

# 1. TRAIN PROBLEMS

- Distance formula

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

- Conversion

$$\text{Km/h} \xrightarrow{\times 5/18} \text{m/s} \quad | \quad \text{m/s} \xrightarrow{\times 18/5} \text{Km/h}$$

- Relativity

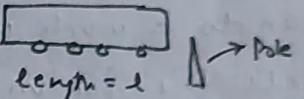
1. Opposite direction:  $|S_1 + S_2|$

2. Same direction:  $|S_1 - S_2|$  (where  $S_1 > S_2$ )

- Time taken by a train to cross a pole/man:

Distance covered = Length of train crossing pole

$$T = \frac{D}{S} = \frac{l}{S}$$



- Time taken by a train to cross a platform/bridge:

Distance covered = length of train + length of platform

$$T = \frac{l+p}{S}$$

- Time taken by 2 train coming from opposite direction to cross each other

$$T = \frac{D}{S} = \frac{a+b}{u+v}$$



- Time taken by 2 train going in same direction to cross each other

$$T = \frac{D}{S} = \frac{a+b}{u-v} \quad u > v$$

- If two trains start at the same time from point A and B toward each other and after crossing they take a and b sec in reaching B & A respect.

A's speed : B's speed =  $\sqrt{b} : \sqrt{a}$



## 2. TIME AND WORK

Q1 If A can complete a work in 10 days & B can complete the same work in 15 days. Then in how many days will they both together complete the work?

Ans  $A \rightarrow 10$  ] Saise phle dono ka LCM nikalo.  
 $B \rightarrow 15$  ]  $\therefore \text{LCM} = 30.$

$$A \text{ per day work} = \frac{30}{10} = 3 \text{ units/day}$$

$$B \text{ per day work} = \frac{30}{15} = 2 \text{ units/day}$$

$$\text{Together, } (A+B) \text{ per day work} = 3+2 = 5 \text{ units/day}$$

$$\text{They will complete the work in, } \rightarrow \frac{\text{total work}}{\text{per day work}} = \frac{30}{5} = 6 \text{ days}$$

Q2 A can do a work in 16 days, B in 12 days. With the help of C, they complete the job in 4 days. How much days does it takes for C to complete the work?

Ans  $A \rightarrow 16$  ]  $A/\text{day} = 48/16 = 3 \text{ units/day}$   
 $B \rightarrow 12$  ]  $B/\text{day} = 48/12 = 4 \text{ units/day}$

$$\text{Now, } \frac{\text{total work}}{\text{per day work}} = 4, \Rightarrow \frac{48}{3+4+C} = 4 \Rightarrow 48 = 28 + 4C$$

$$\Rightarrow 20 = 4C \Rightarrow C = 5$$

$$\therefore C = 5 \text{ units/day.}$$

$$\therefore C \text{ can complete work, } \rightarrow \frac{48}{5} = \underline{\underline{9\frac{3}{5} \text{ days}}}.$$

$$\begin{array}{r} 5 \mid 48 \\ \quad \quad \quad 45 \\ \hline \quad \quad \quad 3 \end{array}$$

Q3 A and B can do a piece of work in 12 days, B and C in 15 days and C and A in 20 days. Find A alone can do that work.

$$\begin{aligned} x: A+B &= 12 \\ y: B+C &= 15 \\ z: C+A &= 20 \end{aligned} \quad \left[ \begin{aligned} x &\rightarrow 60/12 \rightarrow 5u/\text{day} \\ y &\rightarrow 60/15 \rightarrow 4u/\text{day} \\ z &\rightarrow 60/20 \rightarrow 3u/\text{day} \end{aligned} \right]$$

$$x+y+z = 12$$

$$A+B+B+C+C+A = 12$$

$$2(A+B+C) = 12$$

$$A+\underline{B+C} = 6$$

$$A+4 = 6$$

$$A = 2 \text{ u/day}$$

$$\therefore A \rightarrow \frac{60}{2} \rightarrow \underline{\underline{30 \text{ days}}}$$

A = 10 days, B = 20 days, C = 30 days, Total money = ₹600. (3)

C ka share.

First, find ratio of days.

$$10 : 20 : 30 = 1 : 2 : 3$$

Then find ratio of efficiency =  $\frac{1}{\text{days}}$ .

i.e.  $\frac{1}{10} : \frac{1}{2} : \frac{1}{3} \xrightarrow{\text{LCM}} \frac{1}{1} : \frac{1}{2} \times 6 : \frac{1}{3} \times 6 \rightarrow 6 : 3 : 2$

Now C, share

$$\begin{matrix} 6 : 3 : 2 \\ \cancel{x} \end{matrix} \quad \begin{matrix} 11 \\ 6600 \end{matrix}$$

Jiska nikalna hoga,  
work needed.

now,  $2 \times 6600 = 11x$

$$x = \frac{2 \times 6600}{11} = 1200 \text{ Rs}$$

A alone can do a piece of work in 6 days, B alone in 8 days. A and B undertook to do it for ₹3200. With the help of C, they completed the work in 3 days. How much is to be paid to C?

Ans  $A \rightarrow 6$   $B \rightarrow 8$   $\boxed{\text{Lcm} \rightarrow 24}$   $A \rightarrow 24/6 \rightarrow 4 \text{ u/d}$   $B \rightarrow 24/8 \rightarrow 3 \text{ u/d}$

C=?  $\frac{24}{4+3+x} = 3 \Rightarrow 24 = 3(4+3+x) \Rightarrow 24 = 3(7+x)$   
 ~~$24 = 24 + 3x$~~   $8 = 7+x \Rightarrow 8 = 7+x$   
 $\Rightarrow x = 1$

C = 1 u/day, 24 days

Days Ratio  $\rightarrow 6 : 8 : 24 \Rightarrow 3 : 4 : 12$

efficiency  $\rightarrow \frac{1}{3} : \frac{1}{4} : \frac{1}{12} \Rightarrow 4 : 3 : 1$   $\cancel{x} \quad \begin{matrix} 8 \\ 3200 \end{matrix} \quad 3200 = 8x$   
 $x = 400$

B, 4 men & 6 women can complete a work in 8 days. While 3 men and 7 women can complete it in 10 days. In how many days will 10 women complete it.

Ans 1 men work on 1 day =  $x$   
1 woman work on 1 day =  $y$

$$4x+6y = 1/8 \quad \text{--- (1)} \times 3$$

$$3x+7y = 1/10 \quad \text{--- (2)} \times 4$$

$$12x+18y = 3/8 \quad \text{--- (3)}$$

$$12x+28y = 4/10 \quad \text{--- (4)}$$

$$(4) - (3)$$

$$10y = 1/10 - 3/8 = \frac{32-30}{80} = \frac{2}{80} = 1/40$$

$$y = 1/400 \Rightarrow 1 \text{ woman in 1 day}$$

$$\therefore 10y = \frac{1}{400} \times 10 = \frac{1}{40}$$

$$\therefore \underline{\underline{40 \text{ days}}}$$

Q If 6 men and 8 women can do a piece of work in 10 days while 26 men & 48 women can do the same in 2 days, the time taken by 15 men & 20 women in doing the same type of work will be?

Ans  $6n + 8y = 1/10 - \textcircled{1}$      $26n + 48y = 1/2 - \textcircled{2}$      $| n = \frac{1}{100}, y = \frac{1}{200}$

$$\therefore 15n + 20y = 15/100 + 20/200 = 1/4 \Rightarrow \underline{\underline{4 \text{ days}}}$$

Q If 15 men, working 9 hours a day, can reap a field in 16 days. In how many days will 18 men reap the field, working 8 hours a day?

Ans  $15 \times 9 \times 16 = x \times 18 \times 8$

$$\therefore x = \frac{15 \times 9 \times 16}{18 \times 8}$$

$n = 15 \text{ days}$

### 3. PROFIT AND LOSS

(5)

$$1. \text{ Profit} = \text{S.P.} - \text{C.P}$$

$$2. \text{ Loss} = \text{C.P.} - \text{S.P}$$

$$3. \text{ Profit \%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 \quad 4. \text{ Loss \%} = \frac{\text{Loss}}{\text{C.P.}} \times 100$$

Q The ratio of C.P. and S.P. is 4:6. What is profit percentage?

Ans  $\text{C.P. : S.P.} = 4:6 \quad \text{C.P.} = 4, \text{S.P.} = 6 \quad \text{Profit \%} = \frac{\text{Profit}}{\text{C.P.}} \times 100 \Rightarrow \frac{6-4}{4} \times 100$

$$\Rightarrow \frac{2}{4} \times \frac{100}{1} = 50\%$$

$$5. \text{ S.P.} = \frac{100 + \text{Profit \%}}{100} \times \text{C.P}$$

$$6. \text{ C.P.} = \frac{100 - \text{Loss \%}}{100} \times \text{S.P}$$

$$7. \text{ C.P.} = \frac{100}{100 + \text{Profit \%}} \times \text{S.P}$$

$$8. \text{ C.P.} = \frac{100}{100 - \text{Loss \%}} \times \text{S.P}$$

Q A man buys a cycle for 1400 Rs and sells it at a loss of 20%. What is the selling price?

Ans  $\text{C.P.} = \text{Rs} 1400$

$$\text{Loss \%} = 20\%$$

$$\begin{aligned} \text{S.P.} &= \frac{100 - 20}{100} \times 1400 \\ &= \frac{80}{100} \times 1400 \end{aligned}$$

$$\text{S.P.} = 1120$$

Let  $\text{C.P.} = 100\%$ .

$$\begin{array}{r} \text{Rs} \\ 1400 \\ \times 100 \\ \hline 1120 \end{array}$$

$$x = \frac{80}{100} \times 1400 = 1120$$

Q By selling an ipod for Rs 2200 shopkeeper make profit of 40%. Then what will be its profit percentage, if he sold it for Rs 2000?

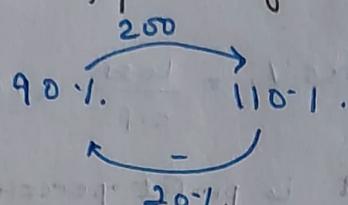
Ans  $\begin{array}{r} \text{C.P.} \quad 100 \\ 2000 \quad \times 140 \\ 2200 \quad x \end{array}$

$$x = \frac{110}{2000 \times 100} \cdot n = 110\% \quad 100 + 10$$

$$\underline{\text{Profit \%} = 10\%}$$

Q An ipod was sold at loss of 10%. If the SP was Rs 200 more, the profit made would have been 10%. What is the actual selling price of the ipod?

Ans



Now find difference of -1.

$$\begin{array}{ccc} 20\% & & 200 \\ 20\% & \xrightarrow{-} & 200 \\ 90\% & & x \end{array}$$

$$x = \frac{200 \times 90}{20} = 900$$

Actual Price = 900

Q Harjit purchased 8 apples for ₹ 20 and sells 7 apples for ₹ 20. Find gain %?

Ans

Rs	Object	140 = 7 x 20	8 ]- 1cm x 7 = 56	160 = 8 x 20 7 x 8 = 56
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$$\begin{array}{lll} CP & 140 & 56 \\ SP & 160 & 56 \end{array}$$

$$P\% = \frac{24}{140} \times 100 = \frac{100}{7} = 14\frac{2}{7}\%$$

Q Paji sold 16 pens at cost of 20 pens. What is the profit or loss percentage made by him.

Ans

$$P/L = \frac{n-m}{m} \times 100$$

then  $n=16$

$m=20$  Total price per dozen

$$\% = \frac{20-16}{16} \times 100 = 25\% \quad \left| \begin{array}{l} \text{If } m \text{ is +, profit} \\ \text{If } m \text{ is -, loss} \end{array} \right.$$

$$\begin{aligned} 16s &= 20c \\ s &= \frac{20}{16} c = \frac{5}{4} c \\ &= 1.25 \end{aligned}$$

$$s = 125c$$

$$100c\% + 25\% \underline{\underline{P.I.}}$$

Q A shopkeeper expect a gain 22.5% on his cost price if his sale was 392. Now find his profit?

Ans

$$\begin{array}{ccc} 150\% & \cancel{x} & n \\ 122.5\% & \cancel{\times} & 392 \end{array}$$

$$CP, n = \frac{392 \times 100}{122.5} = 320 \quad \begin{array}{l} \text{Profit} = 392 - 320 \\ = \underline{\underline{\text{₹ 72}}} \end{array}$$

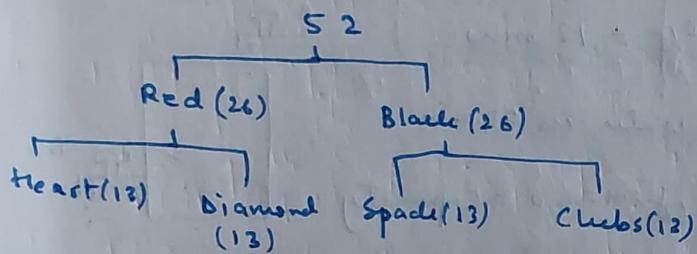
#### 4. PROBABILITY

$$1. P(A) = \frac{n(A)}{n(S)}$$

$n(A) \rightarrow$  no of favourable outcome  
 $n(S) \rightarrow$  sample space

2. In coin problem, sample space ( $n$ ) =  $2^n \rightarrow n = \text{no of coins}$ .

#### 3. Card Problem



Face cards: 12 ( $3 \times 4$ )

Number cards: 36 ( $9 \times 4$ )

Honor cards: 16 ( $4 \times 4$ )

#### 4. Dice Problem

Sum	No. of fav. outcome
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1
	36 → sample space

$$5. 0 \leq P(E) \leq 1 \quad \leftarrow P(S) = 1 \quad 7. P(\emptyset) = 0 \quad \text{or} \quad P(A^-) = 1 - P(A)$$

$$9. P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

## 5. PIPES AND CISTEN

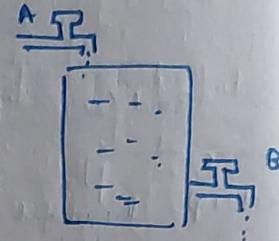
→ Panik tanki

1 Tap A fills tank in 20 min, Tap B in 30 min. A+B=?  
Same as time & work concept.

$$A = 20 \\ B = 30 \\ \text{LCM} = 60 \quad \text{Tank capacity} = 60 \text{ l}$$

$$A = 60/20 = 3 \text{ l/min} \\ B = 60/30 = 2 \text{ l/min}$$

$$A+B = 3+2 = 5 \text{ l/min} \quad \therefore 60/5 = 12 \text{ mins}$$



2 A fills tap in 15 mins, B=discharge water in 20 min.

Time to fill the tank?  $A = 15 \\ B = 20 \\ \text{LCM} = 60 = \text{Capacity}$

$$A = 60/15 = 4 \text{ l/min}$$

$$B = 60/20 = 3 \text{ l/min}$$

$$A-B = 4-3 = 1 \text{ l/min} \quad \underline{\underline{60 \text{ mins}}} / 1 \text{ hour}$$

Q Three pipes A, B & C can fill a tank from empty to full in 80 min, 20 min & 10 min respectively. When the tank is empty all three pipes are opened. A, B, C discharge chemical solution P, Q, R. What is the proportion of the solution R in the liquid in the tank after 3 mins?

Ans  $A = 30 \\ B = 20 \\ C = 10 \\ \text{LCM} = 60$

$$A = 2 \text{ l/min} \quad A+B+C = 11 \text{ l/min}$$

$$B = 3 \text{ l/min} \quad A+B+C \text{ in } 3 \text{ min} = 33 \text{ ltrs in } 3 \text{ min}$$

$$C = 6 \text{ l/min} \quad A+B+C \cdot 3 \text{ min proportion} = \frac{60}{33} = \frac{20}{11}$$

$$\text{in } 3 \text{ min} = 6 \times 3 = 18 \quad \frac{60}{18} = \frac{10}{3}$$

$$A+B+C = \frac{20}{11}, \quad C = 10/3$$

$$20/11 / 10/3 = \frac{2}{11} \times \frac{3}{10} = \underline{\underline{6/11}}$$

Q A pump can fill the tank with water in 2 hours. Because of a leak, it took  $2\frac{1}{3}$  hours to fill the tank. The leak can drain all the water of the tank in?

Ans  $A = 2 \quad A = 7 \text{ l/hr}$

$$A+B = 2\frac{1}{3} = 7/3 \quad \text{LCM} \quad \frac{2}{1} \times \frac{7/3}{1} = \frac{14}{1} \quad \therefore \text{LCM} = 14$$

$$A+B = \frac{14}{7} = 2 \text{ l/hr}$$

$$\therefore A+B = 2$$

$$7+B = 2$$

$$B = -1 \text{ l/hr}$$

↑

Nived shethai

$$\frac{14}{1} = 14 \text{ hours to drain all water}$$

(9)

Two pipes can fill a tank in 20 and 24 mins respectively and a waste pipe can empty 3 gallons per minute. All the three pipe working together can fill the tank in 15 mins. The capacity of tank is ?

Ans

$$\begin{aligned} A &= 20 \text{ min } \\ B &= 24 \text{ min } \\ C = x & \quad \left. \begin{array}{l} \text{Lcm} \\ 120x \end{array} \right\} \rightarrow \text{Capacity assumption} \quad A = \frac{120n}{20} = 6n \\ & \quad A+B+C \rightarrow 1 \text{ hour} \quad B = 5n \\ & \quad 6n + 5n + 120 = 11n + 120 \quad C = 120 \\ \frac{120n}{11n+120} &= 15 \quad n = \frac{18(11n+120)}{120} \\ & \quad 8n = 11n + 120 \quad -120 = 3n \\ 40 \times 3 &= \underline{\underline{120 \text{ gallon}}} \quad n = -40 \end{aligned}$$

$\frac{15}{90}$

Q Two pipe A & B together can fill a cistern in 4 hours. Had they been opened separately than B would have taken 6 hours more than A to fill the cistern. How much time will be taken by A to fill the cistern separately?

Ans

$$\begin{aligned} A &= x \text{ hours} \\ B &= (x+6) \text{ hours} \quad \left. \begin{array}{l} \text{Lcm} \\ x(x+6) \end{array} \right\} \quad A = \frac{(x+6)}{x} \cdot \frac{x(n+x)}{n} \quad A+B = 2x+6 \\ & \quad B = x \quad A+B \text{ in one hour} \\ \therefore A &= \underline{\underline{6 \text{ hours}}} \quad \frac{x(x+6)}{2x+6} = 4 \\ & \quad \Rightarrow x^2 - 2x - 24 = 0 \\ & \quad x = 6, -4 \end{aligned}$$

Q A tank is filled by three pipes with uniform flow. The first 2 pipes operating simultaneously, fill the tank in same time during which the tank is filled by third pipe alone. The second pipe fill the tank 5 hour faster than the first pipe & 4 hour slower than third pipe. The time required by the first pipe is?

Ans

$$\left. \begin{array}{l}
 A, B, C \text{ are } \\
 A = x \text{ hr} \\
 A + B = C \text{ hr} \\
 B = x - 5 \text{ hr} \\
 C = x - 5 - 4 \text{ hr} \\
 = x - 9 \text{ hr}
 \end{array} \right\} \quad \left. \begin{array}{l}
 \text{LCM of } A \text{ & } B \\
 x(x-5) \quad (\cancel{x}) \\
 A + B = 2x - 5
 \end{array} \right\} \quad \left. \begin{array}{l}
 A = (x-5) \\
 B = x \\
 A + B = 2x - 5
 \end{array} \right\}$$

$$\frac{x(x-5)}{2x-5} = (x-9), \quad x^2 - 5x = (2x-5)(x-9)$$

$$x^2 - 5x = 2x^2 - 23x - 845$$

$$x^2 - 18x - 845 = 0$$

$$x = 15, 3$$

$$\underline{x = 15 \text{ hours}}$$

$$\underline{\underline{A = 15}}$$

## 6. Calender Problems

Basics:

In 1 week = 7 days

1 year = 365 days

= 52 weeks + 1 odd day (i.e. extra)

Leap year = 52 weeks + 2 odd days

0 = Sunday

5 = Friday

1 = Monday

6 = Saturday

2 = Tuesday

7 =

3 = Wednesday

4 = Thursday

Cheat Code

O	3	3	6	1	4	6	2	5	0	3	5
J	F	M	A	M	J	Jy	A	S	O	N	D

$$1600 - 1699 \rightarrow 6$$

$$1700 - 1799 \rightarrow 4$$

$$1800 - 1899 \rightarrow 2$$

$$1900 - 1999 \rightarrow 0$$

$$2000 - 2099 \rightarrow 6$$

Q. What was the day of the week on 26<sup>th</sup> Jan 1947?

- A) Monday    B) Sunday    C) Thursday    D) Wednesday

Ans

26 Jan 1947

1) 47    (last 2 digit of year)

$$\begin{array}{r} 11 \\ 4 \sqrt{47} \\ \underline{4} \\ 7 \end{array}$$

2) 11    (Div step 1 by 4, write quotient)

$$\begin{array}{r} 4 \\ 07 \\ \underline{4} \\ 3 \end{array}$$

3) 26    (write date given)

4) 0    (~~Jan~~ Month code)

5) 0    (year code)

84    (Add all)

$$\begin{array}{r} 12 \\ 7 \sqrt{84} \\ \underline{7} \\ 14 \\ \underline{14} \\ 0 \end{array}$$

6) Now divide final sum by 7

$$\begin{array}{r} 1 \\ 7 \sqrt{14} \\ \underline{14} \\ 0 \end{array}$$

7) Note the remainder. Here 0

0 → Sunday.

$$\rightarrow \boxed{0}$$

Q Today is Monday, after 30 days it will be ?

$$7 \overline{) 30} \quad (\text{div by } 7)$$

02

2 add to count  
2 → Mon + Tues + Wed

Q On what dates of April, 2001 did wednesday fall?

A First day on 2 April, 2001

$$\begin{array}{r} 01 \\ 0 \\ 01 \\ 6 \\ 6 \\ \hline 14 \end{array} \quad 7 \overline{) 1} \quad \begin{array}{r} 0 \\ \hline 1 \end{array}$$

→ Sunday

1 April → Monday

2	"	1	0	8	8	0
3	"	M	A	M	T	C
4	"	I	N	J	J	E

4 → wed

4, 11, 18, 25

Q Jan 1, 2008 is Tuesday. What of the week lies on Jan 1, 2009.

A 1 Jan 2008 → leap year → 2 odd days

1 Jan 2009 → non-leap year → 1 odd day (but it will come after Feb 28, so ignore)

Add 2 odd days to Sunday.

T. + W + Th.

## 7. Problems on Ages

(13)

- Q Form linear equations.
- Q The age of man is 4 times of his son. 5 years ago, the man was 9 times as old as his son was at that time. What is the present age of man.

Am Son age =  $n$  } Present  
man age =  $4n$  } Present

$$\begin{aligned} \text{Son age} &= n \\ \text{Man age} &= 4n \end{aligned} \quad \left. \begin{array}{l} \text{5 yrs ago} \\ \text{Man} = 4n - 5 \end{array} \right\}$$

A.T.Q.

$$4n - 5 = 9(n - 5)$$

$$n = 8$$

$$\therefore \text{Man age} = 4n = 32 \text{ yrs}$$

- Q The ratio of father's age to the son's age is 4:1. The product of their ages is 196. What will be the ratio of their ages after five years.

Am

Let Son age =  $n$  A.T.Q.  
Father age =  $4n$

$$4n \cdot n = 196$$

$$4n^2 = 196$$

$$n^2 = \frac{196}{4} = 49$$

$$\underline{n = 7}$$

After 5 yrs.

$$\text{Son} = 7 + 5 = 12$$

$$\text{father} = 28 + 5 = 33 \text{ yrs}$$

$$\text{Ratio} = 33/12 = 11/4 = 11:4$$

- Q One year ago, the ratio b/w Sumers & Raj's age was 4:3. One year hence the ratio of their ages will be 5:4. What is the sum of their present ages in years.

Am

$$\frac{S}{R} = \frac{4n}{3n} \quad \left. \begin{array}{l} \text{1 year ago} \\ \text{Present} \end{array} \right\}$$

$$\frac{S}{R} = \frac{4n+1}{3n+1} \quad \left. \begin{array}{l} \text{Present} \\ \text{1 year later} \end{array} \right\}$$

$$\frac{S}{R} = \frac{4n+2}{3n+2} \pm \frac{5}{4} \quad \left. \begin{array}{l} \text{1 year later} \\ \text{1 year earlier} \end{array} \right\}$$

$$4(4n+2) = 5(3n+2)$$

$$16n+8 = 15n+10$$

$$n = 2$$

 Present  
 $S = 4n + 1 = 9$   
 $R = 3n + 1 = 7$

$$\underline{\text{Sum}} = 16$$

## B. Permutation & Combinations

Q How many 4 digit number are possible with the digits  
 2, 3, 4, 5, 6, 8, 9

- Digits can not be repeated
- Digits can be repeated

Ans a)

$$\begin{array}{cccc} - & - & - & - \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 7 & 6 & 5 & 4 \end{array} = 840 \quad (7 \times 6 \times 5 \times 4) \quad \left\{ \begin{array}{l} \text{choices} \\ \text{4 positions} \end{array} \right.$$

b)

$$\begin{array}{cccc} - & - & - & - \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 7 & 7 & 7 & 7 \end{array} = 7 \times 7 \times 7 \times 7 = 2401$$

Q How many 4 digit number are possible with the digit

1, 2, 3, 4, 5, 6, 0

Beware of 0

- Digits cannot be repeated
- Digits can be repeated

Ans a)

$$\begin{array}{cccc} - & - & - & - \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 6 & 6 & 5 & 4 \\ \uparrow \end{array} = 720 \quad (6 \times 5 \times 4 \times 3)$$

We can't consider 0 at this position

b)

$$\begin{array}{cccc} - & - & - & - \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 6 & 7 & 7 & 7 \end{array} = 6 \times 7 \times 7 \times 7 = 2058$$

Q How many 3 digit number greater than 400 can be made with the digit 2, 3, 4, 0, 5, 6? Digits cannot be repeated

Ans

$$\begin{array}{cccc} - & - & - & - \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 2 \times 4 \checkmark & \rightarrow 3 & 5 & 4 \\ 3 \times 5 \checkmark & & & \\ 0 \times 6 \checkmark & & & \end{array} = 3 \times 5 \times 4 = 60$$

Q How many 3 digit numbers b/w 300 and 700 can be made with the digit 1, 4, 3, 0, 5, 6. Digits cannot be repeated

Ans

$$\begin{array}{cccc} - & - & - & - \\ \uparrow & \uparrow & \uparrow & \uparrow \\ 1 \times 4 \checkmark & \rightarrow 4 & 5 & 4 \\ 0 \times 3 \checkmark & & & \\ 5 \checkmark & & & \\ 6 \checkmark & & & \end{array} = 4 \times 5 \times 4 = 80$$

Q How many 4 digit numbers are possible with the digits 1, 2, 3, 4, 5, 6, 7 which are divisible by 5? Digits are not repeated.

Ans

$$\begin{array}{cccc} \underline{1} & \underline{5} & \underline{4} & \underline{1} \\ | & | & | & | \\ 6 & 5 & 4 & 1 \rightarrow 5 \text{ only} \end{array} = 6 \times 5 \times 4 \times 1 \rightarrow 0/5$$

Same with 0, 1, 2, 3, 4, 5, 6

Ans

$$\begin{array}{cccc} \underline{1} & \underline{5} & \underline{4} & \underline{1} \\ | & | & | & | \\ 6 & 5 & 4 & 0 \rightarrow 0 \end{array} = 120 \quad \begin{array}{cccc} \underline{5} & \underline{5} & \underline{4} & \underline{1} \\ | & | & | & | \\ 5 & 5 & 4 & 1 \rightarrow 5 \end{array} = 100$$

$$\therefore 120 + 100 = 220$$

Q How many 3 digit numbers are possible with the digits 1, 2, 3, 4, 5 which are even/odd? Digits are not repeated

Ans even

$$\begin{array}{ccc} \underline{4} & \underline{3} & \underline{2} \rightarrow 2/4 \\ | & | & | \\ 4 & 3 & 2 \end{array} = 12 \quad \begin{array}{ccc} \underline{4} & \underline{3} & \underline{3} \rightarrow 1/3/5 \\ | & | & | \\ 4 & 3 & 3 \end{array} = 3!$$

$$0!=1, 1!=1, 2!=2, 3!=6, 4!=24, 5!=120, 6!=720, 7!=5040$$

Q In how many different ways letter of the word LEADING can be arranged?

Ans

$$\begin{array}{ccccccccc} \underline{7} & \underline{6} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1} \\ | & | & | & | & | & | & | \end{array} = 7! = 5040$$

No Rep.  
↑

Q In how many different ways letter of the word LEADING can be arranged if it starts with letter [C].

Ans

$$\begin{array}{ccccccccc} \underline{1} & \underline{C} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1} \\ | & | & | & | & | & | & | \end{array} = 1 \times 6! = 720$$

Q In how many different ways can the letter of the word LEADING be arranged in such a way that it starts with vowels / consonants

Ans vowels AEIOU, E, A, I

$$\begin{array}{ccccccccc} \underline{1} & \underline{C} & \underline{5} & \underline{4} & \underline{3} & \underline{2} & \underline{1} \\ | & | & | & | & | & | & | \end{array}$$

$E, A, I \rightarrow 3$

$$3 \times 6! = 3 \times 720 = 2160$$

consonants

$$\begin{array}{ccccccccc} \underline{L, D, N, G} & \underline{1} & \underline{2} & \underline{3} & \underline{4} & \underline{5} & \underline{6} & \underline{7} & \underline{8} \\ | & | & | & | & | & | & | & | & | \end{array}$$

$4 \times 6! = 4 \times 720 = 2880$

$$\begin{array}{r} 5040 \\ 2160 \\ \hline 2880 \end{array}$$

Q In how many different way the letter of word "BANKING" can be arranged?

Ans

$$\frac{\text{Total}}{\text{repeat}} = \frac{7!}{2!} = \frac{5040}{2}$$

b) BANANA KING

$$\begin{array}{l|ll} & B=1 & N=3 \\ & A=3 & K=1 \\ & G=1 & I=1 \end{array} \quad \text{Total}=10$$
$$\frac{10!}{3! \times 3!} = \frac{10 \times 9 \times 8 \times 7 \times 6!}{3 \times 2 \times 1 \times 3 \times 2 \times 1} = 160,800$$

Q In how many different ways can the letter of the word LEADING be arranged in such a way that the vowels always come together.

Ans

L D N G (E A I)

↑ Assume at 1 alphabet

= ~~4!~~ Total = 5 alphabets now

$$\Rightarrow 5! \times 3!$$

$$\Rightarrow 120 \times 6 = 720$$

Q In how many different way can be letter of the word "CORPORATION" be arranged so that the vowels always come together?

Ans

C R P R T N (O O A I O)

$$\frac{7!}{2!} \times \frac{5!}{3!} = \frac{5040}{2} \times \frac{120}{3} = \frac{30240}{2} = 15,120$$

A E I O U

$${}^n C_r = \frac{n!}{r!(n-r)!}$$

Q From group 12 student 5 are to be chosen for competition.

Ans  ${}^{12} C_5 = \frac{12 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3 \times 2 \times 1} = 11 \times 9 \times 8 = \frac{99 \times 8}{2} = 792$

Q From 4 men and 7 women, 5 person are to be selected to form a committee. In how many ways can it be done?

Ans  ${}^{12} C_5 = 792$   ${}^{11} C_5 = \frac{11 \times 10 \times 9 \times 8 \times 7}{5 \times 4 \times 3 \times 2 \times 1} = 11 \times 7 = 66 \times 7 = 462$

Q In how many ways can a group of 5 men & 2 women be made out of a total of 7 men and 3 women?

Ans  ${}^7 C_5 \times {}^3 C_2 = \frac{7 \times 6 \times 5}{2} \times \frac{3}{1} = 7 \times 9 = 63$  and or +

Q From a group of 7 men & 6 women, 5 person are to be selected to form a committee so that at least 3 men are there in the committee. In how many ways can it be done?

Ans  $3M \times 2W / 4M \times 1W / 5M$

$$\begin{aligned} & {}^7 C_3 \times {}^6 C_2 + {}^7 C_4 \times {}^6 C_1 + {}^7 C_5 \\ &= \frac{7 \times 6 \times 5}{3 \times 2} \times \frac{6 \times 5}{2 \times 1} + \frac{7 \times 6 \times 5}{3 \times 2} \times 6 + \frac{7 \times 6}{2 \times 1} \\ &= 35 \times 15 + 7 \times 30 + 21 \\ &= 756 \end{aligned}$$

Q In how many ways can 3 postcards can be posted in 5 post boxes?

$$\underline{5} \quad \underline{5} \quad \underline{5} = 125$$

## 9. Numbers

- Divisible by 2 - Unit place divisible by 2 or not ex 1234 ✓
- Divisible by 3 - Add all digits, if sum div by 3, Then ✓
- Divisible by 4 - Last 2 digits (ex 12324) div by 4, Then ✓
- Divisible by 5 - Unit place 0 or 5, Then ✓
- Divisible by 6 - No must be div by 2 and 3 both, Then ✓
- Divisible by 8 - Last 3 digit (123104) div by 8, Then ✓
- Divisible by 9 - Add all digits, if sum div by 9, Then ✓
- Divisible by 10 - Unit place 0.
- Divisible by 11 - Number the digits from R to L., Take sum of even & odd.  
 Ex.      
$$\begin{array}{r} 699107 \\ \hline 6\ 5\ 4\ 3\ 2\ 1 \end{array}$$

Check result of even-odd.  
if div by 11, Then ✓

Add even	Add odd
$0+9+6$	$7+1+9$
$=15$	$=17$

$$15-17 = -2 \times$$
- Div by 12 - Must be div by 4 and 3. Then ✓
- Div by 14 - " " " " " 7 and 2
- Div by 15 - " " " " " 5 and 3
- Div by 7 - Unit digit of any No mult by 5 and add in remaining part of no.  
 Ex.      3983      ① ~~3983~~  $3 \times 5 = 15$   
 ② 
$$\begin{array}{r} 398 \\ +15 \\ \hline 413 \end{array}$$
      
$$\begin{array}{r} 57 \\ 7 \sqrt{413} \\ \quad 35 \\ \hline \quad 63 \end{array}$$
- Div by 13 - Unit digit of any no mult by 4, add in remaining part of no.
- Div by 17 - " " " " " " 12 ~~and 11~~ " " " " "
- Div by 19 - " " " " " " " 2 " " " " " " "
- Div by 23 - " " " " " " " 7 " " " " " " "
- Div by 29 - " " " " " " " 3 " " " " " " "

## 10. Percentage

Increase formula

Step 1 : Difference

Step 2 :  $\frac{\text{diff}}{\text{Original}} \times 100$

Q Ashok monthly salary was increased by 20%. His new monthly salary is Rs 7200. Find his initial monthly income.

Sol

$$\frac{100}{120} \times n = \frac{7200 \times 100}{1200} = 6000$$

Q A number when decreased by 20% becomes 136. What is the no?

Ans

$$\frac{100}{80} \times n = \frac{136 \times 100}{80} = 170$$

Q Kiran salary was first increased by 30%. Then it was decreased by 30%. If the latest salary is £ 2275. Then what was the original salary of Kiran?

$$100 \xrightarrow{+30\%} 130 \xrightarrow{-30\%} 130 - 39 = 91$$

$$\frac{100}{91} \times n = \frac{2275 \times 100}{91} = 2500$$

Q The price of an article increased by 20% and then increased by 25%. But the latest price is decreased by 10%. What is the effective percentage change in the price?

$$\text{Ans} \quad 100 \xrightarrow{+20\%} 120 \xrightarrow{+25\%} 120 + 30 \rightarrow 150 \xrightarrow{-10\%} 150 - 15 \rightarrow 135$$

$$\text{effec } 135 - 100 = 35\%$$

Q The population of the city increased at the rate of 20% every year for the last three years. If the present population is 203904 then what was the population of city 3 years ago?

$$\text{Ans} \quad 100 \xrightarrow{+20\%} 120 \xrightarrow{+20\%} 144 \xrightarrow{+20\%} 172.8$$

$$\frac{100}{172.8} \times 203904 = x$$

$$\frac{100}{172.8} \times n = 203904$$

$$n = \frac{100 \times 203904}{172.8} =$$

## Bachet Plan

$$100\% \times 100\% = 10000$$

↑              ↑              ↑

Amount      Consumption      Expenditure.

always keep same

Q The price of wheat is increased by 25%. By what percentage the consumption be reduced so that the expenditure on wheat remains same?

Ans

$$125 \times n = 10000$$

$$n = \frac{10000}{125} = 80$$

$\therefore$  Reduced by 20%.

Q The price of tea is decreased by 20%. What percentage should Mohan increase his consumption of tea in order to maintain his expenditure same?

Ans

$$80 \times n = 10000$$

$$n = \frac{10000}{80} = 125$$

25%

Q P & Q wrote test. P got 70% of maximum marks which is 560 and Q got 60% of it. Find marks scored by Q.

Ans

$$70\% \rightarrow 560$$

$$60\% \rightarrow n$$

$$n = \frac{560 \times 60}{70}$$

$$n = \underline{\underline{420}}$$

What percentage of numbers from 1 to 70 have 1 or 9 in the unit digit?

Ans 1, 9, 11, 19, 21, 29, 31, 39, 41, 49, 51, 59, 61, 69,  $\Sigma = 14$

$$= \frac{14}{70} \times 100 = \underline{\underline{20\%}}$$

Q If 20% of a is b, then b% of 20 is same as?

M Let  $a = 100$

$$b = \frac{20}{100} \times 100 = 20$$

20% of 20

$$\frac{20}{100} \times 20 = 4\%$$

4% of a =

Q A student multiplied a number by  $3/5$  instead of  $5/3$ .

What is the percentage error in the calculation?

Sol % error formula =  $\frac{\text{diff}}{\text{original}} \times 100$

$$\text{orig no} = 5/3 \quad \text{diff} = 5/3 - 3/5$$

$$\text{wng no} = 3/5$$

$$= \frac{25-9}{15} = \frac{16}{15}$$

$$= \frac{16/15}{5/3} \times 100 = \frac{16}{15} \times \frac{3}{5} \times \frac{4}{4} \times 100 = \underline{\underline{64\%}}$$

Q A mixture of 20 litre of milk and water contains 20% of water.

The new mixture is formed by adding 5 litre of water. What is the percentage of milk in the new mixture?

M mixture = 20 litre

$$\frac{24}{100} \times 20 = 4 \text{ i.e } 16 \text{ l milk + 4 l water}$$

$$16 \text{ litre milk} + (4+5) \text{ l water} = 25 \text{ l.}$$

$$\frac{16}{25} \times \frac{4}{100} = \text{milk \%}$$

64\% milk

Q Two students appeared at an examination. One of them secured 9 marks more than the other and his marks was 56% of the sum of their marks. The marks obtained by them are?

M I =  $x$  II = 56% of sum of both

$$(x+9) = \frac{56}{100} \times (2x+9)$$

$$25x + 225 = 28x + 126$$

$$I = 33$$

$$II = 42$$

$$\begin{array}{r} 225 \\ 126 \\ \hline 99 \end{array}$$

$$3x = 99$$

$$x = 33$$

## Equivalent Ratios

- 100%  $\rightarrow 1 \star$       12.5%  $\rightarrow 1/8$
- 50%  $\rightarrow 1/2$       11.11%  $\rightarrow 1/9$  \*
- 33.33%  $\rightarrow 1/3$
- 25%  $\rightarrow 1/4$       10%  $\rightarrow 1/10$
- 20%  $\rightarrow 1/5$       9.09%  $\rightarrow 1/11$  \*
- 16.66%  $\rightarrow 1/6$       8.33%  $\rightarrow 1/12$  \*
- 14.28  $\rightarrow 1/7$  \*

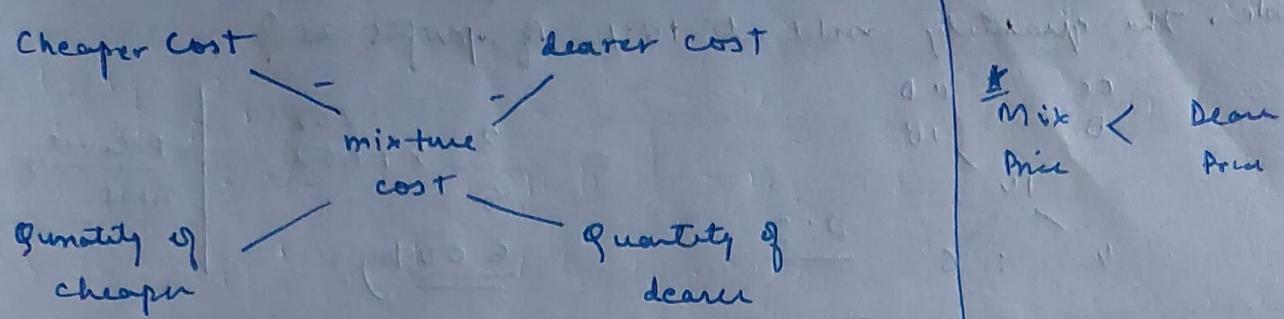
## Split & Merge

45% of 4000

(50-5)% of 4000

50% of 4000 + 5% of 4000

## 11. Alligations' & Mixtures



Q In what ratio must a grocer mix two varieties of wheat at cost £15 and £25 per kg so as to get a mixture worth £21 per kg?

Sol

$$\begin{array}{ccc} \text{CD} & & \text{CC} \\ 15 & & 25 \\ & \swarrow & \searrow \\ & 21 & \\ & \downarrow & \\ 4 & & 6 \\ 4:6 & \rightarrow & \underline{\underline{2:3}} \end{array}$$

Q Find the ratio in which rice at £7.20 a kg be mixed with rice at £5.70 a kg to produce a mixture worth £6.30 Kg.

Sol

$$\begin{array}{ccc} \text{CD} & & \text{CC} \\ 7.20 & & 5.70 \\ & \swarrow & \searrow \\ & 6.30 & \\ & \downarrow & \\ 0.60 & & 0.90 \\ 0.60 : 0.90 & \rightarrow & 2:3 \end{array}$$

\* Pele jo di vilenha  
ha . CC & CD mei

Q The cost of type 1 rice is £15/kg & type 2 rice is £20/kg. If both type 1 & 2 are mixed in the ratio of 2:3, Then the price per kg of the mixed variety of rice is?

Sol

$$\begin{array}{ccc} \text{CC} & & \text{CD} \\ 15 & & 20 \\ & \boxed{18} & \\ 2 & & 3 \\ & & 18/\text{kg} \end{array}$$

Q In what ratio must a grocer mix 2 varieties of tea worth £60/kg & £65/kg so that by selling ~~the~~ the mixture at £68.20/kg he may gain 10%?

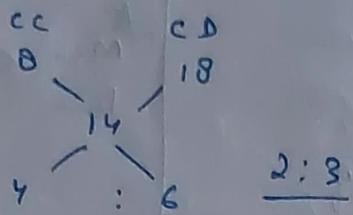
Sol

$$\begin{array}{ccc} \text{CC} & & \text{CD} \\ 60 & & 65 \\ & 62 & \\ 3:2 & & \underline{\underline{3:2}} \end{array}$$

$$\begin{array}{l} x \quad 100 \\ 68.20 \quad 110 \\ \uparrow \\ \text{S.P.} \\ n = \frac{158 \times 68.20}{110} \\ n = 62 \\ \text{C.P.} = 62 \end{array}$$

Q A merchant has 1000 kg of sugar, part of which he sells at 5% of profit & rest at 18% profit. He gains 14%. on the whole. The quantity sold at 18% of profit is?

Ans



$$= \frac{3}{5} \times \frac{200}{600} \times \text{Total Qty}$$

$$= \underline{\underline{600}} \text{ kg.}$$

Q A mixture of certain quantity of milk with 16 litres of water is worth 90p/litre if pure milk be worth £1.08/litre, how much milk is there in the mixture?

Ans Assume price of water = 0p

Given  $m$   $w$   
 $m + 16 \rightarrow 90 \text{ p/L}$   
 $m \rightarrow 1.08 \text{ Rs/L}$   
 $\rightarrow 108 \text{ p/L}$

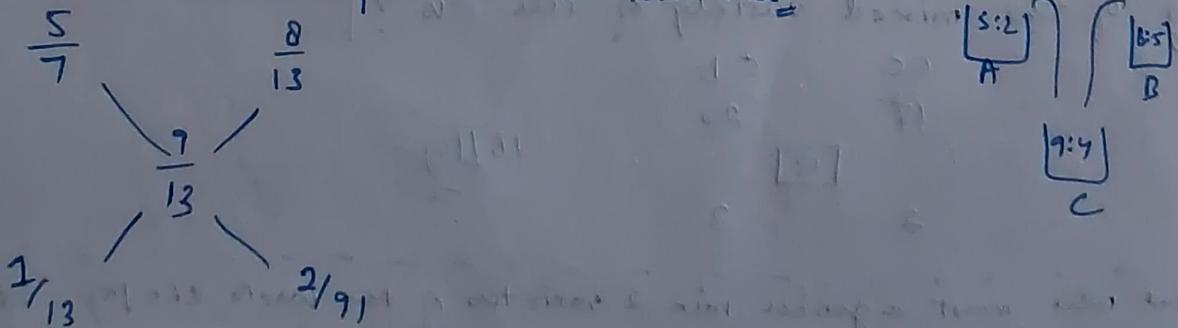
$$\begin{array}{c} m \\ 108 \\ w \\ 0 \end{array}$$

$$108 \rightarrow 16 \rightarrow 90 \text{ p/L}$$

$$16 \rightarrow 16 \times 5 = 80 \text{ litres}$$

Q Milk & water are mixed in a vessel A in the proportion 5:2 and in vessel B in the proportion 8:5. In what proportion should quantities be taken from the two vessels so as to form a mixture in which milk & water will be in the proportion of 9:4?

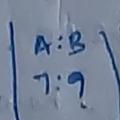
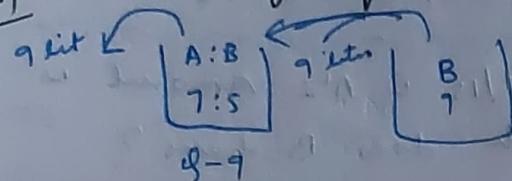
Ans Milk  $\rightarrow$  Take one | Water which be taken



$$\frac{1}{13} : \frac{2}{9}$$

$$7:2$$

A can contains a mixture of two liquids A and B in the ratio 7:5. When 9 litres of mixture was drawn off and the can was filled with B, the ratio of A and B becomes 7:9. How many litres of liquid A was contained by the can initially?



$$\frac{7}{12} : \frac{7}{16} = 1 : ?$$

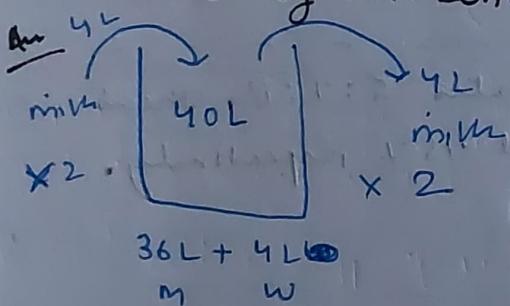
$$\frac{48}{16} : 1 \Rightarrow 3 : 1$$

\* kisi ek ko leuke ka

$$9 = 27 + 7 = \underline{\underline{36 \text{ litres}}}$$

$$\frac{7}{12} \times 36 \\ = 21 \text{ litres}$$

A container contains 40 litres of milk. From this container 4 litres of milk was taken out and replaced by water. This process was repeated further two times. How much milk is now contained by the container?



$n \left( \frac{n-y}{n} \right)^n$   
 $n \rightarrow \text{initial Qty}$   
 $y \rightarrow \text{Nikalne waali}$   
 $n \rightarrow \text{Jism. replicated}$

$$40 \left( \frac{40-4}{40} \right)^3 = 40 \left( \frac{36}{40} \right)^3 = 40 \left( \frac{9}{10} \right)^3 \\ = 29.16$$

## 12. Ratio & Proportion

Ratio

$a : b$   
↓  
consequent  
antecedent

Proportion | the equality of two ratios is proportion

$$\text{eg } 4:5 = 8:10$$

Q A and B together have ₹ 1210. If  $\frac{4}{15}$  of A's amount is equal to  $\frac{2}{5}$  of B's amount, how much does B have?

Ans

$$\begin{aligned} \frac{4}{15} A &= \frac{2}{5} B & A + B &= 1210 \\ \downarrow & & \rightarrow B's \text{ amount} & A = 1210 - B \\ A's \text{ amount} & & & \\ \frac{4}{15} (1210 - B) &= \frac{2}{5} B & \Rightarrow \frac{4840 - 4B}{15} &= \frac{2}{5} B \\ 10B &= 4840 \Rightarrow B = 484 \end{aligned}$$

Q Two numbers are respectively 20% and 50% more than a third number. The ratio of the two numbers is?

Sol Let third no be  $x$

$$\begin{array}{c} 120x \\ 100 \end{array} / \begin{array}{c} 150x \\ 100 \end{array} \quad 4:5$$

Q Seats for 3 subjects in a school is in ratio 5:7:8. There is an increase in seats by 40%, 50% and 75% respectively. What is the ratio of new seats?

Ans

$$\begin{array}{ccc} S_1 & S_2 & S_3 \\ 5x & 7x & 8x \\ \text{Old} \rightarrow & 50 & 70 & 80 \\ \text{New} \rightarrow & 70 & 105 & 140 \end{array} \rightarrow 10:15:20 \rightarrow 2:3:4$$

### 13. Boats & Streams

(27)

- Boat speed in still water =  $U$  kmph
- Stream speed =  $V$  kmph
- downstream speed =  $(U+V)$  kmph
- upstream speed =  $(U-V)$  kmph

Q The speed of boat in still water is 12 kmph and speed of stream is 3 kmph. If the boat is going downstream, how much time it will take to cover 60 kmph?

Sol Boat speed = 12 kmph

Downstream =  $12 + 3$

Stream speed = 3 kmph

= 15 kmph

4 hrs =

Q A boat speed with the current is 15 kmph and speed of the current is 2.5 kmph. The speed against the current is

Sol Downstream speed =  $U+V = 15$

$U + 2.5 = 15$

Current speed,  $V = 2.5$  kmph

$U = 15 - 2.5 = 12.5$

Upstream =  $U-V = 12.5 - 2.5 = 10$  kmph

If downstream speed (a) is given, upstream speed (b) is given, Then

Boat speed =  $\frac{1}{2}(a+b)$

[Note  $a > b$ ]

Stream speed =  $\frac{1}{2}(a-b)$

DS ↑  
↑ US

Q A man rows upstream at 7 km/hr and downstream at 10 km/hr. Find man rate in still water & rate of current?

Sol  $a = 10$

Boat speed =  $\frac{1}{2}(a+b) = \frac{1}{2}(10+7) = \frac{17}{2} = 8.5$

$b = 7$

Stream speed =  $\frac{1}{2}(10-7) = \frac{3}{2} = 1.5$

Distance formula,

$$S = \frac{d}{t}$$

Q A boat running down stream covers a distance of 16 km in 2 hours while for covering same distance upstream it takes 4 hours. What is the speed of the boat in still water?

Sol  $D, S = \frac{D}{T} = \frac{16}{2} = 8 \text{ km/hr}$  |  $U.S = \frac{16}{4} = 4 \text{ km/hr}$

$$B.S = \frac{1}{2} (8+4) = \frac{12}{2} = 6 \text{ km/hr}$$

Q A motor boat whose speed is 15 km/h in still water goes 30 km. down stream and comes back in 4 hours & 30 minutes. What is the speed of stream.

Sol  $B.S (U) = 15$  |  $D.S = 15+n$  |  $\frac{15+n}{15-n} = \frac{30}{30} = 1$  |  $n = 7.5$  |  $U.S = 7.5$

$$t = \frac{d}{s} \quad t_1 + t_2 = 4\frac{1}{2} \Rightarrow \frac{30}{15+n} + \frac{30}{15-n} = \frac{9}{2}$$

$$\frac{100}{(15)^2 - (n)^2} = \frac{9}{2} \Rightarrow 200 = 225 + n^2 \Rightarrow n^2 = 25$$

$$n = \pm 5 \quad \therefore U.S = 5$$

A man can row at 5 kmph in still water if the velocity of current is 1 kmph & takes him 1 hour to row to a place & come back. How far is this place?

Sol  $U=5, V=1 \quad a=5+1=6, b=5-1=4$

Distance  $t_d + t_u = 1 \quad \frac{x}{6} + \frac{x}{4} = 1 \quad x \left( \frac{10}{24} \right) = 1$

$$x = \frac{24}{10} = 2.4 \text{ km}$$

The speed of a boat in still water is 15 km/hr & the rate of current is 3 km/hr. the distance travelled downstream is 12 min is

Sol  $U=15, V=3 \quad a=15+3=18 \text{ km/hr} \quad S=\frac{d}{T}$

$$d = S \times T = 18 \times \frac{12}{60} = \frac{18}{5} = 3.6 \text{ km}$$

## 14. HCF & LCM

LCM: least common multiple  
8, 12

$$\begin{aligned} 8 &: 8, 16, 24, 32, 40, 48 \\ 12 &: 12, 24, 36, 48, 60 \end{aligned}$$

$$\text{LCM of } 8 \text{ & } 12 = \underline{24}$$

HCF: highest common factor  
8, 12

$$\begin{aligned} 8 &: 1, 2, 4, 8 \\ 12 &: 1, 2, 3, 4, 6, 12 \end{aligned}$$

$$\text{HCF of } 8 \text{ & } 12 = \underline{4}$$

Q Find LCM & HCF of  $2^5 \times 3^2 \times 5 \times 7^3$

$$2^5 \times 3 \times 5^3 \times 7 \times 9$$

$$2^5 \times 3^3 \times 5^2 \times 7^2 \times 11$$

(i)  $\text{HCF: } 2^2 \times 3 \times 5 \times 7 = 4 \times 3 \times 5 \times 7 = 20 \times 21 = 420$

(ii)  $\text{LCM: } 2^5 \times 3^3 \times 5^3 \times 7^3 \times 9 \times 11$

Q Three numbers are in the ratio of 2:3:4 and their LCM is 240.

Find their HCF?

Sol Let HCF =  $x$ ,  $2x, 3x, 4x$  LCM =  $12x$

$$12x = 240$$

$$\underline{x = 20}$$

Q Three numbers are in the ratio of 3:4:5 and their LCM is 2400.

Find their HCF?

Am Let HCF =  $x$ ,  $3x, 4x, 5x$  LCM =  $60x$

$$x = \frac{2400}{60} = 80$$

Q The ratio of two numbers is 3:4 and HCF is 4. Find their LCM?

Am  $3x, 4x$

$$x = 4, \therefore 12, 16 \quad \text{LCM of } 12, 16 = \underline{\underline{48}}$$

Q Find the greatest number that will divide 43, 91, 183 so as they get same remainder in each case? A) 4 B) 7 C) 9 D) 13

Sol Jo no digit hai (43, 91, 183) vo badi haas options se, toh HCF nikalna hai vunne LCM.

$$(183 - 43), (183 - 91), (91 - 43)$$

$$\text{HCF} \rightarrow 140, 92, 40$$

$$= 4$$

The HCF of two numbers is 11, and LCM is 7700. If one of the no is 275, find the other no?

HCF = 11, LCM = 7700, no 1(A), no 2(B)

$$\boxed{HCF \times LCM = A \times B}$$

$$11 \times 7700 = 275 \times B$$

$$B = \frac{11 \times 7700}{275} = \underline{\underline{308}}$$

The HCF of  $\frac{9}{10}, \frac{12}{25}, \frac{18}{35}, \frac{21}{40}$  is?

$$\frac{\text{HCF of Num}}{\text{LCM of Den}} = \text{HCF of frac} = \frac{9, 12, 18, 21}{10, 25, 35, 40} = \frac{3}{1400}$$

The least number which should be added to 2497 that the sum is exactly divisible by 5, 6, 4 and 3 is?

$$\text{LCM of } 5, 6, 4, 3 = 60 \quad 60 \overline{)2497} \begin{array}{r} 41 \\ 240 \\ \hline 97 \\ 60 \\ \hline 37 \\ \hline 23 \end{array}$$

What will be least number which when double will be exactly divisible by 12, 18, 21 and 30?

$$\text{LCM} = 1260 \therefore \text{Half of it} = \underline{\underline{630}}$$

A, B and C start at same time in the same direction to run around a circular stadium. A complete round in 252 sec, B in 308 sec, C in 198 sec all starting at same point. After what time will they again meet at the starting point?

$$A = 308, B = 252, C = 198$$

$$\text{LCM of } A, B, C = 2772 \text{ seconds}$$

$$= 46 \text{ min } 12 \text{ seconds.}$$

The greatest possible length which can be used to measure exactly the length 7m, 3m 85cm, 12m 95cm?

Ans HCF

$$\boxed{700}, 385, 1295$$

$$\rightarrow = 35 \text{ cm}$$

The product of two numbers is 4107 if the HCF of these numbers is 37, then the greatest number is?

Sol Let the no be  $37a \& 37b = 4107$

$$1369ab = 4107$$

$$ab = \frac{4107}{1369} = 3$$

$$q.b = 3$$

Co-prime of this no.

$$(1, 3)$$

$$37 \times 1, 37 \times 3$$

$$\begin{array}{r} 37 \\ \underline{\times} \quad 1 \\ \hline 111 \end{array}$$

$$\circlearrowleft \quad \text{111}$$

## 15. Simple & Compound Interest

Simple Interest,  $S.I = \frac{P \times N \times R}{100}$   $N \rightarrow$  always in years

Q What is the interest earned by Raj on an amount of £ 15000 in 10 months at 5% per annum?

Ans  $P = 15000$   $R = 5$

$$N = 10 \text{ months} = \frac{10}{12} \text{ yrs}$$

$$S.I = \frac{15000 \times 10 \times 5}{12 \times 100} = 625$$

Q A man invested some amount in bank at the rate  $16\frac{2}{3}\%$  for 3 years SI is 400, find P?

Ans  $400 = \frac{P \times 50 \times 3}{100 \times 3}$   $P = \frac{400 \times 100 \times 3}{50 \times 3} = 800$

Q A person took some amount with some interest for 2 years but increase the interest for 1%, he paid £ 120 extra then how much amount he took?

Ans  $N = 2$   $1\% \rightarrow 2 \rightarrow 120 \text{ Rs}$

$$2\% = 120 \text{ Rs}$$

$$1\% = 60 \text{ Rs}$$

$$1\% \rightarrow 0 \text{ Rs}$$

$$100\% \rightarrow 2$$

$$x = 6000$$

Q How much time will it take for an amount of £ 900 to yield £ 81 as interest at 45% per annum of S.I?

Ans  $S.I = \frac{900 \times n \times 45}{100}$   $n = \frac{900 \times 100 \times 102}{900 \times 45}$

Q Consider a sum of money at simple interest to £ 955 in 4 years and £ 1055 in 5 years? Find P.

Ans  $955 \text{ in } 4 \text{ yrs} - ①$   
 $1055 \text{ in } 5 \text{ yrs} - ②$   
 $\Rightarrow £ 100 \text{ in } 1 \text{ year}$

$955 \text{ in } 4 \text{ years}$   
 $T.A = P + S.I$   
 $100 \times 4 = 400 \text{ Rs}$   
 $955 = P + 400$   
 $P = 555$

At what rate per annum will a sum of money double in 20 years.

$$N = 20$$

$$T.A. = P + S.I \rightarrow P$$

~~$T.A. = P + S.I \rightarrow P$~~

$R = \frac{100}{N}$

$$S.I = \frac{P.N.R}{100}$$

$$R = \frac{S.I \times 100}{P \times N} = \frac{P \times 100}{P \times 20}$$

$$R = \frac{100}{20} = 5\%$$

Mr Sameer invested an amount of £ 13900 divided into two different Bank A and B at simple interest rate of 14% & 11%. respectively if the total amount of simple interest earned in 2 years be £ 3508, what was the amount invested in Bank B?

Ans,  $A = x, B = 13900 - x$

$$3508 = \frac{x \times 14 \times 2}{100} + \frac{(13900 - x) \times 11 \times 2}{100}$$

$$45000 = 6x + 13900 \times 2 - 11x \\ x = 7500$$

$$B = 6500$$

### Compound Interest

Q. A man borrows £ 1000 from a bank at 10% per annum for 2 years. Find simple & compound interest?

Ans,  $S.I = \frac{1000 \times 10 \times 2}{100} = 200$

$$C.I = P \left(1 + \frac{R}{100}\right)^n - P \\ = 1000 \left(1 + \frac{1}{10}\right)^2 - 1000$$

$$= 1000 \times \frac{121}{100} - 1000$$

$$= 1210 - 1000 = \underline{\underline{210}}$$

### S. I

$$S.I = \frac{P \times N \times R}{100}$$

$$T.A. = P + S.I$$

### C. I.

$$A = P \left(1 + \frac{R}{100}\right)^n$$

$$C.I = P \left(1 + \frac{R}{100}\right)^n - P$$

Q A man borrows ₹ 1000 from a bank for 2 years at the rate of 5% per annum. Find C.I?

$$\text{R.M} \quad CI = P \left(1 + \frac{R}{100}\right)^n - P$$

$$\begin{aligned} &= 1000 \left(1 + \frac{5}{100}\right)^2 - P \\ &= 1000 \left(\frac{21}{20}\right)^2 - 1000 \\ &= \frac{5 \times 21 \times 21 - 2000}{20} \\ &= \frac{225}{2} \quad 1025 \end{aligned}$$

Effective percentage

$$\boxed{\frac{a+b+ab}{100}}$$

$$a = 5 \quad \text{Interest of 1st yr}$$

$$b = 5 \quad " " \quad \text{2nd yr}$$

$$5 + 5 + \frac{25}{100}$$

$$10 + \frac{1}{4} = \frac{41}{4}$$

$$100\% \times \frac{1000}{10.25 \cdot n}$$

$$n = \frac{1000 \times 10.25}{100}$$

$$\underline{n = 102.5}$$

for 3 years

$$\text{first 2 yrs} = 10.25$$

$$a = 10.25, b = 5$$

easy for 2/3 yrs

Q What is the diff. b/w the compound interest on ₹ 5000 for 1 and half years at 4% per annum compounded yearly & half yearly

~~$$\text{Ans yearly} \quad A = 5000 \left(1 + \frac{4}{100}\right)^2$$~~

half

$$\begin{aligned} &= 5000 \times \frac{25}{25} \\ &= 5200 \end{aligned}$$

Q The diff b/w C.I & SI a certain sum at 20% per annum for 2 yrs is ₹ 800. find P?

$$\text{Ans} \quad 800 = P \left(1 + \frac{20}{100}\right)^2 - P - \frac{P \times 2 \times 20}{100} \Rightarrow 800 = \frac{1}{25} \Rightarrow P = 20,000.$$

Q If a sum on compound interest becomes 3 times in 4 yrs then with same rate the sum will become 27 times in?

$$\text{Ans} \quad 3P = P \left(1 + \frac{R}{100}\right)^4 \quad \text{--- ①}$$

$$4\sqrt{3} = 1 + \frac{R}{100}$$

$$27P = P \left(1 + \frac{R}{100}\right)^n \quad \text{--- ②}$$

$$27 = (4\sqrt{3})^n$$

$$(3)^3 = (3)^{n/4}$$

$$n = 12 \text{ yrs}$$

## 16. Clock Problems

Hour

$$360^\circ \rightarrow 12 \text{ hrs}$$

Minute

$$360^\circ \rightarrow 60 \text{ min}$$

$$30^\circ \rightarrow 1 \text{ hr}$$

$$6^\circ \rightarrow 1 \text{ min}$$

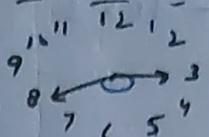
$$\frac{1}{2}^\circ \rightarrow 1 \text{ min}$$

$$\frac{1}{2}^\circ \rightarrow 1 \text{ min}$$

Q. Find the angle b/w  $3:40$ ?

Ans

$$Q = \frac{11}{2} \text{ min} - 30H$$



$$Q = \frac{11}{2} \times 40 - 30 \times 3 = 220 - 90 = 130^\circ$$

Q. At what angle the hands of a clock are inclined in 15 minutes past 5?

Ans

5:15

$$Q = \frac{11}{2} \times 15 - 30 \times 5 = \frac{11 \times 15 - 60 \times 5}{2} = \frac{11(11 - 20)}{2} = \frac{-135}{2} = 67\frac{1}{2}^\circ$$

Ignore

Q. Find the reflex angle b/w the hands of a clock at  $10:25$ ?

Ans

$$Q = \frac{11}{2} \times 25 - 30 \times 10 = \frac{11 \times 25 - 60 \times 10}{2} = \frac{275 - 600}{2} = 162.5$$

$$= 197\frac{1}{2}^\circ$$

$\frac{25}{275} = \frac{600}{275} = \frac{125}{275} = \frac{25}{55}$

Clock coincide concept  $\rightarrow$  Angle  $0^\circ$

$\frac{25}{55} = \frac{5}{11}$

Q. At what time b/w  $3:0$  &  $4:0$  o'clock both the needles will coincide each other?

Ans

$$Q = \frac{11}{2} \text{ min} - 30H$$

$$0^\circ = \frac{11}{2} \text{ min} - 30(3) \quad (\text{consider } \frac{1}{11} \text{ part})$$

$$\text{min} = \frac{90 \times 2}{11} = \frac{180}{11} = 16\frac{4}{11} \text{ min}$$

$$\therefore 3 : 16\frac{4}{11} \text{ min}$$

Opposite direction

$\rightarrow$  Angle  $180^\circ$   
H  $\rightarrow$  Pehla no.

Q. At what time b/w  $4$  &  $5$  o'clock will the hands of a watch point in opp direction?

Ans

$$180 = \frac{11}{2} H + 30 \times 4 \quad \cancel{*} = \frac{60 \times 2}{11} \quad \cancel{*} = \frac{120}{11} = 10\frac{10}{11}$$

$$H = \frac{600}{11} \quad \therefore H = 54\frac{6}{11}$$

$\frac{10}{11} = \frac{10}{11}$

Right angle Angle =  $90^\circ$   
H = Perhln No.

Q What Time b/w 3 & 4 the hands of clock forms right angle?

Ans  $90 = \frac{11}{2}x - 30 \times 3$   $x = \frac{360}{11} = 32 \frac{8}{11}$

$$\boxed{3 : 32 \frac{8}{11}}$$

$$\begin{array}{r} 32 \\ \times 11 \\ \hline 32 \\ 30 \\ \hline 32 \\ \hline 8 \end{array}$$

Q What time b/w 5:30 & 6 the hands of clock forms right angle?

Ans  $90 = \frac{11}{2} \text{ min} - 30H$   $90 = \frac{11}{2}(\text{min} + 30) - 30H$

$$n = \frac{480}{11} - 30$$

$$= \frac{150}{11} = \underline{13\frac{7}{11}}$$

$$\begin{array}{r} 5 : 30 \\ + 13\frac{7}{11} \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ \times 11 \\ \hline 11 \\ 40 \\ \hline 33 \\ \hline 7 \end{array}$$

$$\underline{\underline{5 : 43\frac{7}{11}}}$$

Q A watch gains 5 sec in 3 minutes & was set right at 8 am. What time will it show at 10 pm on the same day?

Ans  $3 \text{ min} \rightarrow 5 \text{ sec}$

for  $\frac{60 \text{ min}}{2 \text{ hr}}$

$3 \times 20 \rightarrow 5 \times 20$

$$1 \text{ hr} \rightarrow 100 \text{ seconds}$$

$$14 \text{ hr} \rightarrow 14 \times 100$$

$$\rightarrow 1400 \text{ seconds} \Rightarrow \frac{1400}{3600} \Rightarrow 23.\underline{33} \text{ sec.}$$

$$8 \text{ am} \rightarrow 10 \text{ pm}$$

14 hours

$$\begin{array}{r} 23.3 \\ \times 40 \\ \hline 920 \end{array}$$

$$\boxed{10 : 23.33 \text{ sec}}$$

Q A watch which gains 5 seconds in 3 mins was set right at 7 am. In the afternoon of the same day, when the watch indicated quarter past 4 o'clock, the true time is

Ans  $3 \text{ min} \rightarrow 5 \text{ sec}$

$$1 \text{ hr} \rightarrow 100 \text{ sec}$$

$$15 \text{ min} \rightarrow 25 \text{ sec}$$

$$7 \text{ am} \rightarrow 4:15$$

$$\text{diff } 9 \text{ hour } 15 \text{ min}$$

$$= 925 \text{ sec ahead}$$

$$= \frac{925}{60} = 15.41 \text{ mins}$$

It is running 15 mins late

4:15 it was actual

True 4

### 17. Chain Rule

a 3 pumps working 8 hours a day, can empty a tank in 2 days. How many hours a day must 4 pumps work to empty the tank in 1 day?

$$3 \text{ pumps} \times 8 \text{ hours} \rightarrow 2 \text{ days}$$

$$4 \text{ pumps} \times ? \rightarrow 1 \text{ day}$$

Indirect prop

$$\begin{matrix} 4 : 3 \\ 1 : 2 \end{matrix} = \underbrace{\qquad}_{12} \qquad 8 : x$$

$$4 \times 1 \times x = 3 \times 2 \times 8$$

$$x = \frac{6 \times 8}{4} = \underline{\underline{12}}$$

Q 39 persons can repair a road in 12 days, working 5 hours a day. In how many days will 30 persons, working 6 hours a day complete the work?

Person      Days      Hour

39            12            5

30            ?            6

$$\begin{matrix} 30 : 39 \\ 6 : 5 \end{matrix} = \underbrace{\qquad}_{12} : x$$

$$30 \times 6 \times 12 = 39 \times 5 \times x$$

$$n = \frac{39 \times 5 \times 12}{30 \times 6} = 13$$

$$\begin{array}{r} 13 \\ 18 \cancel{3} \\ \hline 7 \end{array}$$

Q If 7 spiders make 7 webs in 7 days, then 1 spider will make 1 web in how many days?

spider      webs      day

7            7            7

1            1            ?

$$1 : 7 = 7 : x$$

$$7 : 1 = \underbrace{\qquad}_{?}$$

$$1 \times 7 \times n = 7 \times 1 \times 7$$

$$n = 7$$

Q. 36 men can complete a piece of work in 18 days.  
In how many days will 27 men complete the same work?

Ans

Men	Days
36	18
27	?

$$27 : 36 = 18 : x$$

        4        
        4      

$$27 \times x = 36 \times 18$$

$$x = \frac{36 \times 18}{27} = 24$$