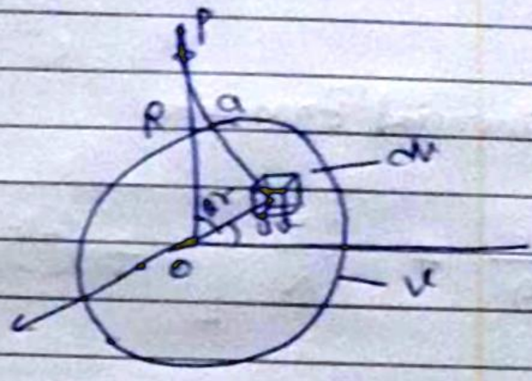
Concept of Multipole :-

"Field due to an arbitary charge distribution and potential at a point"



dv = volume element

∴ charge density $\rho = \frac{q}{dv}$

total charge $q = \rho\, dv$

electric potential at P due to small volume dv

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{q}{a}$$

$$d\phi = \frac{1}{4\pi\epsilon_0}\frac{\rho\, dv}{a} \quad \text{--- (1)}$$

Total electric potential at point P

$$\phi = \int d\phi = \frac{1}{4\pi\epsilon_0}\int\frac{\rho\, dv}{a} \quad \text{--- (2)}$$

<u>triangle formula</u>

$$a^2 = R^2 + r^2 - 2Rr\cos\theta$$

$$a = (R^2 + r^2 - 2Rr\cos\theta)^{1/2}$$

& $$\frac{1}{a} = \frac{1}{(R^2 + r^2 - 2Rr\cos\theta)^{1/2}}$$

$$\frac{1}{a} = (R^2 + r^2 - 2Rr\cos\theta)^{-1/2}$$

$$\frac{1}{q} = \frac{1}{R}\left[1+\left(\frac{r^2}{R^2} - \frac{2r}{R}\cos\theta\right)\right]^{-1/2}$$

using binomial theorem $(1+x)^n = 1 + nx + \frac{n(n-1)}{\lfloor 2}x^2$

From eq^n (2)

$\therefore$

$$\phi = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho\, dv}{(R^2+r^2-2Rr\cos\theta)^{-1/2}} \quad \text{—— (3)}$$

binomial Expansion

$$\frac{1}{q} = \frac{1}{R}\left[1+\left(-\tfrac{1}{2}\right)\left(\frac{r^2}{R^2} - \frac{2r\cos\theta}{R}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{1}{2}-1\right)}{2\cdot 1}\left(\frac{r^2}{R^2} - \frac{2r\cos\theta}{R}\right.\right.$$

$$\frac{1}{q} = \frac{1}{R}\left[1 - \frac{r^2}{2R^2} + \frac{r\cos\theta}{R} + \frac{3/2 \times 1/2}{2}\left(\frac{r^4}{R^4} - \frac{2r^2}{R^2}\frac{2r\cos\theta}{R} + \frac{4r^2\cos^2\theta}{R^2}\right) + \cdots\right.$$

$$\frac{1}{q} = \frac{1}{R}\left[1 - \frac{r^2}{2R^2} + \frac{r\cos\theta}{R} + \frac{3}{8}\frac{r^4}{R^4} - \frac{3}{8_2}\times\frac{4r^3\cos\theta}{R^3} + \frac{3}{8}\times\frac{4r^2\cos^2\theta}{R^2}\right)$$

$$\frac{1}{q} = \left(\frac{1}{R} - \frac{r^2}{2R^3} + \frac{r\cos\theta}{R^2} + \frac{3r^4}{8R^5} - \frac{3}{2}\frac{r^3\cos\theta}{R^4} + \frac{3}{2}\frac{r^2\cos^2\theta}{R^3} + \right.$$

Neglect higher power of R

$$\frac{1}{q} = \left(\frac{1}{R} + \frac{r\cos\theta}{R^2} + \frac{r^2}{2R^3}\left(3\cos^2\theta - 1\right)\right] + \cdots$$

from eq^n (3)

$$\phi = \frac{1}{4\pi\varepsilon_0}\left[\int \frac{\rho\, dv}{R} + \int \frac{\rho\, dv\, r\cos\theta}{R^2} + \frac{1}{2R^3}\int \rho(3\cos^2\theta - 1) r^2\, dv\right] \quad -(4)$$

Let $\int \rho\, dv = P_0$ & $\int \rho\, dv\, r\cos\theta = P_1$

& $\int \frac{\rho(3\cos^2\theta - 1)}{2} r^2 dv = P_2$

from eq^n (4)

$$\phi = \frac{P_0}{4\pi\varepsilon_0 R} + \frac{P_1}{4\pi\varepsilon_0 R^2} + \frac{P_2}{4\pi\varepsilon_0 R^3} + \text{---} \quad -(5)$$

Condition

case 1 net charge present (R is big)

in eq^n (5) 2nd & 3rd will be negligible

$$\phi = \frac{P_0}{4\pi\varepsilon_0 R} \quad \text{but } P_0 = \int \rho\, dv \quad \text{w.r.t. } \rho = \frac{dq}{dv}$$

$$\int dq = \int \rho\, dv$$

$$q = \int dq = \int \rho\, dv$$

$\boxed{P_0 = q}$

monopole moment

$$\boxed{\phi = \frac{q}{4\pi\varepsilon_0 R}}$$

The potential due to charged system (having net charge) is equal to the potential due to point charge on any point at same direction

Date:
P. No:

case - 2 if No net charge (+ve charge = -ve charge

$P_0 = 0$ 2nd = significant

$P_2 = 0$ higher term neg

$$\phi = \frac{P_1}{4\pi\varepsilon_0 R^2}$$ here $P_1 = \int \rho\, dv\, r\cos\theta$

$r\cos\theta = z$

$P_1 = \int \rho z\, dv$

$$\boxed{P_1 = qz}$$

dipole moment

case - 3 $P_0 = 0$ & $P_1 = 0$

P_2 = significant

$$\phi = \frac{P_2}{4\pi\varepsilon_0 R^3}$$ here $P_2 = \int \frac{\rho(3\cos^2\theta - 1) r^2 dv}{2}$

$$\boxed{P_2 = \int \frac{\rho(3z^2 - r^2)\, dv}{2}}$$

Quadrupole moment

Tensor depend on direction

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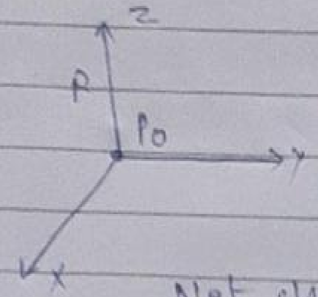
1 monopole



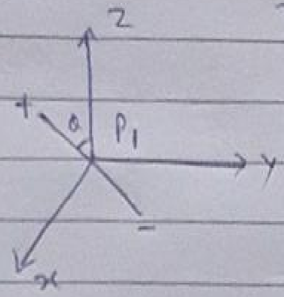
monopole

$\phi \propto \frac{1}{R}$

$E \propto \frac{1}{R^2}$

Net charge

2. Dipole



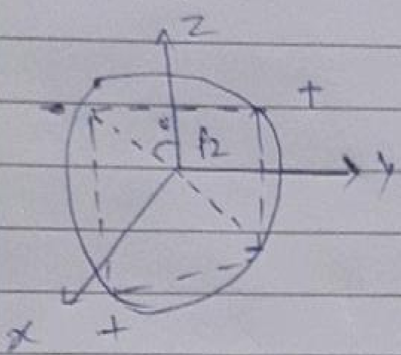
dipole

No net charge

$\phi \propto \frac{1}{R^2}$

$E \propto \frac{1}{R^3}$

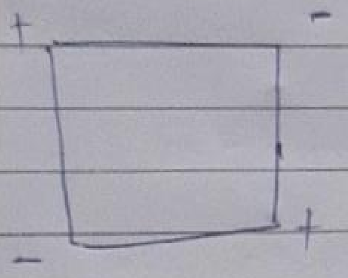
3. Quadrupole



a. sperical $Q = 0$

b. elipse z major $z > r$ +ve

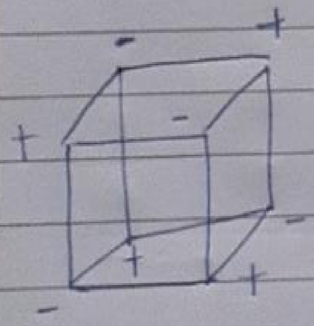
z minor $z < r$ -ve



Quadrupole

$\phi \propto \frac{1}{R^3}$

$E \propto \frac{1}{R^4}$



$\phi \propto \frac{1}{R^4}$

$E \propto \frac{1}{R^5}$