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Electrical Circuit

Current - Rate of flow of charge

Current → $\boxed{I = \frac{dq}{dt}}$

Voltage → Potential difference
↓
work done / energy

Potential → work done / charge

$$\boxed{V_{ab} = \frac{dw}{dq}}$$

$$\boxed{\text{Power } P = \frac{dw}{dt}} \; \boxed{= \frac{dw}{dq} \cdot \frac{dq}{dt} = VI}$$

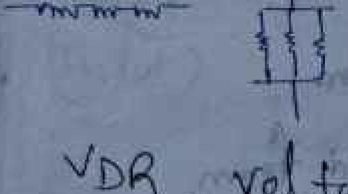
Instantinous Power

Emf = That energy which creats potential difference

Electric circuit = must have atleast one closed path

Electrical network = Connection of elements

Series, Parrallel



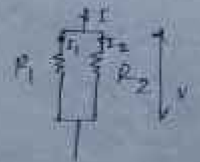
VDR = voltage division rule

CDR = Current division rule

[Apply VDR]

VDR: $V_1 = V \times \frac{R_1}{R_1+R_2}$

$V_2 = V \times \frac{R_2}{R_1+R_2}$



CDR: $I_1 = I \times \frac{R_2}{R_1+R_2}$

$I_2 = I \times \frac{R_1}{R_1+R_2}$

- Active devices = That generate s energy.
  (Voltage current Source, Independent not Independent volt and Current Source)
  eg - Battery, Generator
- Passive devices
  (Resistance, Capacitances, Inductance)
- Resistance is distributed → wire or conductor
  Lumped form / Distributed form
- Continuous and discrete signal
  A signal is continuous w.r.t time

After sampling it becomes discrete

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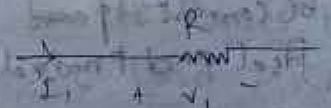
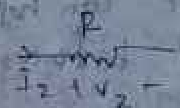
- Linear elements

Conditions - i) Principle of Superposition
ii) Principle of homogeneity
→ If follow this 2 then linear

i) Principle of Superposition - Additive property

ii) Principle of homogeneity - Scaling property





$V_1 = I_1 R$ & $V_2 = I_2 R$

$(I_1 + I_2) R = I_1 R + I_2 R = V_1 + V_2$ → additive property

(Input) (output)

If

Homogeneity
I Property

$I \Rightarrow V = IR$

$\Rightarrow KI \Rightarrow (KI)R = KIR = KV$

(Input) (output)

If we multiply any constant in the input we will get the output is also multiplied with the same constant.

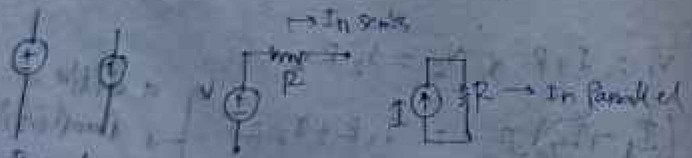
Electronics componets are non linear components. (Transistor)

Linear components (Resistor)

• **Source transformation** [The internal resistance of ideal source is zero]

Ideal source can not be convert.

• The different between ideal and practical source



Ideal current and voltage source

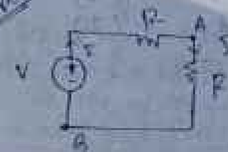
Practical voltage and current source

[R is always in series with voltage and is parallel with current]

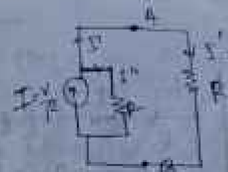
Source transformation

Source transformation

Examples



$I = \frac{V}{R+R_1}$



$I' = \frac{V}{R} \cdot \frac{R}{R+R_1} = \frac{V}{R+R_1}$

For the sources there are no changes in the load. So this circuits of combination are equivalent. The contribution for each of sources (of this picture) to the element R_1 is same. So they are called equivalent.

• **Dependent Sources**

Indicated by dimention diamond

→ Dependent Vol. Source

→ " Current Source

Classification – 4 type of dependent sources

- CCVS → current control voltage source [generally depend on current]
- CCCS → " " current " (I)
- VCVS → voltage " voltage " (V)
- VCCS → " " current " (I)

(CCVS: (V))

• R

$V = IR$ (DC)

$V = V_m \sin\omega t$

$I = I_m \sin\omega t$

$\cancel{V = IR}$ $P = VI$

$= V_m \sin\omega t \, I_m \sin\omega t$

$= V_m I_m \sin^2\omega t$

$P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} P\,dt = \frac{1}{2\pi}\int_0^{2\pi} V_m I_m \sin^2\omega t\,dt$

$= \frac{V_m I_m}{2\pi \times 2}\int_0^{2\pi}(1-\cos 2\omega t)$

$= \frac{V_m I_m}{2\pi \times 2}\left[\omega t - \frac{\sin 2\omega t}{2}\right]_0^{2\pi}$

$= \frac{V_m I_m}{2\pi \times 2}[2\pi - 0 - 0 + 0]$

$= \frac{V_m I_m}{2}$

$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$

$\boxed{P = VI}$

L

- Static emf = when we pass ac current ~~into~~ through a coil, ~~and~~ we get a varying flux. As this flux is varying, there are no physical change. they cancel out each other and

- Dynamic emf –

[Static emf = when we pass ac current through a coil. as the ac current is varying for one cycle it will give a flux and for the another cycle it will give just the opposite flux, these 2 flux cancel out each other, so there will be no physical change. it will give a static effect]

$V \propto \frac{dI}{dt}$

$\Rightarrow V = L\frac{dI}{dt}$ [L = Self inductance]

$= L\frac{d}{dt}(I_m \sin\omega t)$

$= \omega I_m L \cos\omega t$

$= \omega L I_m \cos\omega t$

$\boxed{V = \omega L I_m \sin(\omega t + \frac{\pi}{2})}$

$V = V_m \sin(\omega t + \frac{\pi}{2})$

where $V_m = \omega L \times I_m$ [X_L = Inductive reactance]

$= X_L I_m$

$\frac{V_m}{\sqrt{2}} = X_L \frac{I_m}{\sqrt{2}}$

$\boxed{V = X_L I}$

C 

$C = \frac{q}{v} \Rightarrow q = CV = C\,V_m \sin\omega t$

$I = \frac{dq}{dt}$

$I = \frac{d}{dt}(C V_m \sin\omega t)$

$= \omega C V_m \cos\omega t$

$I = \omega C V_m \cos\omega t = \omega C V_m \sin(\omega t + \frac{\pi}{2})$

$I = I_m \sin\omega t$

where, $I_m = \omega C V_m = \frac{V_m}{(1/\omega C)} = \frac{V_m}{X_C}$ [X_C = capacitive resistance]

$\therefore \boxed{I = X_C V_m \sin(\omega t + \frac{\pi}{2})}$

$I = \frac{dq}{dt}$

$\Rightarrow I_m \sin\omega t = \frac{dq}{dt}$

$\Rightarrow \int dq = \int I_m \sin\omega t \, dt$

$\Rightarrow q = -\frac{I_m \cos\omega t}{\omega}$

$\Rightarrow CV = \frac{I_m}{\omega} \sin(\omega t - \frac{\pi}{2})$

$V = \frac{I_m}{\omega C} \sin(\omega t - \frac{\pi}{2}) = \frac{I_m}{X_C} \sin(\omega t - \frac{\pi}{2})$

$\boxed{V = V_m \sin(\omega t - \frac{\pi}{2})}$

① The total charge entering a terminal is given by $q = 5t \sin 4\pi t$ mC. Calculate the current at $t = 0.5$ s.

$q \rightarrow \quad I = \frac{dq}{dt}$

$= \frac{d}{dt}(5t \sin \omega t \times 10^{-3})$

$= 5 \times 10^{-3} [t \cos 4\pi t \times 4\pi + \sin 4\pi t]$

at $t = 0.5$ s $= 5 \times 10^{-3} [0.5 \cos 4\pi \times 0.5 \times 4\pi + \sin 4\pi \cdot 0.5]$

Here
[$\pi = 3.14$]
[$\pi = 180°$]

$I = 0.0314$ A

$= 31.41$ mA / 1000 A

② Find the power delivered to an element at $t = 3$ ms, if the current entering its positive terminal is $I = 5 \cos 60\pi t$ A and the voltage is a) $V = 3I$ b) $V = 3\frac{dI}{dt}$

Find the power delivered.

a) $P = VI$ $\quad t = 3$ ms $= 3 \times 10^{-3}$

$= 3I \times I$

$= 3 \times (5 \cos 60\pi t)^2$

~~$= 15 \cos 60\pi \times 3 \times 10^{-3}$~~

~~$= 0.0711$ W~~

~~$= 15 \cos 60\pi \times 10^{-3}$~~

$= 53.166$ Ans

b) $v = 3\frac{di}{dt}$

$P = vi$

$= 3\frac{di}{dt} i$

$= 3(5\cos 60\pi t)\frac{d}{dt}(5\cos 60\pi t)$

$= -15 \times 5 \cos 60\pi t \sin 60\pi t \times 60\pi$

at $t = 3$ $= -75 \times 60\pi \times \cos 60\pi t \times \sin \pi \times 60 \times 3$

$[\pi = 180°]$

$= \cancel{-139.52 W}$

$[-4500\pi \sin 60\pi t \cos 60\pi t]$

$= \cancel{-366.45 \times 10^3} W$

$= -\frac{1}{2} \times 4500\pi$

$= -750 \times 60 \times 3.14 \times 0.905$

$2 \sin 60\pi t \cos 60\pi t$

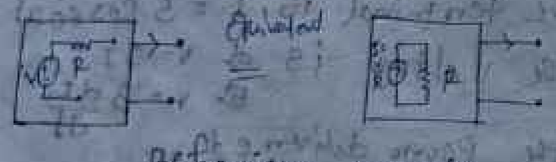
$= -6.39 \times 10^3$

$= -2250\pi \times \sin 120\pi t$

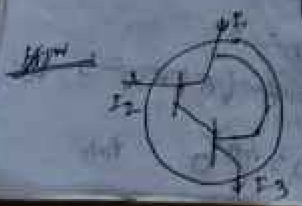
$= -6.39 kW$

rad

- load → deliver/supply = '−'ve
- absorb = '+'ve

- 

Behaviour remain same, whatever we do in through internal



① Current – the fundamental electric property

An electric current is a flow of electrical charge, electric charge flows when there is voltage present accross a conductor.

$\boxed{I(\text{current}) = \frac{dq}{dt}}$

② Voltage / Potential difference – voltage, electric potential difference is the difference in electric potential between two points, which is defined as work needed per unit of charge to move a test charge between the two points.

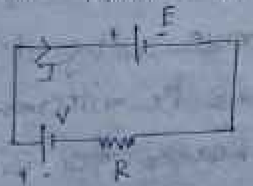
$\boxed{V \text{ or } \frac{dw}{dq}}$

③ Power – The rate of doing work or the rate of using energy is called power.

$\boxed{P(\text{Instantaneous power}) = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = VI}$

④ Instantaneous value – The value of alternating quantity at any particular instant is called instantaneous value. It is denoted by lower letters, such as e for emf, i for current, v for voltage.

(5) Emf- we define emf is the voltage of battery when no current is flowing, while terminal potential difference is voltage across battery when connected to a resistan and current is flowing through it. Emf is that energy which create potential difference.



(6) Electric Circuit - An electronic circuit is composed of individual electronic components such as resistors, capacitors, transistors etc connected by conductive wires through which electric current can flow. It must possess one closed path.

(7) Electric network - It is connection of electric elements, such as resistors, capacitors, transistors etc.

(8) Active devices - An active device is any type of circuit component with the ability to electrically control electric charge flow (electricity controlling electricity). In order for a circuit to be properly called electronic, it must contain at least one active device. Active devices generates energy. Ex voltage, current source (dependent and independent) include active devices. Examples - Battery, generator, transistors, vacuum tubes etc.

(9) Passive devices - This do not generate energy, but can store/consume it or dissipate it. Passive devices are the main components used in electronics such as resistors, inductors, transformers, capacitors which together are required to build any electrical or electronic circuit. Ex - Resistors, capacitors, diodes etc.

(10) lumped network - A lumped network is usually a simplification of distributed networks such as a transmission line. A transmission line has built in resistance, inductance and capacitance for every tiny fraction of it's length.

It would take a lot of calculations to do any calculation on these.

(11) Distributed network - A distributed network is a type of computer network that is spread over different networks. This provides a single data communication network, which can be managed jointly or separately by each network. Besides shared communication within the network, a distributed network often also distributes processing.

(12) Continuous signal - A continuous signal or a continuous-time signal is a varying quantity (a signal) whose domain, which is often time, is a continuum. That is, the function's domain is an uncountable set. The function itself need not to be continuous.

(13) Discrete signal - A discrete signal or discrete time signal is a sequence of values that correspond to particular instants in time. The time instants at which the signal is defined are the signal's sample times and the associated signal values are the signal's samples.

(14) Linear elements - These are elements in which the constituent relation, the relation between voltage and current, is a linear function. They obey superposition principle. Ex - Resistance, capacitance, inductance etc.

(15) Nonlinear elements - In non-linear circuit, the non-linear elements are an electrical element and it will not have any linear relationship between the current and voltage. Ex - Diode, Some of the nonlinear elements are not there in the electric circuit is called a linear circuit.

(16) Principle of Superposition - This is also known as superposition property, states that for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually.

(17) Principle of Homogeneity - It states that dimentions of each of the terms of a dimensional equation on both sides should be the same. This helps to convert Principle

the units from one form to another.

(18) Scaling property - the time scaling property of Fourier transformation states that if a signal is extended in time by a quantity (a), then its Fourier transformation is compressed in frequency by the same amount. (no need)

(19) Additive property - It ~~is a st al~~ (no need)

(20) Source transformation - It is a process of simplifying a circuit solution especially with mixed sources by transforming voltage source into current sources and vice versa using thevenin's theorem and Norton's theorem respectively.

(21) Ideal Source - These are those imaginary electrical sources, that provides constant voltage or current to the circuit regardless of the load current. These ideal sources don't have any internal resistance where it is impossible to build a source with zero internal resistance.

(22) Practical Source - A practical source is a two terminal device having some resistance connected across its terminals. Unlike ideal current source, the output current of practical source depends on the voltage of the source. The more this voltage, the lesser will be the current.

(23) Diff between ideal and practical source -
The ideal source don't have any internal resistance. ~~Ag~~ Again the practical source has internal resistance.
The ideal source provides constant voltage or current regardless of the load current. Again the practical source output current depends on the voltage of the source.

(24) VCVS - where the actual source voltage is controlled by the voltage across any element elsewhere in that circuit.

VCCS - A amplifier that converts a voltage to a current, where the actual source voltage is controlled by the current through any element elsewhere in that circuit

CCVS – A voltage source that depends on a current input

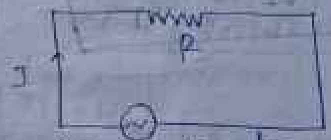
CCCS – A current source that depends on a current input

(25) Static emf – When the conductor is stationary and the magnetic field is changing, the induced EMF in such a way is known as statically induced EMF. It is so called because the EMF is induced in a conductor which is stationary. Ex – transformer

(26) Dynamic emf – When the conductor is moved in a stationary magnetic field so that the magnetic flux linking with it changes in magnitude, as the conductor is subjected to a changing magnetic, therefore an EMF will be induced in it. The EMF induced in this way is known as dynamically induced EMF. Ex – DC or AC generator

(27) Dependent Source – In the theory of electrical networks, a dependent source is a voltage or a current source whose value depends on a voltage or current elsewhere in the network. Dependent sources are useful, for example, in modeling the behaviour of amplifiers. It is represented by diamond shape.

(28) Pure resistance – The circuit which contains a pure resistance or has negligible inductance is called pure resistance circuit.



$V = V_m \sin \omega t$

for pure resistance,

the current in the circuit is $= I = \frac{V}{R}$

$= \frac{V_m \sin \omega t}{R}$

$= \frac{V_m}{R} \sin \omega t$

$\boxed{I = I_m \sin \omega t}$ – (1)

$\left[\because I_m = \frac{V_m}{R}\right]$

The current will be maximum if $\omega t = \frac{\pi}{2}$ and $\sin\omega t = 1$, therefore

$$I_m = \frac{V_m}{R}$$

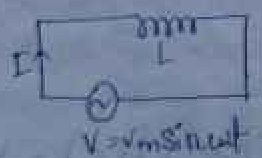
By placing I_{max} in eqn ①, we get

$$\boxed{I = I_m \sin\omega t}$$

The avg power of a pure resistive circuit

$$\boxed{P_{av} = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms} = VI}$$

(ii) Pure inductance-



The voltage is $v = v_m \sin\omega t$

The self induced emf of the coil is $= -L\frac{dI}{dt}$

Now, applied voltage is equal and opposite to self-induced emf. Therefore,

~~v = vm sin ωt~~ net emf,

$$v - L\frac{dI}{dt} = 0 \quad [\because R = 0, IR = 0]$$

$$\therefore v = L\frac{dI}{dt}$$

$$\Rightarrow V_m \sin\omega t = L\frac{dI}{dt}$$

$$\Rightarrow \int dI = \int \frac{V_m}{L}\sin\omega t \, dt$$

$$\Rightarrow I = -\frac{V_m}{\omega L}\cos\omega t$$

$$\Rightarrow I = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$

The current I will be maximum when, $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$

$$\therefore I_m = \frac{V_m}{\omega L}$$

$$\boxed{I = I_m \sin\left(\omega t - \frac{\pi}{2}\right)}$$

[The voltage is leading and the current is lagging, in the case for pure inductive circuit]

$$\boxed{X_L = \omega L}$$

[X_L = Inductive resistance]

$$\boxed{P_{av} = 0}$$

The avg power absorbed over a complete cycle in pure inductor is zero.

(30) Pure capacitor —

$v = V_m \sin\omega t$

The applied voltage is $v = V_m \sin\omega t$
The charge stored in the capacitor is

$q = Cv = C V_m \sin\omega t$

now, $I = \frac{dq}{dt} = \frac{d}{dt}(C V_m \sin\omega t)$

$\Rightarrow I = \omega C V_m \cos\omega t$

$\Rightarrow I = \frac{V_m}{(1/\omega C)} \cos\omega t$

$\Rightarrow I = \frac{V_m}{1/\omega C} \sin(\omega t + \frac{\pi}{2})$

now I will be maximum, when $\sin(\omega t + \frac{\pi}{2}) = 1$

$\therefore I_m = \frac{V_m}{1/\omega C}$

$\therefore \boxed{I = I_m \sin(\omega t + \frac{\pi}{2})}$

$\boxed{X_C = \frac{1}{\omega C}\,\Omega}$ [X_C = capacitive resistance]

The average power absorbed over a complete cycle is zero $\boxed{P_{avg} = 0}$

Hence, the current is leading and voltage is lagging.

(31) $P_{avg} = \frac{1}{2\pi}\int_0^{2\pi} p\,dt$

$= \frac{1}{2\pi}\int_0^{2\pi} vI\,dt$

$= \frac{1}{2\pi}\int_0^{2\pi} V_m \sin\omega t \; I_m \sin\omega t\,dt$ $\left[\because v = V_m \sin\omega t,\ I = I_m \sin\omega t\right]$

$= \frac{V_m I_m}{2\times 2\pi}\int_0^{2\pi} 2\sin^2\omega t\,dt$

$= \frac{V_m I_m}{4\pi}\int_0^{2\pi} (1 - \cos 2\omega t)\,dt$

$= \frac{V_m I_m}{4\pi}\left[t - \frac{\cos 2\omega t}{2\omega}\right]_0^{2\pi}$

$= \frac{V_m I_m}{4\pi}\left[(2\pi - 1) - (0 - 1)\right]$

$= \frac{V_m I_m}{4\pi} \times 2\pi$

$= \frac{V_m I_m}{\sqrt{2}\times\sqrt{2}} = \frac{V_m}{\sqrt{2}} \cdot \frac{I_m}{\sqrt{2}} = V_{rms} I_{rms}$

$\boxed{P = VI}$

(32) $\boxed{P_{av} = V_{rms} I_{rms} \cos\varphi}$ [φ = Angle between V and I]
[φ = phase diff / phasor angle also]

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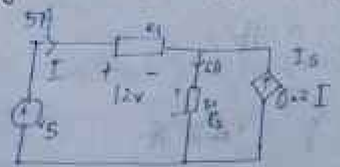
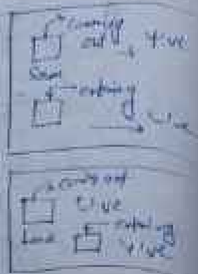
③ Calculate the power supplied or absorbed by each element in the given figure.





For 20V, $P = V_s I = 20\times5 = 100W$

For 12V, $P = 12\times5 = +60W$

For 8V, $P = 8\times6 = +48W$

$I = 5\times\frac{8}{8+8} = 5$

For 0.2I, $P = VI = 8\times(0.2\times5)$ $= 8W$

$\Rightarrow 20 = V\times 0.2$

$\Rightarrow V = \frac{20}{0.2} = 100$ ~~$= 100$~~

~~$P = 100\times0.2$~~

Source is delivering	Load is absorbing
100 W	60 W
+ 8 W	+ 48 W
108 W	108 W

In any circuit Source delivers Power = load absorbs Power

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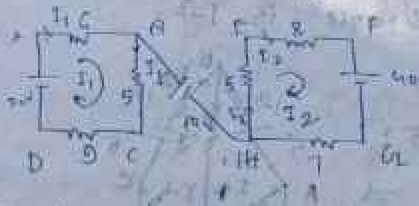
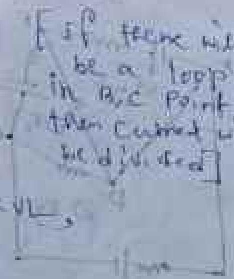
④ Find V_{CE} and V_{AG} in the given circuit.





In loop ABCD writing KVL,

$6I_1 + 5I_1 + 9I_1 = 20$

$\Rightarrow 20I_1 = 20$

$\Rightarrow I_1 = 1A$

In loop EFGH writing KVL,

$8I_2 + 7I_2 + 5I_2 = -40$

$\Rightarrow 20I_2 = -40$

$\Rightarrow I_2 = -2A$

CBHE,

$V_{CE} = -5I_1 + 5I_2 + 10$

$= -5I_1 + 5I_2 + 10$

$= -5\times1 + 5\times(-2) + 10$

$= -5V$

CBHGFE,

$V_{CE} = -5 + 10 + 14 - 40 + 16$

$= -5$

ABHG,

$V_{AG} = 6I_1 + 10 - 7I_2$

$= 6 + 10 + 7\times2$

$= 16 + 14$

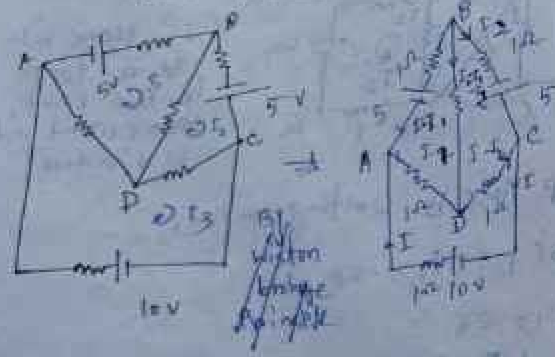
$= 30V$

$V_{AG} = 6\times1 + 40 - 10 - 16 + 10$

$= 30V$

Loop/mesh analysis (Application of KVL)

① Determine the branch currents in the network when the value of each branch resistance is 1Ω.



In loop ABDA applying KVL,

$(I-I_1)+(I-I_1-I_2)-I_1=5$

$\Rightarrow I-I_1+I-I_1-I_2-I_1=5$

$\Rightarrow 2I-3I_1-I_2=5$ — (i)

$\Rightarrow 3I_1-I_2-I_3=5$ — (i)

BCDB,

$(I-I_1-I_2)+I_2=5$

$\Rightarrow I-I_1-2I_2=5$

$\Rightarrow -I_1+3I_2-I_3=5$ — (ii)

~~ABCA~~ ADCA,

$-I_1-I_2+3I_3=10$ — (iii)

Solve by matrix form —

$$A=\begin{bmatrix}3 & -1 & -1\\ -1 & 3 & -1\\ -1 & -1 & 3\end{bmatrix} \quad B=\begin{bmatrix}5\\5\\10\end{bmatrix}$$

$$X=\begin{bmatrix}x\\y\\z\end{bmatrix} \qquad \begin{matrix}x=I_1\\ y=I_2\\ z=I_3\end{matrix}$$

$AX=B$

$X=A^{-1}B$

$$|A|=\begin{vmatrix}3 & -1 & -1\\ -1 & 3 & -1\\ -1 & -1 & 3\end{vmatrix}$$

$=3(9-1)+1(-3-1)-1(1+3)$

$=3\times8-4-1\times4$

$=24-4-4$

$=16$

$$\text{adj}A=\begin{bmatrix}\begin{vmatrix}3 & -1\\ -1 & 3\end{vmatrix} & -\begin{vmatrix}-1 & -1\\ -1 & 3\end{vmatrix} & \begin{vmatrix}-1 & 3\\ -1 & -1\end{vmatrix}\\ -\begin{vmatrix}-1 & -1\\ -1 & 3\end{vmatrix} & \begin{vmatrix}3 & -1\\ -1 & 3\end{vmatrix} & -\begin{vmatrix}3 & -1\\ -1 & -1\end{vmatrix}\\ \begin{vmatrix}-1 & -1\\ 3 & -1\end{vmatrix} & -\begin{vmatrix}3 & -1\\ -1 & -1\end{vmatrix} & \begin{vmatrix}3 & -1\\ -1 & 3\end{vmatrix}\end{bmatrix}^T$$

$$= \begin{bmatrix} (9-1) & -(-3-1) & (1+3) \\ -(-3-1) & (9-1) & -(-3-1) \\ (1+3) & -(-3-1) & (9-1) \end{bmatrix}^T$$

$$= \begin{bmatrix} 8 & 4 & 4 \\ 4 & 8 & 4 \\ 4 & 4 & 8 \end{bmatrix}^T$$

$$= \begin{bmatrix} 8 & 4 & 4 \\ 4 & 8 & 4 \\ 4 & 4 & 8 \end{bmatrix}$$

$$x = \frac{1}{16} \begin{bmatrix} 8 & 4 & 4 \\ 4 & 8 & 4 \\ 4 & 4 & 8 \end{bmatrix} \times \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \\ 3 & -1 & -1 \\ -1 & -1 & 3 \\ -1 & 3 & -1 \end{vmatrix}$$

$$= (27 - 1 - 1) - (3 + 3 + 3)$$

$$= 16$$

$$= 5 \times \frac{1}{16_4} \times \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{5}{4} \begin{bmatrix} 2+1+2 \\ 1+2+2 \\ 1+1+4 \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 40+20+40 \\ 20+40+40 \\ 20+20+80 \end{bmatrix}$$

$$= \frac{1}{16} \begin{bmatrix} 100 \\ 100 \\ 120 \end{bmatrix}$$

$I_1 = \frac{100}{16} = 6.25\ A$

$I_2 = \frac{100}{16} = 6.25\ A$

$I_3 = \frac{120}{16} = 7.5\ A$

The branch current of AB $= I_1 = 6.25$

" " " BC $= I_2 = 6.25$

" " " CD $= I_2 - I_3 = -1.25$

" " " DA $= I_1 - I_3 = -1.25$

" " " CA $= I_3 = 7.5$

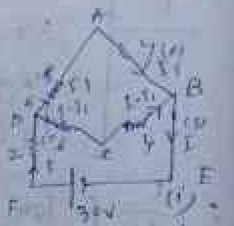
" " " BD $= I_1 - I_2 = 0$

(ii) solved by Cramer's rule

[B] Applying Wheatstone bridge principle $\longrightarrow$ P.T.O.]

HW Find I_0 using mesh analysis.

(3)



By wheatstone bridge principle we know that if $\frac{P}{Q} = \frac{R}{S}$ then the middle resistance will be removed. Here, $\frac{6}{6} = \frac{4}{4}$. So the ~~diagram~~ fig (1) is drawn by us.

Now,

Applying KVL on ABCDA, $6I_1 + 4I_1 - 4(I - I_1) - 6(I - I_1) = 0$

$\Rightarrow 10I_1 - 4I + 4I_1 - 6I + 6I_1 = 0$

$\Rightarrow 20I_1 - 10I = 0$

$\Rightarrow 20I_1 = 10I$

$\Rightarrow 2I_1 = I$

$\Rightarrow I = 2I_1$ — (i)

DCBFED, $6(I - I_1) + 4(I - I_1) + 2I = \frac{30}{10^3}$

$\Rightarrow 6I - 6I_1 + 4I - 4I_1 + 2I = 30 \times 10^{-3}$

$\Rightarrow 12I - 10I_1 = 30 \times 10^{-3}$

$\Rightarrow 12 \times 2I_1 - 10I_1 = 30 \times 10^{-3}$ [∵ from (i)

$\Rightarrow 24I_1 - 10I_1 = 30 \times 10^{-3}$ $I = 2I_1$]

$\Rightarrow 14I_1 = 30 \times 10^{-3}$

$\Rightarrow I_1 = 2.143 \times 10^{-3} A = 2.143 mA$

∴ from (i)

$I = 2 \times I_1$

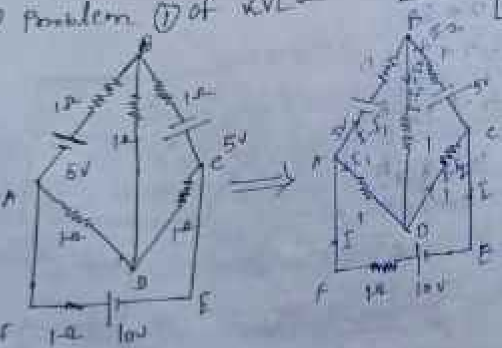
$= 2 \times 2.143$

$= 4.286 A$

$I = I_0 = 4.286 A$

This is not correct

(1) Problem (1) of KVL —



Sol

Applying KVL in ABDA,

$(I - I_1) + (I - I_1 - I_2) - I_1 = 5$

$\Rightarrow I - I_1 + I - I_1 - I_2 - I_1 = 5$

$\Rightarrow 2I - 3I_1 - I_2 = 5$ — (i)

Applying KVL in BCDB,

$I_2 - (I - I_2) - (I - I_1 - I_2) = 5$

$\Rightarrow I_2 - I + I_2 - I + I_1 + I_2 = 5$

$\Rightarrow -2I + I_1 + 3I_2 = 5$ — (ii)

Applying KVL in ADCEFA,

$I_1 + I - I_2 + I = 10$

$\Rightarrow 2I + I_1 - I_2 = 10$ — (iii)

From (i), (ii), (iii) we get a eqn $AI = B \Rightarrow I = A^{-1}B$

Let, $A = \begin{bmatrix} 2 & -3 & -1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$

$|A| = \begin{vmatrix} 2 & -3 & -1 \\ -2 & 1 & 3 \\ 2 & 1 & -1 \end{vmatrix}$

$= 2(-1-3) + 3(2-6) - 1(-2-2)$

$= -8 - 12 + 4$

$= -16$

$\text{adj} A = \begin{bmatrix} \begin{vmatrix} 1 & 3 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} -2 & 3 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix} \\ -\begin{vmatrix} -3 & -1 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} 2 & -3 \\ 2 & 1 \end{vmatrix} \\ \begin{vmatrix} -3 & -1 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ -2 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -3 \\ -2 & 1 \end{vmatrix} \end{bmatrix}^T$

$= \begin{bmatrix} (-1-3) & -(2-6) & (-2-2) \\ -(3+1) & -(-2+2) & -(2+6) \\ (-9+1) & -(6-2) & (2-6) \end{bmatrix}^T$

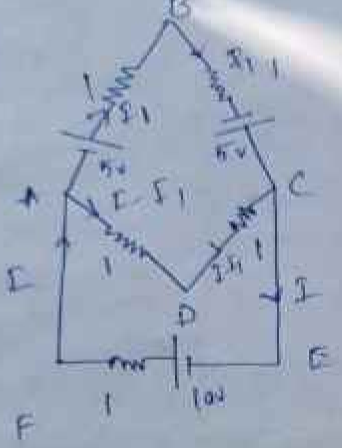
$= \begin{bmatrix} -4 & 4 & -4 \\ -4 & 0 & -8 \\ -8 & 4 & -4 \end{bmatrix}^T$

$= \begin{bmatrix} -4 & -4 & -8 \\ 4 & 0 & 4 \\ -4 & -8 & -4 \end{bmatrix}$

$I = A^{-1}B = \frac{4 \times}{16} \begin{bmatrix} -1 & -1 & -2 \\ 1 & 0 & 1 \\ -1 & -2 & 1 \end{bmatrix} \times \begin{bmatrix} 5 \\ 5 \\ 10 \end{bmatrix}$

$= \frac{4}{16} \begin{bmatrix} -5 - 5 - 20 \\ 5 + 0 + 10 \\ -5 - 10 + 10 \end{bmatrix}$

By wiston brigde Principle -



$ABCDA$, $I_1 + I_1 - (I - I_1) - (I - I_1) = 5 + 5$

$\Rightarrow 2I_1 - I + I_1 - I + I_1 = 10$

$\Rightarrow 4I_1 - 2I = 10$

$\Rightarrow 2I_1 - I = 5$ — (i)

$ADCEFA$,

$I - I_1 + I - I_1 + I = 10$

$\Rightarrow 3I - 2I_1 = 10$

$\Rightarrow -2I_1 + 3I = 10$ — (ii)

$\cancel{2I_1} - I = 5$ — (i)

$\cancel{-2I_1} + 3I = 10$ — (ii)

$\Rightarrow 2I = 15$

$\Rightarrow I = \frac{15}{2} = 7.5$

$2I_1 - 7.5 = 5$

$\Rightarrow I_1 = 6.25$

A| =

The branch currents are,

current on AB = 6.25 A

" " BC = 6.25 A

" " CD = 1.25 A

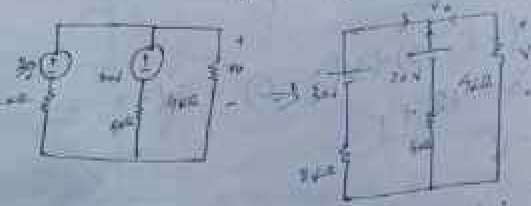
" " DA = 1.25 A

" " CA = 7.5 A

I_3 A

• Nodal analysis

Ex-① Find v_0 by using node analysis



$$\frac{v_0 - 30}{2\times10^3} + \frac{v_0 - 20}{5\times10^3} + \frac{v_0}{4\times10^3} = 0$$

$$\Rightarrow \frac{10(v_0 - 30) + 4(v_0 - 20) + 5v_0}{20\times10^3} = 0$$

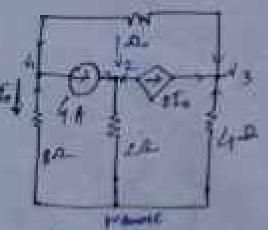
$$\Rightarrow 10v_0 + 4v_0 + 5v_0 - 300 - 80 = 0$$

$$\Rightarrow 19v_0 = 380$$

$$\Rightarrow v_0 = 20V$$

Ex-②

Find I_0 in the given circuit



(4 nodes)

node-1

$$\frac{v_1}{8} + \frac{v_1 - v_3}{1} \cancel{+4 = 0} = -4$$

$$\Rightarrow v_1 + 8v_1 - 8v_3 = -32$$

$$\Rightarrow 9v_1 - 8v_3 = -32 \quad \text{— (1)}$$

node-2

$$\cancel{4} + \frac{v_2}{2} = 4 - 2I_0$$

$$\Rightarrow v_2 = 8 - 4I_0$$

$$\Rightarrow v_2 = 8 - 4\times\frac{v_1}{8}\,2 \qquad \left[\because I_0 = \frac{v_1}{8}\right]$$

$$\Rightarrow 2v_2 = 16 - v_1$$

$$\Rightarrow 2v_2 + v_1 = 16 \quad \text{— (1)}$$

node-3

$$\frac{v_3}{4} + \frac{v_3 - v_1}{1}$$

$$\Rightarrow \frac{v_3}{4} + \frac{v_3 - v_1}{1} = 2\times\frac{v_1}{8}$$

$$\Rightarrow v_3 + 4v_3 - 4v_1 = v_1$$

$$\Rightarrow 5v_3 - 5v_1 = 0$$

$$\Rightarrow v_1 = v_3 \quad \text{— (11)}$$

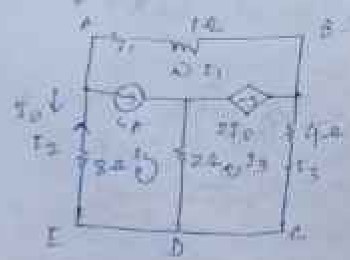
from ①,

$$9v_1 - 8v_1 = -32$$

$$\Rightarrow v_1 = -32$$

$$\therefore I_0 = \frac{v_1}{8} = \frac{-32}{8} = -4A$$

mesh analysis



[when no voltage source then the eqn of the whole loop give the eqn (1)]

In supermesh, ABCDEA

$I_1 + 4I_3 + 8I_2 = 0$ — (i)

$I_2 - I_1 = 4$ — (ii)

$I_3 - I_1 = 2I_0$ — (iii)

$[\because I_0 = -I_2]$

$\Rightarrow I_3 - I_1 = -2I_2$

$\Rightarrow -I_1 + 2I_2 + I_3 = 0$ — (iv)

$\therefore$ Let, $A = \begin{bmatrix} 1 & 8 & 4 \\ -1 & 1 & 0 \\ -1 & 2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$

the eqn is

$AI = B$

$I = A^{-1}B$

$|A| = \begin{vmatrix} 1 & 8 & 4 \\ -1 & 1 & 0 \\ -1 & 2 & 1 \end{vmatrix} = 1(1-0) - 8(-1-0) + 4(-2+1)$

$= 1 + 8 - 4$

$= 5$

adj $A = \begin{bmatrix} \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} -1 & 0 \\ -1 & 1 \end{vmatrix} & \begin{vmatrix} -1 & 1 \\ -1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 8 & 4 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 8 \\ -1 & 2 \end{vmatrix} \\ \begin{vmatrix} 8 & 4 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 1 & 4 \\ -1 & 0 \end{vmatrix} & \begin{vmatrix} 1 & 8 \\ -1 & 1 \end{vmatrix} \end{bmatrix}^T$

$= \begin{bmatrix} (1-0) & -(-1-0) & (-2+1) \\ -(8-8) & (1+4) & -(2+8) \\ (0-4) & -(0+4) & (1+8) \end{bmatrix}^T$

$= \begin{bmatrix} 1 & 1 & -1 \\ 0 & 5 & -10 \\ -4 & -4 & 9 \end{bmatrix}^T$

$= \begin{bmatrix} 1 & 0 & -4 \\ 1 & 5 & -4 \\ -1 & -10 & 9 \end{bmatrix}$

Now, $I = \frac{1}{5}\begin{bmatrix} 1 & 0 & -4 \\ 1 & 5 & -4 \\ -1 & -10 & 9 \end{bmatrix} \times \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix}$

$= \frac{1}{5}\begin{bmatrix} 0+0+0 \\ 0+20+0 \\ 0-40+0 \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 4 \\ -8 \end{bmatrix}$

$\therefore I_2 = 4 = \cancel{I_0}$

we know, $I_0 = -I_2 = -4$

$\therefore I_0 = -4A$

$I_1 = I_2 - 4$ — (IV)

From (I), $I_2 - 4 + 4I_3 + 8I_2 = 0$

$\Rightarrow 9I_2 + 4I_3 - 4 = 0$

$\Rightarrow I_3 = \dfrac{4 - 9I_2}{4}$ — (V)

From (III), applying (IV) and (V)

$-I_2 + 4 + 2I_2 + \dfrac{4 - 9I_2}{4} = 0$

$\Rightarrow I_2 + 4 + \dfrac{4 - 9I_2}{4} = 0$

$\Rightarrow 4I_2 + 16 + 4 - 9I_2 = 0$

$\Rightarrow -5I_2 = -20$

$\Rightarrow I_2 = 4$

$\therefore I_1 = I_2 - 4$

① Nodal analysis - The mathematical method for calculating the distribution of voltage between the nodes in a circuit. This is a application of KCL.

② node - junction point [minimum 1 element should be connected in between 2 nodes]

③ mesh analysis - The method in which the current flowing through a planar circuit is calculated. This is a application of KVL.

④ KCL (~~loop/mesh~~ nodal analysis) - it states that the algebraic sum of the currents entering or leving a junction point at any instant is equal to zero.

$$\Sigma I = 0$$

KCL is also known as Kirchhoff's first law or junction rule. The principle of this law is to conserve the electric charge. The law states that the amount of current flowing into a node is equal to the sum of current out of it. For performing the nodal analysis, we have to go for KCL.

$$I_1 + I_2 + I_3 = I_5 + I_4$$

$$\Rightarrow I_1 + I_2 + I_3 + (-I_4) + (-I_5) = 0$$

The currents I_1, I_2, I_3 entering the node are considered positive values, while the currents I_4 and I_5 leaving the nodes are considered negative values.

⑤ KVL (mesh/loop analysis) - KVL is also known as Kirchhoff's second law or loop law. The principle of this law is to conserve energy. The law states that the sum of voltages is in a closed loop is zero. The total amount of energy gained is equal to the energy lost per unit charge.

$$\Sigma V = 0$$

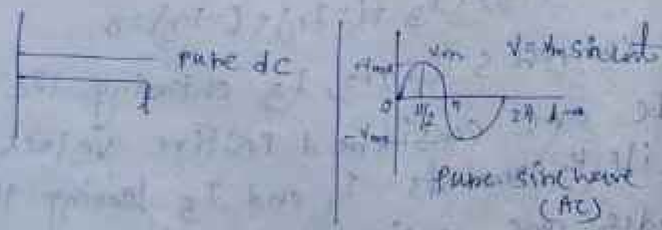
$$\Rightarrow \Sigma IR = \Sigma E$$

$$\Rightarrow \Sigma V = \Sigma E$$ [V and E is same both of them is voltage]

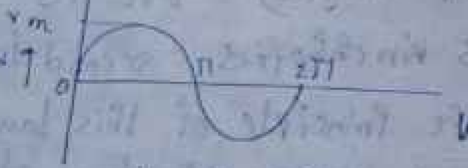
For loop/mesh analysis, we have to go for KVL.

- $P = VI\cos\varphi$ [Angle between voltage and current $= \varphi$]
  [AC]

  $P = VI$ (DC)

- 

- AC fundamentals and circuits —



A sinusoidal wave

The expression for AC voltage

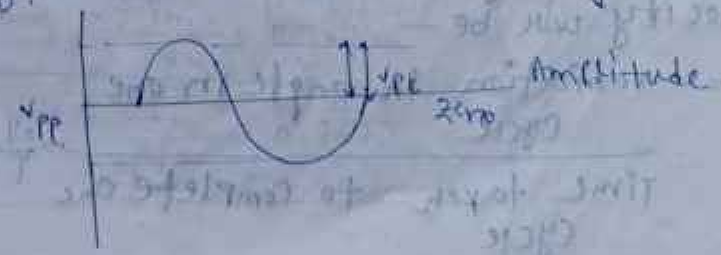
$\boxed{v = V_m \sin\omega t}$ [v = Instantaneous value, V_m = max value]

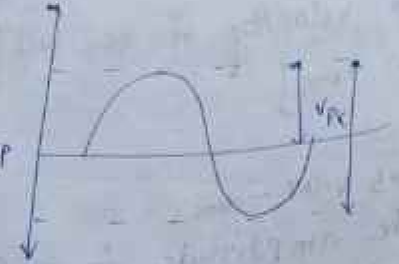
The expression for alternating current is — $\boxed{I = I_m \sin\omega t}$ [I = Instantaneous value, I_m = max value]

[ω = Angular velocity of the coil]

[$I = I_{rms}$, $V = V_{rms}$]

- Instantaneous value — done
- Amplitude — The amplitude of a sine wave is the maximum vertical distance reached in either direction from the center line of the wave. As a sine wave is symmetrical about its center line, the amplitude of the wave is half of the peak to peak value.
- Peak to peak value — The peak to peak value is the vertical distance between the top and bottom of the wave. volts for a voltage waveform.
- Peak value — The peak value of the wave is the highest value the wave reaches above a reference value. The reference value normally used is zero.





• Cycle and Alternation – When an alternating quantity completes one complete set of positive and negative values, it is called one cycle, whereas when it completes only one set of either positive or negative values, that is, half cycle, then it is called one alternation. One alternation corresponds to 180° or π rad while one cycle is 360° or 2π rad.

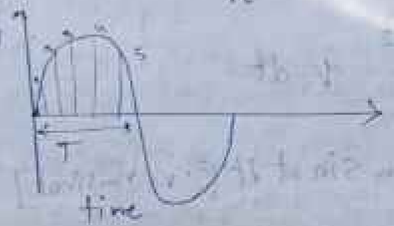
• Angular Frequency or angular velocity – An alternating quantity completes one cycle by 2π rad. The angular velocity will be –

$$\omega = \frac{\text{variation in angle in one cycle}}{\text{Time taken to complete one cycle}} = \frac{2\pi}{T} = \frac{2\pi}{(1/f)}$$

$= 2\pi f$ electrical rad/s

• Graphically –

$$V_{av} = \frac{v_1 + v_2 + \dots + v_n}{n}$$



• Analytically –

$$\boxed{\text{Area of half cycle voltage wave} = 2V_m}$$

V_{av} = Height the equivalent rectangle having the same base $= \frac{\text{Area}}{\text{base}}$

$V_{av} = \frac{2V_m}{\pi} = 0.637\,V_m$	
$I_{av} = \frac{2I_m}{\pi} = 0.637\,I_m$	

•

$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\,V_m$	
$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\,I_m$	

• $v_{rms} = v_{av} \times \frac{\pi}{2\sqrt{2}}$

$$\Rightarrow \boxed{\frac{v_{rms}}{v_{av}} = \frac{\pi}{2\sqrt{2}}}$$

• $$V_{av} = \frac{1}{\left(\frac{T}{2}\right)}\int_0^{T/2} v\,dt$$

$$= \frac{2}{T}\int_0^{T/2} V_m \sin\omega t\,dt \quad [\because v = V_m \sin\omega t]$$

$$= \frac{2V_m}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2}$$

$$= \frac{2V_m}{T\omega}\times\left[-\cos\frac{\omega T}{2}+1\right]$$

$$= \frac{2V_m}{2\pi}\times\left(1-\cos\frac{2\pi}{2}\right) \quad \left[\because T = \frac{2\pi}{\omega},\ \omega T = 2\pi\right]$$

$$= \frac{2V_m}{\pi}$$

$$V_{rms}^2 = \frac{1}{T}\int_0^T v^2\,dt$$

$$= \frac{V_m^2}{T}\int_0^T \sin^2\omega t\,dt$$

$$= \frac{V_m^2}{2T}\int_0^T 2\sin^2\omega t\,dt$$

$$= \frac{V_m^2}{2T}\int_0^T (1-\cos 2\omega t)\,dt$$

$$= \frac{V_m^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{V_m^2}{2T}\left[T - \frac{\sin 2\omega T}{2\omega}\right]$$

$$= \frac{V_m^2}{2T}\left[T - \frac{\sin 4\pi}{2\omega}\right]$$

$$= \frac{V_m^2}{2T}\times T$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

• For a DC circuit, the relationship between voltage and current is simple and is expressed as, $I = \frac{V}{R}$. However, in the case of an AC circuit, the magnetic effect and the electrostatic effect are also taken into account.

• Calculate the rms and average value of a square wave, shown in the fig. If the current is passed through 2Ω resistor, find out average power absorbed by it.

$$I_{rms}^2 = \frac{1}{T}\int_0^T I(t)^2\,dt = \frac{1}{T}\int_0^T I_m^2(t)\,dt$$

$$I_{av} = \frac{1}{T}\int_0^T I(t)\,dt = \frac{1}{T}\int_0^T I_m(t)\,dt$$

[$I_m = I_{max}$ = Amplitude / magnitude of I]

For the time interval $0 < t < 0.2$, $I = 10A$ and for $0.2 < t < 0.4$, $I = 0A$. Time period of the current waveform, $T = 0.4$, the rms value is given by

$$I^2_{rms} = \frac{1}{T}\int_0^T I^2(t)\,dt$$

$$= \frac{1}{0.4}\left[\int_0^{0.2} 10^2\,dt + \int_{0.2}^{0.4} 0^2\,dt\right]$$

$$= \frac{1}{0.4}\left[\int_0^{0.2} 100\,dt\right]$$

$$= \frac{100}{0.4} \times [0.2 - 0]$$

$$= \frac{100}{2} = 50$$

$$I_{rms} = \sqrt{50} = 7.071A$$

$$I_{avg} = \frac{1}{T}\int_0^T I(t)\,dt$$

$$= \frac{1}{0.4}\left[\int_0^{0.2} 10\,dt + \int_{0.2}^{0.4} 0\,dt\right]$$

$$= \frac{10}{0.4}[0.2 - 0]$$

$$= 5$$

Avg power absorbed $= I^2_{rms} R$

$= 50 \times 2$

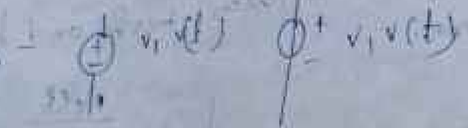
$= 100W$

- Distinguish between mesh and loop of a network.

Loop	Mesh
1) Loop is any closed path through a circuit where no node more than one is encountered.	1) A mesh is a closed path in a circuit with no other paths inside it.
2) To find a loop, start at node in the ckt, from this node, move back to the same node along a path to ensure that no node is encountered more than once.	2) In other terms, a loop with no other loops inside it.

- Use: Independent voltage source – An independent voltage source is an idealized circuit component that fixes the voltage in a branch respectively, to a specified value. The middle symbol is the symbol for a specific type of independent voltage source known

circuit diagram

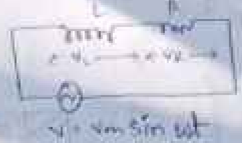


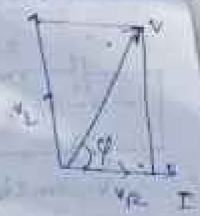
- Independent current source - An independent current source is an energy source that pushes a constant flow of electrons through an electrical circuit regardless of the load presented to it. In other words, a 1 amp current source will maintain a current flow of 1 amp through its terminals if it has an open circuit or short circuit as a load.

symbol - A circle with an arrow inside is the symbol to indicate the direction of the flow of the current.

circuit diagram -

- <u>LR circuit</u>





$V_R = IR$

$V_L = IX_L$

net voltage $(V) = \sqrt{V_L^2 + V_R^2}$

$= \sqrt{I^2 X_L^2 + I^2 R^2}$

$= I\sqrt{R^2 + X_L^2}$

$\Rightarrow \frac{V}{I} = \sqrt{R^2 + X_L^2}$

$\Rightarrow Z = \sqrt{R^2 + X_L^2}$ [Z = impedance]

$$\boxed{Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}}$$

$$\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IV_R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

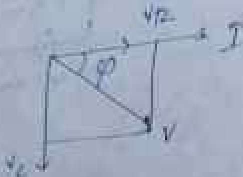
$$\boxed{\phi = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right)}$$

[ϕ = Phasor angle]

In LR circuit current lag behind the voltage by phasor angle $\tan^{-1}\left(\frac{\omega L}{R}\right)$

- CR Circuit :

$V = v_m \sin wt$



$V_C = I X_C$
$V_R = I R$

net voltage $(V) = \sqrt{V_C^2 + V_R^2}$

$= I\sqrt{R^2 + X_C^2}$

$\Rightarrow \frac{V}{I} = \sqrt{R^2 + X_C^2}$

$\Rightarrow Z = \sqrt{R^2 + X_C^2}$

$$\boxed{Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{w^2C^2}}}$$

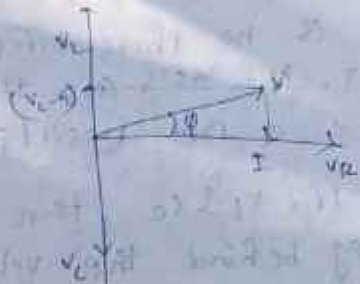
$\tan\varphi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1}{wCR}$

$$\boxed{\varphi = \tan^{-1}\frac{1}{wCR}}$$

In CR circuit the voltage lag behind the current by the phasor angle $\tan^{-1}\left(\frac{1}{wCR}\right)$

- RLC Circuit :

$V = v_m \sin wt$



$V_L = I X_L$
$V_C = I X_C$
$V_R = I R$

net voltage $(V) = \sqrt{(V_L - V_C)^2 + V_R^2}$

$= I\sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}$

$= I\sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$

$\Rightarrow Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(wL - \frac{1}{wC}\right)^2}}$$

$\tan\varphi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R} = \frac{wL - \frac{1}{wC}}{R}$

$$\boxed{\varphi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{wL - \frac{1}{wC}}{R}\right)}$$

Here 3 cases are avialable :

(i) If, $X_L = X_C$, $\phi = 0$ ∴ That means there is no phase difference between V and I. In this case the circuit will act like a pure resistive circuit.

(ii) If, $X_L > X_C$, then the current will lag behind the voltage by an angle $\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$. In this case the circuit will act like a pure inductive circuit.

(iii) If, $X_L < X_C$, then the voltage will lag behind the current by an angle $\tan^{-1}\left(\frac{\frac{1}{\omega C} - \omega L}{R}\right)$. In this case the circuit will act like a pure capacitive circuit.

• The condition of resonance in series RLC circuit —

In series RLC circuit $Z = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

If $\omega L = \frac{1}{\omega C}$, then $Z = Z_{min}$

in this case $Z = Z_{min} = R$

In this situation the highest current flow of this circuit will be maximum. This is called the resonance of series RLC circuit.

• Resonance frequency (অনুনাদী কম্পাঙ্ক) —

The condition for resonance, —

$\omega_0 L = \frac{1}{\omega_0 C}$ [For resonance $\omega = \omega_0$]

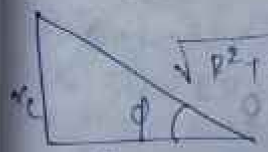
$\Rightarrow \omega_0^2 = \frac{1}{LC}$

$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

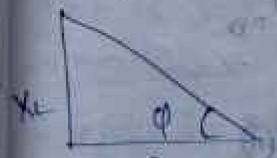
$\Rightarrow 2\pi n = \frac{1}{\sqrt{LC}}$ [n/f = Frequency]

$\Rightarrow \boxed{n = \frac{1}{2\pi\sqrt{LC}}}$

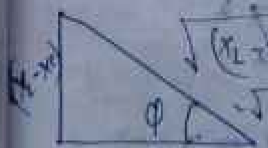
* Impedance triangle —



(CR circuit)



(LR circuit)

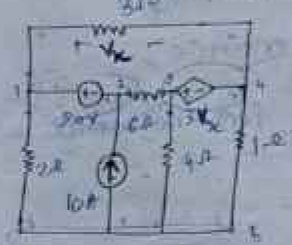


(RLC circuit)

1

3/8/19

(1) Find the node voltages in the given circuit.



supernode

[...]

[5 nodes]

$$\frac{v_1}{2} + \frac{v_1 - v_4}{3} + \frac{v_2 - v_3}{6} = 10$$

$$\Rightarrow \frac{3v_1 + 2v_1 - 2v_4 + v_2 - v_3}{6} = 10$$

$$\Rightarrow 5v_1 + v_2 - v_3 - 2v_4 = 60 \quad \text{— (I)}$$

$$\frac{v_3 - v_2}{6} + \frac{v_3}{4} + \frac{v_4 - v_1}{3} + \frac{v_4}{1} = 0$$

$$\Rightarrow \frac{2v_3 - 2v_2 + 3v_3 + 4v_4 - 4v_1 + 12v_4}{12} = 0$$

$$\Rightarrow -4v_1 - 2v_2 + 5v_3 + 16v_4 = 0 \quad \text{— (II)}$$

$v_1 - v_2 = 20$ — (III)

$v_3 - v_4 = 3v_x$ — (IV)

$v_1 - v_4 = v_x$ — (IV)

$+20 + v_2 - v_1 = 0$

$\Rightarrow v_1 - v_2 = 20$

$v_3 - v_4 = 3v_x$

$v_1 - v_4 = v_x$

$\Rightarrow v_1 - v_3 = -2v_x$

$\Rightarrow v_1 - v_3 = -2(v_1 - v_4)$

$\Rightarrow 3v_1 - v_3 - 2v_4 = 0$ — (V)

from → (III)

$v_1 - v_2 = 20$

$\Rightarrow v_2 = v_1 - 20$ — (VI)

from (IV) → (V)

$\frac{v_3 - v_4}{3} = v_1 - v_4$

$\Rightarrow v_3 - v_4 = 3v_1 - 3v_4$

$\Rightarrow v_3 = 3v_1 - 2v_4$ — (VII)

from (VI)

$5v_1 + v_1 - 20 - 3v_1 + 2v_4 - 2v_4 = 60$ [from (VII) $v_3 = 3v_1 - 2v_4$]

$\Rightarrow 6v_1 - 3v_1 = 60 + 20$

$\Rightarrow 3v_1 = 80$

$\Rightarrow v_1 = 26.67\,V$

$v_2 = 26.67 - 20 = 6.67\,V$

from (I),

$5 \times 26.67 + 6.67 - v_3 - 2v_4 = 60$

$\Rightarrow v_3 + 2v_4 = 80.02$ — (VI)

from (II),

$-4 \times 26.67 - 2 \times 6.67 + 5v_3 + 16v_4 = 0$

$\Rightarrow 5v_3 + 16v_4 = 120.02$ — (VII)

~~$\Rightarrow v_3 + 3v_4 = 24.004$~~ — (VII)

$V_3 + 2V_4 = 80.02$ — (VI)

$-V_3 - 3V_4 = -24.004$ — (VII)

$-V_4 = 56.016$

$\Rightarrow V_4 = -56.016V$

$V_3 + 2 \times 56.016 = 80.02$

$\Rightarrow V_3 = -32.012V$

$5V_3 + 16V_4 = 120.02$ — (VII)

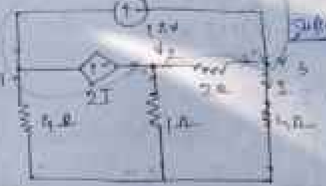
$V_3 + 2V_4 = 80.02$ — (VI)

$\Rightarrow V_4 = -46.68V$

$V_3 - 2 \times (46.68) = 80.02$

$\Rightarrow V_3 = 173.38V$

② Find the V_1, V_2, V_3



Supernode analysis:

[Here, when traversing ... supernode ... [illegible] ... V_1, V_3 ...]

$$\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_3}{4} = 0 \quad \left[\frac{V_1}{4} + \frac{V_2}{1} + \frac{V_2 - V_3}{2} + \frac{V_3}{4} + \frac{V_3 - V_2}{2} = 0 \right]$$

$\Rightarrow V_1 + 4V_2 + V_3 = 0$

$\Rightarrow V_1 + 4V_2 + V_3 = 0$ — (I)

$V_1 - V_3 = 12$ — (V)

$\Rightarrow V_1 = 12 + V_3$ — (VI)

$V_1 - V_2 = 2I$ — (II)

$I = \frac{V_3}{4}$ — (III)

from (II)

$$V_1 - V_2 = \frac{2V_3}{4}$$

$\Rightarrow 4V_1 - V_2 - 2V_3 = 0$ — (IV)

from (I) and (IV), (V)

from (IV),

$4(12 + V_3) - V_2 - 2V_3 = 0$

$\Rightarrow 48 + 4V_3 - V_2 - 2V_3 = 0$

$\Rightarrow 2V_3 - V_2 = -48$ — (VII)

from (I), $12 + V_3 + 4V_2 + V_3 = 0$

$\Rightarrow 4V_2 + 2V_3 = -12$ — (VIII)

$2v_3 - v_2 = -48$ — (vii)

$4v_3 + 4v_2 = -12$ — (viii)

$8v_3 - \cancel{4v_2} = -192$

$v_3 + \cancel{4v_2} = -12$

$\Rightarrow 7v_3 = -204$

$\Rightarrow v_3 = -29.14\ V$

$2 \times 29.14 - v_2 = -48$

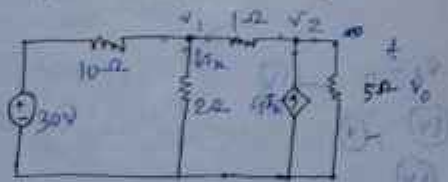
$\Rightarrow v_2 = 106.28\ V$

from (i)

$v_1 + 4 + 106.28 + 29.14 = 0$

$\Rightarrow v_1 = -3\ V$

(3) using nodal analysis determine V_o in this circuit.



$$\frac{v_1 - 30}{10} + \frac{v_1}{2} + \frac{v_1 - v_2}{1} = 0$$

$\Rightarrow v_1 - 30 + 5v_1 + 10v_1 - 10v_2 = 0$

$\Rightarrow 16v_1 - 10v_2 = 30$ — (i)

$$\frac{v_2 - v_1}{1} + \frac{v_2}{5} = 4I_x$$

$\Rightarrow 5v_2 - 5v_1 + v_2 = 20I_x$

$\Rightarrow -5v_1 + 6v_2 = 20I_x$ — (ii)

$$I_x = \frac{v_1}{2}$$

$\Rightarrow -5v_1 + 6v_2 = 20 \times \frac{v_1}{2}$

$\Rightarrow -15v_1 + 6v_2 = 0$ — (iii)

$16v_1 - 10v_2 = 30$

$-15v_1 + 6v_2$

from (iii)

$\Rightarrow 6v_2 = 15v_1$

$\Rightarrow v_2 = \frac{15v_1}{6}$ — (iv) $\Rightarrow v_2 = \frac{15 \times 3.33}{6} = -8.325V$

from (i)

$16v_1 - 10 \times \frac{15v_1}{6} = 30$

$\Rightarrow -9v_1 = 30$

$\Rightarrow v_1 = -\frac{30}{9} = -3.33\ V$

$v_o = v_2 = -8.325\ V$

mesh analysis - don't add direction
Nodal - Add

$I_x = \frac{\ldots}{2}$ $\quad 10$

$\Rightarrow -5V_1 + 6V_2 = 30 \ldots \frac{V_1}{2}$

$\Rightarrow -15V_1 + 6V_2 = 0$ — (iv)

$16V_1 - 10V_2 \ldots$

$V_1 + 4V_2 + V_3 = 0$ — (i)

$\cancel{V_3} - V_3 = 12$

$\Rightarrow V_3 = V_1 - 12$ — (ii)

$V_1 - V_2 = 2I$

$\Rightarrow V_2 = V_1 - 2I$

$= V_1 - 2\times\frac{V_3}{2}\quad [\because I = \frac{V_3}{4}]$

$= V_1 - \frac{(V_1 - 12)}{2}$ [from (ii)]

$= \frac{2V_1 - V_1 + 12}{2} = \frac{V_1 + 12}{2}$ — (iii)

from (i)

$V_1 + 4\left[\frac{V_1 + 12}{2}\right] + V_1 - 12 = 0$

$\Rightarrow V_1 + 2V_1 + 24 + V_1 - 12 = 0$

$\Rightarrow 4V_1 + 12 = 0$

$\Rightarrow 4V_1 = -12$

$\Rightarrow V_1 = -3\ V$

$V_3 = -3 - 12$

$= -15V$

$V_2 = \frac{-3+12}{2}$

$= -4.5V$

...s determine V_o in

5Ω V_o

$\ldots 10V_2 = 0$

mesh analysis - don't add direction

Nodal - Add

5/6/22

1. Applying mesh analysis obtain I_x = 50

[When there is voltage source, then the eqn of whole loop/line eqn (iii) be]

~~ABJHA~~ $I_2 - I_1 = 4$ — (i) (?)

ABCDEFGHA

$4I_1 + 3I_2 + 1I_2 + 2(I_2 - I_4) + 2(I_1 - I_3) = 24$ — (ii)

$I_3 = -2$ — (iii)

$4I_1 + 3I_2 + I_2 + 2I_2 - 2I_4 + 2(I_1 + 2) = 24$

$\Rightarrow 4I_1 + 6I_2 - 2I_4 + 2I_1 + 4 = 24$

$\Rightarrow$ ~~$2I_1 + 6I_2 = 2$~~

$\Rightarrow 6I_1 + 6I_2 - 2I_4 = 20$ — (iii)

$2(I_4 - I_2) + 4I_4 + (I_4 + 2) = -9$

$\Rightarrow 7I_4 - 2I_2 = -11$ — (iv)

~~(iii)/3 $\Rightarrow 6I_2 = 2I_4 - 6I_1$~~

~~$\Rightarrow I_2 = \frac{I_4}{3} - I_1$~~

(iv) $\Rightarrow I_4 = \frac{2I_2 - 11}{7}$

Put eqn (i) and (iv) in (iii)

$6 \times (I_2 - 4) + 6I_2 - 2\left(\frac{2I_2 - 11}{7}\right) = 20$

~~$+ 2(I_2 - 4) = 20$~~

$\Rightarrow 6I_2 - 24 + 6I_2 - \frac{(4I_2 - 22)}{7} = 20$

$\Rightarrow 42I_2 - 168 + 42I_2 - 4I_2 + 22 = 0$

$\Rightarrow$ ~~84I~~ $80I_2 = 146$

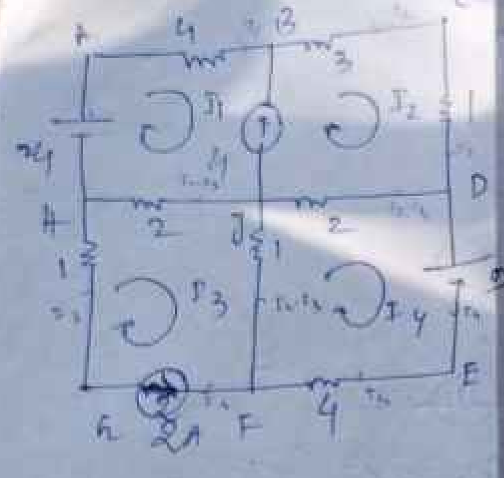
$\Rightarrow I_2 =$ ~~1.825~~ 3.575

$3.575 - I_1 = 4$

$\Rightarrow I_1 = -0.425$

$I_4 = \frac{2 \times 3.575 - 11}{7}$

$= -0.55$ A



[Here we cannot divide the current in normal way (the way I do in KVL). So consider I_1, I_2, I_3, I_4 like this way.
As we are not dividing the current in diagram like normal way, so from where we consider a closed loop, will be the direction of current]

Here, $I_3 = -2$
Here, $I_2 - I_1 = 4$

[I_1, I_2 was major
In, ABCDJHA, I_2 was in the box ABCDJHA
In, JDEFJ, I_4 was in the box JDEFJ
$\therefore I_4 \rightarrow$ major]

[We cannot apply KVL in loop ABJHA, BCDJB, HJFGH as current is present there]

ABCDJHA, KVL,

$4I_1 + 3I_2 + I_2 + 2(I_2 - I_4) + 2(I_1 - I_3) = 24$

$\Rightarrow 6I_2 + 6I_1 - 2I_4 - 2I_3 = 24$

$\Rightarrow 6I_2 + 6I_1 - 2I_4 + 4 = 24 \quad [\because I_3 = 2]$

$\Rightarrow 6I_2 + 6I_1 - 2I_4 = 20$ —— (I)

JDEFJ, KVL

$2(I_4 - I_2) + 4I_4 + (I_4 - I_3) = -9$

$\Rightarrow 6I_4 - 2I_2 - I_3 + I_4 = -9$

$\Rightarrow 7I_4 - 2I_2 + 2 = -9 \quad [\because I_2 = -2]$

$\Rightarrow 7I_4 - 2I_2 = -11$ —— (II)

KCL at A

∴ from (i), => $I_1 = I_2 - 4$

$6I_2 + 6(I_2 - 4) - 2I_4 = 20$

=> $12I_2 - 2I_4 = 44$ — (iii)

$6I_2 - 2I_4 = 22$ — (iii) $\times 2$

$-2I_2 + 7I_4 = -11$ — (i) $\times 6$

$12\cancel{I_2} - 2I_4 = 44$

$-12\cancel{I_2} + 42I_4 = -66$

=> $40I_4 = -22$

=> $\boxed{I_4 = -0.55}$

Now, $I_0 = I_3 - I_4$

$= -2 + 0.55$

$= -1.45\,A$

[The mag of I_4 = -0.55

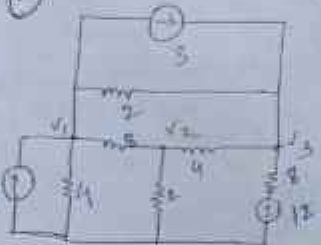
hence direction indicate the mag]

$\therefore I_0 = I_3 - I_4$

$= -2 - (-0.55)$

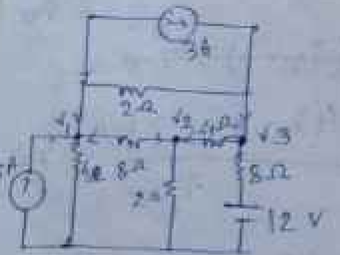
$= -1.45A$

(2)



-find-

V_1, V_2, V_3.



$V_1 \times V_1$

$$\frac{V_1}{4} + \frac{V_1 - V_2}{8} + \frac{V_1 - V_3}{2} = 5 - 3$$

$$\Rightarrow \frac{2V_1 + V_1 - V_2 + 4V_1 - 4V_3}{8} = 2$$

$\Rightarrow 7V_1 - V_2 - 4V_3 = 16$ — (1)

$$\frac{V_2 - V_1}{8} + \frac{V_2}{2} + \frac{V_2 - V_3}{4} + \cancel{\frac{V_2 - 12}{8}} = 0$$

$\Rightarrow V_2 - V_1 + 4V_2 + 2V_2 - 2V_3 = 0$

$\Rightarrow 7V_2 - V_1 - 2V_3 = 0 \Rightarrow -V_1 + 7V_2 - 2V_3 = 0$ — (II)

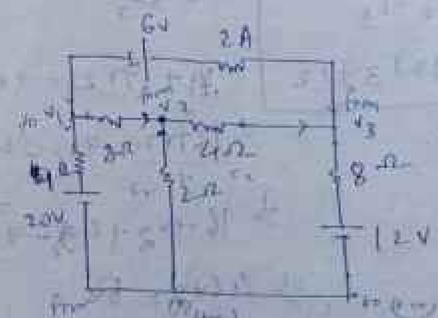
$$\frac{V_3 - V_2}{4} + \frac{V_3 - 12}{8} + \frac{V_3 - V_1}{2} = 3$$

$\Rightarrow 2V_3 - 2V_2 + V_3 - 12 + 4V_3 - 4V_1 = 24$

$\Rightarrow -4V_1 - 2V_2 + 7V_3 = 36$

[This Process - no need]

Source transformation



[nodal analysis]

$$\frac{V_2 - V_1}{8} + \frac{V_2}{2} + \frac{V_2 - V_3}{4} = 0$$

$\Rightarrow V_2 + V_1 + 4V_2 + 2V_2 - 2V_3 = 0$

$\Rightarrow V_1 + 7V_2 - 2V_3 = 0$ — (1)

$$\frac{V_1 + 6}{2} + \frac{V_1 - 20}{4} + \frac{V_1 - V_2}{8} = 0$$

Mesh analysis

$-6 + 2I_1 + 8(I_1 - I_3) + 4(I_1 - I_2) = 0$

$\Rightarrow 14I_1$

$2(I_2 - I_1) + 2(I_2 - I_3) - 20 + 4I_2 = 0$

$4(I_3 - I_1) + 8I_3 + 12$

$2(I_3 - I_2) = 0$

$\Rightarrow \dfrac{4V_1 + 24 + 2V_1 - 30 + V_1 - V_2}{8} = 0$

$\Rightarrow 7V_1 - V_2 = 16$ — (ii)

$\dfrac{V_3 - 6}{2} + \dfrac{V_3 - 12}{8} + \dfrac{V_3 - V_2}{4} = 0$

$\Rightarrow 4V_3 - 24 + V_3 - 12 + 2V_3 - 2V_2 = 0$

$\Rightarrow 7V_3 - 2V_2 = 36$ — (iii)

$14I_1 - 4I_3 - 8I_2 = 6$ — (i)

$14I_2 - 8I_1 - 2I_3 = 20$ — (ii)

$14I_3 - 4I_1 - 2I_2 = -12$ — (iii)

mesh analysis eqn

$V_1 = -4I_2 + 20$

$V_2 = 2(I_2 - I_3)$

$V_3 = +8I_3 + 12$

from (iii): $7V_3 = 36 + 2V_2$

$V_3 = \dfrac{36 + 2V_2}{7}$

from (ii): $7V_1 = 16 + V_2$

$V_1 = \dfrac{16 + V_2}{7}$ — (i)

from (i),

$V_1 + 7V_2 - 2V_3 = 0$

$\Rightarrow \dfrac{16 + V_2}{7} + 7V_2 - 2 \times \dfrac{36 + 2V_2}{7}$

$\Rightarrow 16 + V_2 + 49V_2 - 72 - 4V_2$

$\Rightarrow 46V_2 = 56$

$\Rightarrow V_2 = 1.33\,V$

$V_1 = \dfrac{16 + 1.33}{7} = 16.19\,V$

$V_3 = \dfrac{36 + 2 \times 1.33}{7} = 5.52\,V$

not correct

mesh analysis

(from Pr...)

$14I_1 - 4I_3 - 8I_2 = 6$

$\Rightarrow 7I_1 - 4I_2 - 2I_3 = 3$ — (i)

$-8I_1 + 14I_2 - 2I_3 = 20$

$\Rightarrow -4I_1 + 7I_2 - I_3 = 10$ — (ii)

$-4I_1 - 2I_2 + 14I_3 = -12$

$\Rightarrow -2I_1 - I_2 + 7I_3 = -6$ — (iii)

Let $A = \begin{bmatrix} 7 & -4 & -2 \\ -4 & 7 & -1 \\ -2 & -1 & 7 \end{bmatrix}$, $B = \begin{bmatrix} 3 \\ 10 \\ -6 \end{bmatrix}$

$|A| = \begin{vmatrix} 7 & -4 & -2 \\ -4 & 7 & -1 \\ -2 & -1 & 7 \end{vmatrix}$

$= 7(49 - 1) + 4(-28 - 2) - 2(4 + 14)$

$= 336 - 120 - 36$

$= 180$

adj A $= \begin{bmatrix} \begin{vmatrix} 7 & -1 \\ -1 & 7 \end{vmatrix} & -\begin{vmatrix} -4 & -1 \\ -2 & 7 \end{vmatrix} & \begin{vmatrix} -4 & 7 \\ -2 & -1 \end{vmatrix} \\ -\begin{vmatrix} -4 & -2 \\ -1 & 7 \end{vmatrix} & \begin{vmatrix} 7 & -2 \\ -2 & 7 \end{vmatrix} & -\begin{vmatrix} 7 & -4 \\ -2 & -1 \end{vmatrix} \\ \begin{vmatrix} -4 & -2 \\ 7 & -1 \end{vmatrix} & -\begin{vmatrix} 7 & -2 \\ -4 & -1 \end{vmatrix} & \begin{vmatrix} 7 & -4 \\ -4 & 7 \end{vmatrix} \end{bmatrix}^T$

$= \begin{bmatrix} (49-1) & -(-28-2) & (4+14) \\ -(-28-2) & (49-4) & -(-7-8) \\ (4+14) & -(-7-8) & (49-16) \end{bmatrix}^T = \begin{bmatrix} 48 & 30 & 18 \\ 30 & 45 & 15 \\ 18 & 15 & 33 \end{bmatrix}^T$

$= \begin{bmatrix} 48 & 30 & 18 \\ 30 & 45 & 15 \\ 18 & 15 & 33 \end{bmatrix}$

$I = A^{-1}B = \dfrac{1}{180} \begin{bmatrix} 48 & 30 & 18 \\ 30 & 45 & 15 \\ 18 & 15 & 33 \end{bmatrix} \times \begin{bmatrix} 3 \\ 10 \\ -6 \end{bmatrix}$

$= \dfrac{1}{180} \begin{bmatrix} 336 \\ 450 \\ 6 \end{bmatrix} \Rightarrow$ $I_1 = 1.87$, $I_2 = 2.5$, $I_3 = 0.03$

$V_1 = -4 \times 2.5 + 20 = 10\,V$

$V_2 = 4.94\,V$

$V_3 = 12.24\,V$

(1)

$$\frac{v_1+6-v_3}{2} + \frac{v_1-20}{4} + \frac{v_1-v_2}{8} = 0$$

$$\Rightarrow \frac{4(v_1+6-v_3)+2(v_1-20)+v_1-v_2}{8} = 0$$

$$\Rightarrow 4v_1+2v_1+v_1-4v_3-v_2+24-40=0$$

$$\Rightarrow 7v_1-v_2-4v_3=16 \quad \text{— (I)}$$

$$\frac{v_2-v_1}{8} + \frac{v_2-v_3}{4} + \frac{v_2}{2} = 0$$

$$\Rightarrow \frac{v_2-v_1+2(v_2-v_3)+4v_2}{8} = 0$$

$$\Rightarrow -v_1+v_2+2v_2+4v_2-2v_3=0$$

$$\Rightarrow -v_1+7v_2-2v_3=0 \quad \text{— (II)}$$

$$\frac{v_3-6-v_1}{2} + \frac{v_3-v_2}{4} + \frac{v_3-12}{8} = 0$$

$$\Rightarrow \frac{4(v_3-6-v_1)+2(v_3-v_2)+v_3-12}{8} = 0$$

$$\Rightarrow 4v_3+2v_3+v_3-4v_1-2v_2-24-12=0$$

$$\Rightarrow 7v_3-4v_1-2v_2=36$$

$$\Rightarrow -4v_1-2v_2+7v_3=36 \quad \text{— (III)}$$

(II), $v_1 = 7v_2-2v_3$ — (IV)

(I), $7(7v_2-2v_3)-v_2-4v_3=16$

$\Rightarrow 49v_2-14v_3-v_2-4v_3=16$

$\Rightarrow 48v_2-18v_3=16$

$\Rightarrow 24v_2-9v_3=8$ — (V)

(III), $-4(7v_2-2v_3)-2v_2+7v_3=36$

$\Rightarrow -28v_2-2v_2+8v_3+7v_3=36$

$\Rightarrow -30v_2+15v_3=36$

$\Rightarrow -10v_2+5v_3=12$ — (VI)

$$A = \begin{bmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 7 & -1 & -4 \\ -1 & 7 & -2 \\ -4 & -2 & 7 \end{vmatrix}$$

$= 7(49-4)+1(-7-8)-4(2+28)$

$= 315-15-120$

$= 180$

$$\text{adj}\,A = \begin{bmatrix} \begin{vmatrix} 7 & -2 \\ -2 & 7 \end{vmatrix} & -\begin{vmatrix} -1 & -2 \\ -4 & 7 \end{vmatrix} & \begin{vmatrix} -1 & 7 \\ -4 & -2 \end{vmatrix} \\ -\begin{vmatrix} -1 & -4 \\ -2 & 7 \end{vmatrix} & \begin{vmatrix} 7 & -4 \\ -4 & 7 \end{vmatrix} & -\begin{vmatrix} 7 & -1 \\ -4 & -2 \end{vmatrix} \\ \begin{vmatrix} -1 & -4 \\ 7 & -2 \end{vmatrix} & -\begin{vmatrix} 7 & -4 \\ -1 & -2 \end{vmatrix} & \begin{vmatrix} 7 & -1 \\ -1 & 7 \end{vmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} (49-4) & -(-7-8) & (2+28) \\ -(-7-8) & (49-16) & -(-14-4) \\ (2+28) & -(-14-4) & (49-1) \end{bmatrix}$$

$$= \begin{bmatrix} 45 & 15 & 30 \\ 15 & 33 & 18 \\ 30 & 18 & 48 \end{bmatrix}$$

$$= \begin{bmatrix} 45 & 15 & 30 \\ 15 & 33 & 18 \\ 30 & 16 & 48 \end{bmatrix}$$

$V = A^{-1}B$

$$= \frac{1}{180}\begin{bmatrix} 45 & 15 & 30 \\ 15 & 33 & 18 \\ 30 & 18 & 48 \end{bmatrix} \begin{bmatrix} 16 \\ 0 \\ 36 \end{bmatrix}$$

$$= \frac{1}{180}\begin{bmatrix} 720+1080 \\ 240+648 \\ 480+1728 \end{bmatrix}$$

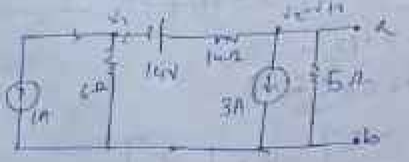
$$= \frac{1}{180}\begin{bmatrix} 1800 \\ 888 \\ 2208 \end{bmatrix}$$

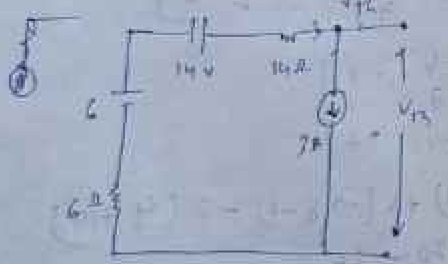
$v_1 = 10$

$v_2 = 4.93$

$v_3 = 12.26$

(1) Find thevenin and norton's equivalent of network from the given circuit.

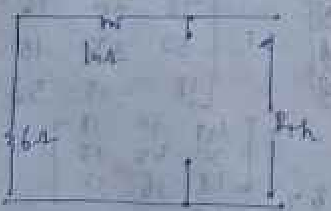




$$\frac{V_{th} - 8}{20} = -3$$

$\Rightarrow V_{th} = -3 \times 20 + 8 = -60 + 8$

$= \cancel{-34V} = -52V$



Inverted

$R_{th} = 20\,\Omega$

$\therefore I_{th} = \frac{-52}{20} = -2.6A$

$$\frac{V_1}{6} + \frac{V_1 + 14 - V_2}{14} = 1$$

$$\frac{V_1 + 1}{6} + \frac{V_1 + 14 - V_2}{14} = 0$$

$\Rightarrow \frac{14V_1 + 6V_1 + 84 - 6V_2}{84} = 1$

$\Rightarrow 20V_1 - 6V_2 = 84 - 84$

$\Rightarrow 10V_1 - 3V_2 = 0$

$\Rightarrow V_1 = \frac{3V_2}{10}$

$$\frac{V_2 - 14 - V_1}{14} + \frac{V_2}{5} = -3$$

$\Rightarrow \frac{5V_2 - 70 - 5V_1 + 14V_2}{70} = -3$

$\Rightarrow 6V_2 - 5V_1 = -210 + 70$

$\Rightarrow 6V_2 - \frac{5 \times 3V_2}{10} = -140$

$\Rightarrow 12V_2 - 3V_2 = -280$

$\Rightarrow 9V_2 = -280$

$\Rightarrow V_2 = -31.11V$

$\therefore V_1 = \frac{3 \times 31.11}{10} = 9.33V$

$\Rightarrow -5V_1 + 19V_2 = -140$

$\Rightarrow -5 \times \frac{3V_2}{10} + 19V_2 = -140$

$\Rightarrow -3V_2 + 38V_2 = -280$

$\therefore V_2 = -8V$

Find Thevenin's and Norton's equ ~~voltage~~ between terms from the given circuit =

⇓

Thevenin's theorem

[Here we will consider the sign when ~~calc~~ calculating the magnitude]

[i.e. $v_{th} = -8V$ that means mag of $v_{th} = 8V$ ... to the magnitude and neg sign]

~~v_{th}~~ As current of 1A entering to the source v_1 So,

$$-1 + \frac{v_1}{6} + \frac{v_1 + 14 - v_2}{14} = 0$$

$$\Rightarrow \frac{-42 + 7v_1 + 3v_1 + 42 - 3v_2}{42} = 0$$

$\Rightarrow 10v_1 = 3v_2$

$\Rightarrow v_1 = \frac{3v_2}{10}$

As current of 3A comming out from source v_2 So,

$$\frac{v_2}{5} + \frac{v_2 - 14 - v_1}{14} + 3 = 0$$

$$\Rightarrow \frac{14v_2 + 5v_2 - 70 - 5v_1 + 210}{70} = 0$$

$\Rightarrow 15v_2 - v_1 = 196$

$\Rightarrow 15v_2 - \frac{3v_2}{10} = 1960$

$\Rightarrow 15v_2 - 3v_2 = 1960$

$\Rightarrow 12v_2 = 196$

$$\Rightarrow \frac{14v_2 + 5v_2 - 70 - 5v_1 + 210}{70} = 0$$

$\Rightarrow 19v_2 - 5v_1 = -140$

$\Rightarrow 19v_2 - \frac{5 \cdot 3v_2}{10} = -140$

$\Rightarrow 38v_2 - 3v_2 = -280$

$\Rightarrow 35v_2 = -280$

$\Rightarrow v_2 = -8V$

$\therefore v_{th} = -8V$

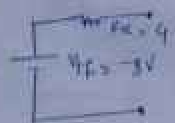
210
−14
196

210
−70
140

For calculating equivalent of the terminals the R would be present when calculating R_{th} (+ve) ... for calculating the ... the ... we be

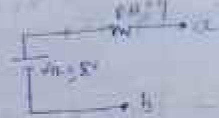
$6 + 14 = 20$

$$R_{eq} = \frac{20 \cdot 5}{25} = 4 = 4\Omega = R_{th}$$



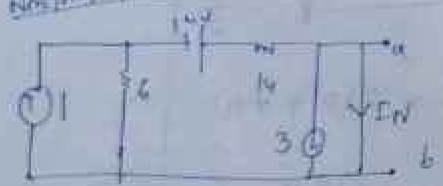
$R_{th} = \frac{20\times5}{25} = 4$

$I_{RL} = \frac{8}{4+5}$

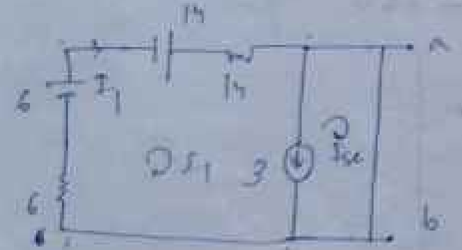


$= 0.88$

Norton's theorem



⇓



abcda KVL → X
as 3A current source
also come to that
closed loop.

$-6 + 6I_1 - 14 + 14I_1 = 0$

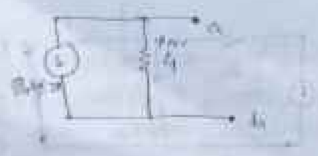
⇒ $20 I_1 = 20$

⇒ $I_1 = 1A$

$I_1 - I_{sc} = 3$

⇒ $I_{sc} = I_1 - 3 = 1 - 3 = -2A$

$R_{th} = 4\Omega$

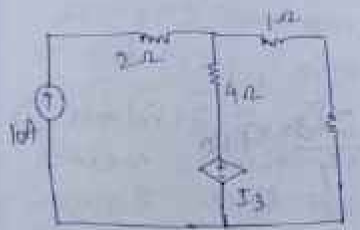


$I_{RL} = \frac{-2 \times 4}{4+5}$

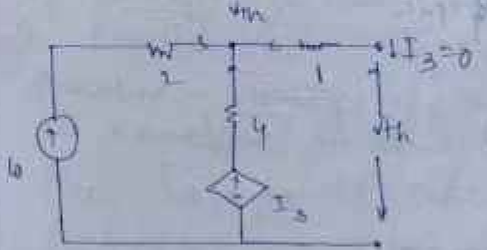
~~= 1.6A~~

$= 0.88\,A$

(3) use thevenin's theorem to find the voltage across 3Ω resister



$I_3 = \frac{12\times1}{5}$



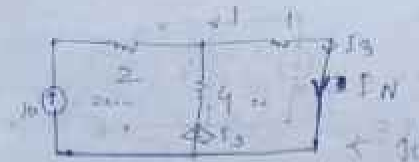
As $I_3 = 0$
I_3 CCVS is short

$\frac{V_{th} - I_3}{4} = 10$

⇒ $V_{th} - \cancel{I_3}^{0} = 40$

⇒ $V_{th} = 40V$

Theorems → method
2nd Sem (EEM)

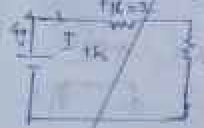


$R_{Th} = \frac{V_{Th}}{I_N}$

I.d I_N by nodal analysis, taking any random V'

$R_{Th} = 2\,\Omega$

$I_{Th} = \frac{40}{20}$



$I_{Th} = \frac{40}{5} = 8A$

$\frac{V_1' - I_3}{4} = 10 - I_N$

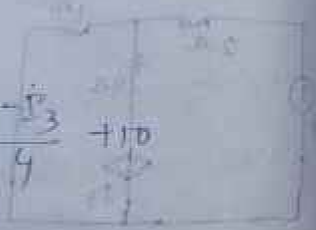
$\Rightarrow I_N = 10 - \frac{V' - I_3}{4}$ — (i)

$\frac{V'}{1} = I_N = I_3$ — (ii)

from (i)

$\Rightarrow I_N = \frac{I_3 - I_3}{4} + 10$

$\Rightarrow I_N = 10$



$\therefore R_{Th} = \frac{40}{10} = 4\,\Omega$

$I_{Th} = \frac{40}{7} = 5.71$

voltage across $3\,\Omega$ $= V_3 = V_{Th}\frac{R_3}{R_3 + R_{Th}}$

$= \frac{40 \times 3}{7}$

$= \frac{120}{7}$

$= 17.14$

(1) Thevenin's theorem's statement – It states that it is possible to simplify any linear circuit, irrespective of how complex it is, for an equivalent of two circuit with a single voltage source and a series resistance. Find R_L, then remove R_L through voltmeter.

Procedure – to find out R_{Th} voltage source should be short circuited with their internal resistance. Current source should be open. Measure equivalent resistance of ckt (R_{Th}), viewed from the load terminal.

(2) Norton's theorem – It states that any linear circuit containing several energy sources and resistance can be replaced by a single constant current generator in parallel with a single resistor.

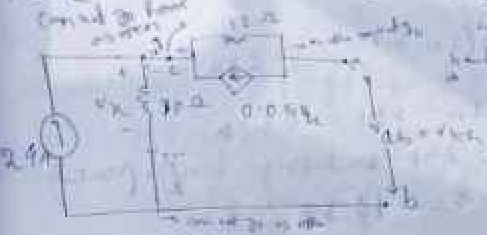
Procedure – Identify R_L, then short R_L through an ammeter, redraw the ckt diagram, then have to measure I scl short ckt current (I_N), find R_{Th}, redraw the ckt diagram.

(3) Superposition theorem – It states that a ckt with multiple voltage and current sources is equal to the sum of simplified circuits using just one of the sources.

only one source will act at one time
more than 1 source should be present in the ckt [voltage/current source — more than 1]

④ maximum Power transformation theorem —
max power will flow in the ckt when load resistance is equal to the R_{th} [$P = I^2 R_L$]. Here ckt efficiency is 50% [Prove 2nd sem — SEE]

① Thevenin's eq at the terminal a, b — 19/8/22



$V_x = 24 \times 10 = 240V = V_{cb}$

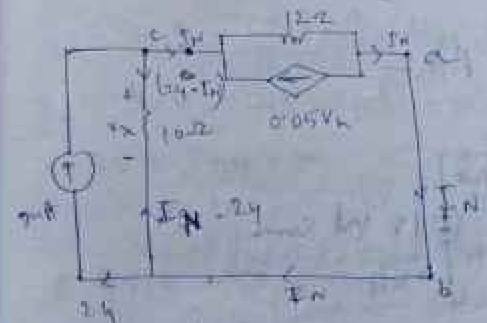
$V_{ac} = -0.05 \times 240 \times 12$

$= -144V$

$V_{ab} = V_{ac} + V_{cb}$

$= -144 + 240$

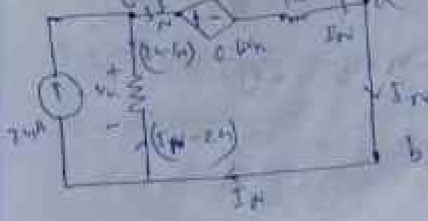
$= 96V = V_{th}$



[In incoming, 24 outgoing] $(I_N - 24)$

I_N outgoing, 24 incoming $(24 - I_N)$

$V_x = 10(24 - I_N)$ — ①



$\Rightarrow 12 I_N + 10(I_N - 24) = -0.6 V_x$

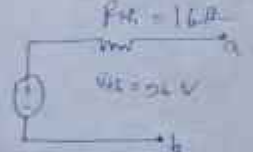
$\Rightarrow 12 I_N + 10 I_N - 240 = -0.6 V_x$

$\Rightarrow 22 I_N = \cancel{240 - 6 - 2.384} \; 0.6[10(24 - I_N)]$

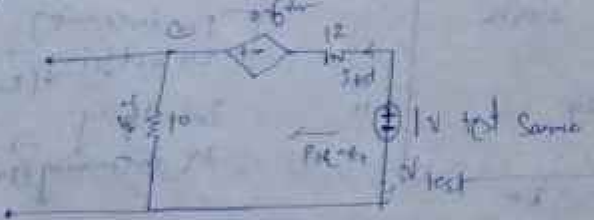
$\Rightarrow I_N = \cancel{10.036} \; 16 \cancel{.88}$ A

$V_x = 10(24 - \cancel{16.88})$
$= \cancel{131.2 V}$
$= 180$ V

$R_{th} = \frac{V_{th}}{I_N} = \frac{96}{6} = 16\,\Omega$



other Process to get R_{th} —



$R_{th} = \frac{V_{test}}{I_{test}}$

$0.6 V_x - 12 I_{test} + 1 - V_x = 0$ — (i)

$V_x = I_{test} \times 10$ — (ii)

$\Rightarrow 0.6 \times 10 I_{test} - 12 I_{test} + 1 = I_{test} \times 10$

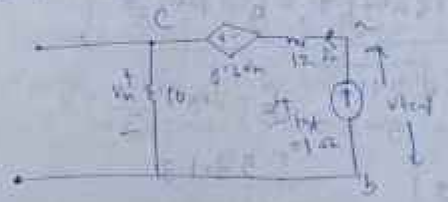
$\Rightarrow -4 I_{test} - 12 I_{test} = -1$

$\Rightarrow -16 I_{test} = -1$

$\Rightarrow I_{test} = \frac{1}{16} = \cancel{0.0625}$

Hence $R_{th} = V_{test} / I_{test} = 1/(1/16)$
$= 16\,\Omega$

3rd Process



$0.6 V_x - 12 I_{test} + V_{test} - V_x = 0$

$\Rightarrow -0.4 V_x - 12 + V_{test} = 0$ — (i)

$V_x = I_{test} \times 10$
$= 10$ — (ii)

$-0.4 \times 10 - 12 + V_{test} = 0$

$\Rightarrow -4 - 12 + V_{test} = 0$

$\Rightarrow V_{test} = 16$ V

$R_{th} = 16$ V

$|A| \angle \theta \rightarrow$ polar $\times$ /

$a + jb \longrightarrow$ Rectangular form 1 — (3)

12/8/22

AC circuit

Polar & Rectangular

$\boxed{|A| \angle \theta \rightarrow A\,[\cos\theta + j \sin\theta]}$

Rectangular $\longrightarrow$ polar

$\boxed{a + jb \longrightarrow A = \sqrt{a^2 + b^2},\ \theta = \tan^{-1}\frac{b}{a}}$

Re: $3 + 4j \longrightarrow A = \sqrt{9 + 16} = 5$, $\theta = \tan^{-1}\frac{4}{3} = 53.13$

polar

Re: $5 \angle 53.13\ (A \angle \theta)$

[in calculator — 1st Pol(3, 4) = 5
then, RCL → tan]

Pol: $5 \angle 53.13\ (A \angle \theta)$

[in calculator — 1st Rec(5, 53.13)
then, Rcl → tan]

Rec: $3 + 4j$

$A = 5,\ \theta = 53.13$

$5\,[\cos 53.13 + j \sin 53.13]$

$5 \angle 53.13 \times 4 \angle 10$

$|5| \times |4| = 20$ [Angle always added for multiplication]

$\angle 53.13 + \angle 10 = \angle 63.13$

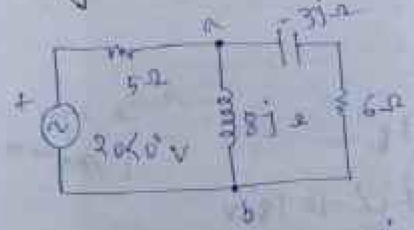
$20 \angle 63.13 \,/\, 10 \angle -45$

$20 / 10 = 10$

$63.13 - (-45) =$ [Angle will subtract for division]

[As it is AC ckt, the Res will be written as ... there is no resistance in current ckt, we are ...]

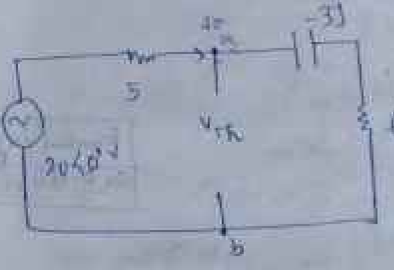
① Determine the voltage across and current through the inductor in the given circuit using thevenin's theorem.



[inductor will always $+j$
capacitor " " $-j$]

[There is 1 AC source, so $+$ $-$ consideration is not considering]

$R_L = 8j$

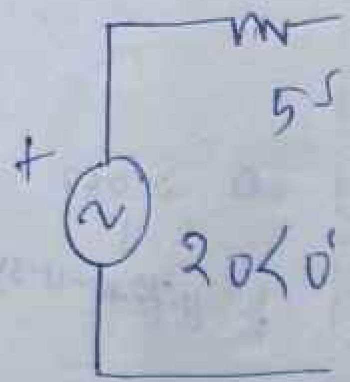


$V_{th} = V_{ab}$

$$\frac{V_a - 20}{5} + \frac{V_a}{6 - 3j} = 0$$

$-3j\,\Omega$

[inductors will always j j]



pol	→	Rec
$50\angle 0^\circ$	=	$50+0.j$
$50\angle 90^\circ$	=	$0+50j$

Rec		pol
$50\angle 90^\circ$	=	$0+50j$
$50\angle 0^\circ$	=	$50+0.j$

ation

ompulsor

$V_{th} = V_{ab}$

$V_a - 20 + V_a$

$\Rightarrow \dfrac{(V_a - 20)(6-3j) + 5V_a}{5(6-3j)} = 0$

$\Rightarrow V_a\left[\dfrac{1}{5} + \dfrac{1}{6-3j}\right] = 4$

$\Rightarrow V_a\left[0.2 + \dfrac{1}{6.71\angle -26.565}\right] = 4$

$\Rightarrow V_a\left[0.2 + 0.149\angle 26.565\right] = 4$

$\Rightarrow V_a\left[0.2 + 0.134 + 0.06j\right] = 4$

$\Rightarrow V_a\left[0.334 + 0.06j\right] = 4$

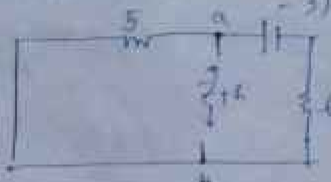
$\Rightarrow V_a\left[0.34\angle 10.18\right] = 4$

$\Rightarrow V_a = \dfrac{4}{0.34\angle 10.18}$

$\Rightarrow V_a = 11.76\angle 10.18$

$V_a = V_{ab} = V_{th} = 11.76\angle -10.18^\circ\ V$

Z_{th}

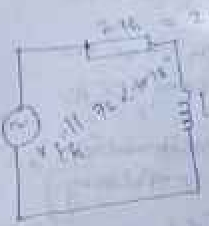


$Z_{th} = \dfrac{(6-3j)\times 5}{(6-3j)+5}$

$= \dfrac{30-15j}{11-3j} = \dfrac{33.54\angle -26.565}{11.4\angle -15.255}$

$Z_{th} = 2.942\angle -11.27^\circ\ \Omega$

$= 2.88 - 0.574j\ \Omega$



$V_L = \dfrac{11.76\angle -10.18 \times 8j}{2.88 - 0.574 + 8j}$

$= \dfrac{(11.76\angle -10.18)\times(8\angle 90)}{2.88 + 7.43j}$

$= \dfrac{94.08\angle 79.82}{2.88 + 7.43j}$

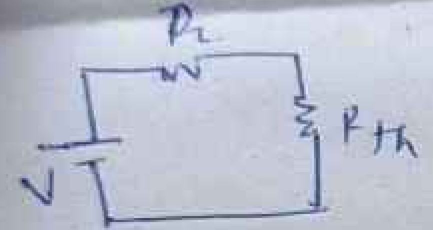
$= \dfrac{94.08\angle 79.82}{7.968\angle 68.81}$

$= 11.81\angle 11.01^\circ\ V$

$I_L = \dfrac{V_L}{8j} = \dfrac{11.81\angle 11.01}{8\angle 90}$

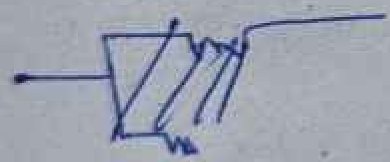
$= 1.476\angle -78.99$

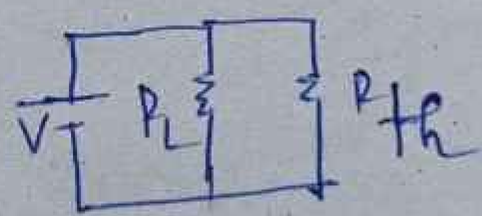
$= 1.476\angle -79^\circ\ A$



$I_L = I_R$

~~V~~ $V_R \neq V_L$ ∴ VDR will be applied





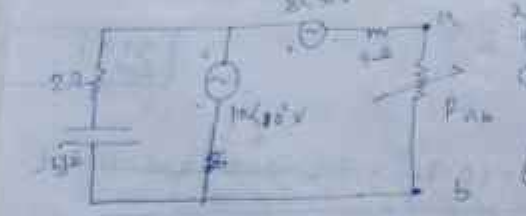
$V_L = V_R$

$I_L \neq I_R$ ∴ CDR will applied

$= 1.476\angle -78.99$

$= 1.476\angle -79^\circ$ A

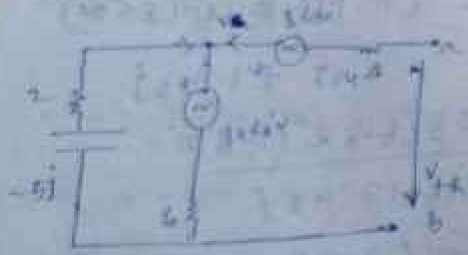
(6) In the network shown determine the variation of current in R_{ab} when it is varried from 2Ω to 10Ω.



[When there are 2 source S(A) then 4 consideration (complalsory)]

[I_1, I_2 ...

1st find thevenin's equivalent network



$$\frac{V_{th}}{2-5j} + \frac{V_{th}-10}{6} = 0$$

$$\Rightarrow \frac{V_{th}}{2-5j} + \frac{V_{th}}{6} = \frac{10}{6}$$

$$\Rightarrow V_{th}\left[\frac{1}{6} + \frac{1}{2-5j}\right] = 1.67$$

$$\Rightarrow V_{th}\left[0.167 + \frac{1}{5.385\angle -68.2}\right] = 1.67$$

$$\Rightarrow V_{th}[0.167 + 0.185\angle 68.2] = 1.67$$

$$\Rightarrow V_{th}[0.167 + 0.068 + 0.17j] = 1.67$$

$$\Rightarrow V_{th}[0.235 + 0.17j] = 1.67$$

$$\Rightarrow V_{th} = \frac{1.67}{0.235 + 0.17j}$$

$$= \frac{1.67}{0.29\angle 35.9}$$

$$\Rightarrow V_{th} = 5.76\angle -35.9$$

V_{th} $V_{ab} = V - 8\angle 30$ [...]

$= 5.75\angle -35.9 - 8\angle 30$

$= \cancel{4.66\angle -3.37}$

$= 4.66 - 3.37j - (6.92 + 4j)$

$= -2.26 - 7.37j = 7.716\angle -107.048$

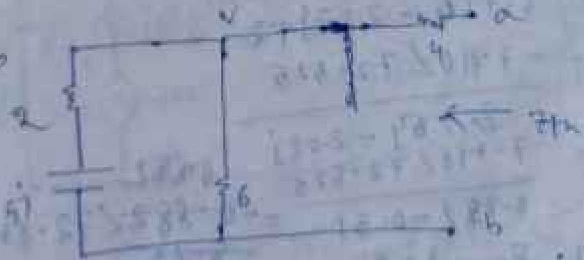
$= -(2.26 + 7.37j)$

$= -7.716\angle 72.95$

$= 7.716\angle(72.95 - 180)$

$= 7.716\angle -107.048$

2nd,



$$\frac{6\times(2-5j)}{2+6-5j} + j4 \quad \frac{12-30j}{8-5j} + j4$$

$= \frac{32.31\angle -68.2}{9.43}$

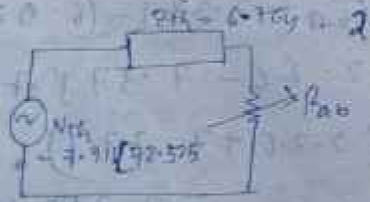
$$= \frac{12-30j+32-20j}{8-5j}$$

$$= \frac{44-50j}{8-5j}$$

$$= \frac{66.6\angle -48.652}{9.43\angle -32}$$

$$= 7.062\angle -16.652$$

$$= (6.765 - 2.02j)\,\Omega$$



(i) When $R_{ab} = 2\,\Omega$

$$I = \frac{7.716\angle 72.525}{6.764 - 2.02j + 2}$$

$$= \frac{-7.716\angle 72.525}{8.764 - 2.02j}$$

$$= \frac{-7.716\angle 72.525}{8.79\angle -0.31} = -0.882\angle 72.835$$

(ii) When $R_{ab} = 10\,\Omega$

$$I = \frac{-7.716\angle 72.525}{6.764 - 2.02j + 10}$$

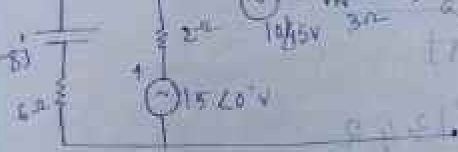
$$= \frac{-7.716\angle 72.525}{16.764 - 2.02j}$$

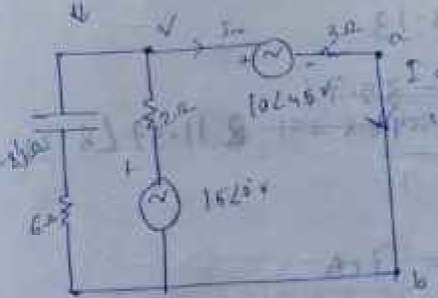
$$= \frac{-7.716\angle 72.525}{16.75\angle -0.59}$$

$$= -0.461\angle 73.115$$

$$= 0.461\angle 106.845$$

(3) Obtain the Nortons equivalent circuit at terminals a,b of the network.





Here I_N is to be calculated only current across any element (Impedance) between the terminals. If it were not then we have to calculated Z_N ...

$$\frac{V}{(6-8j)} + \frac{V-15}{2} + \frac{V-10\angle 45}{3} = 0$$

$$\Rightarrow V\left(\frac{1}{6-8j} + \frac{1}{2} + \frac{1}{3}\right) = \frac{15}{2} + \frac{10\angle 45}{3}$$

$\Rightarrow V\left(\frac{1}{6-8j} + \frac{1}{3}\ 0.5 + 0.3\right) = 7.5 + 3.33\angle 45$

$\Rightarrow V\left(\frac{1}{10\angle -53.13} + 0.8\right) = 7.5 + 3.33\angle 45$

$\Rightarrow V(0.1\angle 53.13 + 0.8) = 7.5 + 3.33\angle 45$

$\Rightarrow V(0.06 + 0.08j) = 7.5 + 2.35 + 2.35j$

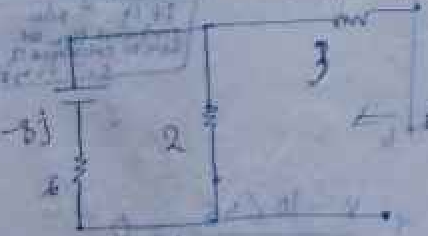
$\Rightarrow V(0.86 + 0.08j) = 9.85 + 2.35j$

$\Rightarrow V = \frac{9.85 + 2.35j}{0.89 + 0.08j}$

$= \frac{10.123\angle 13.42}{0.89\angle 5.13}$

$\frac{101.3\angle 30.71}{}$

$= \cancel{11.304\angle 8.35}\ \ 11.37\angle 8.78$



nohed

$I_N = \frac{V - 10\angle 45}{3}$

$= \frac{11.304\angle 8.35 - 10\angle 45}{3}$

$= \frac{11.189 + 1.64j - (7.07 + 7.07j)}{3}$

$= \frac{4.114 - 5.43j}{3}$

$= \frac{1}{3}(4.11 - 5.43j)$

$= \frac{11.37\angle 8.78 - 10\angle 45}{3}$

$= \frac{11.25 + 1.63j - (7.07 + 7.07j)}{3}$

$= \frac{4.18 - 5.44j}{3}$

$= \frac{6.86\angle -52.46}{3}$

$= 2.28\angle -52.46$

$$V\left(\frac{1}{6-8j}+\frac{1}{2}+\frac{1}{3}\right)=\frac{15}{2}+\frac{10\angle 45}{3}$$

$$\Rightarrow V\left(\frac{1}{10\angle -53.13}+0.5+0.3\right)=7.5+3.33\angle 45$$

$$\Rightarrow V\left(0.1\angle 53.13+0.8\right)=7.5+3.33\angle 45$$

$$\Rightarrow V=\frac{7.5+3.33\angle 45}{0.1\angle 53.13+0.8}$$

$$=\frac{7.5+2.35+2.35j}{0.06+0.08j+0.8}$$

$$=\frac{9.85+2.35j}{0.86+0.08j}$$

$$=\frac{10.12\angle 13.42}{0.86\angle 5.31}$$

$$V=11.76\angle 8.1$$

$$I_N=\frac{V-10\angle 45}{3}$$

$$=\frac{11.76\angle 8.1-10\angle 45}{3}$$

$$=\frac{11.64+1.65j-(7.07+7.07j)}{3}$$

$$=\frac{11.64+1.65j-7.07-7.07j}{3}$$

$$=\frac{4.57-5.42j}{3}$$

$= \cancel{7.08}$

$\cancel{7.1}$

$$\frac{7.1 \angle -49.86}{3}$$

$= 2.37 \angle -49.86$

18/4/22

Maximum Power transfer theorem

(1) What should be the value of Z_L connected across the terminal a, b so that it will draw maximum power. What is the amount of maximum power?

Asignment — Derivation
Pmax —

Theorem — Max power will transfer to the L is (AC circuit) max when the load is complex conjugate of the thevenin's equivalent impedance

Complex conjugate — $(x+jy)^* = x-jy$

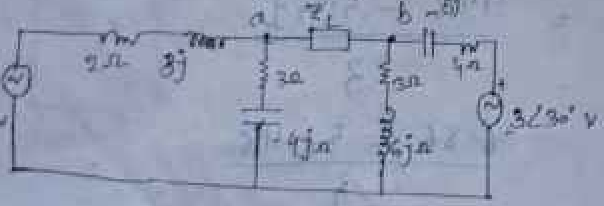
$A\angle\theta \longrightarrow A\angle-\theta$

$Z = \sqrt{R^2+(X_L-X_C)^2}$

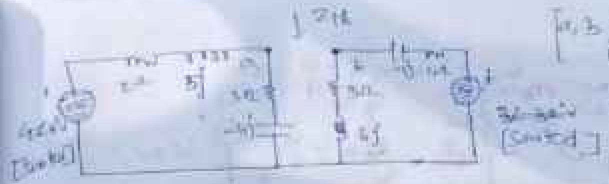
$Z = R + xj$

$+j \longrightarrow X_L$
$-j \longrightarrow X_C$

Real part = R, Img part = X



Z_{th}



[0.3 point intro]

$$Z_{th} = \frac{(2+3j)\times(3-4j)}{2+3j+3-4j} + \frac{(4-5j)\times(3+6j)}{4-5j+3+6j}$$

$$= \frac{6-8j+9j-12j^2}{5-j} + \frac{12+24j-15j-30j^2}{7+j}$$

$$= \frac{3.61\angle56.31 \times 5\angle-53.13}{5.1\angle-11.31} + \frac{6.4\angle-51.34 \times 6.71\angle63.43}{7.07\angle8.13}$$

$$= \frac{18.05\angle3.18}{5.1\angle-11.31} + \frac{42.944\angle12.09}{7.07\angle8.13}$$

$= 3.54\angle14.49 + 6.07\angle3.96$

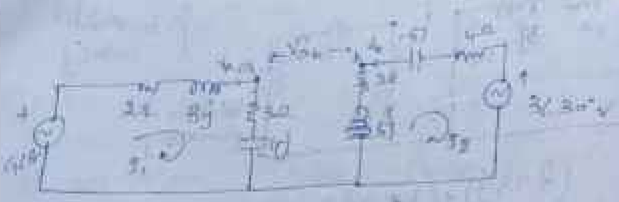
$= 3.43 + 0.88j + 6.055 + 0.42j$

$= 9.486 + 1.3j$

$= 9.55\angle7.8$

$Z_L = Z_{th}^* = 9.47 - 1.3j\ \Omega$ | ~~9.55∠~~ $9.486 - 1.3j = 9.57\angle-7.8$

$$P_{max} = \frac{|V_{th}|^2}{4R_L},\ I_L^2 R_L = I_L^2 Z_L\ \ I_L^2\ R_{th} \cdot I_L^2 Z_{th}$$



$$\frac{V_a - 4\angle 0}{(2+3j)} + \frac{V_a}{(3-4j)} = 0$$

$$\Rightarrow V_a\left[\frac{1}{2+3j} + \frac{1}{3-4j}\right] = \frac{4}{2+3j}$$

$$\Rightarrow V_a\left[\frac{1}{3.4\angle 56.31} + \frac{1}{5\angle -53.13}\right] = \frac{4}{3.6\angle 56.31}$$

$$\Rightarrow V_a\left[0.28\angle -56.31 + 0.2\angle 53.13\right] = 1.11\angle -56.31$$

$$\Rightarrow V_a\left[0.15 - 0.23j + 0.12 + 0.16j\right] = 1.11\angle -56.31$$

$$\Rightarrow V_a\left[0.28 - 0.07j\right] = 1.11\angle -56.31$$

$$\Rightarrow V_a = \frac{1.11\angle -56.31}{0.28 - 0.07j}$$

$$= \frac{1.11\angle -56.31}{0.288\angle -14.03}$$

$$= 3.85\angle -42.28$$

$$= 2.85 - 2.6j$$

$$\frac{V_b - 3\angle 30}{(4-5j)} + \frac{V_b}{(3+6j)} = 0$$

$$\Rightarrow V_b\left[\frac{1}{4-5j} + \frac{1}{3+6j}\right] = \frac{3\angle 30}{4-5j}$$

$$\Rightarrow V_b\left[\frac{1}{6.4\angle -51.34} + \frac{1}{6.7\angle 63.43}\right] = \frac{3\angle 30}{6.4\angle -51.34}$$

$$\Rightarrow V_b\left[0.156\angle 51.34 + 0.15\angle -63.43\right] = 0.47\angle 81.34$$

$$\Rightarrow V_b\left[0.097 + 0.12j + 0.067 - 0.13j\right] = 0.47\angle 81.34$$

$$\Rightarrow V_b = \frac{0.47\angle 81.34}{0.164 - 0.01j}$$

$$= \frac{0.47\angle 81.34}{0.16\angle -3.5}$$

$$= 3\angle 84.84$$

$$= \cancel{-1.32 + 2.7j} = 0.27 + 3j$$

$$V_{ab} = -(V_b - V_a)$$

$$= -\{0.27 + 3j - 2.85 + 2.6j\}$$

$$= -(-2.58 + 5.6j) = V_{th} = 6.16\angle 114.7$$

$$P_{max} = \frac{(-2.58 + 5.6j)^2}{4 \times 0.55\angle 7.8} = \frac{6.16^2 \angle 114.7 + 114.7}{4 \times 0.55\angle 7.8}$$

$$= 1.14\angle 221.6$$

$$I_L = \frac{6.76\angle 114.7}{0.456 - 1.3j + 0.48 + 1.3j}$$

$$= 20.32\angle 114.7$$

$$P_{max} = (0.32\angle 114.7)^2 \times 0.57\angle -7.8$$

$$= 0.07\angle 221.6$$

24/8/22

$$I_1 = \frac{4}{5-j}$$

$$= \frac{4}{5.1\angle -11.31}$$

$$= 0.784\angle 11.31$$

$$I_2 = \frac{3\angle 30}{7+j}$$

$$= \frac{3\angle 30}{7.07\angle 8.13}$$

$$= 0.424\angle 21.87$$

$$V_a = I_1(5-4j) = (0.784\angle 11.31)(5\angle -51.34)$$

$$= 3.92\angle -41.82$$

$$V_b = I_2(3+6j)$$

$$= (0.424\angle 21.87)(3+6j)$$

$$(0.424\angle 21.87)(6.71\angle 63.43)$$

$$= 2.89\angle 85.3$$

$$V_{ab} = V_{th} = V_a - V_b$$

$$= 3.92\angle -41.82 - 2.89\angle 85.3$$

$$= 2.92 - 2.61j - (0.24 + 2.88j)$$

$$= 2.68 - 5.49j$$

$$= 6.11\angle -63.6$$

$$P_{max} = \frac{|V_{th}|^2}{4R_L}$$

$$= \frac{36\angle -126.6}{4\times 9.47}$$

$$= 0.95 \cancel{\angle -126.6}\ W$$

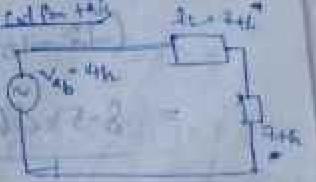
R_L: We have to take the real part of real Z_{th}

$Z_L = Z_{th}^*$ for the pole of

$$I_L = \frac{V_{th}}{Z_{th} + Z_L}$$

$$= \frac{6\angle -63.6}{2\times 9.47}$$

$$= 0.316\angle -63.6\ A$$



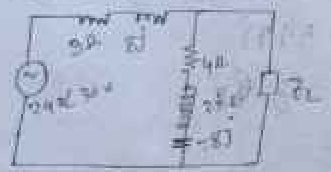
$$P_{L_{max}} = I_L^2 R_L$$

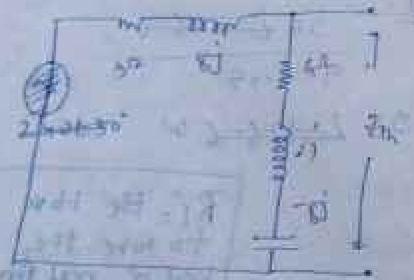
$$= (0.316)^2 \times 9.47$$

$$= 0.95\ W$$

[for P_{max} only all of the mag values will be used no angle will come there]

① Find the circuit shown, what load impedance absorb the max power and what is the power?





$Z_{th} = \dfrac{(3+8j)\times(4-6j)}{3+4+8j-6j}$

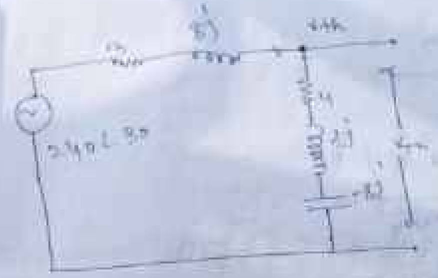
$= \dfrac{12-18j+32j-48}{}$

$= \dfrac{8.54\angle 69.44 \times 7.21\angle -56.31}{7.28\angle 15.94}$

$= 8.45\angle -2.81$

$= 8.44 - 0.41j$

$Z_L = 8.44 + 0.41j$



$\dfrac{V_{th} - 240\angle 30}{(3+8j)} + \dfrac{V_{th}}{(4-6j)} = 0$

$\Rightarrow V_{th}\left[\dfrac{1}{3+8j} + \dfrac{1}{4-6j}\right] = \dfrac{240\angle 30}{3+8j}$

$\Rightarrow V_{th}\left[\dfrac{1}{8.54\angle 69.44} + \dfrac{1}{7.21\angle -56.31}\right] = \dfrac{240\angle 30}{8.54\angle 69.44}$

$\Rightarrow V_{th}\left[0.12\angle -69.44 + 0.138\angle 56.31\right] = 28.1\angle -39.44$

$\Rightarrow V_{th}\left[0.042 - 0.112j + 0.08 + 0.116j\right] = 28.1\angle -39.44$

$\Rightarrow V_{th}\left[0.122 + 0.004j\right] = 28.1\angle -39.44$

$\Rightarrow V_{th} = \dfrac{28.1\angle -39.44}{0.122 + 0.004j}$

$= \dfrac{28.1\angle -39.44}{0.122\angle 1.87} = 230.32\angle -41.31$

$$P_{max} = \frac{|V_{th}|^2}{4R_L}$$

$$= \frac{(2.31 \cdot 0.32)^2}{4 \times 8.04}$$

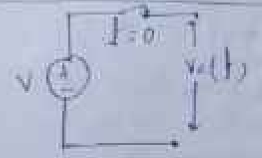
$$= 1.57 \text{ W}$$

26/8/22

Signal

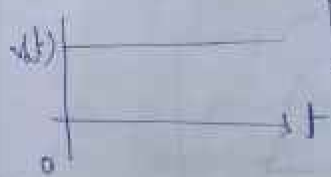
(is a func representing physical quantity)
(Temp, P, voice etc.)
(electrical I, volt)

i) step function / step signal –



$$v_0(t) = \begin{cases} 0, & \text{for } t < 0 \\ V, & \text{for } t \geq 0 \end{cases}$$

$0 \leq t < \infty$



$$\boxed{v_0(t) = V u(t)}$$

[$u(t)$ = unit step func]

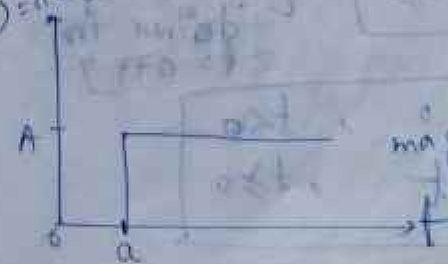
$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

general step function –

$$\boxed{f(t) = A \cdot u(t-a) = \begin{cases} 0, & t < a \\ A, & t \geq a \end{cases}}$$

[A = general amplitude
a = Delay]

$f(t) = A u(t-a)$

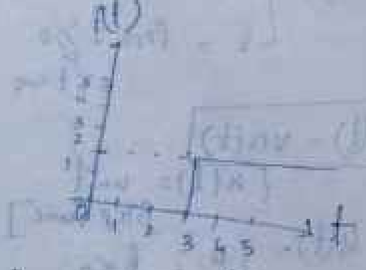


mag = A

$f(t) = Au(t+a)$ [a: progress here]

$f(t) = Au(t+a)$

$f(t) = 2u(t-3)$



Ramp signal/function

$r(t) = \int u(t)\,dt = \int 1\,dt$

$\boxed{r(t) = t}$

$\int u(t)\,dt$ $\int 1\,dt = u(t)\int dt = t\,u(t)$

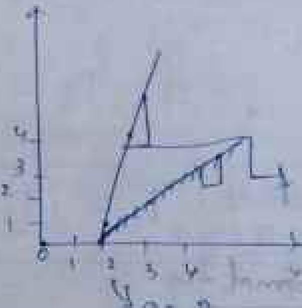
$\boxed{r(t) = t\,u(t)}$ → [this value is defined for $t = 0$ to $+\infty$]

$$r(t) = \begin{cases} 0, & t<0 \\ t, & t \geq 0 \end{cases}$$

(ramp signal)

general ramp function

$$f(t) = A\,r(t-a) = \begin{cases} 0, & t<a \\ At, & t \geq a \end{cases}$$



$A = \frac{2}{1} = 2$

$f(t) = 2\,r(t-2)$

$a = 2$

$$\boxed{\frac{d}{dt}[r(t)] = u(t)}$$

Parabola / parabolic function

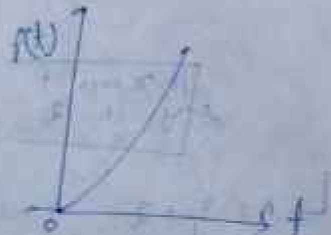
$$\boxed{\int \int u(t)\,dt = \int r(t) = \int t\,dt = \frac{t^2}{2}}$$

$$f(t) = \begin{cases} 0, & t < 0 \\ \frac{t^2}{2}, & t \geq 0 \end{cases}$$

or

$$f(t) = \begin{cases} 0 & t < 0 \\ t^2, & t \geq 0 \end{cases}$$

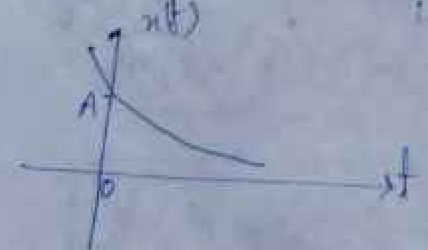
[As constant don't change the shape of signal]

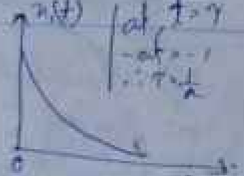


Exponential function / signal -

$$\boxed{x(t) = A e^{-at}}$$

$$\boxed{x(t) = A e^{-at} \cdot u(t)}$$





at, $t = \tau$
$-a\tau = -1$
$\therefore \tau = \frac{1}{a}$

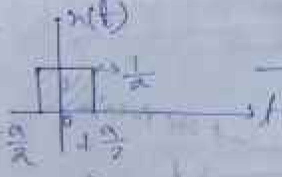
Theoritically $x(t)$ can become equal to 0 at $t = \infty$

But practically $x(t)$ is almost '0' or negligibly small $x(t) \simeq 0$ at $t = 5\tau$ (time)

Where, $\tau = \frac{1}{a}$ = time constant. [at this value the exponent is equal to -1]

Impulsive signal / function or Dirac delta function

Pulse



→ (finite / rectangular pulse)

$$x(t) = \begin{cases} \frac{1}{a}, & -\frac{a}{2} < t < \frac{a}{2} \\ 0 & \text{otherwise} \end{cases}$$

Now,

$$\boxed{\delta(t) = \lim_{a \to 0} x(t)}$$ → area

$$= \begin{cases} 1, & t = 0 \\ 0, & t \neq 0 \end{cases}$$

Base = a
height = $\frac{1}{a}$
area = 1

[For any value of a the area is 1]

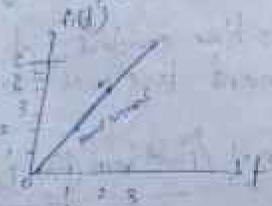
So, A unit impulse is a pulse with infinite height, 0 ~~with~~ width and area = 1 (that is the uniqueness)

Composite signals

(1) $f(t) = 2t\,u(t-3)$

$f_1 \quad f_2$

$f_1(t) =$



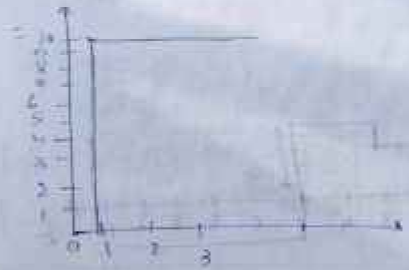
$f_2(t) =$

$f(t) = f_1(t) \cdot f_2(t)$

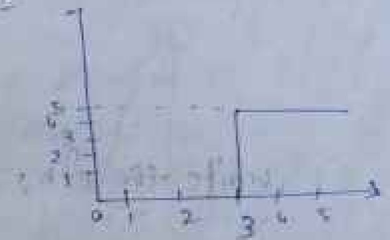
$f(t) = 10\,u(t-1) + 5\,u(t-3) - 20\,u(t-5)$

$f_1 \qquad f_2 \qquad f_3$

$f_1(t) =$



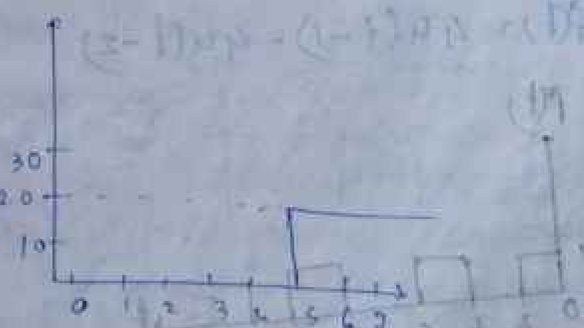
$f_2(t) =$

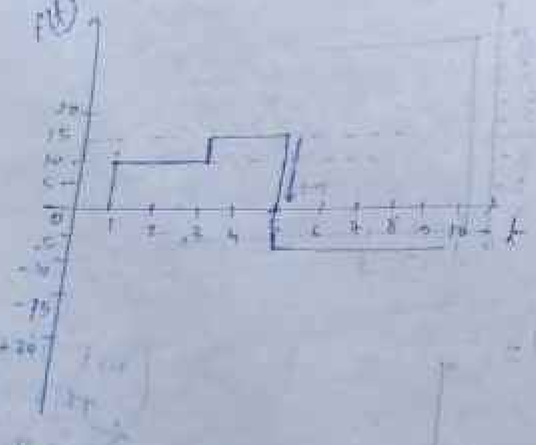


(not like this)

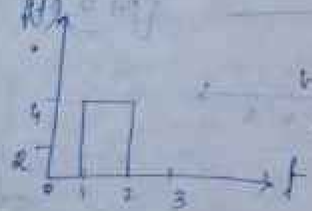
[p+0]

$f_3(t) =$





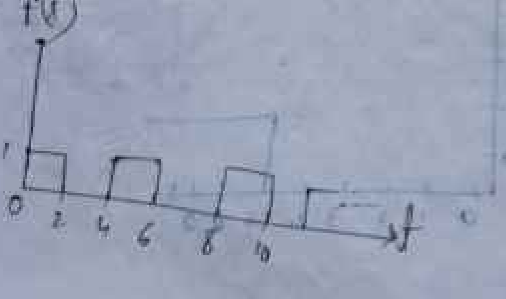
②



write the eqn,

$f(t) = 4u(t-1) - 4u(t-2)$

③



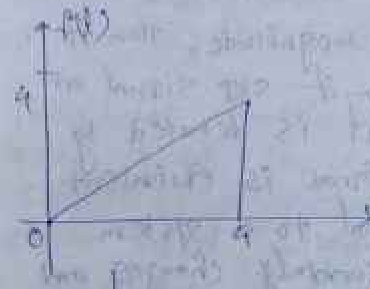
$f(t) = u(t) - u(t-2) + u(t-4) - u(t-6)$

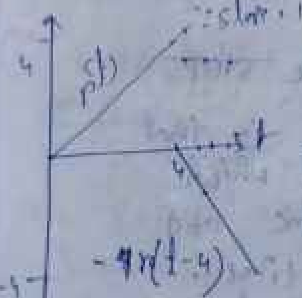
$= \cancel{u(t) + u(t-2) - \{u(t) + u(t-6)\}}$

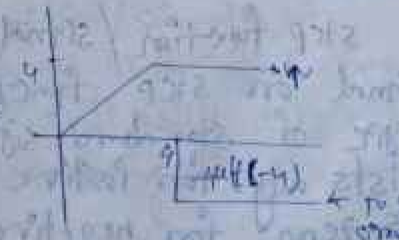
$= \sum_{n=0}^{\infty} (-1)^n u(t-2n)$

③ $\boxed{\begin{array}{l} r(t) = t\,u(t) \\ r(t-a) = (t-a)\,u(t-a) \end{array}}$

④







$f(t) = r(t) - 4r(t-4) - 4u(t-4)$

$= t\,u(t) - 4(t-4)u(t-4) - 4u(t-4)$

① signal - In electronics and telecommunications, signal refers to any time varying voltage, current or electromagnetic wave that carries information. In signal processing, signals are analog and digital representations of analog physical quantities.

②. Step function/signal - If a step signal has unity magnitude, then it is known as the unit step signal or unit step function. It is denoted by u(t). The step signal is equivalent to applying a signal to a system whose magnitude suddenly changes and remains constant forever after application.

③ step function/signal - the step signal or step function is that type of standard signal which exists only for positive time and it is zero for negative time. In order words, a signal x(t) is said to be step signal if and only if it exists for t > 0 and zero for t < 0. The step signal is an important signal used for analysis of many systems.

④ Ramp function/signal - The ramp signal/function is a unary real function, whose graph is shaped like a ramp. It can be expressed by numerous definitions, for example "0 for negative inputs, output equals input for non-negative inputs".

⑤ Parabolic function/signal - When a signal gives the constant acceleration distinction of actual input signal, such a signal is know as parabolic signal or parabolic function. It is also known as unit acceleration signal.

⑥ Exponential function/signal - An exponential signal or exponential function is a function that literally represents an exponentially increasing or decreasing series.

⑦ Unit impulse function/signal - Continuous-time unit impulse signal. Hence, by the defination, the unit impulse signal has zero amplitude everywhere except at $t=0$. At the origin $(t=0)$ the amplitude of impulse signal is infinity so that the area under the curve is unity.

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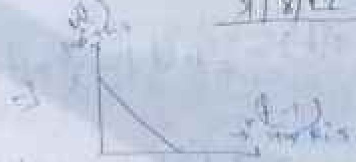
①

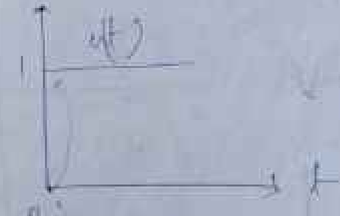


$A = -1$ one method

$x(t) = A\,r(t-1)$

$= [r(t-1)]\ u(t) + r(t-1)$

③



$x(t) = u(t) - r(t) + r(t-1)$

$= u(t) - t\,u(t) + (t-1)\,u(t-1)$

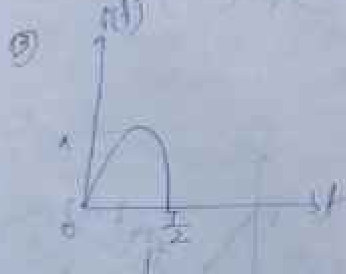
②

$+\frac{3}{2} u(t-6)$ $-\frac{3}{2} r(t-?)$

$r(t) = \frac{3}{2} r(t) - \frac{3}{2} r(t-2) - \frac{3}{2} r(t-2)$
$+ \frac{3}{2} r(t-8)$

$= \frac{3}{2} t\, u(t) - \frac{3}{2}(t-2)\, u(t-2)$
$- \frac{3}{2}(t-4)\, u(t-2) + \frac{3}{2}(t-8)\, u(t-8)$

(5)



$A \sin\left[\frac{2\pi}{T} t\right] \times u(t) + A \sin\left[\frac{2\pi}{T}\left(t - \frac{T}{2}\right)\right] \times u\left(t - \frac{T}{2}\right)$

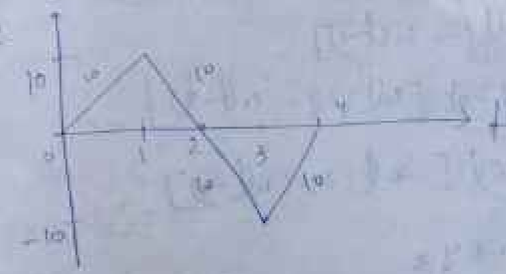
$- A \cos\left[\frac{2\pi}{T} t - \frac{\pi}{2}\right] \times u(t) + A \cos\left[\frac{2\pi}{T}\left(t - \frac{T}{2}\right) - \frac{\pi}{2}\right] \times u(t)$

$= A \cos\left[\frac{\pi}{2} - \frac{2\pi t}{T}\right] \times u(t) + A \cos\left[\frac{\pi}{2} - \frac{2\pi}{T}\left(t - \frac{T}{2}\right)\right] \times u\left(t - \frac{T}{2}\right)$

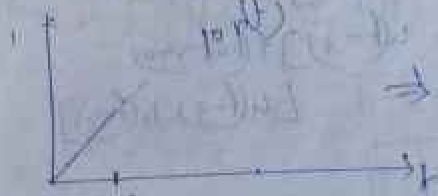
$= A \cos\left[\frac{2\pi}{T}\left(t - \frac{T}{4}\right)\right] \times u(t) + A \cos\left[\frac{2\pi}{T}\left(t - \frac{3T}{4}\right)\right] \times u\left(t - \frac{T}{2}\right)$

$= A \cos\left[\frac{2\pi}{T}\left(t - \frac{T}{4}\right)\right] \cdot u(t) + A \cos\left[\frac{2\pi}{T}\left(t - \frac{3T}{4}\right)\right] \times u\left(t - \frac{T}{2}\right)$

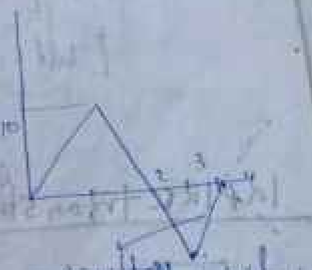
(7)



(8)



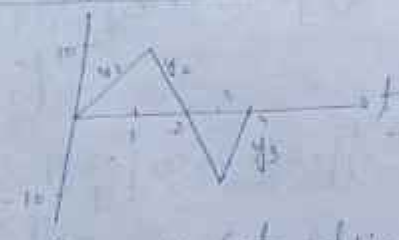
$\Rightarrow$



$-20\, r(t-1)$ $\quad +20\, r(t-3)$ $\quad 10\, r(t-4)$

$10\, r(t) - 20\, r(t-1) + 20\, r(t-3) - 10\, r(t-4)$

other method



$y = mx + c$ (straight line / ...)

$y_1 = 10t\,[u(t) - u(t-1)]$

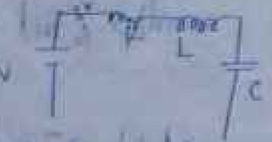
$y_2 = (-10t + 20)\,[u(t-1) - u(t-3)]$

$y_3 = (10t - 40)\,[u(t-3) - u(t-4)]$

$f(t) = y_1 + y_2 + y_3$

$= 10t\,[u(t) - u(t-1)] + (-10t+20)\,[u(t-1) - u(t-3)] + (10t-40)\,[u(t-3) - u(t-4)]$

laplace transform



$v = iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt$ (integro differential eqn)

$\mathcal{L}[f(t)] = F(s) = \int_{-\infty}^{\infty} f(t)\,e^{-st}\,u(t)\,dt$

$= \int_{0^-}^{\infty} f(t)\,e^{-st}\,dt$ [∵ $u(t)$ is multiplied] 0^- is the initial condition

Here $s = \sigma + j\omega$, the complex frequency variable

Inverse laplace transform

$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j}\int_{\sigma_0 - j\infty}^{\sigma_0 + j\infty} F(s)\,e^{st}\,ds$

Here σ_0 is a real positive constant

(1) $f(t) = u(t)$

then, $\mathcal{L}\,f(t) = \int_0^{\infty} u(t)\,e^{-st}\,dt$

$= \left[\frac{e^{-st}}{-s}\right]_0^{\infty}$

$= \boxed{+\frac{1}{s}}$

(2) $f(t) = e^{-at}$

$\mathcal{L}[f(t)] = \int_0^{\infty} e^{-at}\,e^{-st}\,dt$

$= \int_0^{\infty} e^{-(s+a)t}$

$= -\frac{1}{s+a}\left[e^{-(s+a)t}\right]_0^{\infty}$

$$= \boxed{\frac{1}{s+a}}$$

$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{j\omega t} = \cos\omega t + j\sin\omega t$

$e^{-j\omega t} = \cos\omega t - j\sin\omega t$

③ $\sin\omega t$

$$\mathcal{L}[\sin\omega t] = \int_0^\infty \sin\omega t \, e^{-st} \, dt$$

$$= \frac{1}{2j} \int_0^\infty \left(e^{j\omega t} - e^{-j\omega t}\right) e^{-st} \, dt$$

$$= \frac{1}{2j} \int_0^\infty \left[e^{j\omega t - st} - e^{-(j\omega t + st)}\right] dt$$

$$= \frac{1}{2j} \left[\frac{e^{(j\omega - s)t}}{(j\omega - s)} \right]_0^\infty + \left[\frac{e^{-(j\omega + s)t}}{j\omega + s} \right]_0^\infty$$

$$\cancel{\frac{1}{2j}\left[\frac{1}{j\omega - s}\right]}$$

$$\frac{1}{2j} \int_0^\infty \left[e^{-(s-j\omega)t} - e^{-(s+j\omega)t} \right]$$

$$= \frac{1}{2j} \left\{ \left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)} \right]_0^\infty + \left[\frac{e^{-(s+j\omega)t}}{s+j\omega} \right]_0^\infty \right\}$$

$$= \frac{1}{2j} \left[\frac{1}{s-j\omega} - \frac{1}{s+j\omega} \right]$$

$$= \frac{1}{2j} \cdot \frac{s+j\omega - s + j\omega}{s^2 - j^2\omega^2}$$

$$= \frac{1}{j} \times \frac{j\omega}{s^2 - j^2\omega^2}$$

$$= \boxed{\frac{\omega}{s^2+\omega^2}}$$

④ $\cos\omega t$

$$\mathcal{L}[\cos\omega t] = \int_0^\infty \cos\omega t \, e^{-st} \, dt$$

$$= \frac{1}{2} \int_0^\infty \left(e^{j\omega t} + e^{-j\omega t}\right) e^{-st} \, dt$$

$$= \frac{1}{2} \int_0^\infty \left[e^{-(j\omega - s)t} + e^{-(j\omega + s)t} \right] dt$$

$$= \frac{1}{2} \int_0^\infty \left[e^{-(s-j\omega)t} + e^{-(s+j\omega)t} \right] dt$$

$$= \frac{1}{2} \left\{ \left[\frac{e^{-(s-j\omega)t}}{-(s-j\omega)} \right]_0^\infty + \left[\frac{e^{-(s+j\omega)t}}{-(s+j\omega)} \right]_0^\infty \right\}$$

$$= \frac{1}{2} \left[\frac{1}{(s-j\omega)} + \frac{1}{s+j\omega} \right]$$

$$\cancel{= \frac{1}{2} \left[\frac{1}{s+j\omega} + \frac{1}{s-j\omega} \right]} = \frac{1}{2} \left[\frac{s - j\omega + s + j\omega}{s^2 + \omega^2} \right]$$

$$\cancel{= \frac{1}{2} \left[\frac{s - j\omega + s + j\omega}{s^2 + \omega^2} \right]} = \frac{1}{2} \times \frac{2s}{s^2+\omega^2}$$

$$\cancel{= \frac{1}{2} \cdot \frac{2j\omega}{s^2+\omega^2}} = \boxed{\frac{s}{s^2+\omega^2}}$$

Few important properties on operation of transforms

$f(t)$	$F(s)$
1) $k\,f(t)$	$k\,F(s)$
2) $f_1(t)+f_2(t)$	$F_1(s)+F_2(s)$
3) $\frac{d}{dt}f(t)$	$sF(s)-f(0)$
4) $\int_0^t f(t)$	$\frac{F(s)}{s}$
5) $e^{-at}f(t)$	$F(s+a)$
6) $f(t-a)\,u(t-a)$	$e^{-as}F(s)$

Few transform pairs

$f(t)$ $\longleftrightarrow$	$F(s)$
1) $\delta(t)$	1
2) $u(t)$	$1/s$
3) t	$1/s^2$
4) e^{-at}	$1/s+a$
5) $\sin\omega t$	$\frac{\omega}{s^2+\omega^2}$
6) $\cos\omega t$	$s/s^2+\omega^2$

7) $t e^{-at}$ $\quad \frac{1}{(s+a)^2}$

Initial value theorem

$$\lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s) \quad [\text{if } f(t)/\text{whose } F'(s) \text{ exists}]$$

Proof

$$L\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0)$$

$$\Rightarrow \int_0^\infty \frac{d}{dt}f(t)e^{-st}\,dt = sF(s) - f(0)$$

Applying $\lim_{s\to\infty}$ on both sides, we gets

$$\Rightarrow \lim_{s\to\infty}\int_0^\infty \frac{d}{dt}f(t)e^{-st}\,dt = \lim_{s\to\infty}[sF(s) - f(0)]$$

$$\Rightarrow \int_0^\infty \lim_{s\to\infty}\frac{d}{dt}f(t)e^{-st}\,dt = \lim_{s\to\infty}[sF(s) - f(0)]$$

$$\Rightarrow 0 = \lim_{s\to\infty}[sF(s)] - f(0)$$

$$\Rightarrow f(0) = \lim_{s\to\infty} sF(s)$$

$$\Rightarrow \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$$

Final value theorem

$f(t)$ and $f'(t)$ should exist

As well as $F(s)$ and $sF(s)$.

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

$$L\left[\frac{d}{dt} f(t)\right] = sF(s) - f(0)$$

$$\Rightarrow \int_0^\infty \frac{d}{dt} f(t)\, e^{-st}\, dt = sF(s) - f(0)$$

Applying $\lim_{s\to 0}$ on both sides we get

$$\Rightarrow \int_0^\infty \lim_{s\to 0} \frac{d}{dt} f(t)\, e^{-st}\, dt = \lim_{s\to 0}\left[sF(s) - f(0)\right]$$

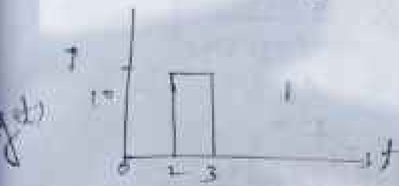
$$\Rightarrow \int_0^\infty d\,f(t) = \lim_{s\to 0}\left[sF(s) - f(0)\right]$$

$$\Rightarrow \left[f(t)\right]_0^\infty = \lim_{s\to 0} sF(s) - f(0)$$

$$\Rightarrow f(\infty) - \cancel{f(0)} = \lim_{s\to 0} sF(s) - \cancel{f(0)}$$

$$\Rightarrow \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$$

① Find the laplace transform of the get function -



Sol: $g(t) = 10\left[u(t-2) - u(t-3)\right]$

Applying laplace on both side

$$G(s) = 10\, L\left[e^{-2s}\cdot\frac{1}{s} - e^{-3s}\cdot\frac{1}{s}\right]$$

using this relation — $L\left[f(t-a)\,u(t-a)\right] = e^{-as}F(s)$

$$= \frac{10}{s}\left[e^{-2s} - e^{-3s}\right]$$

② Find $f(t)$ if $F(s) = \dfrac{6(s+2)}{(s+1)(s+3)(s+4)}$

Let, $\dfrac{6(s+2)}{(s+1)(s+2)(s+4)} = \dfrac{A}{s+1} + \dfrac{B}{s+3} + \dfrac{C}{s+4}$ ← Partial fraction

$\Rightarrow 6(s+2) = A(s+2)(s+4) + B(s+1)(s+3) + C(s+1)(s+2)$

$s=-2$

$\omega = 0 = B(-1)(2)$

$\Rightarrow B(-2) = 0$

$\Rightarrow B = 0 \quad B = 3$

$S = -1$

$A(1)\times(3) = 1$

$\Rightarrow A = \frac{1}{3} \quad A = 1$

$S = -4$

$C(-3)\times(-2) =$

$\Rightarrow C = \frac{2}{3} = 4$

$$\left.\frac{(s+1)\,6(s+2)}{(s+1)(s+3)(s+4)}\right|_{s=-1} = \frac{6}{2} = 3 \quad [A=1]$$

$$\left.\frac{(s+3)\,6(s+2)}{(s+1)(s+3)(s+4)}\right|_{s=-3} = \frac{6\times(-1)}{(-2)\times(1)} = 3$$

$$\left.\frac{(s+4)\,6(s+2)}{(s+1)(s+3)(s+4)}\right|_{s=-4} = -4$$

$$C = \left.\frac{(s+4)\,6(s+2)}{(s+1)(s+2)(s+4)}\right|_{s=-4} = -4$$

$$F(s) = \frac{1}{s+1} + \frac{3}{s+2} - \frac{4}{s+4} \quad \text{(Laplace domain)}$$

$= [e^{-1t} + 3e^{-2t} - 4e^{-4t}] \quad [\because e^{-at}u(t) \leftrightarrow \frac{1}{s+a}]$

$[e^{-t} + 3e^{-2t} - 4e^{-4t}]\,u(t) \rightarrow$ [time domain]

$(e^{-t} + 3e^{-2t} - 4e^{-4t})\,u(t)$

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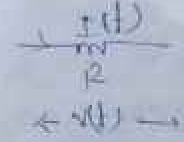
$k f(t) \longleftrightarrow k F(s)$

$e^{-at} \longleftrightarrow \frac{1}{s+a}$

$\frac{d}{dt} f(t) \longleftrightarrow sF(s) - f(0)$

Conversion from t domain to s domain

R



$V = RI$

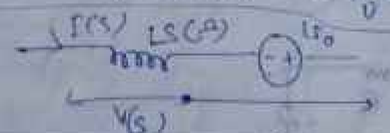
$\boxed{V(s) = R\,I(s)}$

L

$v(t) = L\frac{di(t)}{dt}$

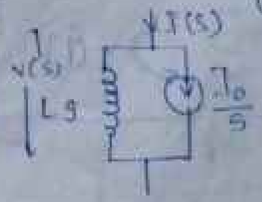
$V(s) = L[sI(s) - I(0)]$

$\boxed{V(s) = LsI(s) - LI_0}$ $[I_0 = I(0)]$



V, I_0 → series
I, I_0 → parallel

$$\boxed{I(s) = \frac{V(s)}{Ls} + \frac{I_0}{s}}$$

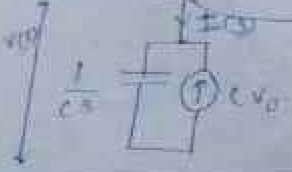


C

$$I(t) = C\frac{dv(t)}{dt}$$

$$I(S) = C[SV(S) - V(0)]$$

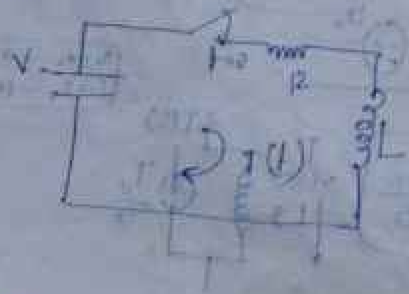
$$\boxed{I(S) = CSV(S) - CV_0} \quad [V(0) = V_0]$$



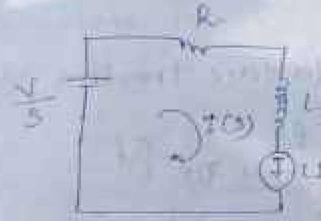
$$\boxed{V(S) = \frac{I(S)}{CS} + \frac{V_0}{S}}$$

① Consider the given circuit and when switch is closed $t=0$ find $I(t)$?

RL)



transforming this ckt into (from t domain) to laplace (S domain) form



$$\boxed{V(t) \to \frac{V}{S}}$$

$$\frac{V}{S} = RI(S) + LSI(S) - LI_0$$

$$\Rightarrow \frac{V}{S} + LI_0 = I(S)[R + LS]$$

$$\Rightarrow I(S) = \frac{V}{S(R+LS)} + \frac{LI_0}{(R+LS)}$$

$$= \frac{V/L}{S(S+\frac{R}{L})} + \frac{LI_0}{L(\frac{R}{L}+S)}$$

$$\frac{V/L}{S(S+\frac{R}{L})} = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$$

$$\frac{V}{L} = A\left(S+\frac{R}{L}\right) + BS$$

$$A = \left.\frac{\cancel{S}\, V/L}{\cancel{S}(S+\frac{R}{L})}\right|_{S=0} = \frac{V \times L}{L \times R} \Rightarrow \frac{V}{R}$$

$$B = \left.\frac{\cancel{(S+\frac{R}{L})}\, V/L}{S\cancel{(S+\frac{R}{L})}}\right|_{S=-\frac{R}{L}} \Rightarrow \frac{V \times L}{L \times R} \Rightarrow -\frac{V}{R}$$

$I(s) = \frac{V}{Rs} - \frac{V}{R(s+\frac{R}{L})} + \frac{I_0}{s+\frac{R}{L}}$

Applying inverse laplace transform

$i(t) = \frac{V}{R} - \frac{V}{R}e^{-\frac{Rt}{L}} + I_0 e^{-\frac{Rt}{L}}$

$= \frac{V}{R}\left[1 - e^{-\frac{t}{\tau}}\right] + I_0 e^{-t/\tau}$

[we can write $\frac{V}{R}e^{\frac{Rt}{L}}u(t) + I_0 e^{-\frac{Rt}{L}}u(t)$

where $\tau = \frac{L}{R}$ = time constant of this circuit]

If $I_0 = 0$

then, $i(t) = \frac{V}{R}\left[1 - e^{-\frac{t}{\tau}}\right]$

$= I_{max}\left(1 - e^{-\frac{t}{\tau}}\right)$

where $I_{max} = \frac{V}{R}$

[As $t = \infty$, $i(t) = \frac{V}{R} = I_{max}$]

[In this ckt (L) the I will max at $t = \infty$]

at $t = \tau$

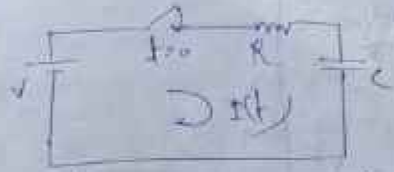
$i(t) = 0.632\frac{V}{R}$

i.e at time constant value of R

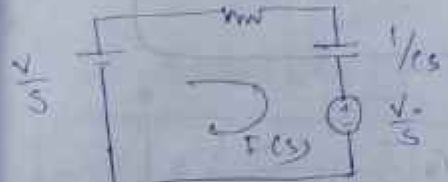
will increase $0.632\frac{V}{R}$

(2) Similar for RC circuit (previous)

RC



Transforming this ckt into laplace form —



$\frac{V}{s} - \frac{V_0}{s} = I(s)R + I(s)\frac{1}{cs}$

$\Rightarrow \frac{V}{s} - \frac{V_0}{s} = I(s)\left[R + \frac{1}{cs}\right]$

$\Rightarrow I(s) = \frac{V}{s(R+\frac{1}{cs})} - \frac{V_0}{s(R+\frac{1}{cs})}$

$\frac{V}{s(R+\frac{1}{cs})} = \frac{A}{s} + \frac{B}{R+\frac{1}{cs}}$

$= \frac{V}{s\cdot\frac{1}{s}(Rs+\frac{1}{c})} - \frac{V_0}{s\cdot\frac{1}{s}(\ldots)}$

$A = s \times \frac{V}{s(R+\frac{1}{cs})}\Big|_{s=0}$

$$I(S) = \frac{V}{RS + \frac{1}{C}} - \frac{V_0}{RS + \frac{1}{C}}$$

$$\frac{V}{RS + \frac{1}{C}} = \frac{A}{RS + \frac{1}{C}}$$

$$A = \frac{V \times (RS + \frac{1}{C})}{(RS + \frac{1}{C})}$$

$$= V$$

$$I(S) = \frac{V}{RS + \frac{1}{C}} - \frac{V_0}{RS + \frac{1}{C}}$$

Applying inverse laplace transform

$I(t) =$

$$I(S) = \frac{V}{R(S + \frac{1}{CR})} - \frac{V_0}{R(S + \frac{1}{CR})}$$

$$\frac{V}{R(S + \frac{1}{CR})} = \frac{A}{R} + \frac{B}{S + \frac{1}{CR}}$$

$$A = \frac{V \times R}{R(S + \frac{1}{CR})}$$

Applying inverse laplace transform

$$I(t) = \frac{V}{R} e^{-\frac{t}{CR}} - \frac{V_0}{R} e^{-\frac{t}{CR}}$$

$$= \frac{V}{R} e^{-\frac{t}{\tau}} - \frac{V_0}{R} e^{-\frac{t}{\tau}} = \left(\frac{V - V_0}{R}\right) e^{-\frac{t}{\tau}}$$

we can write $\frac{V}{R} - V_0 e^{-\frac{t}{\tau}}$ values

[where, $\tau = CR$ = time constant]

If no initial change,

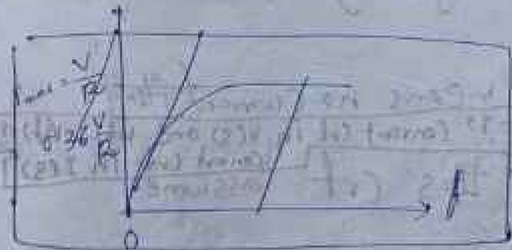
i.e. if $V_0 = 0$

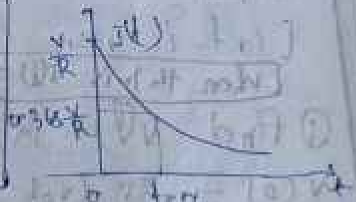
then $I(t) = \frac{V}{R} e^{-\frac{t}{\tau}}$ [In this case (1) the I will max at $t = 0$]

$I(t)$ will be max when $t = 0$

i.e., If $t = 0$, then $I(t) = I_{max} = \frac{V}{R}$

$$\therefore I(t) = I_{max} e^{-\frac{t}{\tau}}$$





at $t = \tau$

$$I(t) = 0.368\frac{V}{R}$$

i.e at time constant τ, value will ~~increase~~ decrease

$0.36\frac{V}{R}$

L — $i(t) = \frac{V}{R}(1 - e^{-\frac{t}{\tau}})$

C — $i(t) = \frac{V}{R} e^{-\frac{t}{\tau}}$

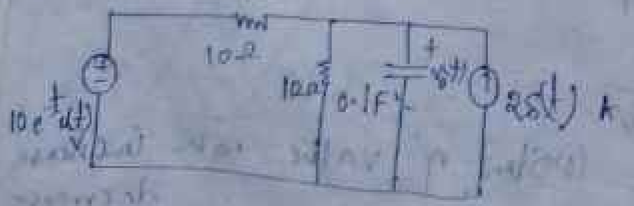
L ⟶ $I(0) = 0$, $I(\infty) = \frac{V}{R}$

C ⟶ $I(0) = \frac{V}{R}$, $I(\infty) = 0$

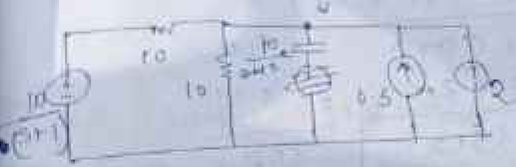
- In initial the inductor is open. At infinity (steady point) the inductor is short [As $\frac{V}{R}$, there are no effects of inductors]
- In capacitor the initially it is short. At infinity (steady point) the capacitor is open. [ckt is open means no current flow]

[when there is $i(t) = ??$ convert ckt in V(s) and when V(t) convert ckt in I(s)]

① Find V(t) in this ckt assume V(0) = 5V (V₀)



transforming this ckt in laplace domain



$\left[u(t) = \frac{1}{s},\ e^{-at} x(t) = \frac{1}{s+a} \right]$

VCL,

$$\frac{V - \frac{10}{s+1}}{10} + \frac{V}{10} + \frac{V}{\frac{10}{s}} = 0.5 + 2$$

$$\Rightarrow V\left[\frac{1 - \frac{10}{s+1}}{10} + \frac{1}{10} + \frac{1 \times s}{10}\right] = 2.5$$

$$\Rightarrow \frac{V}{10}\left[1 - \frac{10}{s+1} + 1 + s\right] = 2.5$$

$$\Rightarrow \frac{V}{10}\left[\frac{s+1-10}{s+1} + 1 + s\right] = 2.5$$

$$\Rightarrow V\left[\frac{1}{10} + \frac{1}{10} + \frac{s}{10}\right] - \frac{1}{s+1} = 2.5$$

$$\Rightarrow V\left[\frac{2}{10} + \frac{s}{10}\right] - \frac{1}{s+1} = 2.5$$

$$\Rightarrow \frac{V}{10}(s+2) - \frac{1}{s+1} = 2.5$$

$$\Rightarrow \cancel{\frac{V \times 10}{s+2} - \frac{1}{s+1} = 2.5}$$

$$\Rightarrow V = \frac{\left[2.5 + \frac{1}{s+1}\right] \times 10}{s+2}$$

$$= \frac{2.5(s+1) + 1}{(s+1)(s+2)} \times 10$$

$$\frac{v}{10} - \frac{1}{s+1} + \frac{v}{10} + \frac{vs}{10} = 2.5$$

$$\Rightarrow \frac{2v}{10} + \frac{vs}{10} - \frac{1}{s+1} = 2.5$$

$$\Rightarrow v\left(\frac{2}{10} + \frac{s}{10}\right) - \frac{1}{s+1} = 2.5$$

$$\Rightarrow \frac{v}{10}(s+2) - \frac{1}{s+1} = 2.5$$

$$\Rightarrow \frac{v}{10} = \frac{\left[2.5 + \frac{1}{s+1}\right]}{s+2}$$

$$\Rightarrow \frac{v}{10} = \frac{2.5}{s+2} + \frac{1}{(s+1)(s+2)}$$

$$\Rightarrow v = \frac{25}{s+2} + \frac{10}{(s+1)(s+2)}$$

$\frac{10y}{3} - \frac{10x}{3^2} = \frac{2}{3}$

$\Rightarrow \frac{10y}{3} + \frac{10}{3^2} \cdot \frac{2}{3} = \frac{2}{3} \cdot \frac{10}{3}$

$\Rightarrow \frac{10y}{3} = \frac{2}{3}$

$\Rightarrow \boxed{y = 0}$

$3h - 0 = 1 - \frac{x}{5}$

$\Rightarrow 3h = 1 + \frac{7}{5} = \frac{12}{5}$

$\Rightarrow \boxed{h = \frac{12}{15}}$

(s) and

assume

$\Rightarrow V = \frac{\left[2.5 + \frac{1}{S+1}\right] \times 10}{S+2}$

$= \frac{2.5(S+1) + 1}{(S+1)(S+2)} \times 10$

$$\Rightarrow \frac{2.5s + 2.5 + 1}{(s+1)(s+2)} \quad s+0$$

$$= \frac{2.5s + 3.5}{(s^2+3s+2)} \quad = \frac{2.5 + \frac{10}{s+1}}{s+2}$$

$$\frac{2.5s + 3.5}{s^2+3s+2}$$

$$= \frac{2.5}{s+2} + \frac{10}{(s+1)(s+2)}$$

$$\frac{10}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \left.\frac{10\,(s+1)}{(s+1)(s+2)}\right|_{s=-1}$$

$$= \frac{10}{1}$$

$$= 10$$

$$B = \left.\frac{10\,(s+2)}{(s+1)(s+2)}\right|_{s=-2}$$

$$= -10$$

Hence

$$V \text{ fn } V(s) = \frac{2.5}{s+2} + \frac{10}{s+1} - \frac{10}{s+2}$$

$$= \frac{15}{s+2} + \frac{10}{s+1}$$

$$v(t) = \left(15e^{-2t} + 10e^{-t}\right) u(t) - \text{ans}$$

10/3/22

(1) Find the unit step response for I_L in the given network. It is given that $I_L(0^-) = 0$

Excitation / main source $= u(t)$ [As we have to find the unit step response]

From the fn $I_S = u(t)$

Transforming the given ckt into laplace domain

[Have to find out $I_L(t)$ But It is not given so $I_L(0^-) = 0$ Please find $I_L(s)$]

[As in qn they told to find unit response for I_L, so no need for $v(t)$ no $I_L(t)$ conversion]

$$I_L(s) = \frac{1}{s} \cdot \frac{3}{3+4+5s} \quad [\text{by CDR}]$$

$$= \frac{1}{s} \cdot \frac{3}{\cancel{12}\,4} \quad = \frac{3}{s(5s+7)}$$

$$\frac{1}{4s}$$

$$\frac{3}{s(5s+7)} = \frac{A}{s} + \frac{B}{5s+7}$$

$$A = \left.\frac{3 \cdot s}{s(5s+7)}\right|_{s=0}$$

$= \frac{3}{7}$

$= \frac{3}{7}$

$B = \frac{3\cancel{(5s+7)}}{s(5s+7)}\Big|_{s=-\frac{7}{5}}$

$= \frac{3\times5}{-7}$

$= -\frac{15}{7}$

$= \frac{3}{7s} - \frac{15}{7(5s+7)}$

$= \frac{3}{7s} - \frac{15}{7\times5(s+\frac{7}{5})}$

Taking inverse laplace

$I_L(t) = \frac{3}{7}u(t) - \frac{3}{7}e^{-\frac{7t}{5}}u(t)$

$= \frac{3}{7}\cancel{u(t)}$

$= \frac{3}{7}u(t)\left[1 - e^{-\frac{7t}{5}}\right]$

(2) ~~4~~ has A coil has a resistance of 2.5Ω and its time constant is 1.6s. Determine a) the current once is after 10V is applied b) the time taken for the current to attain half it's final value.

[when time constant (τ) is given there is no need to ~~find~~ of V(t) no I(t) formation]

$I_L(t) = \frac{V}{R}(1 - e^{-\frac{t}{\tau}})$ $[\tau = \frac{L}{R}]$

$\tau = \frac{L}{R} = 1.6s$

a) $I_L(1) = \frac{10}{2.5}(1 - e^{-\frac{1}{1.6}})$ [no info about t, so assume t = 1]

$= 1.858\,A$

$\approx 1.86\,A$ [as we assume t=1]

b) Final value of current $= \frac{10}{2.5} = 4A$ [As I(t) isn't given we assume t=1]

Half of final value = 2 A

$2 = \cancel{\frac{10}{2.5}}^{4}\left(1 - e^{-\frac{t}{1.6}}\right)$

$\Rightarrow \frac{1}{2} = 1 - e^{-\frac{t}{1.6}}$

$\Rightarrow e^{-\frac{t}{1.6}} = 1 - \frac{1}{2} = \frac{1}{2}$

$\Rightarrow \frac{1}{1.6} = \ln\left(\frac{1}{2}\right)$ $\left[10^{x} = y \Rightarrow x = \log y,\ e^{x} = y,\ \ln_{e} = \ln y\right]$

$\Rightarrow \frac{t}{1.6} = 0.692$

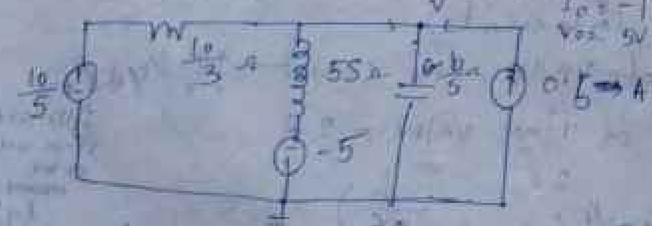
~~$t = 0.4$~~

$\Rightarrow t = 1.1$

(3) Find the value of the voltage across the capacitor assuming that the value of $V_S(t) = 10u(t)$ and at $t = 0$, -1 A flows through the inductor and $+5$ V is across the capacitor.

Given, $I_0 = -1$, $V_0 = 5$ V

Transforming the ckt into Laplace domain



$$\frac{V - \frac{10}{s}}{\frac{10}{3}} + \frac{V + 5}{5s} + \frac{V}{\frac{10}{s}} = 0.5$$

$$\Rightarrow \frac{3V}{10} - \frac{1}{s} + \frac{V}{5s} + \frac{1}{s} + \frac{Vs}{10} = 0.5$$

$$\Rightarrow \frac{3V}{10} + \frac{V}{5s} + \frac{Vs}{10} = 0.5 + \frac{1}{3s} + \frac{1}{5s}$$

$$\Rightarrow V\left[\frac{3}{10} + \frac{1}{5s} + \frac{s}{10}\right] = 0.5 + \frac{4}{s}$$

$$\Rightarrow \frac{V}{10s}\left[3s + 2 + s^2\right] = 0.5 + \frac{4}{s}$$

$$\Rightarrow \frac{V}{10s}(s^2 + 3s + 2) = \frac{0.5s + 4}{s}$$

$$\Rightarrow V(s) = \frac{5s + 40}{s^2 + 3s + 2} = \frac{5s + 40}{(s+1)(s+2)}$$

~~$\frac{5s + 40}{(s+1)(s+2)} = \frac{A}{(s+1)} + \frac{B}{s+2}$~~

~~$A = \frac{5s + 40}{(s+1)(s+2)} \times (s+1) \Big|_{s=-1}$~~

~~$s = -1$~~

~~$= \frac{5/5}{s+1}$~~

$$\frac{5s + 40}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$A = \frac{(5s+40) \times (s+1)}{(s+1)(s+2)}\Bigg|_{s=-1} = \frac{5 \times (-1) + 40}{1} = 35$$

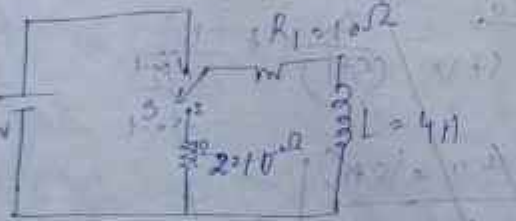
$$B = \frac{(5s+40) \times (s+2)}{(s+2)(s+1)}\Bigg|_{s=-2} = \frac{-10 + 40}{-1} = -30$$

$\frac{5s+10}{(s+1)(s+2)} = V(s) = \frac{35}{s+1} - \frac{30}{s+2} = V$

taking inverse laplace —

$V(t) = [35 e^{-t} - 30 e^{-2t}]$ v/t

(4) This ckt in the figure is initially under steady state condition. The switch is moved from position 1 to position 2 at t=0. find the current after switching



Before switching the Inductor was in steady state, so

$R_1 = R_2 = R = 10$

$I_L(0^-) = \frac{V}{R} = \frac{20}{10} = 2A$

$[0^- = $ Before switching$]$

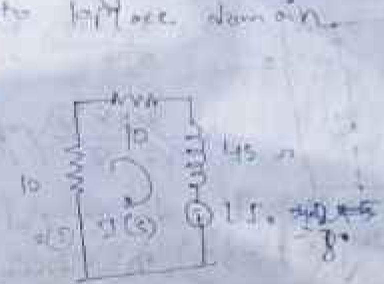
After switching

$[0^+$ After switching$]$

$I_L(0^+) = I_0 = 2A$ [As the current of Inductors will not change so quickly]

[As before switching the value of I is initial and it is I_0]

Converting the after switching ckt into laplace domain.



$I(s) = \frac{8}{4s+20} = \frac{2}{s+5}$ (KVL)

taking inverse laplace transform

$I_L(t) = 2e^{-5t}$ u(t)

(5) Consider this ckt shown the switch was in closed position for a long time. It is opened at t=0. Find the current I(t) for t>0 (after switching)

[opening $t=0$ = 0^+ after switching]

Before switching, $t = 0^-$

generally [$t = 0^-$ before switching]

[Switching = we move the switch from one state (position) to another state (position)]

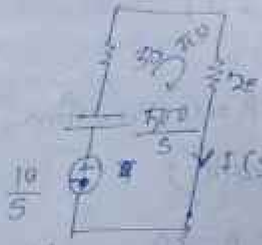
As before switching (0^- $-\infty$ to 0^-) there was no effect of capacitor in ckt i.e. C.C.E. ~~same as before~~

$$\frac{V_c - 35}{500} + \frac{V_c}{200} = 0$$

$$V_c = \frac{\cancel{350} \times 50}{\cancel{2350}} \;//\; 7$$

[we have to find V_c as the after the switching] transfer ckt

After switching ($t = 0^+$) in S-domain



$$\left[\frac{1}{2 \times 10^{-3} S} = \frac{500}{S}\right]$$

[Before switching the voltage = After switching initial voltage]

[Here before switching =

after =]

$\Rightarrow$ ———

$$\frac{V_c}{500} + \frac{V_c}{200} = \frac{35}{500}$$

$$\Rightarrow V_c\left[\frac{1}{500} + \frac{1}{200}\right] = \frac{35}{500}$$

$$\Rightarrow V_c\left[2 \times 10^{-3} + 5 \times 10^{-3}\right] = \frac{35}{500}$$

$$\Rightarrow V_c = \frac{35}{500 \times 7 \times 10^{-3}}$$

$$= 10\,V \;(= V_{0^-})$$

Before switching the vol. of this ckt is 10V, the initial voltage of this ckt = $V_0 = 10V$

$$I(S) = \frac{10}{S\left(\frac{500}{S} + 500 + 200\right)}$$

$$= \frac{10}{S\left(\frac{500}{S} + 250\right)}$$

$$= \frac{10 \times S}{S(500 + 250 S) \times 10}$$

$$= \frac{1}{50 + 25 S}$$

$$= \frac{1}{25(S+2)}$$

$$i(t) = 0.04\, e^{-2t} \;\; u(t)$$

① Laplace transform - In mathematics the laplace transform, named after its discoverer Pierre-Simon Laplace, is an integral transform that converts a function of a real variable to a function of a complex variable s. the transform has many applications in science and engineering because it is a tool for solving differential equations.

② Inverse laplace transform -

The inverse laplace transform is the transformation of a laplace transform into a function of time. If then f(t) is the inverse laplace transform of F(s) the inverse being written as:

[13] The inverse can generally be obtained by using standard transforms.

③ Shifted function - A shift is an addition or subtraction to the x of f(x) component & when you shift a function, you're basically changing the position of the graph of the function. A vertical shift raises or lowers the function as it adds or subtracts a constand to each y coordinate while the x coordinate remains the same.

④ Initial and final value theorem - Initial and final value theorems are basic properties of Laplace transform. these theorem's were given by French mathematician and physicist Pierre Simon marquis De Laplace. Initial and final value theorem are collectively called Limiting theorems.

⑤ Introduction to transient - Transient analysis calculates a circuit's response over a period of time defined by the user. The accuracy of the transient analysis is dependent on the size of internal time steps, which together make up the complete simulation time known as the Run time or Stop time. X

⑥ Transient disturbance :- The term transient usually originates from electric circuit theory where it describes voltage and current variation from its present steady state to next steady state.

2) In the power system the term the transient is usually used in a different way it denotes abrupt change in voltages and current for short duration.

3) Usually for duration less than the period of power system voltage and current signal (50-60 Hz)

⑦ Cause of transient disturbance-

1) switching operation faults, and other disturbances like lightning.

2) Shutdown of heavily loaded ckts

3) necessary communication of a high powered network (e.g. pf connection)

⑧ How can it be resolved :- for better power quality assessment we have to define new indices (other than conventional) based on signal processing technique. As present indices for transient disturbance analysis have limitations.

2) Indices can be also defined on basis of time frequency analysis.

3) Indices based on time domain analysis are crest factor, peak and RMS value.

4) Indices based on frequency domain are THD. Distortion Index, K factor. These indices are defined in terms of Fourier series coefficient

⑨

23/9/22

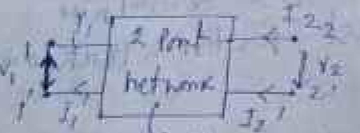
Two port networks

Terminal - It is a point where a connection can be given.
(in electronics or electrical)

Port - Pair of terminals where the current can enter and leave. It is the port or an network.

Two port network is the basic building block of any electrical or electronic ckt.

Two port network - 4 terminals, 2 input, rest of them outputs.



response side

linear passive ckt

Source / excitation side = this positions can be interchanged

Port condition is that the current entering the port is equal to the current coming out from the port.
must be

i.e $I_1 = I_2$, $I_1' = I_2'$

V_1 I_1 V_2 I_2

2 - independent variables
2 - dependent variables
→ It varies for different parameters

So, that we can have 2 eqns.
4 variables in group of 2.
So, there are $^4C_2 = 6$ possible set of eqns.

1. Z parameters
2. Y - Parameters
3. T - Parameters
4. h - Parameters
5. Inverse T net Parameters
6. Inverse h (or g) Parameters

1 Open ckt Parameters or Z-Parameters or Impedance parameters or open ckt impedance parameters

$(V_1, V_2) = f(I_1, I_2)$

The defining eqns of Z-parameters

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$
$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

matrix form

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ [I_2 is independent]

[Z_{11}, Z_{22} = Driving point] [I_2 is taken 0 to find out Z_{11} (dependent parameter)]

[Z_{12}, Z_{21} = transfer "]

[Z_{11} - Input driving point impedance, Z_{22} = Output "] [I_1, I_2 are two port. we can set any condition 0, I]

[Z_{12}, Z_{21} = I/P, I/P/O/P = Reverse transfer imp, O/P/I/P = Forward transfer imp]

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$ → I/p driving point impedance (Ω)

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$ → Forward transfer impedance (Ω)

$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$ → Reverse transfer impedance (Ω)

$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$ → output driving point impedance (Ω)

Equivalent ckt of Z-parameters

(2) short ckt of parameters or Y-parameters or Admittance parameters

$(I_1, I_2) = f(V_1, V_2)$

$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

matrix form

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

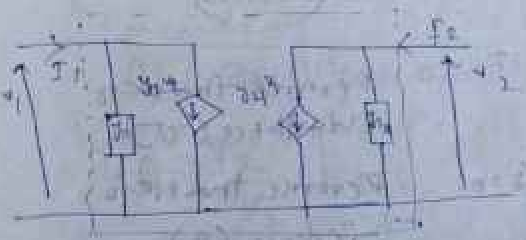
$y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$ (℧/s) I/P port driving point admittance

$y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0}$ (℧/s) Forward transfer admittance

$y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$ (℧/s) Reverse transfer admittance

$y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$ (℧/s) O/P port driving point admittance

Equivalent ckt of y-parameters



the change will be into the network the ports will be same.

$V = IZ$

$\Rightarrow I = \frac{1}{Z}V = yV$

$\frac{1}{\text{impedance}}$ = Admittance

for admittance $V = IZ$

(3) ABCD Parameter or transmission line Parameters or T-Parameters or Chain Parameters.

$(V_1, I_1) = f(V_2, I_2)$

$V_1 = AV_2 + B(-I_2)$

$I_1 = CV_2 + D(-I_2)$

matrix form

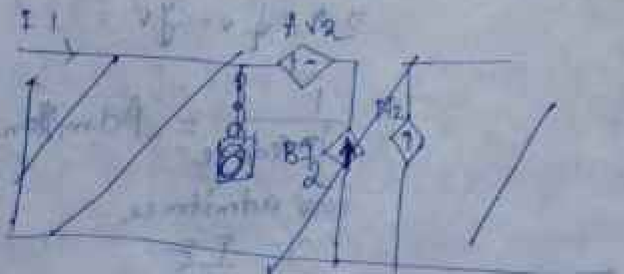
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$A = \frac{V_1}{V_2}\Big|_{I_2=0}$ Reverse voltage gain (unitless)

$C = \frac{I_1}{V_2}\Big|_{I_2=0}$ Reverse transfer admittance (℧)

$B = \frac{V_1}{-I_2}\Big|_{V_2=0}$ Reverse transfer impedance (Ω)

$D = \frac{-I_1}{I_2}\Big|_{V_2=0}$ Reverse current gain (unitless)



equivalent ckt is not possible because no relation between I_1, I_2

(4) Hybrid parameter or h-parameter

$(V_1, I_2) = f(I_1, V_2)$

$V_1 = h_{11}I_1 + h_{12}V_2$

$I_2 = h_{21}I_1 + h_{22}V_2$

matrix form

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

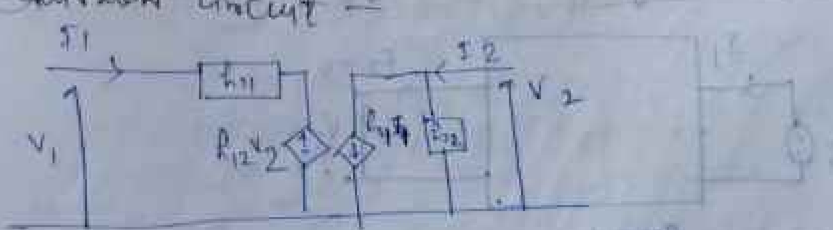
$h_{11} = \frac{V_1}{I_1}\Big|_{V_2=0}$ I/p driving point impedance (Ω)

$h_{21} = \frac{I_2}{I_1}\Big|_{V_2=0}$ ~~Forward transfer impedance (Ω)~~ Current gain (no unit)

$h_{12} = \frac{V_1}{V_2}\Big|_{I_1=0}$ Reverse voltage gain (no unit)

$h_{22} = \frac{I_2}{V_2}\Big|_{I_1=0}$ ~~Forward transfer~~ O/P driving point admittance (℧)

equivalent circuit —



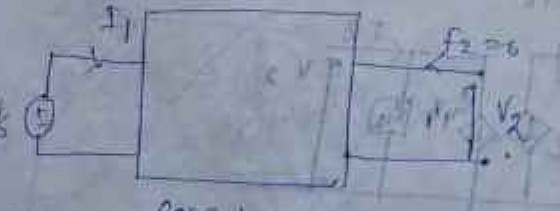
Conditions for symmetricity and reciprocity

A 2 port network will be called reciprocity if there excitation to response remain same even if their positions are interchanged.

i.e., $\frac{x}{y} = \frac{w}{z}$ (response ratio is same, not means $x = w$ or $y = z$)

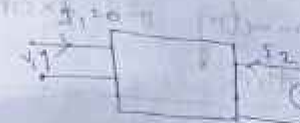
If the ports are interchanged then the behavior of the network same i.e the impedance seen by i/p side is same to the impedance seen by o/p side, that type of network is called symmetric.

Symmetry



case 1

$\frac{V_S}{I_1}$



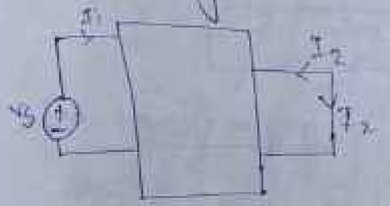
case 2

$\frac{V_S}{I_2}$

If, $\boxed{\frac{V_S}{I_1} = \frac{V_S}{I_2}}$ ← the condition for this network to be symmetric

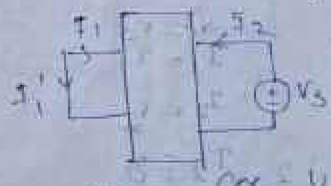
then it is symmetric

Reciprocity



case - 1

$\frac{V_S}{I_2'}$



case - 11

$\frac{V_S}{I_1}$

if, $\boxed{\frac{V_S}{I_2'} = \frac{V_S}{I_1'}}$ ← the condition for this network to be reciprocal

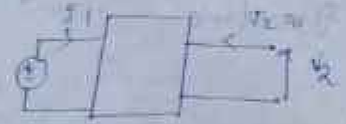
then it is reciprocal

Condition for symmetry and reciprocity for Z-parameters

$$V_1 = Z_{11} I_1 + Z_{12} I_2 \quad \text{—— (i)}$$
$$V_2 = Z_{21} I_1 + Z_{22} I_2 \quad \text{—— (ii)}$$

for symmetry.

Case-1



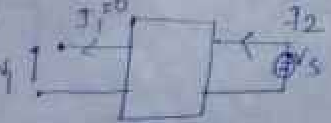
$V_1 = V_S$
$I_1 = I_1$
$I_2 = V_2$
$I_2 = 0$

Putting this in defining eqns.

$V_S = Z_{11} I_1 +$

$\cancel{V_1}$ $\dfrac{V_S}{I_1} = Z_{11}$ —— (A)

Case – II



$V_1 = V_1$
$I_1 = 0$
$V_2 = V_S$
$I_2 = I_2$

Putting this in defining eqn.

$V_1 = Z_{12} I_2$

$V_S = Z_{22} I_2$

$\Rightarrow \dfrac{V_S}{I_2} = Z_{22}$ —— (B)

For the network to be symmetrical eqn (A) must be equal to eqn (B)

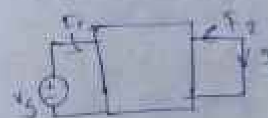
i.e. $\dfrac{V_S}{I_1} = \dfrac{V_S}{I_2}$

∴ the condition for the n to be symmetrical in terms of Z-parameters is –

$$\boxed{Z_{11} = Z_{22}}$$

for reciprocity.

Case - 1



$I_1 = I_1$
$V_1 = V_S$
$I_2 = -I_2'$
$V_2 = 0$

∴ $V_S = Z_{11} I_1 - Z_{12} I_2'$ —— (III)

$\Rightarrow \dfrac{V_S}{I_1} =$

$0 = Z_{21} I_1 - Z_{22} I_2'$

$\Rightarrow I_1 = \dfrac{Z_{22} I_2'}{Z_{21}}$ —— (IV) from eqn (1)

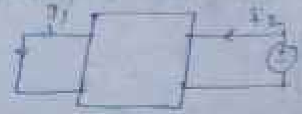
$$V_S = Z_{11} \times \frac{Z_{22} I_2'}{Z_{21}} - Z_{12} I_2'$$

$$\Rightarrow V_S = \left(\frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12}\right) I_2' \quad \text{(II)}$$

$$\Rightarrow \frac{V_S}{I_2'} = \frac{Z_{11} Z_{22}}{Z_{21}} - Z_{12} = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

$$= \frac{\Delta Z}{Z_{21}} \Rightarrow \frac{V_S}{I_2'} = \frac{\Delta Z}{Z_{21}} \left[\Delta Z = \begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}\right] \quad \text{(C)}$$

Case - II



$I_1 = -I_1'$

$V_1 = 0$

$I_2 = I_2$

$V_2 = V_S$

$$0 = Z_{11}(-I_1') + Z_{12} I_2$$

$$\Rightarrow I_2 = \frac{Z_{11} I_1'}{Z_{12}} \quad \text{(V)}$$

$$V_S = -Z_{21} I_1' + Z_{22} I_2$$

$$\Rightarrow V_S = -Z_{21} I_1' + Z_{22} \times \frac{Z_{11} I_1'}{Z_{12}} \quad \text{(from (V))}$$

$$\Rightarrow V_S = \left(\frac{Z_{22}Z_{11} - Z_{21}Z_{12}}{Z_{12}}\right) I_1'$$

$$\Rightarrow \frac{V_S}{I_1'} = \frac{\Delta Z}{Z_{12}} \quad \text{(D)}$$

∴ the condition for reciprocity in terms of Z-parameters is $\frac{V_S}{I_1'} = \frac{V_S}{I_2'}$

$$= \boxed{\frac{\Delta Z}{Z_{12}} = \frac{\Delta Z}{Z_{21}}} \quad (3)$$

28/9/22

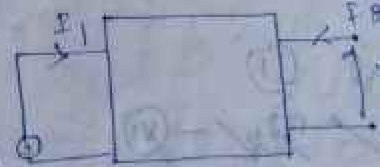
Find the conditions for reciprocity and symmetricity for T-parameters

$$V_1 = AV_2 + B(-I_2) \quad \text{(I)}$$

$$I_1 = CV_2 + D(-I_2) \quad \text{(II)}$$

Symmetricity

Case I-



$V_1 = V_S$

$I_1 = I_1$

$V_2 = V_2$

$I_2 = 0$

From (I) and (II)

$$I_1 = CV_2 + D \times 0 \quad \text{(III)} \Rightarrow V_2 = \frac{I_1}{C} \quad \text{(IV)}$$

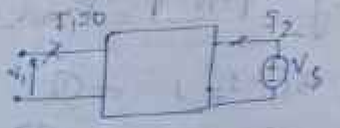
$V_S = A V_{S}/\cdots$ — (IV)

(IV)/(VI)

$\frac{V_S}{I_1} = \frac{A V_S}{C V_S}$

$= \frac{A}{C}$ — (V)

Case – II



$V_1 = V_1$

$I_1 = 0$

$I_2 = I_2$

$V_2 = V_S$

from (I) and (II)

~~$V_1 = A V_S - B I_2$ — (VI)~~

$0 = C V_S - D I_2$

$\Rightarrow C V_S = D I_2$

$\Rightarrow \frac{V_S}{I_2} = \frac{D}{C}$ ——— (VII)

∴ the condition is

$\frac{V_S}{I_1} = \frac{V_S}{I_2}$

$\Rightarrow \boxed{\frac{A}{C} = \frac{D}{C}}$ or $\boxed{A = D}$

for reciprocity.

Case – I

$I_1 = I_1$

$V_1 = V_S$

$I_2 = I_2'$

$V_2 = 0$

from (I) and (II)

$V_S = B I_2'$

$\Rightarrow \frac{V_S}{I_2'} = B$

Case – II

$V_2 = V_S$

$I_2 = I_2$

$I_1 = -I_1'$

$V_1 = 0$

from (I) and (II)

$0 = A V_S - B I_2$

$\Rightarrow I_2 = \frac{A V_S}{B}$

$-I_1' = C V_S - D I_2$

$\Rightarrow -I_1' = C V_S - \frac{D A V_S}{B}$

⇒ $V_S\left(C - \frac{AD}{B}\right) = -I_1'$

⇒ $V_S \frac{(BC-AD)}{B} = -I_1'$

⇒ $V_S \frac{(AD-BC)}{B} = I_1'$

⇒ $\frac{V_S}{I_1'} = \frac{B}{AD-BC}$

∴ the condition is

$$\frac{V_S}{I_1'} = \frac{V_S}{I_2'}$$

⇒ $\boxed{\frac{B}{AD-BC} = B}$

⇒ $\boxed{\frac{1}{AD-BC} = 1}$ $\boxed{AD-BC = 1}$

Find the conditions for reciprocity and symmetricity for y-parameters

$I_1 = y_{11}V_1 + y_{12}V_2$ — (i)

$I_2 = y_{21}V_1 + y_{22}V_2$ — (ii)

Symmetric

Case-1

$V_1 = V_S$

$I_1 = I_1$

$I_2 = 0$

$V_2 = V_2$

from (ii),

$0 = y_{21}V_S + y_{22}V_2$

⇒ $y_{22}V_2 = -y_{21}V_S$

⇒ $V_2 = \frac{-y_{21}}{y_{22}}V_S$ — (iii)

from (i).

$I_1 = y_{11}V_S - \frac{y_{21}y_{12}}{y_{22}}V_S$ [from (iii)]

⇒ $I_1 = V_S\left(\frac{y_{22}y_{11} - y_{12}y_{21}}{y_{22}}\right)$

⇒ $\frac{V_S}{I_1} = \frac{y_{22}}{y_{22}y_{11} - y_{12}y_{21}}$

$= \frac{y_{22}}{\Delta y}$ — (A) [where $\Delta y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22}\end{vmatrix}$]

Case-II

$V_1 = V_1$

$I_1 = 0$

$V_2 = V_S$

$I_2 = I_2$

from (i),

$0 = y_{11}V_1 + y_{12}V_S$

⇒ $y_{11}V_1 = -y_{12}V_S$

⇒ $V_1 = -\frac{y_{12}}{y_{11}}V_S$ — (IV) from (11)

$I_2 = -\frac{y_{21}y_{12}}{y_{11}} V_S + y_{22} V_S$

$\Rightarrow I_2 = V_S\left(y_{22} - \frac{y_{21}y_{12}}{y_{11}}\right)$

$\Rightarrow I_2 = V_S\left(\frac{y_{11}y_{22} - y_{12}y_{21}}{y_{11}}\right)$

$\Rightarrow \frac{V_S}{I_2} = \frac{y_{11}}{\Delta y}$ — (B) $\left[\text{where, } \Delta y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}\right]$

the condition is-

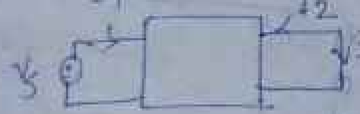
$\frac{V_S}{I_1} = \frac{V_S}{I_2}$

$\Rightarrow \frac{y_{22}}{\cancel{\Delta y}} = \frac{y_{11}}{\cancel{\Delta y}}$

$\Rightarrow \boxed{y_{11} = y_{22}}$

Reciprocity,

Case-1



$V_1 = V_S$

$I_1 = I_1$

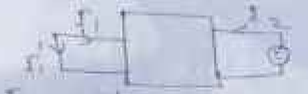
$I_2 = -I_2'$

$V_2 = 0$

from (11), $-I_2' = y_{21} V_S$

$\Rightarrow \frac{V_S}{I_2'} = -\frac{1}{y_{21}}$ — (A)

Case-II



$I_1 = -I_1'$

$V_1 = 0$

$I_2 = I_2$

$V_2 = V_S$

from (1),

$-I_1' = y_{12} V_S$

$\Rightarrow \frac{V_S}{I_1'} = -\frac{1}{y_{12}}$ — (B)

the condition is

$\frac{V_S}{I_1'} = \frac{V_S}{I_2'}$

$\Rightarrow -\frac{1}{y_{12}} = -\frac{1}{y_{21}}$

$\Rightarrow \boxed{y_{12} = y_{21}}$

Symmetricity and reciprocity

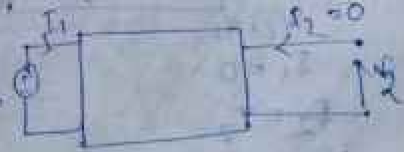
Find the conditions for h-Parameters

$V_1 = h_{11}I_1 + h_{12}V_2$ — (I)

$I_2 = h_{21}I_1 + h_{22}V_2$ — (II)

for symmetric,

Case-1



$V_1 = V_S$

$I_1 = I_1$

$I_2 = 0$

$V_2 = V_2$

from (I),

$0 = h_{21} I_1 + h_{22} V_2$

$\Rightarrow h_{22} V_2 = -h_{21} I_1$

$\Rightarrow V_2 = -\frac{h_{21}}{h_{22}} I_1$ — (III)

from (I),

$V_S = h_{11} I_1 - \frac{h_{12} h_{21}}{h_{22}} I_1$

$\Rightarrow V_S + \frac{h_{12} h_{21}}{h_{22}} V_S = h_{11} I_1$

$\Rightarrow V_S \left(\frac{h_{22} + h_{12} h_{21}}{h_{22}} \right) = h_{11} I_1$

$\Rightarrow \frac{V_S}{I_1} = \frac{h_{22} h_{11}}{h_{22} + h_{12} h_{21}}$ — (A)

$\Rightarrow V_S = \left(\frac{h_{22} h_{11} - h_{12} h_{21}}{h_{22}} \right)$

$\Rightarrow \frac{V_S}{I_1} = \frac{\Delta h}{h_{22}}$ — (A)

where, $\left[\Delta h = \begin{vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{vmatrix} \right]$

Case - II

$V_1 = V_1$

$I_1 = 0$

$I_2 = I_2$

$V_2 = V_S$

from (II)

$I_2 = h_{22} V_S$

$\Rightarrow \frac{V_S}{I_2} = \frac{1}{h_{22}}$ — (B)

the condition is

$\frac{V_S}{I_1} = \frac{V_S}{I_2}$

$\Rightarrow \boxed{\frac{h_{22} h_{11}}{h_{22} + h_{12} h_{21}} = \frac{1}{h_{22}}}$ $\Rightarrow \frac{\Delta h}{h_{22}} = \frac{1}{h_{22}}$

$\Rightarrow \boxed{\Delta h = 1}$

for reciprocity

Case - I

$V_1 = V_S$

$I_1 = I_1$

$I_2 = -I_2'$

$V_2 = 0$

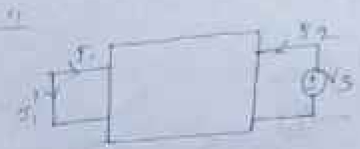
from, (II) $-I_2' = h_{21} I_1$

$\Rightarrow I_1 = -\frac{I_2'}{h_{21}}$ — (IV)

from (I), $V_S = -\frac{h_{11}}{h_{21}} I_2'$ [from (IV)]

$\Rightarrow \frac{V_S}{I_2'} = -\frac{h_{11}}{h_{21}}$ — (A)

Case - II



$I_1 = -I_1'$

$V_1 = 0$

$I_2 = I_2$

$V_2 = V_S$

from ①,

$0 = h_{11} I_1' + h_{12} V_S$

$\Rightarrow h_{12} V_S = h_{11} I_1'$

$\Rightarrow \frac{V_S}{I_1'} = \frac{h_{11}}{h_{12}}$ — ⑨

The condition is

$\frac{V_S}{I_1'} = \frac{V_S}{I_1'}$

$\Rightarrow \frac{h_{11}}{h_{12}} = \frac{-h_{11}}{h_{21}}$

$\Rightarrow \boxed{h_{12} = -h_{21}}$

⑥ h-Parameters - Hybrid parameters are know as 'hybrid' P. as they use Z-P, Y-P, voltage ratio, and current ratios to represent the relationship between voltage and current in a two port network.

① Terminal - In electronics or electrical it is a coil where a connection can be given. A two-port network (a kind of four-terminal network or quadripole) is an electrical network (circuit) or device with two pairs of terminals to connect to external circuits.

② Port - In electrical circuit theory, a port is a pair of terminals connecting an electrical network or circuit to an external circuit, as a point of entry or exit for electrical energy.

③ Z-Parameters - ~~are~~ They are also known as open-circuit impedance parameters as they are calculated under open circuit conditions. i.e. $I_x = 0$, where $x = 1, 2$ refers to input and output currents flowing through the ports (of a two-port network in this case) respectively.

④ Y Parameters - Y parameters of two port network is a 2×2 admittance matrix. Since admittance is the ratio of circuit current and voltage, therefore this admittance matrix gives the relationship between the input and output current and voltage of the network. It is also known as short circuit admittance parameters.

⑤ T - Parameters - They are called inverse transmission parameters or A'B'C'D' parameters. The parameters A' and D' do not have any units, since those are dimensionless.

The units of parameters B and C are ohm and mho respectively. We can calculate the ... between the parameters.

Inter relationship between the parameters

Z in terms of T

$V_1 = z_{11} I_1 + z_{12} I_2$ — (I)

$V_2 = z_{21} I_1 + z_{22} I_2$ — (II)

$V_1 = AV_2 + B(-I_2)$ — (III)

$I_1 = CV_2 + D(-I_2)$ — (IV)

Z in terms of T $\Rightarrow Z = f(A, B, C)$

from (IV)

$I_1 = CV_2 - DI_2$

$\Rightarrow I_1 + DI_2 = CV_2$

$\Rightarrow V_2 = \frac{1}{C}(I_1 + DI_2)$ — (V)

from (III),

$V_1 = AV_2 - BI_2$

$\Rightarrow V_1 = \frac{A}{C}(I_1 + DI_2) - BI_2$ [from (V)]

$\Rightarrow V_1 = \frac{AI_1}{C} + \frac{ADI_2}{C} - BI_2$

$\Rightarrow V_1 = \frac{A}{C} I_1 + \left(\frac{AD}{C} - B\right) I_2$ — (VI)

Comparing (VI) and (I) we get,

$\boxed{z_{11} = \frac{A}{C}}$, $\boxed{z_{12} = \frac{AD}{C} - B = \frac{AD - BC}{C} = \frac{\Delta T}{C}}$

$\boxed{\text{where } \Delta T = \begin{vmatrix} A & B \\ C & D \end{vmatrix}}$

from (IV)

$I_1 = CV_2 - DI_2$

$\Rightarrow -CV_2 = -I_1 - DI_2$

$\Rightarrow CV_2 = I_1 + DI_2$

$\Rightarrow V_2 = \frac{1}{C} I_1 + \frac{D}{C} I_2$ — (VII)

Comparing (VII) and (II) we get

$\boxed{z_{21} = \frac{1}{C}}$, $\boxed{z_{22} = \frac{D}{C}}$

h in terms of y

$V_1 = h_{11} I_1 + h_{12} V_2$ — (I)

$I_2 = h_{21} I_1 + h_{22} V_2$ — (II)

$I_1 = y_{11} V_1 + y_{12} V_2$ — (III)

$I_2 = y_{21} V_1 + y_{22} V_2$ — (IV)

from (III)

$I_1 = y_{11} V_1 + y_{12} V_2$

$\Rightarrow -y_{11} V_1 = -I_1 + y_{12} V_2$

$\Rightarrow y_{11} V_1 = I_1 - y_{12} V_2$

$\Rightarrow V_1 = \frac{1}{y_{11}} I_1 - \frac{y_{12}}{y_{11}} V_2$ — (V)

Comparing (V) and (I) we get,

$\boxed{h_{11} = \frac{1}{y_{11}}}$, $\boxed{h_{12} = -\frac{y_{12}}{y_{11}}}$

Comparing (I) and (IV)

$\boxed{h_{21} = y_{21}}$, $\boxed{h_{22} = y_{22}}$

Ans (III)

$I_1 = y_{11} V_1 + y_{12} V_2$

$\Rightarrow I_1 - y_{12} V_2 = y_{11} V_1$

$\Rightarrow V_1 = \frac{1}{y_{11}} (I_1 - y_{12} V_2)$ — (V)

Put (V) in (IV)

$I_2 = y_{21} \frac{1}{y_{11}} (I_1 - y_{12} V_2) + y_{22} V_2$

$\Rightarrow \frac{y_{21}}{y_{11}} I_1 - \frac{y_{21} y_{12} V_2}{y_{11}} + y_{22} V_2$

$= \frac{y_{21}}{y_{11}} I_1 + \left(y_{22} - \frac{y_{21} y_{12}}{y_{11}}\right) V_2$

$\Rightarrow I_2 = \frac{y_{21}}{y_{11}} I_1 + \left(\frac{y_{11} y_{22} - y_{21} y_{12}}{y_{11}}\right) V_2$ — (VI)

Comparing (VI) and (II) we get

$\boxed{h_{21} = \frac{y_{21}}{y_{11}}}$, $\boxed{h_{22} = \frac{y_{11} y_{22} - y_{21} y_{12}}{y_{11}}}$

or $= \frac{\Delta y}{y_{11}}$

[Where $\Delta y = \begin{vmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{vmatrix}$]

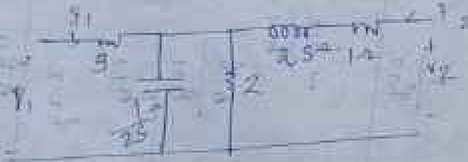
Parameters	z	y	h	T
z	z_{11} z_{12} z_{21} z_{22}	$\frac{y_{22}}{\Delta y}$ $\frac{-y_{12}}{\Delta y}$ $\frac{y_{21}}{\Delta y}$ $\frac{y_{11}}{\Delta y}$	$\frac{\Delta h}{h_{22}}$ $\frac{h_{12}}{h_{22}}$ $-\frac{h_{21}}{h_{22}}$ $\frac{1}{h_{22}}$	$\frac{A}{C}$ $\frac{\Delta T}{C}$ $\frac{1}{C}$ $\frac{D}{C}$
y	$\frac{Z_{22}}{\Delta Z}$ $-\frac{Z_{12}}{\Delta Z}$ $-\frac{Z_{21}}{\Delta Z}$ $\frac{Z_{11}}{\Delta Z}$	y_{11} y_{12} y_{21} y_{22}	$\frac{1}{h_{11}}$ $-\frac{h_{12}}{h_{11}}$ $\frac{h_{21}}{h_{11}}$ $\frac{\Delta h}{h_{11}}$	$\frac{D}{B}$ $-\frac{\Delta T}{C}$ $-\frac{1}{B}$ $\frac{A}{B}$
h	$\frac{\Delta Z}{Z_{22}}$ $\frac{Z_{12}}{Z_{22}}$ $-\frac{Z_{21}}{Z_{22}}$ $\frac{1}{Z_{22}}$	$\frac{1}{y_{11}}$ $-\frac{y_{12}}{y_{11}}$ $\frac{y_{21}}{y_{11}}$ $\frac{\Delta y}{y_{11}}$	h_{11} h_{12} h_{21} h_{22}	$\frac{B}{D}$ $\frac{\Delta T}{D}$ $-\frac{1}{D}$ [illegible]
T	$\frac{Z_{11}}{Z_{21}}$ $\frac{\Delta Z}{Z_{21}}$ $\frac{1}{Z_{21}}$ $\frac{Z_{22}}{Z_{21}}$	$-\frac{y_{22}}{y_{21}}$ $-\frac{1}{y_{21}}$ $-\frac{\Delta y}{y_{21}}$ $-\frac{y_{11}}{y_{21}}$	$-\frac{\Delta h}{h_{21}}$ $-\frac{h_{11}}{h_{21}}$ $-\frac{h_{22}}{h_{21}}$ $-\frac{1}{h_{21}}$	A B C D

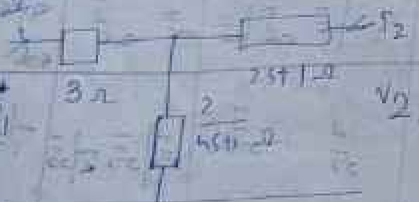
12/10/22

① Find Z-parameters

Z is open ckt parameter so open ckt

Transform into S domain





$V_1 = Z_{11} I_1 + Z_{12} I_2$

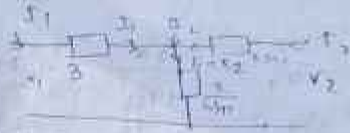
$V_2 = Z_{21} I_1 + Z_{22} I_2$

$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$, $Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$

$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$, $Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$

Case 1:

$I_2 = 0$



$V_1 = 3I_1 + \left(\frac{2}{4s+1}\right) I_1$

$\Rightarrow \frac{V_1}{I_1} = 3 + \frac{2}{4s+1}$

$\Rightarrow \frac{V_1}{I_1} = \frac{12s+5}{4s+1}$

$\Rightarrow \boxed{Z_{11} = \frac{12s+5}{4s+1}}$

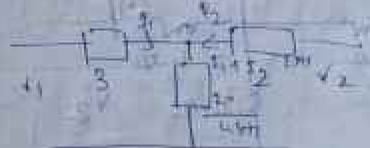
$V_2 = \frac{2}{4s+1} I_1$

$\Rightarrow \frac{V_2}{I_1} = \frac{2}{4s+1}$

$\Rightarrow \boxed{Z_{21} = \frac{2}{4s+1}}$

Case 2:

$I_1 = 0$



$\frac{V_1}{I_2} = \frac{2}{4s+1}$

$\Rightarrow \boxed{Z_{12} = \frac{2}{4s+1}}$

$$\frac{V_2}{I_2} = \frac{2s}{4s+1} + 2s + 1$$

$$= \frac{2+4s+1}{4s+1} + 2s$$

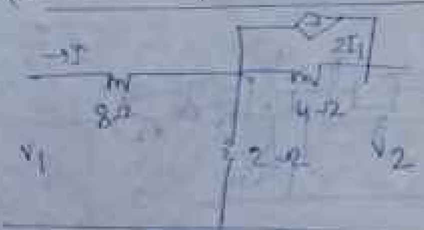
$$= \frac{3+4s}{4s+1} + 2s$$

$$= \frac{3+4s+8s^2+2s}{4s+1}$$

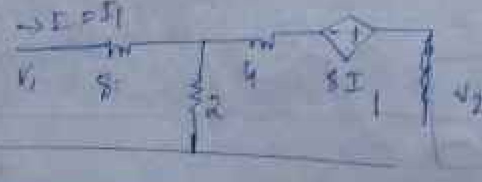
$$= \frac{8s^2+6s+3}{4s+1}$$

$$\boxed{z_{22} = \frac{8s^2+6s+3}{4s+1}}$$

② Find y- parameters



Source transformation



$I_1 = y_{11}V_1 + y_{12}V_2$

$I_2 = y_{21}V_1 + y_{22}V_2$

It is short ckt parameter so, short.

Case 1: $V_2 = 0$

$V_1 = 8I_1 + 2(I_1 + I_2)$ (KVL1)

$V_1 = 10I_1 + 2I_2$ — ①

$V_1 = 8I_1 + -4I_2 - 8I_1$ (KVL2)

$V_1 = -4I_2$

$\Rightarrow \boxed{\frac{I_2}{V_1} = y_{21} = -\frac{1}{4}}$

$\Rightarrow I_2 = -\frac{V_1}{4}$ — (ii)

from ①,

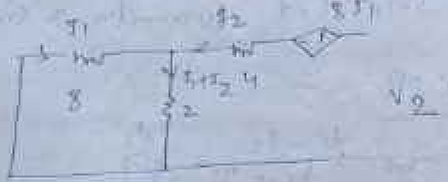
~~$I_1 = y_{11}V$~~ $V_1 = 10I_1 - \frac{V_1}{2}$

$\Rightarrow \frac{3}{2}V_1 = 10I_1$

$\Rightarrow \frac{I_1}{V_1} = \frac{3}{20}$

$\Rightarrow \boxed{y_{11} = \frac{3}{20}}$

case 2:
$V_1 = 0$



$V_2 = 4I_2 + 2(I_1 + I_2)$
$= 6I_2 + 2I_1$ — (1)

$V_2 = 4$

$V_2 = 8I_1I_2 + 4I_2 + 2(I_1 + I_2)$
$= 8I_1I_2 + 6I_2 + 2I_1$ — (1)

$V_2 = 8I_1I_2 + 4I_2 - 8I_1$
$= 8I_1I_2 + 4I_2 - 8I_1$ — (2)

$\Rightarrow V_2 = 8I_1I_2 + 4I_2 - 8I_1$
$\Rightarrow 4I_2 = V_2 + 8I_1I_2 + 8I_1$
$\Rightarrow I_2 = \boxed{\frac{1}{4}(V_2 + 8I_1I_2 + 8I_1)}$

from (1)

$V_2 = 8I_1I_2 + 16 - \frac{1}{4}(V_2 + 8I_1I_2$

$V_2 = 8I_1 + 4I_2 + 2(I_1 + I_2)$
$= 10I_1 + 6I_2$ — (1)

$V_2 = 8I_1 + 4I_2 - 8I_1$

$I_2 = \frac{V_2}{4} \Rightarrow \frac{I_2}{V_2} = \frac{1}{4}$
$\Rightarrow \boxed{I_2 = \frac{V_2}{4}}$

from (1), $V_2 = 10I_1 + 6\frac{V_2}{4}$
$\Rightarrow V_2 - \frac{3V_2}{2} = 10I_1$
$\Rightarrow -\frac{V_2}{2} = 10I_1$
$\Rightarrow \frac{V_2}{I_1} = -20$
$\Rightarrow \frac{I_1}{V_2} = -\frac{1}{20}$
$\Rightarrow \boxed{y_{12} = -\frac{1}{20}}$

from (9),

$V_1 = 2I_1 - 2I_3 \Rightarrow 2I_3 = 2I_1 - V_1$

$\Rightarrow I_3 = \frac{1}{2}(2I_1 - V_1)$ — (9)

putting in (8),

$V_2 = -3V_1 + 2I_1 - \frac{3}{2}(2I_1 - V_1)$

$\Rightarrow V_2 = -3V_1 + 2I_1 - 3I_1 + \frac{3V_1}{2}$

$\Rightarrow V_2 = -3V_1 + \frac{3V_1}{2} - I_1$

$\Rightarrow V_2 = -\frac{3V_1}{2} - I_1$

$\Rightarrow I_1 = -\frac{3V_1}{2} - V_2$ — (C)

comparing (C) with (3)

$\boxed{y_{11} = -\frac{3}{2},\ y_{12} = -1}$

from (8),

$V_2 = -3V_1 + \cancel{2I_1} - 2I_3 - I_3$

$= -3V_1 + (2I_1 - 2I_3) - \frac{1}{2}(2I_1 - V_1)$ (from (9))

$= -3V_1 + V_1 - I_1 + \frac{V_1}{2}$ [from (5)]

$= -2V_1 - I_1 + \frac{V_1}{2}$

$= -2V_1 + \frac{V_1}{2} - I_1$

(*)

(5) − 2×(6)

$V_1 - 2V_1 = 2I_1 + I_2$

$\Rightarrow -V_1 = 2I_1 + I_2 \Rightarrow V_1 = -(2I_1 + I_2)$

from (8)

$V_2 = -3(2I_1 + I_2) + 2I_1 - \frac{3}{2}(2I_1 - V_1)$

$\Rightarrow V_2 = 6I_1 + 3I_2 + 2I_1 - 3I_1 + \frac{3}{2}V_1$

(3) Determine the Z and Y parameters for the network shown

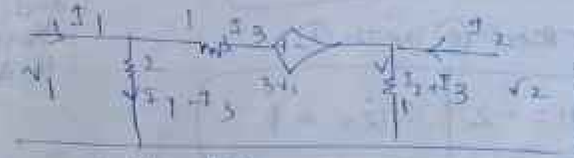
$V_1 = Z_{11}I_1 + Z_{12}I_2$ — (1)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ — (2)

$I_1 = y_{11}V_1 + y_{12}V_2$ — (3)

$I_2 = y_{21}V_1 + y_{22}V_2$ — (4)

defining equations



KVL applied ... supernode ...

from KVL $= 2(I_1 - I_3)$

$V_1 = 2I_1 - 2I_3$ — (5)

~~In these, there are 3 parameters I_3, V_1 ... so we can't compare ... case-I, case-II, here~~

$V_1 = I_3 + 3V_1 + I_2 + I_3$

$\Rightarrow V_1 = 2I_3 + 3V_1 + I_2$ — (6)

~~$\Rightarrow -2V_1 = 2I_3 + I_2$~~

$\Rightarrow -2V_1 = 2I_3 + I_2$

$\Rightarrow V_1 = -\frac{1}{2}(2I_3 + I_2)$ — (6)

Here KVL applied on supernode as dependent VS as it is supernode.

$V_2 = (I_2 + I_3) \times 1$ — (7)

$V_2 = -3V_1 - I_3 + 2I_1 - 2I_3$

$= -3V_1 + 2I_1 - 3I_3$ — (8)

$V_1 = 2(I_1 - I_3)$

$V_1 = 2I_1 - 2I_3$ — (I)

$V_1 = I_3 + 3V_1 + I_2 + I_3$

$\Rightarrow -2V_1 = I_2 + 2I_3$ — (II)

(I) + (II)

$\Rightarrow -V_1 = 2I_1 + I_2$

$\Rightarrow V_1 = -2I_1 - I_2$

$\boxed{Z_{11} = -2, \quad Z_{12} = -1}$

$V_2 = I_2 + I_3$ — (III)

$V_2 = -3V_1 - I_3 + 2(I_1 - I_3)$

$\Rightarrow V_2 + 3V_1 = -I_3 + 2I_1 - 2I_3$

$\Rightarrow V_2 + 3V_1 = -3I_3 + 2I_1$ — (IV)

$\Rightarrow V_2 + 3\left(\frac{I_2}{2} - I_3\right) = -3I_3 + 2I_1$

$\Rightarrow V_2 - \frac{3}{2}I_2 - \cancel{3I_3} = -\cancel{3I_3} + 2I_1$

$\Rightarrow V_2 = 2I_1 + \frac{3}{2}I_2$

$\boxed{Z_{21} = 2, \quad Z_{22} = \frac{3}{2}}$

(II) × 2 + (I)

$\cancel{2V_1 = 2I_2 + 4I_3 + 2I_1 - 2I_3}$

$\cancel{\Rightarrow V_1 + 2V_2 = 2I_1 + 2I_2}$

(II) − 2×(III)

$-2V_1 - 2V_2 = I_2 + 2I_3 - 2I_2 - 2I_3$

$-2V_1 - 2V_2 = -I_2$

$\Rightarrow 2V_1 + 2V_2 = I_2$

$\Rightarrow I_2 = 2V_1 + 2V_2$

$\boxed{y_{21} = 2, \quad y_{22} = 2}$

from (I) $\cancel{-\frac{1}{2}V_1 + 2I_3 = 2I_1}$ — (V)

from (IV), putting (V)

$\cancel{\Rightarrow V_2 + 3V_1 = -3I_3 + V_1 + 2I_3}$

$\cancel{\Rightarrow V_2 + 3V_1 - V_1 = I}$

from (I), $-\frac{1}{2}(V_1 - 2I_1) = I_3$ — (V)

from (IV), putting (V),

$V_2 + 3V_1 = +\frac{3}{2}(V_1 - 2I_1) + 2I_1$

$\Rightarrow V_2 + 3V_1 = \frac{3V_1}{2} - 3I_1 + 2I_1$

$\Rightarrow V_2 + 3V_1 - \frac{3V_1}{2} = -I_1$

$\Rightarrow -I_1 = \frac{3V_1}{2} + V_2$

$\Rightarrow I_1 = -\frac{3V_1}{2} - V_2$, $\boxed{y_{11} = -\frac{3}{2}, \quad y_{12} = -1}$

from (4)

$\Rightarrow -2I_3 = 2V_1 + I_2$

$\Rightarrow I_3 = -\frac{1}{2}(2V_1 + I_2)$

Putting it in (5)

$V_1 = 2I_1 + 2 \cdot \frac{1}{2}(2V_1 + I_2)$

$\Rightarrow 2I_1 + 2V_1 + I_2$

$\Rightarrow \boxed{V_1 = -(2I_1 + I_2)}$ — (A)

Comparing (A) with (1)

$\boxed{Z_{11} = -2 \quad Z_{12} = -1}$

[Here, after solving all eqns, then calculate all parameters by $I_1 = 0$, $I_2 = 0$ and V_1 and V_2 = short]

[or after solving all eqns you can also complete them with defining eqn and then calculate the parameters]

(5) + 2 × (7)

~~$2I_1 + 2I_3 = 2I_2 + 2I_3$~~

$V_1 + 2V_2 = 2I_1 + 2I_2$

$\Rightarrow -2I_1 - I_2 + 2V_2 = 2I_1 + 2I_2$ (from A)

$\Rightarrow 2V_2 = 4I_1 + 3I_2$

$\Rightarrow V_2 = 2I_1 + \frac{3}{2}I_2$

$\boxed{Z_{21} = 2 \quad Z_{22} = \frac{3}{2}}$

(ii) for y-p

(5) − 2 × (6)

$V_1 - 2V_1 = 2I_1 + I_2$

$\Rightarrow V_1 = -(2I_1 + I_2)$

$\Rightarrow V_1 = -2I_1 - I_2$

$\Rightarrow -\frac{1}{2}(V_1 + I_2) = I_1$

from (C) $\Rightarrow -\frac{1}{2}(V_1 + I_2) = -\frac{3V_1}{2} - V_2$

$\Rightarrow -\frac{V_1}{2} - \frac{1}{2}I_2 = -\frac{3V_1}{2} - V_2$

$\Rightarrow -\frac{1}{2}I_2 = -\frac{3V_1}{2} + \frac{V_1}{2} - V_2$

$\Rightarrow -\frac{1}{2}I_2 = -V_1 - V_2$

$\Rightarrow I_2 = 2V_1 + 2V_2$

$\therefore \boxed{y_{21} = 2 = y_{22}}$

from (6),

$2V_1 = -2I_3 - I_2$

$\Rightarrow 2I_3 = -I_2 - 2V_1$

$\Rightarrow I_3 = -\frac{1}{2}(I_2 + 2V_1)$ — (10)

Putting (10) in (8)

$V_2 = -3V_1 + 2I$

Z - m

$$\begin{bmatrix} -2 & -1 \\ 2 & \frac{3}{2} \end{bmatrix}$$

y in z

$$y_{11} = \frac{\frac{3}{2}}{(-3+2)} = -\frac{3}{2}, \quad y_{12} = \frac{-(-1)}{-1} = -1$$

$$y_{21} = -\frac{(2)}{-1} = 2 \qquad y_{22} = \frac{-2}{-1} = 2$$

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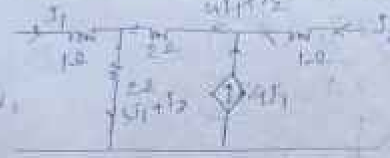
5/ h - parameters

(1)



$V_1 = I_1 + 2(5I_1 + I_2)$

$= 11I_1 + 2I_2$ ← (1)

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

$V_2 = 2(4I_1 + I_2) + 2(5I_1 + I_2) + I_2$

$= 8I_1 + 10I_1 + 2I_2 + 2I_2 + I_2$

$V_2 = 18I_1 + 5I_2$ — (ii)

$\Rightarrow V_2 - 5I_2 = 18I_1 \Rightarrow I_2 = \frac{1}{5}(V_2 - 18I_1)$

$\Rightarrow I_2 = -\frac{18}{5}I_1 + \frac{1}{5}V_2$ (iii)

$\Rightarrow I_1 = \frac{1}{18}(-5I_2 + V_2)$

$\therefore$ $h_{21} = -\frac{5}{18}$; $h_{22} = \frac{1}{18}$

$\Rightarrow h_{21} = -\frac{18}{5}$ | $\Rightarrow h_{22} = \frac{1}{5}$ ℧

from (ii) $\frac{1}{5}(V_2 - 18I_1) = I_2$

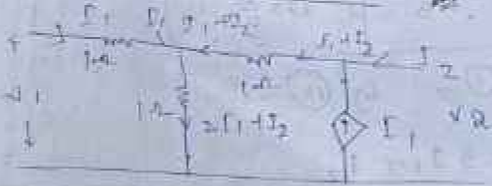
from (i), $V_1 = 11I_1 + \frac{2}{5}(V_2 - 18I_1)$

$= 11I_1 - \frac{36}{5}I_1 + \frac{2}{5}V_2$

$V_1 = \frac{19I_1}{5} + \frac{2}{5}V_2$

$h_{11} = \frac{19}{5}$ Ω | $h_{12} = \frac{2}{5}$

(2) Find Z and T parameters



$V_1 = I_1 + 2I_1 + I_2$

$V_1 = 3I_1 + I_2$ — (1)

$V_2 = 1 \cdot I_2 + 2I_1 + I_2$

$V_2 = 3I_1 + 2I_2$ — (ii)

for Z.P.

$V_1 = Z_{11} I_1 + Z_{12} I_2$

$V_2 = Z_{21} I_1 + Z_{22} I_2$

from (1): $Z_{11} = 3$ Ω | $Z_{12} = 1$ Ω

from (2): $Z_{21} = 3$ Ω | $Z_{22} = 2$ Ω

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

from (ii), $V_2 = 3I_1 + 2I_2$

$\Rightarrow 3I_1 = V_2 - 2I_2$

$$= 11 I_1 + 2 I_2 \quad - (1)$$

$$V_1 = h_{11} I_1 + h_{12} V_2$$

from (III),

$$V_1 = 11 I_1 + 2 I_2$$

$$= 11 I_1 + \frac{2}{5}\left(V_2 - 18 I_1\right)$$

$$= 11 I_1 - \frac{36}{5} I_1 + \frac{2}{5} V_2$$

$$V_1 = \frac{19 I_1}{5} + \frac{2}{5} V_2$$

$$\begin{array}{r} 55 \\ -36 \\ \hline 19 \end{array}$$

$$\therefore h_{11} = \frac{19}{5}, \quad h_{12} = \frac{2}{5}$$

$$Z = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix} \qquad T = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

T in Z

$A = \frac{3}{3} = 1, \quad B = \frac{6-3}{3} = 1, \quad C = \frac{1}{3} = \frac{1}{3}$

$D = \frac{2}{3} = \frac{2}{3}$

$$T = \begin{bmatrix} 1 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

Z in T

$Z_{11} = \frac{A}{C} = 1 \times 3 = 3$

$Z_{12} = \frac{\Delta T}{C} = \frac{\frac{2}{3} - \frac{1}{3}}{\frac{1}{3}} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$

$Z_{21} = \frac{1}{C} = 3$

$Z_{22} = \frac{D}{C} = \frac{2}{3} \times \frac{3}{1} = 2$

T in Z

$V_1 = AV_2 + B(-I_2)$ —(i)

$I_1 = CV_2 - DI_2$ —(ii)

$V_1 = Z_{11}I_1 + Z_{12}I_2$ —(iii)

$V_2 = Z_{21}I_1 + Z_{22}I_2$ —(iv)

from (iv)

$V_2 = Z_{21}I_1 + Z_{22}I_2$

$\Rightarrow V_2 - Z_{22}I_2 = Z_{21}I_1$

$\Rightarrow I_1 = \frac{1}{Z_{21}}(V_2 - Z_{22}I_2)$ —(A)

Comparing (A) with (ii)

$C = \frac{1}{Z_{21}}, \quad D = \frac{Z_{22}}{Z_{21}}$

from (iii),

$V_1 = \frac{Z_{11}}{Z_{21}}(V_2 - Z_{22}I_2) + Z_{12}I_2$

$= \frac{Z_{11}}{Z_{21}}V_2 - \frac{Z_{11}Z_{22}I_2}{Z_{21}} + Z_{12}I_2$

$= \frac{Z_{11}}{Z_{21}}V_2 - I_2\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$

$\therefore A = \frac{Z_{11}}{Z_{21}}, \quad B = \frac{\Delta Z}{Z_{21}}$

$\Rightarrow 3I_1 = V_2 - 2I_2$

$\Rightarrow I_1 = \frac{1}{3}V_2 - \frac{2}{3}I_2$ — (ii)

$\boxed{C = \frac{1}{3}\ \mho \quad D = \frac{2}{3}}$

from (i), by (ii)

$V_1 = 3I_1 + I_2$

$\Rightarrow V_1 = 3\left(\frac{1}{3}V_2 - \frac{2}{3}I_2\right) + I_2$

$= V_2 - 2I_2 + I_2$

$\Rightarrow V_1 = V_2 - I_2$

$\boxed{A = 1\ \Omega \quad B = 1\ \Omega}$

(2) The A, B, C, D parameters of this 2-port network are given like this

$$T = \begin{bmatrix} \frac{5}{2} & \frac{1}{2}\ \Omega \\ \frac{1}{2}\ \mho & \frac{1}{2} \end{bmatrix}$$

Determine Z, Y and h parameters.

$$T = \begin{bmatrix} \frac{5}{2}(A) & \frac{1}{2}\ \Omega\ (B) \\ \frac{1}{2}\ \mho\ (C) & \frac{1}{2}\ (D) \end{bmatrix}$$

for Z,

$Z_{11} = \frac{A}{C} = \frac{5/2}{1/2} = 5\ \Omega$

$Z_{12} = \frac{\Delta T}{C} = \frac{AD - BC}{C} = \frac{\frac{5}{4} - \frac{1}{4}}{\frac{1}{2}} = 2\ \Omega$

$Z_{21} = \frac{1}{C} = 2\ \Omega$

$Z_{22} = \frac{D}{C} = 1\ \Omega$

$$Z = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$$

for Y,

$Y_{11} = \frac{D}{B} = 1\ \mho$

$Y_{12} = -\frac{\Delta T}{C} = \frac{AD - BC}{C} = \frac{\frac{5}{4} - \frac{1}{4}}{\frac{1}{2}} = 2\ \mho$

$Y_{21} = -\frac{1}{B} = -2\ \mho$

$Y_{22} = \frac{A}{B} = \frac{5/2}{1/2} = 5\ \mho$

$$Y = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$$

for h,

$h_{11} = \frac{B}{D} = 1\ \Omega$

$h_{12} = \frac{\Delta T}{C} = \frac{AD - BC}{C} = 2$

$h_{21} = -\frac{1}{D} = -2$

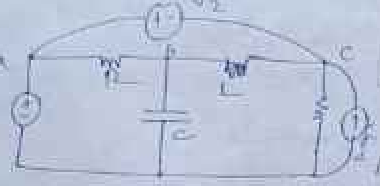
$h_{22} = \frac{1}{D} = 1\ \mho$

$$h = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$$

Graph Theory

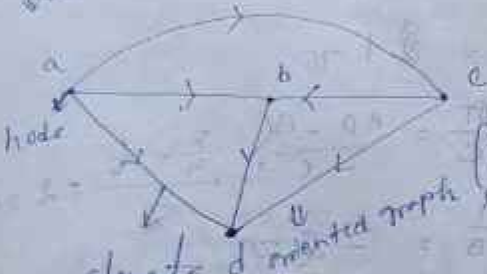
Graph is nothing but a skeleton / structure.

significance of graph theory



Here
Voltage source short
(line segment)
Current source open

Graph



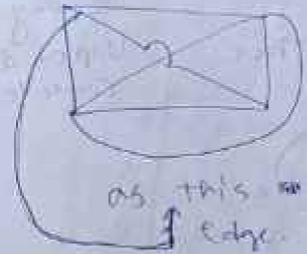
[Current source open = no drawing
ie = open]

The line segments are elements here.
The junctions ~~are~~ vertices of the segment ~~or~~ elements are node here

If assign direction = oriented graph
All the directions are arbitary.

no edge crosses = planary graph

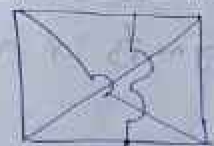
If it possible to draw a graph where there are no edges crossing, then this type of graph is called planers graph



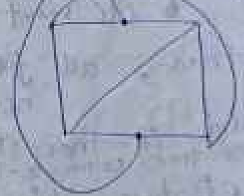
⇒ not a planers graph

as this line will cross ~~the~~ edge.

Non planners graph



⇓ not planers



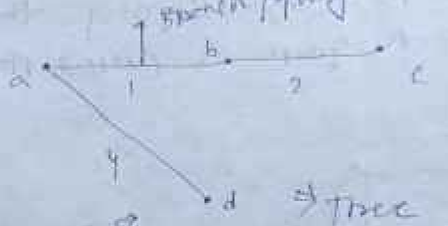
Tree

Connected graph – is a graph where there exist atleast one path between a pairs of nodes of that graph

Tree

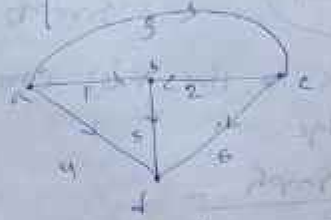
Tree of a connected graph is a connected sub graph that contains all the nodes of original graph but not contains ~~its~~ any loop.

Branch / twig



→ Tree

Contain minimum branches such that there will not be any loop and connected.



The elements / line segments of a tree = Branch / Twig

Co-tree - Except those elements that need to form a tree, all remains sd elements are called co-tree of

Co-tree is the complement of tree. co-tree - T

The elements of co-tree = link / chord

No of elements of a graph = e

nodes = n

twigs = tree = b

links - cotree l

$\boxed{b = n-1}$

$\boxed{l = e-b}$ $\boxed{= e-n+1}$

Incidence and Incident matrix

In a graph if an element is connected to a node then it is incident on the node.

Augmented incidence / complete incedence matrix

$$= A_a = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} +1 & 0 & +1 & +1 & 0 & 0 \\ -1 & -1 & 0 & 0 & +1 & 0 \\ 0 & +1 & -1 & 0 & 0 & +1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

$a_{ij} = +1$

$= -1$

$= 0$

incoming = -ve

outgoing = +ve

column sum = 0 (always)

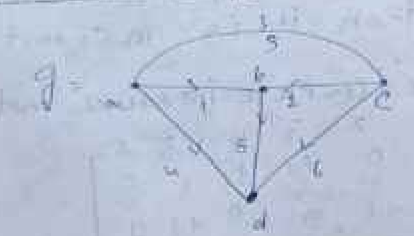
If we want to eliminate one row we can simply do that, nothing will be change.

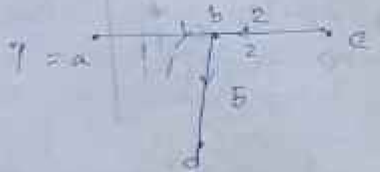
Reduced incedente matrix =

$$A_a = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} +1 & 0 & +1 & +1 & 0 & 0 \\ [illegible] & -1 & 0 & 0 & +1 & 0 \\ 0 & +1 & -1 & 0 & 0 & +1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \end{array}$$

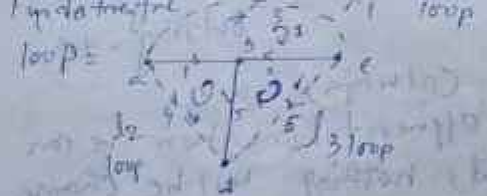
eliminate row d

Th^m Fundamental loops and fundamental matrix or Tie-set matrix





Fundamental loops = l_1 loop



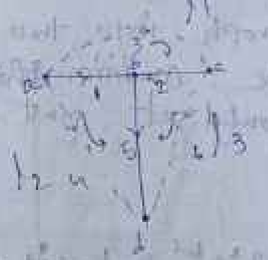
Condition of fundamental loop

In that loop there must be only 1 fundamental loop. i.e the loop must contain only one link. Further the direction of that loop will be in the direction of the defining link.

$$B = \begin{array}{c} \\ l_1 \\ l_2 \\ l_3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} -1 & +1 & +1 & 0 & 0 & 0 \\ -1 & 0 & 0 & +1 & +1 & 0 \\ 0 & -1 & 0 & 0 & -1 & +1 \end{bmatrix} \end{array}$$

10/10/21

2nd



Branch voltages- $v_1, v_2, \ldots v_6$

l_1: $v_3 + v_2 - v_1 = 0 \Rightarrow -v_1 + v_2 + v_3 + 0 \cdot v_4 + 0 \cdot v_5 + 0 \cdot v_6 = 0$

l_2: $-v_1 - v_5 + v_4 = 0 \Rightarrow -v_1 + v_4 - v_5 + 0 \cdot v_6 + 0 \cdot v_2 + 0 \cdot v_3 = 0$

l_3: $v_6 - v_5 - v_2 = 0 \Rightarrow -v_2 - v_5 + v_6 + 0 \cdot v_1 + 0 \cdot v_3 + 0 \cdot v_4 = 0$

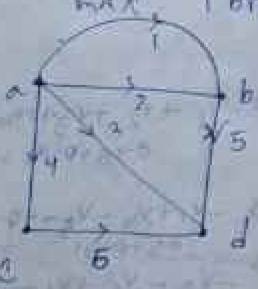
$$\begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{bmatrix} = 0$$

$\Rightarrow \boxed{[B][V_b] = 0}$ [in the matrix is same as [B]]

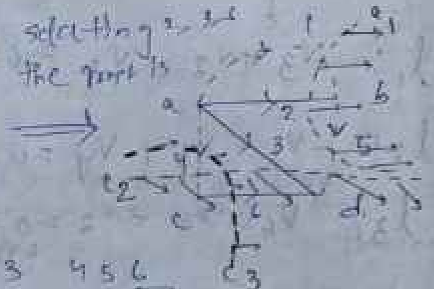
$\boxed{\text{Tie-set matrix} \cdot \text{branch vol. matrix} = 0}$

cut-set matrix

cut set = It is such a set of element which may remove or cut that divides the graph into two parts satisfy the condition that the element should set must contain max 1 branch.



selecting 2, 3, 5 the part is



$$[C] = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} +1 & +1 & 0 & 0 & -1 & 0 \\ 0 & 0 & +1 & +1 & +1 & 0 \\ 0 & 0 & 0 & -1 & 0 & +1 \end{bmatrix} \end{array}$$

$[Q]$ or $[C]$

Direction = defining branch direction

C_1 : $I_1 + I_2 - I_5 = 0$ (at b) $\Rightarrow I_1 + I_2 + 0I_3 + 0I_4 - I_5 + 0I_6 = 0$

$I_3 + I_5 + I_6 = 0$ (at d)

$I_6 = I_4$

So, $I_3 + I_5 + I_6 = 0$

$C2$: $\Rightarrow I_3 + I_5 + I_4 = 0 \Rightarrow 0I_1 + 0I_2 + I_3 + I_4 + I_5 + 0I_6 = 0$

$C3$: $I_4 - I_6 = 0$ (at c) $\Rightarrow 0I_1 + 0I_2 + 0I_3 + I_4 + 0I_5 - I_6 = 0$

$$\begin{bmatrix} 1 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = 0$$

$\Rightarrow \boxed{[Q][I_b] = 0}$

$I_1, I_2 \ldots I_6$ Branch current

H/W: verify

$\boxed{[A][I_b] = 0}$

$[A]$ = Incidence matrix

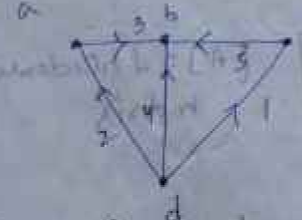
① The reduced incidence matrix of an oriented graph is given as

$$A = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

i) Draw the graph.
ii) How many trees are possible for this graph
iii) Write the tie-set matrix.
iv) Write the cut-set matrix.

i) node

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline a & 0 & -1 & 1 & 0 & 0 \\ b & 0 & 0 & -1 & -1 & -1 \\ c & -1 & 0 & 0 & 0 & 1 \\ d & 1 & 1 & 0 & 1 & 0 \end{array}$$



→ graph [A]

(ii) No of possible trees = det{[A][A^T]}
[A^T → transpose]

$$A^T = \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$A \cdot A^T = \begin{bmatrix} 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -1 \\ -1 & 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & -1 \\ -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & -1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+1+0+0 & 0+0-1+0+0 & 0+0+0+0+0 \\ 0+0-1+0+0 & 0+0+1+1+1 & 0+0+0+0-1 \\ 0+0+0+0+0 & 0+0+0+0-1 & 1+0+0+0+1 \end{bmatrix}$$

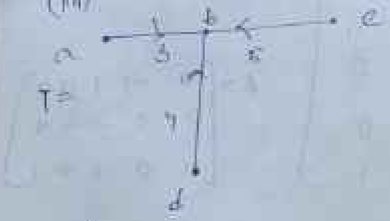
$$= \begin{bmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{bmatrix}$$

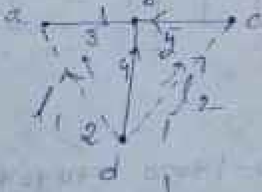
$$\begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{vmatrix} = 2(6-1) + 1(-2-0)$$

$= 10 + 2$

$= 8$ no of possible trees.

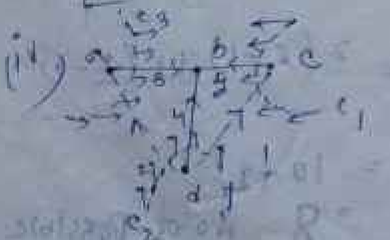
(iii)





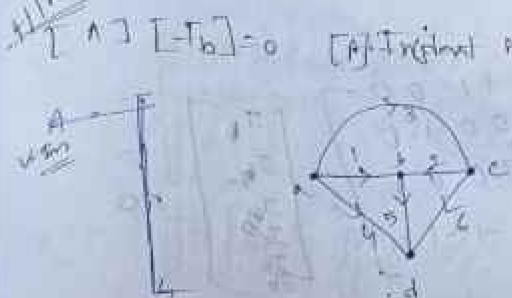
$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} l_1 \\ l_2 \end{matrix} & B = \begin{bmatrix} 0 & +1 & +1 & -1 & 0 \\ +1 & 0 & 0 & -1 & +1 \end{bmatrix}
\end{array}
$$

(iv)



$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\
\begin{matrix} c_1 \\ c_2 \\ c_3 \end{matrix} & C = \begin{bmatrix} -1 & 0 & 0 & 0 & +1 \\ +1 & +1 & 0 & +1 & 0 \\ 0 & -1 & +1 & 0 & 0 \end{bmatrix}
\end{array}
$$

$[A][I_b] = 0$ [A] Incidental matrix



Branch ch currents $= [I_1, I_2, \ldots I_6]$

a: $-I_3 - I_1 - I_4 = 0$ (at a)

$\Rightarrow I_1 + I_3 + I_4 = 0$

$\Rightarrow I_1 + 0I_2 + I_3 + I_4 + 0I_5 + 0I_6 = 0$

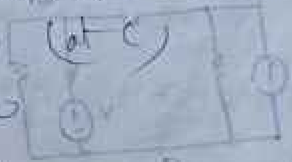
b: $I_1 - I_2 = 0$ (at b)

$\Rightarrow I_2 - I_1 - I_5 = 0$

$\Rightarrow -I_1 + I_2 + 0I_3 + 0I_4 + I_5 + 0I_6 = 0$

$\Rightarrow -I_1 - I_2 + 0I_3 + 0I_4 + I_5 + 0I_6 = 0$

c: $I_3 - I_2 - I_6 = 0$ (at c)



$\Rightarrow -I_2 + I_3 - I_6 = 0$

$\Rightarrow I_2 - I_3 + I_6 = 0$

$\Rightarrow 0I_1 + I_2 - I_3 + 0I_4 + 0I_5 + I_6 = 0$

(at d)

d: $I_4 + I_5 + I_6 = 0$

$\Rightarrow -I_4 - I_5 - I_6 = 0$

$\Rightarrow 0I_1 + 0I_2 + 0I_3 - I_4 - I_5 - I_6 = 0$

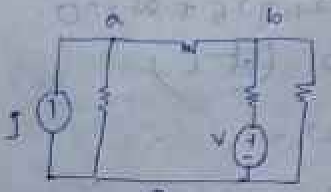
$$\begin{bmatrix} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 & -1 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = 0$$

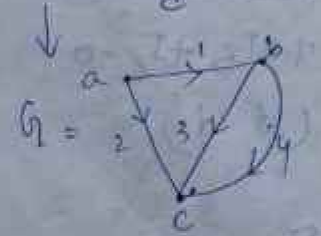
$\Rightarrow [A][I_b] = 0$

$\Rightarrow$ Augmented incidented matrix × branch current matrix = 0

21/10/22

Q2 Draw the graph of the network and draw and indicate the no. of all possible trees of the network.





$$A_a = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline a & +1 & +1 & 0 & 0 \\ b & -1 & 0 & +1 & +1 \\ c & 0 & -1 & -1 & -1 \end{array}$$

$$A^T = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

No of possible trees =

$$A \cdot A^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+0+0 & -1+0+0+0 \\ -1+0+0+0 & 1+0+1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 \\ -1 & 3 \end{bmatrix}$$

$$\det [A \cdot A^T] = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5$$

(home task)

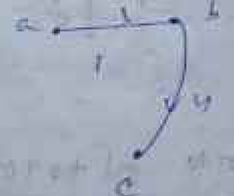
$b = n - 1$

$= 3 - 1 = 2$

$t_1 =$ 

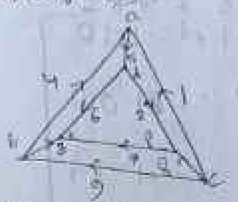
$t_2 =$ 

$t_3 =$ 

$t_4 =$ 

$t_4 =$ 

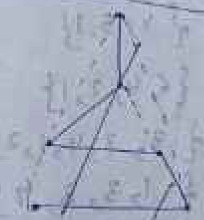
③ For the oriented graph shown in figure and for the tree indicated obtain all the tie-set and cut-set matrices.

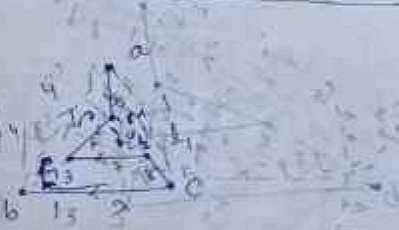


Tree : {5, 6, 7, 8, 9}

$b - n - 1 = 6 - 1 = 5$ = no. of fundamental loops

$l = e - b = 9 - 5 = 4$ = no. of fundamental cutsets





l_1 : {1, 5, 6, 7, 8}

l_2 {2, 6, 7}

l_3 : {3, 7, 8, 9}

l_4 : {4, 5, 6, 7, 8, 9}

~~matrix - B~~

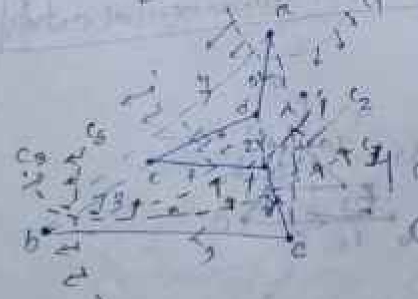
l_1 : $v_1 + v_5 + v_6 - v_7 - v_8 = 0$, $1v_1 + 0v_2 + 0v_3 + 0v_4 + v_5 + v_6 - v_7 - v_8 = 0$

l_2 : $v_2 + v_7 - v_6 = 0$, $0v_1 + v_2 + 0v_3 + 0v_4 + 0v_5 - v_6 + v_7 + 0v_8 + 0v_9 = 0$

l_3 : $v_3 - v_7 - v_8 + v_9 = 0$, $0v_1 + 0v_2 + v_3 + 0v_4 + 0v_5 + 0v_6 - v_7 - v_8 + v_9 = 0$

J_4: $v_4 + v_5 + v_6 - v_7 - v_8 + v_9 = 0v_1 + 0v_2 + 0v_3 + v_4 + v_5 + v_6 - v_7 - v_8 + v_9 = 0$

$$B = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix} \\ \begin{matrix} J_1 \\ J_2 \\ J_3 \\ J_4 \end{matrix} & \begin{bmatrix} +1 & 0 & 0 & 0 & +1 & +1 & -1 & +1 & 0 \\ 0 & +1 & 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & +1 & 0 & 0 & 0 & -1 & -1 & +1 \\ 0 & 0 & 0 & +1 & +1 & +1 & -1 & -1 & +1 \end{bmatrix} \end{array}$$



c_1 : {4, 5, 1}

c_2 : {6, 4, 2, 1}

c_3 : {7, 3, 4, 1, 2}

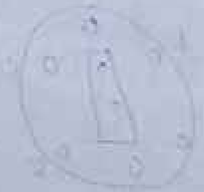
c_4 : {1, 3, 8, 4}

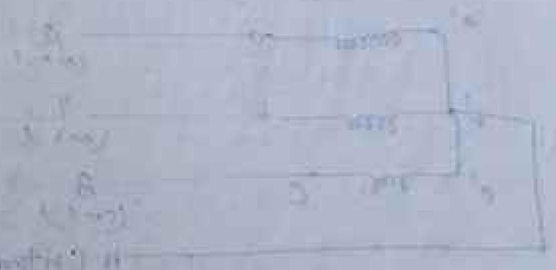
c_5 : {4, 3, 9}

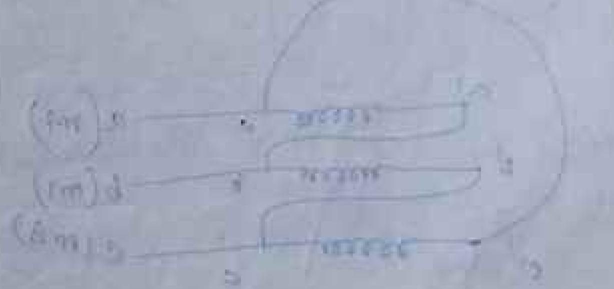
[9 is comming towards b so, c_3's direction should be towards b]

~~matrix C~~ =

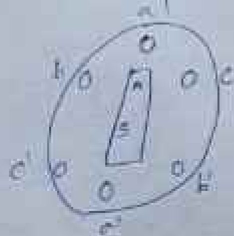
$$C = \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{matrix} \\ \begin{matrix} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{matrix} & \begin{bmatrix} -1 & 0 & 0 & -1 & +1 & 0 & 0 & 0 & 0 \\ -1 & +1 & 0 & -1 & 0 & +1 & 0 & 0 & 0 \\ -1 & +1 & -1 & +1 & 0 & 0 & +1 & 0 & 0 \\ +1 & 0 & +1 & +1 & 0 & 0 & 0 & +1 & 0 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & +1 \end{bmatrix} \end{array}$$



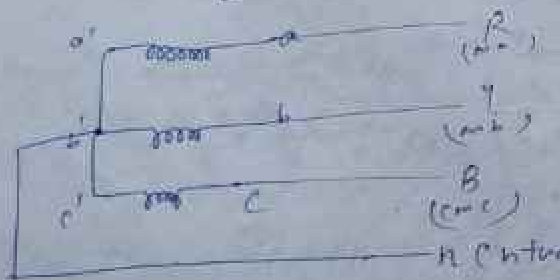




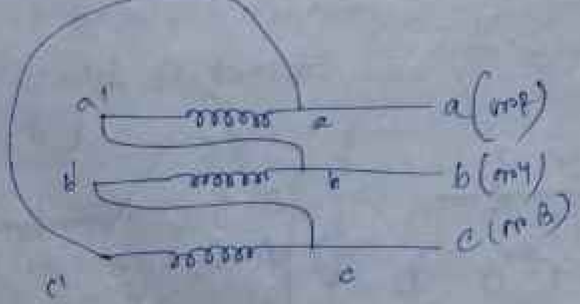
3-phase (Poly phase)

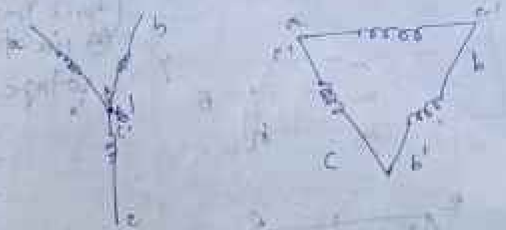


RYB or ABC
phase diff = 120° $\left[\frac{360}{3} = 120\right]$



→ Star connection / YY connection



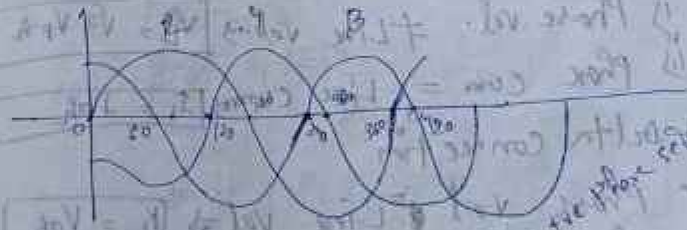


$E_a = |E| \angle 0° = E$

$E_b = |E| \angle -120° = E\angle -120°$

$E_c = |E| \angle -240° = E\angle 120°$

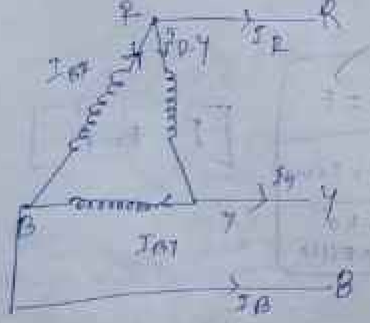
[E = Emf]



phase sequence → RYB → +ve phase sequence
→ RBY → -ve phase sequence
2 possibilities only

Line to phase / Line voltages = V_{RY}, V_{YB}, V_{BR}

phase voltage = V_{RN}, V_{YN}, V_{BN} (or) line to neutral voltage



this is the active direction

this is positive reference direction. If there is a 2nd wire, then it can be there but the direc. of the 2nd w will be opposite

(1) Star connection

i) Phase vol $\neq$ Line vol $\Rightarrow$ $\boxed{V_L = \sqrt{3}\, V_{ph}}$

ii) phase curr = Line curr $\boxed{I_L = I_{ph}}$

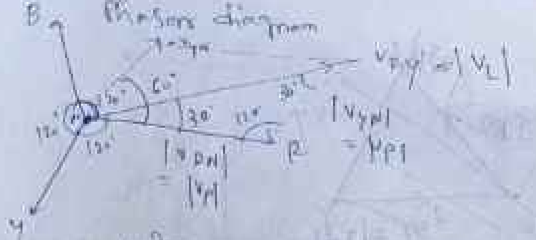
(2) Delta connection

i) phase vol = Line vol $\Rightarrow$ $\boxed{V_L = V_{ph}}$

ii) phase curr $\neq$ Line curr $\Rightarrow$ $\boxed{I_L = \sqrt{3}\, I_{ph}}$

H.W Same, solve (ii) method

Phasors diagram



$$V_L^2 = V_P^2 + V_P^2 - 2 V_P V_P \cos 120^\circ$$

$$= 2V_P^2 + 2 V_P^2 \cdot \frac{1}{2}$$

$$= 3V_P^2$$

$$\boxed{|V_L| = \sqrt{3}\, |V_P|}$$

$\therefore$ Line voltage is $\sqrt{3}$ time of phase voltage in magnitude. Line vol is leading phase vol by 30°.

$$E_{ab} = E_a - E_b$$

$$= E\angle 0 - E\angle -120^\circ$$

$$= E\,[1 - 1\angle -120^\circ]$$

$$= E\,[1 - (-0.5 - 0.866j)]$$

$$= E\left[1 + \left(\frac{1}{2} + \frac{\sqrt{3}}{2}j\right)\right]$$

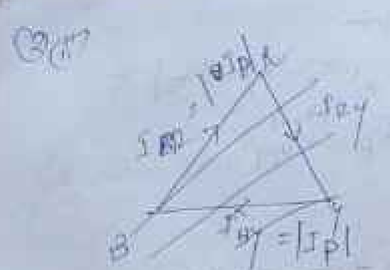
$$= E\left(\frac{3}{2} + \frac{\sqrt{3}}{2}j\right)$$

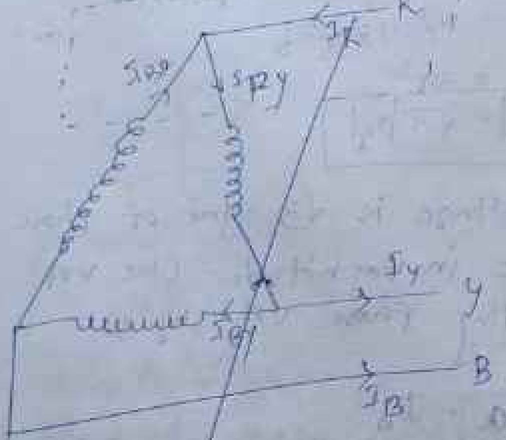
$$= \frac{1}{2}E\,(3 + \sqrt{3}j)$$

$$= \sqrt{3}E\angle 30^\circ$$

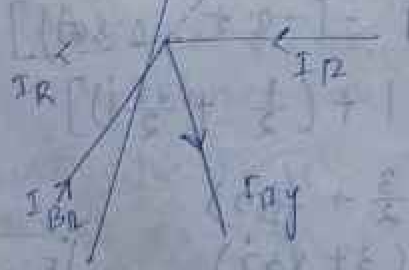
$$= 1.73E\angle 29.9$$

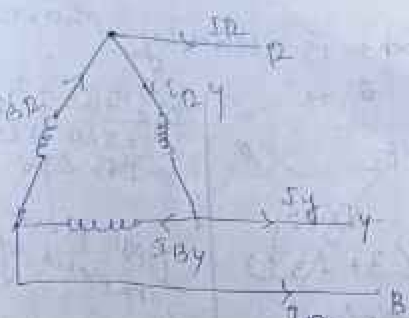
$\therefore \boxed{E_{ab} = \sqrt{3}E\angle 30^\circ}$



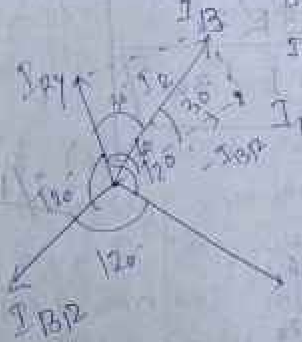


I_R is resultant of I_{BR} and I_{RY}





I_R is the resultant of I_{BR} and I_{RY}



I_R, I_Y, I_B line current $= I_L$

I_{RY}, I_{BY}, I_{BR} = phase current $= I_{ph}$

Now,

$$I_R = \sqrt{(I_{RY})^2 + (I_{BR})^2 + 2 I_{RY} \cdot I_{BR} \cos 60^\circ}$$

$$\Rightarrow I_L = \sqrt{2 I_{ph}^2 + 2 I_{ph}^2 \times \frac{1}{2}}$$

$[\because I_R = I_L,\ I_{RY} = I_{ph},\ I_{BR} = I_{ph}]$

$$= \sqrt{3 I_{ph}^2}$$

$$\boxed{|I_L| = \sqrt{3}\, |I_{ph}|}$$

∴ Line current is $\sqrt{3}$ times of phase current in magnitude.

Line current is leading phase current by 30°

$E_{bc} = E_b - E_c$

$= E\angle 0 - E\angle 120^\circ$

$= \frac{1}{2} E\,(3 + \sqrt{3}\,j)$

$= 1.73\,E\angle 30^\circ$

$\boxed{E_{bc} = \sqrt{3}\,E\angle 30^\circ}$

$I_B = I_{BR} - I_{YB}$
$= I\angle 0 - I\angle +120$
$= I\sqrt{3}\angle 30^\circ$

[All E are replaced by I]

2/11/22

$E_a = |E|\angle 0^\circ$

$E_b = |E|\angle -120^\circ$

$E_c = |E|\angle +120^\circ$

$I_a + I_b + I_c$

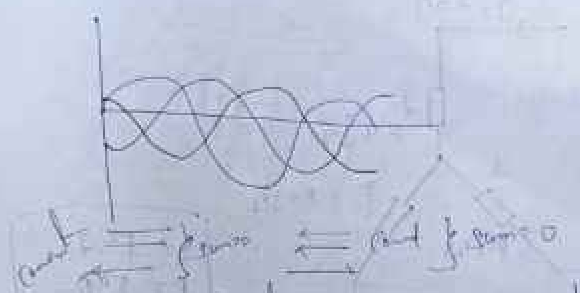
$= |I|\,[1\angle 0 + 1\angle -120 + 1\angle +120^\circ]$

$= |I|\,[1 + (-0.5 - 0.866j) + (-0.5 + 0.866j)]$

$= |E|\,[1 - 2\times 0.5]$

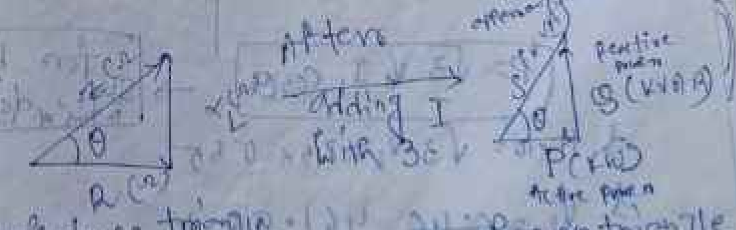
$= 0$

Sum of 3ph balance quantity = 0

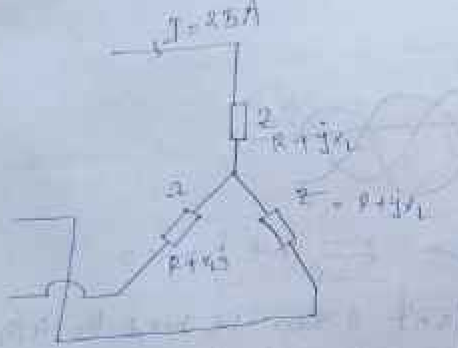


If sum isn't 0 then we need the 4th wire and that is neutral = n

(1) The load connected to a 3-ph supply comprises 3 similar coils connected in star, the line currents are 25 A, and KVA and KW inputs are 20 and 11 respectively. Find the line and phase voltages, KVAR input and the resistance and reactance of each coil. ~~Every~~ If coils are now connected in delta to this same 3-ph supply calculate the line currents and the powers taken.



Impedance triangle — Power triangle



$S = I^2 Z$
$P = I^2 R$
$Q = I^2 X_L$

$P = 11$ kW
$S = 20$ kW

$Q = \sqrt{S^2 - P^2}$
$= \sqrt{(20)^2 - (11)^2}$
$= 16.7$ kVAR

(iii) $\tan\theta = \frac{Q}{P}$
$= \frac{16.7}{11}$
$= 56.63$

(ii) $\cos\theta = \frac{P}{S}$
$= \frac{11}{20}$
$= 0.55$
$= 56.63°$

$\boxed{P = \sqrt{3}\, V_L I_L \cos\theta}$ → for both Star, delta tau in terms of line

$\Rightarrow 11 \times 10^3 = \sqrt{3}\, V_L \times 25 \times 0.55$

$\Rightarrow V_L = \cancel{0.46}$ 461.88 V

$V_{ph} = \frac{V_L}{\sqrt{3}} = 266.67$ V

$Z = \frac{V_{ph}}{I}$ $\left[\because \boxed{V_{ph} = ZI}\right]$
$= \frac{266.67}{25}$
$= 10.67\ \Omega$

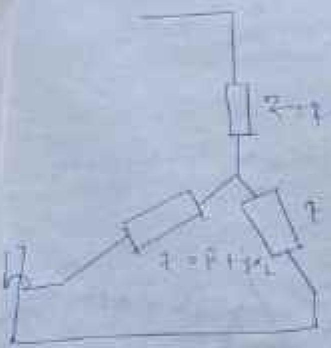
$\boxed{R = Z\cos\theta} = 10.67 \times 0.55 = 5.87\ \Omega$
$\boxed{X_L = Z\sin\theta} = 10.67 \times 0.83 = 8.01\ \Omega$

(iv) $P_{ph} = \frac{P}{3}$ $\left[\because \boxed{P = 3P_{ph}}\right]$ 3 ph Power in terms of ph
$= \frac{11}{3}$
$= 3.67$ kW
$= 366.67$ W

$\boxed{P_{ph} = I_{ph}^2 \times R_{ph}}$ $[R_{ph} = R]$
$\Rightarrow 366.67 = (25)^2 \times R$
$\Rightarrow R = 5.87\ \Omega$

$\frac{Q}{3} = Q_{ph} = 5.56 \times 10^3$ $\left[\boxed{Q = 3Q_{ph}}\right]$

$\Rightarrow \boxed{Q_{ph} = I_{ph}^2 X_L}$
$\Rightarrow \cancel{16.7 \times 10^3}$ $5.56 \times 10^3 = (25)^2 \times X_L$
$\Rightarrow X_L = 8.01\ \Omega$

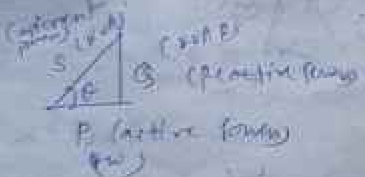


as it is coil r_L

and 3 coils are similar so all Z will be same

now, given, line currents = 25A

$I_{ab} = I_{bc} = I_{ca} = I = 25A$



given, S = 20 kVA
P = 11 kW

$\therefore Q = \sqrt{S^2 - P^2} = 16.7$ kVAR (Ans)

we have to get line vol = V_{ab} V_{bc} V_{ca} = V_L
phase vol = V_a, V_b, V_c = V_{ph}

Now as I and Z are similar for all coils, ie, the coils are similar $V_a = V_b = V_c = V_{ph}$
$V_{ab} = V_{bc} = V_{ca} = V_L$

we have to get also R, X_L

Now, we know that

$\boxed{R = Z\cos\theta}$, $\boxed{X_L = Z\sin\theta}$

from power triangle,

$\tan\theta = \frac{Q}{P} = \frac{16.7 \times 10^3}{11 \times 10^3}$

$\Rightarrow \theta = 56.62°$

we know that $\boxed{V_{ph} = Z I_{ph}}$

$\therefore$ we have to get V_{ph}

we also know that

$\boxed{P_L = \sqrt{3} V_L I_L \cos\theta}$

$\Rightarrow 11 \times 10^3 = \sqrt{3} \times V_L \times 25 \times \cos(56.62)$

$\Rightarrow V_L = 461.72$ V (Ans)

$V_L = \sqrt{3} V_{ph}$

$\Rightarrow V_{ph} = \frac{V_L}{\sqrt{3}}$
$= \frac{461.72}{\sqrt{3}}$

$V_{ph} = 266.57$ V (Ans)

$\therefore 266.57 = Z \times 25$

$\Rightarrow Z = 10.67\,\Omega$ (Ans)

$\therefore R = 10.67 \times \cos(56.62)$
$= 5.867\,\Omega$ (Ans)

$X_L = 10.67 \times \sin(56.62)$
$= 8.91\,\Omega$ (Ans)

[For only 1 coil]

$\left[P_{ph} = \frac{P_L}{3}\right]$

$\left[Q_{ph} = \frac{Q_L}{3}\right]$

[i.e. $P, Q_L = P_T$]

$\left[P_{ph} = I_{ph}^2 \times R_{ph}\right]$

$\left[Q_{ph} = I_{ph}^2 \times X_{L_{ph}}\right]$

$[\because I_{ph} = I_L]$

For delta connection

$V_L = V_{ph} = 461.88\,V$

$I_{ph} = \frac{461.88}{Z}$ [$I_{ph} = \frac{V}{Z}$]

$= \frac{461.88}{10.67}$

$= 43.28$

$I_L = \sqrt{3} \times 43.28$ [$\because I_L = \sqrt{3} I_{ph}$]

$= 74.97$

$= 75A$

$P = \sqrt{3}\, V_L I_L \cos\theta$

$= \sqrt{3} \times 461.88 \times 75 \times 0.55$

$= 32.99$ kW

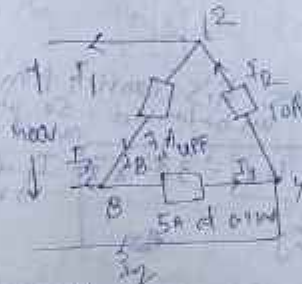
(or) $P = 3(I_{ph}^2 R)$ [$\because P = 3P_{ph} = 3(I_{ph}^2 R)$]

$= 3 \times (43.28^2 \times 5.87)$

$= 33$ kW

9/11/22

Q Find the currents and power consume by the unbalanced delta connection as shown.



Power factors = UPF, lead, Lag
(PF)
↓
unity power factor

$\boxed{PF = \cos\theta = \frac{R}{Z} = \frac{kW}{kVA}}$

UPF ⇒ $\theta = 0$

$\therefore \cos\theta = 1$ → it is called unity PF

in Lag, θ = (+)ve

Lag → I is lagging V

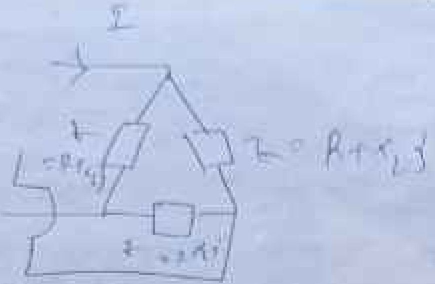
$I = \frac{V}{Z\angle\theta}$ [V is reference so the angle of V = 0, if θ will +ve, I will be negative]

In lead, θ = (−)ve

Lead → I is leading V

$\boxed{UPF = \angle 0^\circ}$, $\boxed{Lag = \angle -\theta}$, $\boxed{Lead = \angle\theta}$

$\boxed{\angle -\theta = \angle -\cos^{-1}\theta}$



Every thing will be same like
16.7 kVAR, Z, X_L, R

but here $V_{ph} = V_L$ [as $\cos\theta$ will remain same]

$V_L = 461.88$

∴ $V_{ph} = 461.88$

~~Now, we know $V_{ph} = Z I_{ph}$~~

~~Now, $I_L = I_{ph} \times \sqrt{3}$~~

~~$\Rightarrow I_{ph} = \frac{I_L}{\sqrt{3}} = \frac{25}{\sqrt{3}} = 14.43$ A~~

~~∴ $461.88 = Z \times 14.43$~~

~~$\Rightarrow Z = 32$~~

we have to get line currents $I_a = I_b = I_c = I_L = ?$

and P = ?

∴ we know that

$V_{ph} = Z I_{ph}$

$\Rightarrow \frac{461.88}{10.67} = I_{ph}$

$\Rightarrow I_{ph} = 43.28$ A

∴ $I_L = \sqrt{3} \times 43.28 = 75$ A (Ans)

$P = \sqrt{3} V_L I_L \cos\theta$
$= \sqrt{3} \times 461.88 \times 75 \times \cos 56.6$
$= 330$ kW (Ans)

$I_R = 10\angle -\cos^{-1}0.8 = 10\angle -36.87$

$I_y = 5\angle +\cos^{-1}0.7 = 5\angle 45.57$

$I_B = 7\angle 0°$

KCL at R, [As this connection is unbalanced so we cannot apply $I_1 = I_2 = I_3$]

$I_1 = -7\angle 0 + 10\angle -36.87$

$= -7 + 8 - 6j$

$= 1 - 6j$

$= 6.08\angle -80.5$

$I_2 = -I_R + I_y$

$= 5\angle 45.57 - 10\angle -36.87$

$= 3.5 + 3.57j - (8 - 6j)$

$= -4.5 + 9.57j$

$= 10.57\angle 115.18$ A

$I_3 = I_B - I_y$

$= 7 - 5\angle 45.57$

$= 7 - (3.5 + 3.57j)$

$= 3.5 - 3.57j = \cancel{4.9\angle -0.24}$ $-5\angle 45.57$

$P_{R\,phase} = |V_{Ry}||I_R|\cos\theta_R$ [magnitude of I_R]

$= 400 \times 10 \times 0.8$ [$\cos\theta_R = 0.8$]

$= 3200$ W

$P_{y\,phase} = |V_{yB}||I_y|\cos\theta_y$ [$\cos\theta_y = 0.7$]

$= 400 \times 5 \times 0.7$

$= 1400$ W

[$\cos\theta_B = 1$]

$P_{B\,phase} = |V_{BR}||I_B|\cos\theta_B$

$= 400 \times 7 \times 1$

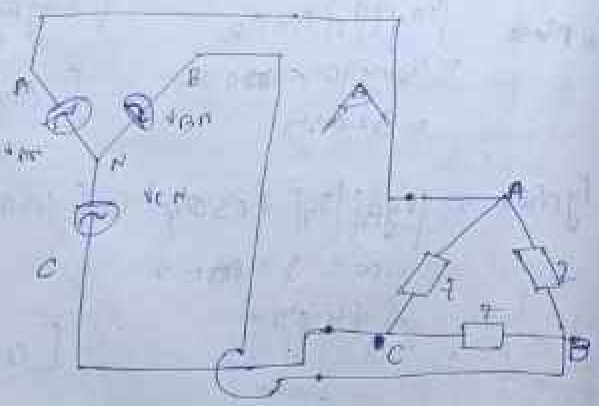
$= 2800$

[as it is unbalanced ...]

Total power $= 3200 + 1400 + 2800$

$= 7400$ W

② One line voltage of a balanced star connected source is $V_{AB} = 240\angle -20°$ V. If this source is connected to a delta connected load of $20\angle 40°\ \Omega$. Find the phase and line currents. Assume A B C sequence.

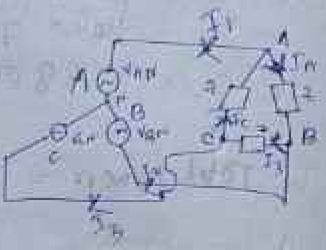


$V_{AB} = 240\angle -20^\circ V$

$Z = 20\angle 40^\circ \Omega$

$V_{BC} = 240\angle -140^\circ V$

$V_{CA} = 240\angle 100^\circ V$



$\boxed{I_A = \frac{V_{AB}}{Z}} = \frac{240\angle -20^\circ}{20\angle 40^\circ} = 12\angle -60 A$

$\boxed{I_B = \frac{V_{BC}}{Z}} = \frac{240\angle -140}{20\angle 40} = 12\angle -180 A$

$\boxed{I_C = \frac{V_{CA}}{Z}} = \frac{240\angle 100}{20\angle 40} = 12\angle 60 A$

$I_1 = I_C - I_A = 12\angle 60 - 12\angle -60^\circ$

$= 6+10.39j - 6+10.39j$

$= 12[1\angle 60 - \angle -60^\circ]$

$= 20.78j = 20.78\angle 90^\circ A$

$I_2 = I_A - I_B$

$= 12\angle -60 - 12\angle -180^\circ$

$= 6 - 10.39j + 12 \; [12\angle -180^\circ = -12]$

$= 18 - 10.39j$

$= 20.78\angle -30$

$= 12\sqrt{3}\angle -30^\circ A$

$I_3 = I_B - I_C$

$12\angle -180 - 12\angle 60^\circ$

$= -12 - (6+10.39j)$

$= -18 - 10.39j$

$= -(18+10.39j)$

$= -12\sqrt{3}\angle 30 A$

$= 12\sqrt{3}\angle -150 A$

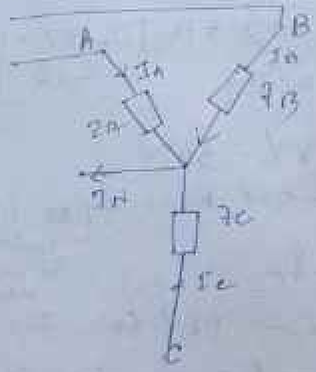
[As I is balanced so we can adjust $I_1 = \sqrt{3} I_A$ $= \sqrt{3}\,12\angle ...$ As line current (I_1) is leading phase by 30°]

(3) An unbalanced ~~three~~ four wire star connected load has balanced line voltages of 400 V. The load impedances are $Z_A = (10+3j)\Omega$, $Z_B = (3-8j)\Omega$, $Z_C = (10+4j)\Omega$. Calculate the line currents, neutral current and the powers in each phase. Phase sequence is A, C, B

$V_{AB} = 400\angle 0°$

$V_{BC} = 400\angle -120°$ [∵ sequence A, C, B; $V_{BC} = -120°$, $V_{CA} = -(+120)$]

$V_{CA} = 400\angle -120°$



Y →
Same
$V_L = \frac{V_L}{\sqrt{3}}$
Δ V same
$I_L = \frac{I_L}{\sqrt{3}}$

Anticlockwise direction

$Z_A = 10 + 3j = 10.44\angle 16.7°$

$Z_B = 5 - 8j = 8.54\angle -58.4°$

$Z_C = 10 + 4j = 10.77\angle 21.8°$

Line current = I_A, I_B, I_C
Phase

$$I_A = \frac{V_{AN}}{Z_A} = \frac{\frac{400}{\sqrt{3}}\angle -30°}{10.44\angle 16.7} = 22.12\angle -46.7\ A$$

[∵ phase voltage (V_{AN}) is lagging by 30° by line voltage (V_{AB})]

$$I_B = \frac{V_{BN}}{Z_B} = \frac{\frac{400}{\sqrt{3}}\angle 120 - 30}{8.54\angle -58.4} = 27.04\angle 150.4\ A$$

$$I_C = \frac{V_{CN}}{Z_C} = \frac{\frac{400}{\sqrt{3}}\angle -120 - 30}{10.77\angle 21.8} = 21.44\angle -171.8\ A$$

$I_N = I_A + I_B + I_C$

$= 22.12\angle -46.7 + 27.04\angle 150.4 + 21.44\angle -171.8$

$= 15.17 - 16.1j + -23.3 + 0.51j$

$21.22 - 3.03j$

$= -31.35 - 2.54j$

$= 30.9\angle -172.4$

$= 30.9\angle 184.76$

Power in each phase – [$P = I_{ph}^2 R$]

$P_A = |I_A|^2 R_A = (22.12)^2 \times 10 = 4892.9\ W = 4.89\ kW$

$P_B = |I_B|^2 R_B = (27.03)^2 \times 3 = 2.2\ kW$

$P_C = |I_C|^2 R_C = (21.44)^2 \times 10 = 4.59\ kW$

(or)

$P_A = |V_{AN}|\,|I_A| \cos\theta_A = \frac{400}{\sqrt{3}} \times 22.12 \times \cos(46.7) = 3503.93\ W$

$P_B = |V_{BN}|\,|I_B| \cos\theta_B = \frac{400}{\sqrt{3}} \times 27.04 \times \cos(150.4) = 5845.3\ W$

$P_C = |V_{CN}|\,|I_C| \cos\theta_C = \frac{400}{\sqrt{3}} \times 21.44 \times \cos(171.8) = -4900.73\ W$

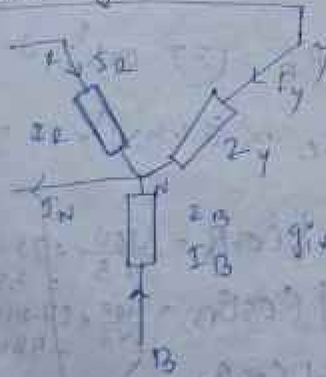
$P_A = 231 \times 22.12 \times \cos(63.67) = 4.9$ kW

$P_B = 231 \times 27.04 \times \cos(69.45) = 2.2$ kW

$P_C = 231 \times 21.45 \times \cos(28.8) = 4.6$ kW

H.W.

① A 3-phase 4-wire system having a 230V phase voltage as the following loads connected between the respective lines and neutral, $Z_R = 15\angle 30^\circ\ \Omega$, $Z_Y = 15\angle 45^\circ\ \Omega$, $Z_B = 15\angle 60^\circ\ \Omega$. Calculate the current in the neutral wire and power taken by each load.



$Z_R = 15\angle 30 = 13 + 7.5j$

$Z_Y = 15\angle 45 = 10.61 + 10.61j$

$Z_B = 15\angle 60 = 7.5 + 13j$

given, phase vol. =

$V_{RN} = 230\angle 0$

$V_{YN} = 230\angle -120$

$V_{BN} = 230\angle +120$

To calculate current $= I_{RY}, I_{YB}, I_{BR}$ (no necessary)

$= I_R, I_Y, I_B$

I_N, P_{ph}

$I_R = \frac{V_{RN}}{Z} = \frac{230\angle 0}{15\angle 30} = 15.33\angle -30$

$I_Y = \frac{V_{YN}}{Z} = \frac{230\angle -120}{15\angle 45} = 15.33\angle -165$

$I_B = \frac{V_{BN}}{Z} = \frac{230\angle 120}{15\angle 60} = 15.33\angle 60$

$I_N = I_R + I_Y + I_B$

$= 13.28 - 7.665j - 14.81 - 4j + 7.665 + 13.28j$

$= 6.135 + 1.615j$

$= 6.34\angle 14.75^\circ$

~~$P_{ph} = P_{Rph} + P_{Yph} + P_{Bph}$~~

for star connection $I_{ph} = I_L$

$\therefore I_{RY} = I_R = 15.33\angle -30$

$I_{YB} = I_Y = 15.33\angle -165$

$I_{BR} = I_B = 15.33\angle 60$

no need

$\therefore P_{ph} = P_{Rph} + P_{Yph} + P_{Bph}$

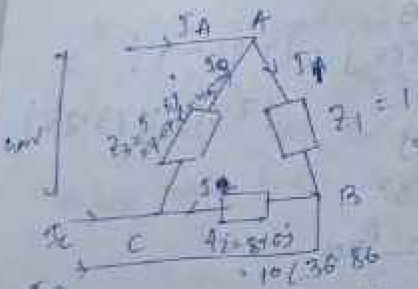
no need $\rightarrow = |I_{RY}|^2 \times R_R + |I_{YB}|^2 \times R_Y + |I_{BR}|^2 \times R_B$

$= |I_R|^2 \times R_R + |I_Y|^2 \times R_Y + |I_B|^2 \times R_B$

$= (15.33)^2 [13 + 10.61 + 7.5]$

$= 7.3 \times 10^3$ W $= 7.3$ kW

(4) 3 impedances Z_1, Z_2 and Z_3 are delta connected to a symmetrical 3-ph 400V 50Hz supply of phase sequence ABC. $Z_1 = 10 + j0\ \Omega$ connected between lines A and B, $Z_2 = 8 + 6j\ \Omega$ connected between lines B and C and Z_3 $5 - 5j\ \Omega$ connected between line C and A. Calculate the phase and line currents and the total active power consume. Draw the phasor diagram.



Z1 nothing (R L) then consider it phase val KADA

$V_{AB} = 400\angle 0°\ V$

$V_{BC} = 400\angle -120°\ V$

$V_{CA} = 400\angle +120°\ V$

$I_1 = \frac{V_{AB}}{Z_1} = \frac{400\angle 0}{10} = 40\ A$

$I_2 = \frac{V_{BC}}{Z_2} = \frac{400\angle -120}{10\angle 36.86} = 40\angle -156.86\ A$
$= -36.78 - 15.72j$

$I_3 = \frac{V_{CA}}{Z_3} = \frac{400\angle +120}{7.07\angle -45} = 56.57\angle 165\ A$
$= -54.65 + 14.6j$

(All not balanced)

$I_A = I_1 - I_3 = 40 - 56.57\angle 165$
$= 40 + 54.65 - 14.6j$
$= 94.65 - 14.6j = 95.77\angle -8.8\ A$

$I_B = I_2 - I_1$
$= -36.78 - 15.72j - 40$
$= -76.78 - 15.72j$
$= -78.37\angle 11.57\ A$

$I_C = I_3 - I_2 =$
$-54.65 + 14.6j + 36.78 + 15.72j$
$= -17.87 + 30.32j$
$= 35.19\angle 120.5\ A$

Total 3-ph Powers

$P_A = |I_1|^2 R_A + |I_2|^2 R_B + |I_3|^2 R_C$

P_B

P_C

$= (40)^2 \times 10 + (40)^2 \times 8 + (56.57)^2 \times 5$

$= 16000 + 12800 + 16000$

$= 44.8\ kW$

(4)

[If said, get the power then calculate by default calculate P phase]

Note

To identify if nothing is said (whether by laplace or by ac circuit)

(1) For A.C ckt → ac source will be there, pd and the form of voltage, capacitor, inductor will be given.

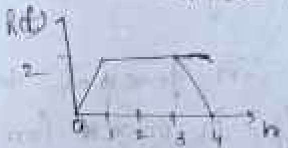
(2) For Laplace trans → there will not be any a.c source, there will be dc source, the C will be given like — 0.1 F, inductance will be given like — 5H. The terms those are used in laplace will be given there it's $u(t)$, $\delta(s)$, $v(t)$, $i(t)$, $u(t)e^{-at}$, $I(t)$ etc. [Switch may be given]

(3) For 3-phase, all the terms of 3-ph like star connection, delta connection, 4-wire system, load, phase volt, phase curr, line vol, line curr, neutral current, power consume etc term will be there

(4) For 2-Ports, you can easily identify them by the term Z, Y, h, g parameters etc

EEN Previous year Qn

Q) 1. a) Find the Laplace transformation of $f(t)$.

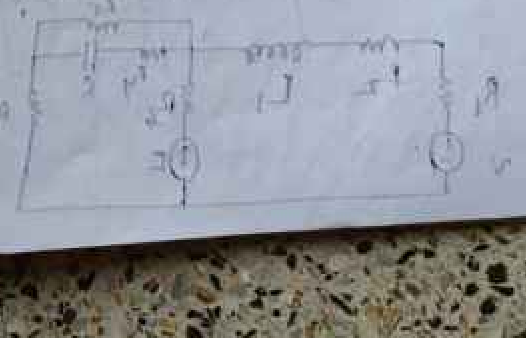


$f(t) = 2r(t) - 2r(t-1) - 2r(t-3) + 2r(t-4)$

$f(t) = 2t\,u(t) - 2(t-1)\,u(t-1) - 2(t-3)\,u(t-3) + 2(t-4)\,u(t-4)$

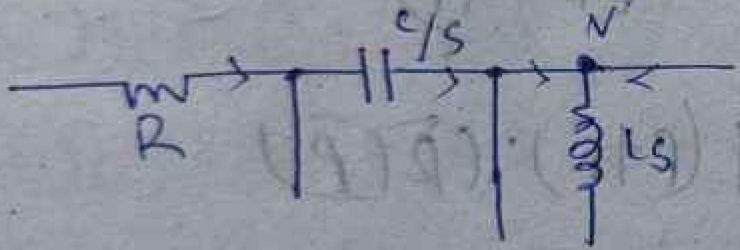
Applying laplace on both side,

$$F(s) = 2\left[\frac{1}{s^2} - \frac{1}{(s-1)^2}e^{-s}\cdot\frac{1}{s} - \frac{1}{(s-3)^2}e^{-3s}\frac{1}{s} + \frac{1}{(s-4)^2}e^{-4s}\cdot\frac{1}{s}\right]$$



$$-\frac{1}{(s-3)^2}e^{-3s}\frac{1}{s}+2\frac{1}{(s-4)^2}$$

Normal KCL will be applicable for ac crt for laplace t. KCL will apply, but we will consider all resistance, inductance, capacitance.



$\rightarrow \frac{V}{R+\frac{C}{S}+LS}$, like that

$$\cancel{p\bar{q} + \bar{p} + p\bar{r} + q}$$

$$= p\bar{q} + p\bar{r} + \bar{p} + qr$$

$$= \cancel{(p\bar{q} + p)} \, ((p\bar{q} + qr) + (p\bar{r} + p)$$

$$= (p\bar{q} + q) \cdot (p\bar{q} + r) + (p + \bar{p}) \cdot (\bar{r} + \bar{p})$$

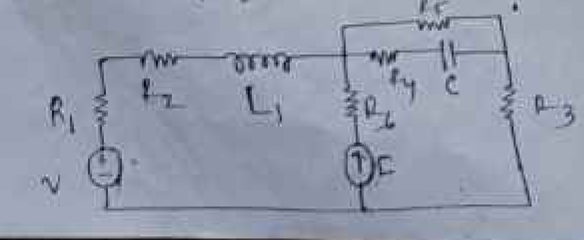
$$= (p + q) \cdot (\bar{q} + r)(p + r) + \bar{r} + \bar{p}$$

1(b) State Dirichlet's conditions for a function to be expanded as a Fourier series.

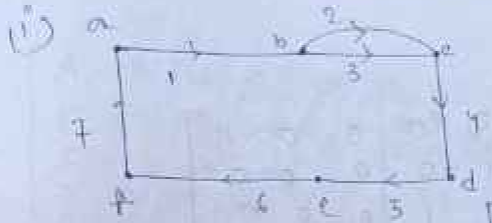
Fourier series in general tends to converge slowly. In order for a function to be expanded properly, it must satisfy the following Dirichlet conditions: A piecewise function must be periodic with at most a finite number of discontinuities and/or a finite number of minima or maxima within one period.

~~f(x) ha~~

1(c) For the ckt shown in fig (i) draw the oriented graph and write the (i) incidence matrix, (ii) f cutset matrix, (iv) How many trees are possible for the graph of networks?



(i)



[the orientations is are arbitrary]

(ii) incidence matrix,

$$A_a = \begin{array}{c} \\ a \\ b \\ c \\ d \\ e \\ f \end{array} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ +1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & +1 & +1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & +1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & +1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & +1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & +1 \end{array}$$

(iv) no. of trees possible $\det\{[A][A^T]\}$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$[B] = [A+I] =$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & -1 \\ -1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$

$$\times \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1 & -1 & 0 & 0 & 0 \\ -1 & 1+1+1 & -1-1 & 0 & 0 \\ 0 & -1-1 & 1+1+1 & -1 & 0 \\ 0 & 0 & -1 & 1+1 & -1 \\ 0 & 0 & 0 & -1 & 1+1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ -1 & 3 & -2 & 0 & 0 \\ 0 & -2 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -1 & 0 & 0 & 0 \\ 0 & 5 & -4 & 0 & 0 \\ 0 & -2 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \quad [R_2' = 2R_2 + R_1]$$

$\rightarrow$ not need of this step

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -2 & 3 & -2 & 0 & 0 \\ -2 & -2 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \quad [C_1' = C_1 + C_2]$$

$$= \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 5 & -2 & 0 & 0 \\ 0 & -4 & 3 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix} \quad \begin{bmatrix} R_2' = R_2 - 2R_1 \\ R_3' = R_2 + 2R_1 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & -2 & 0 & 0 \\ -4 & 3 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= -\begin{bmatrix} -2 & 5 & 0 & 0 \\ 3 & -4 & -1 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 5 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 3 & -4 & -1 & 0 \\ 0 & 0 & -1 & -2 \end{bmatrix}$$

$$= -\begin{bmatrix} -1 & 0 & 2 & -1 \\ -2 & 5 & 0 & 0 \\ 3 & -4 & -1 & 0 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -2 & 1 \\ 2 & -5 & 0 & 0 \\ -3 & 4 & 1 & 0 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$\boxed{\begin{aligned} R_2' &= R_2 - 2R_1 \\ R_3' &= R_3 + 3R_1 \end{aligned}}$$

$$= \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & -5 & 4 & -2 \\ 0 & 4 & -5 & 3 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & 4 & -2 \\ 4 & -5 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

$$\begin{vmatrix} -5 & 4 & -2 \\ 4 & -5 & 3 \\ 0 & 1 & -2 \end{vmatrix}$$

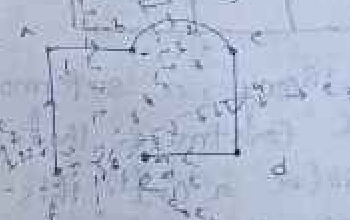
$= -5((10-3)) + 4(-8-0) - 2(4-0)$

$= -5 \times 7 + 32 - 8$

$= -11$

11 no. of trees are possible.

(ii) T =

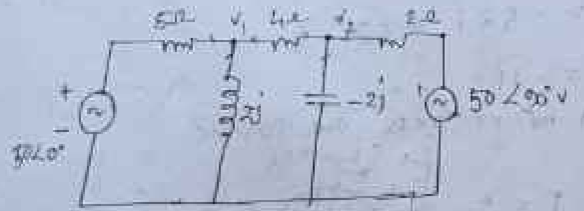


$$C = \begin{array}{c} \\ c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{array} \begin{array}{c} \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\ \begin{bmatrix} +1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & +1 \\ 0 & 0 & 0 & +1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & +1 & +1 & 0 \\ 0 & +1 & +1 & 0 & 0 & -1 & 0 \end{bmatrix} \end{array}$$

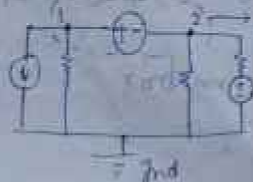
this is the cut-set matrix

Un-8

2) What is super node in electrical circuit? In the network shown in the fig: find the node voltages v_1 and v_2 and current in the branch with the resistor 4Ω



In ckt theory, a supernode is a theoretical construct that can be used to solve a ckt. this is done by viewing a voltage source on a wire as a point source voltage in relation to other point voltages located at various nodes in the ckt, relative to a ground node assigned a zero or negative charge



$$\frac{v_1 - 50\angle 0}{5} + \frac{v_1}{2j} + \frac{v_1}{4} = 0$$

$$\Rightarrow \frac{v_1 - 50}{5} + \frac{v_1}{2j} + \frac{v_1}{4} = 0$$

$$\Rightarrow v_1\left(\frac{1}{5} + \frac{1}{2j} + \frac{1}{4}\right) = \frac{50}{5} = 10$$

$$\Rightarrow v_1\left(0.45 + \frac{1}{2\angle 90}\right) = 10$$

$$\Rightarrow v_1(0.45 + 0.5\angle -90) = 10$$

$$\Rightarrow v_1 = \frac{10}{0 - 0.5j + 0.45}$$

$$= \frac{10}{0.67\angle -48}$$

$$15\angle 48°$$

$$v_1 = 10.03 + 11.1j$$

$$v_2 = \frac{v_2}{4} + \frac{v_2}{-2j} + \frac{v_2 - 50\angle 90}{2} = 0$$

$$\Rightarrow \frac{v_2}{4} + \frac{v_2}{-2j} + \frac{v_2 - 50j}{2} = 0$$

$$\Rightarrow v_2\left(\frac{1}{4} - \frac{1}{2j} + \frac{1}{2}\right) = \frac{50j}{2} \text{ or } \frac{50\angle 90}{2}$$

$$\Rightarrow v_2\left(0.75 - \frac{1}{2\angle 90}\right) = 25\angle 90$$

$$\Rightarrow v_2(0.75 - 0.5\angle -90) = 25\angle 90$$

$$\therefore V_2 = \frac{7.5\angle 90^\circ}{0.75 + 0.5j}$$

$$= \frac{25\angle 90^\circ}{0.9\angle 33.69}$$

$$= 27.78\angle 56.31$$

$$V_2 = 15.41 + 23.11j$$

$$V_{th} = V_{12} = V_1 - V_2$$

$$= (10.03 + 11.1j) - (15.41 + 23.101j)$$

$$= -5.38 - 12.01j$$

$$= -(5.38 + 12.01j)$$

$$Z_{th} = (5 + 2j) + (2 - 2j)$$

$$= 7\Omega$$

$\therefore$ Equivalent ckt,

[$\because$ Load ~~resistance~~ Impedance (Z_L) $= 4\Omega$]

$$I_L = \frac{V_{th}}{Z_L + Z_{th}}$$

$$= \frac{-(5.38 + 12.01j)}{4 + 7}$$

$$= \frac{-27.78\angle 56.31}{11}$$

$$= 7.52\angle$$

$$= \frac{13.164\angle 65.86}{-11}$$

$$= 1.19\angle 65.86$$

$$= 0.48 + 1.08j$$

$I_L = \frac{-(18.17\ 7.81)}{4+7}$

$= 19.73\angle 23.3$

$= 1.79\angle 23.3$

$= 1.64 + 0.71j$

Q) Write the difference between independent and dependent v/c source.

Independent vol/~~current~~ Source	Dependent vol/~~cur~~ Source
1) Definition - done	1) definition - done
2) It is denoted normally the middle symbol is the symbol for a specific type of inde. v. s.	2) It is denoted or represented by diamond shape.
3) It is also termed as "constant source" or time variant source".	3) Here, there is no such terms.

3) It has no types

4) It has 2 types of dependent voltage source - CCVS, VCVS.

Independent Curr Source

1) Defination - done

2) The symbol of this a circle with an arrow inside is the symbol to indicate the direction of the current flow

3) It has no types

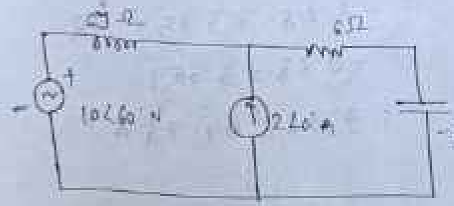
Dependent Curr Source

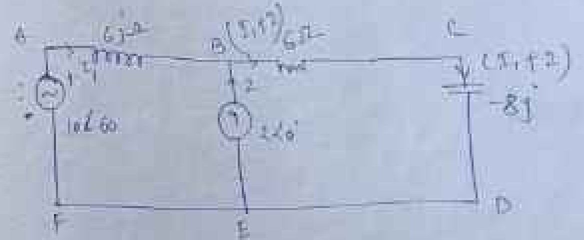
1) Defination - done

2) It is represented by diamond shape.

3) There are 2 types of dependent current source - CCCS, VCCS.

3) b) Find the current through 6Ω resistor shown in fig using the mesh analysis. Here, voltage source is $10\angle 60°$ v and current source is $2\angle 0$ A.





ABCDEFA, KVL,

$6j \times I_1 + 6(I_1+2) + -8j(I_1+2) = 10\angle 60$

$\Rightarrow 6I_1 j - 8I_1 j + 6I_1 + 12 - 16j = 10\angle 60$

$\Rightarrow -2I_1 j + 6I_1 - 16j + 12 = 10\angle 60$

$\Rightarrow I_1(6-2j) + 12 - 16j = 10\angle 60$

$\Rightarrow I_1(6-2j) = 5 + 8.67j - 12 + 16j = -7 + 24.67j$

$\Rightarrow I_1 \cancel{(6-2j)} = \dfrac{-7+24.67j}{6-2j}$

$= \dfrac{25.64\angle 105.84}{6.32\angle -18.43}$

$= 9.056 \angle 124.27$

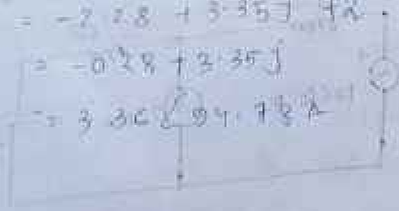
the current flowing 6Ω resistor

$9.72 = 11.056 \angle 124.72 + 2$

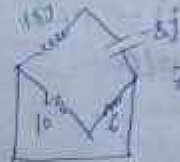
$= -2.28 + 3.35j + 2$

$= -0.28 + 3.35j$

$= 3.36 \angle 94.78 A$



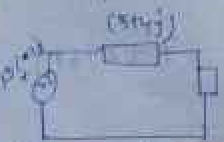
(d)



$Z_{th} = \frac{7j \times 16}{16 + 7j} = \frac{7 \angle 90 \times 16}{17.46 \angle 23.62} = \frac{112 \angle 90}{17.46 \angle 23.62}$

$= 6.414 \angle 66.38$

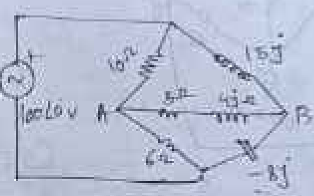
$= 2.56 + 5.87j$



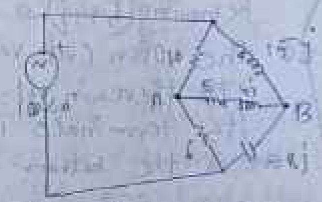
the current through $(5+4j)$ impedance $= \frac{151.78}{5 + 2.56 + 4j + 5.87j}$

$= \frac{151.78}{7.56 + 9.87j} = \frac{151.78}{12.43 \angle 52.54}$

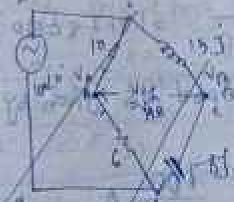
4) State Thevenin's theorem. Find the current in the $(5+4j)\Omega$ impedance connected between A and B in the ckt shown in Fig using Thevenin's theorem



a) done



⇓



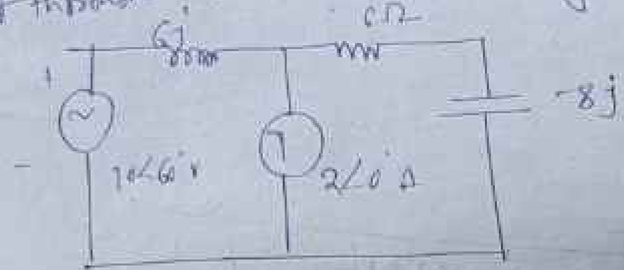
~~applying KVL in abAcda~~, $\Rightarrow$ $\cancel{\frac{V_A}{10} + \frac{V_A}{6} = 0}$

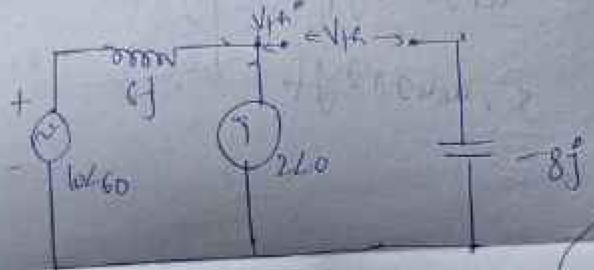
$\cancel{V_A\left(\frac{1}{10} + \frac{1}{6}\right)} = 100$

$= 11.92 \angle -52.54 A = 7.24 - 9.46j A$

current in the ... between A and B in the cir... in Fig using thevenin's theorem

curr through 6Ω resistors using thevenin's theorm





$$\frac{V_{th} - 10\angle 60}{6j} = 2\angle 0$$

$$\Rightarrow V_{th}\left(\frac{1}{6j}\right) = 2\angle 0 + \frac{10\angle 60}{6j}$$

$$\Rightarrow V_{th}\left(\frac{1}{6\angle 90}\right) = 2 + \frac{10\angle 60}{6\angle 90}$$

$$\Rightarrow V_{th} \times 0.167\angle -90 = 2 + 1.67\angle -30$$

$$\Rightarrow V_{th} \times 0.167\angle -90 = 2 + 1.45 - 0.835j$$

$$\Rightarrow V_{th} \times 0.167\angle -90 = 3.45 - 0.835j$$

$$\Rightarrow V_{th} = \frac{3.45 - 0.835j}{0.167\angle -90}$$

$$= \frac{3.54\angle -13.61}{0.167\angle -90}$$

$$= 21.19\angle 76.39$$

$$= 5 + 20.6j$$

$$Z_{th} = 6j - 8j$$

$$= -2j$$

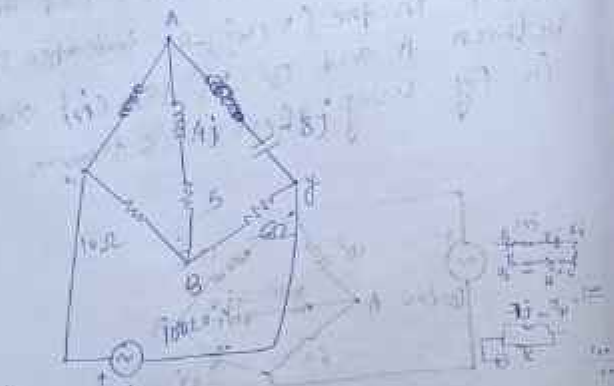
$$\Rightarrow I_L = \frac{V_{th}}{6 + Z_{th}}$$

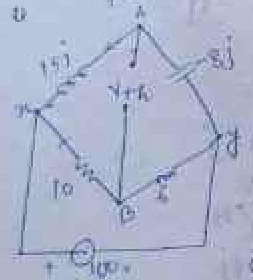
$$= \frac{21.19\angle 76.39}{6 - 2j}$$

$$= \frac{21.19\angle 76.39}{6.32\angle -18.43}$$

$$= 3.35\angle 94.82$$

$= 11.92\angle -52.54$ A $= 7.24 - 9.46j$ A





Removing $(5+4j)\,\Omega$ impedance the open cut voltage / the thevenin vol. across its terminals is found out in the network of fig.

The current through $15j$ load is obtained as

$$= \frac{100}{7j} = \frac{100}{7\angle 90^\circ} = 14.28\angle 90^\circ \text{ A}$$

vol. across ~~to~~ $15j$ load is given by $= \frac{100}{7j} \times 15j = \frac{1500}{7}\text{ V} = V_{AM}$

Current ...... $10\,\Omega$ ...... $= \frac{100}{16}$ A

vol. ...... $10\,\Omega$ ...... $= \frac{100}{16} \times 10 = \frac{1000}{16}\text{ V} = V_{BM}$

$\therefore V_{AB} = V_{AM} - V_{BM} = \frac{1500}{7} - \frac{1000}{16} = 151.78 = V_{th}$

P.T.O

5) a) Evaluate Laplace Transform of $\sin^3(t)$

$$\sin \omega t = \frac{1}{2j}\left[e^{j\omega t} - e^{-j\omega t}\right]$$

Here $\sin t$, $\omega = 1$

$$\therefore \sin t = \frac{1}{2j}\left(e^{jt} - e^{-jt}\right)$$

(or) $\sin^3 t = \frac{1}{3}(1 - \cos 3t) = \frac{1}{3}[u(t) - \cos 3t]$

$$\mathcal{L}[\sin^3 t] = \frac{1}{3}\left[\frac{1}{s} - \frac{s}{s^2+9}\right]$$

$\therefore$ the Laplace transformation of $\sin^3 t = \frac{1}{3}\left[\frac{1}{s} - \frac{s}{s^2+9}\right]$

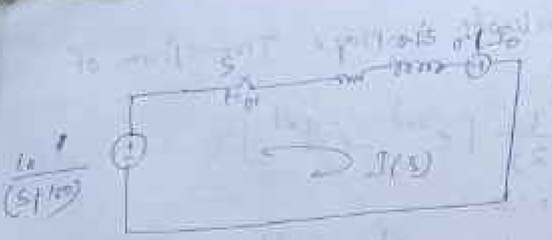
$\left[\because \cos\omega t = \frac{s}{s^2+\omega^2},\ \therefore \omega = 3\right]$

5) b)

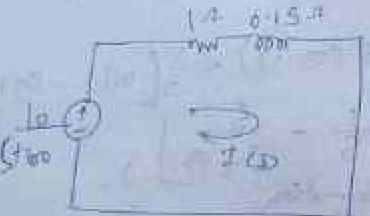
A series R-L circuit shown in fig experiences an exponential voltage $V = 10e^{-100t}$ V after closing the switch $t = 0$. Find the expression for current $i(t)$ using Laplace transform. Assume zero initial inductor current before switching.

$i_0 = 0$ (before switching at $t = 0^-$)

~~Before Swi~~ After switching at $t = 0^+$ transforming the ckt into Laplace domain —



$\frac{10}{(S+100)}$



$$I(s) = \frac{10}{(S+100)(1+0.15s)}$$

$$= \frac{10 \times 10}{(S+100)\left(\frac{1}{0.1}+S\right)}$$

$$= \frac{100}{(S+100)(10+S)}$$

$$= \frac{100}{(S+100)(S+10)}$$

$$\frac{100}{(S+100)(S+10)} = \frac{A}{S+100} + \frac{B}{S+10}$$

$$A = \frac{100 \times (S+100)}{(S+100)(S+10)}\Bigg|_{S=-100}$$

$$= \frac{100}{(-100+10)}$$

$$= \frac{100}{-90} = -1.11$$

$$B = \frac{100\,(S+10)}{(S+100)(S+10)}\Bigg|_{S=-10}$$

$$= \frac{100}{-10+100}$$

$$= \frac{100}{90}$$

$$= 1.11$$

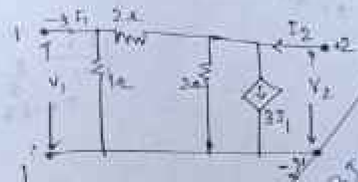
$$I(s) = \frac{100}{(S+100)(S+10)} = \frac{-1.11}{S+100} + \frac{1.11}{S+10}$$

Converting into time domain

$$i(t) = -1.11\, e^{-100t} u(t) + 1.11\, e^{-10t} u(t)$$

$$= -1.11\left[e^{-100t} - e^{-10t}\right] u(t)$$

6) Why do we need two port network? For the network shown in the fig. find Z-parameters.



a) This allows the response of the network to signals applied to signals at the ports to be calculated easily, without solving for all the internal voltages and currents in the network. It also allows similar circuits or devices to be compared easily.

shift + Home → whole line

shift + ↑ → select previous lines

unselect one ...

$V_1 = 2I_1 - 2I/3$

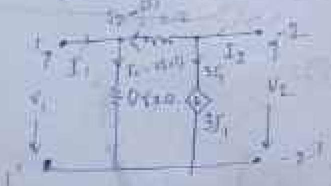
$2V_1 = 2 - I/3$

laplace

post-day

3 | 8/22

for the cct is,



defining eq's of z-Parameters,

$(V_1, V_2) = f(I_1, I_2)$

$V_1 = z_{11} I_1 + z_{12} I_2$ — (I)

$V_2 = z_{21} I_1 + z_{22} I_2$ — (II)

$V_1 = 0.67 (I_2 - 2 I_1)$ — (ii)

$V_1 = -1.34 I_1 + 0.67 I_2$ — (III)

$V_2 = 2 (I_2 - 3 I_1) + 0.67 (I_2 - 2 I_1)$

$= -6 I_1 - 1.34 I_1 + 2 I_2 + 0.67 I_2$

$V_2 = -7.34 I_1 + 2.67 I_2$ — (IV)

Compareing (I) and (III) we get,

$z_{11} = -1.34\,\Omega$, $z_{12} = 0.67\,\Omega$

Comparing (II) and (IV) we get,

$z_{21} = -7.34\,\Omega$, $z_{22} = 2.67\,\Omega$

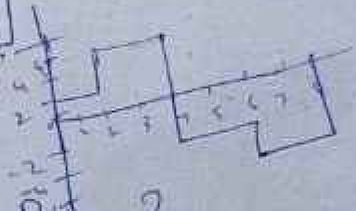
$$z = \begin{bmatrix} -1.34 & 0.67 \\ -7.34 & 2.67 \end{bmatrix}$$

superposition
Thevenin's theorem , done by s/m

$$\boxed{I = \frac{2(6+6j)}{6-6j+6j} = \frac{2\times6\angle0}{6} = 0+2j}$$

$$\frac{6(1+\frac{a}{c})}{s^2+2s+1+1}$$

$(s+1)^2$



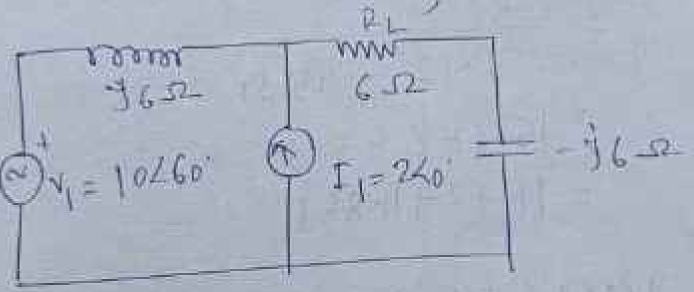
For only, $R_{th} = ?$
We can't short any dependent vol. source and remove any dependent current source. Otherwise we can do it.

EEN Pmr yr 9n solve ①

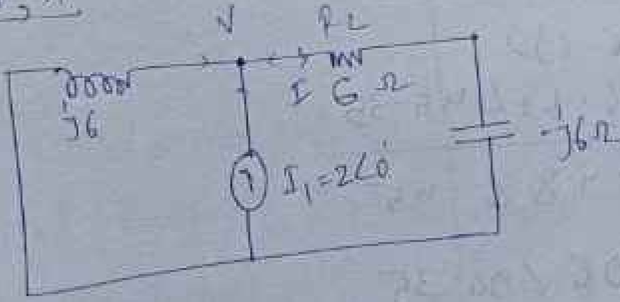
1) $\underline{\text{AnA}}$

Find the current in the resistor R_L using the Principle of superposition in Fig. below. [Superposition isn't in syllabus. I not done by sir]

By ~~thevenin's~~ superposition theorem



short, V_1



$$\frac{V}{6j} + \frac{V}{(6-j6)} = 2$$

$$\Rightarrow V\left[\frac{1}{0+6j} + \frac{1}{6-6j}\right] = 2$$

$$\Rightarrow V\left[\frac{1}{6\angle 90} + \frac{1}{8.48\angle -45}\right] = 2$$

(2)

$\Rightarrow V\left[0.167\angle -90 + 0.118\angle 45\right] = 2$

$\Rightarrow V = \dfrac{2}{0 - 0.167j + 0.083 + 0.083j}$

$= \dfrac{2}{0.083 - 0.084j}$

$= \dfrac{2}{0.12\angle -45.34}$

$= 16.67\angle 45.35$

$\Rightarrow V = 11.72 + 11.86j$

$I = \dfrac{16.67\angle 45.35}{(6-6j)}$

$= \dfrac{16.67\angle 45.35}{8.48\angle -45}$

$= 1.96\angle 90.35$

$= -0.01 + 2j$

∴ The current through $R_L = 1.96\angle 90.35$

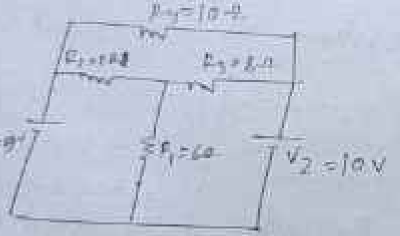
(3)

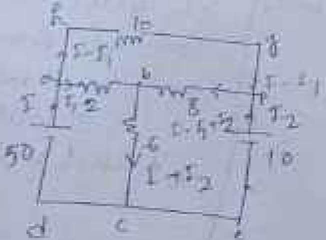
2(b) Distinguish between unilateral & bilateral circuit elements.

Unilateral network	Bilateral network
1) A ckt whose operation, behaviour is dependent on the direction of the current through various elements is called unilateral network.	1) A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called bilateral network.
2) ckt consisting diodes is good example of unilateral ckt	2) circuit consisting only resistance is good example of bilateral network.
3) The networks which have elements that allows the current to flow only in one direction.	3) the network consist of elements that allows the current can flow in either direction.

④

3) Determine current through R₂ using mesh analysis shown fig.





abcda → KVL,

$2I_1 + 6(I + I_2) = 50$

$\Rightarrow 2I_1 + 6I + 6I_2 = 50$

$\Rightarrow 3I + I_1 + 3I_2 = 25$ — ⑦

hgfbah → KVL,

$10(I - I_1) + 8(I - I_1 + I_2) - 2I_1 = 0$

$\Rightarrow 10I - 10I_1 + 8I - 8I_1 + 8I_2 - 2I_1 = 0$

$\Rightarrow 18I - 20I_1 + 8I_2 = 0$

⑤

$\Rightarrow 9I - 10I_1 + 4I_2 = 0$ — (ii)

bfecb → KVL,

$-8(I - I_1 + I_2) - 6(I + I_2) = -10$

$\Rightarrow 8(I - I_1 + I_2) + 6(I + I_2) = 10$

$\Rightarrow 4(I - I_1 + I_2) + 3(I + I_2) = 5$

$\Rightarrow 4I + 3I - 4I_1 + 4I_2 + 3I_2 = 5$

$\Rightarrow 7I - 4I_1 + 7I_2 = 5$ — (iii)

now from (iii)

$\Rightarrow 7I = 5 + 4I_1 - 7I_2$

$\Rightarrow I = \frac{1}{7}(5 + 4I_1 - 7I_2)$ — (iv)

$7I = 5 + 4I_1 - 7I_2$

$\Rightarrow I = \frac{1}{7}(5 + 4I_1 - 7I_2)$ — (iv)

from (ii) & (iv)

$\Rightarrow \frac{9}{7}(5 + 4I_1 - 7I_2) = 0$

from (ii) (iv)

$\Rightarrow \frac{9}{7}(5 + 4I_1 - 7I_2) - 10I_1 + 4I_2 = 0$

$\Rightarrow \frac{45}{7} + \frac{36}{7}I_1$

$\Rightarrow 45 + 36I_1 - 63I_2 - 70I_1 + 28I_2 = 0$

$\Rightarrow -34I_1 - 35I_2 + 45 = 0$

$\Rightarrow 35I_2 + 34I_1 - 45 = 0$

$\Rightarrow 35I_2 = 45 - 34I_1$

$\Rightarrow I_2 = \frac{1}{35}(45 - 34I_1)$ — (v) from ①, ⑫ we ②

$$\frac{3a}{7}\left\{5+4I_1-\frac{2}{35}(45-34I_1)\right\}+I_1+\frac{3}{35}\left\{45-34I_1\right\}=25$$

$$\Rightarrow \frac{3}{35}\left\{(5+4I_1)\times 5-45+34I_1\right\}+I_1+\frac{3}{35}(45-34I_1)=25$$

$$\Rightarrow \frac{3}{35}\left[25+20I_1-45+34I_1\right]+\frac{35I_1}{3}+(45-34I_1)=25$$

$$\Rightarrow \left[(-20+54I_1)+\frac{35I_1}{3}+45-34I_1\right]=\frac{25\times 35}{3}$$

$$\Rightarrow 3(54I_1-20)+35I_1+3(45-34I_1)=25\times 35$$

$$\Rightarrow 162I_1-60+35I_1+135-102I_1=875$$

$$\Rightarrow 95I_1=875+60-135=800$$

$$\Rightarrow I_1=8.42\,A$$

the current through $R_2 = 8.42A$.

⑦

…ers theorem

$$I=\begin{bmatrix} I \\ I_1 \\ I_2 \end{bmatrix},\quad A=\begin{bmatrix} 3 & 1 & 3 \\ 9 & -10 & 4 \\ 7 & -4 & 7 \end{bmatrix},\quad B=\begin{bmatrix} 25 \\ 0 \\ 5 \end{bmatrix}$$

$$AI=B$$

$$\Rightarrow I=A^{-1}B$$

$$|A|=\begin{vmatrix} 3 & 1 & 3 \\ 9 & -10 & 4 \\ 7 & -4 & 7 \end{vmatrix}$$

$$=3(-70+16)-1(63-28)+3(-36+70)$$

$$=-162-35+102$$

$$=-95$$

$$\text{adj } A=\begin{bmatrix} \begin{vmatrix} -10 & 4 \\ -4 & 7 \end{vmatrix} & -\begin{vmatrix} 9 & 4 \\ 7 & 7 \end{vmatrix} & \begin{vmatrix} 9 & -10 \\ 7 & -4 \end{vmatrix} \\ -\begin{vmatrix} 1 & 3 \\ -4 & 7 \end{vmatrix} & \begin{vmatrix} 3 & 3 \\ 7 & 7 \end{vmatrix} & -\begin{vmatrix} 3 & 1 \\ 7 & -4 \end{vmatrix} \\ \begin{vmatrix} 1 & 3 \\ -10 & 4 \end{vmatrix} & -\begin{vmatrix} 3 & 3 \\ 9 & 4 \end{vmatrix} & \begin{vmatrix} 3 & 1 \\ 9 & -10 \end{vmatrix} \end{bmatrix}^T$$

$$=\begin{bmatrix} (-70+16) & -(63-28) & (-36+70) \\ -(7+12) & (21-21) & -(-12-7) \\ (4+30) & -(12-27) & (-30-9) \end{bmatrix}^T$$

$$= \begin{bmatrix} -54 & -35 & 34 \\ -19 & 0 & 19 \\ 34 & 15 & -39 \end{bmatrix}^T$$

$$= \begin{bmatrix} -54 & -19 & 34 \\ -35 & 0 & 15 \\ 34 & 19 & -39 \end{bmatrix}$$

$$I = \frac{1}{-95} \begin{bmatrix} -54 & -19 & 34 \\ -35 & 0 & 15 \\ 34 & 19 & -39 \end{bmatrix} \times \begin{bmatrix} 25 \\ 0 \\ 5 \end{bmatrix}$$

$$= \frac{1}{95} \begin{bmatrix} 54 & 19 & -34 \\ 35 & 0 & -15 \\ -34 & -19 & 39 \end{bmatrix} \times \begin{bmatrix} 25 \\ 0 \\ 5 \end{bmatrix}$$

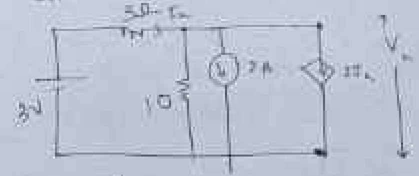
$$= \frac{1}{95} \begin{bmatrix} 1180 \\ 800 \\ -655 \end{bmatrix}$$

$$\therefore I_1 = \frac{800}{95} = 8.42A$$

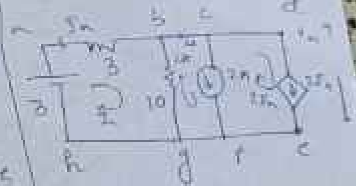
(7)

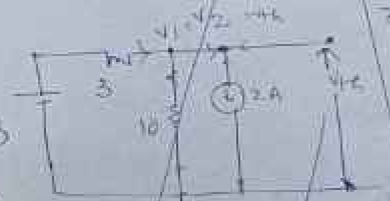
3) (a) State Thevenin's theorem - done

(b) Find V_x and I_x for the circuit shown in fig below.



By Thevenin's theorem





$$\frac{V_1 - 3}{3} + \frac{V_1}{10} = 0$$

$$\Rightarrow 10(V_1 - 3) + 3V_1 = 0$$

$$\Rightarrow 10V_1 + 3V_1 = 3$$

$$\Rightarrow V_1 = \frac{3}{13} = 0.23$$

$$V_1 = V_2 = V_{th} = 0.23$$

$$I_x = \frac{3 - 0.23}{3} = 0.92A$$

abgha, KVL

$$3I_x + 10(I_x - 2) = 3$$

$$\Rightarrow 13I_x - 20 = 3$$

$$\Rightarrow 13I_x = 23$$

$$\Rightarrow I_x = 1.76A$$

$$2I_x = 2 \times 1.76 = 3.53A$$

4) a) Find Laplace transform of $f(t) = e^{-3t}\cos 5t$

$$\mathcal{L}[f(t)] = \int_0^\infty e^{-st} \cos 5t \, e^{-3t} \, dt = F(S)$$

$$= \int_0^\infty e^{-(s+3)t} \cos 5t \, dt$$

Let the Laplace transformation of function of $f(t)$ is $\mathcal{L}[f(t)] = F(S)$

By using first shifting rule,

If $\mathcal{L}[f(t)] = F(S)$, then

$\mathcal{L}[e^{-at} f(t)] = F(s+a)$

(11)

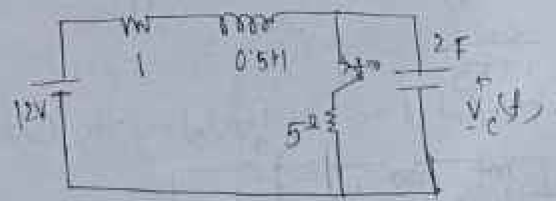
$$\mathcal{L}(\cos at) = \frac{s}{s^2+a^2}$$

$$\therefore \mathcal{L}(\cos 5t) = \frac{s}{s^2+5^2}$$

By applying the property of shifting,

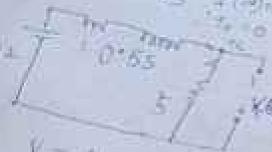
$$\mathcal{L}(e^{-3t}\cos 5t) = \frac{(s+3)}{(s+3)^2+5^2} = \frac{(s+3)}{(s^2+6s+34)}$$

b) Consider the circuit shown in fig below. The switch was closed for a very long time and at $t=0$, the switch is opened. Find the expression of $v_c(t)$ for $t>0$



before switching

as $I(t)=0$ (before switching)



$$\frac{V_c-12}{1+0.5S}+\frac{V_c}{5}=0$$

$$\Rightarrow \frac{V_c}{1+0.5S}+\frac{V_c}{5}=\frac{12}{1+0.5S}$$

$$\Rightarrow V_c\left[\frac{1}{1+0.5S}+\frac{1}{5}\right]=\frac{12}{1+0.5S}$$

$$\Rightarrow V_c\left[\frac{5+1+0.5S}{(1+0.5S)5}\right]=\frac{12}{1+0.5S}$$

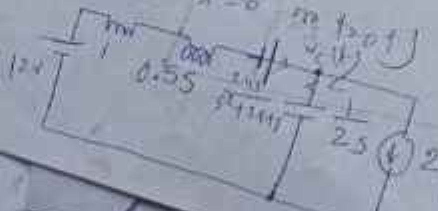
$$\Rightarrow V_c=\frac{60}{6+0.5S}$$

$$V_0=\frac{60}{6+0.5S}\text{ before switching}$$

$$=\frac{60}{6+0.5S}$$

V_0 (V_c initial)

after switching (at $t=0$) the ckt



$$\frac{V}{2}=\frac{12}{1+0.5S}$$

$$=\frac{24S}{S^2+2S+1}$$

(13)

Now,

$$\frac{V_c-12-\frac{24S}{S^2+2S+1}}{(1+0.5S)+\frac{24S}{S^2+2S+1}}+\frac{V_c}{\left(\frac{1}{2S}\right)}=\frac{-120}{6+0.5S}$$

$$\Rightarrow \frac{V_c}{(1+0.5S)}-\frac{12}{1+0.5S}-\frac{24S}{(S+1)^2\times(1+0.5S)}+V_c\times 2S=\frac{-12\times 20}{12(1+0.083S)}$$

$$\Rightarrow \frac{V_c}{\left(1+\frac{S}{2}\right)}-\frac{12}{\left(1+\frac{S}{2}\right)}-\frac{24}{(S+1)^2\left(1+\frac{S}{2}\right)}+V_c\times 2S=\frac{-20}{1+0.083S}$$

$$\Rightarrow V_c\left[\frac{2}{(2+S)}+2S\right]=\frac{-20}{1+0.083S}+\frac{12\times 2}{S+2}+\frac{24\times 2}{(S+1)^2(S+2)}$$

$$\Rightarrow V_c\left[\frac{2+4S+2S^2}{2+S}\right]=\frac{-20}{1+0.083S}+\frac{24}{S+2}\left[1+\frac{1}{(S+1)^2}\right]$$

$$\Rightarrow V_c\frac{2S^2+4S+2}{S+2}=\frac{-20}{1+0.083S}+\frac{24}{S+2}\left[\frac{S^2+2S+2}{(S+1)^2}\right]$$

$$\Rightarrow V_c(S)=\frac{-20}{(1+0.083S)}\times\frac{(S+2)}{2(S^2+2S+1)}+\frac{24}{S+2}\times\frac{(S^2+2S+2)\times(S+2)}{(S+1)^2\times 2(S+1)^2}$$

$$=\frac{-10(S+2)}{0.083(12+S)(S+1)^2}+\frac{12(S^2+2S+2)}{(S+1)^4}$$

$$=\frac{-120.4(S+2)}{(S+12)(S+1)^2}+\frac{12(S^2+2S+2)}{(S+1)^4}$$

$$= \frac{-120(s+2)}{(s+12)(s+1)^2} + \frac{12}{(s+1)^2}\left[1+\frac{1}{(s+1)^2}\right]$$

$$Y(s) = \frac{-120(s+2)}{(s+12)(s+1)^2} + \frac{12}{(s+1)^2} + \frac{12}{(s+1)^4}$$

Now

$$\frac{-120(s+2)}{(s+12)(s+1)^2} = \frac{A}{s+12} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2}$$

$$A = \left.\frac{-120(s+2)\cancel{(s+12)}}{\cancel{(s+12)}(s+1)(s+1)}\right|_{s=-12}$$

$$= \frac{-120\times(-12+2)}{-11\times(-11)}$$

$$= \frac{1200}{121} = 109.1$$

$$B = \left.\frac{-120(s+2)\cancel{(s+1)}\cancel{(s+1)}}{(s+12)\cancel{(s+1)}\cancel{(s+1)}}\right|_{s=-1}$$

$$= \frac{-120\times 1}{11}$$

$$= -10.91$$

(15)

$$C = \left.\frac{-(120)(s+2)\cancel{(s+1)^2}}{\cancel{(s+1)^2}(s+12)}\right|_{s=-1}$$

$$= \frac{-120\times 1}{11}$$

$$= -10.91$$

$$\therefore \frac{-120(s+2)}{(s+12)(s+1)^2} = \frac{109.1}{s+12} - \frac{10.91}{s+1} - \frac{10.91}{s+1}$$

$$= \frac{109.1}{s+12} - \frac{21.82}{s+1}$$

$$\cancel{\frac{12}{(s+1)^2} = \frac{A}{s+1} + \frac{B}{s+1}}$$

$$\cancel{A = \left.\frac{12(s+1)^2}{(s+1)^2}\right|_{s=-1} = 12 = B}$$

$$\cancel{\frac{12}{(s+1)^2}}$$

$$\frac{12}{(s+1)^2\times(s+1)^2} = \frac{A}{(s+1)^2} + \frac{B}{(s+1)^2}$$

$$A = B = \left.\frac{12\,\cancel{(s+1)^4}}{\cancel{(s+1)^2}(s+1)^2}\right|_{s=-1}$$

$$= 12$$

$$\therefore \frac{12}{(s+1)^4} = \frac{12}{(s+1)^2} + \frac{12}{(s+1)^2}$$

(16)

$$Y(s) = \frac{109.1}{s+12} - \frac{21.82}{s+1} + \frac{12}{(s+1)^2} + \frac{12}{(s+1)^2}$$

$$= \frac{109.1}{s+12} - \frac{21.82}{s+1} + \frac{36}{(s+1)^2}$$

$$y(t) = 109.1\, e^{-12t} u(t) - 21.82\, e^{-t} u(t) + 36\, t\, e^{-t} u(t)$$

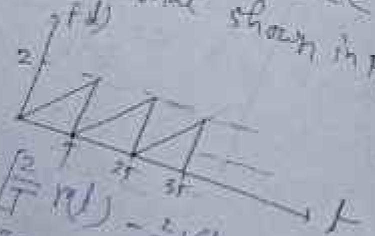
$$= [109.1\, e^{-12t} - 21.82\, e^{-t} + 36\, t\, e^{-t}]\, u(t)$$

Ans – $[109.1 e^{-12t} - 21.82 e^{-t} + 36 t e^{-t}]\, u(t)$

5) a) Prove that $\lim_{t \to \infty} f(t) = \lim_{s \to 0} s F(s)$

done

b) Find the Laplace transform of the signal shown in Fig. below.



slope $= \frac{2}{T}$

$$f(t) = \left[\frac{2}{T} r(t) - \frac{2}{T} r(t-T) - 2u(t-T)\right] + \left[\frac{2}{T} r(t-T) - \frac{2}{T} r(t-2T) - 2u(t-T)\right] + \left[\frac{2}{T} r(t-2T) - \frac{2}{T} r(t-2T) - 2u(t-2T)\right] - \frac{2}{T} r(t-3T) - 2u(t-3T)$$

(17)

$$f(t) = \frac{2}{T} r(t) - 2u(t-T) - 2u(t-2T) - 2u(t-3T) - \dots$$

$$= \frac{2}{T} t u(t) - 2u(t-T) - 2u(t-2T) - 2u(t-3T) \quad \left[\frac{2}{T} r(t-3T)\right.$$

will also cancle out by the next $+\frac{2}{T} r(t-3T)$

$$\Downarrow$$

$F(s)$

$$= \frac{2}{T}\frac{1}{s^2} - \frac{1}{s} - 2\left[e^{-Ts}\cdot\frac{1}{s} + e^{-2Ts}\cdot\frac{1}{s} + e^{-3Ts}\cdot\frac{1}{s}\right]$$

$$= \frac{2}{Ts^2} - \frac{2}{s}\left[e^{-Ts} + e^{-2Ts} + e^{-3Ts}\right]$$

$$= \frac{2}{Ts^2} - \frac{1}{s}\left(e^{-Ts} + e^{-2Ts} + e^{3Ts}\right)$$

6) (a) Find inverse Laplace transform of $F(s) = \frac{s+2}{(s+1)^2(s+3)}$

$$\frac{s+2}{(s+1)^2(s+3)} = F(s) = \frac{A}{(s+1)^2} + \frac{B}{(s+3)}$$

$$A = \left.\frac{(s+2)(s+1)^2}{(s+1)^2(s+3)}\right|_{s=-1} = \frac{1}{2}$$

$$B = \left.\frac{(s+2)(s+3)}{(s+1)^2(s+3)}\right|_{s=-3} = \frac{-1}{4}$$

$F(s) = \frac{(s+2)}{(s+1)^2(s+3)} = \frac{1}{2(s+1)^2} - \frac{1}{4(s+3)}$

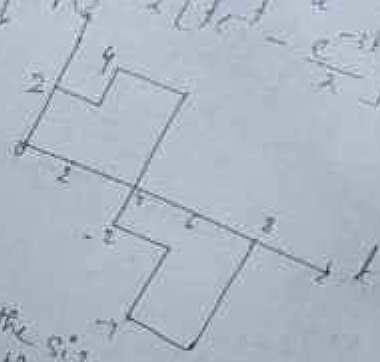
Inverse Laplace transform

$f(t) = \frac{1}{2} t e^{-t} u(t) - \frac{1}{4} e^{-3t} u(t)$

$= \frac{1}{2}\left[t e^{-t} - \frac{1}{2} e^{-3t}\right] u(t)$

$= \frac{1}{2}\left[t e^{-t} - \frac{e^{-3t}}{2}\right] u(t)$

b)



Consider the signal shown in fig. below. Represent the signal in terms of unit step function.

$f(t) = 2u(t) + 2u(t-2) - 6u(t-4) - 2u(t-6) + 6u(t-8)$

(15)

Part-B

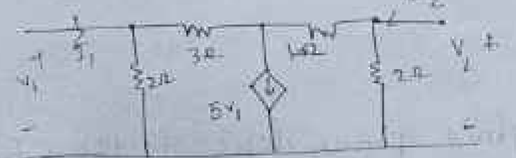
7) (a) Derive the condition for reciprocity and symmetry for T-parameters.

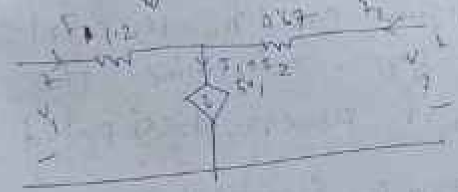
Done

b) Derive h-parameters in terms of Y parameters, and T parameters in terms of h-parameters.

Done

8) Find open ckt impedance for the given circuit.





$\frac{2 \cdot 3}{5} = \frac{6}{5} = 1.2$

$\frac{(2 \cdot 1)}{3} = \frac{2}{3} = 0.67$

$V_1 = z_{11} I_1 + z_{12} I_2$ — (1)

$V_2 = z_{21} I_1 + z_{22} I_2$ — (2)

$\frac{V_1}{I_1} = z_{11}, \frac{V_2}{I_1} = z_{21}$

$\frac{V_1}{I_2} = z_{12}, \frac{V_2}{I_2} = z_{22}$

$\underline{I_1 = 0},$

$\cancel{I_1 + I_2 = 5V_1}$

$\Rightarrow I_1 + I_2 = 5V_1$

$\Rightarrow I_2 = 5V_1$

$\Rightarrow \frac{V_1}{I_2} = \frac{1}{5} = 0.2 = z_{12}$

$V_2 = 0.67 I_2$

$\Rightarrow \frac{V_2}{I_2} = z_{22} = 0.67$

$z_{11} = 1.2$, $z_{12} = 0.2$, $z_{21} = 0$, $z_{22} = 0.67$

Q) (a) The hybrid parameters of the network 'N' shown in figure are: $h_{11} = 1\Omega$, $h_{12} = 1\Omega$, $h_{21} = -4$, $h_{22} = 5\mho$. Determine the supply voltage V_S, if power dissipated in the load resistor R_L ($= 8\Omega$) is 30W and $R_S = 2\Omega$.

$R_L = 8\Omega$, $P = 30W$, $R_S = 2\Omega$

$P = I_2^2 R_L$

$\Rightarrow 30 = (I_2)^2 \times 8$

$\Rightarrow I_2 = 1.936$

$V_1 = h_{11} I_1 + h_{12} V_2$ — (i)

$I_2 = h_{21} I_1 + h_{22} V_2$ — (ii)

$h_{11} = 1$, $h_{12} = 1$, $h_{21} = -4$, $h_{22} = 5$

(21)

Now,

$I_1 = I_1$

$V_1 = V_S$

$I_2 = 1.936$

$V_2 = 0$

~~we know that~~

from (i),

$\Rightarrow V_S = 1 \times I_1 + 1 \times 0$

$\Rightarrow V_S = I_1$

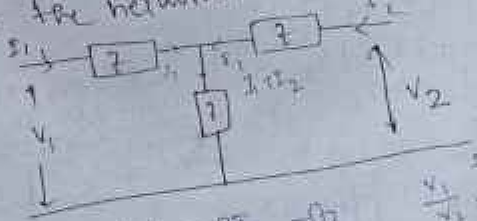
from (ii)

$\Rightarrow 1.936 = (-4) \times V_S + 5 \times 0$

$\Rightarrow \frac{1.936}{-4} = V_S$

$\Rightarrow V_S = -0.484\ V$

Q) Find the transmission parameters for the network.



$V_1 = AV_2 - BI_2$ — (i)

$I_1 = CV_2 - DI_2$ — (ii)

$I_2 = 0$: $\frac{V_1}{V_2} = A$, $\frac{I_1}{V_2} = C$

$V_2 = 0$: $-\frac{V_1}{I_2} = B$, $-\frac{I_1}{I_2} = D$

Case-1
$I_2 = 0$

$V_1 = ZI_1 + Z(I_1 + I_2)$
$= ZI_1 + ZI_1$
$\Rightarrow V_1 = 2ZI_1$ —(i)

$V_2 = 0 \times Z + Z(0 + I_1)$
$\Rightarrow \frac{V_2}{I_1} = Z$
$\Rightarrow \frac{I_1}{V_2} = \frac{1}{Z}$ —(ii)
$\Rightarrow C = \frac{1}{Z}$ —(iii)

from (i)
$I_1 = \frac{V_2}{Z}$
$\Rightarrow V_1 = 2Z \frac{V_2}{Z}$
$\Rightarrow \frac{V_1}{V_2} = 2$
$\Rightarrow A = 2$

Case-II
V_2 - short

$V_1 = ZI_1 + Z(I_1 + I_2)$
$= ZI_1 + ZI_1 + ZI_2$
$V_1 = 2ZI_1 + ZI_2$
$V_1 = ZI_1 - ZI_2$ —(i)
$\Rightarrow -\frac{V_1}{I_2} = Z$ —(ii)
$B = Z$

$\therefore A = 2,\ B = Z,\ C = \frac{1}{Z},\ D = 2$

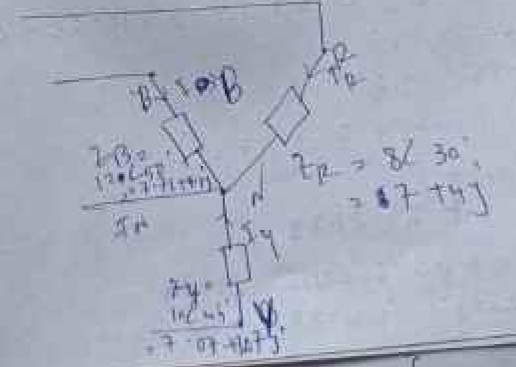
Same process by eqn solving

from (i)
$\Rightarrow -ZI_2 = 2ZI_1 + ZI_2$
$\Rightarrow -2ZI_2 = 2ZI_1$
$\Rightarrow \frac{-I_1}{I_2} = 1$
$\Rightarrow D = 1$

(10) A 3-phase, 4-wire system having a 230 V phase voltage has the following loads connected between the respective lines & neutral: $Z_R = 15\angle 30^\circ\ \Omega$, $Z_Y = 15\angle 45^\circ\ \Omega$ & $Z_B = 15\angle 60^\circ\ \Omega$. Calculate the current in the neutral wire and power taken by each load.

done

(11) A symmetrical 3-phase, 420 V system supplies a star connected load of $Z_R = 8\angle 30^\circ\ \Omega$, $Z_Y = 10\angle 45^\circ\ \Omega$, $Z_B = 12\angle 50^\circ\ \Omega$. Assuming the neutral of the supply is earthed, calculate the voltage of the star point to earth.



I_{BR}, I_{RY}, I_{YR} = phase current

w.r.t symmetrical line voltages

$V_{RY} = 420\angle 0$

$V_{YB} = 420\angle 120^\circ$

$V_{BR} = 420\angle -120$

Phase vol,

$V_{RN} = \frac{400 \angle 0}{\sqrt{3}}$, $V_{YN} = \frac{400 \angle -120}{\sqrt{3}}$, $V_{BN} = \frac{400 \angle 120}{\sqrt{3}}$

(24)

$I_R = \frac{V_{RN}}{Z_R} = \frac{400 \angle 0}{\sqrt{3} \times 8 \angle 30}$

$= 25.41 \angle -6$ A

$I_B = \frac{V_{BN}}{Z_B} = \frac{400 \angle -150}{\sqrt{3} \times 15 \angle 0}$

$= 15.74 \angle -20$

$I_Y = \frac{V_{YN}}{Z_Y} = \frac{400 \angle 90}{\sqrt{3} \times 10 \angle 45}$

$= 23.1 \angle 45$

$I_N = I_R + I_B + I_Y$

$= 14.33 - 29j$

$= 12.56 - 2.17j - 18.1 + 6.5j + 16.32 + 16.35j$

$= 12.74 \angle -9.8$

$R = 7.47 + 9.1j + 7.47j + 7.07 + 7.07j$

$= 21.78 + 20.17j$

$= 20 \angle 42.8$ and $R = 21.78$

$V_N = 12.74 \angle -9.8 \times 21.78$

$= 382 \angle 33$

$= 404.88 \angle -9.8$

(25)

) Take an arbitrary network of your choice (minimum 4 nodes). Draw oriented graph and select a tree. From there on express the following:

(i) Relation between incidence matrix and branch current matrix.

(ii) Relation between tie-set matrix and branch voltage matrix.

(iii) Relation among branch currents matrix, fundamental loop currents matrix and tie-set matrix.

(i), (ii) done

(iii)

Branch vol = Same

$I_1 \ldots I_6$

$l_1 = \{3, 2, 1\}$

$l_2 = \{3, 4, 1\}$

$l_3 = \{6, 4, 2\}$

at a, $I_3 - I_1 = 0$

l_1: $-I_1 + 0I_2$ $~~-I_3 + 0I_4 + 0I_5 + 0I_6~~ = 0$

at b, l_1: $-I_1 + I_2$

$-I_2 - I_1 = 0$ l_3: $V_6 - V_5 - V_2 = 0$

l_1: $I_3 - I_5 - I_4 = 0$

l_2: $-I_1 - I_5$

$$\Rightarrow \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = 0$$

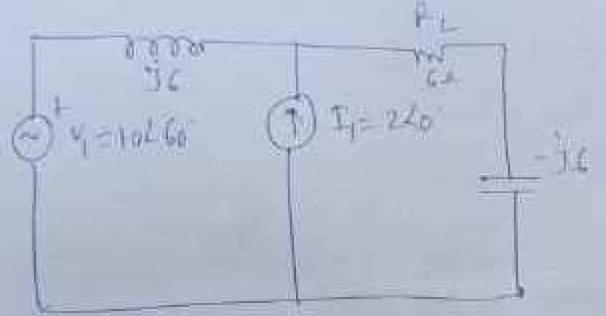
$$\Rightarrow [B] \times [I_b] = 0$$

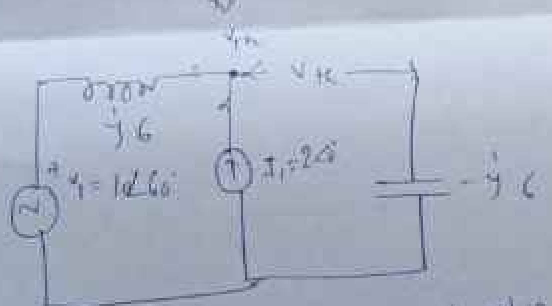
tieset - matrix × Branch current matrix = 0

∴ fundamental loop currents = branch currents ~~matrix~~.

(3)

b) thevenin's theorem.





$$\frac{V_{th} - 10\angle 60}{j6} = 2$$

$$\Rightarrow \frac{V_{th}}{j6} = 2 + \frac{10\angle 60}{0+6j}$$

$$\Rightarrow \frac{V_{th}}{6\angle 90} = 2 + \frac{10\angle 60}{6\angle 90}$$

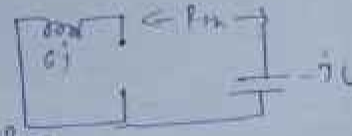
$$\Rightarrow \frac{V_{th}}{6\angle 90} = \frac{12\angle 90 + 10\angle 60}{6\angle 90}$$

$$\Rightarrow V_{th} = 0 + 12j + 5 + 8.66j$$

$$= 5 + 13.66j$$

$$= 14.55\angle 69.89$$

R_{th}



$R_{th} = 0$

4

$$I_{th} = \frac{14.55 \angle 69.89}{0 + 6}$$

$$= 2.4 \angle 69.89$$