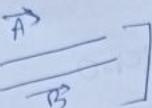


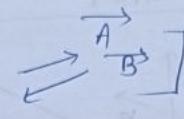
Physics (n main)

2/12/21

Some kind of vectors -

① Equal vectors $\Rightarrow |\vec{A}| = |\vec{B}|$ []
 (i) $\vec{A} = \vec{B}$

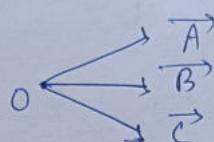
Same mag in same direction

② Negative Vectors $\Rightarrow |\vec{A}| = |\vec{B}|$ []
 (i) $\vec{A} = -\vec{B}$

Same mag in opposite direction

③ (Co-)Planar vectors \Rightarrow

$\vec{A}, \vec{B}, \vec{C}$



They will lie in a same line, then they will
 called co-planer vectors.

④ Unit vectors \Rightarrow

magnitude is unity (that means one)

$$\hat{n} \rightarrow \frac{\vec{P}}{|\vec{P}|}$$

If $\vec{B} = 3\hat{i} + \hat{j} + 2\hat{k}$, find the unit vector along \vec{B} .

$$\hat{n}_2 \quad \underline{3\hat{i} + \hat{j} + 2\hat{k}}$$

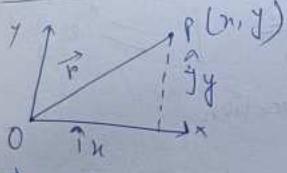
$$\sqrt{9+1+4}$$

$$\rightarrow \frac{1}{\sqrt{14}} (3\hat{i} + \hat{j} + 2\hat{k})$$

(5) zero vectors

mag = zero

(6) position vectors



$$\vec{OP} = \vec{r}$$

$$\vec{r}, i\text{-}h + j\text{-}h$$

$$\boxed{\vec{r} = i\text{-}h + j\text{-}h + k\text{-}h}$$

$$|\vec{r}| = \sqrt{i^2 + j^2 + k^2}$$

↓ This is position vector

The position vectors of two points ~~a and b~~ a and b are

$$\begin{aligned} \vec{a} &= 10\hat{i} - 12\hat{j} + 16\hat{k} \\ \vec{b} &= 4\hat{i} - 5\hat{j} + 12\hat{k} \end{aligned} \quad \text{calculate } |\vec{a}| + |\vec{b}|$$

$$|\vec{a}| = \sqrt{100 + 144 + 256} = \sqrt{500}$$

$$|\vec{b}| = \sqrt{16 + 25 + 144} = \sqrt{185}$$

~~$$|\vec{a}| + |\vec{b}| = \sqrt{500} + \sqrt{185}$$~~

$$|\vec{a} + \vec{b}| = |14\hat{i} - 17\hat{j} + 28\hat{k}|$$

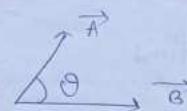
$$\begin{aligned} &\sqrt{196} \\ &= 35.62 \end{aligned}$$

$$\begin{pmatrix} 14 \\ -17 \\ 28 \end{pmatrix}$$

Product of vector

scalar product

$$\vec{A}, \vec{B}$$



$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$$

Characteristics (Scalar)

i) if $\theta = 90^\circ$, that is (i.e.) they are perpendicular

$$\therefore \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \frac{90}{2} = 0$$

They are called orthogonal vectors

ii) if they are parallel, then $\theta = 0$

$$\therefore \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 0$$

$$\begin{aligned} &\bullet \vec{AB} \\ &\text{then } \vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos 0 \\ &\Leftrightarrow \hat{i} \cdot \hat{i} = 1, \hat{j} \cdot \hat{j} = 1, \hat{k} \cdot \hat{k} = 1 \end{aligned}$$

$$\vec{A} = iA_x + jA_y + kA_z$$

$$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{A} \cdot \vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}) \cdot (B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$$

$$= A_x B_x + A_y B_y + A_z B_z$$

iii) distributive property

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\begin{matrix} \vec{A} \cdot \vec{B} \\ \downarrow \\ m \quad n \\ (m\vec{A} \cdot n\vec{B}) \end{matrix}$$

if we find
then the ans is $= mn(\vec{A} \cdot \vec{B})$

① If $\vec{A} = 2\hat{i} + 2\hat{j} - \hat{k}$
 $\vec{B} = 6\hat{i} + 3\hat{j} - 2\hat{k}$

Find the angle between the vectors

$$\begin{aligned} \vec{A} \cdot \vec{B} &= 12 + 6 + 2 \\ &= 20 \end{aligned}$$

$$20 = (2\hat{i} + 2\hat{j}) \cdot \sqrt{4+4+1} \cdot \sqrt{36+9+4} \cos \theta$$

$$\therefore \vec{A} \cdot \vec{B} = 18\sqrt{10} \cos \theta$$

$$\cos \theta = 1.0/6$$

$$\therefore 20 = 3 \cdot 7 \cos \theta$$

$$\therefore \cos \theta = \frac{20}{21}$$

② Show that the vectors,

$$\vec{A} = \hat{i} + 4\hat{j} + 3\hat{k}$$

$$\vec{B} = 4\hat{i} + 2\hat{j} - 4\hat{k}$$
 are mutually perpendicular

$$4+8-12 = \sqrt{1+16+9} \cdot \sqrt{16+4+16} \cdot \cos \theta$$

$$\therefore 0 = \cos \theta$$

$$\therefore \theta = 90^\circ \therefore \vec{A} \perp \vec{B}$$

• vector product

$$\vec{A} \times \vec{B} = AB \sin \theta \hat{n} \quad [\hat{n} \text{ unit vector}]$$

• Properties

- (i) if $\vec{A} \times \vec{B} = 0, \theta = 0^\circ$ [∴ Parallel]
- (ii) if $\theta = \pi$, then they (vec) are anti-parallel
- (iii) vector product of two equal vec = 0
 $\therefore \vec{R} \times \vec{R} = 0$

• distributive properties

$$\vec{A} \times (\vec{B} + \vec{C})$$

$$\Rightarrow \vec{A} \times \vec{B} + \vec{A} \times \vec{C} \quad (\text{Prove})$$

$$\vec{A} = A_1\hat{i} + A_2\hat{j} + A_3\hat{k}$$

$$\vec{B} = B_1\hat{i} + B_2\hat{j} + B_3\hat{k}$$

$$\vec{C} = C_1\hat{i} + C_2\hat{j} + C_3\hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Q force $\vec{F} = (5\hat{i} + 2\hat{j} + 3\hat{k}) N$ acting on a particle
 displaces it's from initial position $2\hat{i} + 5\hat{j} + 5\hat{k}$
 to it's new position $12\hat{i} + 15\hat{j} + 18\hat{k}$. How
 much work is done.

$$\begin{aligned} |\vec{s}| &= ((12\hat{i} - 2\hat{i}) + (15\hat{j} - 5\hat{j}) + (18\hat{k} - 5\hat{k})) \\ &\Rightarrow |\vec{s}| = 10\hat{i} + 10\hat{j} + 13\hat{k} \end{aligned}$$

$$\vec{w} = (5\hat{i} + 2\hat{j} + 3\hat{k}) \cdot (10\hat{i} + 10\hat{j} + 10\hat{k}) \quad [\vec{w} = \vec{P} \cdot \vec{Q}]$$

\Rightarrow LEO 79

- ② If two vectors are $6\hat{i} + 0.3\hat{j} - 5\hat{k}$ and $\hat{i} + 4.2\hat{j} - 2.5\hat{k}$. Find their vector product and by calculation prove that the new vector is perpendicular to the two.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 6 & 0.3 & -5 \\ 1 & 4.2 & -2.5 \end{vmatrix}$$

$$= (-0.15 + 2)\hat{i} - (-15 + 5)\hat{j} + (2.5 \cdot 2 - 0.3)\hat{k}$$

$$= 20.25\hat{i} + 10\hat{j} + 24.9\hat{k}$$

The vector product is $20.25\hat{i} + 10\hat{j} + 24.9\hat{k}$

$$\text{Let, } \vec{A} = 6\hat{i} + 0.3\hat{j} - 5\hat{k}, \vec{B} = \hat{i} + 4.2\hat{j} - 2.5\hat{k}$$

$$\vec{C} = 20.25\hat{i} + 10\hat{j} + 24.9\hat{k}$$

$\vec{A} \cdot \vec{C}$, $A \cos \theta$

$$\Rightarrow (12.15 + 3 - 124.5) = \sqrt{36 + 0.09 + 25} \cdot \sqrt{410 + 100 + 620}$$

$$\Rightarrow \cos \theta = 0 \therefore \theta = 90^\circ \therefore \vec{A} \text{ and } \vec{C} \text{ are perpendicular.}$$

$\vec{B} \cdot \vec{C}$, $B \cos \theta$

$$\Rightarrow (20.25 + 42 - 62.25) = \sqrt{1 + 17.64 + 625} \cdot \sqrt{410 + 100 + 620}$$

$$\Rightarrow \cos \theta = 0 \therefore \vec{B} \perp \vec{C} \perp \vec{B}$$

$\boxed{\text{CSE 214002}}$

- ③ Find the area of the parallelogram determined by the vectors $\vec{A} = 3\hat{i} + 2\hat{j}$ and $\vec{B} = 2\hat{j} - 4\hat{k}$.

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 2 & 0 \\ 0 & 2 & -4 \end{vmatrix}$$

$$\Rightarrow (-8 - 0)\hat{i} - (-12 - 0)\hat{j} + (6 - 0)\hat{k}$$

$$= -8\hat{i} + 12\hat{j} + 6\hat{k}$$

the area of parallelogram (समाप्तिक)

$$\begin{aligned} |\vec{A} \times \vec{B}| &= \sqrt{64 + 144 + 36} \\ &= \frac{15.62}{\sqrt{11}} \text{ unit}^2 \\ &\Rightarrow 7.81 \text{ unit}^2 \end{aligned}$$

• Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$$\text{④ Prove that } -\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$(i) \vec{a} \times (\vec{b} \times \vec{c}) + \vec{b} \times (\vec{c} \times \vec{a}) + \vec{c} \times (\vec{a} \times \vec{b}) = 0$$

$$\Rightarrow \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b}) + \vec{c}(\vec{b} \cdot \vec{a}) - \vec{a}(\vec{b} \cdot \vec{c}) +$$

$$\Rightarrow (\vec{a} \cdot \vec{b}) - \vec{b}(\vec{c} \cdot \vec{a})$$

$\Rightarrow 0$

• Triple Product

$$\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

$$\begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

• show that central force is conservative

$$\vec{F} = m\vec{f}(r)$$

$$\vec{r} \times \vec{F}(r) = 0$$

$$\vec{r} \times m\vec{f}(r) = 0$$

$$\vec{r} \times \vec{f}(r) = 0$$

$$\vec{r} \times \frac{d^2\vec{r}}{dt^2} = 0$$

∴

$$\begin{aligned} & \frac{d}{dt} \left(\vec{r} \times \frac{d\vec{r}}{dt} \right) \\ &= \left(\frac{d\vec{r}}{dt} \times \frac{d\vec{r}}{dt} \right) + \\ & \quad \vec{r} \times \frac{d^2\vec{r}}{dt^2} \\ &= 0 + \vec{r} \times \frac{d^2\vec{r}}{dt^2} \end{aligned}$$

$$\int \frac{d}{dt} \left(\vec{r} \times \frac{d\vec{r}}{dt} \right) = 0$$

$$\Rightarrow \frac{h \times d\vec{r}}{dt} = \text{constant}$$

$$\begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x)\frac{x}{h} & f(y)\frac{y}{h} & f(z)\frac{z}{h} \end{vmatrix}$$

$$= i \left(\frac{z}{h} \frac{\partial f}{\partial y} - \frac{y}{h} \frac{\partial f}{\partial z} \right)$$

$$= j \left(\frac{z}{h} \frac{\partial f}{\partial x} - \frac{x}{h} \frac{\partial f}{\partial z} \right) + k \left(\frac{y}{h} \frac{\partial f}{\partial x} - \frac{x}{h} \frac{\partial f}{\partial y} \right)$$

→ (1)

✓ Prove

$$\text{curl } \vec{F} = 0$$

$$\text{curl } \vec{f}(r) = 0$$

$$\vec{r} \times \vec{f}(r) = 0$$

$$\vec{r} \times \vec{f}(r) \cdot \vec{r} = 0$$

$$\vec{r} \times \frac{f(r) \vec{r}}{r} = 0$$

$$\vec{r} \times \frac{f(r)}{r} (x\hat{i} + y\hat{j} + z\hat{k}) = 0$$

$$(x\hat{i} + y\hat{j} + z\hat{k}) \cdot (x\hat{i} + y\hat{j} + z\hat{k}) = 0$$

$$(x^2 + y^2 + z^2) = 0$$

$$r^2 = x^2 + y^2 + z^2$$

$$\frac{\partial r}{\partial y} = \frac{\partial r}{\partial x} \cdot \frac{\partial x}{\partial y}$$

$$\frac{\partial r}{\partial y} = \frac{y}{h} \Rightarrow \frac{\partial r}{\partial x} \cdot \frac{\partial x}{\partial y} = \frac{y}{h}$$

$$\frac{\partial r}{\partial z} = \frac{z}{h} \cdot \frac{\partial r}{\partial x}$$

$$\frac{\partial r}{\partial x} = \frac{x}{h} \cdot \frac{\partial r}{\partial x}$$

from (1)

$$i \left[\frac{z}{h} \cdot \frac{\partial r}{\partial x} \cdot \frac{y}{h} - \frac{y}{h} \cdot \frac{\partial r}{\partial x} \cdot \frac{z}{h} \right] = i \times 0$$

✓ Proved,

$$j \times 0 = \vec{0}$$

$$k \times 0 = -j \times 0 + i \times 0$$

= 0 (Proved)

• Coefficient of viscosity - (definition)

$$f \propto \frac{dv}{dx}$$

$$\Rightarrow f = \eta \frac{dv}{dx} \quad [\eta = \text{dyn}] \quad [f = \text{per unit area}]$$

• viscosity η cgs unit - Poise

• dimension of η -

$$\cancel{\eta = \frac{f}{\frac{dv}{dx}}} \quad \eta = \frac{f}{\frac{dv}{dx}} \cdot \frac{F/A}{\frac{dv}{dx}}$$

$$= \frac{[T M^{-2} L^2]}{[L T^{-1} / V]}$$

$$= [m L^{-1} T^{-1} F]$$

• Reynolds numbers

$$V = \frac{K h}{\rho r}$$

critical
 V_c = velocity
 r = tube radius
 K = Reynolds number
 ρ = density of the liquid

derivation in part formula form

• Poiseuille's equation (পোয়েলিল সমীক্ষণ)

$$V = \frac{\pi (P_1 - P_2) a^4}{8 \eta l}$$

a = radius of the tube
 l = length of the tube
 P_1, P_2 = Pressures at the two ends of the tube

• equation of continuity

$\rho V_x S_y S_z$ and the mass flowing out through the opposite face

$$= \left\{ \rho V_x + \frac{\partial}{\partial x} (\rho V_x) S_x \right\} S_y S_z$$

Then mass flowing out per sec. parallel to the x -axis -

$$(\nabla \cdot \rho V) S_y S_z$$

• Bernoulli's Eqn (Not Prove, only equation)

$$\frac{P}{\rho g} + \frac{\rho V^2}{2g} + h = H \text{ (constant)}$$

where, $\frac{P}{\rho g}$ = pressure heads, $\frac{V^2}{2g}$ = kinetic energy
 h = gravitational head and H = total head

Elasticity

- definition of Stress and Strain
- Hook's Law
- Poisson's ratio (definition) ✓
- Poisson's ratio

$$\sigma = \frac{\delta / D}{l / L}$$

where,
 σ = Poisson's ratio
 δ = change (ক্রসেক্ষন) of radius
 D = initial diameter (কার্যকর ব্যাস)
 l = change of length
 L = initial length

• Hook's Law

Stress (ক্রসেক্ষন)

Strain (ক্রসফো) = Constant

that means, Stress \propto Strain

- Definition of Stress and Strain
- Coefficient of viscosity - The coefficient of viscosity is defined as the force of friction that is required to maintain a difference of velocity of 1cm/s between parallel layers of fluid.
- Poisson's ratio - The ratio of the proportional decrease in a lateral measurement to the proportional increase in length in a sample of material that is elastically stretched.
- (ii) Within the elastic limit, the ratio of lateral strain to the longitudinal strain is called Poisson's ratio.
- What is central force - In classical mechanics, a central force on an object is a force that is direct towards or away from a point called center of force.
- Nuclear force: $A^{14} \rightarrow$ Atomic No.
↓
Neutron No.
- Isotope \rightarrow Same Proton no.
- Isotope \rightarrow Same neutron no.
- Isobar \rightarrow Same nuclear mass
(definition)
(difference between them), (problem)
- Binding energy: amount of energy required to separate a particle from a system
- Successive Disintegration: Particles of the system.

If a parent radioactive nucleus A has number of atoms No at time t_0 . After disintegration it converts a nucleus B which is further radioactive. Then such type of disintegration is known as successive disint.

Bernoulli's equation and conservation of energy

$$\frac{1}{2} v^2 + \frac{P}{\rho} + U = \text{constant}$$

This is Bernoulli's equation, which expresses the principle of conservation of mechanical energy, since $\frac{1}{2} v^2$ is the K.E per unit mass, $U = gh$ is the gravitational P.E. per unit mass, a height h above the datum level, and $(U + \frac{P}{\rho})$ is the P.E. function per unit mass.

For stationary flow $\frac{\partial U}{\partial t} = 0$ and,

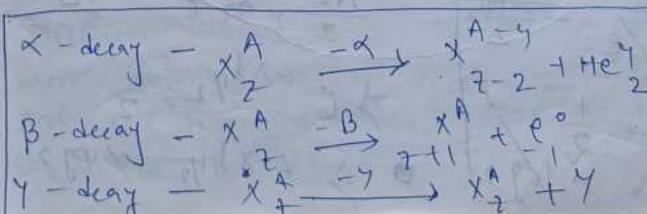
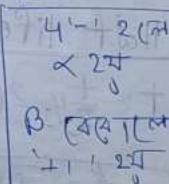
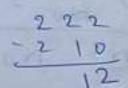
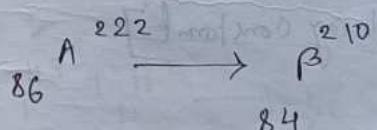
$$\frac{1}{2} \text{grad } v^2 - |\vec{v} \cdot \text{curl } \vec{v}| = \vec{F} - \frac{1}{\rho} \text{grad } p$$

not necessary

Radius of nucleus

$$R = R_0 A^{\frac{1}{3}}$$

$R_0 = 1.2 \times 10^{-15}$ m



• Radioactive decay constant -

$$\frac{dN}{dT} \propto N$$

$\Rightarrow \frac{dN}{dt} = -\lambda N$

[\therefore decay $2T_{1/2}^{(50)} = 6.05 \times 10^3$ sec]

[$N = \text{Number of atoms}$]

$$\frac{dn}{dt} = -\lambda N$$

$$\Rightarrow \frac{dn}{dt} = -\lambda dt$$

$$\begin{array}{ccc} t=0 & \longrightarrow & N>N_0 \\ t=t & \longrightarrow & N=N \end{array}$$

$$\int_{x_0}^x \frac{dx}{x} = -\ln|x| + C$$

$$\Rightarrow [\log^n]_{n=0}^{\infty} = -\lambda [t]_{0}^t$$

$$\Rightarrow \log\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\rightarrow \frac{N}{N_0} = e^{-\lambda t}$$

• Half life

$$t = \frac{1}{2} \ln 2 C^m \quad N = \frac{N_0}{2}$$

$$\frac{V_0}{2} = \frac{-\Delta t}{e^{\frac{-\Delta t}{2}} - 1}$$

$$N^0 \rightarrow e^- \bar{\nu}_e$$

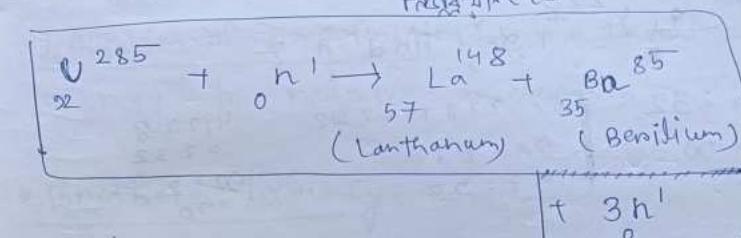
$$\Rightarrow e^{-\lambda t} \parallel_2 = \frac{1}{2}$$

$$\Rightarrow f(x+1/2) = f^{\log 2}$$

$$\rightarrow \lambda t_{1/2} = \log 2$$

$$t_{1/2} = \frac{0.693}{\lambda}$$

Nuclear fission (def) [नियन्त्रित प्रक्रिया से बड़ी ऊर्जा उत्पन्न करना]



四

$\frac{1}{2} \Delta E_{\text{f}} =$ (nuclear fission),

nuclear fission, subdivision of a heavy atomic nucleus.

[वृंदा बत्ते फूल]

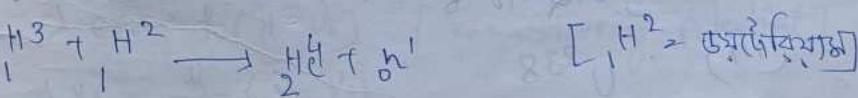
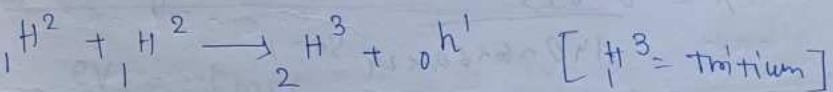
22 V²⁸⁵ (3)

hit এবং

9 ए neutron

$$276AT - 276 = 276$$

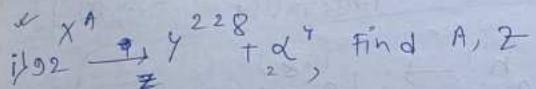
nuclear fusion [नियुक्टीय संश्लेषण]



3.7 MeV energy release 2×10^2 sats,

Nuclear fusion is a reaction in which two or more atomic nuclei are combined to form one or more different atomic nuclei and subatomic particles.

- uses of radioactive isotopes - carbon dating
- medical diagnosis, research involving biology and genetics.

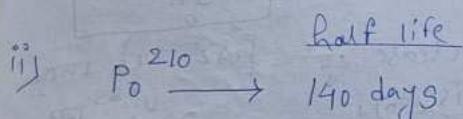


$$A = 232$$

$$Z = 90$$

$$\begin{aligned} A &= 228 + 4 = 232 \\ 92 &= Z + 2 \\ 37 &= Z \end{aligned}$$

$$\begin{aligned} 4 + 228 &= 232 \\ 92 - 2 &= 90 \\ 37 &= Z \end{aligned}$$



find the decay constant

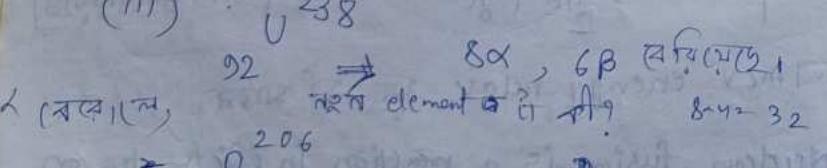
$$210 =$$

$$\frac{0.693}{\lambda}$$

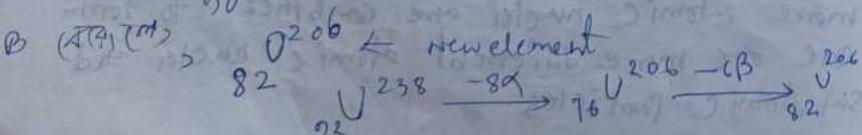
$$\lambda = 0.673$$

$$140 = 6.60 \times 24^{-1}$$

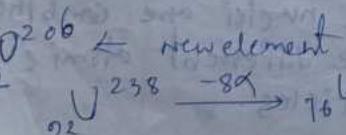
$$= 5.875 \times 10^{-8} \text{ s}^{-1}$$



$$\approx 0^{206}$$



$$82 \rightarrow 80$$



- photoelectric effect [definition] not so imp
for all

$$E_{\max} = \frac{1}{2} mv_{\max}^2$$

$$= h\nu - W$$

-① [W = work function]

definition of work function — min amount of energy to release an electron.

- threshold frequency

$$h\nu_0 = W \quad [\nu_0 = \text{threshold frequency}]$$

from ①,

$$E_{\max} = h\nu = h\nu_0$$

$$E_{\max} = h(\nu - \nu_0)$$

- stopping potential / cutoff potential

$$eV_0 = E_{\max}$$

$$\Rightarrow eV_0 = h(\nu - \nu_0) \quad [\nu_0 = \text{stopping potential}]$$

$$\nu_0 = \frac{h}{e} (\nu - V_0)$$

is the work function of a metal is 3.45 eV.
what is the maximum wavelength of a photon
that can eject an electron from the metal?

$$w = 3.45 \text{ eV}$$

$$\begin{aligned} & h(v - v_0) \\ &= h\left(\frac{c}{\lambda} - \frac{c_0}{\lambda}\right) \\ E_{max} &= hv - w \\ &= \frac{hc}{\lambda} - w \end{aligned}$$

$$hv = w$$

$$\frac{hc}{\lambda} = 3.45$$

$$\begin{aligned} \lambda &= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{3.45 \times 1.6 \times 10^{-19}} \\ &= 5.6 \times 10^{-7} \text{ m} \end{aligned}$$

(ii) A metal of work function 3 eV is illuminated by light of wavelength 3000 Å. calculate the threshold frequency and the E_{max} .

$$\begin{aligned} & hv_0 = w \\ & \frac{hc}{\lambda} = w \\ & \lambda = 3000 \text{ Å} = 3000 \times 10^{-8} \text{ cm} \\ & \quad = 3000 \times 10^{-10} \text{ m} \\ & \Rightarrow 6.626 \times 10^{-34} \times 3000 \times 10^{-10} \end{aligned}$$

$$E_{max} = \frac{hv}{h} \cdot h$$

$$\begin{aligned} \Rightarrow v_0 &= \frac{w}{h} \\ &= 3 \times 1.6 \times 10^{-19} \end{aligned}$$

$$6.626 \times 10^{-34}$$

$$= 4.82 \times 10^{14} \text{ Hz}$$

$$\begin{aligned} E_{max} &= \frac{6.626 \times 10^{-34} \left(\frac{3 \times 10^8}{3000 \times 10^{-10}} - 7.2 \times 10^{-19} \right)}{h} \\ &= -1.84 \frac{hc}{\lambda} - w \left[\frac{v - c_0}{\lambda} \right] \\ &= \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{3000 \times 10^{-10}} \quad w = 6.626 \times 10^{-34} \text{ eV} \\ &= 1.826 \times 10^{17} - 3 \times 10^{-10} \end{aligned}$$

(ii) Find the frequency of the light which ejects from a metal surface electrons fully stopped by a retarding potential 3 volt. The photoelectric effect begins in this metal atom frequency of $6 \times 10^{14} \text{ sec}^{-1}$. Find the work function for this metal.

$$v_0 (\text{stopping potential}) = 3 \text{ V}$$

$$v_0 (\text{threshold frequency}) = 6 \times 10^{14}$$

$$\begin{aligned} w &= hv_0 \\ &= 6.626 \times 10^{-34} \times 6 \times 10^{14} \\ &= 3.97 \times 10^{-19} \text{ J} \end{aligned}$$

$$h = \frac{c}{\lambda}$$

frequency
c = hλ

We know,

$$eV_0 = h\nu - W \quad [\nu = \text{frequency of the light}]$$

W = work function

V_0 = Stopping Potential

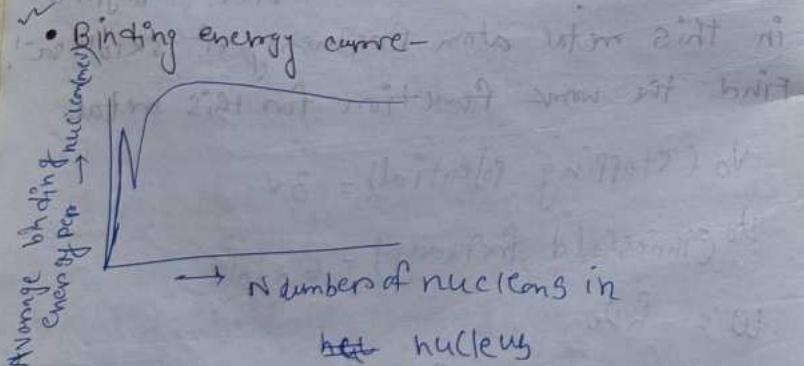
e = electronic charge

$$\frac{eV_0 + W}{h} = \nu$$

$$\frac{eV_0 + W}{h} = \frac{1.6 \times 10^{-19} \times 3 + 4 \times 10^{-19}}{6.626 \times 10^{-34}}$$

$$= 1.328 \times 10^{15} \text{ Hz}$$

- Photoelectric effect - It is a phenomenon in which electrically charged particles are released from or within a material when it absorbs electromagnetic radiation.



• De-broglie •

- De-broglie hypothesis - De Broglie proposed that $P = \frac{h}{\lambda}$ relation applies to the material particles as well as the photons.

$$E = h\nu$$

$$P^2 = \frac{E}{c} = \frac{h\nu}{c}$$

$$P = \frac{h\nu}{c}$$

$$\lambda = \frac{h}{P}$$

$$\lambda = \frac{h}{mv} \rightarrow [\text{For a particle of mass } m \text{ moving with speed } v]$$

$$\lambda = \frac{h}{P} \quad [\lambda = \text{de Broglie wavelength}]$$

$$P = \frac{h \text{ (cut)}}{\text{ (not temp)}} K \rightarrow [K = \text{propagation vector}]$$

$$[h \text{ (cut)} = h \text{ (single)}]$$

$$K = \frac{2\pi}{\lambda}$$

$$\begin{aligned} & \text{Basic relations of quantum theory} \\ & \therefore P = hK \\ & E = h\nu \end{aligned}$$

$$E = \frac{P^2}{2m} \rightarrow [E = \text{kinetic energy}]$$

$$\lambda = \sqrt{2mE}$$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

$$\begin{aligned} & m = \text{mass} \\ & \lambda = \text{wavelength} \\ & E = e^- \text{ charge} \\ & V = \text{potential difference} \end{aligned}$$

• Heisenberg theory (Statement)

$$\Delta x \Delta p \geq h \text{ (cut)} \rightarrow \text{Heisenberg relation for position and momentum}$$

$$\Delta x \Delta p \geq h$$

$$E = \frac{p^2}{2m}$$

$$AE = \frac{2PAP}{2m} = \frac{PAP}{m} \\ \Rightarrow V \cdot AP [P = mV]$$

$$\Delta V = \frac{\Delta n}{A t}$$

$$\Rightarrow \Delta t = \frac{\Delta K}{\Delta V}$$

$$\Delta E \times \Delta t = \underline{y \cdot AP \cdot AK}$$

$$\Delta E A t = \underline{A P A K} \geq h^{\text{cut}}$$

Hizenburg → relation for position

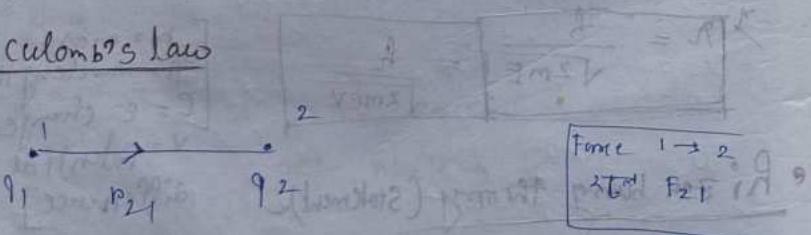
and time.

- Statement - The position and the velocity of an object cannot both be measured exactly, at the same time.
 - Electrostatics

↳ Deals with Static charge

9

- Coulomb's Law



$$\vec{r}_{21} = |\vec{r}_{21}| \hat{r}$$

$$\frac{\vec{F}}{r_{21}^2} \propto \frac{\vec{r}_{12}}{r_{21}^2} \quad \text{--- (1)} \quad \text{By Combining (1) and (1)}$$

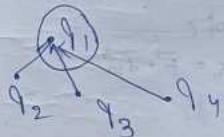
$$\Rightarrow \boxed{P_{2|1} = \hat{r}_{21} \frac{q_1 q_2}{r_{21}^2} \cdot K} \quad [Proportional \text{ Newton 7}]$$

$$V = \frac{1}{4\pi \epsilon_0} \quad [\epsilon_0 = \text{Permittivity constant}]$$

$$\vec{r}_{21} = -\vec{r}_{12}$$

$$\vec{F}_{12} = -k \cdot \frac{q_1 q_2}{r_{12}^2} \cdot \hat{r}_{12}$$

• Principle of Superposition



$$F_i = \frac{\sum_{j=2}^n q_i q_j}{r_{ij}} \hat{r}_{ij}$$

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• Electrostatic Potential

Workdone by an external agency in bringing a unit positive charge from infinity to that point.

$$\text{Point 1: } \oint_{\infty}^b \mathbf{E} \cdot d\mathbf{r}$$

[agency] $\xrightarrow{\text{কোর্টে}}$ field $\xrightarrow{\text{কোর্টে}}$
 গোচারি ক্ষেত্র গোচারি
 পোড়া বন্দু গোচারি
 মেঘ গোচারি sing]

• Electrostatic field

$$\vec{E} = \frac{q}{4\pi\epsilon_0 r^2} \hat{r}$$

relation between E.f and potential

$$\vec{E} = -\nabla \phi$$

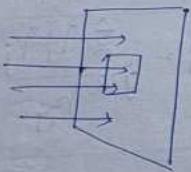
$$\nabla = \frac{\partial}{\partial x} \hat{i} + \frac{\partial}{\partial y} \hat{j} + \frac{\partial}{\partial z} \hat{k}$$

$$\vec{E} = \frac{\partial}{\partial r} \hat{r}$$

∇ = gradient

• Flux

$$\text{Flux} = \iint_S \vec{E} \cdot d\vec{A}$$



$$\text{electric flux} = \vec{E} \cdot \vec{dA}$$

For closed surface

If the surface is close,

$$\text{Flux} = \iint_S \vec{E} \cdot d\vec{A}$$

• Gauss theorem

$$\iint_S \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$



$$\iint_S \vec{E} \cdot d\vec{s} = E \cdot 4\pi r^2$$

$$E \cdot 4\pi r^2 = \frac{q}{\epsilon_0}$$

$$\therefore E = \frac{q}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow \phi = \int \frac{q}{4\pi\epsilon_0 r^2} dr$$

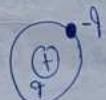
$$= -\frac{q}{4\pi\epsilon_0 r} + C_1$$

$$\text{at } r=0, \phi=0$$

$$C_1 = 0$$

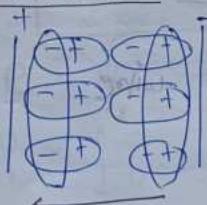
$$q = -\frac{\phi}{4\pi\epsilon_0 r}$$

$$\bullet \text{Dipole} \rightarrow +q - -q$$



dielectric material - water

• Polarisation



$$\vec{P} \propto \chi \vec{E}$$

$$\vec{P} = \epsilon_0 \chi \vec{E}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad [\vec{D} = \text{Displacement Current}]$$

Field of Polarisation

Susceptibility

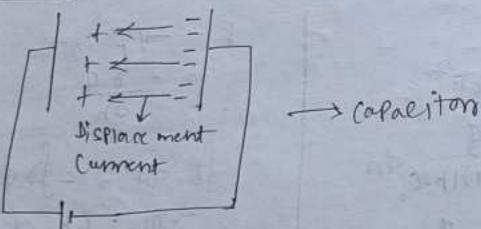
$$\vec{D} = \epsilon_0 \vec{E} + \epsilon_0 \chi \vec{E}$$

$$= (\epsilon_0 + \epsilon_0 \chi) \vec{E}$$

$$\boxed{\vec{D} = \epsilon_0 \vec{E} (1 + \chi)} \quad [\epsilon_0 (1 + \chi) = \epsilon]$$

(displacement quantity)

Capacitor



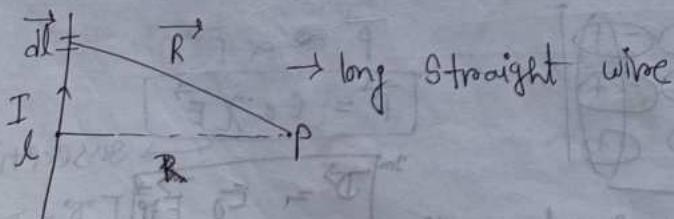
$$C = \frac{\epsilon A}{d} \rightarrow \square \rightarrow \text{Rectangular capacitors}$$

$$C = \frac{2\pi\epsilon}{\ln b/a} \rightarrow \text{Cylindrical capacitors}$$

- magnetic effect
- Bio's Savart law

$$dB = \frac{\mu_0}{4\pi} I \frac{dl \times R}{R^3}$$

[where
 μ_0 = Permeability of free space
 dB = magnetic field at a point]



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

Amperes circuital Law

• magnetic moment = current \times area

$$M = IA$$

Faraday's Law

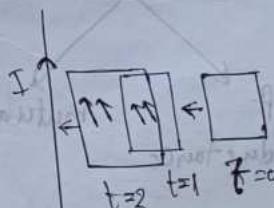
• Induced emf in a cut is \propto to the rate of change of magnetic flux linked with the cut

• The direction of induced emf is such that it means to oppose the cause of its generation.

$$e = -\frac{d\phi}{dt}$$

$$e = \oint E \cdot dl \quad \text{--- (1)}$$

$$\phi = \int_S B \cdot dS \quad \text{--- (2)}$$



$$\text{So } \oint E \cdot dl = - \frac{d}{dt} \int B \cdot ds$$

- mmmm ϕ = flux linked with coil

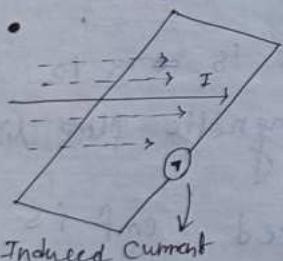
$$\phi = LI \rightarrow$$

$$e = - \frac{d\phi}{dt} = - \frac{d}{dt} (LI) \quad [\text{from } ①]$$

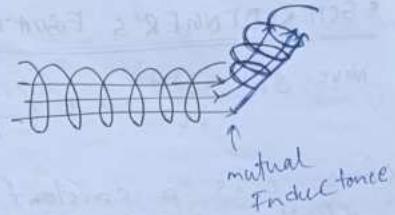
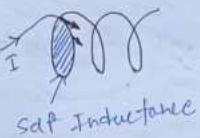
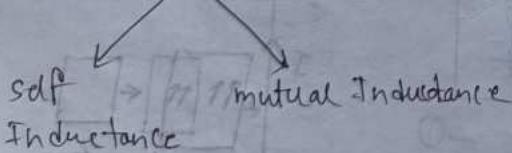
$$e = -L \frac{dI}{dt}$$

$L = \text{Self inductance}$

$$\textcircled{m} e = - \frac{d\phi}{dt}$$



Inductance



suggestion

- 1) coulomb's law \rightarrow statement, mathematical form
- 2) electric field, electrostatic potential (V) \rightarrow definition, (E)

Relation between E and V , math related it.

$$\text{Ex } \vec{E} = -\nabla V \quad V \text{ given}$$

$x^2 + y^2 \rightarrow \text{Find } E$

- 3) Gauss's law \rightarrow statement, mathematical form, math related it.

$$\text{Imp } E, V$$

(The previous math by Amman (pgy))

- 4) Definition of dipole and dipole moment

- 5) Parallel Plate capacitor, cylindrical capacitor \rightarrow formula, math related it.

- 6) Bio Savart Law \rightarrow statement, mathematical form

- 7) Faraday's Law \rightarrow statement

- 8) self inductance \rightarrow about it

- 9) mutual inductance

- 10) Maths of this madam's class

SCHRODINGER'S EQUATION (N man)

$$\text{Wave eqn } \frac{\partial^2 \psi}{\partial t^2} = c^2 \left(\frac{\partial^2 \psi}{\partial x^2} \right)$$

where c is a constant with the dimensions of velocity. This is a differential equation.

In wave mechanics, we introduce a new wave function (ψ), which at every point in space and time can be written as $\psi(x, y, z, t)$

Properties

(i) ψ , is general, is not real but a complex function, i.e. it has a real part and an imaginary part.

The square of ψ is interpreted as the probability (per unit volume) that the particle will be found at that point

$$\boxed{\int \psi^2 dv = 1}$$

(ii) ψ is not a measure of any simple physical quantity such as displacement.

(iii) ψ is continuous to in space.

In the Schrödinger eq

Consider a particle of momentum p and energy E .

Its wavelength is $\lambda = \frac{h}{p}$ and frequency $\nu = \frac{E}{h}$

$$\text{wave number } K = \frac{2\pi}{\lambda} = \frac{p}{h} \quad \left[\frac{h}{2\pi} = k \right]$$

$$\text{angular frequency } \omega = 2\pi\nu = \frac{E}{h}$$

$$\text{For a free particle } E = \frac{p^2}{2m} \Rightarrow \frac{K^2 h^2}{2m}$$

$$\Rightarrow K^2 = \frac{2mE}{h^2}$$

$$\Rightarrow K = \frac{\sqrt{2mE}}{h}$$

$$[h = \text{Planck's const.} = \frac{h}{2\pi}]$$

Now if the particle travels in x -dirn then the wave will also travel in the same dirn the harmonic wave meeting this cond'n is-

$$\psi(x, t) = e^{-i(Et - Kx)} \text{ in the } x\text{-dirn}$$

$$\text{and } \psi(x, t) = e^{-i(Et + Kx)} \quad \text{in -ve } x\text{-dirn}$$

$$\psi \rightarrow e^{-i(Et - Kx)} = e^{-i\left(\frac{Et}{h} - \frac{x}{h}\sqrt{2mE}\right)} \quad \text{(2)}$$

Differentiate eqn (2) once with respect to time.

$$\frac{\partial \psi}{\partial t} \rightarrow -i\frac{E}{h}\psi \quad \text{(3)}$$

differentiate eqn ③ twice w.r.t x^2

$$\frac{\partial \psi}{\partial x} = i \sqrt{\frac{2mE}{\hbar}} \psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2mE}{\hbar^2} \psi \quad \text{--- (4)}$$

Comparing eqn ③ and ④

$$\left[\frac{-\hbar^2}{2m} \cdot \frac{\partial^2}{\partial x^2} \psi(x,t) = i\hbar \frac{\partial}{\partial t} \psi(x,t) \right]$$

↓
The time dependent Schrodinger eqn
in one dimension.

If the particle is restricted to
presence of a force represented by
some given potential energy function $V(x,t)$
then -

$$i \frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m} \cdot \frac{\partial^2 \psi}{\partial x^2} - V(x,t) \psi$$

• Energy and momentum operator

Consider a particle of mass m moving in the
x direction. The potential energy of the
particle is V , momentum is p and total energy
is E . For the free particle wave eqn is
 $\psi = Ae^{-iEt - kx} = A e^{-i(Et - px)} \quad \text{--- (1)}$

differentiate partially w.r.t t

$$\frac{\partial \psi}{\partial t} = -\frac{iE}{\hbar} \psi$$

$$\Rightarrow E\psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}; \Rightarrow E\psi = \frac{i^2 \hbar^2}{\hbar} \frac{\partial \psi}{\partial t}$$

$$\Rightarrow E\psi = i\hbar \frac{\partial \psi}{\partial t} \quad \text{--- (2) [Eqn ② energy operator]}$$

Similarly partially differentiating ① w.r.t x

$$\frac{\partial \psi}{\partial x} = +\frac{ip}{\hbar} \psi$$

$$\Rightarrow p\psi = \frac{\hbar}{i} \frac{\partial \psi}{\partial x}; \Rightarrow p\psi = \frac{-1 \cdot \hbar}{-i} \frac{\partial \psi}{\partial x}; \Rightarrow p\psi = \frac{i^2 \hbar \psi}{-1}$$

$$\Rightarrow p\psi = -i\hbar \frac{\partial \psi}{\partial x} \quad \text{--- (3)}$$

• Time independent Schrodinger's eqn

According to classical mechanics, total energy
of the particle is

$$\begin{aligned} E &= K.E + P.E \\ &= \frac{1}{2}mv^2 + V \\ &= \frac{p^2}{2m} + V \end{aligned}$$

multiplying both sides by ψ

$$E\psi = \left(\frac{p^2}{2m} + V\right)\psi \quad | \quad p\psi = -i\hbar \frac{\partial\psi}{\partial x}$$

$$E\psi = \frac{-\hbar^2}{2m} \cdot \frac{\partial^2\psi}{\partial x^2} + V\psi \quad | \quad \frac{\partial^2\psi}{\partial x^2} = \frac{i^2 p^2}{\hbar^2} \psi$$

$$\frac{\hbar^2}{2m} \cdot \frac{\partial^2\psi}{\partial x^2} = V\psi - E\psi \quad | \quad \frac{\partial^2\psi}{\partial x^2} = \frac{-p^2}{\hbar^2} \psi$$

$$\frac{\hbar^2}{2m} \frac{\partial^2\psi}{\partial x^2} + (E-V)\psi = 0 \quad | \quad -\hbar^2 \frac{\partial^2\psi}{\partial x^2} = p^2\psi$$

$$\boxed{\frac{\partial^2\psi}{\partial x^2} + \frac{2m}{\hbar^2} (E-V)\psi = 0}$$

↓
time independent Schrodinger eqn

for 1D particle.

$$\boxed{\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} \right) + \frac{2m}{\hbar^2} (E-V)\psi = 0}$$

↓
This is for 3D particle

Particle in a box

The walls of the box are assumed to be perfectly elastic, and rigid and the particle moves freely between these walls



If the particle has energy E and momentum $p = \sqrt{2mE}$

then the harmonic waves travelling to the right and left along the x-axis are respectively $= e^{-iC} \frac{Et}{\hbar} - \frac{n}{\hbar} \sqrt{2mE}$

and $e^{-iC} \frac{Et}{\hbar} + \frac{n}{\hbar} \sqrt{2mE}$

From superposition principle

$$\psi(x,t) = Ae^{-iC} \frac{Et}{\hbar} - \frac{n}{\hbar} \sqrt{2mE} + Be^{-iC} \frac{Et}{\hbar} + \frac{n}{\hbar} \sqrt{2mE} \quad \text{--- (1)}$$

A, B are constants,

At $x=0$ and $x=L$, ψ must be zero at all times

$$\begin{aligned} \text{Thus } \psi(0,t) &= 0 \text{ at } x=0 \quad \text{--- (2)} \\ \psi(L,t) &= 0 \text{ at } x=L \quad \text{--- (3)} \end{aligned} \quad \text{Boundary cond'}$$

These two are the boundary cond'.

From eqn (2) and (3), we obtain from eqn (1)

$$Ae^{-iEt/\hbar} + Be^{-iEt/\hbar} = 0 \text{ at } x=0 \quad \text{--- (4)}$$

$$\text{and } Ae^{-i(Et/\hbar - \frac{L}{\hbar} \sqrt{2mE})} + Be^{-i(Et/\hbar + \frac{L}{\hbar} \sqrt{2mE})} = 0$$

$$\text{at, } x=L \quad \text{--- (5)}$$

From eqn (4) we obtain $B = -A$
and then eqn (5) reduces to

$$Ae^{-iEt/\hbar} \left(e^{\frac{iL}{\hbar} \sqrt{2mE}} - e^{-\frac{iL}{\hbar} \sqrt{2mE}} \right) = 0$$

this equation is equal to

$$2iA \sin \left(\frac{L}{\hbar} \sqrt{2mE} \right) = 0$$

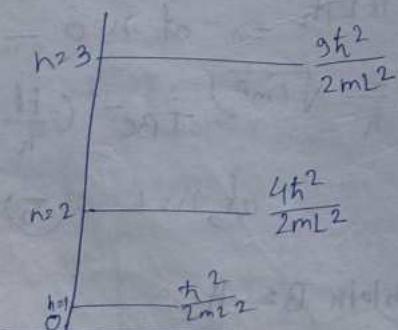
$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}$$

this demands that $(\sin \theta = 0 \text{ if } \theta = 0, \pi, 2\pi \text{ sin function be an integer multiple of } \pi)$

$$\frac{L}{\hbar} \sqrt{2mE} = n\pi$$

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2mL^2} \quad (n=1, 2, 3)$$

eqn ⑥ shows that the energies of the particle in a box are quantized. The lowest energy is obtained by putting $n=1$, called the energy of the ground state. $E_1 = \frac{\pi^2 \hbar^2}{2mL^2}$



∴ since we have $B = -A$ eqn ⑤ can be rewritten as-

$$\begin{aligned} \psi(x,t) &= Ae^{-i\frac{Et}{\hbar}} \left[e^{-\frac{x}{\hbar} \sqrt{2mE}} - e^{-\frac{x}{\hbar} \sqrt{2mE}} \right] \\ &= A e^{-i\frac{Et}{\hbar}} + e^{\frac{ix}{\hbar} \sqrt{2mE}} - Ae^{-i\frac{Et}{\hbar}} e^{-\frac{ix}{\hbar} \sqrt{2mE}} \\ &= A e^{-i\frac{Et}{\hbar}} \left[e^{\frac{ix}{\hbar} \sqrt{2mE}} - e^{-\frac{ix}{\hbar} \sqrt{2mE}} \right] \\ &= Ae^{-\frac{i n^2 \pi^2 \hbar^2 t}{2mL^2}} \left[e^{\frac{ix\sqrt{2mE}}{L^2}} - e^{-\frac{ix\sqrt{2mE}}{L^2}} \right] \\ &= Ae^{-\frac{i n^2 \pi^2 \hbar^2 t}{2mL^2}} \left[e^{\frac{ix\cdot n\pi}{L}} - e^{-ix\cdot \frac{n\pi}{L}} \right] \\ &= Ae^{-\frac{i n^2 \pi^2 \hbar^2 t}{2mL^2}} \sin \left(\frac{n\pi x}{L} \right) \cdot 2i \end{aligned}$$

$$\boxed{\psi(x,t) = 2iAe^{-\frac{i n^2 \pi^2 \hbar^2 t}{2mL^2}} \sin \left(\frac{n\pi x}{L} \right)}$$

contains a time-dependent part and space dependent part.

If we consider only the spatial part

$$\psi(x) = A \sin \frac{n\pi x}{L}$$

From normalization condition

$$\int_0^L \psi^2(x) dx = 1$$

Integration over the box

$$|A|^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1$$

wave function for a particle

$$|A|^2 \cdot \frac{L}{2} = 1$$

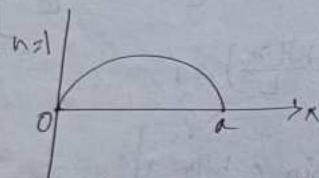
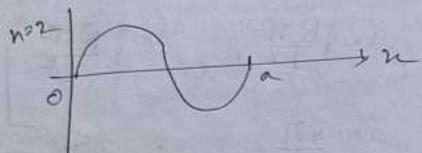
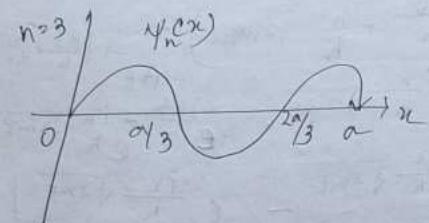
expression for a particle

$$A = \sqrt{\frac{2}{L}}$$

in the box

$$\therefore \boxed{\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}}$$

$$\psi(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)$$



① A proton is confined in an infinite square well of width 10 fm. calculate the energy of the photon emitted when the proton undergoes a transition from the first excited state ($n=2$) to the ground state ($n=1$)

$$E_n = \frac{n^2 \hbar^2}{2mL^2}$$

$$E = E_2 - E_1 = \frac{4\hbar^2}{2mL^2} - \frac{\hbar^2}{2mL^2} \\ = \frac{3\hbar^2}{2mL^2}$$

Q2 A particle with mass m is in an infinite square well potential with the walls at $x = -L/2$ and $x = L/2$. write down the wave functions for the states $n=1, n=2$ and $n=3$

$$\text{at } n=1 \quad \frac{L}{2}$$

the wave functions are,
for $n=1$

$$\psi\left(-\frac{L}{2}\right) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{-1 \cdot \pi \cdot k}{2 \cdot L}\right) \\ = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{\pi}{2}\right)$$

$$\text{for } n=2 \\ \psi\left(-\frac{L}{2}\right) = \sqrt{\frac{2}{L}} \cdot \sin\left(-\frac{2 \cdot \pi \cdot k}{2 \cdot L}\right) \\ = 0$$

$$\text{for } n=3$$

$$\psi\left(-\frac{L}{2}\right) = \sqrt{\frac{2}{L}} \cdot \sin\left(-\frac{3 \cdot \pi \cdot k}{2 \cdot L}\right) \\ = \sqrt{\frac{2}{L}}$$

$$\text{at } n=2 \quad \frac{L}{2}$$

the wave functions are,

$$\text{for } n=2$$

$$\psi\left(\frac{L}{2}\right) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{1 \cdot \pi \cdot k}{2 \cdot L}\right), \quad \psi\left(\frac{L}{2}\right) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{2 \cdot \pi \cdot k}{2 \cdot L}\right) \\ = 0$$

$$\text{for } n=3$$

$$\psi\left(\frac{L}{2}\right) = \sqrt{\frac{2}{L}} \cdot \sin\left(\frac{3 \cdot \pi \cdot k}{2 \cdot L}\right) = \sqrt{\frac{2}{L}}$$

Kinetic energy (A man)

Assumptions of the kinetic theory

- 1) A gas consists of a large number of identical molecules which are like minute hard elastic spheres constantly moving about in all possible directions with different velocities in a random fashion.
- 2) During the motion, the molecules collide with one another and also with the walls of the container, the collisions being perfectly elastic, in other words, there is no loss of K.E. when the collision occurs. As the chance of collision in all directions is the same, the collisions do not affect the molecular density.
- 3) The collisions are essentially instantaneous, that is, the duration of a collision is insignificant compared to the time between collisions.
- 4) The collisions are esmolecules exert no forces (attraction or repulsion) on one another except when they actually collide; that is, between two successive collisions they move in straight lines with uniform.
- 5) Since the molecules are like geometrical mass-points, the actual volume occupied by them is negligible.

compared to the total volume of the gas (that is, of the containing)

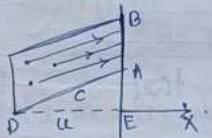
Basic eqn for kinetic energy

Perfect gas eqn

any velocity c of a particle in space can be resolved into components u, v, w along the three axes such that

the limit of u, v and w are from $-\infty$ to ∞ , those of c are from 0 to ∞ .

Now, u suffers a change in direction on reflection from the wall BAE, the other two components v and w remain unchanged by this type of reflection. The change in momentum per reflection of a molecule is $mu - (-mu) = 2mu$.



If n_u be the number of molecules per unit volume moving with velocity u , the number striking unit area of the wall in time dt would be contained in a cylinder of cross-sectional area unity and a vertical height $u dt$.

- volume of the cylinder $= u dt$
- No. of molecules in the cylinder $= n_u dt$
- change in momentum/area suffered by the above molecules in time dt is $-2mu \times n_u dt$
- The above quantity for all the molecules moving in the positive x -direction, that is, total change in momentum per unit area is $2m \sum_{u=0}^{\infty} n_u u^2 dt$

If the above change in momentum results in average force \overrightarrow{SF} , then $\overrightarrow{SF} dt = 2m \sum_{u=0}^{\infty} n_u u^2 dt \Rightarrow \overrightarrow{SF} = 2m \sum_{u=0}^{\infty} n_u u^2$

Since the area involved is unity, \overrightarrow{SF} is the pressure

$$P. \text{ Therefore, } P = 2m \sum_{u=0}^{\infty} n_u u^2$$

where u_i is function of x . So that
Let \bar{u}^2 be the mean square velocity along x , so that

$$\text{we may write, } \bar{u}^2 = \frac{n_1 u_1^2 + n_2 u_2^2 + \dots + n_m u_m^2}{n_1 + n_2 + \dots + n_m} = \frac{\sum n_i u_i^2}{\sum n_i}$$

The factor $\frac{1}{2}$ arises due to the fact that only the molecules in the positive x direction are being considered.

$$\therefore \sum n_i u_i^2 = \frac{1}{2} n \bar{u}^2, \text{ where } n = \text{total no. of molecules per unit volume.}$$

$$\therefore P_x = \frac{2 m_1}{2 n \bar{u}^2} = m \bar{u}^2, \text{ similarly, } P_y = m \bar{u}^2 \text{ and } P_z = m \bar{u}^2$$

The expression for the pressure is

$$P = P_x = P_y = P_z = m \bar{u}^2 = m \bar{v}^2 = m v^2$$

But $\bar{u}^2 = \bar{v}^2 = \bar{w}^2 = \frac{1}{3} \bar{c}^2$ where \bar{c}^2 is the mean square velocity of the molecules. $\therefore P = \frac{1}{3} m n \bar{c}^2 = \frac{1}{3} m n c^2$

where $c = (\bar{c}^2)$ is p.m.s velocity of the molecules. The above expression may also be written as $P = \frac{1}{3} \rho c^2$ where $\rho = m/n$ = density of gas.

• Boyle's law - From the pressure equation

$$\text{we have } P = \frac{1}{3} \rho c^2$$

If m' be the mass of a gas that occupies a volume V , $\rho = m'/V$

$$P = \frac{1}{3} \frac{m'}{V} c^2$$

$$\Rightarrow P V = \frac{1}{3} m' c^2$$

At constant temperature T , c^2 is a constant

$$\therefore P V = \text{constant}$$

which is Boyle's law. This formula is [Pg-31] called the eqn of an Isotherm. So graphs of P vs V for different T called Isotherms, are rectangular hyperbolae.

• Clapeyron's equation

We have Pressure

$$P = \frac{1}{3} m n c^2 = \frac{1}{3} \frac{n}{N_A} \cdot m N_A c^2$$

Assumptions,
deduction of
perfect gas
etc.
Pressure &
Boyle's law
charles law/gray
Clapeyron's eqn [Pg-31]

$$= \frac{1}{3} \frac{n}{N_A} \cdot M c^2 \quad (N_A = \text{Avogadro no.})$$

$$\therefore M = m N_A = \text{mol. weight}$$

$$\therefore P = \frac{1}{3} \frac{n}{N_A} \cdot M c^2 = \frac{n}{N_A} P V = \frac{n}{N_A} R T$$

$$(\because M = m N_A = \text{mol. weight} \text{ and } P V = \frac{1}{3} M c^2 = R T)$$

[Avogadro No. N_A ; Boltzmann's constant - values Pg-64]

$$\therefore R / N_A = K = \text{Boltzmann constant}$$

which is Clapeyron's eqn, an useful expression.

• Gay-Lussac law (Charles' law)

$$P V \propto T; \text{ but } C \propto T$$

$\therefore P V \propto T \Rightarrow V/T = \text{constant}$, at constant P . This equation, called an eqn of an Isober, expresses the well-known Gay-Lussac law (or Charles' law). The graph of the Isober is plainly a straight line emerging from the origin.

$$\rho = N \left(\frac{m}{2 \pi k T} \right)^{3/2} e^{-mc^2/2kT} c^3 dc \quad [2.5.2]$$

(Concept of Probability)

\uparrow

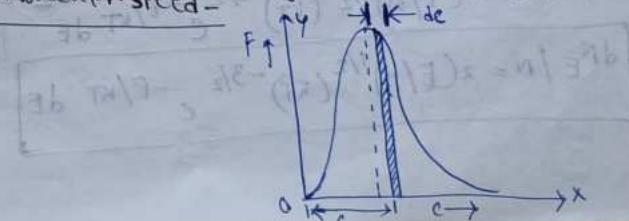
N = Total no. of molecules
 m = mass of single molecules
 k = Boltzmann Constant
 T = Temperature in Kelvin]

$$dN_c = \rho \cdot 4 \pi c^2 \cdot dc$$

$$dN_c = 4 \pi N \left(\frac{m}{2 \pi k T} \right)^{3/2} e^{-mc^2/2kT} c^3 dc$$

Gaussian distribution / Maxwell's distribution law
velocity on Speed

• Plot of the function F against C : Distribution curve of molecular speed - [where $dN_c = \text{no. of molecules}$]



• Ratio of the three speeds.

$$\bar{c} : c : c_m = \sqrt{\frac{8KT}{\pi m}} : \sqrt{\frac{3KT}{m}} : \sqrt{\frac{2KT}{m}}$$

$$= \sqrt{8/\pi} : \sqrt{3} : \sqrt{2}$$

\bar{c} = average velocity

c = r.m.s velocity

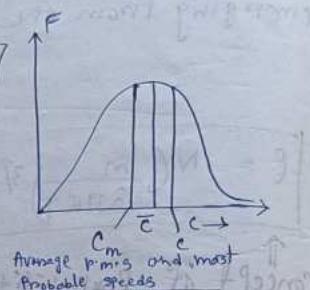
c_m = most probable velocities

$$\text{So, } \frac{\bar{c}}{c} = \sqrt{\frac{8}{3\pi}} = 0.921$$

$$\frac{c_m}{c} = \sqrt{\frac{2}{3}} = 0.817$$

Thus, the ratio of the speeds in the ascending order is

$$c_m : \bar{c} : c = 1 : 1.128 : 1.224$$



• The distribution of Kinetic energy

$$dN_c / N \propto \frac{4\pi}{2\pi KT} \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mc^2/2KT} c^2 dc$$

$$dN_E / N \propto \frac{4\pi}{2\pi KT} \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-E/KT} \cdot \frac{2E}{m} dE$$

$$= 2N \left(\frac{E}{\pi}\right)^{1/2} (KT)^{-3/2} e^{-E/KT} dE$$

$$\boxed{dN_E / N = 2(E/\pi)^{1/2} (KT)^{-3/2} e^{-E/KT} dE}$$

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2-11-2-6

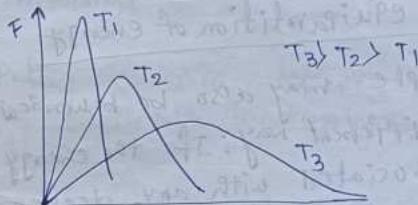
2-11-3

• momentum distribution

$$dn_p = 4\pi N (2\pi m KT)^{-3/2} e^{-mp^2/2KTm^2} p dp$$

• velocity distribution: temperature dependence and reduced form

2-12-1



Broadening of Maxwell speed distribution curve with temperature

Temperature dependence - If we integrate the expression for the fraction of molecules having a specified velocity between c and $c+dc$, i.e.

$$dN_c = 4\pi \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mc^2/2KT} c^2 dc$$

w.r.t. c for all the possible values of it, we should get 1.

$$\therefore \int_0^\infty 4\pi \left(\frac{m}{2\pi KT}\right)^{3/2} e^{-mc^2/2KT} c^2 dc = 1$$

This implies that the integral $\int_0^\infty F dc = 1$. This means that the area under the distribution curve of molecular velocity i.e. F vs c plot is unity.

105-217

- Degrees of freedom of a dynamical system

The no. of independent coordinates necessary for specifying the position and configuration in space of a dynamical system.

- Principle of equipartition of energy (classical)

From the principle may also be derived in a slightly different way: If the energy of a system associated with any degree of freedom is a quadratic function of the variable specifying the degree of freedom (that is proportional to the square of coordinate or component of a velocity), then in a state of thermal equilibrium of the system at a temperature T , the mean value of the corresponding energy equals $\frac{1}{2} kT$.

Ex - Monoatomic gas - For a gas whose molecules are monoatomic and for which the energy is wholly kinetic energy of translation, the degrees of freedom $f = 3$

$$C_V = \frac{1}{2} f k = \frac{3}{2} R$$

$$\gamma = 1 + \frac{2}{f} = 1 + \frac{2}{3} = \frac{5}{3} = 1.67$$

Diatom gas - Number of degrees of freedom $f = 3N - m = 3 \times 2 - 1 = 5$
the molecule has two rotational degrees

of freedom.

$$\therefore C_V = \frac{1}{2} f R = \frac{5}{2} R = 2.5 R$$

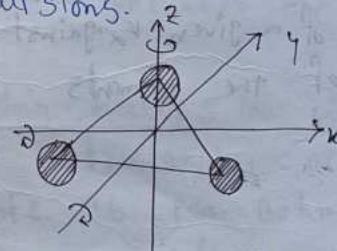
$$\gamma = 1 + \frac{2}{5} = \frac{7}{5} = 1.40$$

For a diatomic atom degrees of freedom is 5.

" " triatomic " " " " "

Free Path - distance between the two successive collisions traversed by the molecule.

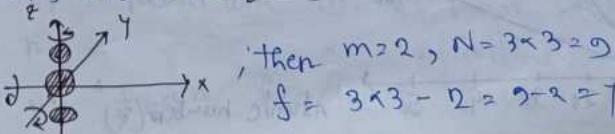
Mean free path - A of a mean free path λ of a molecule is the average distance traversed by it between two successive collisions.



$$m = 3, N = 3 \times 3 = 9$$

$$\therefore f = 3N - m = 9 - 3 = 6$$

If, these balls [●] are in the same line such that,



$$\text{then } m = 2, N = 3 \times 3 = 9 \\ f = 3 \times 3 - 2 = 9 - 2 = 7$$

- Ques
- (1) Assumptions of K.T.
 - (2) Reduce the eqn of perfect gas - maths related this
 - (3) Boyle's law, Charles law, Clapeyron's law from the gas eqn - prove that. - maths related this.

(4) values of -

- (5) three distributions and corresponding plot,
- (6) the ratio of three
- (7) T.m. dependence on velocity distribution
- (8) Degrees of freedom - definition
- (9) Equpartition of energy - definition
- (10) many d, tri atomic's degrees of freedom -

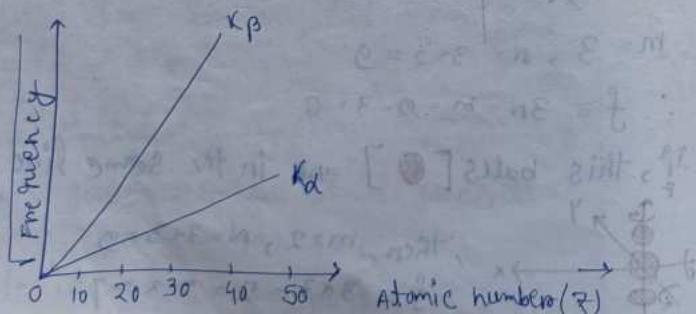
Ans

- (11) Free path, mean free path - definition.

Solid state Physics (N man)

Mosley's Law

In 1913 Mosley plotted the square root of the frequencies ($\sqrt{\nu}$) of a given K_{α} against the atomic number (Z) of the elements emitting that line.



The same linear relation was found to hold good for any line in any series. Therefore he concluded that atomic numbers in the fundamental property of elements.

Mosley's Law - The frequency of a spectral line in X-ray spectrum varies as the square of the atomic no. of the element emitting it

$$\text{Mosley's Law} \quad \nu \propto Z^2$$

$$\text{Ans} \quad \boxed{\sqrt{\nu} = a(Z - b)}$$

Here 'Z' is the atomic no. of the element and 'a' and 'b' are constants depending upon the particular line.

Explain Mosley's law with Bohr's theory

Let a transition occur from n_1 to n_2 state as per Bohr's theory. The energy of an emitted photon is -

$$h\nu = R \frac{1}{n_1^2} - \frac{1}{n_2^2}$$

n_1 = quantum no. of final energy level

n_2 = " " " initial "

ν = freqn for the K_{α} lines

$Z = \text{atomic no}$

$R = \text{Rydberg const}$

$$v = R(Z^2) \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

$$\Rightarrow R C \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = \frac{v}{Z^2}$$

$$\Rightarrow v = a^2 Z^2 \quad \left[\because \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) = a^2 \right]$$

$$\Rightarrow \sqrt{v} = aZ$$

[for $K\alpha$ emission, one of the two K electrons is first emitted leaving a single electron in the K-shell. The negative charge of this residual electrons partially screens the nuclear charge ($Z \cdot e$) of the atom].

Now due to the effective Coulomb force, it becomes $(Z-1)e$

$$\text{Now } Z^2 \rightarrow (Z-1)^2$$

$$\text{So, } v = a^2 (Z-1)^2$$

For $K\alpha$ emission, $n_1 = 1, n_2 = 2$

$$v = R C \left(\frac{1}{1^2} - \frac{1}{2^2} \right) (Z-1)^2$$

$$\Rightarrow v = \frac{3}{4} R C (Z-1)^2$$

$$\Rightarrow \sqrt{v} = \sqrt{\frac{3 R C}{4}} (Z-1) \quad \text{--- (1)}$$

We know, $\sqrt{v} = a(Z-1) \quad \text{--- (2)}$ (approximately according to Moseley's law)

$$\text{Comparing (1) and (2)} \quad a^2 \sqrt{\frac{3 R C}{4}}$$

① If λ_{Cu} is the wavelength of $K\alpha$ x-ray line of copper ($Z=29$) and λ_{Mo} is the wavelength of the $K\alpha$ x-ray line of molybdenum ($Z=42$). Find the ratio of $\lambda_{Cu}/\lambda_{Mo}$

⇒ we know for $K\alpha$ line

$$v = \frac{3}{4} R C (Z-1)^2$$

$$\frac{1}{\lambda} = \frac{3}{4} R (Z-1)^2$$

$$\frac{\lambda_{Cu}}{\lambda_{Mo}} = \frac{(Z_{Mo}-1)^2}{(Z_{Cu}-1)^2} = \frac{(42-1)^2}{(29-1)^2}$$

$$= \frac{41^2}{41^2} = \frac{41^2}{28^2}$$

$$= 2.14$$

Importance of Moseley's law

Moseley's law is imp. as it proved that the atomic no. is a more fundamental property of elements and not atomic mass.

② Characteristic x-rays of frequency $4.2 \times 10^{18} \text{ Hz}$ are produced when transitions from L to K-shell take place in a certain target material

$$\frac{1}{\lambda} = R (Z-1)^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad \boxed{K \rightarrow L \rightarrow M \rightarrow N}$$

$$\Rightarrow \frac{1}{\lambda} = 1.1 \times 10^7 \times (Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right)$$

$$\Rightarrow Z = 42$$

$$\begin{aligned} & [1, 3 \times 10^8] \\ & [1, 4.2 \times 10^{18} \text{ Hz}] \end{aligned}$$

→ (short in)

Contribution from Mosley's Law

Mosley's work helped to perfect the periodic table by -

(i) The discovery of new elements hafnium(72), mium(61), rhenium (75) etc.

(ii) the determination of the atomic no. of rare earths and fixing their positions in the periodic table.

Crystal

A solid of definite shape with its atoms, ions or molecules arranged in some regular repetitions three dimensional pattern is termed as a crystal

Crystalline Solid

The crystalline state of a solid is characterised by regular and periodic arrangement of atoms or molecules.

Amorphous Solid - without any regular form

• Lattice - Periodic arrangements of crystals in atom.

• why are most solids crystalline?

The energy released during formation of an ordered structure is more than that released during the formation of disordered structure thereby making the crystalline state and low energy state.

• Difference between Crystalline and amorphous Solid -

(i) crystalline solid has long range order whereas an amorphous solid has only a short range order.

(ii) crystalline solid is anisotropic in character whereas amorphous solid is isotropic in character.

(iii) crystalline solids have sharp melting points whereas in amorphous solids there is a gradual softening of the material.

• Why X-rays are used for crystal structure analysis?

X-rays are used for producing diffraction effect in crystals due to the fact that wavelength of X-rays is comparable to the interatomic distances in actual crystals.

Example -

$$EV = h\nu = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{EV}$$

$$\text{For } V = 10 \text{ KV}, \lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 10 \times 10^3}$$

$$= 1.24 \times 10^{-10} \text{ m}$$

$$> 1.24 \text{ Å}$$

This wavelength is comparable to the interatomic spacing of an actual ~~bill~~ crystal.

(in case short λ)

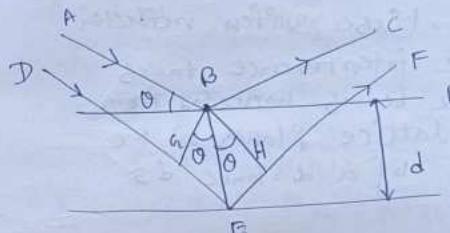
① Visible light has

a larger wavelength. Those waves give rise to familiar effects of optical reflection and refraction.

Therefore, cannot be used for exploring the crystal.

② The wavelength of gamma rays is much smaller than that of x-rays, therefore γ -rays are diffracted through very small angles which cannot be conveniently measured.

Bragg's Law



Consider parallel lattice planes equidistant from one another in a crystal structure, separated by a distance d .

Path difference between the two wave fronts
now in ABGE

$$GE = BE \sin \theta = d \sin \theta$$

$$\text{and } d \sin ABH, EH = BE \sin \theta = d \sin \theta$$

$$\therefore \text{Path difference} = GE + EH = 2d \sin \theta$$

If this path difference is an integral multiple of wavelength λ , constructive interference of wavelength will take place between the reflected beams. Thus the intensity will be maximum if -

$$2d \sin \theta = n\lambda \quad n = 1, 2, 3$$

↓

Bragg's eqn and its represents Bragg's law.

Bragg's equation: When x-rays of wavelength λ are incident on a crystal surface making an angle θ with the surface of the crystal, these suffer reflection and constructive interference takes place between the beams reflected from two consecutive lattice planes of the crystal separated by a distance d . Then the relation

$$2ds\sin\theta = n\lambda$$

where n is the order of the diffraction spectrum maxima, is known as Bragg's equation.

The law represented by Bragg's equation

$$2ds\sin\theta = n\lambda$$

is known as Bragg's law.

(1st part is the statement of Bragg's law)

Ex-

① The spacing between successive (100) planes in NaCl is 2.82 \AA . X-rays incident on the surface of the crystal is found to give rise to first order Bragg reflection at glancing angle 8.8° . Calculate the wavelength of x-rays.

According to Bragg's law, $2ds\sin\theta = n\lambda$

$$d = 2.82 \text{ \AA} = 2.82 \times 10^{-10} \text{ m}$$

$$\theta = 8.8^\circ \quad \therefore \sin\theta > 0.153$$

$$n = 1$$

$$2ds\sin\theta = n\lambda$$

$$\Rightarrow 2 \times 2.82 \times 10^{-10} \times 0.153 = \lambda$$

$$\Rightarrow \lambda = 0.863 \times 10^{-10} \text{ m} = 0.863 \text{ \AA}$$

② X-rays of wavelength 1.5 \AA make a glancing angle of 16° in the first order when diffracted from NaCl crystal. Find the lattice constant of NaCl.

$$2ds\sin\theta = n\lambda \quad \lambda = 1.5 \times 10^{-10} \text{ m}$$

$$n = 1$$

$$\theta = 16^\circ \quad \therefore \sin\theta > 0.2756$$

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1 \times 1.5 \times 10^{-10}}{2 \times 0.2756}$$

$$\Rightarrow 2.72 \times 10^{-10} \text{ m}$$

$$= 2.72 \text{ \AA}$$

*(1st form 3 below
and from the next
topic)*

- magnetic properties (Rsin)

- magnetism

↳ magnetism is a phenomenon by which a material exerts either attractive or repulsive force on another.

2) Basic source of magnetic force

is movement of electrically charged particles. Thus magnetic behaviour of a material can be traced to the structure of atoms.

3) Electrons in atoms have a

planetary motion in that they go around the nucleus. This orbital motion and its own spin spin cause separate magnetic moments, which contribute to the magnetic behaviour of materials. Thus every material can respond to a magnetic field.

4) However, the manner in which a material responds depend much on its atomic structure, and determines whether a material will be strongly

or weakly magnetic

- Bohr magneton

- magnetic moment due to spin of an electron is known as Bohr magneton m_B .

$$m_B = \frac{q\hbar}{4\pi m_e} = 9.274 \times 10^{-24} \text{ A m}^2$$

- where q is the charge on the electron, \hbar - Planck's constant, m_e - mass of e-

- Bohr magnetons is the most fundamental magnetic moment.

- why not all materials are magnets?

- As every material consists spinning electrons each of them could be a magnet. Fortunately not so.

- There are two reasons for it.

- First: according to Pauli exclusion rule, two electrons with same energy level must have opposite spins - thus so are their magnetic moments, which cancel out each other.

- Second: orbital moments of electrons also cancel out each other - thus no net magnetic moments if there is no unpaired electrons.

- Some e^- such as transition elements, lanthanides and actinides have a net magnetic moment since some moments of their energy levels have an unpaired electron.
- magnetic dipoles
- magnetic dipoles are found to exist in magnetic materials analogous to electric dipoles.
- A magnetic dipole is a small magnet composed of north and south poles instead of positive and negative charges.
- within a magnetic field, the force of field exerts a torque that tends to orient the dipoles with the field.
- magnetic field, the force of field exerts magnetic forces are generated by moving electrically charged particles. These forces are in addition to any electrostatic forces that may already exist.
- It is convenient to think magnetic forces in terms of distributed field which is represented by imaginary

lines. these lines also indicate the direction of the force.

magnetic field

- If a magnetic field is generated by passing current I through a coil of length l and number of turns n then the magnetic field strength H (units A/m) is given by

$$H = \frac{nI}{l}$$

magnetic flux density (induction) is the measure of lines within a medium. It has units as weber (Wb)/ m^2 or tesla and it is defined as

$$\downarrow B = \mu H$$

- where μ - permeability. It is a specific property of the medium, and has units $Wb/A.m$ or henry (H)/m

- Relative magnetic Permeability, is defined as $\mu_r = \frac{\mu}{\mu_0}$
- μ_r is a measure of the degree to which the material can be magnetized.
- where μ_0 - magnetic permeability of vacuum.

If m = magnetization defined as
 m is defined as $m = \chi H$ then

$$B = \mu_0 H + \mu_0 M \\ \Rightarrow \mu_0 \mu_r H$$

mechanics of particle (N man) for gram)

~~vector cross product, dot product~~

- ① Center of mass (def.)
- ② Conservation of linear momentum
- ③ Conservation of angular momentum
- ④ Total energy (2.9.8 expression) derivation

① Coulomb's law - The magnitude of the electrostatic force of attraction or repulsion between two point charges is directly proportional to the product of the magnitudes of charges and inversely proportional to the square of the distance between them.

② Electrostatic Potential - The amount of work energy needed to move a unit of electric charge from a reference point to the specific point in an electric field. (done)
[Another definition]

Electric field - An electric property associated with each point in space when charge is present in any form.

③ Gauss law statement - It states that the net flux of an electric field in a closed surface is directly proportional to the enclosed electric charge. Yet another statement of Gauss's law states that the net flux of a given electric field through a given surface divided by the enclosed charge should be equal to a constant.

④ Dipole moment - The product of internuclear distance between bonded atoms and charge present in bonded atoms.

$P = qd$
Dipole - Two equal and opposite charges separated by a very small distance are said to constitute an electric dipole.

⑤ Bio & Savart law - The magnetic intensity at any point due to a steady current in an infinitely long straight wire is directly proportional to the current and inversely proportional to the distance from point to wire.

⑥ Self-inductance - It is the tendency of a coil to resist changes in current in itself.

⑦ Mutual inductance - If two coils of wire are brought into close proximity with each other so the magnetic field from one links with the other, a voltage will be generated in the second coil as a result. This is called

mutual inductance

- ① centre of mass - The centre of mass of the system is defined as a point or whose position vector \vec{R} is given by

$$m\vec{R} = \sum m_i \vec{r}_i$$

$$\Rightarrow \vec{R} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

$$= \frac{\sum m_i \vec{r}_i}{\sum m_i}$$

- ②, ③ - done in mechanics

$$\textcircled{3} - \frac{dP^3}{dt} > 0, \Rightarrow P^3 = \text{constant}$$

- ④ Total energy -

$$T + V = \frac{1}{2} \sum' v_{ij} = \text{constant}$$

where T = total energy of the system

V = Potential energy

The total mechanical energy of the system is constant in time, that is it is conserved.

- ⑤ Show that the gravitational force is conservative.

$$\text{Ansatz } \vec{F} = -G \frac{m_i m_j \vec{r}}{r^3} = -\frac{K}{r^3} \vec{r}$$

We have to prove -

$$\nabla \times \vec{F} = 0 \quad \rightarrow, \vec{i} + \vec{j} + \vec{k}$$

$$\nabla \times \vec{r} = \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k}$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

$$\vec{i} \left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z} \right) - \vec{j} \left(\frac{\partial z}{\partial x} - \frac{\partial x}{\partial z} \right) + \vec{k} \left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y} \right)$$

$$\therefore \vec{r} \times \left(-\frac{K}{r^3} \vec{r} \right) = \vec{r} \left(-\frac{K}{r^3} \times \vec{r} \right)$$

$$= \frac{3K}{r^4} \vec{r} \times \vec{r}$$

$$= 0 \quad [\because \vec{r} \times \vec{r} = 0]$$

$\therefore \vec{r} \times \vec{F} = 0$. Thus the gravitational force is conservative.

$$(i) \vec{F} = \frac{-\hat{r}}{r^2}$$

$$= -\frac{\vec{r}}{r^2}$$

$$= \frac{\vec{r}}{r^3}$$

R.T.P

$$\nabla \times \vec{F} = 0$$

$$\Rightarrow \nabla \times \left(-\frac{\vec{r}}{r^3} \right) = 0$$

$$\text{Now } \vec{r} \times \left(-\frac{\vec{r}}{r^3} \right) = \vec{r} \left(\frac{-1}{r^3} \right) \times \vec{r} = -\frac{1}{r^3} (\vec{r} \times \vec{r})$$

$$\vec{r} \times \vec{r} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix} = 0$$

$$\begin{aligned}\vec{B} \times \left(-\frac{\vec{n}}{hnn} \right) &= \vec{B} \cdot \left(-\frac{1}{hn} \right) \times \vec{n} \\ &= \frac{h+1}{n+2} \vec{B} \times \vec{n} \\ &> 0\end{aligned}$$

$$\therefore \vec{B} \times \left(-\frac{\vec{n}}{hnn} \right) = 0$$

• Dia-magnetism -

(i) very weak, only exists in presence of an electric field, non-permanent.

(ii) The induced magnetic moment is small and the magnetization direction is opposite to the direction of applied field (H).

(iii) Thus the relative permeability is less than unity.

(iv) magnetic susceptibility is negative and its value is small (-10^{-3}).

(v) Eg. - Cu, Ag, Si etc.

• Para-magnetism -

(i) slightly stronger, exists only in presence of an external magnetic field, not permanent.

(i) The dipoles do not interact.
(ii) materials which exhibit a small positive magnetic susceptibility in the presence of a magnetic field are called para-magnetic and the effect is termed as para-magnetism.

(iii) The magnetization is positive.

(iv) However, because the dipoles don't interact extremely large magnetic fields are required to align all of the dipoles.

(v) The increases of temperature decreases para-magnetic effect.

(vi) Eg - Co, Al, Ti, Al₂O₃ etc.

(vii) Relative permeability is greater than unity.

• Ferro-magnetism -

(i) This is permanent and exists in the absence of external field.

(ii) magnetic susceptibility is very large.

(iii) Relative permeability is greater than unity very.

(iv) Above the Curie temperature ferromagnetic materials behave like para-magnetic materials, magnetic susceptibility is given by the Curie-Weiss law, defined as,

$$\chi = \frac{C}{T - T_C}$$

(\rightarrow material constant C , temperature, T_C = Curie temperature.)

Ir, Eg - Co, Ni, Fe etc,