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Roll - CSE 214002 sec - B

Course code - MENUGES01

Course Title - Engineering mechanics

Year - 1st Semester - 1st

A) (i) The coefficient of friction depends on – nature of surface

2) The resultant of two equal forces P making an angle θ is given by – $2P\cos\frac{\theta}{2}$

3) All of these are vector quantities.

4) A force given by $F = 3\hat{i} + 2\hat{j} - 4\hat{k}$ is applied at the point $P(1, -1, 2)$. The magnitude of the moment of the force F about the point $O(2, -1, 3)$

$$\overline{OP} = (\hat{i} - \hat{j} + 2\hat{k}) - (2\hat{i} - \hat{j} + 3\hat{k})$$

$$\Rightarrow \vec{r} = -\hat{i} - \hat{k}$$

$$\vec{M} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 0 & -1 \\ 3 & 2 & -4 \end{vmatrix}$$

$$= \hat{i}(0+2) - \hat{j}(4+3) + \hat{k}(-2-0)$$

$$= 2\hat{i} - 7\hat{j} - 2\hat{k}$$

$$|\vec{M}| = \sqrt{4+49+4} = \sqrt{57}$$

Ans = $\sqrt{57}$

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5) A force $F = (-5\hat{i} + 10\hat{j})N$ causes a displacement $S = (4\hat{i} + 6\hat{j})m$. The magnitude of the force and workdone are-

$$|\vec{F}| = \sqrt{25 + 100} = \sqrt{125} = 5\sqrt{5}\ N$$

$$\vec{W} = \vec{F} \cdot \vec{S} = (-5\hat{i} + 10\hat{j}) \cdot (\cancel{8}4\hat{i} + 6\hat{j})$$

$$= -20\hat{i} + 60\hat{j}$$

$$\therefore |\vec{W}| = \sqrt{400 + 3600} = 63.24\ J$$

The magnitude of the force $= 5\sqrt{5}$ N and workdone $= 63.24$ J

6) The horizontal range of a projectile is maximum when the angle of projection is - 45. ~~90~~

incorrect statement -

7) The c.g G of a semicircle at a distance r/2 from the center.

To design the trusses -

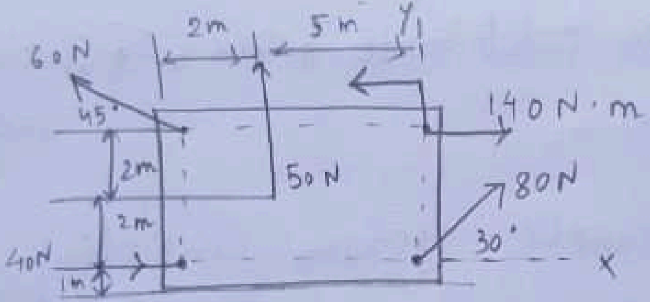
8) ~~All~~ the loads are not applied at the joints.

9) coplanar non-concurrent forces are those ~~that~~ forces which -

do not meet at one point, but their lines of action lie on the same plane

10) According to d' Alembert's Principle, the external forces acting on a body in equilibrium with - the resultant inertia forces on the body

B) 11 (b)



we know,

$$\Sigma F_x = 8\cos 30 + 40 - 60\cos 45$$
$$= 66.85 N$$

$$\Sigma F_y = 80\sin 30 + 50 + 60\sin 45$$
$$= 132.4 N$$

$$\therefore R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$
$$= \sqrt{(66.85)^2 + (132.4)^2}$$
$$= 148.3 N$$

the resultant of four forces = 148.3 N

Roll

direction -

$$\tan\theta = \frac{132.4}{66.85}$$

$\Rightarrow \theta = 63.2°$ [makes an angle θ (63.2°) with x-axis]

one couple act on the plate, ~~st~~

$\Sigma M_0 = +140 \cancel{+ 80 \sin 30} - 50 \times 5 - 60 \sin 45 \times 7 + 60 \cos 45 \times 4$

$= -237$ N-m

The force couple system consisting R and M_0.

we now determine the final line of action of R such that R alone represents the original system

$[Rd = |M_0|]$, $148.3d = 237$

$\Rightarrow d = 1.600$ m

Here, the resultant R ~~ana~~ may applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point ~~A to~~ A to a circle of 1600m radius with center O.

Now, $R_y b = |M_O|$ and $b = \frac{237}{132.4} = 1.792$

Alternatively, the y-intercept could have been obtained by noting that the moment about O would be due to R_x only.

A more formal approach in determining the final line of action of R is to use the vector expression –

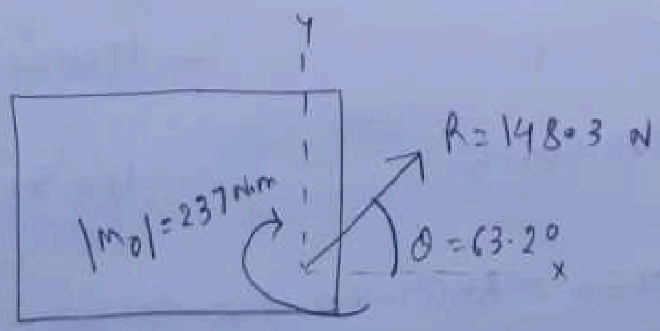
$$r \times R = M_O$$

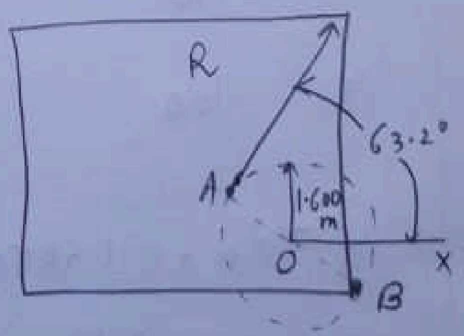
Now, $(x\hat{i} + y\hat{j}) \times (66.9\hat{i} + 132.4\hat{j}) = -237\hat{k}$

$$\Rightarrow (132.4x - 66.9y)\hat{k} = -237\hat{k}$$

Thus, the desired line of action,

$$132.4x - 66.9y = -237$$





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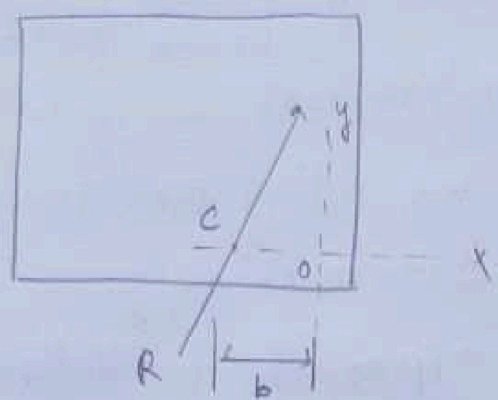
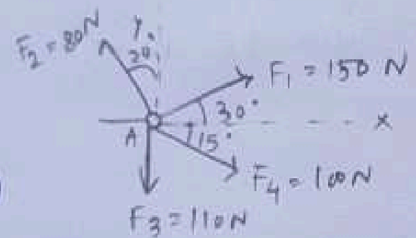


2)(a)



Force	mag.	x-comp	y-comp
F_1	150	$150\cos30$	$150\sin30$
F_2	80	$-80\sin20$	$80\cos20$
F_3	110	0	-110
F_4	100	$100\cos15$	$-100\sin15$

$\therefore \Sigma F_x = 150\cos30 - 80\sin20 + 100\cos15$

$= 199.1\ N$

$\Sigma F_y = \cancel{14.3\ N} = 150\sin30 + 80\cos20 - 110 - 100\sin15$

$= 14.3\ N$

$R = \sqrt{(199.1)^2 + (14.3)^2}$

$= 199.6 N$

$\therefore$ Resultant force $= 199.6 N$

direction,

$\theta = \tan^{-1} \frac{14.3}{199.1}$

$= 4.11°$

~~4.11°~~ Resultant makes an angle $\theta = 4.11°$ with the x - axis.

(b) $S = 2t^3 - 30t + 7$

$V = \frac{ds}{dt} = 6t^2 - 30$

$\therefore$ Now,

(i) $6t^2 - 30 = 60$

$\Rightarrow 6t^2 = 90$

$\Rightarrow t^2 = \frac{90}{6}$

$\Rightarrow t = 3.87$ sec

$\therefore$ The time required for the particle to reach

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a velocity of 60 m/s from it initial position is 3.87 sec.

(ii) $\alpha = \frac{dv}{dt} = 12t$

Let, the particle takes time T to reach the velocity v = 30 m/s

$\therefore$ Now, $6T^2 - 30 = 30$

$\Rightarrow 6T^2 = 60$

$\Rightarrow T^2 = 10$

$\Rightarrow T = 3.2$ sec

$\therefore$ In this time the acceleration of the particle $= 12 \times 3.2 = 38.4$ m/sec^2

(iii) $S = 2t^3 - 30t + 7$

at $t = 2s$, $S = 2 \times (2)^3 - 30 \times 2 + 7$

$= 2 \times 8 - 30 \times 2 + 7$

$= -37$ m

at $t = 8s$, $S = 2 \times (8)^3 - 30 \times 8 + 7$

$= 2 \times 512 - 30 \times 8 + 7$

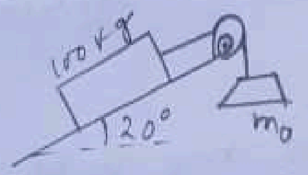
$= 791$

$\therefore$ displacement $= \frac{791 - (-37)}{8 - 2} = 138$ m

4)(a) virtual work principle - work done by an external active force on any ideal mechanical system in equilibrium is zero for all and any virtual displacement consistant with the system constrains.

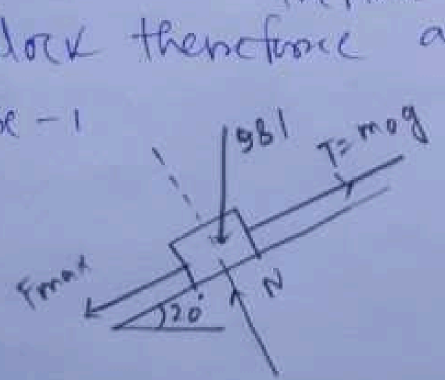
$$\delta U = 0$$

(b)



The miximam value of mo will be given by the requirement of for the motion impending up. The frictional force on the block therefore acts down the plane -

Case - 1



[$\because 100 \times 9.81 = 981 N$]

$[\Sigma F_y = 0]$

$N - 981 \cos 20 = 0$

$\Rightarrow N = 922\,N$

$[F_{max} = \mu N]$ $\Rightarrow F_{max} = 0.30 \times 922$

$= 276.6\,N$

$[\Sigma F_x = 0]$

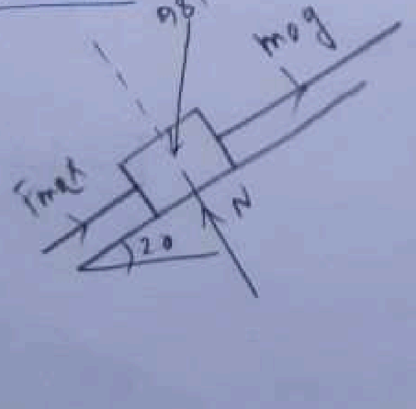
$m_0 g - F_{max} - 981 \sin 20 = 0$

$\Rightarrow m_0 (9.81) - 276.6 - 981 \sin 20 = 0$

$\Rightarrow m_0 = 62.3\,kg$

The minimum value of m_0 will be given by the ~~ba~~ requirement for the motion impending down the plane. The frictional force on the block therefore acts up the plane.

Case-II



$F_{max} = 276.6 N$

$[F_n = 0]$ $\quad m_0 g + F_{max} - 981 \sin 20 = 0$

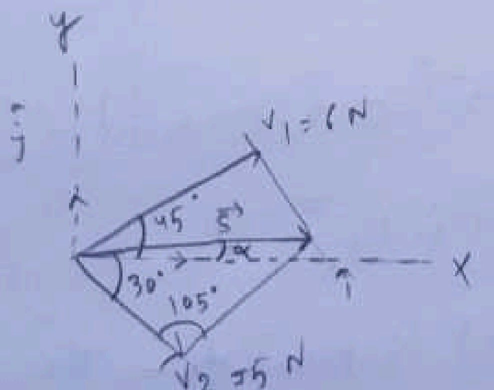
$\Rightarrow (9.81) m_0 + 276.6 - 981 \sin 20 = 0$

$\Rightarrow m_0 = 6 kg$

the value of m_0 will be between 6kg to 62.3kg.

6) (i) $S^2 = 6^2 + 5^2 + 2 \cdot 6 \cdot 5 \cdot \cos 105$ $[\vec{S} = \vec{v_1} + \vec{v_2}]$

$\vec{S} = 6.74$ units



Let the angle between positive x-axis and $\vec{S}$ be α.

$\therefore$ Now, $\dfrac{\sin(\alpha + 30)}{6} = \dfrac{\sin 105}{6.74}$

$\Rightarrow \alpha = 29.3°$

(iii) Now,

$$\vec{S} = S(\hat{i}\cos\alpha + \hat{j}\sin\alpha)$$
$$= 6.74(\hat{i}\cos 29.3 + \hat{j}\sin 29.3)$$
$$= 5.8\hat{i} + 3.3\hat{j}$$

Now, $\hat{n} = \frac{\vec{S}}{|\vec{S}|} = \frac{(5.8\hat{i} + 3.3\hat{j})}{6.67}$

$$\hat{n} = 0.86\hat{i} + 0.49\hat{j}$$

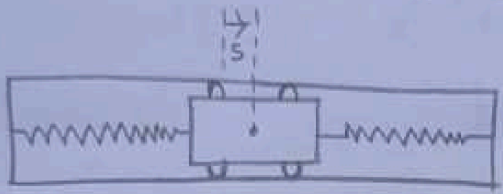
(iv) $\vec{S} = \vec{v_1} - \vec{v_2}$

$$= (6\cos 45\hat{i} + 6\sin 45\hat{j}) - (5\cos 30\hat{i} - 5\sin 30\hat{j})$$
$$= -0.1\hat{i} + 6.74\hat{j}$$

7)(a) D' Alembert's Principle - The second law states that the force F acting on a body is equal to the product of the mass m and acceleration a of the body, or F = ma; in D'Alembert's form, the force F plus the negetive of the mass m times acceleration a of the body is equal to zero: $F - ma = 0$.

(b) newton's 2nd law of motion is $F = ma$; i.e force acting on a body is equal to the product of mass m and acceleration a of the body. But in ~~this~~ D'Alembert's Principle $F - ma = 0$.

(b)



Now,

$a = -k^2 S$

we know $a = v \frac{dv}{ds}$

$\therefore v \frac{dv}{ds} = -k^2 S$

$\Rightarrow \int v\,dv = -k^2 \int S\,ds$

$\Rightarrow \frac{v^2}{2} = \frac{-k^2 S^2}{2} + \frac{C^2}{2}$

$\Rightarrow v^2 = -k^2 S^2 + C^2$

When $S = 0$, $v = 0$

$\therefore C = 0$

$\therefore v^2 = -k^2 S^2$

Let, $-k^2 = P^2$

$\therefore v^2 = P^2 S^2$

$\Rightarrow v = PS$

$\Rightarrow \frac{ds}{dt} = RS$

$\Rightarrow \int_{S_0}^{S} \frac{ds}{S} = R\int_0^t dt \quad [\text{at } t=0,\ S=S_0]$

$\Rightarrow [\log|S|]_{S_0}^{S} = R[t]_0^t$

$\Rightarrow \log \frac{S}{S_0} = Rt$

$\Rightarrow S = S_0 e^{Rt}$

$\Rightarrow S = S_0 e^{(-K^2)^{1/2} t} \quad [\because R^2 = -K^2]$

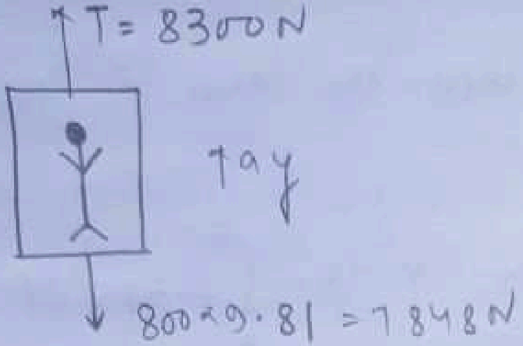
$\therefore$ Displacement, $S = S_0 e^{(-K^2)^{1/2} t}$

Expression for

" " velocity, $v = (-K^2)^{1/2} S \quad [\because R^2 = -K^2]$

or $v^2 = -K^2 S^2$

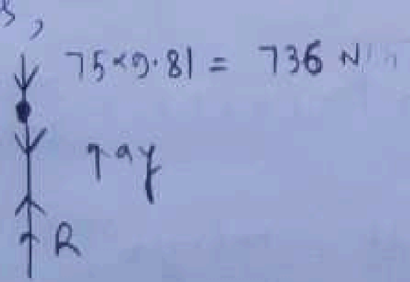
8) (b) The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free body diagram of the elevator scale and the man taken together, the acceleration is found to be,



$[\Sigma F_y = m a_y]$ $\quad \therefore$ $8300 - 7848 = \cancel{m a_y}\ 800 a_y$

$\Rightarrow a_y = 0.565\ m/s^2$

the scale reads the downward force exerted on it by the man's feet. the equal and opposite reaction R to this section is shown on the free body diagram of the man along together, with his weight and the equaltion of motion for him gives,



$[\Sigma F_y = m a_y]$ $\quad$ $R - 736 = 75 \times 0.565$

$\Rightarrow R = 778$ N

The velocity reached the end of the 3 sec. is—

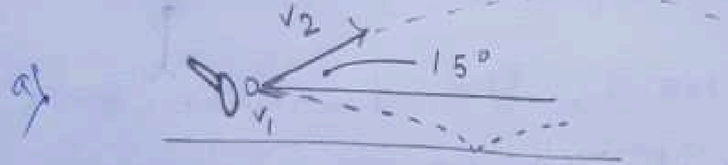
$[\because v = \int a\,dt]$, $v - 0 = \int_0^3 0.565\,dt$

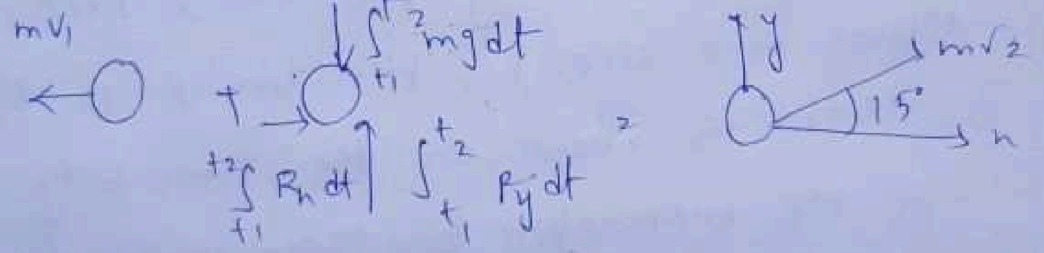
$\Rightarrow v = 0.565 \times [t]_0^3$

$\Rightarrow v = 0.565 \times 3$

$\Rightarrow v = 1.695$ m/s

a)





$[m(v_x)_1 + \int_{t_1}^{t_2} \Sigma F_x\,dt = m(v_x)_2]$

$-\frac{2/16}{32.2}(50) + R_x(0.02) = \frac{2/16}{32.2}(70\cos 15)$

$[m(v_y)_1 + \int_{t_1}^{t_2} \Sigma F_y\,dt = m(v_y)_2]$

$\frac{2/16}{32.2}(0) + R_y(0.02) - (2/16) \times (0.02) = \frac{2/16}{32.2}(70\sin 15)$

we can now solve for the impact forces as,

$R_x = 22.8$ lb

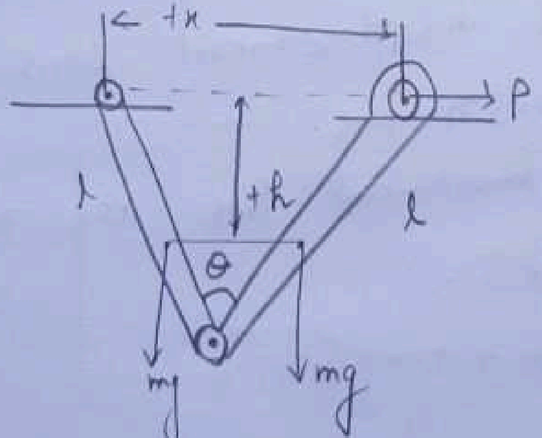
$R_y = 3.64$ lb

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(22.8)^2 + (3.64)^2}$

$= 23.1$ lb

$\beta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{3.64}{22.8} = 9.06°$

5)



The active force diagram for the system composed of the two members is shown separately and includes the weight mg of each bar in addition to the force P. All others forces acting externaly on the system are reactive forces which do not work during a virtual movement δx and are therefore not shown.

The Principle of ~~valle~~ virtual work requires that the total work of all external active forces be zero for any virtual displacement consistent with the constrains.

Thus, for a movement δx the virtual work becomes,

$[\delta U = 0]$ $P\delta x + 2mg\,\delta h = 0$

we now express each of these virtual displacements in terms of the ~~virtual~~ variable θ, the required quantity.

Hence, $x = 2l\sin\frac{\theta}{2}$ and $\delta x = l\cos\frac{\theta}{2}\,\delta\theta$

similarly,

$h = \frac{l}{2}\cos\frac{\theta}{2}$ and $\delta h = -\frac{l}{4}\sin\frac{\theta}{2}\delta\theta$

Substitution into the equation of virtual work gives as-

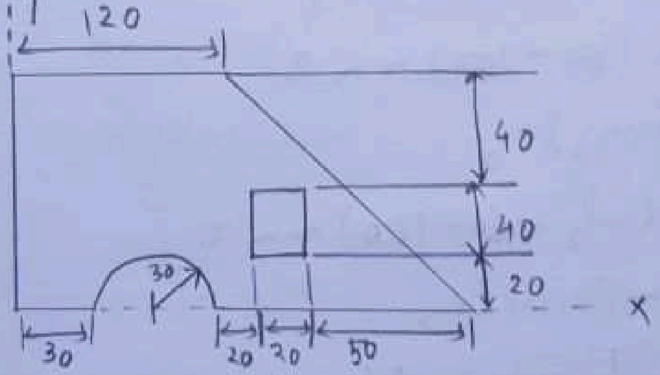
$$Pl\cos\frac{\theta}{2}\delta\theta - 2m\frac{l}{4}\sin\frac{\theta}{2} = 0$$

from which we get,

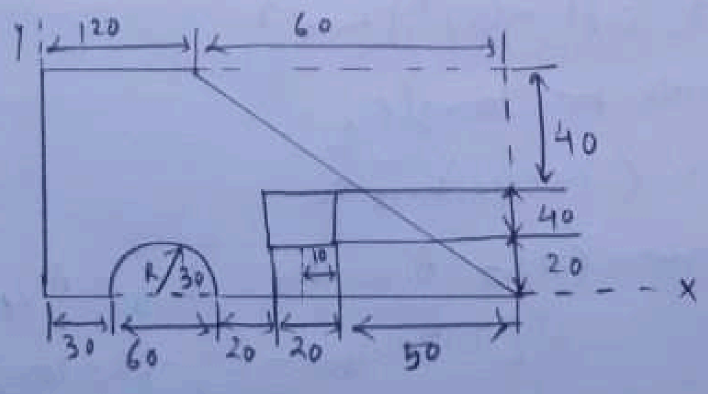
$$\tan\frac{\theta}{2} = \frac{2P}{mg}$$

$$\Rightarrow \theta = 2\tan^{-1}\frac{2P}{mg}$$

1) (a)



~~Ans~~



Now,
~~at joint H~~,

This section can be devided int a big rectangle(A_1), a triangle (A_2), a semicircle (A_3), a small rectangle (A_4).

Areas,

For big rectangle,

$A_1 = (180 \times 100)\ mm^2$

For triangle,

$A_2 = (\frac{1}{2} \times 60 \times 100)\ mm^2$

For semicircle,

$A_3 = \{\frac{1}{2}\pi \times (3)^2\}\ mm^2$

For small rectangle,

$A_4 = (20 \times 40)\ mm^2$

Total ~~A~~ area (A) $= A_1 - A_2 - A_3 - A_4$

$= [(180 \times 100) - (\frac{1}{2} \times 60 \times 100) - \{\frac{\pi}{2} \times (30)^2\} - (20 \times 40)]\ mm^2$

$= (18000 - 3000 - 1413.7 - 800)\ mm^2$

$A = 12786.3\ mm^2$

Now,

Centroid of A_1, $g_1 (x_1, y_1)$

$x_1 = 90\ mm$, $y_1 = 50\ mm$

Centroid of A_2, $g_2 = (x_2, y_2)$

$x_2 = \left(\frac{60}{3} + 120\right) mm$, $y_2 = \frac{100}{3}\ mm$

Centroid of A_3, $g_3 = (x_3, y_3)$

$x_3 = (30 + 30)$, $y_3 = \frac{4R}{3\pi} = \frac{4 \times 30^{10}}{3\pi} = \frac{40}{\pi}\ mm$

Centroid of A_4, $g_4 = (x_4, y_4)$

$x_4 = (10 + 20 + 60 + 30)\ mm$, $y_4 = (20 + 20)\ mm$

Now,

$\bar{x} = \frac{\Sigma A_i x_i}{A}$ [here, $i = 1, 2, 3 \ldots$]

$$= \frac{180 \times 100 \times 90 - \frac{1}{2} \times 60 \times 100 \times \left(\frac{60}{3} + 120\right) - \frac{\pi}{2} \times (30)^2 \times (30 + 30) - 20 \times 40 \times (10 + 20 + 60 + 30)}{12786.3}$$

$$= \frac{180 \times 100 \times 90 - 30 \times 100 \times 140 - \frac{\pi}{2} \times \overset{450}{\cancel{900}} \times 60 - 20 \times 40 \times 120}{12786.3}$$

$= 79.7\ mm$

$\therefore \bar{x} = 79.7\ mm$

Similarly,

$$\bar{y} = \frac{\sum A_i y_i}{A} \quad [\text{Here } i = 1, 2, 3 \ldots]$$

$$= \frac{180 \times 100 \times 50 - \frac{1}{2} \times 60 \times 100 \times \frac{100}{3} - \frac{\pi}{2} \times (30)^2 \times \frac{40}{\pi} - 20 \times 40 \times (20+20)}{12786.3}$$

$$= \frac{180 \times 100 \times 50 - 30 \times 100 \times 33.33 - \cancel{\pi} \times 450 \times \frac{40}{\cancel{\pi}} - 20 \times 40 \times 40}{12786.3}$$

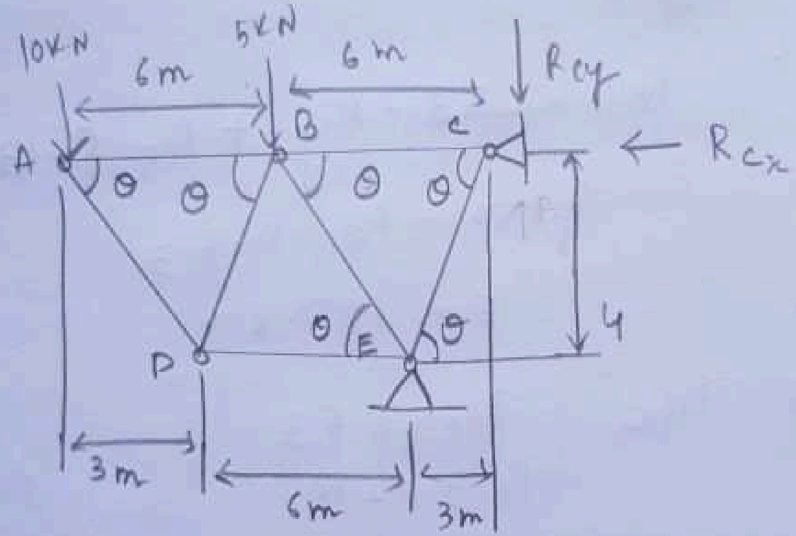
$= 58.65\ mm$

$\therefore \bar{y} = 58.65\ mm$

∴ the centroid of the shaded areas =

(79.7, 58.65)

3)



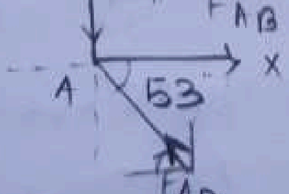
$M_C = 6 \times 5 + 10 \times 12 - R_{cy} = 0$

$\Rightarrow R_{cy} \Rightarrow 150N$

$\theta = \tan^{-1} \cancel{\frac{3}{4}}\ \frac{4}{3}$

$\theta = 53°$

at joint A,

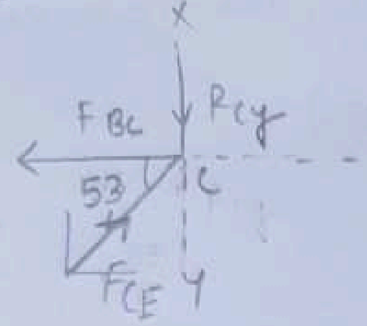


$\Sigma F_y = 0$, $F_{AD} \sin \cancel{48}\, 53 - 10 = 0$

$\Rightarrow F_{AB} = \cancel{31.5N}$ 12.52 N

$\Sigma F_x = 0$ $F_{AB} - F_{AD} \cos \cancel{48}.53 = 0$

$\Rightarrow F_{AB} = \cancel{30N}$ 7.53 N at joint C ,



$\Sigma F_y = 0$, $\cancel{F_{CE}\sin 53} - R_{cy} + F_{CE}\sin 53 = 0$

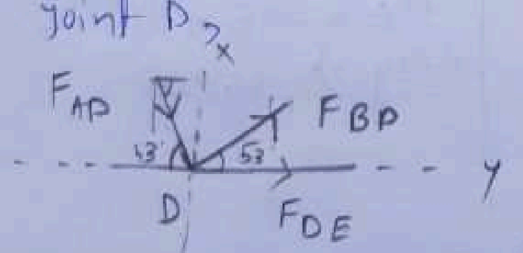
$\Rightarrow 150 = F_{CE}\sin 53$

$\Rightarrow F_{CE} = 187.8\,N$

$\Sigma F_x = 0$ $F_{BC} = F_{CE}\cos 53$

$= 113.02\,N$

at joint D ,



$\Sigma F_y = 0$, $-F_{AD}\sin 53 + F_{BD}\sin 53 = 0$

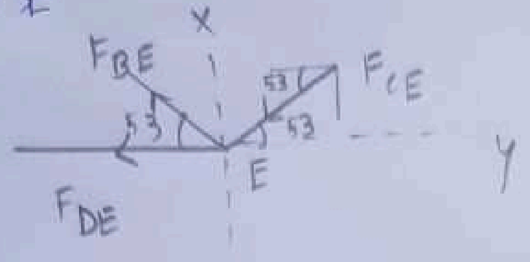
$\Rightarrow F_{AD}\cancel{\sin 53} = F_{BD}\cancel{\sin 53}$

$\Rightarrow F_{BD} = F_{AD} = 12.52\,N$

$\Sigma F_x = 0$, $F_{DE} + F_{BD}\cos 53 - F_{AD}\cos 53 = 0$

$\Rightarrow F_{DE} = 0\,N$ [$\because F_{BD} = F_{AB}$]

at joint E



$\Sigma F_y = 0$, $F_{BE} \sin 53 - F_{CE} \sin 53 = 0$

$\Rightarrow F_{BE} = F_{CE} = 187.8N$

∴ the forces in each member -

$F_{AB} = 7.53N$ (tension)

$F_{BL} = 113.02N$ (tension)

$F_{AD} = 12.52N$ (compression)

$A_{DE} = 0N$ (No force is acting on DE)

$f_{DB} = 12.52N$ (Tension)

$F_{BE} = 187.8N$ (Tension)

$F_{CE} = 187.\ N$ (compression)