SHYAM STEEL

Algorithm

1. • $f(n^2) = O(f(n)^2)$, for polynomial.

Let $f(n) = n$.

$n^2 = n^2$ True.

• for exponential.

Let $f(n) = 2^n$.

2^{n^2} $(2^n)^2$

$> \hookrightarrow 2^{2n}$ ~~f~~false statement.

2. $f(n^2) = \Omega(f(n)^2)$ False statement

Let $f(n) = \log n$.

$\log n^2$ $(\log n)^2$

$2\cancel{\log n} < (\log n)^{\cancel{2}}$

3. a=1, b=1.

while (a<=n)

b=b+1;

a=a+b;

}.

start

$b = 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \ldots$

$a = 1 \rightarrow 3 \rightarrow 6 \rightarrow 10 \ldots$

$1 \quad 1+2 \quad 1+2+3 \quad 1+2+3+4 \ldots$

$1+2+3+\ldots \underline{K} = \frac{K(K+1)}{2} = n$

$K = \sqrt{n}$

$\therefore O(\sqrt{n})$

Top goals	Things to do	Notes

1. • $f(n^2) = O(f(n)^2)$, for polynomial.
let $f(n) = n$.

4. for $i \rightarrow \leq n$; $i = i+1$
for $j \rightarrow \leq n$; $j = j+i$

$T(n) = 2T(\sqrt{n}) + n^2.$

M.T

$T(n) = 2T(n^{1/2}) + n^2$

Let $n = 2^k \longrightarrow \log_2 n = k$

$T(2^k) = 2T(2^{k/2}) + 2^{2k} \rightsquigarrow S(k) = 2S(k/2) + \underset{4^k}{\underset{\downarrow}{2^{2k}}}$

$a = 2$, $b = 2$. $f(n) = 2^{2k}$.

Case 1

$4^k \in O(k^{\log_2 2 - \epsilon})$ ×.

Case 2.

$4^k \in \Theta(k^{\log_2 2} (\log k)^2)$ ×

Case 3.

$4^k \in \Omega(k^{\log_2 2 + \epsilon})$ ✓

$\therefore \Theta(4^k) \rightsquigarrow \Theta(4^{\log_2 n}) \rightsquigarrow \Theta(n^{2\log_2 2})$

$\rightsquigarrow \Theta(n^2)$

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4. for i $\rightarrow$ ~~x~~n i=i+1
   for j $\rightarrow \le n$; j=j+i

∴ Nested and dependent.

i	j
1	n
2	$n/2$ (1+3+5 ...)
3	$n/3$
n	n/n

combine of i & j

∴ $n + \frac{n}{2} + \dots \frac{n}{n}$.

$O(n \log n)$

5. for j $\rightarrow \le n$, $j = j \times 5$. $n/5^k$ $= O(\log_5 n.)$

6. $T(n) = 2T(\sqrt{n}) + \log n.$

$T(n) = 2T(n^{1/2}) + \log n.$

Let $n = 2^k$, $k = \log_2 n.$

$T(2^k) = 2T(n^{k/2}) + k$

Let $T(2^k) = S(k)$

$S(k) = 2S(k/2) + k.$

case 2

$S(k) = \Theta(k \log k)$ $k = \log_2 n$

$T(2^k) = \Theta($ ~~...~~ $\log n \log \log n)$

$T(n)$ ~~...~~

Top goals | Things to do | Notes

SHYAM STEEL

7. Tower of Hanoi, $T(n) = 2T(n-1) + 1$

#8. Swap count = inversion count
is same in Bubble sort & insertion sort

9. DFS, BFS $\rightarrow O(m+n)$

• can determine if simple graph is Bipartite.

10. $\log(\log^* n)$, $2^{\log^* n}$, $\sqrt{2}^{\log n}$, n^2, $n!$, $(\log n)!$

☆ 1. $\log(\log^* n) \Rightarrow \log(\log^{\log^{\log^{\cdots}}} n)$.

2. $n! = n^n$

3. $(\log n)! \quad 2^{n \log n}$

factorial is greater

3. $\sqrt{2}^{\log n}$.
$= 2^{\frac{1}{2}\log n}$.
$= \sqrt{n}^{\log 2}$
$= \sqrt{n}$.

∴ $\log(\log^* n)$, $2^{\log^* n}$, $\sqrt{n}$, n^2, $(\log n)!$, $n!$

Top goals	Things to do	Notes

	Reflexive	Symmetric	Transitive	Transpose Symmetric
O	✓	×	✓	✓
Ω	✓	×	✓	✓
θ	✓	✓	✓	×
o	×	×	✓	✓
ω	×	×	✓	✓

- **Bubble Sort** [Inplace & stable, n-1 iteration]

	No. of compair.	swap.	Complexity.
Best-Case	$n-1$	0	$O(n)$
worst case.	$(n-1)^2$	$\frac{n(n-1)}{2}$	$O(n^2)$

- **Selection Sort** [inplace & stable, n-1 iteration]

Best case	$(n-1)(n-1)$	$n-1$	$O(n^2)$
worst case	$(n-1)(n-1)$	$n-1$	$O(n^2)$

- **Insertion. Sort** [inplace & stable]

Best case	$n-1$	0	$O(n)$
worst case	$\frac{(n-1)(n)}{2}$	$\frac{(n-1)n}{2}$	$O(n^2)$

$O(n + \text{inversions})$ when input is not known,

- **Radix Sort** [non comparison].

$TC = O(n \times d)$ → no. of digit in max. element

No. of passes req. = $\lfloor \log_{10} \text{max valued digit} \rfloor + 1$

or = no. of digits in Max Valued element

- **Merge Sort** [not inplace but stable].

Space Complexity = $O(n + \log_2 n)$

- **Quick Sort.** [not stable & not inplace]

S.C. Best case = $O(\log_2 n)$

worst case = $O(n)$ ← skewed,

T.C	B.C	A.C	W.C.
	$O(n \log n)$	$O(n \log n)$	$O(n^2)$

Min Max. $T(n) = 2T(n/2) + 2$

	without D&C.	with D&C
B.C	$n-1$ comparison.	
W.C	$2(n-1)$ "	
A.C	$\frac{3n}{2} - 1$	$\frac{3n}{2} - 2$.

- **Matrix Chain. Multiplication.** normal case S.C. $O(\log n)$

D&C. S.C

→ $8T(n/2) + O(n^2) = O(n^3)$ $O(1)$

→ $7T(n/2) + O(n^2) = O(n^{2.81})$ S.C $O(\log n + n^2)$

(Strassen)

- **Longest Integer** ...

→ $4T(n/2) + bn$ $\theta(n^2)$ $n^{\log_2 4}$

→ $3T(n/2) + bn$ $\theta(n^{1.58})$ $n^{\log_2 3}$

Anatoly Karatsuba.

- **K way split** Let → x.

D&C → $(K^2)\,T(n/K) + bn$

AK → $(K^2 - 1)\,T(n/K) + bn$ $\quad O(n^{\log_K x})$

Toom & Cook → $(2K-1)\,T(n/K) + bn$.

$T(n) = a \times T(n/b) + F(n)$ $\quad a \geq 1,\ b > 1,\ F(n) = +ve$

1. $F(n) \in O(n^{(\log_b a - \epsilon)})$, $\epsilon > 0$

   $T(n) = \theta(n^{\log_b a})$

2. $F(n) \in \theta(n^{\log_b a} \times (\log n)^K)$, for some K.

   i. $K \geq 0$, $T(n) = \theta(n^{\log_b a} (\log n)^{K+1})$

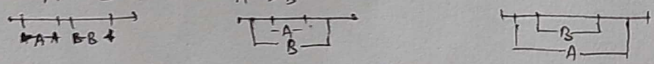
   ii. $K = -1$, $T(n) = \theta(n^{\log_b a} \log \log n)$

3. $F(n) \in \Omega(n^{(\log_b a + \epsilon)})$,

   for some $\epsilon > 0$ & $a f(n/b) \leq \delta \times F(n)$, $\delta < 1$

   $T(n) = \theta(F(n))$

1. optimal file merging. (n+m record movements) [For comparison m+n-1]
   1. with linear L.L $O(n^2)$ 2. Heap. $O(n \log n)$
2. Prim's Algo: Non heap $O(n^2)$ | Heap: $(n+e)\log n$
3. Kruskal Algo: $O(e \log e)$
4. DSSSP: • Matrix Based • Spanning tree based.
   non Heap = $O(n^2)$ Heap: $O((n+e)\log n)$
5. DFS. Directed Graphs.
   • Cross edge • Back edge • forward edge.
   A → B A → B A → B



6. Strongly Connected Component. [V] → [Q ⇄ S] → [P ⇄ R]

1. Heap.
   - Insertion Method. $O(\log n)$ 1 element. • Heapify

   B.C $O(n)$ | W.C $O(n \log n)$ Bottom up → $O(n)$ | Top Down → $O(n \log n)$

   → Heap Sort: TC: $\theta(n \log n)$ SC: $O(1)$ Inplace But not stable.

DP.

1. Bellman ford (n-1 iteration) $O(n \ast e)$
2. Floyd warshall TC: $O(n^3)$ S.C $O(n^2)$
3. 0/1 Knapsack: TC & S.C $O(n \times M)$ n = no. of product, M = scale capacity.
4. LCS: TC SC $O(n \times m)$
5. MCM: TC $O(n^3)$ S.C $O(n^2)$.

$(A_1 A_2)(A_3 A_4)$ $\quad ((A_1 A_2) A_3) A_4$ $\quad A_1(A_2(A_3 A_4))$

$(A_1(A_2 A_3)) A_4$ $\quad A_1((A_2 A_3) A_4)$

$\therefore f(0)f(4) < 0.$

18. ~~Data~~ structure 2021 [Tree]

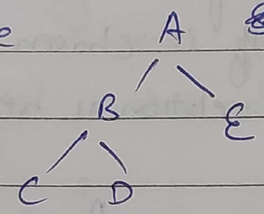
- Consider the following statement

S1: The seq. of procedure calls corresponds to a preorder traversal of the activation tree.

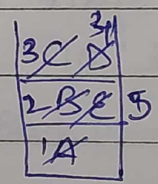
S2: The seq. of procedure return corresponds to a postorder traversal of the activation tree.

Which of the statement(s) are correct?

Let the tree be



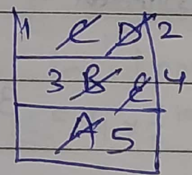
for S1: procedure of call.



$A \to B \to C \to D \to B \to$

preorder.

for S2: Procedure of return



$C \to D \to B \to E \to A$

postorder.

Both statements are true.

Teacher's Signature........

- Let S be the set of composite integer n, where $4 \le n \le 20$. Let R be a "divide relation" on S. i.e iff a divides b. How many minimal elements are there in relation R?

$S = \{4, 6, 8, 9, 10, 12, 14, 15, 16, 18, 20\}$
✓ ✓ ✗ ✓ ✓ ✗ ✓ ✓ ✗ ✗ ✗

[Composite no: can be written as:
$4 \to 2 \times 2$
$6 \to 2 \times 3$
$8 \to 4 \times 2$.

$= 6$ Ans.

27. Algorithm. 2024 [Back Substitution].

- $T(n) = \begin{cases} \sqrt{n}\, T(\sqrt{n}) + n & \text{for } n \ge 1 \\ 1 & \text{for } n = 1 \end{cases}$

$T(n) = n^{1/2}\, T(n^{1/2}) + n.$

$\Rightarrow T(n^{1/2}) = (n^{1/2})^{1/2}\, T((n^{1/2})^{1/2}) + n^{1/2} \Rightarrow n^{1/4}\, T(n^{1/4}) + n^{1/2}$

$\Rightarrow T(n) = n^{1/2+1/4}\, T(n^{1/4}) + n^{1/2+1/2} + n \Rightarrow n^{3/4}\, T(n^{1/4}) + 2n$

$T(n^{1/4}) = (n^{1/4})^{1/2}\, T((n^{1/4})^{1/2}) + n^{1/4} \Rightarrow n^{1/8}\, T(n^{1/8}) + n^{1/4}$

$\Rightarrow T(n) = n^{1/8+3/4}\, T(n^{1/8}) + n^{1/4+3/4} + 2n \Rightarrow n^{1-\frac{1}{8}}\, T(n^{1/8}) + 3n$

$\Longrightarrow T(n) = n^{1-\frac{1}{2^k}}\, T(n^{1/2^k}) + kn$

$n^{1/2^k} = 1 \Rightarrow k = \log\log n$. $\therefore \underbrace{\frac{n^1}{n^{1/2^k}}}_{n} \times 1 + n \log\log n$

$\therefore \theta(n \log\log n)$

Teacher's Signature.......................

Partial Mesh Topology.

[TCP]

- TCP uses ______ Socket and UDP uses ______ Socket

  Stream , Datagram.

C Programming Test Series [Array] [illegible]

```
{
int arr [5] = {1, 2, 3, 5, 7};
 int (*ptr) [5] = &arr +1;
printf ("%d %d \n", *(arr+2) , *(*(ptr-1)) );
return 0 ;
}
```

arr [1 , 2, 3, 5, 7]

1000

&arr

∴ *ptr ∴ Ptr $\boxed{1000 + 1 \times \text{array size}}$

$*(arr+2) = 3$, $*(ptr-1) \Rightarrow arr$, $*(arr) = 1$

∴ 3, 1.

Date
Page

Algorithm. Test Series [Master Theorem]

- Consider the following recurrence relation.

$$T(n) = \sqrt{3}\,(n/2) + O(\sqrt{n}\log n)$$

Here, $a = \sqrt{3}$, $b = 2$, $f(n) = \sqrt{n}\log n$.

$\therefore \frac{1}{2}\log_2 3 = 0.792$

Case I:

is $\sqrt{n}\log n \in O(n^{0.792 - e})$

$\cancel{n^{0.5}} \log n \qquad \cancel{n^{0.5}} {}^{+0.292}$

$\therefore \log n \qquad n^{0.292}$

$\uparrow \qquad \uparrow$

logarithmic < polynomial.

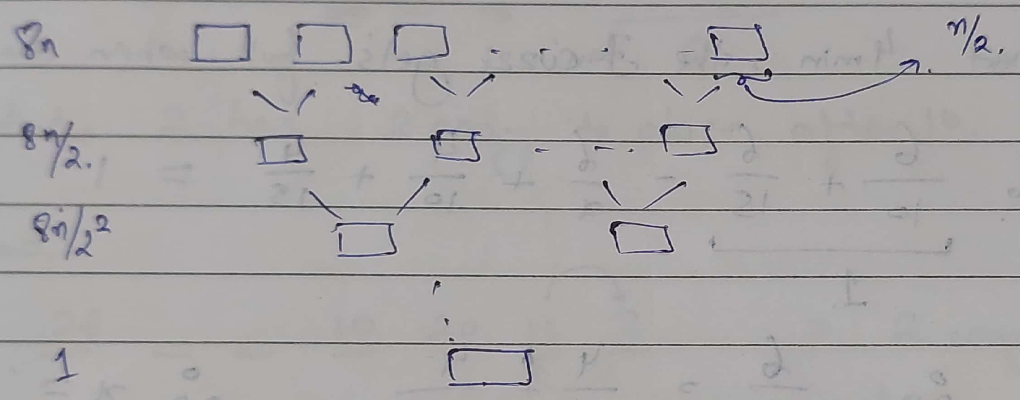
$\therefore O(n)$ Ans.

COA Super 1500 [DMA]

- An 8 bit DMA device is operating in Cycle Stealing Mode. Each DMA transfer takes 4 clock cycles and DMA clock is 2MHz. Intermediate CPU machine cycle takes 8 μsec. Determine the DMA data transfer rate?

64. Algorithm Test Series. [TC]

- Assume that there are $8n$ sorted list each of size $n/2$ then, what is the time complexity of merging them into single sorted list?



$\therefore$ levels $= \log_2 8n = 3 + \log_2 n$.

no. of element at each level $=$ no. of lists $\times$ size of each list

$= 8n \times n/2$.

$\therefore TC = 4n^2 (3 + \log_2 n)$

$\therefore O(n^2 \log_2 n)$.

65. Apptitude Test Series. [Numerical Ability]

- A jaccuzi can be filled by cold water pipe in 10 minutes and by hot water pipe by 15 minutes. Both pipes are opend, and j when the jaccuzzi should have been full, it was discovered that the drain pipe was also open. Once the drain pipe w

$\therefore$ 3600 s $\longrightarrow$

1 s $\longrightarrow$ 0.02 req.

$\therefore$ 500 stations = 0.02 × 500 = 10 req/sec.

A. Slot time is 250 μsec

$\therefore$ no. of slots in 1 sec = 4000 slot/sec.

$\therefore$ Total channel load = $\frac{10}{4000} \times 100 = 0.25\%$.

3. Algorithm. Super Mock

- For an algorithm with quadratic time complexity, it takes 30 sec to process an input of size 1000. What is the maximum size of a problem that can be solved within 2 minutes?

Let, $O(n^2)$

$\therefore$ 30 sec $= C_1 \times (1000)^2$

$\therefore$ $C_1 = \frac{30}{(1000)^2}$

Now, 2 min $= C_1 n^2$

120 sec $= \frac{30}{(1000)^2} n^2$

$n^2 = 120 \times \frac{(1000)^2}{30}$

$n = 2 \times 1000 = 2000$ Ans.

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26 _ _ 10 20 15 2 = 2 way.

26 _ _ 10 2 20 15 = + 1

3 ways.

$\therefore$ 30 + 12 + 3 = 45 Ans.

Algorithm Test Series. [Max heap]

- How many Comparison are needed to find 10th max. element in max heap?

For finding 1st maximum, we need 0 comparison.
" " 2nd " " " 0+1 " .
3rd " " " 0+1+2 " .
⋮
Kth " , " " $\sum K-1$ Comparison.

$\therefore$ 10th maximum, we need. $\frac{9 \times 10}{2}$ = 45 Ans.

Teacher's Signature.....................

Date ____
Page ____

10. Algorithm 2013 [Time Complexity]

- The number of elements that can be ~~sto~~ sorted in $\Theta(\log n)$ time using heap sort is.

A). $\Theta(1)$ B) $\Theta(\sqrt{\log n})$ c) $\Theta\left(\frac{\log n}{\log \log n}\right)$ d) $\Theta \log n$.

We know, for k element $O(k \log k)$ time is needed.

$\therefore$ Let, $k = \frac{\log n}{\log \log n}$.

$$\Rightarrow \therefore \Theta\left(\frac{\log n}{\log \log n} \log \frac{\log n}{\log \log n}\right)$$

$$\Theta\left(\frac{\log n}{\log \log n}\left[\log \log n - \log \log \log n\right]\right)$$

$$\Theta\left(\log n - \underbrace{\frac{\log \log \log n}{\log \log n}}_{\text{very very small}}\right)$$

$\therefore \Theta(\log n)$

$\therefore$ C Ans

Appitude 2013

- The current erection cost of a structure is Rs 13200. If the labour wages per day increases by 1/5 of the current wages. and the working hours decreases by 1/24 of current period, then the new cost of erection in Rs is.