Digital Logic

Logic Gate → NOT, AND, OR, NAND, NOR, X-OR, X-NOR

Minimization → Boolean Algebra, ~~K~~ K-MAP

Combination Circuit → Comparator, MUX, DE-MUX, Encorder, decorder, (without memory) ~~H~~ Adder, subadder, Half sub, Full sub, Serial adder, Pl adder, Look ahead carry adder, Multiplier, code converter.

Sequential Circuit → Latches, flip flop, Register, Counters.
(with memory)

Number System → Base conversion, Magnitude Representation,

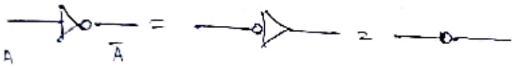
Lec-1

Byte → 8 Bit, Nible → 4 bit.

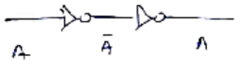
501 → LSB → Least Significant Bit

→ MSB → most significant bit

1. NOT gate [Inverter, Negation, Complement logic]

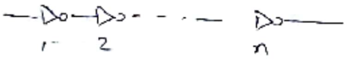


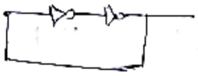
τ_{pd} → Propagation Delay



If Input (A) → output (A)

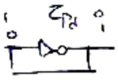
Buffer → To provide delay





1) Basic Memory element.
2) Bistable multi vibrator
3) DC generator

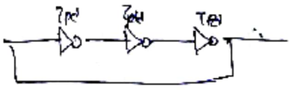
If n = odd = ~~Invert~~ inverter
n = even = buffer.

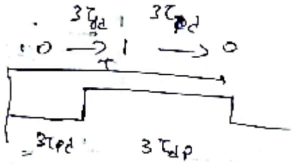


0 → 1 → 0 -

1) Astable Multivibrator
2) Square wave generator
3) Clock generator.
4) Free Running Circuit
5) Ring Oscillator.

$f = \frac{1}{T} = \frac{1}{2\tau_{pd}}$



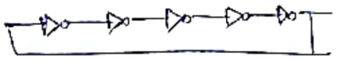


$f = \frac{1}{6\tau_{pd}} = \frac{1}{2\times3\tau_{pd}} = \frac{1}{2N\tau_{pd}}$

Here N must be odd

If N is even f = 0 Hz

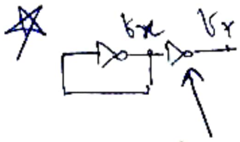
If all the NOT Gates are identical & Having τ_{pd} = 2 us. Find the freq at y



$f = \frac{1}{2\times5\times2\times10^{-6}} Hz = \frac{10^6}{10\times2} Hz = \frac{\overset{50}{\cancel{1000}}\times10^{3}}{10\times\cancel{2}}$

= 50 KHz

τ_{pd} = 5ps
↳ 10^{-12}



freq. of x = freq of Y

This will give no impact on frequency, It is just a time delay

And (Intersection)

$0 \cdot 0 = 0$
$0 \cdot 1 = 0$
$1 \cdot 0 = 0$
$1 \cdot 1 = 1$
$1 \cdot A = A$
$A \cdot A = A$
$A \cdot 0 = 0$
$A \cdot \bar{A} = 0$

$$\boxed{\begin{array}{l} 1+A = 1 \\ A \cdot 1 = A \end{array}}$$

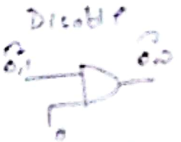
$$\boxed{\begin{array}{l} A \cdot 0 = 0 \\ A+0 = A \end{array}}$$

OR (Union)

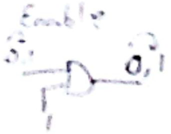
$0+0 = 0$
$0+1 = 1$
$1+0 = 1$
$1+1 = 1$
$0+A = A$
$A+0 = A$
$A+1 = 1$
$A+\bar{A} = 1$

Floating Terminal
Control input
strobe

Enable / Disable



Control '0' Disable



Control '1' enable

Commutative law

$A \cdot B = B \cdot A$

Associative law

$(A \cdot B) \cdot C = A \cdot (B \cdot C)$

OR gate

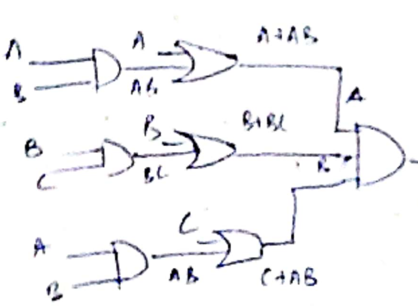
$A+B = B+A$

$(A+B)+C = A+(B+C)$

TTL → Transistor Transistor Logic

This is empty
always means 1 High

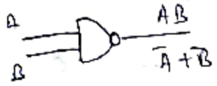
ECL → Emitter Coupled Logic
Floating terminal = 0 low

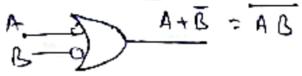


$A+AB$
$A[1+B] \rightarrow A \cdot 1$

$(AB)(C+AB)$
$ABC+AB$
$AB(1+C)$
AB Ans.

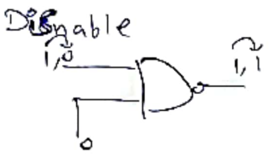
Nand.

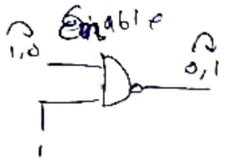




Bubbled OR gate = NAND

0 · 0	1
0 · 1	1
1 · 0	1
1 · 1	0

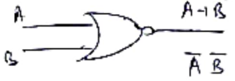


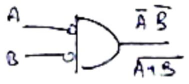


Follows Commutative law.

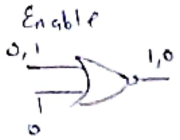
But Doesn't follow Associative law.

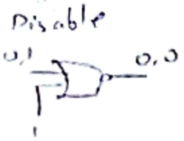
NOR.





0+0	=	1
0+1	=	0
1+0	=	0
1+1	=	0



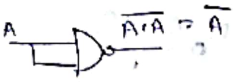


Follows Commutative law

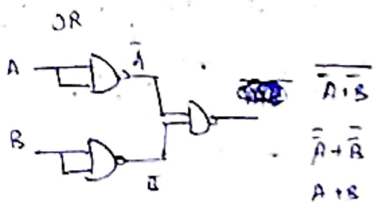
NAND, NOR are called Universal logic

NAND

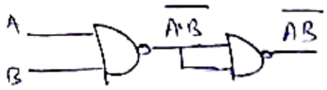
OR



OR

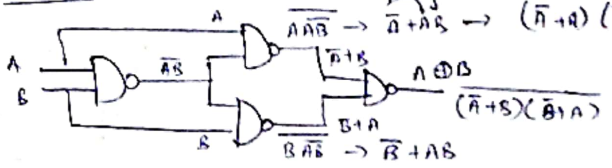


AND



X-OR

$A\oplus B = \bar A B + A\bar B$



$A\overline{AB} \rightarrow \bar A + AB \rightarrow (\bar A+A)(\bar A+B) = 1\cdot(\bar A+B)$

$B\overline{AB} \rightarrow \bar B + AB$

$(\overline{\bar A+B})\cdot(\overline{\bar B+A})$

$(A\cdot\bar B)+(B\cdot\bar A)$

$(A+B)\cdot(\underbrace{\bar B+B}_{1})\cdot(\bar B+\bar A)\cdot(\underbrace{\bar A+A}_{1})$

$(A+B)\ (\bar B+\bar A)$

$(A\bar B)+(\underbrace{\bar A A}_{0})+(\underbrace{\bar B B}_{0})+(\bar A B)$

$A\bar B+\bar A B$

4

X-NoR. $\rightarrow A\odot B = \overline{A\oplus B}$

5.

	NAND	NOR
NOT	1	1
AND	2	3
OR	3	2
X-OR	4	5
X-NOR	5	4
NAND	1	4
NOR	4	1

$\boxed{\text{NAND}}$ if 1 option to be ch

4 Case 1

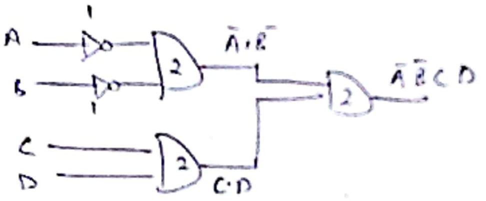
$A \cdot B \cdot \bar{C} \cdot D \cdot \ldots$

n = Total no. of variables   K = Total no. of complement vari

NAND	NOR
$(2n-2)+K$	$(3n-3)-K$

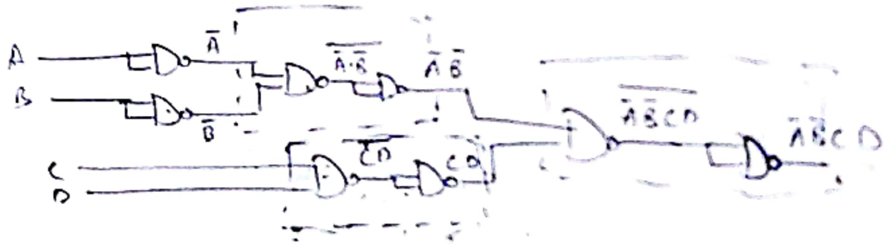
$\bar{A} \cdot \bar{B} \cdot C \cdot D$

NAND



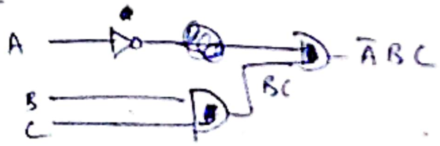
3 Nand

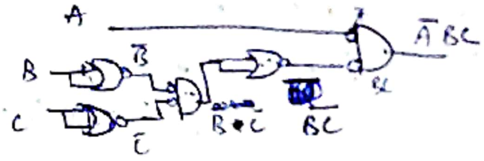
$(2 \times 4 - 2) \cdot$
$= 8$



$\bar{A} \cdot B \cdot C$

NOR





5

$(3 \times 3 - 3) -$
$9 - 3$
$6 - 1$
5

Case 2 $\bar{A} + B + \bar{C} + \bar{D} + ---$.

NAND	NOR
$(3n-3)-x$	$(2n-2)+k$

$f_1 = \bar{A}\bar{B}\bar{C}D$ $f_2 = \underbrace{\bar{A}\bar{B}\bar{C}D}_{6} + \boxed{\bar{B}}$

The no. of NOR gate req.

$f_1 = (3\times4-3)-3$

$= 12 - \cancel{6}$

$= 6$

~~$f_2 = 7 + (2\times1-2)+1$~~ ~~$6+1+2$~~

$= 9$

$f_2 = \bar{B}[\bar{A}\bar{C}\bar{D}+1]$

$= \bar{B} = 1$

$1+0=1$
$0+1=1$

$\therefore f_1 - f_2 = 6-1 = 5.$

$f = A + A\bar{B}C + ABC$

Min. NAND gate req.

~~AC~~ $\rightarrow$ ~~$(2\times2-2)+0$~~

~~$= 2.$~~

~~$A[1+\bar{B}C+BC]$~~

~~$AC[\bar{B}C+BC]$~~

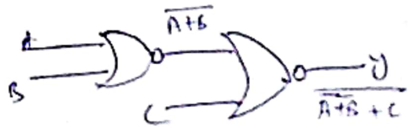
~~AC~~

$A+\bar{A}=1$
$A\cdot\bar{A}=0$

$A[\underbrace{1+\bar{B}C+BC}_{1}]$

$A \longrightarrow (2\times1-2)+0 = 0$ Ans

#



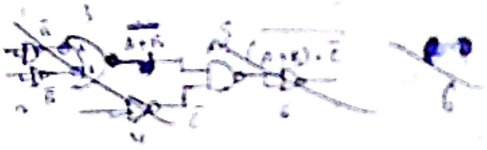
$\overline{\overline{A+B}+C}$

$\overline{\overline{A+B}}\cdot\bar{C}$

$(A+B)\cdot\bar{C}$

$A\bar{C}+B\bar{C}$

How many nand gate.



5

Case 3

$j = AB + CD$

$\therefore 3$ NAND

AND $\rightarrow$ OR

NAND $\rightarrow$ NAND

SOP $\rightarrow$ Sum of product

POS $\rightarrow$ product of sum

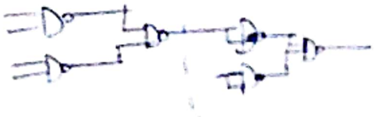
$f = AB + C$

$= AB + CC$

$f = A\bar{B} + C$

$\underbrace{AB}_{x} + CD + E$

$x + EE$

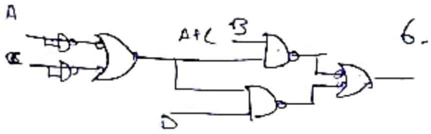


#

$AB + BC + CD + DA$

$B(A+C) + D(A+C)$

$Bx + Dx$

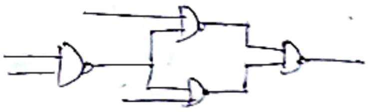


Case 4

4

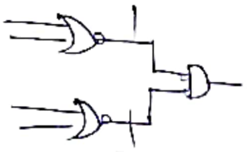
$\bar{A}B + A\bar{B} = A \oplus B$

$(A+B)(\bar{A}+\bar{B})$



$(A+B)(C+D)$

NOR?



OR ──→ AND

NOR ──→ NOR

XOR

$A \oplus B$

$\bar{A}B + A\bar{B}$

0	0	0
0	1	1
1	0	1
1	1	0

$\Sigma m(1,2,4,7)$

↑

3 input → $A \oplus B$

$A = B \quad y = 0$

$A = \bar{B} \quad y = 1$

$A \oplus A = 0$

$A \oplus 0 = A$

$A \oplus A = 0$

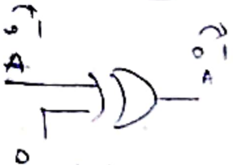
$A \oplus A \oplus A = A$

$A \oplus \bar{A} = 1$

$A \oplus 1 = \bar{A}$

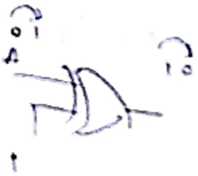
odd A	=	A
Even A	=	0

6

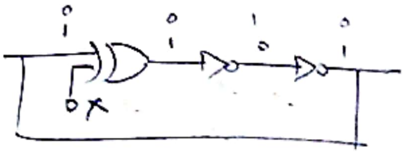


Disable.

Buffer



Interter



Astable multivibrator x = 1

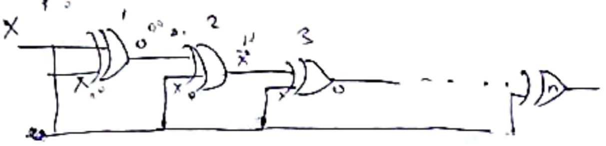
Bistable → x = 0

XOR follows. Commutative & associative

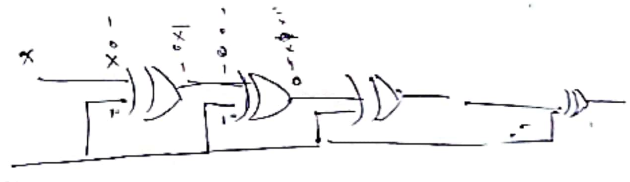
$A \oplus B \oplus C = \Sigma m(1,2,4,7)$

001
010
100
111

7)

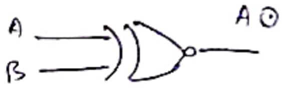


n = even = $\bar{X}$
n = odd = 0



odd = $\bar{X}$
even = X

<u>XNOR</u>



$A \odot B = \overline{A \oplus B}$
$= \bar{A}\bar{B} + AB$

0	0	1
0	1	0
1	0	0
1	1	1

Even parity detector.
even '1' → 1
Equivalence logic
Coincidence "

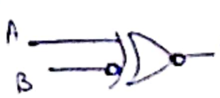
$A \odot \bar{A} = 1$

$A \odot A = 0$

Commutative & Associative

$A \odot B \odot C = (A \odot B) \odot C$

For even input in XNOR = $\overline{XOR}$ $A \odot B = \overline{A \oplus B}$
odd " " " = XOR $A \odot B \odot C = A \oplus B \oplus C$



$A \odot \bar{B} = \bar{A}B + A\bar{B} = A \oplus B$

$A \oplus \bar{B} = \bar{A}\bar{B} + AB = A \odot B$

4) $f(A, B) = A \oplus B \oplus AB$

- OR gate.

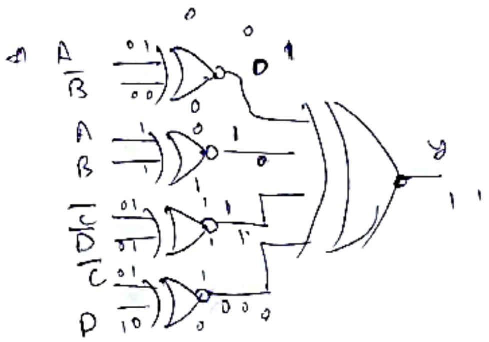
And OR gate req. 3 NAND to implement

A	B	AB	$A \oplus B \oplus AB$
0	0	0	0
1	0	0	1
0	1	0	1
1	1	1	1

5) $Y = A \oplus (A+B)$

$\bar{A} \cdot B$

A	B	A+B	$A \oplus (A+B)$
0	0	0	0
0	1	1	1
1	0	1	0
1	1	1	0



A B $\bar{B}$ 1 2
0 0 1 0 1

B
1

Minimization

Boolean Algebra

Boolean function,
↳ Combination of inputs on which output will depends

SOP	POS	
Sum of product (Minterm)	Product of sum (Max term)	minterm = $\overline{\text{maxterm}}$

$f(A,B) = \underbrace{\bar{A}\bar{B}}_{} + \underbrace{AB}_{}$ 1 minterm

$f(A,B) = \underbrace{(\bar{A}+B)(A+B)}_{\text{maxterm}}$

Literals. → combination of $A \to A, \bar{A}$
$B \to B, \bar{B}$

$f(A,B,C) = \underbrace{\bar{A}\bar{B} + \bar{C} + A\bar{B}}_{\text{4 literals}}$

Decimal	ABC	Minterm	Maxterm
0	000	$\bar{A}\bar{B}\bar{C}$	$A+B+C$
1	001	$\bar{A}\bar{B}C$	$A+B+\bar{C}$
2	010	$\bar{A}B\bar{C}$	
3	011	$\bar{A}BC$	
4	100	$A\bar{B}\bar{C}$	
5	101	$A\bar{B}C$	
6	110	$AB\bar{C}$	
7	111	ABC	$\bar{A}+\bar{B}+\bar{C}$

Minterm means. $A=1$ $\bar{A}=0$

Maxterm means $A=0$ $\bar{A}=1$

				function
0	0 0	$\bar{A}\bar{B}$	$A+B$	1
1	0 1	$\bar{A}B$	$A+\bar{B}$	0
2	1 0	$A\bar{B}$	$\bar{A}+B$	0
3	1 1	AB	$\bar{A}+\bar{B}$	1

SOP

$f(A,B) = \bar{A}\bar{B}\cdot 1 + \underbrace{\bar{A}B\cdot 0}_{\to 0} + \underbrace{A\bar{B}\cdot 0}_{\to 0} + AB\cdot 1$

variables.

$= \bar{A}\bar{B} + AB = m_0 + m_3$

$= \Sigma m(0,3)$

POS

$F(A,B) = \underbrace{(A+B+1)}_{1}(A+\bar{B}+0)(\bar{A}+B+0)\underbrace{(\bar{A}+\bar{B}+1)}_{1}$

$(A+\bar{B})(\bar{A}+B)$

$\Pi M(1,2)$

Standard Canonical

Contains all the form of variable.

Practice Sheet.

1 $f = (A+B)(A+C)(A+\bar{C})$

$= (A+B)(\underset{A}{AA} + A\bar{C} + AC + \underset{0}{C\bar{C}})$

$= (A+B)(A + A\bar{C} + AC)$

$= (A+B)\,A\,(\underbrace{1+\bar{C}+C}_{1})$

$= AA + AB$

$= A(\underbrace{1+B}_{1})$

$= A$ Ans.

* Distridution Law

$A+BC$

$\downarrow$

$(A+B)(A+C)$

$(A+B)\underbrace{(A+C)(A+\bar{C})}$ DL

$\downarrow$

$(A+B)\,(A+\underbrace{C\bar{C}}_{A})$ $= 0$

$(A+B)A$

2. $f(A,B,C,D) = B\bar{C}[A + B\bar{C}D + \underbrace{\bar{B}CD}_{0} + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}]$

$B\bar{C}\,B\bar{C}$

$= AB\bar{C} + B\bar{C}D + \bar{A}B\bar{C}$

$= [\underbrace{A + D + \bar{A}}_{1}]\,B\bar{C}$

$= B\bar{C}$

4 $f(A,B,C) = (A+B)(\bar{B}+C)(A+C)$ $\bar{f}$?

$= \overline{(A+B)(\bar{B}+C)(A+C)}$

$= (\overline{A+B}) + (\overline{\bar{B}+C}) + (\overline{A+C})$

$= \bar{A}\bar{B} + B\bar{C} + \bar{A}\bar{C}$

* Consisis theorm. → ~~The~~ when 2 variables will repeat in 3 terms and the one having both forms of literal is the ans.

∴ Ans $\bar{A}\bar{B} + B\bar{C}$

Example

$AB + \bar{A}C + BC$

Ans. $AB + \bar{A}C$

$AB + \bar{A}CD + BCD$

$AB + \bar{A}CD$

Transpose Theorem.

☆ $(A+B)(\bar{A}+C)$

$\underbrace{A\bar{A}}_{0} + BC + \bar{A}B + AC$

$BC + \bar{A}B + AC$

$\bar{A}B + AC$

So $(A+B)(\bar{A}+C)$

$AC + \bar{A}B$

$(\bar{A}+\bar{B})(A+B)$

$\bar{A}B + A\bar{B} = A \oplus B$

5. $f(A,B,C,D) = AB + \bar{A}\underline{CD} + \bar{B}\underline{CD}$

$= AB + (\bar{A}+\bar{B})CD$

$= \underbrace{AB}_{x} + \underbrace{\overline{AB}}_{\bar{x}}\,CD$

$= x + \bar{x}CD$

$= (x+\bar{x})(x+CD)$

$= AB + CD$

6 $f(A,B,C) = \bar{A}\bar{B} + \bar{A}BC + \bar{A}B\bar{C}$

$= \bar{A}\bar{B} + (C+\bar{C})\bar{A}B$

$= \bar{A}\bar{B} + 1\,\bar{A}B$

$= \bar{A}(\bar{B}+B)$

$= \bar{A}$

⇒ $\overline{\bar{A}B + A\bar{B}} = (\overline{\bar{A}B})(\overline{A\bar{B}})$

$= (A+\bar{B})(\bar{A}+B)$

$\overline{AB\,\overline{CD}} = \overline{AB} + CD$

$= \bar{A} + \bar{B} + CD$

$\overline{\bar{A}\bar{B}C} = A\bar{B} + \bar{C}$

$= (A+\bar{C})(\bar{B}+\bar{C})$

\# $A \oplus \bar{B} = A \odot B = \bar{A} \oplus B$ ☆

$\overline{A \oplus \bar{B}} = A \odot \bar{B} = A \oplus B$

$\overline{\bar{A} \odot \bar{B}} = \bar{A} \oplus \bar{B} = A \oplus B$

1 bar remove → sign changes.

$\overline{\bar{A} \oplus \bar{B}} = \bar{A} \odot \bar{B} = A \odot B$

Duality

$A \cdot B \to A + B$

$A \oplus B \to A \odot B$

$\overline{AB} \to \overline{A+B}$

$\bar{A} \to \bar{A}$

$A \to A$

2 variable → 4

3 " ⇒ 256

Total expression $= 2^{2^n}$

$\hookrightarrow 2^{2^{n-1}} \longrightarrow$ Self Dual

If $f^D = f$ → Duality

than self dual $A \to A$

$f = AB + BC + AC$

$f^D = (A+B)(B+C)(A+C)$

$= (A+BC)(B+C)$

$= AB + AC + BCB + BCC$

$= AB + AC + BC \longrightarrow$ Self Dual.

→ Minimization techique.