ENGINEERING MECHANICS

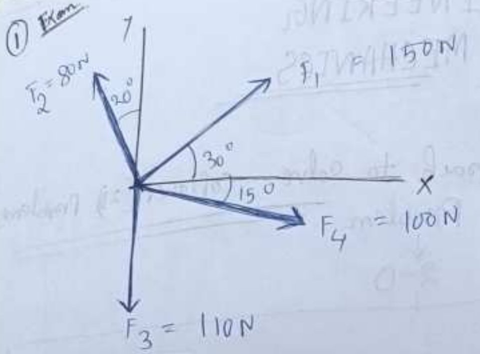
- method of approach to solve coplaner (2D) Problems

Problem
↓
2-D

- Concurrent
  - Not equilibrium (Resultant)
    - 1) Parralal Law
    - 2) Triangle law
    - 3) Polygon low
    - 4) method of Projections
  - Equilibrium (unknowns)
    - 1) $F_x = 0$
    - 2) $F_y = 0$
- Non concurrent
  - Not equilibrium (Resultant)
    - 1) Chose a reference Point
    - 2) Shift all the forces to a Point
    - 3) Find the resultant force and couple at the Point
    - 4) Reduce the force-couple system to a single force
  - Equilibrium (unknown)
    - 1) $\Sigma F_x = 0$
    - 2) $\Sigma F_y = 0$
    - 3) $\Sigma M_z = 0$

Problems - 2D - Concurrent (Resultant) ← (i)

① Exam



Force	mag	x-component	y component
$\vec{F_1}$	150N	150 cos 30	150 sin 30
$\vec{F_2}$	80 N	− 80 sin 20	80 cos 20
$\vec{F_3}$	110 N	0	0
$\vec{F_4}$	100 N	100 cos 15	− 100 sin 15

$\Sigma F_x = 150\cos30 - 80\sin20 + 100\cos15$
$= 199.1$

$\Sigma F_y = 150\sin30 + 80\cos20 - 100\sin15$
$= 124.2$

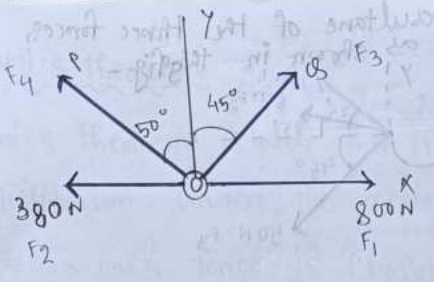
R (Resultant) -

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$= \sqrt{199.1^2 + 124.2^2}$

$= 234.8 N$

② ← (ii) (unknown)

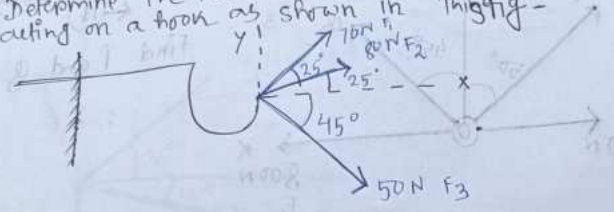


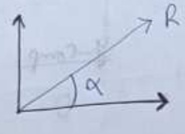
Find P and Q

Force	mag	x-comp	y-comp
$\vec{F_1}$	800	800	0
$\vec{F_2}$	380	− 380	0
$\vec{F_3}$	Q	Q sin 45	Q cos 45
$\vec{F_4}$	P	− P sin 50	P cos 50

$\Sigma F_x = 0$, $\Sigma F_x = 800 - 380 + Q\sin45 - P\sin50 = 0$

(Exam) ③ Determine the resultane of the three forces acting on a hook as shown in this fig -





Force	mag	x - comp	y - comp
$\vec{F_1}$	70	70 cos 50	70 sin 50
$\vec{F_2}$	80	80 cos 25	80 sin 25
$\vec{F_3}$	50	50 cos 45	- 50 sin 45

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$\Sigma F_x = 70 \cos 50 + 80 \cos 25 + 50 \cos 45$
$= 152.8$

$\Sigma F_y = 70 \sin 50 + 80 \sin 25 - 50 \sin 45$
$= 52.1$

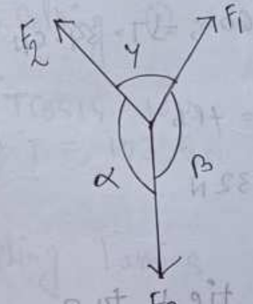
$R = \sqrt{152.8^2 + 52.1^2}$
$= 161.5$ N

$\alpha = \tan^{-1} \dfrac{\Sigma F_y}{\Sigma F_x} = \tan^{-1} \dfrac{52.1}{152.8} = 18.83°$

• Lami's theorem

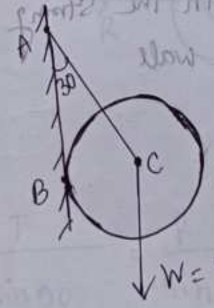
Lami's theorem states that if a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the others two forces.

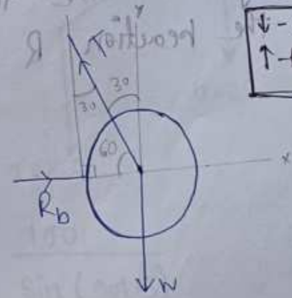
$$\boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}}$$



④

① Find T, R_b





R_b = প্রতিক্রিয়া বল

↓ - clock wise
↑ - Anticlock wise

Since the body is in equilibrium and the forces are concurrent —

$\Sigma F_x = 0$, $T\cos 30 - W = 0$ — (I)

$\Sigma F_y = 0$, $-T\sin 30 + R_b = 0$ — (II)

or $\Sigma F_x = 0$, $R_b - T\cos 60 = 0$ — (I)

$\Sigma F_y = 0$ $T\sin 60 - W = 0$ — (II)

$T\cos 30 - W = 0$ — (I)

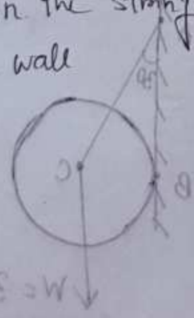
$-T\sin 30 + R_b = 0$ — (II)

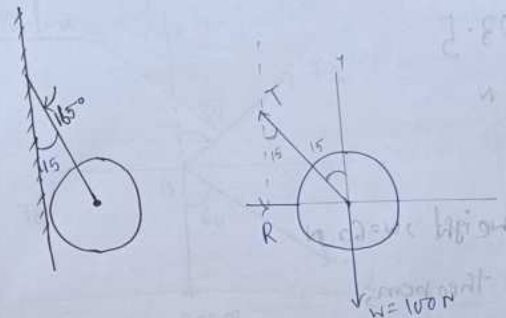
$\frac{T\sqrt{3}}{2} - 30 = 0$, $\Rightarrow T\sqrt{3} = 60$, $\Rightarrow T = 34.6N$

$-\frac{T}{2} + R_b = 0$; $\Rightarrow \frac{34.6}{2} = +R_b$

$\Rightarrow R_b = 17.32N$

(5) A sphere weighing 100N is tied to a smooth wall by a string as shown in that fig. Find the tension T in the string and the reaction R from the wall





$T\cos 15 - 100 = 0$ — (I)

$R - T\sin 15 = 0$ — (II)

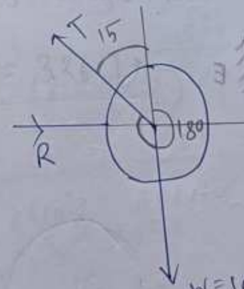
by solving (I) and (II) we get

$T\cos 15 = 100$ $R = T\sin 15$

$\Rightarrow T = 103.5N$ $\Rightarrow R = 103.5 \times \sin 15$

$= 26.7N$

Appling Lami's theorem,



$\frac{T}{\sin 90} = \frac{R}{\sin(360-195)} = \frac{100}{\sin(90+15)}$

$\Rightarrow \frac{T}{1} = \frac{R}{\sin 165} = \frac{100}{\sin 105} = 103.5$

$180 + 15 = 195$

$$\frac{R}{0.25} = 103.5$$

$\Rightarrow R = 26.7$ N

$T = 103.5$ N

(6) (7) same, but weight, $w = 60$ N

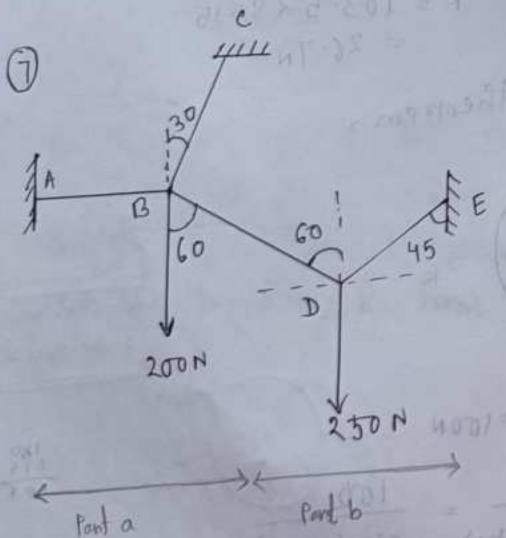
By lami's theorem,

$$\frac{R}{\sin 165} = \frac{T}{\sin 90} = \frac{60}{\sin 105} = 62.1$$

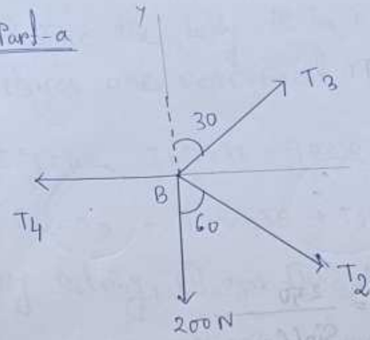
$$\frac{R}{0.25} = 62.1 \quad ; \quad \frac{T}{1} = 62.1$$

$\Rightarrow R = 15.5 \quad \Rightarrow T = 62.1$

(7)



Part-a



$T_3 \cos 30 - T_2 \cos 60 - 200 = 0$ —(i) $[\because \Sigma F_y = 0]$

$T_3 \sin 30 + T_2 \sin 60 - T_4 \cos 0 = 0$ —(ii) $[\because \Sigma F_x = 0]$

By solving (i) and (ii)

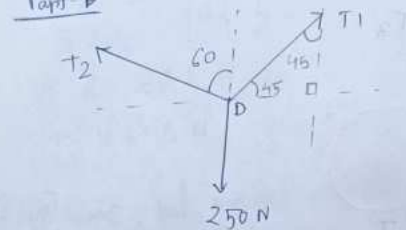
$T_3 \cos 30 - 183 \times \cos 60 - 200 = 0$

$\Rightarrow T_3 = 336.5$ N

$336.5 \times \sin 30 + 183 \times \sin 60 = T_4$

$\Rightarrow T_4 = 326.7$ N

Part-b



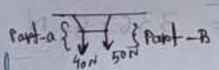
$$\frac{T_1}{\sin(90+30)} = \frac{T_2}{\sin(90+45)} = \frac{250}{\sin(60+45)}$$

$$\Rightarrow \frac{T_1}{\sin 120} = \frac{T_2}{\sin 135} = \frac{250}{\sin 105} = 258.8$$

$T_1 = 258.8 \times \sin 120$
$= 224.12\ N$

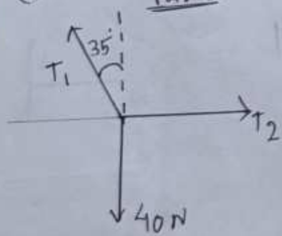
$T_2 = 258.5 \times \sin 135$
$= 183.01\ N$

$\begin{array}{r} 180 \\ +35 \\ \hline 215 \end{array}$



(8) Find T_1, T_2, T_3, θ

Part-a



By Lami's Rule,

$$\frac{T_1}{\sin 90} = \frac{T_2}{\sin(360-215)} = \frac{40}{\sin(35+90)}$$

$$\Rightarrow \frac{T_1}{\sin 90} = \frac{T_2}{\sin 145} = \frac{40}{\sin 125} = 48.8$$

$T_1 = 48.8$

$T_2 = 48.8 \times \sin 145$
$= 28$

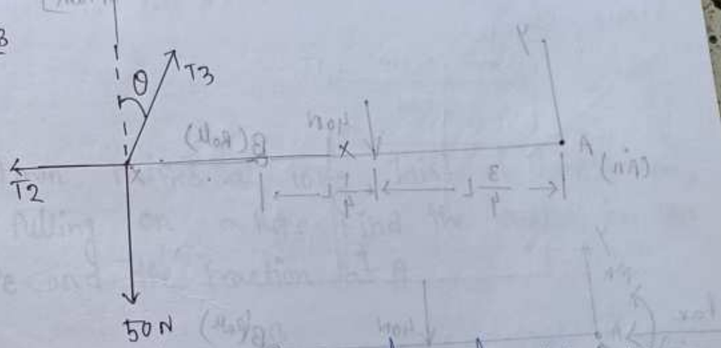
(8a) Since the body is in equilibrium and the forces are concurrent

$\Sigma F_y = 0, \quad T_1 \cos 35 - 40 = 0$ — (i)

$\Sigma F_x = 0, \quad -T_1 \sin 35 + T_2 = 0$ — (ii)

By solving (i) and (ii) we get, $T_1 = 48.8\ N$, $T_2 = 28.0\ N$

Part-B



[As θ is not given we can not solve this by appling Lami's Rule]

Since the body is in equ. and the forces are concurrent

$\Sigma F_x = 0, \quad T_3 \cos\theta - 50 = 0$ — (i)

$\Sigma F_y = 0, \quad T_3 \sin\theta - T_2 = 0$ — (ii)

(ii) ÷ (i)

$$\frac{\cancel{T_3}\sin\theta}{\cancel{T_3}\cos\theta} = \frac{50}{T_2} = \frac{50}{28}$$

$\tan\theta = 0.56$

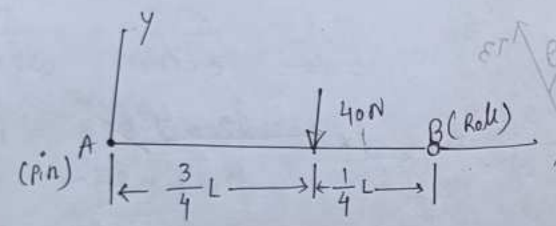
$\Rightarrow \theta = \tan^{-1} 0.56$

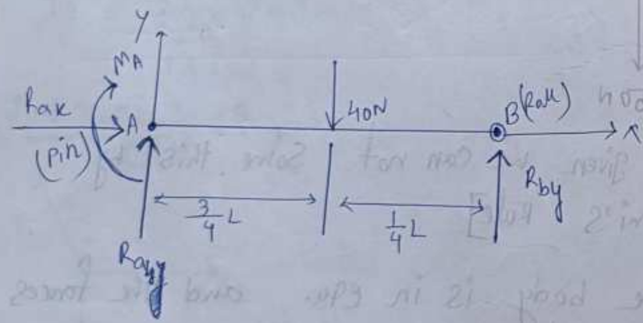
$= 29.2^\circ$

$T_3 \times 0.87 = 50 \quad [\because \cos 29.2 = 0.87]$

$\Rightarrow T_3 = 57.3\ N$

⑨ Determine the reactions at A and B
[Problem – 2D – Non Concurrent – Equilibrium]





$\Sigma F_x = 0,\quad R_{ax} = 0$ — (i)

$\Sigma F_y = 0,\quad R_{ay} + R_{by} - 40 = 0$ — (ii)

$\Sigma M_A = 0,\quad R_{by} \times L - 40 \times \frac{3}{4} L = 0$ — (iii)

from (iii), $L(R_{by} - 30) = 0$

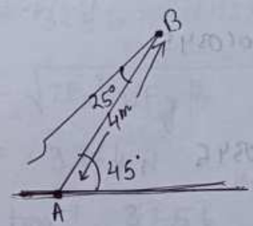
$\Rightarrow R_{by} = 30N$

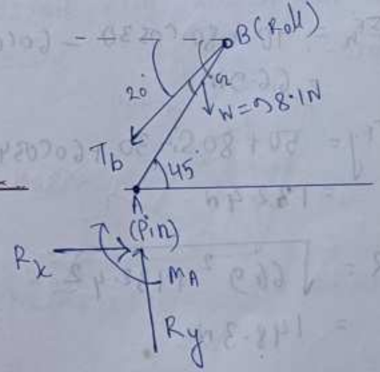
from (i) $R_{ax} = 0\ N$

from (ii) $R_{ay} + 30 - 40 = 0$

$\Rightarrow R_{ay} = 10\ N$

⑩ A man raises a long joist, of length 4m, by pulling on a rope. Find the tension in the rope and the reaction at A.



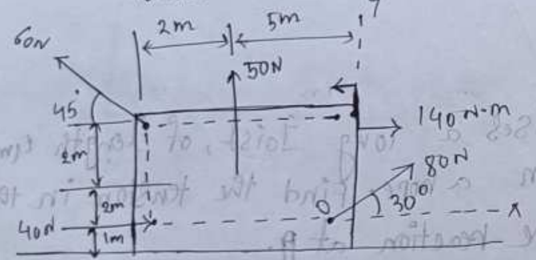


$\Sigma F_h = 0,\quad R_x - T_b \cos 20 = 0$ — (i)

$\Sigma F_y = 0,\quad R_y - 98.1 - T_b \sin 20 = 0$ — (ii)

$\Sigma M_A = 0$

(11) Determine the resultant of the four forces and one couple which act on the plane shown.



$\Sigma F_x = 40 + 80\cos 30 - 60\cos 45$
$= 66.9N$

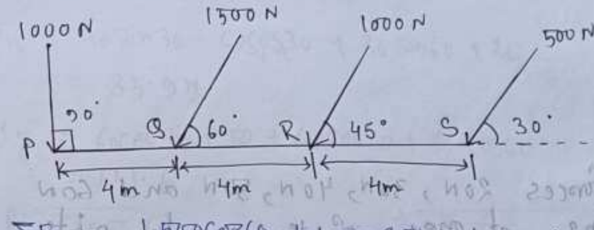
$\Sigma F_y = 50 + 80\sin 30 + 60\cos 45$
$= 132.4N$

$\therefore R = \sqrt{66.9^2 + 132.4^2}$
$= 148.3N$

$\theta = \tan^{-1}\dfrac{132.4}{66.9} = 63.2°$

$M_O = 140 - 50\times5 + 60\cos45\times4 - 60\sin45\times7$
$= -237 N\cdot m$

(12) A horizontal line PQRS is 12m long where PQ = QR = RS = 4m. Forces of 1000N, 1500N, 1000N and 500N act at P, Q, R and S respectively with downward direction. The lines of action



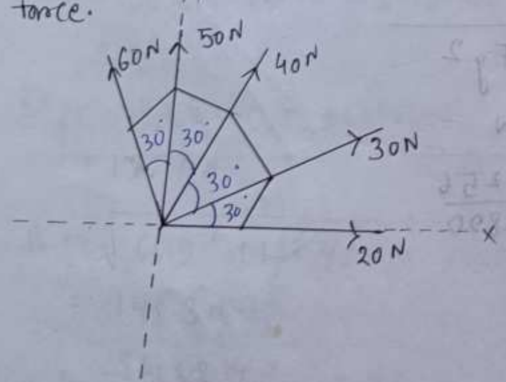
$\Sigma F_x = 1500\cos60 + 1000\cos45 + 500\cos30$
$= 1890.1N$

$\Sigma F_y = 1000 + 1500\sin60 + 1000\sin45 + 500\sin30$
$= 3256$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$
$= 3764 N$

$\theta = \tan^{-1}\dfrac{3256}{1890}$
$\Rightarrow \theta = 59.8°$

(13) The forces 20 N, 30 N, 40 N, 50 N and 60 N are acting at one of the angular points of a regular hexagon, towards the other five angular points, taken in order. Find the magnitude and direction of the resultant force.



F	m	x	y
$\vec{F_1}$	60	$-60\sin30$	$60\cos30$
$\vec{F_2}$	50	0	50
$\vec{F_3}$	40	$40\sin30$	$40\cos30$
$\vec{F_4}$	30	$30\sin60$	$30\cos60$
$\vec{F_5}$	20	20	0

$$\Sigma F_x = 40\sin30 - 60\sin30 + 30\sin60 + 20$$
$$= 35.9\ N$$

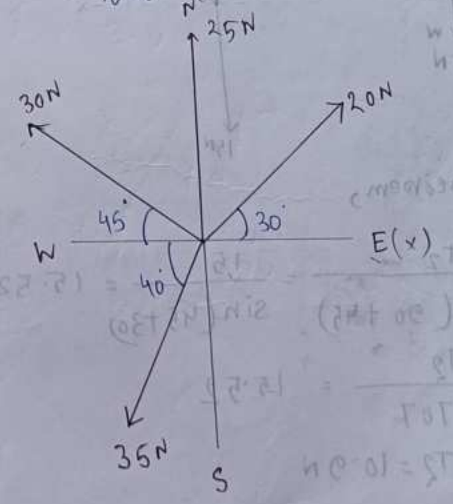
$$\Sigma F_y = 60\cos30 + 50 + 40\cos30 + 30\cos60$$
$$= 151.6$$

$R = 155.7\ N$

$\theta = \tan^{-1}\dfrac{151.6}{35.9}$

$= 76.6°$

(14)



F	m	x	y
$\vec{F_1}$	20 N	20 cos 30	20 sin 30
$\vec{F_2}$	25 N	0	25
$\vec{F_3}$	30 N	− 30 cos 45	30 sin 45
$\vec{F_4}$	35 N	− 35 cos 40	−35 sin 40

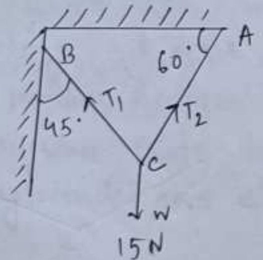
$\Sigma F_x = -30.7$

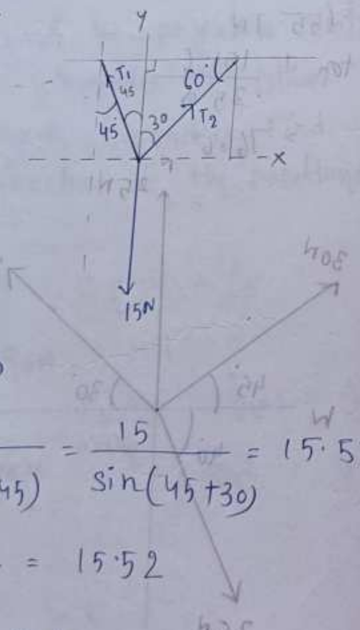
$\Sigma F_y = 33.7$

$\therefore R = 45.5$

$\tan^{-1} \frac{33.7}{30.7} = 47.6°$

(15)





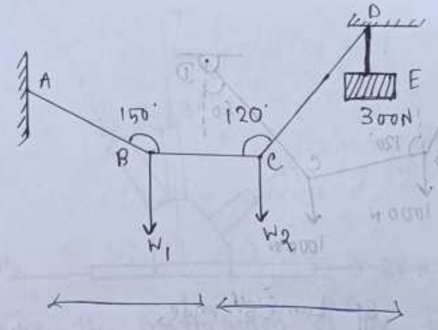
Appling Lami's theorem,

$$\frac{T_1}{\sin(90+60)} = \frac{T_2}{\sin(90+45)} = \frac{15}{\sin(45+30)} = 15.52$$

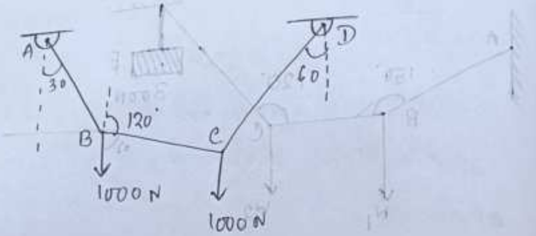
$$\Rightarrow \frac{T_1}{0.85} = \frac{T_2}{0.707} = 15.52$$

$T_1 = 7.765$ N $\quad T_2 = 10.9$ N

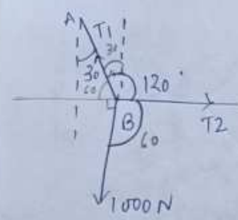
(16)



17



By Lami's rule,

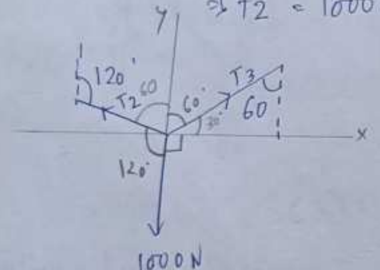


$$\frac{T_1}{\sin 60} = \frac{T_2}{\sin(90+60)} = \frac{1000}{\sin(120+30)}$$

$$\Rightarrow \frac{T_1}{\sin 60} = \frac{T_2}{\sin 150} = \frac{1000}{\sin 150} = 2000$$

$\Rightarrow T_1 = 1732\,N$

$\Rightarrow T_2 = 1000\,N$



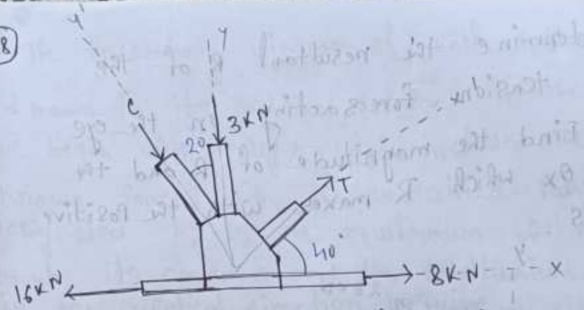
$$\frac{T_2}{\sin(90+30)} = \frac{T_3}{\sin 120} = \frac{1000}{\sin 120} = 1154.7$$

$$\frac{T_3}{\sin 120} = 1154.7$$

$\Rightarrow T_3 = 1000\,N$

$T_1 = 1154.7$
$T_2 = 577.3$
$T_3 = 2000$

18



Determine the magnitudes of the forces C and T, which along with the other three forces shown, act on the bridge-truss joint

For the x-y axes as shown we have.

$\Sigma F_h = 0 \quad T\cos 40 + 8 - 16 + C\sin 20 = 0 \quad$ — (i)

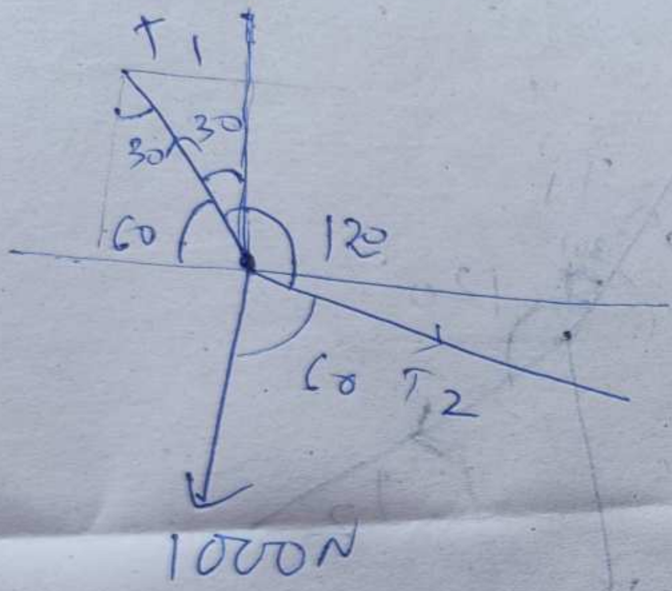
$\Sigma F_y = 0 \quad T\sin 40 - C\cos 20 - 3 = 0 \quad$ — (ii)

$0.766T + 0.342C = 8$ — (i)

$0.643T - 0.940C = 3$ — (ii)

By solving (i) and (ii) we get

$T = 9.09\,kN, \quad C = 3.03\,kN$



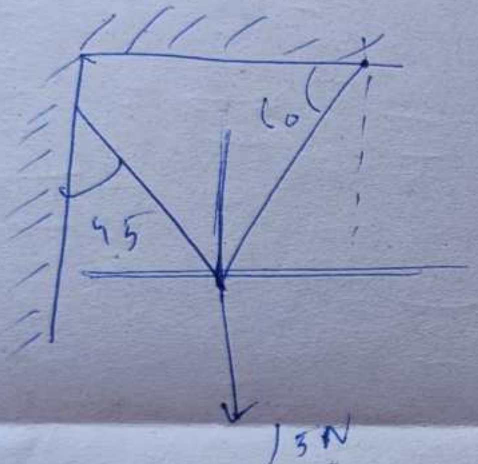
$$\frac{T_1}{\sin 60} = \frac{T_2}{\sin(60+90)} = \frac{1000}{\sin 120}$$

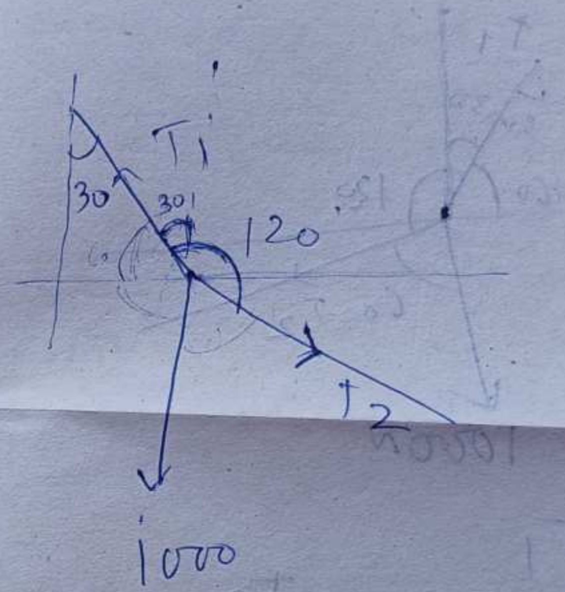
$T_1 = 1000N$

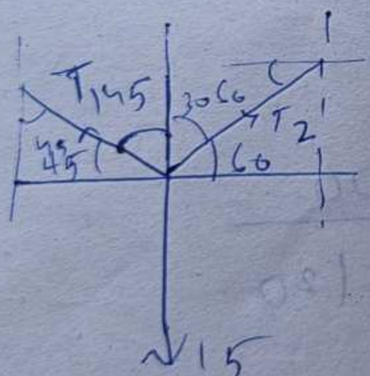
$T_2 = 577.3\ N$

which along with the other three forces shown

... truss joint



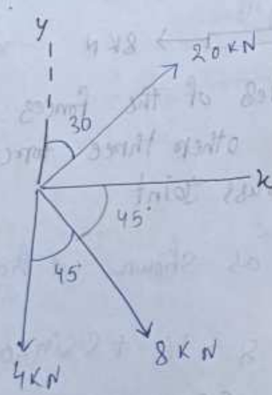




$$\frac{T_1}{\sin 60} = \frac{T_2}{\sin 150} = \frac{1000}{\sin 120}$$

$$\frac{T_1}{\quad} = \frac{T_2}{\quad} = \frac{15}{\quad} = 15.53$$

(19) Determine the resultant R of the three tension forces acting on the eye bolt. Find the magnitude of R and the angle θ_x which R makes with the positive x-axis.



F	m	x	y
$\vec{F_1}$	20 KN	20 sin 30	20 cos 30
$\vec{F_2}$	8 KN	8 sin 45	− 8 sin 45
$\vec{F_3}$	4 KN	0	− 4

$\therefore \Sigma F_x = 20 \sin 30 + 8 \sin 45 + 0$

$= 15.6 N$

$\Sigma F_y = 20 \cos 30 - 8 \cos \sin 45 - 4$

$= 7.66 N$

$R = \sqrt{15.6^2 + 7.66^2}$

$= 17.38 N$

$\theta = \tan^{-1} \frac{7.66}{15.6}$

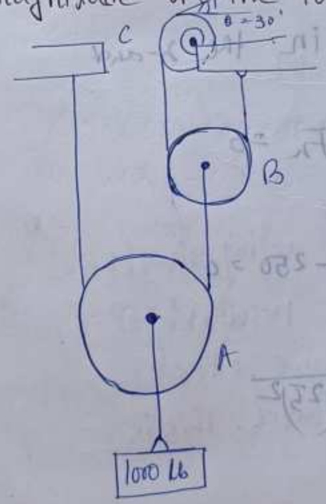
$= 26.15°$

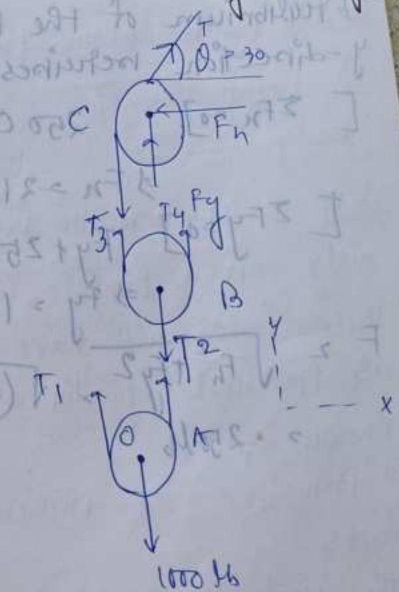
(20) The free-body diagram of each pulley is drawn in its relative position to the others.

Sol^n

We begin with pully A, which included the only known force with the unspecified pully radius designated by r, the equilibrium of moments about its center o and the equilibrium of forces in the vertical direction require.

Calculate the tension T in the cable which supports the 1000-lb load with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C.





$[\Sigma M_o = 0]$ $T_1 r - T_2 r = 0 \Rightarrow T_1 = T_2$

$[\Sigma F_y = 0]$ $T_1 + T_2 - 1000 = 0$

$\Rightarrow 2T_1 = 1000$

$\Rightarrow T_1 = 500\,lb = T_2$

From the example of pulley A we may write the equilibrium of forces on pulley B by inspection as

$T_3 = T_4 = T_2/2 = 250\,lb$

for pulley C the angle $\theta = 30°$ in no way affects the moment of T about the center of the pulley, so that moment equilibrium requires

$T = T_3$ or $T = 250\,lb$

Equilibrium of the pulley in the x- and y-direction requires

$[\Sigma F_x = 0]$ $250 \cos 30 - F_x = 0$

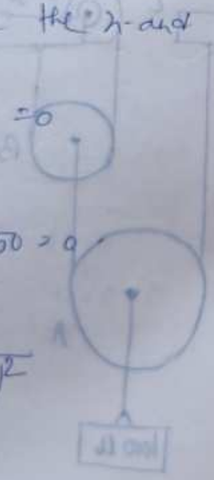
$\Rightarrow F_x = 217\,lb$

$[\Sigma F_y = 0]$ $F_y + 250 \sin 30 - 250 = 0$

$\Rightarrow F_y = 125\,lb$

$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(217)^2 + (125)^2}$

$= 250\,lb$



(21) The uniform long I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable at C it is desired to elevate end B to a position 3m above end A. Determine the required tension P, the reaction at A, and the angle θ made by the beam with the horizontal in the elevated position.

done

• OBJECTIVE Questions

1) Which of the following state is correct?

both a and b

(a) A force is an agent which produces or tends to produce motion.

(b) A force is an agent which stops or tends to stop motion.

3) In order to determine the effects of a force acting on a body, we must know

a) Its magnitude and direction of the line along which it acts.

b) Its nature (where push or pull)

c) Point through which it acts on the body.

3) If a number of forces are acting simultaneously on a particle, then the resultant of these forces will have the same effect as produced by the all the forces. This is known as -

(a) Principle of physical independence of forces

4) The vector method, for the resultant force, is also called polygon law of forces -

Incorrect

(5) The resultant of two forces P and Q acting at an angle θ is equal to -

$\sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

(6) If the resultant of two forces P and Q acting at an angle (α) with P, then -

$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta}$

• Laws for the resultant force - The resultant force of a given system of forces, may also be found out by the following laws -

① Triangle law of forces

② Polygon law of forces

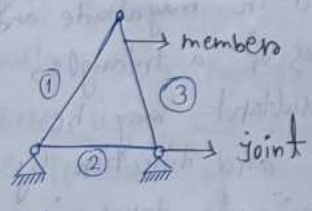
① Triangle law of forces -

It states, "If two forces acting simultaneously on a particle, be represented in magnitude and direction by the two sides of a triangle, taken in order; their resultant may be represented in magnitude and direction by the third side of the triangle, taken in opposite order."

③ Polygon law of forces:-
It is an extension of triangle Law of forces for more than two forces, which states, "If a number of forces acting simultaneously on a particle, be represented in magnitude and direction by the sides of a polygon taken in order; then the resultant of all these forces may be represented, in magnitude and direction by the closing side of the polygon, taken in opposite order.

- Analysis of Pin-joined Plane Frames

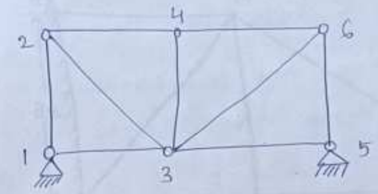
$\boxed{m = 2j - 3}$ [where, j = no. of joint
m = no. of member]



3 joints, 3 members

$\therefore m = 3$, $2j - 3$
$= 2\times3 - 3$
$= 6 - 3$
$= 3$

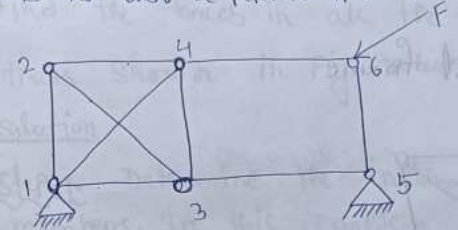
LHS = RHS $\therefore m = 2j - 3$ this equation is satisfied. This is a perfect ~~joi~~ frame.



$m = 9$, $j = 6$

$\therefore 2j - 3 = 2\times6 - 3 = 9$

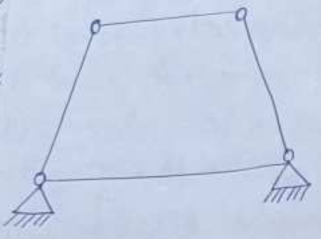
This is also a perfect frame.



$m = 9$
$j = 6$ This fram is not capable of retengular shape, if loaded at joint 6. Hence the necessary and sufficient condition for a perfect frame that it should retain its shape when load is applied in the plane of frame at any point.

- $m = 2j - 3$ — If this equation is not satisfied and if $2j - 3 > m$, then this type of frame is called deficient frame and when $2j - 3 < m$, then this type of frame is called redundant frame.

③

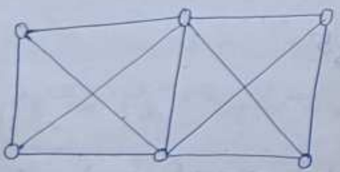


$m = 4,\ j = 4$

$2j - 3 = 2\times4 - 3 = 5$

$m < 2j - 3$

This is a deficient frame



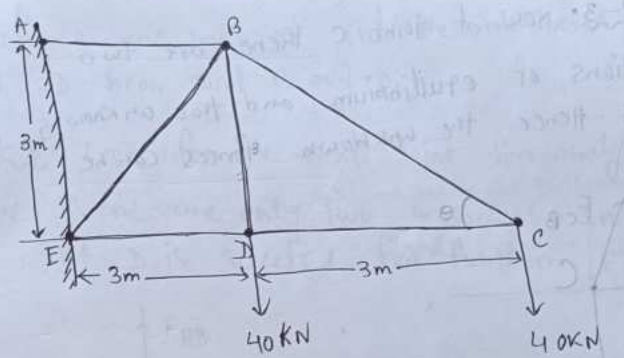
$m = 11,\ j = 6$

$2j - 3 = 2\times6 - 3 = 9$

$\therefore m > 2j - 3$

This is redundant frame

①



Find the forces in all the members of the truss shown in Fig. tabulate the results

Solution

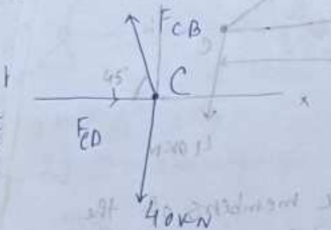
Step-1. Determine the inclination of all inclined members. In this problem

$\tan\theta = \frac{3}{3} = 1$

$\theta = 45^\circ$

Step-2. Look for a joint where there are only two unknowns. If such a joint is not ~~availa~~ available, determine the reactions at supports and then the unknown at one of the support may reduce to only two. In this case at joint C, there are only two unknown forces, forces in members CB and CD say F_{CB} and F_{CD}

(3) Step-3. Now at joint C there are two equations of equilibrium and two unknowns forces. Hence the unknown forces can be found.



At joint C,

$F_{CB} \sin 45 - 40 = 0$ —(i) [$\because \Sigma F_V = 0$]

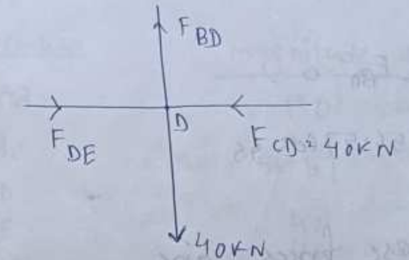
$F_{CD} - F_{CB} \cos 45 = 0$ —(ii) [$\because \Sigma F_H = 0$]

$\therefore$ From (i), $F_{CB} = 56.57 KN$

From (ii) $F_{CD} = 56.57K \frac{1}{\sqrt{2}}$
$= 40KN$

Step-4 On the diagram of the truss, mark arrow on the members near the joint analyse to indicate the forces on the joint. At the other end, mark the arrows in the reverse direction. In the present case, arrows are marked at joint C in member CB and CD. Then reversed directions are marked in the members CB and CD near joint B and D.

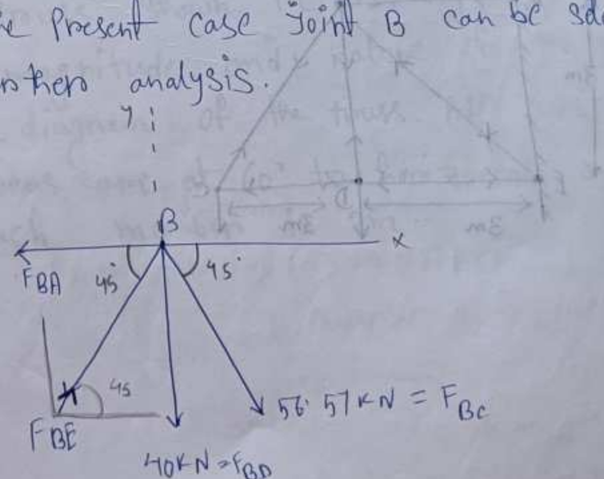
Step-5. Look for the next joint for analysis where there are only two unknowns. In this case, joint D is selected for further analysis.



$\therefore F_{DE} = 40KN$, $F_{BD} = 40KN$ [$\because$ Balance করছে]

Step-6 - Repeat step 4 and 5 till forces in all members are determined.

In the present case joint B can be selected for further analysis.



(3) $\Sigma F_V = 0$

$\therefore F_{BE} \sin 45 - F_{BD} - F_{BC} \sin 45 = 0$ —(1)

From (1),

$\Rightarrow F_{BE} \times \sin 45 - 40 - 56.57 \times \frac{1}{\sqrt{2}} = 0$

$\Rightarrow F_{BE} = 113.14$ KN

b. $\Sigma F_H = 0$

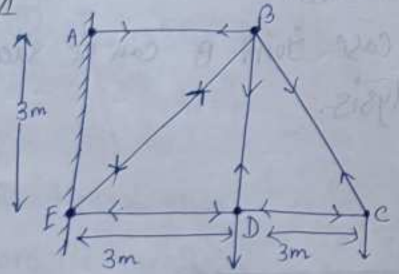
$\Rightarrow F_{BE} \cos 45 + F_{BC} \cos 45 - F_{BA} = 0$

$\Rightarrow F_{BA} = 113.14 \cos 45 + 56.57 \cos 45$

$= 120$ KN

This direction of these forces are marked on the diagram. Now the analysis complete since the forces in all members are found.

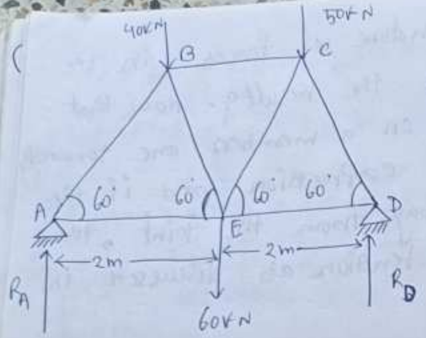
Step-7



step-7 Determine the nature of forces in the members and tabulate the results. Now that the arrows marked on a member are towards joints the member is compression and if the arrows are going away from the joint, then the members is in tension as discussed in Art.

member	magnitude of force in KN	nature
AB	120	Tension
BC	56.57	T
CD	40	Compression
DE	40	C
BE	113.14	C
BD	40	T

(2) Determine the forces in all members of the truss shown in this fig. and indicate the magnitude and nature of the forces on the diagram of the truss. All indicates members are at 60° to horizontal and length of each member is 2m.



Sol
In this Problem we don't find a joint with only 2 unknowns forces stpaifed way. Hence let us first find the support reactions.

Considering the equilibrium of entire frame.

$\Sigma M_A = 0$

$-R_D \times 4 + 40 \times 1 + 60 \times 2 + 50 \times 3 = 0$

$\Rightarrow 4R_D - 40 - 60 \times 2 - 50 \times 3 = 0$

$\Rightarrow 4R_D - 40 - 120 - 150 = 0$

$\Rightarrow R_D = 77.5\,KN$.

$\Sigma F_H = 0$

$\Rightarrow H_A = 0$ [∵ লম্ব দিকে কোনো বৈশাখ্য থাকে না]

∴ Reaction at A is vertical.

$\Sigma F_V = 0$

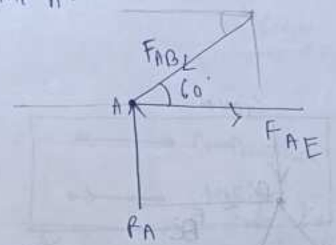
$R_A + R_D - 40 - 60 - 50 = 0$

$\Rightarrow R_A + R_D = 150$

$\Rightarrow R_A + 77.5 = 150$

$\Rightarrow R_A = 150 - 77.5$
$= 72.5\,KN$

Now at joint A there are only two unknown forces. Hence consider the equilibrium of joint A.



$\Sigma F_V = 0$

$-F_{AB} \sin 60 + R_A = 0$

$\Rightarrow F_{AB} \sin 60 - R_A = 0$

$\Rightarrow F_{AB} \sin 60 = 72.5$

$\Rightarrow F_{AB} = 83.72\,KN$ (Compression)

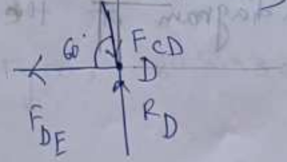
$\Sigma F_H = 0$

$-F_{AB} \cos 60 + F_{AE} = 0$

$\Rightarrow 83.72 \times \cos 60 - F_{AE} = 0$

$\Rightarrow F_{AE} = 41.86\,KN$ (Tension)

Joint D,



$\Sigma F_V = 0$

$\Rightarrow -F_{DC} \sin 60 + R_D = 0$

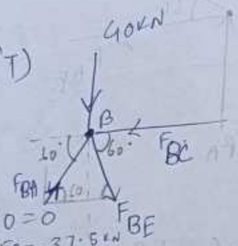
$\Rightarrow F_{DC} = \dfrac{77.5}{\sin 60} = 89.5\,\text{kN}\ (C)$

$\Sigma F_H = 0$

$\Rightarrow -F_{DE} + F_{DC} \cos 60 = 0$

$\Rightarrow F_{DE} = 89.5 \cos 60$
$= 44.75\,\text{kN}\ (T)$

Joint B,

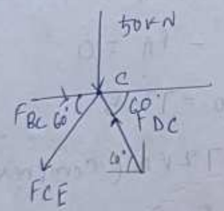


$\Sigma F_V = 0$

$F_{AB} \sin 60 - 40 - F_{BE} \sin 60 = 0$ $\quad \therefore F_{BE} = 37.5\,\text{kN}$

$\Rightarrow F_{BC} = 60.62\,\text{kN}\ (C)$

Joint C,

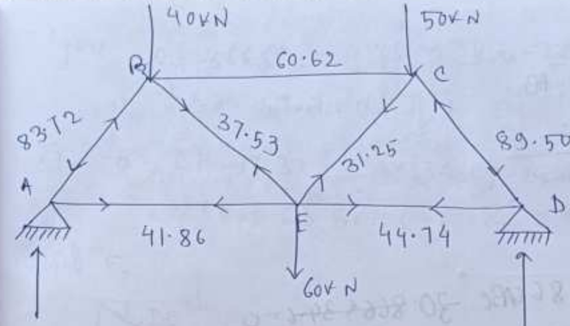


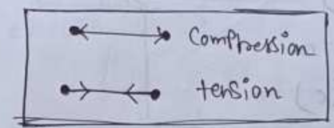
$\Sigma F_V = 0$

$\Rightarrow F_{DC} \sin 60 - 50 - F_{CE} \sin 60 = 0$

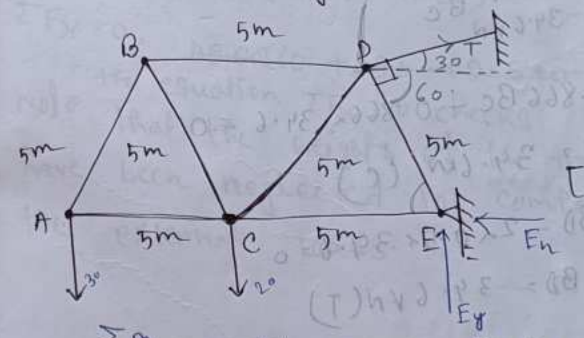
$\Rightarrow F_{CE} = 31.76\,\text{kN}\ (T)$

Now the forces in all the members are known. If joint E is analysed it will give check from the analysis. The results are shown on diagram of the truss in fig.





(3) (Exam) Compute the force in each members of the loaded cantilever truss by the method of joints.



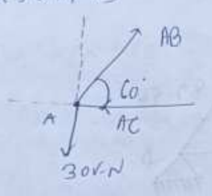
[Like elder Problem]

$\Sigma M_E = 0, \ 5T - 20 \times 5 - 30 \times 10 = 0$ — (1) $\Rightarrow T = 80\,\text{kN}$

$\Sigma F_x = 0, \ 80 \cos 30 - E_H = 0, \ E_H = 69.3\,\text{kN}$

$\Sigma F_y = 0, \ 80 \sin 30 + E_y - 20 - 30 = 0, \ E_y = 10\,\text{kN}$

at joint A,



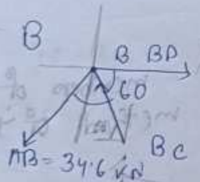
$\Sigma F_y = 0$, $0.866\,AB - 30\ \cancel{0.866 \times 34.6} = 0$

$\Rightarrow AB = 34.6\,kN\,(T)$

$\Sigma F_x = 0$ $\quad AC - 0.5 \times 34.6 = 0$

$\Rightarrow AC = 17.32\,kN\,(C)$

at joint B



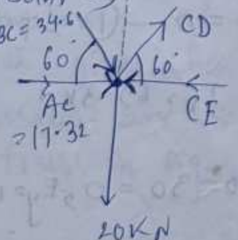
$\Sigma F_y = 0$, $0.866\,BC - 0.866 \times 34.6 = 0$

$\Rightarrow BC = 34.6\,kN\,(C)$

$\Sigma F_x = 0$ $\quad BD - 2 \times 0.5 \times 34.6 = 0$

$\Rightarrow BD = 34.6\,kN\,(T)$

at joint C,



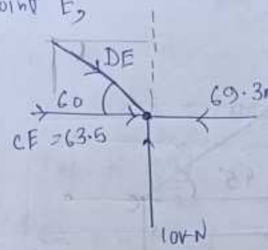
$\Sigma F_y = 0$, $0.866\,CD - 0.866 \times 34.6 - 20 = 0$

$\Rightarrow CD = 57.7\,kN\,(T)$

$\Sigma F_x = 0$, $CE - 17.32 - 0.5 \times (34.6) - 0.5 \times 57.7 = 0$

$\Rightarrow CE = 63.5\,kN\,(C)$

Joint E,



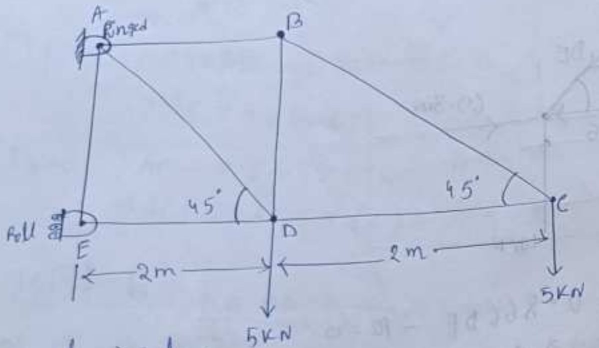
$\Sigma F_y = 0$, $0.866\,DE - 10 = 0$

$\Rightarrow DE = 11.55\,kN\,(C)$

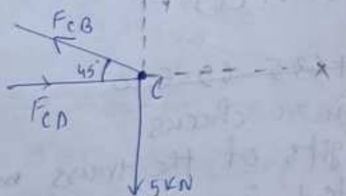
$\Sigma F_x = 0$, $\cancel{DE \cos 60 + 63.5 - 69.5 = 0}$

Note: the equation $\Sigma F_x = 0$ checks that the weights of the truss members have been neglected in comparison with the external loads.

(4) A cantilever braced truss supported on rollers at E and hinged at A is loaded as shown in the fig. Determine graphically or otherwise, the forces in the members of the truss, also determine the reactions at A and E.



Now at joint C,



$\Sigma F_V = 0$

$\Rightarrow F_{CB} \sin 45 - 5 = 0$

$\Rightarrow F_{CB} = 5\times\sqrt{2} = 7.07 KN$

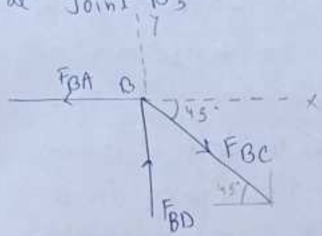
$\Sigma F_H = 0$

$\Rightarrow F_{CD} - F_{CB} \cos 45 = 0$

$\Rightarrow F_{CD} = 7.07 \times \frac{1}{\sqrt{2}}$

$= 4.99 KN$

Now at joint B,



$\Sigma F_H = 0$

$\Rightarrow F_{BC} \cos 45 - F_{BA} = 0$

$\Rightarrow F_{BA} = \frac{7.07}{\sqrt{2}}$

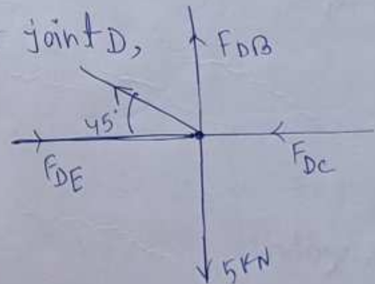
$\Rightarrow F_{BA} = 4.99 KN$

$\Sigma F_Y = 0$

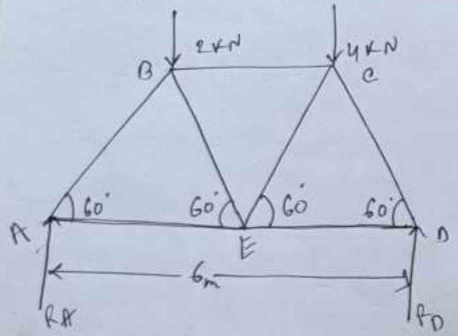
$\Rightarrow -F_{BC} \sin 45 + F_{BD} = 0$

$\Rightarrow F_{BD} = 4.99 KN$

Now at joint D,



(5) This fig shows a warren girder consisting of seven members each of 3m length freely supported at its end points. The girder is loaded at B and C as shown. Find the forces in all the members of the girder, indicating wheather the force is compressive or tensile.



$\Sigma M_A = 0$

$\Rightarrow R_D \times 6 - 2 \times 1.5 - 4 \times 4.5 = 0$

$\Rightarrow 6R_D = 21$

$\Rightarrow R_D = 3.5$ KN

$\Sigma F_H = 0$

$\therefore H_A = 0$

∴ Reaction at A is vertical

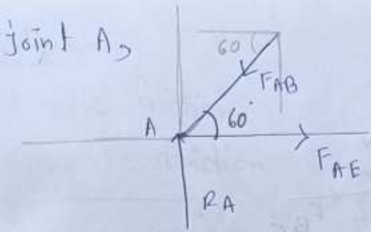
$\therefore \Sigma F_V = 0$

$\Rightarrow R_D + R_A - 2 - 4 = 0$

$\Rightarrow R_A = 6 - 3.5$

$\Rightarrow R_A = 2.5$ KN

at joint A,



$\Sigma F_V = 0$

$R_A - F_{AB} \sin 60 = 0$

$\Rightarrow 2.5 - F_{AB} \sin 60 = 0$

$\Rightarrow F_{AB} = 2.88$ KN (C)

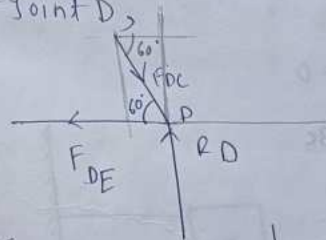
$\Sigma F_H = 0$

$\Rightarrow F_{AE} - F_{AB} \cos 60 = 0$

$\Rightarrow F_{AE} = 2.88 \times \cos 60$

$= 1.44$ KN (T)

at joint D,



$\Sigma F_V = 0$

$R_D - F_{DC} \sin 60 = 0$

$\Rightarrow 3.5 - F_{DC} \sin 60 = 0$

$\Rightarrow F_{DC} = 4.04$ KN (C)

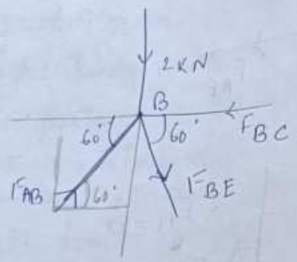
$\Sigma F_H = 0$

$\Rightarrow -F_{DE} + F_{DC} \cos 60 = 0$

$\Rightarrow F_{DE} = 4.04 \cos 60$

$\Rightarrow F_{DE} = 2.02$ KN (T)

at joint B,



$\Sigma F_V = 0$

$\Rightarrow -2 - F_{BE}\sin 60 + F_{AB}\sin 60 = 0$

$\Rightarrow F_{BE} = 0.57\,KN$ (T)

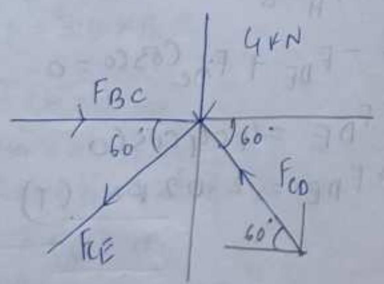
$\Sigma F_H = 0$

$F_{BE}\cos 60 + F_{AB}\cos 60 - F_{BC} = 0$

$\Rightarrow 0.5 \times \cos 60 + 2.88 \cos 60 = F_{BC}$

$\Rightarrow F_{BC} = 1.69\,KN$ (C)

at joint C,

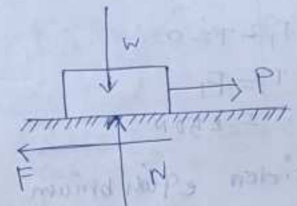


$\Sigma F_V = 0$

$\Rightarrow -F_{CE}\sin 60 + F_{CD}\sin 60 - 4 = 0$

$\Rightarrow F_{CE} = 8.65\,KN$ (T)

- Friction
  - Static friction
  - Dynamic friction
    - Sliding friction
    - Rolling friction



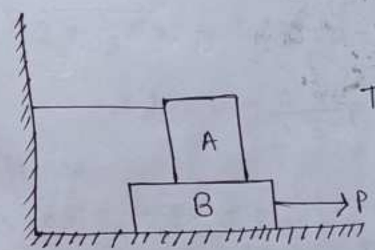
$\boxed{\text{Coefficient of friction} = \frac{F}{N}}$

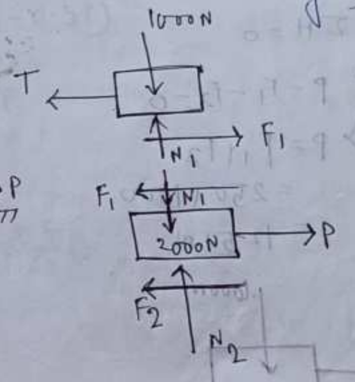
[where F is limiting friction and N is normal reaction between the constant surface]

$\boxed{\mu = \frac{F}{N}}$

[Coefficient of friction is denoted by μ]

•





$\Sigma V = 0$

$\Rightarrow N_1 - 1000 = 0$

$\Rightarrow N_1 = 1000\,N$

$\because F_1$ is limiting friction,

$\mu = \frac{F_1}{N_1}, \Rightarrow 0.25 = \frac{F_1}{1000}, \Rightarrow F_1 = 250N$ [$\because \mu = 0.25$ for A]

$\Sigma H = 0$

$\Rightarrow F_1 - T = 0$

$\Rightarrow T = F_1$

$= 250N$

consider equilibrium of block B

$\Sigma V = 0$

$\Rightarrow N_2 - 2000 - N_1 = 0$

$\Rightarrow N_2 = 2000 + 1000 = 3000\ N$

$\because F_2$ is limiting friction,

$F_2 = \mu N_2$

$= 0.33 \times 3000 \quad \left[\mu = \frac{1}{3}, \text{for B}\right]$

$= \frac{1}{3} \times 3000$

$= 1000\ N$

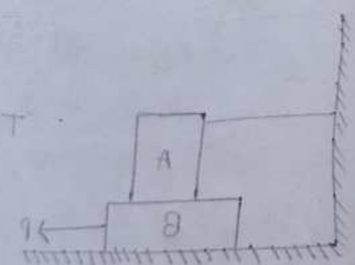
$\Sigma H = 0$

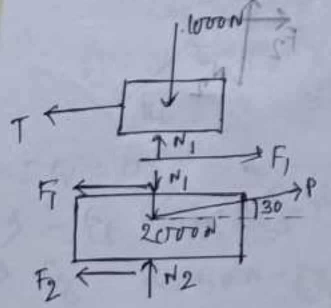
$\Rightarrow P - F_1 - F_2 = 0$

$\Rightarrow P = F_1 + F_2$

$= 250 + 1000$

$= 1250\ N$





(b) when P is inclined

Free body diagram for this case is shown in the fig.

As in the previous ~~friction~~ case,

$F_2 = \mu N_1 = 1000\ N$

and $F_1 = 250N$

consider the equilibrium of block B

$\Sigma V = 0$

$\Rightarrow N_2 - 2000 - N_1 + P \sin 30 = 0$

$\Rightarrow N_2 + P\sin 30^\circ = 2000 + N_1$

$\Rightarrow N_2 + 0.5P = 2000 + 1000$

From law of friction

$F_2 = \frac{1}{3} N_2 = \frac{1}{3}(3000 - 0.5P)$

$= 1000 - \frac{0.5}{3} P$

$\Sigma H = 0$

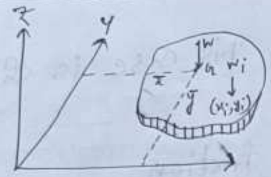
$P\cos 30 - F_1 - F_2 = 0$

$\Rightarrow P\cos 30 - 250 - (1000 - \frac{0.5}{3}P) = 0$

$\Rightarrow P(\cos 30 + \frac{0.5}{3}) = 1250$

$\Rightarrow P = 1210.43N$

Centroid and moment of Inertia



Consider a flat plate of thickness t as shown in the fig. Let w_i be the weight of any elemental portion acting at a point (x_i, y_i). Let W be the total weight of the plate acting at the point $(\bar{x}, \bar{y})$. According to defination of centre of gravity, the point $(\bar{x}, \bar{y})$ is the centre of gravity. Now,

Total weight, $W = \Sigma w_i$

taking moment about x axis and equating moment of resultant to moment of component forces, we get

$$W\bar{y} = w_1y_1 + w_2y_2 + w_3y_3 + \cdots = \Sigma w_i y_i$$

$$\therefore \bar{y} = \frac{\Sigma w_i y_i}{W}$$

Similarly, taking moment about y axis we get,

$$W\bar{x} = w_1x_1 + w_2x_2 + w_3x_3 + \cdots = \Sigma w_i x_i$$

$$\bar{x} = \frac{\Sigma w_i x_i}{W}$$

- **centroid**

Let A_i be the area of the ith element of plate of uniform thickness in the plate. If γ is the unit weight of the material of plate and t its uniform thickness, then

$$w_i = \gamma A_i t \quad \text{— (III)}$$

$$\therefore \text{Total weight, } W = \Sigma \gamma A_i t = \gamma t \Sigma A_i = \gamma t A \quad \text{— (IV)}$$

where, $A = \Sigma A_i$, is total area

we know, $\bar{y} = \frac{\Sigma w_i y_i}{W}$ — (I)

$\bar{x} = \frac{\Sigma w_i x_i}{W}$ — (II)

from (I) and (II), we get

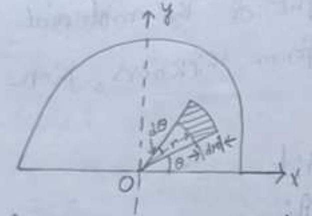
$$\bar{y} = \frac{\Sigma A_i \gamma t \, y_i}{\gamma t A} = \frac{\Sigma A_i y_i}{A} \quad \text{[from (III) and (IV)]}$$

$$\bar{x} = \frac{\Sigma A_i \gamma t \, x_i}{\gamma t A} = \frac{\Sigma A_i x_i}{A}$$

[$\because$ γ and t are constants]

■ the centroid of a triangle is at a distance $\frac{h}{3}$ from the base (or $\frac{2h}{3}$ from the apex) of the triangle where h is the height of the triangle. [apex = শীর্ষ]

• centroid of a semicircle – [centroid = ভরকেন্দ্র]



Consider the semicircle of radius R as shown in the fig. Due to symmetry centroid must lie on y axis. Let its distance from diametral axis be $\bar{y}$. To find $\bar{y}$, consider an element at a distance r from the centre O of the semicircle, radial width being dr and bound by radii at θ and $\theta + d\theta$.

(ব্যাস রেখা)

(ব্যাসার্ধ)

Area of element $= r\,d\theta\,dr$

Its moment about diametral x axis is given by: $r\,d\theta \times dr \times r\sin\theta$

$$= r^2 \sin\theta \, dr\, d\theta$$

∴ Total moment of area about diametral axis.

$$= \int_0^{\pi}\int_0^{R} r^2 \sin\theta \, dr\, d\theta$$

$$= \int_0^{\pi} \left[\frac{r^3}{3}\right]_0^R \sin\theta\, d\theta$$

$$= \frac{R^3}{3}\left[-\cos\theta\right]_0^{\pi}$$

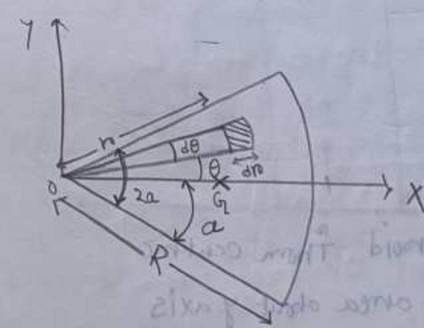
$$= \frac{2R^3}{3}$$

Area of semicircle $A = \frac{1}{2}\pi R^2$

$$\bar{y} = \frac{\text{moment of area}}{\text{total area}} = \frac{\frac{2R^3}{3}}{\frac{1}{2}\pi R^2} = \frac{4R}{3\pi}$$

(বৃত্তকলা)

• centroid of sector of a circle –



Consider the sector of a circle of angle 2α as shown in the figure.

Due to symmetry, centroid lies on x axis. To find its distance from the centre O, consider the element area shown.

Area of the element $= r\,d\theta\,dr$

It's moment about y axis =

$r\,d\theta \times dr \times r\cos\theta = r^2\cos\theta\,dr\,d\theta$

$\therefore$ Total moment of area about y axis —

$= \int_{-a}^{a}\int_{0}^{R} r^2\cos\theta\,dr\,d\theta$

$= \int_{-a}^{a}\left[\frac{r^3}{3}\right]_0^R \cos\theta\,d\theta$

$= \frac{R^3}{3}\left[\sin\theta\right]_{-a}^{a}$

$= \frac{R^3}{3}\,2\sin a$

Total area of the sector,

$= \int_{-a}^{a}\int_{0}^{R} r\,dr\,d\theta$

$= \int_{-a}^{a}\left[\frac{r^2}{2}\right]_0^R d\theta$

$= \frac{R^2}{2}\cdot 2a$

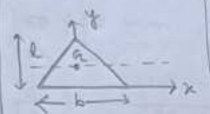
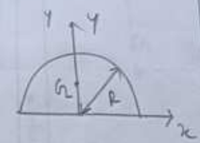
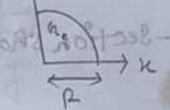
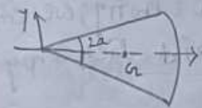
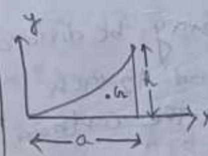
$= R^2 a$

$\therefore$ The dist of centroid from centre O

$= \dfrac{\text{moment of area about y axis}}{\text{Area of the figure}}$

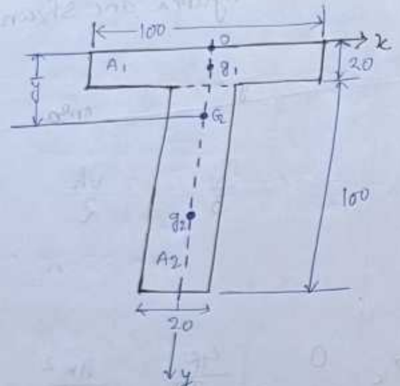
$\bar{x} = \dfrac{\frac{2R^3}{3}\sin a}{R^2 a} = \frac{2R}{3a}\sin a$

• centroids of some common figures are shown in this table —

Shape	fig	$\bar{x}$	$\bar{y}$	Area
Triangle		–	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi R^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi R^2}{4}$
Sector of a circle		$\frac{2R}{3a}\sin a$	0	$R^2 a$
Parabolic Spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$

• Centroid of composite sections

(1)



All dimensions in mm

- Locate the centroid of the T-section shown in the fig.

Selection the axis as shown in fig, we can say due to symmetry centroid lies on y axis, i.e. $\bar{x}=0$

Now the given T-Section may be divided into two rectangles A_1 and A_2 each of size 100×20 and 20×100. The centroid of A_1 and A_2 are $g_1(0,10)$ and $g_2(0,70)$ respectively.

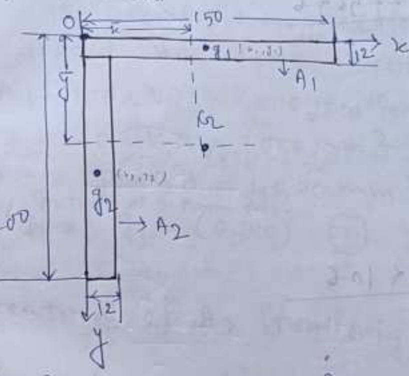
∴ The distance of centroid from the top is given by:

$$\bar{y} = \frac{\Sigma a_i y_i}{\Sigma a_i} = \frac{100\times20\times10 + 20\times100\times70}{100\times20 + 20\times100}$$

$$= 40\,mm$$

Hence, centroid of T-section is on the symmetric axis at a distance 40 mm from the top.

(2) Find the centroid of unequal angle 200×150×12 mm, shown in fig.



The given composite fig. can be divided into two rectangles.

$A_1 = 150\times12 = 1800\,mm^2$

$A_2 = 12(200-12) = 2256\,mm^2$

Total area $= A_1 + A_2 = 1800 + 2256 = 4056\,mm^2$

Selection referance axis x and y as shown in the fig. The centroid of A_1 is $g_1(75, 6)$ and that of A_2 is $g_2\left[6, 12 + \frac{(200-12)}{2}\right]$

$= g_2(6, 106)$

$\bar{x} = \frac{\sum a_i x_i}{\sum a_i} =$

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$

$= \frac{1800 \times 75 + 2256 \times 6}{4056}$

$= 36.62$ mm

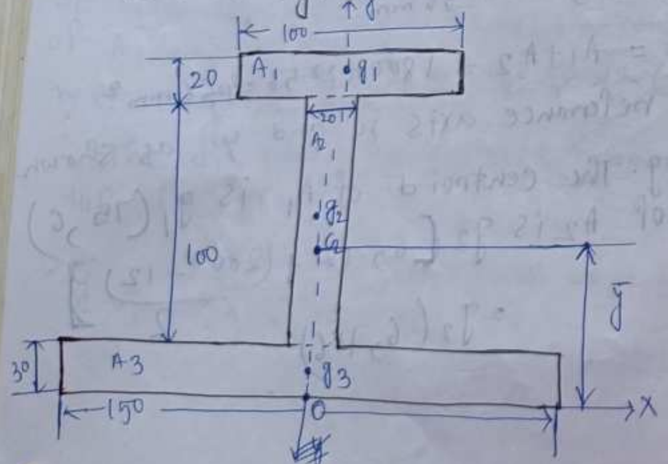
$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{1800 \times 6 + 2256 \times 106}{4056}$

$= 61.62$ mm

Thus the centroid is at $\bar{x} = 36.62$ mm and $\bar{y} = 61.62$ mm as shown in the fig.

③ Locate the Centroid of the I-Section shown in fig.



selecting the co-ordinate system as shown in the fig, due to symmetry centroid lies must on Y axis,

i.e, $\bar{x} = 0$

Now, the composition section (I-section) may be divided into three rectangles.

$A_1 = 100 \times 20 = 2000$ mm^2

$A_2 = 20 \times 100 = 2000$ mm^2

$A_3 = 150 \times 30 = 4500$ mm^2

Centroid of A_1 from origin

$g_1 = (0, 140)$ or $y_1 = 30 + 100 + \frac{20}{2} = 140$ mm

Centroid of A_2 from origin

$g_2 = (0, 80)$ or $y_2 = 30 + \frac{100}{2} = 80$ mm

Centroid of A_3 from origin

$g_3 = (0, 15)$ or $y_3 = \frac{30}{2} = 15$ mm

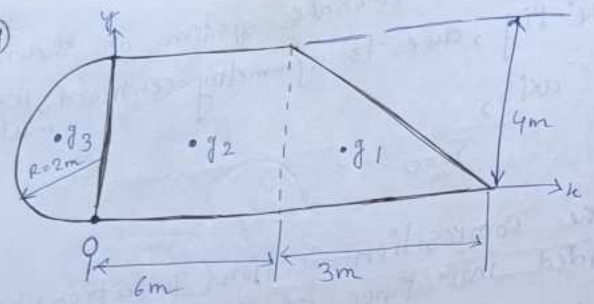
∴ Now,

$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_3 + A_2 + A_1}$

$= \frac{2000 \times 140 + 2000 \times 80 + 4500 \times 15}{2000 + 2000 + 4500}$

$= 59.71$ mm

(4)



Determine the centroid of the area shown in this fig. with respect to the axes shown.

The composite section is divided into three simple figures, a triangle, a rectangle and a semicircle.

Now area of triangle, $A_1 = \frac{1}{2} \times 3 \times 4$
$= 6 \, m^2$

Area of rectangle, $A_2 = 6 \times 4 = 24 \, m^2$

Area of semicircle, $A_3 = \frac{1}{2} \times \pi 2^2$
$= 2\pi$
$= 6.2832 \, m^2$

$\therefore$ Total area $= A = 6 + 24 + 6.2832$
$= 36.2832 \, m^2$

The coordinates of centroids of these three simple figures are,

$x_1 = 6 + \frac{1}{3} \times 3 = 7m$ [$6 + \frac{3}{3}$ হলো কারণ y অক্ষের সাপেক্ষে বলা হয়েছে এবং ত্রিভুজের ভরকেন্দ্র calculate করার সময় 3 দিয়ে ভাগ করা হয়]

$y_1 = \frac{4}{3}$ [যেই কারণে $\frac{1}{3} \times 3$ হলো অর্থাৎ $\frac{\text{ভূমি}}{3}$ সেই কারণে $\frac{1}{3} \times 4$ হলো, $\frac{\text{লম্ব/উচ্চতা}}{3}$]

$x_2 = \frac{6}{2} = 3$

$y_2 = \frac{4}{2} = 2$

$x_3 = -\frac{4R}{3\pi}$ [(-ve) কারণ neg. quad এ আছে]

[$\because$ centroid টি x অক্ষের negative দিকে আছে তাই $x_3 = -\frac{4R}{3\pi}$]

$= -\frac{4 \times 2}{3\pi}$
$= -0.8488m$

$y_3 = \frac{(2+2)}{2} = 2m$

$\bar{x} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A}$

$= \frac{6 \times 7 + 24 \times 3 + 6.2832 \times (-0.8488)}{36.2832}$

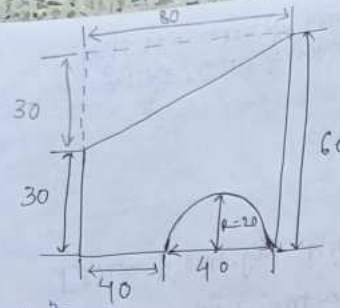
$\bar{x} = 2.995$

$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A}$

$= \frac{6 \times \frac{4}{3} + 24 \times 2 + 6.2832 \times 2}{36.2832}$

$\bar{y} = 1.890$

(5) Determine the centroid of the remaining area (shown hatched)



Big rectangle,

$A_1 = 80 \times 60 = 2400 mm^2$

Triangle,

$A_2 = \frac{1}{2} \times 80 \times 30$

$= 1200 mm^2$

semicircle,

$A_3 = \frac{1}{2} \times \pi \times (20)^2$

$= \frac{1}{2} \times \pi \times 400$

$= 628.31 mm^2$

Total area $(A) = 2400 - 1200 - 628.31$

$= 571.69$

Now,

$x_1 = 40, \quad y_1 = 30$

$x_2 = \frac{80}{3}$
$= 26.67, \quad y_2 = 30 + \frac{30}{3}$
$= 40$

$x_3 = 40 + 20$
$= 60, \quad y_3 = \frac{4R}{3\pi}$
$= \frac{4 \times 20}{3\pi} = 8.488$

$\bar{x} = \frac{2400 \times 40 - 1200 \times 26.67 - 628.31 \times 60}{571.69}$

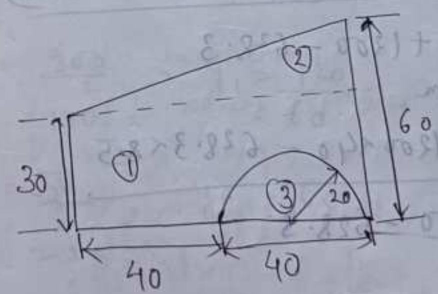
$= 45.99 mm$

$\bar{y} = \frac{2400 \times 30 - 1200 \times 40 - 628.31 \times 8.488}{571.69}$

$= 32.6 mm$

or (Book)

Sol- As the section is not symmetrical about any axis, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the area. Split up the area into three parts as shown in fig. Let left face and base of the trapezium be the axes of reference.



(i) Rectangle,

$a_1 = 80 \times 30 = 2400 \text{ mm}^2$

$x_1 = \frac{80}{2} = 40$, $y_1 = \frac{30}{2} = 15 \text{ mm}$

(ii) Triangle,

$a_2 = \frac{80 \times 30}{2} = 1200 \text{ mm}^2$

$x_2 = \frac{80 \times 2}{3} = 53.3$, $y_2 = 30 + \frac{30}{3} = 40$

(iii) semicircle,

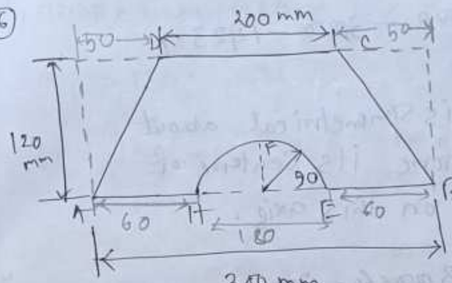
$a_3 = \frac{\pi}{2} \times (20)^2 = 628.3 \text{ mm}^2$

$x_3 = 40 + \frac{40}{2} = 60 \text{ mm}$, $y_3 = \frac{4 \times 20}{3\pi} = 8.5 \text{ mm}$

$$\bar{x} = \frac{(2400 \times 40) + (1200 \times 53.3) - (628.3 \times 60)}{2400 + 1200 - 628.3} = 41.1 \text{ mm}$$

$$\bar{y} = \frac{2400 \times 15 + 1200 \times 40 - 628.3 \times 8.5}{2400 + 1200 - 628.3} = 26.5 \text{ mm}$$

(6)



Find the position of the centre of gravity of the figure.

Big ~~rect~~ Rectangle, $A_1 = 300 \times 120 = 36000$

Triangle (1), $A_2 = \frac{1}{2} \times 50 \times 120 = 3000$

Triangle (2), $A_3 = \frac{1}{2} \times 50 \times 120 = 3000$

Semicircle, $A_4 = \frac{1}{2} \times \pi \times (90)^2 = \frac{1}{2} \times \pi \times 8100 = 12723.4$

$x_1 = \frac{300}{2} = 150$, $y_1 = \frac{120}{2} = 60$

$x_2 = \frac{50}{2} = 25$, $y_2 = 60$

$x_3 = 50 + 200 + 25 = 275$, $y_3 = 60$

$x_4 = 60 + 90 = 150$, $y_4 = \frac{4 \times 90}{3\pi} = 38.2$

$A = 36000 - 3000 - 3000 - 12723.4$

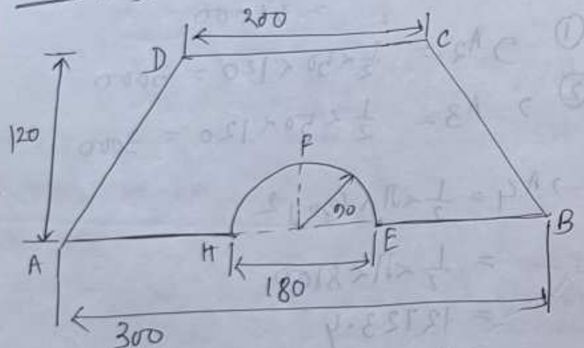
$= 17276.6$

$\therefore$ The section is symmetrical about y-y axis, therefore its centre of gravity will lie on this axis.

$$\therefore \bar{y} = \frac{36000 \times 60 - 3000 \times 60 - 3000 \times 60 - 12723.4 \times 38.2}{17276.6}$$

$= 76.05$ mm

(Q) (book)



As the section is symmetrical about y-y axis, therefore its centre of gravity will lie on this axis. Now consider two portions of this figure viz, trapezium ABCD and semicircle EFH. Let base of the trapezium AB be the axis of reference.

(i) Trapezium ABCD,

$$a_1 = 120 \times \frac{200+300}{2} = 30000 \text{ mm}^2$$

$$y_1 = \frac{120}{3} \times \left(\frac{300 + 2 \times 200}{300+200}\right) = 56$$

(ii) Semicircle,

$$a_2 = \frac{1}{2} \times \pi \times (90)^2 = 4050\pi$$

$$y_2 = \frac{4 \times 90}{3\pi} = \frac{120}{\pi}$$

We know that distance between centre of gravity of the section AB

$$\bar{y} = \frac{(30000 \times 56) - \left(4050\pi \times \frac{120}{\pi}\right)}{30000 - 4050\pi}$$

$= 69.1$ mm

(ii) semicircle, $a_2 = \frac{1}{2}\times\pi\times(90)^2 = 4050\pi$

$\rho^2\alpha \quad \alpha\rho^2 \quad \alpha\rho^2$

Ans = 82.7

$A_1 = 36000,\ A_2 = 3000,\ A_3 = 3000,\ A_4 = 12723.4$

$x_1 = 150,\ y_1 = 60$

$\bar{x} = \dfrac{36000\times150 - 3000\times16.67 - 3000\times266.6 - 12723.4\times150}{36000 - 3000 - 3000 - 12723.4}$

$x_2 = \frac{50}{3},\ y_2 = \frac{120}{3} = 40$

$= 16.67$

$\bar{y} = \dfrac{36000\times60 - 3000\times40\times2 - 12723.4\times38.2}{36000 - 3000 - 3000 - 12723.4}$

$x_3 = 50 + 200 + \frac{50}{3}$ $\quad y_3 = \frac{120}{3} = 40$

$= 266.6$

$= \dfrac{1.43\times10^6}{17276.6}$

$x_4 = 60 + 90 = 150$ $\quad y_4 = \frac{4\times90}{3\pi} = 38.2$

rity

• Force analysis

• Force is a vector quantity (it depends on both magnitude, direction and follow parallelogram laws of addition.

• External and Internal Effects–

1. External effects – Forces can be either applied forces or reactive forces.
2. Internal effects – Deformation throughout the materials.

→ Relation between External effects and Internal effects depends on material properties.

• Virtual work

Previous concept and new concept–

• System are in equilibrium–

→ Then only we are able to apply summation of forces and summation of moments.

• Real problem: many equilibrium configurations can be possible when a connected bodies/members are in relative movement to each others.

→ Although force and moment-equillibrium equations are valid and adequate, are often not the most convenient approach.

→ A method of "work done" is more direct. Also, it enable us to examine the stability of systems in equillibrium.

• vector form (Linear)

$$\boxed{dU = F \cdot dr}$$

$$\boxed{U = \int F \cdot dr = \int (F_x\,dx + F_y\,dy + F_z\,dz)}$$

• vector form (Angular)

$$\boxed{dU = M\,d\theta}$$

$dr_{A/B}$ = displacement of A with respect to B

Work done, $F \cdot dr_{A/B} = F b\,d\theta = M\,d\theta$ (in Joule N-m)
[dU]

• virtual work

• For any assumed and arbitrary small displacement or away from its static equillibrium position and consistent with the system constraints is called a "virtual displacement".

→ virtual work (linear) $\delta U = F \cdot \delta r$

$= \Sigma F \, \delta r = 0$ (equillibrium cond.)

→ virtual work (angular), $\delta U = M \cdot \delta\theta$

$= \Sigma M \, \delta\theta$

$= 0$ (equillibrium cond.)

dr = actual infinitesimall change in position and can be integrated.

δr = virtual infinitesimall change in position and cannot be integrated.

→ mathematically = both quantities are first order differentials.

• Equillibrium of a particle

$$\delta U = F_1 \cdot \delta r + F_2 \cdot \delta r + F_3 \cdot \delta r \ldots$$
$$= \Sigma F \cdot \delta r$$
$$= (i\Sigma F_x + j\Sigma F_y + k\Sigma F_z) \cdot (i\,\delta x + j\,\delta y + k\,\delta z)$$
$$= \Sigma F_x \,\delta x + \Sigma F_y \,\delta y + \Sigma F_z \,\delta z$$
$$= 0$$

$\therefore \boxed{\delta U = 0 \text{ and } \Sigma F = 0}$

• Principle of virtual work

The virtual work done by external active forces on an ideal mechanical system in equillibrium is zero for any and all virtual displacements consistent with the constraints.

$\boxed{\delta U = 0}$

• Equillibrium of ideal systems of Rigid Body

Ideal systems of Rigid Body → two or more rigid members linked together by mechanical connections which are incapable of absorbing energy through elongation and compression, and in which friction is small enough to be neglected.

Active forces → capable of doing virtual work during possible virtual displacements.

Reactive forces → only reactive forces exist, no virtual displacements takes place in the direction of the force.

Internal forces → Forces in the connections between members. The net work done by the internal forces at the connections is zero.

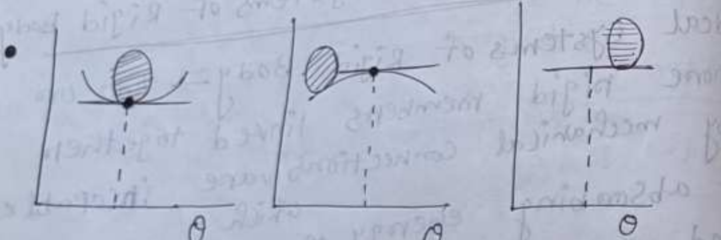
$\boxed{\delta(V_e + V_g) = 0}$, $\boxed{\delta V = 0}$

→ Potential energy, V has a stationary value.

→ For a 1 dof system

$$\boxed{\frac{dV}{dx} = 0}$$

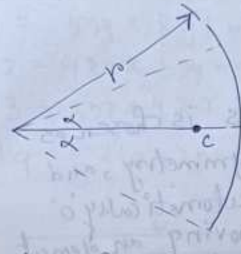
→ For mdof, we need partial derivatives



(a) Stable equillibrium (b) unstable equillibrium (c) Neutral equillibrium

$$\boxed{\begin{array}{ll} \text{Equillibrium} & \frac{dV}{dx} = 0 \\ \text{Stable} & \frac{d^2V}{dx^2} > 0 \\ \text{Unstable} & \frac{d^2V}{dx^2} < 0 \end{array}}$$

- SAMPLE PROBLEM 5/1 (Exam)



Locate the centroid of a circular arc as shown in the figure.

Sol – Choosing the axis of symmetry as the x-axis makes $\bar{y} = 0$. A differential element of area has the length $dL = r\,d\theta$ expressed in polar coordinates, and the x-coordinate of the element is $r\cos\theta$.

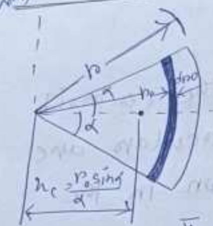
Applying the first of Eqs. 5/4 and substituting $L = 2\alpha r$ give

$[L\bar{x} = \int x\,dL]$ $\quad (2\alpha r)\bar{x} = \int_{-\alpha}^{\alpha} (r\cos\theta)\, r\,d\theta$

$$\Rightarrow 2\alpha r\bar{x} = 2r^2 \sin\alpha$$

$$\Rightarrow \bar{x} = \frac{r\sin\alpha}{\alpha}$$

• Sample Problem 5/3



The x-axis is chosen as the axis of symmetry, and $\bar{y}$ is therefore automatically 0.
We may cover the area by moving an element in the form of a partial circular ring, as shown in the fig. from the center to the outer periphery. The radius of the ring is r_0 and its thickness is dr_0, so that its area is $dA = 2r_0 \alpha\, dr_0$.
The x-coordinate to the centroid of the element from Sample Problem 5/1 is $x_c = \frac{r_0 \sin\alpha}{\alpha}$, where r_0 replaces r in the formula. Thus, the first of Eqs. 5/5a gives

$$\left[A\bar{x} = \int x_c\, dA\right] \quad \frac{2\alpha}{2\pi}(\pi r^2)\bar{x} = \int_0^r \left(\frac{r_0 \sin\alpha}{\alpha}\right)(2r_0\alpha\, dr_0)$$

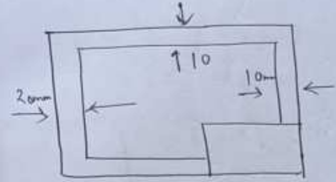
$$\Rightarrow r^2 \alpha \bar{x} = \frac{2}{3} r^3 \sin\alpha$$

$$\Rightarrow \bar{x} = \frac{2}{3}\frac{r\sin\alpha}{\alpha}$$

• Elements of vectors (ভেক্টর) 20/12/21

Length of A_0 Paper = 841 × 1189 mm

A_1 = 594 × 841 mm
A_2 = 420 × 594 mm
A_3 = 297 × 420 mm
A_4 = 210 × 297 mm



① Two vectors are given $\vec{F_1} = 6\hat{i} + 10\hat{j} - 5\hat{k}$ and $\vec{F_2} = 5\hat{i} + 2\hat{j} + 10\hat{k}$. Prove that two vectors are perpendicular to each other.

$$\vec{F_1}\cdot\vec{F_2} = |\vec{F_1}||\vec{F_2}|\cos\theta$$

$$\Rightarrow \cos\theta = \frac{30 + 20 - 50}{\sqrt{36+100+25}\cdot\sqrt{25+4+100}}$$

$$= 0 \quad \therefore \theta = 90°$$

$\therefore F_1 \perp F_2$ (Proved)

② Determine the cross product of the vector and angle between them. The vectors are —

$\vec{F_1} = 4\hat{i} - 5\hat{j} - 3\hat{k}$
$\vec{F_2} = 4\hat{i} + 5\hat{j} - 6\hat{k}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -5 & -3 \\ 4 & 5 & -6 \end{vmatrix}$$

$= (30 + 15)\hat{i} - (-24 + 12)\hat{j} + (20 + 20)\hat{k}$

$= 45\hat{i} + 12\hat{j} + 40\hat{k}$

$$\cos\theta = \frac{16 - 25 + 18}{\sqrt{16+25+9}\cdot\sqrt{16+25+36}}$$

$$= \frac{9}{7.07\times\sqrt{77}}$$

$\theta = 81.934^\circ$

(4) $\vec{A} = \hat{i} + \hat{j} - 2\hat{k}$

$\vec{B} = 4\hat{i} + 3\hat{j} - 2\hat{k}$

$\hat{C} = 2\hat{i} + \hat{j} + \hat{k}$

Resultain vector and unit vector.

$\hat{A} = \frac{\vec{A}}{|A|}$ } → এটা নয়

$\Rightarrow \frac{\hat{i} + \hat{j} - 2\hat{k}}{1} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

$x^2 + y^2 + z^2 = 1$

$x = 1, y = 1, z = -2$

$\hat{B} = \frac{\vec{B}}{|B|}$

$\Rightarrow 4\hat{i} + 3\hat{j} - 2\hat{k} = \frac{\vec{B}}{|B|}$

$\Rightarrow \frac{4\hat{i} + 3\hat{j} - 2\hat{k}}{1} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

$x = 4, y = 3, z = -2$

$\hat{c} = \frac{\vec{C}}{|C|}$

$\Rightarrow \frac{2\hat{i} + \hat{j} + \hat{k}}{1} = \frac{x\hat{i} + y\hat{j} + z\hat{k}}{\sqrt{x^2+y^2+z^2}}$

$x = 2, y = 1, z = 1$

$$\begin{vmatrix} 1 & 1 & -2 \\ 4 & 3 & +2 \\ 2 & 1 & 1 \end{vmatrix}$$

$= 1(3+2) + 1(4+4) - 2(4-6)$

x

$\vec{R} = \vec{A} + \vec{B} + \vec{C}$

$= (\hat{i} + \hat{j} - 2\hat{k}) + (4\hat{i} + 3\hat{j} - 2\hat{k}) + (2\hat{i} + \hat{j} + \hat{k})$

$= 7\hat{i} + 5\hat{j} - 3\hat{k}$

Unit vector on the direction of the resultant R is

$$= \frac{7\hat{i} + 5\hat{j} - 3\hat{k}}{\sqrt{49+25+9}} = \frac{7\hat{i} + 5\hat{j} - 3\hat{k}}{\sqrt{83}}$$

③ $\vec{P} = 4\hat{i} - 2\hat{j} + 3\hat{k}$
$\vec{Q} = 2\hat{i} + 4\hat{j} + 5\hat{k}$
$\vec{R} = 7\hat{i} - \hat{j} + x\hat{k}$
Determine the value of x for which all the vectors are co-planer.

cross product = 0

$\vec{A} \times (\vec{B} \times \vec{C})$
$= \vec{P} \times (\vec{Q} \times \vec{R})$
$\Rightarrow (\vec{P} \cdot \vec{R})\vec{Q} - (\vec{P} \cdot \vec{Q})\vec{R} = 0$

$= -\cancel{(28 + 2 + 3x)}\ \cancel{(2\hat{i} + 4}$ The condition of co-planer,

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & 3 \\ 2 & 4 & 5 \\ 7 & -1 & x \end{vmatrix} = 0$$

$\Rightarrow 4(4x + 5) + 2(2x - 35) + 3(-2 - 28) = 0$
$\Rightarrow 16x + 4x + 20 - 70 - 90 = 0$
$\Rightarrow 20x = 140$
$\Rightarrow x = 7$

④ Two forces are given by the equations $\vec{F_1} = 2\hat{i} - 3\hat{j} - \hat{k}$ and $\vec{F_2} = 6\hat{i} + 9\hat{j} + 3\hat{k}$. Prove that two forces are parallel.

If $\vec{F_1} = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$ and $\vec{F_2} = a_2\hat{i} + b_2\hat{j} + c_2\hat{k}$
then $\vec{F_1} \parallel \vec{F_2}$, when $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$.

So, now,
$$\frac{2}{-6} = \frac{-3}{9} = -\frac{1}{3}$$
$$\Rightarrow -\frac{1}{3} = -\frac{1}{3} = -\frac{1}{3}$$
$\therefore \vec{F_1} \parallel \vec{F_2}$

Other process

$$\vec{F_1} \times \vec{F_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & -1 \\ -6 & 9 & 3 \end{vmatrix}$$
$= \hat{i}(-9 + 9) - \hat{j}(6 - 6) + \hat{k}(18 - 18)$
$= 0$

$\therefore \sin\theta = 0;\ \theta = 0°$
$\therefore$ Hence two forces are parallel to each other.

⑤ A force vector F is equal to $2\hat{i} + 3\hat{j} - \hat{k}$. The point of application of this force moves from the point $2\hat{i} - \hat{j} + \hat{k}$ to the point $3\hat{i} + \hat{j} - \hat{k}$. Determine the work done by the force.

Distance moved by the force $= (3\hat{i} + \hat{j} - \hat{k}) - (2\hat{i} - \hat{j} + \hat{k})$
$= \hat{i} + 2\hat{j} - 2\hat{k}$

workdone by the force is given by the dot product of force applied and distance moved.

$\therefore$ Work $= \vec{F} \cdot \vec{S}$

$= (2\hat{i} + 3\hat{j} - \hat{k}) \cdot (\hat{i} + 2\hat{j} - 2\hat{k})$

$= 2+6+2$

$= 10$ units

⑥ A vector $\vec{A}$ is equal to $3\hat{i} - 2\hat{j} + 2\hat{k}$. Find the projection of this vector on the line joining the points $P(1, 2, -3)$ and $Q(-1, -2, 2)$

Here given $\vec{A} = 3\hat{i} - 2\hat{j} + 2\hat{k}$

The line joining points P and Q is given by a vector

$\vec{R} = (-\hat{i} - 2\hat{j} + 2\hat{k}) - (\hat{i} + 2\hat{j} - 3\hat{k})$

$= -2\hat{i} - 4\hat{j} + 5\hat{k}$

We know $\vec{A} \cdot \vec{R} = |\vec{A}|$ times projection of $\vec{R}$ on $\vec{A}$

$= |\vec{R}|$ times projection of $\vec{A}$ on $\vec{R}$

$\therefore$ Projection of $\vec{A}$ on $\vec{R} = \frac{\vec{A} \cdot \vec{R}}{|\vec{R}|}$

Now $\vec{A} \cdot \vec{R} = -6 + 8 + 10 = 12$

$|\vec{R}| = \sqrt{4 + 16 + 25} = 6.7$

$\therefore$ Projection of $\vec{A}$ on $\vec{R} = \frac{12}{6.7} = 1.79$

⑦ A force $\vec{F} = 4\hat{i} + 2\hat{j} + 3\hat{k}$ is applied at a point $P(1, 2, -2)$. Determine the moment of the force $\vec{F}$ about the point $O(2, -1, 2)$.

The position vector $\vec{r}$ of the point P with respect to (short form - w.r.t) O

= Position vector of point P - Position vector of point O

$= (\hat{i} + 2\hat{j} - 2\hat{k}) - (2\hat{i} - \hat{j} + 2\hat{k})$

$= -\hat{i} + 3\hat{j} - 4\hat{k}$

The moment m is given by

$$\vec{m} = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 3 & -4 \\ 4 & 2 & 3 \end{vmatrix}$$

$= 17\hat{i} - 13\hat{j} - 14\hat{k}$

⑧ A point is acted upon by a set of three forces given by $(3\hat{i} - 5\hat{j} - 2\hat{k})$, $(2\hat{i} + 7\hat{j} + 3\hat{k})$ and $(-\hat{i} + 2\hat{j} + 5\hat{k})$. Determine the magnitude of the resultant force and its direction cosines.

$\vec{R} = (3\hat{i} - 5\hat{j} - 2\hat{k}) + (2\hat{i} + 7\hat{j} + 3\hat{k}) + (-\hat{i} + 2\hat{j} + 5\hat{k})$

$= 4\hat{i} + 4\hat{j} + 6\hat{k}$

$\therefore$ magnitude of resultant $= \sqrt{16+16+36}$
$= 8.24$ units

Direction cosines of the resultant are

$l = \frac{4}{8.24} = 0.485$

$\Rightarrow \cos\alpha = 0.485$

$\Rightarrow \alpha = 60.95°$

$m = \frac{4}{8.24} = 0.485$

$\Rightarrow \cos\beta = 0.485$

$\Rightarrow \beta = 60.95°$

$n = \frac{6}{8.24} = 0.728$

$\Rightarrow \cos\gamma = 0.728$

$\Rightarrow \gamma = 43.26°$

(9) A force $\vec{P}$ is directed from a point A (4, 1, 4) metres towards a point B(-3, 4, -1) meters. If it causes a moment $M_z = 1900$ Nm, determine the moment of P about x and y axes.

Position vector of point $A = 4\hat{i} + \hat{j} + 4\hat{k}$ and

position vector of point $B = -3\hat{i} + 4\hat{j} - \hat{k}$

Position vector of force $\vec{F}$ is given by

$r = (-3\hat{i} + 4\hat{j} - \hat{k}) - (4\hat{i} + \hat{j} + 4\hat{k})$

$= -7\hat{i} + 3\hat{j} - 5\hat{k}$

So, unit vector in the direction of force,

$= \frac{-7\hat{i} + 3\hat{j} - 5\hat{k}}{\sqrt{(-7)^2 + (3)^2 + (-5)^2}}$

$= \frac{-7\hat{i} + 3\hat{j} - 5\hat{k}}{9.11}$

So force vector $\vec{F}$ = magnitude of force times the unit vector in the direction of force

$= \frac{P(-7\hat{i} + 3\hat{j} - 5\hat{k})}{9.11}$

$\therefore$ moment of the force about point A is given by cross product of position vector of point A with the force vector (same result will come if we consider the position vector of point B)

m_0 = position vector of point $\vec{A} \times$ force vector $\vec{F}$

$= (4\hat{i} + \hat{j} + 4\hat{k}) \times \frac{P(-7\hat{i} + 3\hat{j} - 5\hat{k})}{9.11}$

$= \frac{P}{9.11} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 1 & 4 \\ -7 & 3 & -5 \end{vmatrix}$

$= \frac{P}{9.11}(-17\hat{i} - 8\hat{j} + 19\hat{k})$

$\therefore$ moment of the given force about z-axis $\frac{P}{9.11} \times 19$

(The coefficient of 'k' gives moment about z-axis.)

But moment of given force about z-axis = 1900 Nm

$\therefore \frac{P}{9.11} \times 19 = 1900$

$\Rightarrow P = \frac{1900 \times 9.11}{19}$

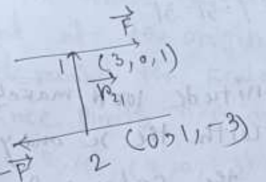
$= 911\ N$

$\therefore$ The moment of the given forces about x and y-axes is given by

$M_x = \frac{P}{9.11}(-17) = \frac{911}{9.11}(-17) = -1700\ Nm$

$M_y = \frac{P}{9.11}(-8) = \frac{911}{9.11}(-8) = -800\ Nm$

(10) A force $\vec{F} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ acts at a position (3,0,1) meters. At point (0,1,-3) meters an equal but opposite force $-\vec{F}$ acts. Determine the couple moment and also determine the direction cosines normal to the plane of the couple.



We choose two points 2 and 1 on lines of action of $-\vec{F}$ and $\vec{F}$ respectively as shown in the figure.

So the position vector

$\vec{r_{21}} = (3-0)\hat{i} + (0-1)\hat{j} + (1+3)\hat{k}$

$= 3\hat{i} - \hat{j} + 4\hat{k}$

So moment of the couple,

$\vec{m} = \vec{r_{21}} \times \vec{F}$

$= (3\hat{i} - \hat{j} + 4\hat{k}) \times (6\hat{i} + 3\hat{j} + 2\hat{k})$

$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 4 \\ 6 & 3 & 2 \end{vmatrix}$

$\vec{m} = -14\hat{i} + 18\hat{j} + 15\hat{k}$

magnitude of moment of the couple ~~are~~ $|\vec{m}| = \sqrt{14^2 + 18^2 + 15^2} = 27.29$

D.C are

$l = \cos\alpha = \frac{-14}{27.29} = -0.51 \Rightarrow \alpha = 120.8°$

$m = \cos\beta = \frac{18}{27.29} = 0.45 \Rightarrow \beta = 49.45°$

$n = \cos\gamma = \frac{15}{27.29} = 0.54 \Rightarrow \gamma = 57.31$

(11) A space force of magnitude 100 N makes angles of 30° and 75° with the x and y axes respectively. Determine the scalar components of the force along the x, y and z axes and write the vector equation for the force.

$l = \cos\alpha = \cos 30° = 0.866$

$m = \cos\beta = \cos 75° = 0.258$

we know,

$l^2 + m^2 + n^2 = 1$

$\cos\gamma = n = \sqrt{1 - l^2 - m^2}$

$= \sqrt{1 - (0.866)^2 - (0.258)^2}$

$= 0.428$

$\gamma = \cos^{-1}(0.428) = 64.63°$

So the scalar components of the force are:

$F_x = F\cos\alpha = 100 \cos 30 = 86.6\ N$

$F_y = F\cos\beta = 100 \cos 75 = 25.88 N$

$F_z = F\cos\gamma = 100 \cos 64.63 = 42.84\ N$

The vector equation of the force -

$F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$= 86.6\hat{i} + 25.88\hat{j} + 42.84\hat{k}$

(13) The lines of action of three forces concurrent at the origin passes respectively. Determine the scalar components of the force three points A, B and C having coordinates (−1, 2, 4), (3, 0, −3) and (2, −2, 4). The magnitude of the forces are $F_A = 50N$, $F_B = 20N$ and $F_C =$ 30 N. Determine the magnitude and direction of the resultant.

magnitude of forces are $F_A \cdot 50 N$, $F_B = 20 N$,

$F_C = 30 N$

The position vector of,

$F_A = (-1-0)\hat{i} + (2-0)\hat{j} + (4-0)\hat{k}$

$= -\hat{i} + 2\hat{j} + 4\hat{k}$

The unit vector in the direction force

$F_A = \frac{-\hat{i} + 2\hat{j} + 4\hat{k}}{\sqrt{(-1)^2 + (2)^2 + (4)^2}} = \frac{-\hat{i} + 2\hat{j} + 4\hat{k}}{\sqrt{21}}$

The unit vector in direction force -

$F_A = \frac{-\hat{i} + 2\hat{j} + 4\hat{k}}{4.58}$

$\therefore$ Force vector $\vec{F_A}$ = magnitude of F_A times unit vector

$= \frac{50(-\hat{i} + 2\hat{j} + 4\hat{k})}{4.58}$

$= -10.91\hat{i} + 21.83\hat{j} + 43.66\hat{k}$

Similarly, the position vector of force

$F_B = 3\hat{i} - 3\hat{k}$

The unit vector in the direction of force

$$F_B = \frac{3\hat{i} - 3\hat{k}}{\sqrt{3^2 + (-3)^2}} = \frac{3\hat{i} - 3\hat{k}}{4.24}$$

$\therefore$ Force vector $\vec{F_B}$ = magnitude of F_B times unit vector in the direction of force $F_B = \frac{20(3\hat{i} - 3\hat{k})}{4.24} = 14.15\hat{i} - 14.15\hat{k}$

Similarly, force vector $\vec{F_C}$ = magnitude of F_C times units vector in the direction of F_C

$$= \frac{30(2\hat{i} - 2\hat{j} + 4\hat{k})}{\sqrt{4 + 4 + 16}}$$

$= 12.26\hat{i} - 12.26\hat{j} + 24.53\hat{k}$

$\therefore$ Resultant force vector

$\vec{R} = \vec{F_A} + \vec{F_B} + \vec{F_C}$

$= 15.5\hat{i} + 9.57\hat{j} + 54.04\hat{k}$

mag. of the resultant force

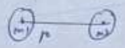
$\sqrt{(15.5)^2 + (9.57)^2 + (54.04)^2} = 57.02\ N$

$\therefore$ unit vector in the direction of resultant force vector $= \frac{15.5\hat{i} + 9.57\hat{j} + 54.\hat{k}}{57.02}$

$= 0.27\hat{i} + 0.16\hat{j} + 0.94\hat{k}$

- Mechanics
- Dymics

(i) Rigid body - No deformation of the body.



$F = G\frac{m_1 m_2}{r^2}$

$F_{net} = ma$

$\Rightarrow F = G\frac{m\, m_e}{R^2}$

$\Rightarrow m a = G\frac{m\, m_e}{R^2}$

$\Rightarrow a = g_0 = \frac{G m_e}{R^2}$

$g = \frac{G m_e}{(R+h)^2}$

$$\boxed{g = g_0 \frac{R^2}{(R+h)^2}}$$

[g = this is g when the hight is h]
is a

[$g_0 = 9.806$
$= 9.81$]

[g_0 = at sea leavel at latitude of 45°]

- Moment of Inertia (? Sir)

- moment of Inertia — moment & perpendicular distance

- $I_{AB} = I_G + ah^2$ [Theorem of parallel axis]
- $I_{ZZ} = I_{xx} + I_{yy}$ [Theorem of perpendicular axis]

- MOMENT OF INERTIA OF A PLANE AREA

The moment of inertia of the area,

$I = a_1 r_1^2 + a_2 r_2^2 + a_3 r_3^2 + \ldots$
$= \sum a r^2$

$\therefore \boxed{I = \sum a r^2}$ [a_1, a_2, a_3 = Areas of small elements
r_1, r_2, r_3 = Corresponding distances of the elements from the line about which the moment of inertia is required to be found out]

- units of Moment of inertia

[Momenet of inertia of a plane area depends upon the unit of the area and the length]

1) If area is in m^2 and the length is also in m. The moment of inertia is expressed in m^4.

2) If area in mm^2 and the length is also in mm, then moment of inertia is expressed in mm^4.

• methods for moment of inertia

1) By Routh's rule

2) By Integration

NOTE The Routh's Rule is used for finding the moment of inertia of a plane area or a body of uniform thickness.

• moment of inertia by Routh's Rule

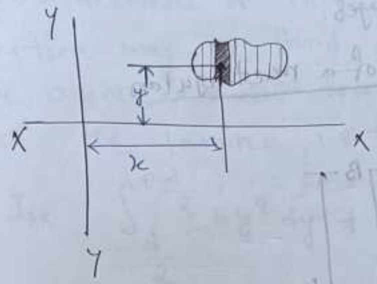
The Routh's Rule states, if a body is symmetrical about three mutually perpendicular axes, then the moment of inertia, about any one axis passing through its centre of gravity is given by:

$$\boxed{I = \frac{A (\text{or } M) \times S}{3}}$$ — (For a square or Rectangular lamina)

$$\boxed{I = \frac{A (\text{or } M) \times S}{4}}$$ — (For a circular or Elliptical lamina)

• moment of inertia by integration

The moment of inertia of an area may also be found out by the method of integration as discussed below:



Consider a plane figure, whose moment of inertia is required to be found out by about x-x axis and y-y axis as shown in fig. Let us divide the whole area into a no. of strips. Consider one of these strips.

Let dA = Area of the strip

x = Distance of the centre of gravity of the strip on x-x axis and

y = Distance of the c.g. of the strip on y-y axis.

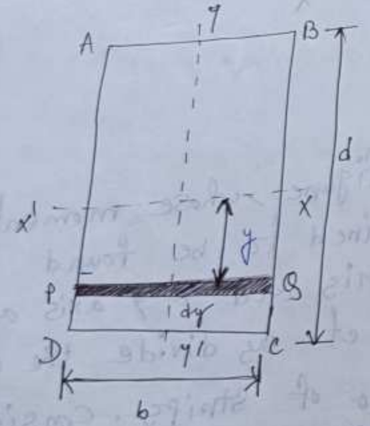
We know that the moment of inertia of the strip about y-y axis $= dA \cdot x^2$

Now the moment of inertia of the whole area may be found by integrating above equation, i.e

$$\boxed{\begin{aligned} I_{yy} &= \Sigma dA \cdot x^2 \\ \text{Similarly } I_{xx} &= \Sigma dA \cdot y^2 \end{aligned}}$$

In the following pages.

- moment of inertia of a rectangular section



Consider a rectangular section ABCD as shown in fig. whose moment of inertia is required to be found out

b = width of the section

d = depth of the section

Now consider a strip PQ of thickness dy parallel to x-x axis and at a distance y from it as shown in the fig.

∴ Area of the strip

$= b \cdot dy$

we known that moment of inertia of the strip about x-x axis,

$= \text{Area} \times y^2 = (b \cdot dy) y^2 = b \cdot y^2 dy$

Now moment of inertia of the whole section may be found out by integrating the above equation for the whole length of the lamina, i.e from $-\frac{d}{2}$ to $+\frac{d}{2}$.

$$I_{xx} = \int_{-\frac{d}{2}}^{+\frac{d}{2}} b y^2 dy = b \int_{-\frac{d}{2}}^{+\frac{d}{2}} y^2 dy$$

$$= b \left[\frac{y^3}{3} \right]_{-\frac{d}{2}}^{+\frac{d}{2}}$$

$$= b \left[\frac{(d/2)^3}{3} - \frac{(-d/2)^3}{3} \right]$$

$$\boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly

$$\boxed{I_{yy} = \frac{db^3}{12}}$$

NOTE - Cube is to be taken of the side, which is at right angles to the line of reference.

Ex-① Find the moment of inertia of a rectangular section 30mm wide and 40 mm deep about X-X axis and Y-Y axis.

$d = 40mm$, $b = 30\ mm$

$$I_{XX} = \frac{bd^3}{12}$$

$$= \frac{30 \times (40)^3}{12}$$

$$= 160 \times 10^3\ mm^4$$

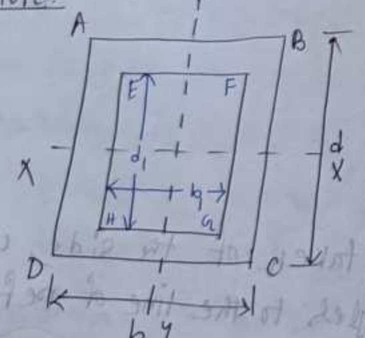
similarly,

$$I_{YY} = \frac{db^3}{12}$$

$$= \frac{40 \times (30)^3}{12}$$

$$= 90 \times 10^3\ mm^4$$

• moment of inertia of a hallow Rectangular section



b = Breadth of the outer rectangle
d = Depth of the outer rectangle
and
b_1, d_1 = corresponding values for the cut out rectangle

we known that the moment of inertia of the outer rectangle ABCD about X-X axis

$$= \frac{bd^3}{12} \quad \text{--- (i)}$$

and moment of inertia of the cut out rectangle EFGH about X-X axis

$$= \frac{b_1 d_1^3}{12} \quad \text{--- (ii)}$$

∴ M.I. of the hollow rectangular section about X-X axis,

I_{XX} = M.I. of rectangle ABCD − M.I. of rectangle EFGH

$$\boxed{I_{XX} = \frac{bd^3}{12} - \frac{b_1 d_1^3}{12}}$$

$$\boxed{I_{YY} = \frac{db^3}{12} - \frac{d_1 b_1^3}{12}}$$ Similarly

Note- This relation holds good only if the centre of gravity of the main section as well as that of the cut out section coincide with each other.

• Theorem of perpendicular axis

It states, If I_{xx} and I_{yy} be the moments of inertia of plane section about two perpendicular axis meeting at G, the moment of inertia I_{zz} about the axis z-z, perpendicular to the plane and passing through the intersection of x-x and y-y is given by:

$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$

• Moment of inertia of a circular section

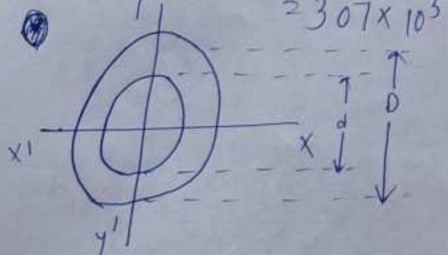
$$\boxed{I_{xx} = I_{yy} = \frac{I_{zz}}{2} = \frac{\pi}{64}(d)^4}$$ [d = diameter]

Ex – ① Find the moment of inertia of a circular section of 50 mm diameter about an axis passing through its centre.

Given diameter (d) = 50 mm

$\therefore I_{xx} = \frac{\pi}{64} \times (d)^4 = \frac{\pi}{64} \times (50)^4$

$= 307 \times 10^3 \ mm^4$



• Moment of inertia of a hollow circular section

I_{xx} = moment of inertia of main circle – moment of inertia of cut out circle

$= \frac{\pi}{64}(D)^4 - \frac{\pi}{64}(d)^4$

$$\boxed{I_{xx} = \frac{\pi}{64}(D^4 - d^4)}$$ [D = Diameter of main circle, d = Diameter of cut out circle]

Similarly

$$\boxed{I_{yy} = \frac{\pi}{64}(D^4 - d^4)}$$

• Theorem of Parallel axis

It states, If the moment of inertia of a plane area about an axis through its centre of gravity is denoted by I_G then moment of inertia of the area about any other axis AB, parallel to the first and at a distance h from the centre of gravity is given by:

$$\boxed{I_{AB} = I_G + ah^2}$$

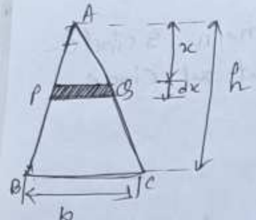
where, I_{AB} = moment of inertia of the area about an axis AB,

I_G = moment of Inertia of the area about its centre of gravity

a = Area of the section and

h = Distance between centre of gravity of section and axis of AB

• moment of inertia of a triangle section



BC = Base = b

$$\boxed{I_{BC} = \frac{bh^3}{12}}$$

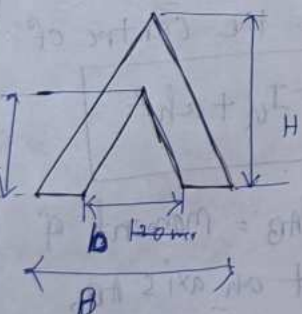
[b = Base of the triangular section and
h = Heigh of the triangular section]

we know the distance between centre of gravity of triangle section and base BC, $d = \frac{h}{3}$

∴ moment of inertia of the triangular section about an axis through its centre of gravity and parallel to X-X axis,

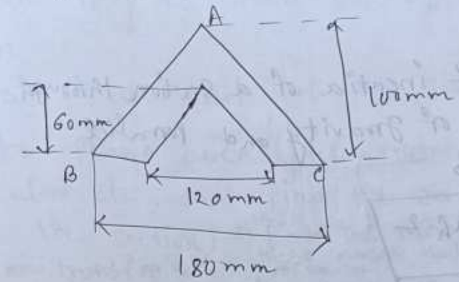
• moment of inertia of a hollow triangle section

$$\boxed{I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12}}$$



[B = Base width of main triangle
b = Base width of cut out triangle
H = Height of main triangle
h = Height of cut out triangle]

Ex – ① A hollow triangle section show in fig. is symmetrical about its vertical axis.



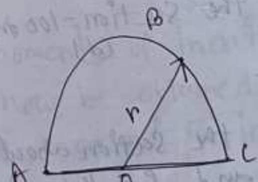
find the moment of inertia of the section about the base BC.

B = 180 mm, b = 120mm, H = 100 mm, h = 60mm

$$I_{BC} = \frac{BH^3}{12} - \frac{bh^3}{12} = \frac{180 \times (100)^3 - 120 \times (60)^3}{12}$$

$$= 12.84 \times 10^6 \text{ mm}^4$$

• moment of inertia of a semicircular section



[r = radius of the semicircle]

$$I_{AC} = \frac{1}{2} \times \frac{\pi}{64} \times (d)^4 = 0.393 \times r^4$$

we also know that area of semicircular section, $a = \frac{1}{2} \times \pi r^2 = \frac{\pi r^2}{2}$

and distance between centre of gravity of the section and the base AC,

$h = \frac{4r}{3\pi}$

∴ moment of inertia of a section through its centre of gravity and parallel to x-x axis,

$\boxed{I_G = I_{AC} - ah^2}$

$\boxed{I_G = 0.11r^4}$

The moment of inertia about y-y axis will be the same as that about the base AC, i.e. $0.393r^4$

Ex-① Determine the moment of inertia of a semicircular section of 100mm diameter about its centre of gravity and parallel to x-x and y-y axes.

Solⁿ: Given diameter of the section = 100 mm (d)

∴ radius (r) = 50 mm

Moment of inertia of the section about its centre of gravity and parallel to x-x axis = $0.11 \times r^4$

$= 0.11 \times (50)^4$

$= 687.5 \times 10^3 \ mm^4$

" " " y-y axis = $0.393 \times (r)^4 = 0.393 \times (50)^4$

$= 2.46 \times 10^6 \ mm^4$

• Moment of inertia of a composite section

The moment of inertia of a composite section may be found out by the following steps:

1) First of all, split up the given section into plane areas (i.e. rectangular, triangular, circular etc., and find the centre of gravity of the section) 2) Find the moments of inertia of these areas about their respective centres of gravity.

3) Now transfer these moment of inertia about the required axis (AB) by the Theorem of parallel axis, i.e.,

$I_{AB} = I_G + ah^2$

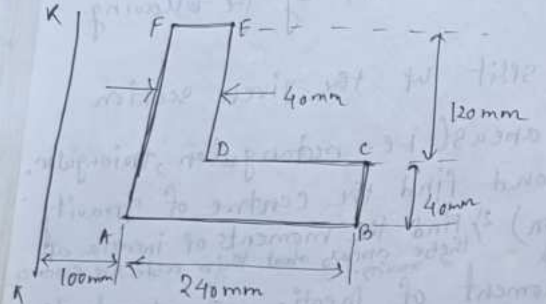
[I_G = moment of inertia of a section about its centre of gravity and parallel to the axis

a = Area of the section

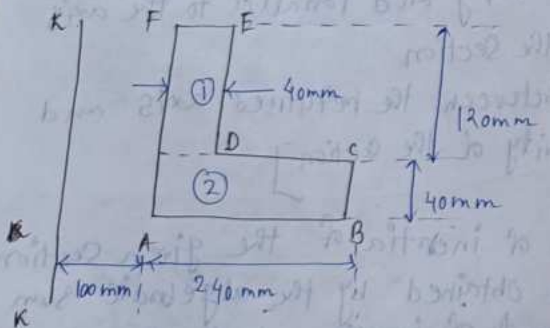
h = Distance between the required axis and centre of gravity of the section]

4) The moments of inertia of the given section may now be obtained by the algebraic sum of the moment of inertia about the required axis.

Ex-① This Fig. shows an area ABCDEF. Compute the moment of inertia of the above area about axis K-K.



Soln: As the moment of inertia is required to be found out about the K-K axis, therefore there is no need of finding out the centre of gravity of the area.



Let us split up the area into two rectangles ① and ②, as shown in fig.

We know that the moment of inertia of section ① about its centre of gravity and parallel to axis K-K

$b_1 = 40\ mm$

$d_1 = 120\ mm$

$$I_{G_1} = \frac{b_1 d_1^3}{12}$$

$$= \frac{120 \times (40)^3}{12}$$

$$= 640 \times 10^3\ mm^4$$

and distance between centre of gravity of section ① and axis K-K

$$h_1 = 100 + \frac{40}{2} = 120\ mm$$

∴ moment of inertia of section ① about axis K-K =

$$I_{G1} + a_1 h_1^2 = (640 \times 10^3) + [(120 \times 40)(120)^2]$$

$$= 69.76 \times 10^6\ mm^4$$

Similarly, moment of inertia of section ② about its centre of gravity and parallel to axis K-K,

$$I_{G_2} = \frac{40 \times (240)^2}{12} = 46.08 \times 10^6\ mm^4$$

and distance between centre of gravity of section ② and axis K-K,

$$h_2 = 100 + \frac{240}{2} = 220\ mm$$

∴ moment of inertia of section ② about the axis K-K

$= I_{G2} + a_2 h_2^2$

$= (46.08 \times 10^6) + [(240 \times 40) \times (220)^2]$

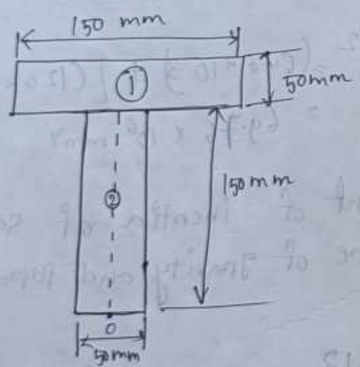
$= 510.72 \times 10^6 \ mm^4$

Now moment of inertia of the whole area about axis K-K

$I_{KK} = (69.76 \times 10^6) + (510.72 \times 10^6)$

$= 580.48 \times 10^6 \ mm^4$

② Find the moment of inertia of a T-section with flange as 150 mm × 50 mm and web as 150 mm × 50 mm about x-x and y-y axes through the centre of gravity of the section.



Sol^n: The given T-section is shown in fig. First of all, let us find out centre of gravity of the section. As the section is symmetrical about y-y axis, therefore its centre of gravity will lie on this axis. Split up the whole section int two rectangles viz, ① and ② as shown in figure. Let bottom of the web be the axis of reference.

(i) Rectangle ①

$a_1 = 150 \times 50 = 7500 \ mm^2$

and $y_1 = 150 + \frac{50}{2} = 175 \ mm$

(ii) Rectangle ②

$a_2 = 150 \times 50 = 7500 \ mm^2$

and $y_2 = \frac{150}{2} = 75 \ mm$

$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{(7500 \times 175) + (7500 \times 75)}{7500 + 7500}$

$= 125 \ mm$

∴ The distance between centre of gravity of the section and bottom of web = 125 mm

moment of inertia about x-x axis

we also know that M.I of rectangle ① about an axis through its centre of gravity and parallel to x-x axis.

$I_{G1} = \frac{150 \times (50)^3}{12} = 1.5625 \times 10^6 \ mm^4$

$[I_{xx} = \frac{bd^3}{12}$
$b = 150, d = 50 \ mm]$

and distance between centre of gravity of rec ① and x-x axis

$h_1 = 175 - 125 = 50 \ mm$

moment of inertia of the rec (1) about x-x axis

$I_{G_1} + a_1 h_1^2$

$= (1.5625\times10^6) + [(150\times50)\times(50)^2]$

$= 20.31\times10^6\ mm^4$

$I_{G_2} = \cancel{db^3}\ \dfrac{150\times(50)^3}{12}$ $\left[I_{yy} = \dfrac{db^3}{12},\ d = 150,\ b = 50\right]$

$= 1.5625\times10^6\ mm^4$

and the distance between centre of gravity of rec (2) and X-X axis -

$h_2 = 125 - 75$

$= 50\ mm$

∴ moment of inertia of the rec (2) about x-x axis $= I_{G_2} + a_2 h_2^2$

$= (1.5625\times10^6) + [(50\times150)\times(50)^2]$

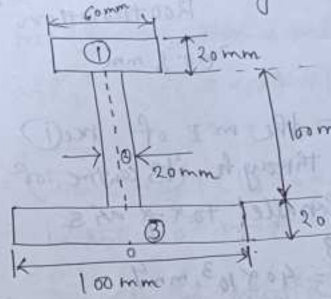
$= 20.31\times10^6\ mm^4$

∴ Now the moment of inertia of the whole section about x-x axis -

$I_{xx} = 20.31\times10^6\times2$

$= 40.62\times10^6\ mm^4$

(3) An - I - section is made up of three rectangle as shown in fig. Find the M.I. of the section about the horizontal axis passing through the centre of gravity of the section.



Sol^n: First of all, let us find out centre of gravity of the section. As the section is symmetrical about yy axis, therefore its centre of gravity will lie on this axis. Split up the whole section into three rectangles (1), (2) and (3) as shown in fig. Let bottom face of the bottom flange be the axis of reference.

(i) Rectangle (1)

$a_1 = 60\times20 = 1200\ mm^2$

and $y_1 = 20 + 100 + \frac{20}{2} = 130\ mm$

(ii) Rectangle (2)

$a_2 = 100\times20 = 2000\ mm^2$

$y_2 = 20 + \frac{100}{2} = 70\ mm$

(iii) Rectangle (3)

$a_3 = 100\times20 = 2000\ mm^2$

and $y_3 = \frac{20}{2} = 10\ mm$

we know that the distance between c.g of the section and bottom face,

$\bar{y} = \frac{a_1y_1 + a_2y_2 + a_3y_3}{y_1+y_2+y_3} = \frac{(1200\times 130)+(2000\times 70)+(2000\times 10)}{1200+1200+2000}$ mm

$= 60.8$ mm

we know that the m.I of rec ① about an axis through its centre of gravity and parallel to x-x axis

$I_{G1} = \frac{60\times(20)^3}{12} = 40\times 10^3\ mm^4$

and distance between centre of gravity of rec ① and x-x axis,

$h_1 = 130 - 60.8 = 69.2$ mm

∴ moment of inertia of rec ① about x-x axis,

$= I_{G1} + a_1h_1^2 = (40\times 10^3) + [1200\times(69.2)^2]$

$= 5786\times 10^3\ mm^4$

Similarly, $I_{G2} = \frac{20\times(100)^3}{12} = 1666.7\times 10^3\ mm^4$

$h_2 = 70 - 60.8 = 9.2$ mm

∴ moment of inertia of rec ② about x-x axis,

$= I_{G2} + a_2h_2^2$

$= (1666.7\times 10^3) + [2000\times(9.2)^2]$

$= 1836\times 10^3\ mm^4$

similarly,

$I_{G3} = \frac{100\times(20)^3}{12} = 66.67\times 10^3\ mm^4$

$h_3 = 60.8 - 10 = 50.8$ mm

∴ m.I of rec ③ about x-x axis,

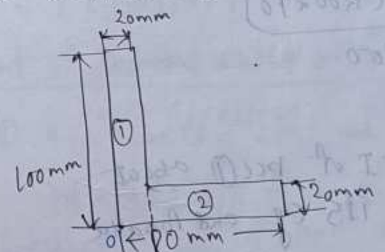
$= I_{G3} + a_3h_3^2 = (66.7\times 10^3) + [2000\times(50.8)^2]$

$= 5228\times 10^3\ mm^4$

∴ $I_{xx} = (5786\times 10^3) + (1836\times 10^3) + (5228\times 10^3)$

$= 12850\times 10^3\ mm^4$

④ Find the moment of inertia about the centroidal x-x and y-y axis of the angle section shown in fig.



Soln: First of all, let us find the c.g of the section. As the section is not symmetrical about any section, therefore we have to find out the values of $\bar{x}$ and $\bar{y}$ for the angle section. Split up the section into two rectangles ① and ② as shown in fig.

M.I about centroidal x-x axis,

Let bottom face of the angle section be the axis of reference.

Rectangle ①

$a_1 = 100 \times 20 = 2000\ mm^2$

and $y_1 = \frac{100}{2} = 50\ mm$

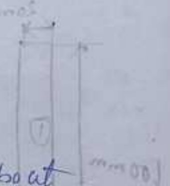
Rectangle ②

$a_2 = (80-20) \times 20 = 1200\ mm^2$

and $y_2 = \frac{20}{2} = 10\ mm$

we know that distance between the c.g of the section and bottom face —

$$\bar{y} = \frac{(2000 \times 50) + (1200 \times 10)}{2000 + 1200}$$



$= 35\ mm$

We know that M.I of rec① about an axis through its c.g and parallel to x-x axis,

$$I_{G1} = \frac{20 \times (100)^3}{12} = 1.667 \times 10^6\ mm^4$$

and distance of c.g of rec① from x-x axis —

$h_1 = 50 - 35 = 15 mm$

∴ M.I of rec① about x-x axis,

$= (1.667 \times 10^6) + [2000 \times (15)^2]$

$= 2.117 \times 10^6\ mm^4$

Similarly,

$$I_{G2} = \frac{60 \times (20)^3}{12} = 0.04 \times 10^6\ mm^4$$

and $h_2 = 35 - 10 = 25\ mm$

∴ M.I. of rec② about x-x axis,

$= (0.04 \times 10^6) + [1200 \times (25)^2]$

$= 0.79 \times 10^6\ mm^4$

Now moment of inertia of the whole section about x-x axis,

$I_{xx} = (2.117 \times 10^6) + (0.79 \times 10^6) = 2.907 \times 10^6\ mm^4$

moment of inertia about centroidal y-y axis

Rec① $a_1 = 100 \times 20 = 2000\ mm^2$

$x_1 = (80-20) + \frac{20}{2} = 60 + 10 = 10\ mm$

Rec②

$a_2 = (80-20) \times 20 = 1200\ mm^2$

$x_2 = 20 + \frac{(80-20)}{2} = 20 + \frac{60}{2} = 20 + 30\ mm = 50$

$$\bar{x} = \frac{(2000 \times 10) + (1200 \times 50)}{2000 + 1200}$$

$= 25$

we know that the M.I of rec① about an axis through its c.g and parallel to Y-Y axis,

$$I_{G_1} = \frac{100 \times (20)^3}{12}$$

$$= 66.67 \times 10^3 \ mm^4$$

and the distance of c.g of rec ① from Y-Y axis

$h_1 = 25 - 10 = 15 \ mm$

M.I of rec ① about Y-Y axis =

$$(66.67 \times 10^3) + [2000 \times (15)^2]$$

$$= 516.67 \times 10^3 \ mm^4$$

$$= 0.517 \times 10^6 \ mm^4$$

Similar,

$$I_{G_2} = \frac{20 \times (60)^3}{12}$$

$$= 0.36 \times 10^6 \ mm^4$$

$h_2 = 50 - 25 = 25 \ mm$

∴ M.I. of rec ② about Y-Y axis,

$$= (0.36 \times 10^6) + [1200 \times (25)^2]$$

$$= 1.11 \times 10^6 \ mm^4$$

∴ Now moment of inertia of the whole section about Y-Y axis =

$$I_{YY} = (0.517 \times 10^6) + (1.11 \times 10^6)$$

$$= 1.627 \times 10^6 \ mm^4$$

• Elements of vectors (? sin)

rest numericals

(13) A force $\vec{F}$ acts through the origin of a coordinate system in a direction defined by the angle $\theta_x = 70°$ and $\theta_z = 58°$. The component of force $\vec{F}$ along y-direction is −120 N. Determine (a) magnitude of $\vec{F}$ (b) components of force along x and z direction and (c) component of force on a line that passes through the origin at point (2, 3, 1).

The direction cosines along the x and z directions are –

$$l = \cos\theta_x = \cos 70 = 0.342$$

$$n = \cos\theta_z = \cos 58 = 0.529$$

From the relation

$l^2 + m^2 + n^2 = 1$ we get

$$m = \cos\theta_y = \pm\sqrt{1 - l^2 - n^2} = \pm\sqrt{1 - (0.342)^2 - (0.529)^2}$$

$$= \pm 0.776$$

$\Rightarrow \theta_y = \cos^{-1}(0.776)$ and $\theta_y = \cos^{-1}(-0.776)$

$= 39.10$ $= 140.67$

(a) From the relation $F_y = F\cos\theta_y$

so, the magnitude of the force $F = \frac{F_y}{\cos\theta_y} = \frac{-120}{\cos(140.67)}$

$= 154.65 \ N$

(b) scalar components of force in the x and z direction are:

$F_x = F\cos\theta_x = 154.65\cos 70 = 52.89N$

$F_z = F\cos\theta_z = 154.65\cos 58^\circ = 81.95N$

vector equation force

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$= 52.89\hat{i} - 120\hat{j} + 81.95\hat{k}$

Position vector $\vec{OA} = 2\hat{i} + 3\hat{j} + \hat{k}$

unit vector along $\vec{OA} = \frac{2\hat{i}+3\hat{j}+\hat{k}}{\sqrt{4+9+1}}$

$= \frac{2\hat{i}+3\hat{j}+\hat{k}}{3.74}$

$\therefore$ Force vector $\vec{F}$ = magnitude of force x unit vector along the direction OA

$= 154.65 \times \frac{2\hat{i}+3\hat{j}+\hat{k}}{3.74}$

$\therefore$ ~~Force~~ $= 82.7\hat{i} + 124.05\hat{j} + 41.35\hat{k}$

(c) So the components of force on a line that passes through the origin and Point (2,3,1) are $F_A = 82.7N$, $F_y = 124.05N$

$F_z = 41.35N$

(14) A force of 200N is directed along the line drawn from point P(5,2,4) to the Point Q(3,-5,6). Determine the moment of this force about a point A(4,3,2). The distances are in meters.

Position vector $\vec{PQ} = (3-5)\hat{i} + (-5-2)\hat{j} + (6-4)\hat{k}$

$= -2\hat{i} - 7\hat{j} + 2\hat{k}$

unit vector along PQ (along the direction of force) $= \frac{-2\hat{i} - 7\hat{j} + 2\hat{k}}{\sqrt{(-2)^2+(-7)^2+(2)^2}}$

$= \frac{-2\hat{i} - 7\hat{j} + 2\hat{k}}{7.54}$

$\therefore$ Force vector $\vec{F}$ = magnitude of force x unit vector along the direction of force

$= 200 \times \frac{(-2\hat{i} - 7\hat{j} + 2\hat{k})}{7.54}$

$= -53.05\hat{i} - 185.67\hat{j} + 53.05\hat{k}$

Position vector -

$\vec{PA} = (4-5)\hat{i} + (3-2)\hat{j} + (2-4)\hat{k}$

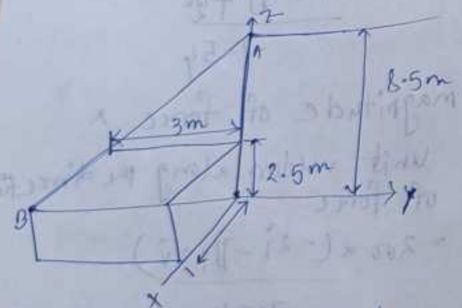
$= -\hat{i} + \hat{j} - 2\hat{k}$

$\therefore$ moment of force vector about Point A

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & 1 & -2 \\ -53.05 & -185.67 & 53.05 \end{vmatrix}$$

$$= (318.29\hat{i} + 159.15\hat{j} + 238.72\hat{k}) \text{ Nm}$$

(15) A man pulls on the cord AB with a force of 70 N as shown in fig. Represent this force acting on the support A as a cartesian vector and determine its direction.



The coordinates of points A and B are;

A (0, 0, 8.5 m) ; B (2m, -3m, 2.5m)

Position vector

$$\vec{AB} = (2-0)\hat{i} + (-3-0)\hat{j} + (2.5-8.5)\hat{k}$$
$$= 2\hat{i} - 3\hat{j} - 6\hat{k}$$

unit vector along AB

$$= \frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{\sqrt{4+9+36}}$$
$$= \frac{2\hat{i} - 3\hat{j} - 6\hat{k}}{7}$$

Force vector $\vec{F}$ = magnitude of force × unit vector along the direction AB

$$= 70 \times \frac{(2\hat{i} - 3\hat{j} - 6\hat{k})}{7}$$
$$= 20\hat{i} - 30\hat{j} - 60\hat{k}$$

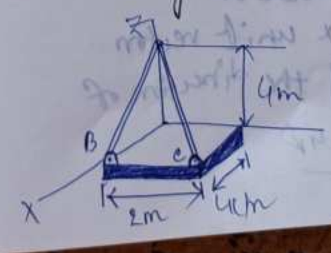
Direction cosines are

$l = \cos\alpha = \frac{2}{7}$ $\quad \alpha = 73.39°$

$m = \cos\beta = -\frac{3}{7}$ $\quad \beta = 115.37°$

$n = \cos\gamma = \left(-\frac{6}{7}\right)$ $\quad \gamma = 148.99°$

(16) The cables exert forces $F_{AB} = 100$ N and $F_{AC} = 120$ N the ring at A as shown in. Determine the magnitude of the resultant force acting at A.



Position vector $\vec{AB} = (4-0)\hat{i} + (0-0)\hat{j} + (0-4)\hat{k}$

$= (4\hat{i} - 4\hat{k})$ m

$\therefore$ unit vector along AB

$= \frac{4\hat{i} - 4\hat{k}}{\sqrt{16+16}}$

$= \frac{4\hat{i} - 4\hat{k}}{4\sqrt{2}}$

$= \frac{\hat{i} - \hat{k}}{\sqrt{2}}$

Force vector

$\vec{F_{AB}}$ = magnitude of force × unit vector along the direction of AB

$= 10 \times \frac{\hat{i} - \hat{k}}{\sqrt{2}} = 70.7\hat{i} - 70.7\hat{k}$

Position vector of $\vec{AC}$

$= (4-0)\hat{i} + (2-0)\hat{j} + (0-4)\hat{k}$

$= 4\hat{i} + 2\hat{j} - 4\hat{k}$

unit vector along

$AC = \frac{4\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{16+4+16}} = \frac{4\hat{i} + 2\hat{j} - 4\hat{k}}{6}$

$\therefore$ Force vector $\vec{F_{AB}}$ = magnitude of force × unit vector along the direction of AC

$= 120 \times \frac{4\hat{i} + 2\hat{j} - 4\hat{k}}{6}$

$= 80\hat{i} + 40\hat{j} - 8\hat{k}$

$\therefore$ The resultant force is

$\vec{F_R} = \vec{F_{AB}} + \vec{F_{AC}}$

$= (70.7\hat{i} - 70.7\hat{k}) + 80\hat{i} + 40\hat{j} - 80\hat{k}$

$= 150.7\hat{i} + 40\hat{j} - 150.7\hat{k}$

magnitude of the resultant force

$|\vec{F_R}| = \sqrt{(150.7)^2 + (40)^2 + (-150.7)^2}$

$= 216.84$ N

- **Dynamics** $\boxed{\text{retardation} - \text{मंदन}}$

(1) A ~~particules~~ particle is moving with a uniform retardation of $0.1\ m/s^2$ has travelled 15m. If the initial velocity of the particle is 2 m/s. Find out the time taken by the particle to travle distance of 15m.

$u = 2\ m/s$

$t = ?$

$a = 0.1\ m/sec^2$

$S = 15\ m$

$S = ut - \frac{1}{2} at^2$ (a is -ve because retardation)

$15 = 2t - \frac{1}{2} \times 0.1 \times t^2$; $\Rightarrow (t-10)(t-30) = 0$

$t = 10,\ t = 30$ sec.

(2) (Exam)

$S = 20t + 3t^2 - 2t^3$

$v = \frac{ds}{dt} = 20 + 6t - 6t^2$

$\alpha = \frac{dv}{dt} = 6 - 12t$

$V = \left(\frac{ds}{dt}\right)_{t=2} = 20 + 6 \times 2 - 8 \times 4$

$= 20 + 12 - 324$

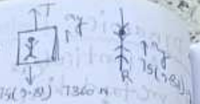
$= 8$

- ~~Rectilinear~~ ~~mot~~ Linear motion

$\boxed{\text{Impulse = very Short distance and large duration}}$

Ex (3)/1 (Exam)

A 75 kg man stands on a Spring scale in an elevator. During the first 3 seconds of motion from rest, the tension T in hoisting cable is 8300 N. Find the reading R of the scale in newtons during this interval and upward velocity v of the elevator at the end of the 3 sec. the

total mass of the elevator man, 
and scale in 750 kg.
The force registered by the scale and the velocity both depend on the acceleration of the elevator, which is constant during the interval for which the forces are constant. From the free-body diagram of the elevator, scale, and man taken together, the acceleration is found to be

• ~~Angular momentum~~

$[\Sigma F_y = ma_y] \quad 8300 - 7360 = 750 a_y$

$\Rightarrow a_y = 1.257 \text{ m/s}^2$

The scale reads the downward force exerted on it by the man's feet. The equal and opposite reaction R to this section is shown on the free-body diagram of the man alonge together with his weight, and the equation of motion for him gives

$[\Sigma F_y = ma_y] \quad R - 736 = 75(1.257)$

$\Rightarrow R = 830 \text{ N}$

The velocity reached at the end of the 3 seconds is

$[\Delta v = \int a\,dt] \quad v - 0 = \int_0^3 1.257\,dt$

$\Rightarrow v = 3.77 \text{ m/s}$

• ~~Angular~~ momentum (Basics)

$$\boxed{a = v\frac{dv}{dx}}$$

$$\boxed{\frac{d(P \times Q)}{du} = \frac{dP}{du} \times Q + P \times \frac{dQ}{du}}$$

$$\boxed{\frac{dP}{du} = \frac{dP_x}{du}\hat{i} + \frac{dP_y}{du}\hat{j} + \frac{dP_z}{du}\hat{k}}$$

• motion relative to a frame in translation

$$\boxed{\begin{aligned} r_B &= r_A + r_{B/A} \\ v_B &= v_A + v_{B/A} \\ a_B &= a_A + a_{B/A} \end{aligned}}$$

• Linear momentum

(vector representation)
(Rectangular components / Rectilinear motion)

$$\boxed{\begin{aligned} r &= x\hat{i} + y\hat{j} \\ v &= \dot{r} = \dot{x}\hat{i} + \dot{y}\hat{j} \\ a &= \dot{v} = \ddot{r} = \ddot{x}i + \ddot{y}j \end{aligned}}$$

• Equations of motion

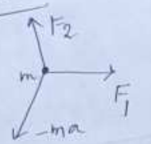
$$\boxed{\sum F = ma}$$

• Rectangular components:

$$\boxed{\sum(F_x i + F_y j + F_z k) = m(a_x i + a_y j + a_z k)}$$

- Dynamic Equilibrium

$\sum F - ma = 0$



$= 0$

→ Here 'ma' is called inertia vector and it applied opposite to the direction of the acceleration.

- Linear impulse – momentum Principle

The effect of resultant force "ΣF" on the linear momentum of the particle over a finite period of time (t_1 to t_2)

$$\Sigma F = m\frac{dv}{dt} = \frac{d}{dt}(mv) = \frac{d}{dt}(G)$$
$$= \frac{d(G)}{dt}$$
$$= \dot{G}$$

$$\Rightarrow \int_{t_1}^{t_2} \Sigma F\, dt = G_2 - G_1$$

$$\Rightarrow G_1 + \int_{t_1}^{t_2} \Sigma F\, dt = G_2$$

$$\Rightarrow \boxed{G_1 + \int_{t_1}^{t_2} \Sigma F\, dt = G_2}$$

- Linear momentum

$\Sigma F = \dot{G} = 0$

$G =$ constant

- Conservation of linear momentum: If the resultant force acting on a particle is zero, the linear momentum of the particle remains constant, in both magnitude and direction.

- Angular momentum

Moment of Linear momentum –

$$\boxed{H_O = r \times mv}$$

$\boxed{\Sigma M_O = \dot{H}_O}$ $\boxed{\dot{H} = r \times ma}$

- Conservation of angular momentum: If the resultant moment acting on a particle is zero the angular momentum of the particle remains constant, in both magnitude and direction.

Ex-3/24

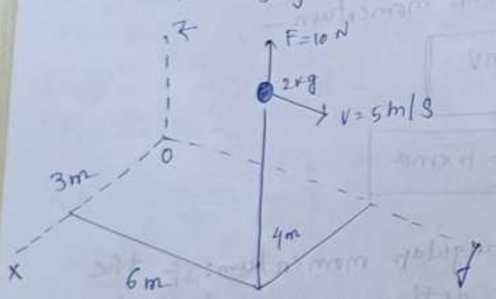
A small sphere has the position and velocity indicated in the figure and is acted upon by the force F. Determine the angular momentum H_O about point O and the time derivative $\dot{H}_O$.

we begin with the definition of angular momentum and write

$$H_o = r \times mv$$
$$= (3i + 6j + 4k) \times 2(5j)$$
$$= -40i + 30k \text{ N·m/S}$$

From Eq. 3/31

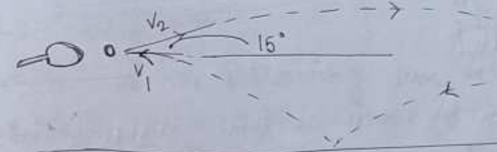
$$\dot{H}_o = M_o$$
$$= r \times F$$
$$= (3i + 6j + 4k) \times 10k$$
$$= 60i - 30j \text{ N·m} \quad \text{Ans}$$

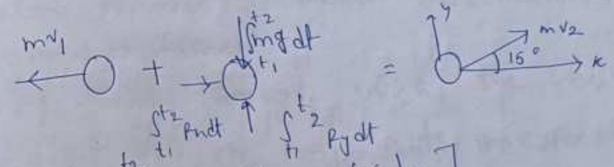


As with moments of forces, the position vector must run from the reference point (O in this case) to the line of action of the linear momentum mv. Here r runs directly to the particle.

Ex 3/2 (Exam)

A tennis player strikes the tennis ball with her racket when the ball is at the uppermost point of its trajectory. The horizontal velocity of the ball just before impact with the racket is v_1 = 50 ft/sec and just after impact its velocity is v_2 = 70 ft/sec directed at the 15° angle as shown. If the 2-oz ball is in contact with the racket for 0.02 sec, determine the magnitude of the average force R exerted by the racket on the ball. Also determine the angle β made by R with the horizontal.





$$\left[m(v_x)_1 + \int_{t_1}^{t_2} \Sigma F_x \, dt = m(v_x)_2\right]$$

$$-\frac{2/16}{32.2}(50) + R_x(0.02) = \frac{2/16}{32.2}(70\cos 15)$$

$$\left[m(v_y)_1 + \int_{t_1}^{t_2} \Sigma F_y \, dt = m(v_y)_2\right]$$

$$\frac{2/16}{32.2}(0) + R_y(0.02) - (2/16)\times(0.02) = \frac{2/16}{32.2}(70\sin 15)$$

we can now solve for the impact forces as

$R_x = 22.8$ lb

$R_y = 3.64$ lb

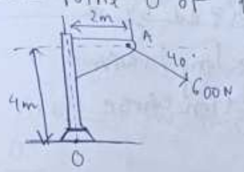
$R = \sqrt{R_x^2 + R_y^2} = \sqrt{22.8^2 + 3.64^2}$

$= 23.1$ lb

$\beta = \tan^{-1} \frac{R_y}{R_x} = \tan^{-1} \frac{3.64}{22.8} = 9.06°$

Previous year question (2021-March-end sem)

A) 1. The magnitude of the moment about the base point O of the given force is –



$M_O = \vec{r} \times \vec{F} = (2\hat{i} + 4\hat{j}) \times 600(\hat{i}\cos 40 - \hat{j}\sin 40)$

$= -2610\,\hat{k}$ N m

The (-ve) sign indicates that the vector is in the negetive z-direction. The magnitude of vector expression is –

$M_O = 2610$ N m

2) which of the following are vector quantities (ies)?

Linear velocity, Linear acceleration, Linear displacement

3) The point through which the whole weight of the body acts, irrespective of its position, is known as – center of gravity.

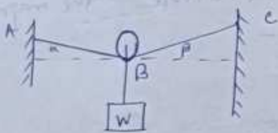
4) If the resultant of two forces P and Q acting at an angle θ makes an angle α with P, then the resultant should be – $\tan\alpha = \frac{Q\sin\alpha}{P + Q\cos\theta}$

5] If two forces each equal to P in magnitude act at right angles, their effect may be neutralized by a third force acting along their bisector in opposite direction whose magnitude is — 2P

6) A framed ~~AQ angle α~~ structure is perfect if it contains members equal to_

2n - 3

7) In figure, angle α compared to β will be—



a) Greater b) Lower c) same d) unpredictable

8) Forces $5\hat{i} + 6\hat{j} + 7\hat{k}$ N and $-8\hat{i} - 9\hat{k}$ N are acting at a point. The resultant force will be of magnitude—

$\vec{R} = -3\hat{i} + 6\hat{j} - 2\hat{k}$

$|\vec{R}| = \sqrt{9 + 36 + 4}$

$= \sqrt{49}$

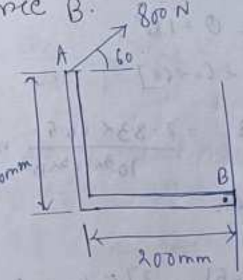
$= 7$ N

9) The three forces keep acting at a point are in equilibrium, then — The forces must be co-planer.

10) A mass of 10 kg is resting on a rough table. The friction force acting on it is — 0

B) 1. The forces F_1, F_2 and F_3, all of which act on point A of the bracket, as shown in the figure. Determine the resultant force and its direction. done

2. A force of 800N acts on a bracket as shown in fig. Determine the moment of the force B.



$M_B = \vec{r} \times \vec{F} = (200\hat{i} + 160\hat{j}) \times (800\cos 60\,\hat{i} + 800\sin 60\,\hat{j})$

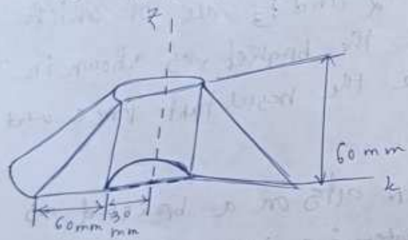
$= \cancel{200\cos 60}\ 200 \times 800 \sin 60\,\hat{k} - 160 \times 800 \cos 60\,\hat{k}$

$= 74564\,\hat{k}$

∴ The moment is in positive 'z' direction and its magnitude is — 74564

3) Locate the centroid of a circular sector with respect to vertex-done.

4) calculate the volume V of the solid generated by revolving the 60 mm right-triangular area through 180° about z-axis. If this body were constructed of steel, what would be its mass "m"?



with the angle of revolution $\theta = 180°$,

$V = \theta \bar{r} A = \pi \left[30 + \frac{1}{3} \times 60\right]\left[\frac{1}{2} \times 60 \times 60\right]$

$= 2.83 \times (10^5) mm^3 = \frac{2.83 \times 10^5}{10^3 \times 10^3 \times 10^3} m^3$

The mass of the body is then

$m = \rho V = \left[7830 \frac{kg}{m^3}\right]\left[2.83(10^5) mm^3\right]\left[\frac{1 m}{1000 mm}\right]^3$

$= 2.21 kg$

(or) $d = \frac{m}{V}$ [$d = 7830 kg/m^3$ given]

$\therefore m = d \times V$

$= \frac{7830 \times 2.83 \times 10^5}{(1000)^3} = 2.21 kg$

5) The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 25t + 7$, where is measured in meters from a convenient origin and t is in seconds. Determine -

a) The time required for the particle to reach a velocity of 72 m/s from its initial condition, at $t = 0$

b) The acceleration of the particle when $v = 30 m/s$

c) The net displacement of the particle during the interval from $t = 1s$, to $t = 4s$

$s = 2t^3 - 25t + 7$

$v = \frac{ds}{dt} = 6t^2 - 25$

a) $6t^2 - 25 = 72$

$\Rightarrow 6t^2 = 97$

$\Rightarrow t = 4.02 sec$

b) $\alpha = \frac{dv}{dt} = 12t$

when $v = 30 m/s$, then t,

$30 = 6t^2 - 25$

$\Rightarrow t = 3.02 s$

$\therefore$ at $t = 3.02s$, $\alpha = 12 \times 3.02$
$= 36.33 m/s^2$

c) $s = 2t^3 - 25t + 7$

at $t = 1$,

$s = 2 - 25 + 7$

$= -16$

at $t = 4$

$s = 2 \times 64 - 25 \times 4 + 7$

$= 35$

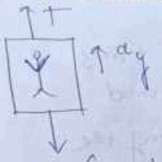
$\therefore$ net displacement during the interval from $t = 1\,s$ to $t = 4\,s$

$\dfrac{-16 - 35}{1 - 4}$

$= \dfrac{51}{3}$

$= 17\,m$

6) A 75-kg man stands on a spring scale in an elevator. During the first 3-seconds of motion from reset, the tension 'T' is the hoisting cable in 8300 N. Find the reading R of the scale in newton during this interval and the upward velocity 'v' of the elevator at the end of the 3-seconds. The total mass of the elevator, man and scale is 800 kg.

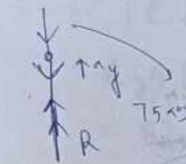


$800\,(9.81) = 7848$

$[\Sigma F_y = m a_y]$ $T - 7848 = m a_y$

$\Rightarrow 8300 - 7848 = 800\, a_y$

$\Rightarrow a_y = 0.565$



75×9.81

$= 736$

$[\Sigma F_y = m a_y]$ $R - 736 = 75\, a_y$

$\Rightarrow R - 736 = 75 \times 0.565$

$\Rightarrow R = 778.3$

$[\nabla v = \int a\,dt]$ $v - 0 = \int_0^3 0.565\,dt$

$\Rightarrow v = [0.565\, t]_0^3$

$= 1.695\, m/s$

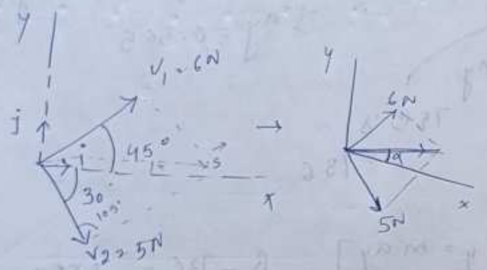
Q 1. For the vectors $\vec{v}_1$ and $\vec{v}_2$ shown in the fig-,

a) Determine the magnitude s of their vector sum $\vec{S} = \vec{v}_1 + \vec{v}_2$

b) Determine the angle α between between $\vec{S}$ and positive x-axis.

c) write $\vec{s}$ as a vector in terms of the unit vectors $\hat{i}$ and $\hat{j}$ and then write a unit vector $\hat{n}$ along the vector sum $\vec{s}$.

d) Determine the vector difference $\vec{D} = \vec{v}_1 - \vec{v}_2$



a) $s^2 = 6^2 + 5^2 + 2 \cdot 6 \cdot 5 \cdot \cos 105°$

$= 45.48$

$\vec{S} = 6.74$ units

b) using the law of sines for the lower triangle, we have

$$\frac{\sin 105°}{6.74} = \frac{\sin(\alpha + 30)}{6}$$

$\Rightarrow \sin(\alpha + 30) = 0.692$ | $\alpha = 29.3$

$\Rightarrow (\alpha + 30) = 43.8$

$\alpha = -24.31$

c) with knowledge of both S and α, we can write vector $\vec{S}$ as

$\vec{S} = S(\hat{i} \cos\alpha + \hat{j} \sin\alpha)$

$= 6.74 [\hat{i} \cos(24.31) + \hat{j} \sin(24.31)]$

$= 5.423\, \hat{i} - 2.778\, \hat{j}$ | $6.74(\hat{i} \cos 29.3 + \hat{j} \sin 29.3)$
$= 5.8\hat{i} + 3.29\hat{j}$

~~Then $\hat{n} = \frac{\vec{S}}{|\vec{S}|} = \frac{5.43\hat{i} + 1.328\hat{j}}{5.59} = 0.971\hat{i} - 0.238\hat{j}$~~

Then $\hat{n} = \frac{\vec{S}}{|\vec{S}|} = \frac{6.14\hat{i} - 2.77\hat{j}}{6.674} = 0.91\hat{i} - 0.41\hat{j}$

$\hat{n} = \frac{5.8\hat{i} + 3.29\hat{j}}{6.7} = 0.86\hat{i} + 0.49\hat{j}$

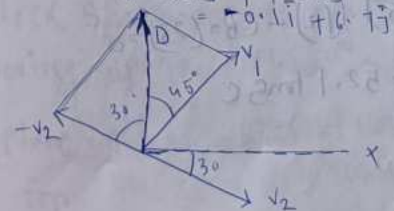
d) The vector difference $\vec{D}$ is

$\vec{D} = \vec{v}_1 - \vec{v}_2 = 6(\hat{i} \cos 45 + \hat{j} \sin 45) -$

$5(\hat{i} \cos 30 - \hat{j} \sin 30)$

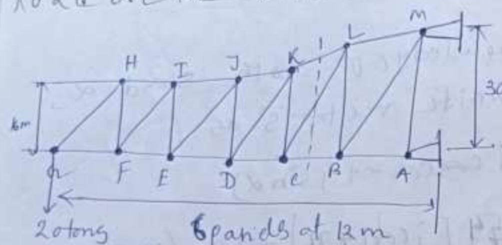
$= \cancel{0.230\hat{i} + 4.33\hat{j}}$ units

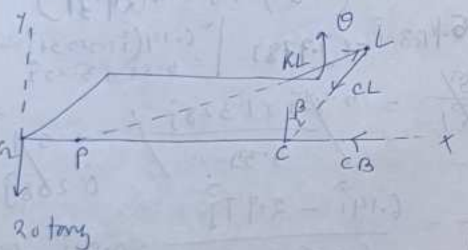
$= -0.1\hat{i} + 6.7\hat{j}$



2) done

3) compute the forces induced in member KL, CL and CB by 20 ton load on the cantilever truss.





summing moments about L requires finding the moment arm

$$\overline{BL} = 16 + \frac{(30-16)}{2} = 23 ft$$

Thus,

$[\Sigma M_L = 0]$ $20 \times (5) \cdot (12) - CB \times (23) = 0$

$\Rightarrow CB = 52.1$ tons C

next we take moments about C, which requires a calculation of $\cos\theta$ from the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos\theta = 12/13$

Therefore,

$[\Sigma M_C = 0]$ $20 \times (4) \times 12 - \frac{12}{13} KL (6) = 0$

$\Rightarrow KL = 65$ tones T

Finally, we may find CL by a moment sum about P, whose distance from C is given by $\overline{PC}/16 = 24(26-16)$ or $\overline{PC} = 38.4$ ft

we also need β, which is given by $\beta = \tan^{-1}(CB/BL) = \tan^{-1}(12/21) = 29.7°$ and $\cos\beta = 0.868$ we now have -

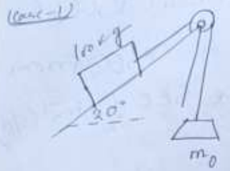
$[\Sigma M_P = 0] \Rightarrow 20(48-38.4) - CL(0.868)(38.4) = 0$

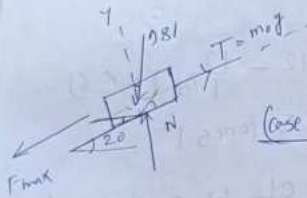
$\Rightarrow CL = 5.76$ C

4) Determine the range of values which the mass m_0 may have so that the 100 kg block shown in the fig will neither start moving up the plane nor slip down the plane. the coefficient of static friction for the contact surface is 0.20.

The max value of m_0 will be given by the requirement for motion impending up the plane. The friction force on the block therefore acts down the plane, as shown in the free body diagram of the block for case 1 with the weight $mg = 100 \times 9.81 = 981 N$ the equations of equilibrium give -

(case-1)





$\Rightarrow N - 981 \cos 20 = 0$ $[\Sigma F_h = 0]$

$\Rightarrow N = 922$ N

$\mu = \frac{F_{max}}{N}$

$\Rightarrow 0.20 = \frac{F_{max}}{922}$

$\Rightarrow F_{max} = 184.4 N$

$m_0(9.81) - F_{max} - 981 \sin 20 = 0$ $[\Sigma F_y = 0]$

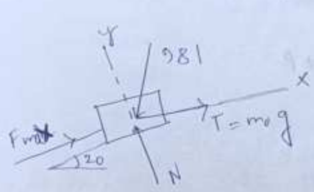
$\Rightarrow m_0 (9.81) - 184.4 - 981 \sin 20 = 0$

$\Rightarrow m_0 = \frac{112}{53}$ kg

(case-II)

The minimum value of m_0 is ~~defendin~~ determined when motion is impending down the plane. The friction force on the block will act up the plane to oppose the tendency to move, as shown in the free body diagram for case

II · Equilibrium in the x-direction requires-



(case-II)

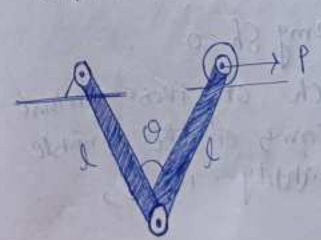
$[\Sigma F_h = 0]$ $m_0(9.81) + 184.4 - 981 \sin 20 = 0$

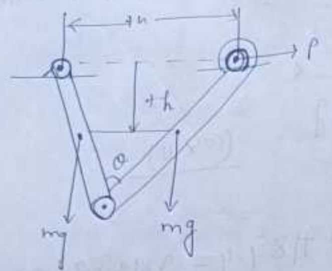
$\Rightarrow m_0 = 15.404$ kg

Thus, m_0 may have any value from 15.404 kg to $\frac{112}{53}$ kg and the block will remain at rest.

[In both cases equilibrium requires that the resultant of F_{max} and N be concurrent with the 981 N weight and the tension T.]

5) Each of the two uniform hinged bars has a mass m and length l and is supported and loaded as shown. For a given force P, determine the angle θ for equilibrium.





The active force diagram for the system composed of the two members is shown separately and includes the weight mg of each bar in addition to the force P. All other forces acting externally on the system are ~~therefore~~ reactive forces which do not work during a virtual movement δx and are therefore not shown.

The principle of virtual work requires that the total work of all external active forces be zero for any virtual displacement consistent with the constrains. Thus, for a movement δx the virtual work becomes-

$[\delta U = 0] \quad P\delta x + 2mg\,\delta h = 0$

we now express each of these virtual displacements in terms of the variable θ, the required quantity. Hence,

$x = 2l \sin\frac{\theta}{2}$ and $\delta x = l\cos\frac{\theta}{2}\,\delta\theta$

Similarly,

$h = \frac{l}{2}\cos\frac{\theta}{2}$ and $\delta h = -\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta$

Substitution into the equation of virtual work gives us —

$Pl\cos\frac{\theta}{2}\,\delta\theta - 2mg\,\frac{l}{4}\sin\frac{\theta}{2}\,\delta\theta = 0$

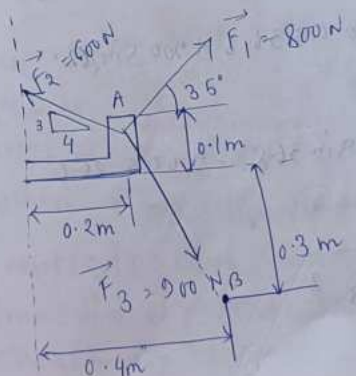
from which we get,

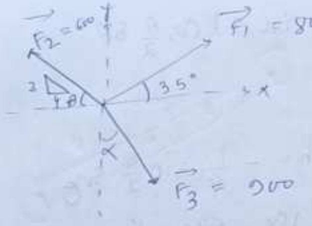
$\tan\frac{\theta}{2} = \frac{2P}{mg}$ or $\theta = 2\tan^{-1}\frac{2P}{mg}$

(2019 – Odd sem. – December)

A)1–10 – done

B)1. The forces F_1, F_2 and F_3, all of which act on a point A of the bracket are specified in three different ways as shown in the fig. Determine the resultant force and its direction.





$$\alpha = \tan^{-1}\left(\frac{0.4/2}{0.1+0.3}\right)$$

$$= \tan^{-1}\left(\frac{0.2}{0.4}\right)$$

$$= 26.6^\circ$$

$$\beta = \tan^{-1}\left(\frac{3}{4}\right)$$
$$= 36.8^\circ$$

(or) Hypotenuse = 5
$\cos\beta = \frac{4}{5}$, $\sin\beta = \frac{3}{5}$ (No need to calculate β)

Force	magnitude	x-com	y-comp
$\vec{F_1}$	800 N	800 cos 35	800 sin 35
$\vec{F_2}$	600 N	$-600 \times \frac{4}{5}$ / $-600 \cos 36.8$	$600 \times \frac{3}{5}$ / $600 \sin 36.8$
$\vec{F_3}$	900 N	900 sin 26.6	−900 cos 26.6

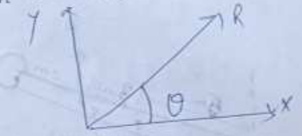
$$\Sigma F_x = 800 \cos 35 - 600 \cos 36.8 + 900 \sin 26.6$$
$$= 577.8 \text{ N}$$

$$\Sigma F_y = 800 \sin 35 + 600 \sin 36.8 - 900 \cos 26.6$$
$$= 13.53 \text{ N}$$

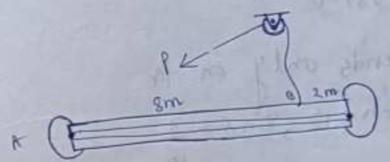
$$\therefore R = \sqrt{577.8^2 + 13.53^2}$$
$$= 578 \text{ N}$$

Let R make the angle θ with x-axis)

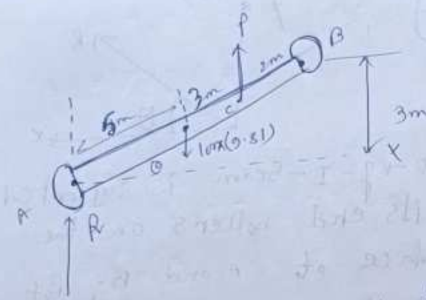
$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$
$$= 1.34^\circ$$



(2) A uniform 100-kg-I-beam is supported initially by its end rollers on the horizontal surface at A and B. By means of the cable at point C, it is desired to elevate end B to a position 3m above end A. Determine the required tension P, the reaction at A and the angle θ made by the beam with the horizontal in the elevated position.



In constructing the force-body diagram, we note that the reaction on the roller at A and the weight are verticle forces. Consequently, in the absence of other horizontal forces, P must also be verticle. From sample problem 3/2 we see immediately that the tension P in the cable equals the tension P applied to the beam at C.



moment equilibrium about A eleminates force R and gives-

$[\Sigma M_A = 0]$ $P(8\cos\theta) - 981(5\cos\theta) = 0$

$\Rightarrow P(8\cos 17.45) - 981(5\cos 17.45) = 0$

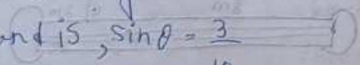
$\Rightarrow P = 613N$

$[\Sigma F_y = 0]$ $P + R - 981 = 0$

$\Rightarrow 613 + R - 981 = 0$

$\Rightarrow R = 368$

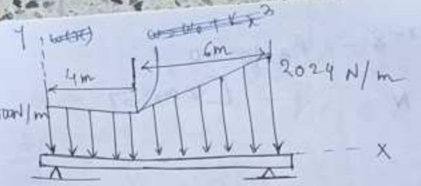
The angle θ depends only on the specified geometry and is, $\sin\theta = \frac{3}{10}$



$\theta = 17.45°$

3) done, 5) done

4) Determine the equivalent cocentrated load(s) and external reactions for the simply supported beam which is subjected to the distributed load as shown in the fig



The area associated with the load distribution is divided into the rectangular and triangular areas shown.

The concentrated load values are determined by computing the areas, and these loads are located at the centroids of the respective areas.

Once the concentrated loads are determined, they are placed on the free body diagram of the beam along with the external reactions at A and B. Using principles of equilibrium, we have

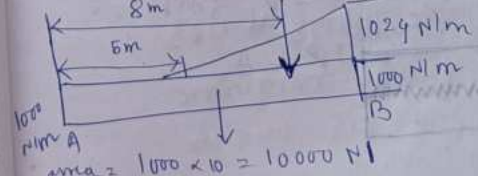
$1200 \times 5 + 480 \times 8 - R_B(10) = 0$ $[\Sigma M_A = 0]$

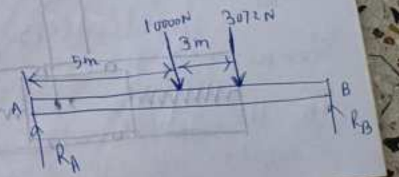
$\Rightarrow R_B = 984$ lb

$R_A \times 10 - 1200 \times 5 - 480 \times 2 = 0$ $[\Sigma M_B = 0]$

$\Rightarrow R_A = 696$ lb

$\frac{1}{2} \times 1024 \times 6 = 3072$ N





area = $1000 \times 10 = 10000$ N

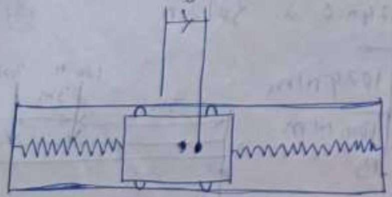
$10000 \times 5 + 3072 \times 8 - R_B \times 10 = 0$ $[\Sigma M_A = 0]$

$\Rightarrow R_B = 7457$ N

$R_A \times 10 - 10000 \times 5 - 3072 \times 2 = 0$ $[\Sigma M_B = 0]$

$\Rightarrow R_A = 5614$ N

c) The spring-mounted slider moves in the horizontal guide with negligible friction and has a velocity v_0 in the s-direction as it crosses the mid-position where $s=0$ and $t=0$. The two springs together exert a retarding force to the motion of the slider, which gives it an acceleration proportional to the displacement but oppositely directed and equal to $a = -k^2 s$, where k is constant. Determine the expressions for the displacement s and velocity v as functions of the time t as shown in the figure.



Incorrect

$a = -K^2 S$

$\Rightarrow \frac{dv}{dt} = -K^2 S$

$\Rightarrow \int_{v_0}^{v} dv = -\int_0^t K^2 S\,dt$

$\Rightarrow [v]_{v_0}^{v} = -K^2 S[t]_0^t$

$\Rightarrow v - v_0 = -K^2 St + c$

$t = 0,\ v = v_0$

$\therefore 0 = c$

$\Rightarrow c = 0$

$\therefore v - v_0 = -K^2 St$

$v = v_0 - K^2 St$

$\Rightarrow \frac{ds}{dt} = v_0 - K^2 St$

$\Rightarrow \int_0^s ds = \int_0^t (v_0 - K^2 St)\,dt$

$\Rightarrow S = \left[v_0 t - \frac{K^2 St^2}{2}\right]_0^t$

$\Rightarrow S = v_0 t - \frac{K^2 St^2}{2}$

$a = -K^2 S$

$a = \frac{dv}{dt} = \frac{dv}{ds} \cdot \frac{ds}{dt} = \frac{dv}{ds} \cdot v$

$v \frac{dv}{ds} = -K^2 S$

$\Rightarrow \int v\,dv = -K^2 \int S\,ds$

$\Rightarrow \frac{v^2}{2} = -\frac{K^2 S^2}{2} + \frac{C}{2}$

$\Rightarrow v^2 = -K^2 S^2 + C$

when $S = 0$, $v = 0$

$\therefore C = 0$

$\therefore v^2 = -K^2 S^2$

Let $-K^2 = R^2$

$\therefore v^2 = R^2 S^2$

$\Rightarrow v = RS$

$\Rightarrow \frac{ds}{dt} = RS$

$\Rightarrow \int_{S_0}^{S} \frac{ds}{S} = R\int_0^t dt$

$\Rightarrow \log \frac{S}{S_0} = Rt$ [at $t = 0$, $S = S_0$]

$\Rightarrow \frac{S}{S_0} = e^{Rt}$

$\Rightarrow S = S_0 e^{Rt}$

$\Rightarrow S = S_0 e^{-K^2 t}$

$\therefore$ Ans $\Rightarrow V = -K^2 S^2$

$S = S_0 e^{-K^2 t}$

c) 3, 4, 5, 6 – done .

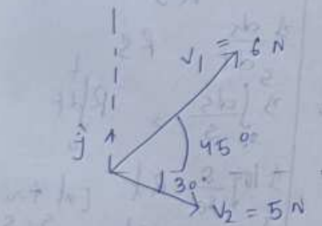
1) For the vectors $\vec{v}_1$ and $\vec{v_2}$ shown in the figure:

a) Determine the magnitude s of their vector sum $\vec{S} = \vec{v_1} + \vec{v_2}$

b) Determine the angle α between $\vec{s}$ and the positive x-axis.

c) write s as a vector in terms of the unit vectors $\hat{i}$ and $\hat{j}$ and then write a unit vector $\hat{n}$ along the vector sum $\vec{s}$.

d) Determine the vector difference $\vec{D} = \vec{v_1} - \vec{v_2}$



~~a) $s^2 =$~~ done

2) The trap door OA is raised by the cable AB, which passes over the small frictionless guide pulleys at B as shown in figure. The tension everywhere in the cable is T, and this tension applied at point A causes a moment M_O about the hinge at O. Determine the relation between M_O/T with the door elevation angle θ. Find the maximum and minimum value of T.