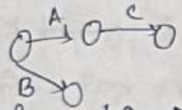
Activity	A	B	C	D	E	F	G	H	I	J	K
Predecessor	–	–	A	A	I,J,K	B, D	B, D	F	A	G,H	F

A & B are the starting activity (predecessor)

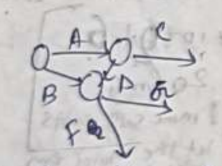


End & Start point will be always one, i.e 1 start point, 1 end point.

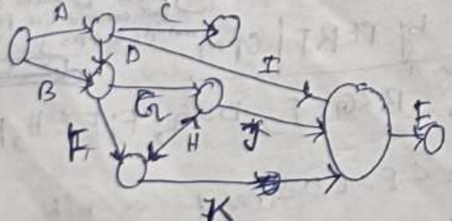
C has the predecessor A



D has the predecessor A & B, D are the predecessor of F & G

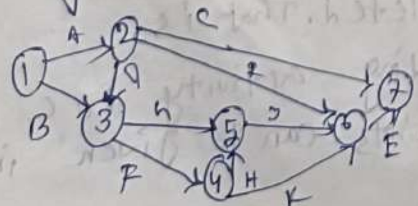


H has the predecessor F & G, H are predecessor of J

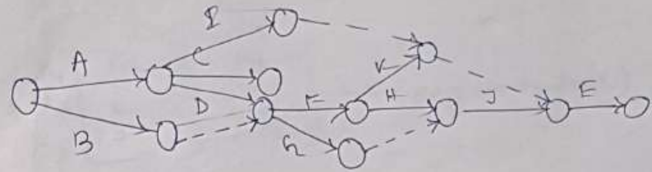


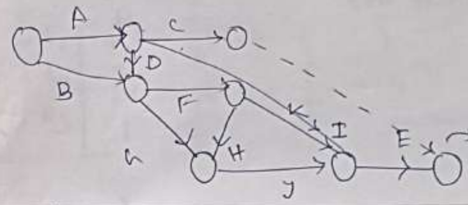
not like this

Finally,



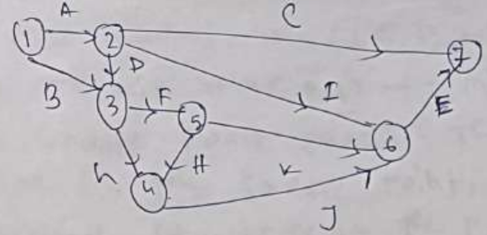
• Game Theory





(2 ending points can't be there)

Finally,



A_1	-5	2	0	7	-5
A_2	5	6	4	8	4
A_3	4	0	2	-3	-3

Col minima: 5 6 4 8

• Game Theory

Eg 12.1 Solve the game where the pay-off matrix is given by

$$
\begin{array}{cc}
 & \text{Player B} \\
\text{Player A} & \begin{array}{c|ccc} & B_1 & B_2 & B_3 \\ \hline A_1 & 1 & 3 & 1 \\ A_2 & 0 & -4 & -3 \\ A_3 & 3 & 5 & -1 \end{array}
\end{array}
\rightarrow \text{(pay off matrix)}
$$

Soln -

$$
\begin{array}{cc}
 & \text{Player B} \\
\text{Player A} & \begin{array}{c|ccc|c} & B_1 & B_2 & B_3 & \text{Row minima} \\ \hline A_1 & 1 & 3 & 1 & 1 \\ A_2 & 0 & -4 & -3 & -4 \\ A_3 & 1 & 5 & -1 & -1 \\ \hline \text{col. } \cancel{\text{minima}} \text{ maxima} & 1 & 5 & 1 & \end{array}
\end{array}
$$

$\max_i$ (Row minima) $= \max(1, -4, -1) = 1$

$\min_j$ (col maxima) $= \min(1, 5, 1) = 1$

~~maxmin~~ maxmin value $\underline{v} = 1 =$ minmax value $\bar{v}$

Hence, saddle point exists. The value of the game is the saddle point, which is 1.

The optimal strategy is the position of the saddle point is given by (A_1, B_1).

Exercise: Determine the optimal strategy for each player in the following game:

$$
\begin{array}{c|cccc|c}
 & B_1 & B_2 & B_3 & B_4 & \text{Row minima} \\ \hline
A_1 & -5 & 2 & 0 & 7 & -5 \\
A_2 & 5 & 6 & 4 & 8 & 4 \\
A_3 & 4 & 0 & 2 & -3 & -3 \\ \hline
\text{col minima:} & 5 & 6 & 4 & 8 &
\end{array}
$$

$\max_i$ (Row minima) = $\max(-5, 4, -3) = 4$

$\min_j$ (col minima) = $\min(5, 4, 8) = 4$

maxmin value $\underline{v} = 4$ = minmax value $\overline{v}$. Hence, Saddle point exists. The value of the game is the saddle point, which is 4. The optimal strategy is the position of the saddle point is given by (A_2, B_3).

Eg-15.2

For what value of λ, is the game with the following matrix strictly determinable?

$$\begin{array}{cc} & \text{Player } B \\ & \begin{array}{ccc} B_1 & B_2 & B_3 \end{array} \\ \text{Player } A \begin{array}{c} A_1 \\ A_2 \\ A_3 \end{array} & \begin{bmatrix} \lambda & 6 & 2 \\ -1 & \lambda & -7 \\ -2 & 4 & \lambda \end{bmatrix} \end{array}$$

Soln Ignoring the value of λ, the pay-off matrix is given by -

		Player B		
	B_1	B_2	B_3	Row minima
Player A A_1	λ	6	2	2
A_2	-1	λ	-7	-7
A_3	-2	4	λ	-2
Col maxima	-1	6	2	

The game is strictly determinable if,

$\underbrace{\underline{v}}_{\text{maximin}} = \underbrace{v}_{\text{value}} = \underbrace{\overline{v}}_{\text{minimax}}$

Hence $\underline{v} = 2$, $\overline{v} = -1$

$\therefore -1 \le \lambda \le 2$.

(one person is facing win & one person is facing loss & win loss = 0)

Eg 15.3 Determine which of the following two-person zero-sum games are strictly determinable & fair. Given the optimum strategy for each player in the case of strictly determinable games.

(i)
$$\begin{array}{cc} & \text{Player } B \\ & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{Player } A \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} -5 & 2 \\ -7 & -4 \end{bmatrix} \end{array}$$

(ii)
$$\begin{array}{cc} & \text{Player } B \\ & \begin{array}{cc} B_1 & B_2 \end{array} \\ \text{Player } A \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix} \end{array}$$

maximin $= \underline{v} = \max(-5, -7) = -5$

minimax $= \overline{v} = \min(-5, 2) = -5$

Soln

	Player B		
	B_1	B_2	Row minima
Player A A_1	-5	2	-5
A_2	-7	-4	-7
Column maxima	-5	2	

$\therefore$ $\underline{v} = \overline{v} = -5 \neq 0$, the game is strictly determinable. There exists a saddle point = -5. Hence, the value of the game is -5. The optimal strategy is the position of the saddle point given by (A_1, B_1).

ii)

$$
\begin{array}{ccc}
 & \text{Player B} & \\
 & \begin{array}{cc} B_1 & B_2 \end{array} & \text{Row minimum} \\
\text{Player A} \begin{array}{c} A_1 \\ A_2 \end{array} & \begin{bmatrix} 1 & 1 \\ 4 & -3 \end{bmatrix} & \begin{array}{c} 1 \\ -3 \end{array} \\
\text{col maximum} & \begin{array}{cc} 4 & 1 \end{array} &
\end{array}
$$

maxi (Row minimum) $= 1$

mini (col maximum) $= 1$

$\therefore \underline{V} = \overline{V} = 1 \neq 0$, the game is strictly determinable. value of the game is 1. the optimal strategy is (A_1, B_2). (3)

$\therefore$ Both are strictly determinable, no one is fair. $(\underline{V} = \overline{V} = 0 \therefore \text{fair})$

North West Corner Rule – Steps (Brief):

1. Start at the top-left (north-west) cell of the transportation table.
2. Allocate the minimum of supply and demand to that cell.
3. Adjust supply and demand by subtracting the allocated value.
4. Move right if demand is zero, move down if supply is zero.
5. Repeat steps until all supply and demand are fulfilled.
6. Ensure $(m + n - 1)$ allocations for a basic feasible solution.

1. Identify the cell with the lowest cost in the transportation table.
2. Allocate the minimum of supply and demand to that cell.
3. Adjust the supply and demand by subtracting the allocated value.
4. Cross out the row or column where supply or demand becomes zero.
5. Repeat the process with the next lowest cost cell in the reduced table.
6. Continue until all supply and demand are fulfilled.
7. Ensure $(m + n - 1)$ allocations for a basic feasible solution.

Vogel's Approximation Method (VAM) – Steps (Brief)

1. Calculate penalties for each row and column:
    - Penalty = difference between the lowest and second-lowest costs in that row/column.
2. Identify the highest penalty among all rows and columns.
3. In the selected row or column, allocate the minimum of supply and demand to the lowest cost cell.
4. Adjust supply and demand, and cross out the satisfied row or column.
5. Recalculate penalties for the reduced table.
6. Repeat the process until all supply and demand are fulfilled.
7. Ensure $(m + n - 1)$ allocations for a basic feasible solution.

Let me know if you'd like a side-by-side comparison of all three methods!

MODI Method – Steps (Brief with Details)

1. Start with an initial basic feasible solution (from NWCR, Least Cost, or VAM).
2. Assign potentials u_i and v_j such that $u_i + v_j = c_{ij}$ for all occupied cells.
   - Start by assigning 0 to any row or column with the most allocations.
3. Compute net evaluation values for unoccupied cells using:
   $\Delta_{ij} = c_{ij} - (u_i + v_j)$.
4. Check optimality:
   - If all $\Delta_{ij} > 0$: optimal and unique solution.
   - If all $\Delta_{ij} \geq 0$: optimal but alternate solution exists

- If all $\Delta_{ij} \geq 0$: optimal but alternate solution exists.
- If any $\Delta_{ij} < 0$: not optimal — improve solution.

5. Identify the most negative Δ_{ij} and form a closed loop through occupied cells using horizontal and vertical lines.
   - Alternate + and – signs.
   - Find the minimum allocation in the (–) positions.
6. Adjust allocations by adding and subtracting the minimum value along the loop to improve total cost.
7. Repeat from Step 2 until all $\Delta_{ij} \geq 0$ — this gives the optimal s on.

Hungarian Method – Steps (Brief with Details)

1. Prepare the cost matrix.
   - If it's not square, add dummy rows/columns with zero cost.
2. Row reduction:
   - Subtract the smallest value in each row from every element of that row.
3. Column reduction:
   - Subtract the smallest value in each column from every element of that column.
4. Cover all zeros with a minimum number of lines (horizontal/vertical).
   - If number of li order of matrix, go to

4. Cover all zeros with a minimum number of lines (horizontal/vertical).
    - If number of lines = order of matrix, go to assignment.
    - If not, proceed to Step 5.
5. Find the smallest uncovered element.
    - Subtract it from all uncovered elements.
    - Add it at intersections of lines.
    - Repeat Steps 4 and 5 until optimal condition is met.
6. Make assignments:
    - In each row/column, find a single zero and assign it (circle it).
    - Cross out other zeros in the same row/column.

condition is met.

6. Make assignments:
   - In each row/column, find a single zero and assign it (circle it).
   - Cross out other zeros in the same row/column.
   - Repeat until all tasks are assigned.
7. Final assignment:
   - The positions of the circled zeros give the optimal solution.

Let me know if you'd like all methods compiled into a PDF or table!