

Gaze Angle Estimate and Correction in Iris Recognition

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Abstract—Conventional iris recognition using a full frontal iris image has reached a very high accuracy rate. In this paper, we focus on processing off-angle iris images. Previous research has shown that it is possible to correct off-angle iris images, but knowledge of the angle was needed. Very little work has focused on iris angle estimation which can be used for angle correction. In this paper, we describe a two-phase angle estimation based on the geometric features of the ellipse. Angle correction is accomplished by projective transformation. Evaluation of this angle estimation and correction method includes a 3D eyeball simulator, and performance test on the West Virginia University Off-Angle Dataset.

I. INTRODUCTION

Iris recognition is considered to be one of the most reliable, accurate biometric approaches for human identification. However, high accuracy is typically achieved on a frontal view full iris recognition in a controlled or “constrained” environment. In contrast, iris images which have occlusions or blur or are captured when the iris is off-angle are called “*non-ideal* iris images”. In many cases, non-ideal iris images are taken in an unconstrained environment. To develop a more robust iris recognition system for more complex scenarios, research has focused on the recognition of *non-ideal* iris images.

Off-angle iris images are when the gaze direction is not directly towards the capturing device. From the perspective of the camera, the shape of the iris and the pupil is no longer a circle, but an ellipse or oval and the center of the pupil is not in the center of the iris region. In that case, a part of the iris texture, is often distorted and usually occluded by eyelids and eyelashes.

Several methods have been developed to process frontal view iris images. Frontal view iris images, because the pupillary and limbic boundary are very much close to a circle, have the benefit to use polar coordinates. The iris region is “two concentric circles” shape that can be transformed to be a rectangle region in the polar coordinates. Methods that are used to solve the off angle iris recognition fall into two categories: 1)transform the off-angle iris into the frontal

view iris, and process it as a frontal view iris [1][2][3]. 2) rather than correcting the angles, processing the iris image with alternative normalization and matching methods [4][5]. To fully utilize all the existing frontal view iris recognition methods, “correcting” or “rectifying” off angle iris images to frontal view could be easily adapted by all other iris recognition processing methods. The angle estimation and correction component could work as a stand-alone component before other processing steps carry on. This paper focuses on estimating the angle from a single iris image, and correct the angle with projective transformation. Compared to other angle estimation, our proposed method is more generic and easier to be adopted by other existing iris recognition methods. The evaluation also shows the proposed method can improve the iris recognition performance significantly.

A. Hardware supported angle estimation

Eye gaze tracking and gazing estimation firstly raise up as the problems in human computer interface research[6][7][8][9]. People can use the eye gaze estimation system to interact with the computer system like other input devices such as keyboard and mouse. As a device for human computer interaction, eye gaze estimation system must not only record the position and movement of the eyeball, but also the position of the head. To accurately track the movement of the eyeball, a certain hardware device, such as stereo glasses, are often explored to keep the track of eyeball movement. Low cost eye tracking devices need to calibrate the eye movement before capturing the image.[10].

B. Angle estimation based on an iris image

In contrast to hardware-facilitated angle estimation, the gaze angle estimation method we designed for iris recognition only needs one iris image as the input, by estimating the angle based on shape of the elliptical iris region. Wang *et.al*[6][7] also has an approach to estimate the accurate angle of the eyeball movement. This approach needs the subjects to look at a few preset fixed points, in order to take the data for camera calibration. Our method does not need the camera settings for

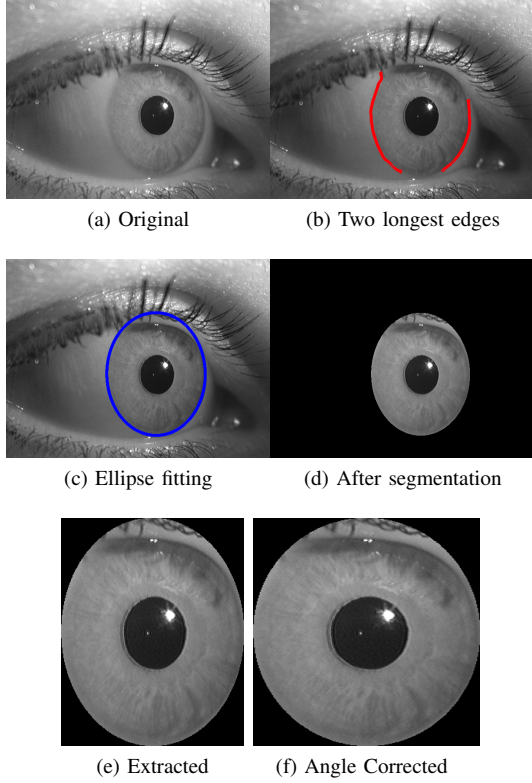


Fig. 1: The processing steps of off-angle iris images: a) The original off-angle iris image; b) Two longest vertical edges detected by Canny edge detection; c) Applied ellipse fitting algorithm to recover a complete iris boundary; d) After the segmentation ; e) the iris region for angle estimation and correction; f) After angle correction, the frontal view full iris recovered from the off-angle iris image.

the captured images, and estimates the angle only based on one image input. The goal of the angle estimation is to recover the off-angle iris to the frontal view iris, without the support of hardware.

Computational complexity is also an important factor that affects the portability of the angle estimation method. The early work in Clarkson University and West Virginia University[3] proposed to use the Daugman's integrodifferential equation as the objective function in order to compensate the angle. The search for the maximum of the objective function is a computation-consuming procedure. Compared to the early work, the method in this paper will try to reduce the brute force search time by giving an initial range, so that the overall time complexity is improved.

Recent research from Oakridge National Laboratory also estimates the angle with a single input iris image. Since there are difference between 3D eyeball and rotating the 2D plane images, direct compensating the angles may not reflect the eyeball rotation [1]. Not only taking account of 3D eyeball structure in angle correction[11], ORNL's method also consider limbus effect and cornea refraction. The ORNL biometric eye model was created based on human eye anatomy model[12]. Taking account of limbus effect and cornea refraction[2], the biometric eye model can "reconstruct" a frontal view iris image

rather than "correct" an off-angle iris image[13]. The ellipse fitting algorithm [14] is used to trace both iris and pupil boundaries. The ellipse parameters, such as major axis, minor axis, and orientation, are compared to the Look up Table(LuT) generated with parameters from the ORNL biometric eye model in each angles.

The proposed angle estimation method in this paper and the ORNL method are inspired by the hardware-facilitating angle estimation[6][7]. Different from the ORNL method, our method estimates the angle based on the geometric features in each iris image instead of applying a general model to every image. A detailed comparison between our proposed method and ORNL method will be introduced in Section II, and the experiment results will be provided in Section III.

II. METHOD

A. Geometry-based angle estimation

In ideal frontal view iris images, the shape of both iris and pupil region are almost circles. While the off-angle iris images, the shape of iris region and pupil region become ellipse due to the prospective projection. Our proposed gaze angle estimation method is based on the correspondence between circle boundary and its projective ellipse and try to recover the circle from the ellipse.

The gaze direction has been studied for the human-computer interaction. The devices are used to capture and record the various gaze direction and movement as an input, and respond the information accordingly. To precisely detect the gaze direction, most devices require a pose calibration in the initialization of the system between human and the device. Wang *et.al* [6][7] simplified the human eyeball anatomy structure to be a geometric model. The Figure 2a shows a top down view of an eyeball model. The angle between the rotated iris plane and its original position is θ . θ also can be represented by the ratio of the long axis and short axis of the ellipse (Figure 2b). The following equations are used to compute θ , ψ . θ stands for the yaw angle(left and right rotation). ψ stands for the pitch angle(up and down rotation).

$$\theta = \arccos(R_y/R_x) \quad (1)$$

$$\psi = \arccos(R_x/R_y) \quad (2)$$

With the above formula, we are able to find the angle of the deformation of the iris region.

In order to accurately determine the shape of the ellipse, we need to find the precise boundary between iris region and sclera. We define this boundary to be the "iris boundary". Since the eyelid and eyelashes occlude a portion of the iris, the boundary is not necessarily a complete circle or ellipse. In our promised method, we use the Direct Least Square Fitting of Ellipse[14] to recover the ellipse with a few scatter points on the iris boundary. It has been used to segment the ellipse boundary of the iris region in other papers, e.g., [15][16][17].

To obtain a more accurate iris boundary, the research in Oak Ridge National Laboratory also considers the limbus effect in off angle iris images. The limbus effect is a component

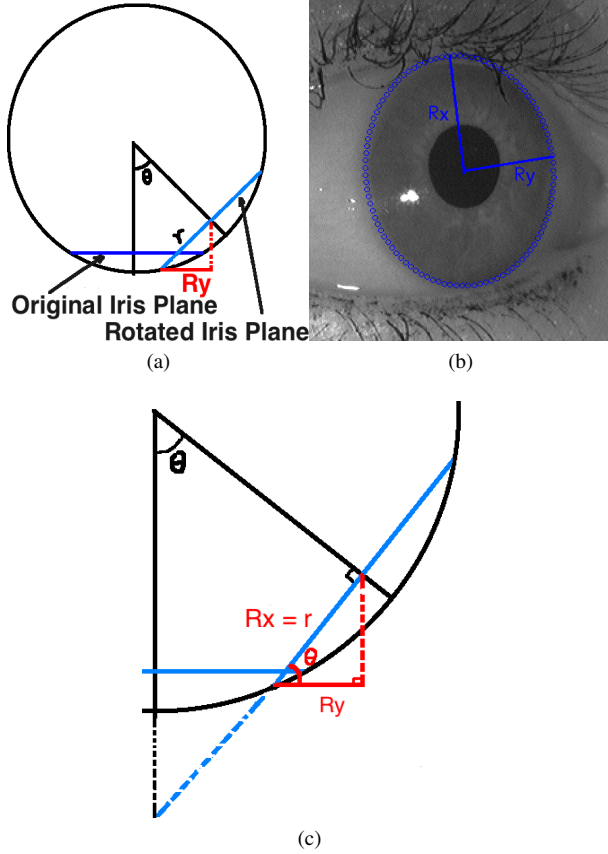


Fig. 2: a) The original iris plane and rotated iris plane from the top-down perspective of the eyeball. The angle difference between the original position and the position after the rotation can be computed by ratio of long axis and short axis in an ellipse. Here θ is the angle difference in yaw direction, r is the radius of the iris region. In the yaw rotation, R_y is the short axis of the ellipse, R_x is the long axis of the ellipse, and the length is equal to the r . The pitch rotation ψ is similar. b) The blue labels the actual boundary of the iris region given by the ellipse fitting algorithm. The red are a demonstration of the long axis and short axis of this iris region. The angle estimated from this figure is 35.6° . c) A zoomed-in version of the trigonometric relation between the original iris plane and rotated iris plane used to estimate θ .

in the ORNL biometrics model[12]. Additionally, the pupil boundary in distorted conditions has also been considered in ORNL biometrics eye model.

B. Processing procedure

The proposed gaze estimation method and the consecutive angle correction are a pre-processing step in order to process the image before the other conventional iris recognition steps. The processing pipeline can be concluded in a few major steps, shown in Figure 3.

- *Data input and eye location*

Before the Two-phase angle estimation begins, the eye location algorithm is applied to locate iris region in a whole image. There are multiple features in the images that we can leverage to find the approximate region,

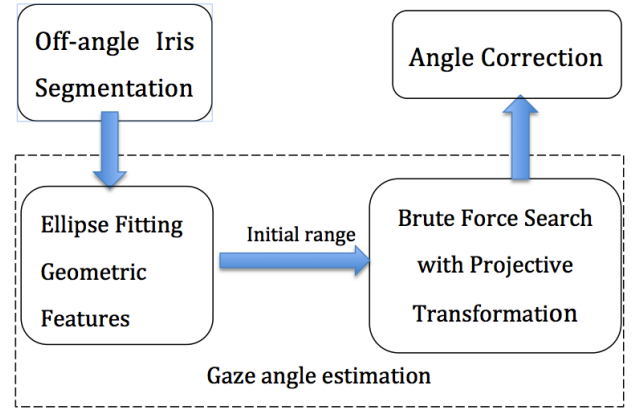


Fig. 3: Two-phase angle estimation

such as the grayscale threshold, the circle shape edges, and the spectrum reflection. The output of the eye location algorithm is an approximate circle region that can be used to filter out the noise outside this region.

- *Segmentation*

Different from the other segmentation methods, the shape of iris region in off-angle iris images is not a circle, and also not necessarily an ellipse, the process begins with a basic edge detection method. At first, our own implementation of Canny edge detection was used to find all the edges within a range of orientation. In particular, to eliminate the interference of the occlusion of the eyelids and eyelash, the algorithm only searches for the vertical edges in the iris images. Next the eye location information is used to filter out the noise edges outside the iris region. We assume the longest vertical edges are left and right boundary of the iris. The edge detection provides the two longest vertical edges to the ellipse fitting algorithm. As we introduced in the last subsection, a complete ellipse is generated that tracks the boundary of the iris region.

- *Two-phase angle estimation*

The angle estimation procedure is composed of two steps. 1) In the previous step, the ellipse fitting algorithm returns a set of the parameters of the ellipse, including the long axis and short axis value, the center of the ellipse, and the orientation of the ellipse. By examining the orientation of the ellipse, we can decide whether the yaw angle, pitch angle or both need to be involved in the angle estimation. If the orientation is within $[80^\circ, 100^\circ]$, the program will process as the yaw angle only. If the orientation locates within $[-10^\circ, 10^\circ]$, the program will process as the pitch angle only. If the orientation is not in these ranges, it is a combination of both yaw and pitch angle. The initial range $< \theta, \psi >$ as an input for the next phase would be computed with Equation 1. 2) The second phase estimation will use projective transformation to rotate the iris region we extracted in the segmentation step. The brute force search will start with the initial range from the first phase. The program will use projective transformation to alter

the iris region, in order to search the maximum area in yaw angle $[\theta - 10^\circ, \theta + 10^\circ]$ for each degree in $[\psi - 10^\circ, \psi + 10^\circ]$. If there are multiple positions which have an approximate same area value, then the program will compute the difference between long axis and short axis, and select the position with minimum long axis and short axis difference. Since the brute force search is a computationally consuming work, the initial range provided by the first phase is the initial value for the adjustment in order to reduce the computation complexity. The figure 3 shows the pipeline of the two-phase angle estimation method.

- *Angle correction*

Once the angle estimation is complete, the off angle iris region can be corrected by projective transformation with the angles $\langle \theta, \psi \rangle$. In this step, the off-angle iris will be restored to the frontal view. Figures 1(e)(f) show the off angle iris image before and after angle correction.

- *Iris pattern encoding and matching* After angle correction, we can use existing encoding and matching methods as if they were frontal view images. In our performance experiment, the encoding method is a modified version of an implementation of the Daugman's concentric circle models [18]. The final performance is also evaluated by the Hamming Distance.

C. ORNL method

Compared to our proposed method, the method of Oakridge National Laboratory is using a similar segmentation method, but the mechanism of estimation is different. Both of the two methods used an ellipse fitting algorithm based on points from the edge detection algorithm. ORNL's method also takes account of the refraction of cornea, and the limbus effect of the off angle iris. The research at ORNL also built a 3D eyeball model and uses this model to generate a Look up Table (LuT) which maps the ellipse parameters to a corresponding angle value. Instead of gradually probing the angle to recover the off-angle iris to frontal view, ORNL's method efficiently obtains the estimation by using the Look up Table. Without consider the time to generate the LuT from the ORNL biometrics model, the LuT has more advantage in computational complexity.

The novelty of our proposed method directly estimates the angles based on the iris image, which we think will be more general for cases where the LuT may not be available or appropriate for a specific iris database.

In the next section, we discuss the performance evaluation in our angle estimation and correction method.

III. EXPERIMENT AND DISCUSSION

The performance evaluation of the angle estimation and correction method includes a test using 3D eyeball simulator developed in our laboratory and iris recognition performance test based on the WVU off-angle dataset.

A. Virtual eye

To test the accuracy of the angle estimation approaches, a 3D eyeball simulator *virtualeye* is developed to simulate the rotation of the human eyeball. We assume the basic eyeball model is a sphere. According to a study [19], the radius of the eyeball is very close to an anatomical constant. The radius of human eyeball ranges from 12mm to 13mm. In our experiment, the radius of the sphere was set to 12.5mm. The radius of the iris region is acquired from[8]. In our experiment, the radius of the iris region was set to 7mm. The purpose of this simulator is to generate off-angle snapshots with ground truth angle value.

Set	← 15°	← 25°	↓ 15°	↓ 25°	↓15°, ←15°
Estimate 1	0°, ← 18°	0°, ← 30°	↓ 18°, 0°	↓ 30°, 0°	↓26°, 0°
Estimate 2	0°, ← 18°	0°, ← 30°	↓ 17°, 0°	↓ 31°, 0°	↓15°, ←15°
Set	→ 15°	→ 25°	↑ 15°	↑ 25°	↑15°, ←15°
Estimate 1	0°, → 17°	0°, → 29°	↑ 17°, 0°	↑ 29°, 0°	↑ 25°, 0°
Estimate 2	0°, → 17°	0°, → 28°	↑ 17°, 0°	↑ 30°, 0°	↑15°, ←15°

Fig. 4: Angle estimation for our method tested on *virtualeye* screenshots. The estimation is represented in pitch and yaw angles. The arrow means the direction of the angle, the number behind the arrow is the value estimated by the method.

The program of *virtualeye* is implemented in OpenGL. Users can use the keyboard to control the rotation of the eyeball degree by degree, either up-and-down or left-and-right. Figure 4 shows the screenshots of the *virtualeye* program, the ground truth pitch and yaw angle. In Figure 4, the second row shows the actual angle we set in the *virtualeye*. The other two rows are the estimations.

To generate a sequence of snapshots, we adjusted the pitch angles (up-and-down) and yaw (left-and-right) in the *virtualeye* simulator. Then we processed the screenshots with our angle estimation program. The result is shown in Figure 4.

Figure 4 shows the results of the angle estimation method tested on the *virtualeye* simulator. After the second phase, the result is more accurate than the result of the first phase. We noticed the angles we detected with our angle estimation method have a small offset than the ground truth angle. The difference was caused by the 3D model and the transformation when it was projected to be a 2D image. To recover the frontal view, rotating a 2D image is different with rotating a 3D model. The 2D image has to turn more degrees to compensate the angle, compared to the angles on the 3D model. This difference also exists in other angle estimation and correction methods that correct an angle from a 2D plane. In the later performance experiment, the difference between ground-truth angle and the angle we estimated impacts performance.

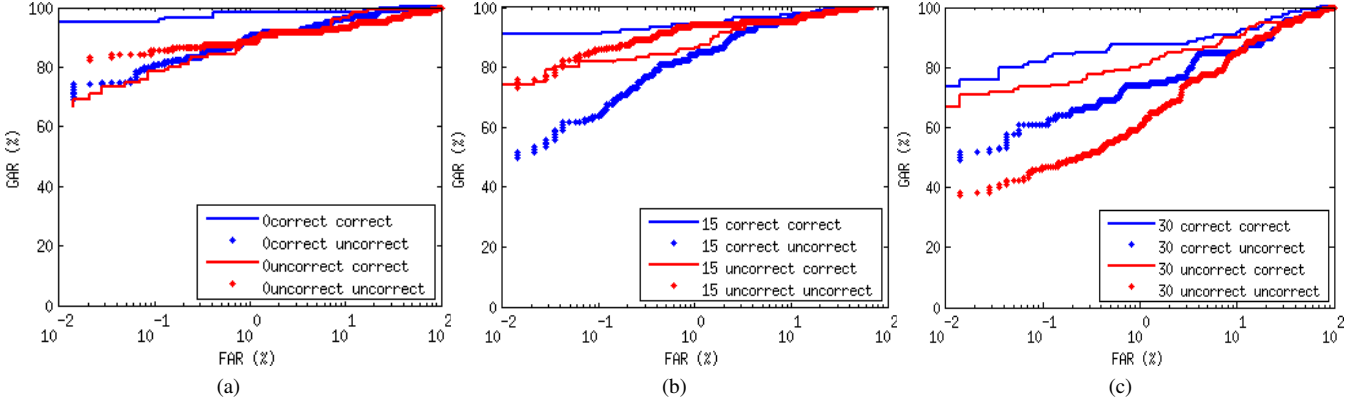


Fig. 5: ROC curves for each degrees (Two-phase angle estimation and correction method)

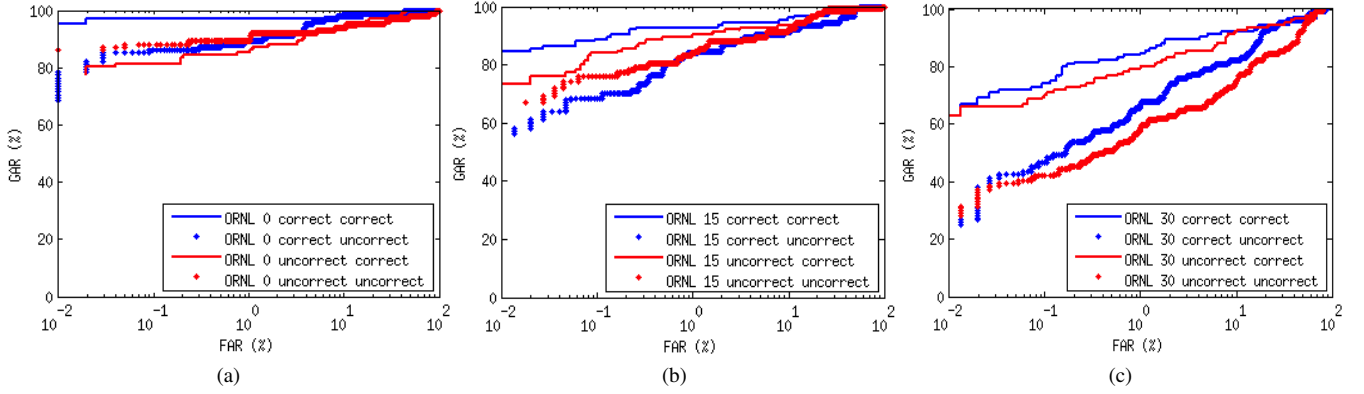


Fig. 6: ROC curves for each degrees (ORNL angle estimation and correction method)

B. Recognition Performance

The performance evaluation is conducted on West Virginia University off-angle monochrome iris dataset. The WVU off-angle dataset contains 73 subjects in total. Each subject has both right eye and left eye data. The images were taken with a chin rest that can help the subjects to control their head movement. So that all the angles we detected come from the eyeball movement. Here we treated both right eye and left eye to be two subjects. Each eye has four iris images taken from four different angles 0° , 15° , 30° , 0° . We treated one of the 0° image as the gallery image.

In the matching stage, the experiments were conducted to evaluate the impact of our angle estimation and correction method in terms of iris recognition performance. In addition to correcting the 15 and 30 degree images, we also corrected the 0 degree gallery and probe images. One 0 degree image was used as the gallery for all tests. We examined four types of comparison: angle corrected probe images vs. angle corrected gallery images, corrected probe images vs. uncorrected gallery images, uncorrected probe images vs. corrected gallery images, and uncorrected probe images vs. uncorrected gallery images. We used the Hamming Distance scores as our matching scores. The ROC (receiver operating characteristic) curves were plotted for set of images in figure 5.

In Figure 5, the corrected vs corrected curve for 0° , 15° , and 30° shows the highest recognition performance among all these four curves. To compare the result, we measure the Genuine Accept Rate when the False Accept Rate is 0.1%. The result is plotted in Table I. The Figure 5 shows our proposed angle estimation method can increase the recognition rate comparing to the original recognition method without using the angle estimation and correction methods.

Two-phase angle estimation and correction	0°	15°	30°
corrected vs. corrected	95%	91%	82%
corrected vs. uncorrected	80%	61%	60%
uncorrected vs. corrected	75%	81%	72%
uncorrected vs. uncorrected	85%	84%	45%
Look-up-Table angle estimation and correction	0°	15°	30°
corrected vs. corrected	97%	89%	73%
corrected vs. uncorrected	85%	68%	45%
uncorrected vs. corrected	80%	83%	67%
uncorrected vs. uncorrected	88%	75%	40%

TABLE I: Genuine Accept Rate comparison between Two-phase angle estimation method and the ORNL method, when the False Accept Rate is equal to 0.1%.

The WVU off angle dataset contains 73 subjects in total. Segmentation is a major difficulty in the entire process. The

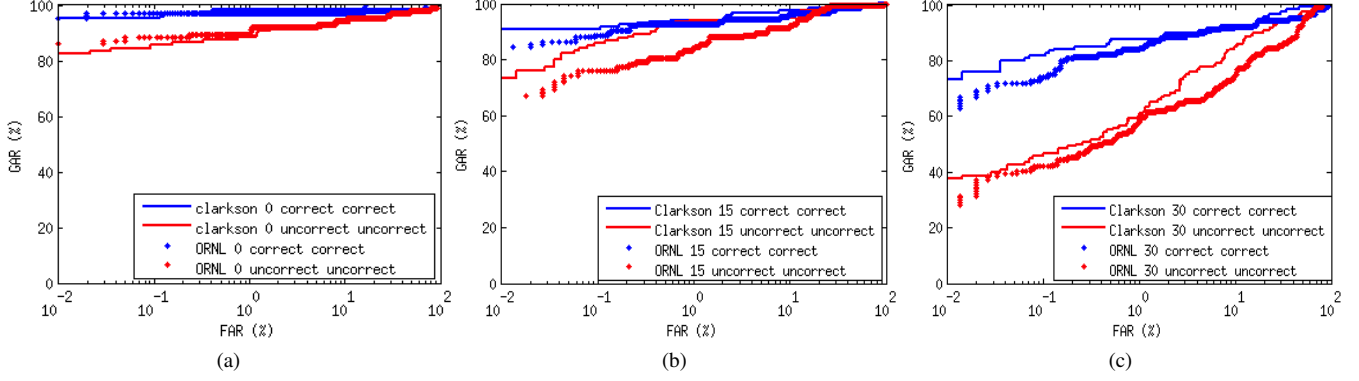


Fig. 7: ROC curves comparison between Two-phase angle estimation and correction method and ORNL angle estimation and correction method.

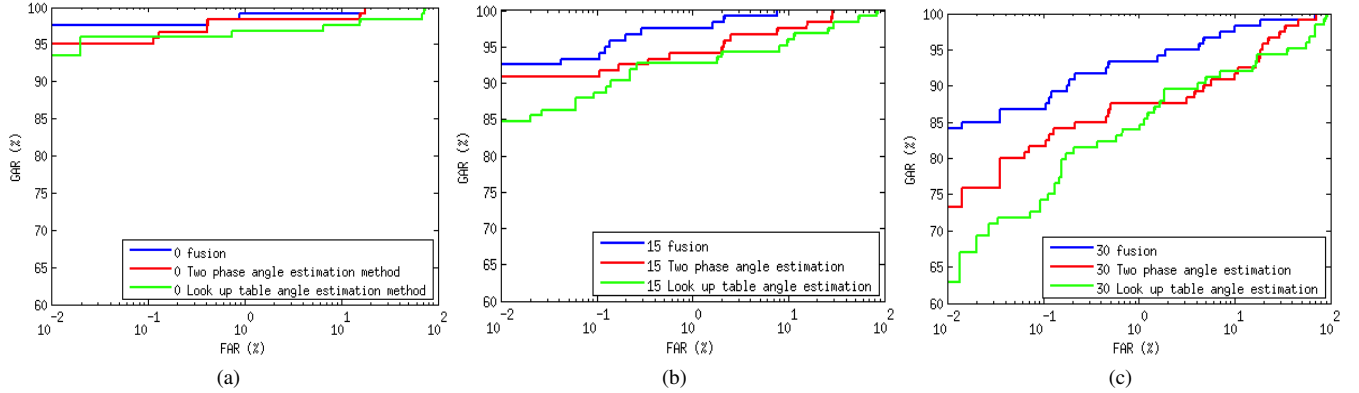


Fig. 8: ROC curves comparison among two angle estimation methods and the fusion of these two methods by selecting the best matching in matching step.

parameters for the edge detection method are not invariant, and need to be adjusted during the process procedure to seek the more successful segmentation. The same edge detection method was applied to both angle estimation methods. In the final iris recognition step, there is some tuning in the segmentation with Hough transformation. If there is an unacceptable segmentation in one image in a subject's data, the data of the subject is removed from the later process. Since after the angle correction, the corrected iris images will be processed by the modified version Libor Masek iris recognition code, there are also many images which failed in this step. In that case, 60 subjects out of 73 are involved in the performance experiment for Two-phase angle estimation method, and 64 subjects are involved in the experiment for Look up Table angle estimation method.

The original experiment in ORNL papers were conducted in their own ORNL off angle iris dataset, and the results were presented in terms of VeriEye [20] matching scores. To make a comparison, we adapted their angle estimation method to our iris recognition code, measured the similarity with Hamming Distance scores. The result is also plotted in the form of ROC curves and in Table I

Comparing our Two-phase method and ORNL look up

table angle estimation and correction method, our method has a better recognition performance according to the result. In Table I, the recognition rate is especially better in 15° and 30°. The performance improvement can be confirmed in a larger scale performance evaluation. Without considering 3D eyeball surface and optics theory, our two-phase method also can achieve the same good performance. The novelty of our method is it is an alternative angle estimation and correction method achieving good recognition performance and acceptable computation complexity.

Since the ORNL's estimation method is based on an ideally predefined LuT(Look-up-Table), searching the corresponding value with the ellipse parameters cost less computation time in angle estimation phase. Without using the ideal 3D model and Look-up-Table, our program estimates the angle directly from the image, searching for the proper angle for the projective transformation. The slight performance difference may come from the dataset, the current Look-up-Table is generated from the generic biometrics model. If the ORNL's LuT (Look-up-Table) could consider the information from the dataset during its building up procedure, the efficiency of the algorithm will be maintained and the performance will be improved.

Achieving higher accuracy performance with less compu-

tation complexity is always the aim for designing a system. To be specific, our angle estimation and correction methods are written in Matlab 2012, and the performance test is conducted on a server with Quad-core Xeon 2.0GHz CPU. If there is no compound angles(both yaw and pitch angles) involved, one direction angle estimation costs less than 10 seconds for one image. The compound angles means larger searching space and 3-5 minutes to process. There is hardly other discussions about the computation complexity in other biometrics literatures. We presume the computation complexity issue could be compensated by implementation and other optimization approaches, so it is not the main concentration of the biometrics study.

The matching score level fusion experiment is also conducted upon these two angle estimation methods. The fusion matching scores are defined to be the best matching scores of these two methods. The result of matching score level fusion is shown in figure 8. The red and green are the Two-phase angle estimation method and Look up Table angle estimation method, the blue is the fusion result of these two by picking the best matching score in their corrected vs corrected comparison. The result shows the score level fusion of the two approaches can improve iris recognition performance.

IV. CONCLUSION AND FUTURE WORK

In this paper, we have presented the gaze angle estimation method working as a pre-processing component to other conventional iris recognition method. We tested the accuracy and performance of our method in the West Virginia University off-angle iris dataset. The same performance evaluation is also conducted in ORNL angle estimation method. The performance comparison shows our method has a slightly better recognition performance than ORNL's method.

The dataset from West Virginia University is still a cooperative dataset, the off angle iris images were taken in a control environment. The maximum angle is set to 30 degrees. In non-cooperative environment the angle is very random and may be larger than 30 degree. Since a part of the iris pattern may be over distorted, the larger off angle iris images may not be simply corrected by projective transformation. Further research could focus on the quality of the over distorted off angle images.

The *virtualeye* simulator source code, iris images, segmented off-angle images and the edge detection segmentation source will be available by request at the CITeR website. <http://clarkson.edu/citer/research/collections/index.html>

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