Some Helper Function:

Softmax Function:

```
import numpy as np

def softmax(z):


    # Your Code Here.
    z_exp = np.exp(z - np.max(z, axis=1, keepdims=True))
    return z_exp / np.sum(z_exp, axis=1, keepdims=True)
```

Softmax Test Case:

This test case checks that each row in the resulting softmax probabilities sums to 1, which is the fundamental property of softmax.

```
# Example test case
z_test = np.array([[2.0, 1.0, 0.1], [1.0, 1.0, 1.0]])
softmax_output = softmax(z_test)

# Verify if the sum of probabilities for each row is 1 using assert
row_sums = np.sum(softmax_output, axis=1)

# Assert that the sum of each row is 1
assert np.allclose(row_sums, 1), f"Test failed: Row sums are {row_sums}"

print("Softmax function passed the test case!")
```

```
Softmax function passed the test case!
```

Prediction Function:

```
def predict_softmax(X, W, b):
    logits = np.dot(X, W) + b
    probabilities = softmax(logits)
    return np.argmax(probabilities, axis=1)
```

Test Function for Prediction Function:

The test function ensures that the predicted class labels have the same number of elements as the input samples, verifying that the model produces a valid output shape.

```
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])
b_test = np.array([0.1, 0.2, 0.3])

y_pred_test = predict_softmax(X_test, W_test, b_test)

assert y_pred_test.shape == (3,), f"Test failed: Expected shape (3,), got {y_pred_test.shape}"
print("Predicted class labels:", y_pred_test)
```

```
Predicted class labels: [1 1 0]
```

Loss Function:

```
def loss_softmax(y_pred, y):

    return -np.sum(y * np.log(y_pred + 1e-9))
```

Test case for Loss Function:

This test case Compares loss for correct vs. incorrect predictions.

- Expects low loss for correct predictions.
- Expects high loss for incorrect predictions.

```
import numpy as np

# Define correct predictions (low loss scenario)
y_true_correct = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])  # True one-hot labels
y_pred_correct = np.array([[0.9, 0.05, 0.05],
                           [0.1, 0.85, 0.05],
                           [0.05, 0.1, 0.85]])  # High confidence in the correct class

# Define incorrect predictions (high loss scenario)
y_pred_incorrect = np.array([[0.05, 0.05, 0.9],  # Highly confident in the wrong class
                             [0.1, 0.05, 0.85],
                             [0.85, 0.1, 0.05]])

# Compute loss for both cases
loss_correct = loss_softmax(y_pred_correct, y_true_correct)
loss_incorrect = loss_softmax(y_pred_incorrect, y_true_correct)

# Validate that incorrect predictions lead to a higher loss
assert loss_correct < loss_incorrect, f"Test failed: Expected loss_correct < loss_incorrect, but got {loss_correct:.4f} >= {loss_incorrect:.4f}"

# Print results
print(f"Cross-Entropy Loss (Correct Predictions): {loss_correct:.4f}")
print(f"Cross-Entropy Loss (Incorrect Predictions): {loss_incorrect:.4f}")
```

```
Cross-Entropy Loss (Correct Predictions): 0.4304
Cross-Entropy Loss (Incorrect Predictions): 8.9872
```

Cost Function:

```
def cost_softmax(X, y, W, b):

    logits = np.dot(X, W) + b
    probabilities = softmax(logits)
    total_loss = np.sum(-y * np.log(probabilities + 1e-9))
    return total_loss / X.shape[0]
```

Test Case for Cost Function:

The test case assures that the cost for the incorrect prediction should be higher than for the correct prediction, confirming that the cost function behaves as expected.

```
import numpy as np

# Example 1: Correct Prediction (Closer predictions)
X_correct = np.array([[1.0, 0.0], [0.0, 1.0]])  # Feature matrix for correct predictions
y_correct = np.array([[1, 0], [0, 1]])  # True labels (one-hot encoded, matching predictions)
W_correct = np.array([[5.0, -2.0], [-3.0, 5.0]])  # Weights for correct prediction
b_correct = np.array([0.1, 0.1])  # Bias for correct prediction

# Example 2: Incorrect Prediction (Far off predictions)
X_incorrect = np.array([[0.1, 0.9], [0.8, 0.2]])  # Feature matrix for incorrect predictions
y_incorrect = np.array([[1, 0], [0, 1]])  # True labels (one-hot encoded, incorrect predictions)
W_incorrect = np.array([[0.1, 2.0], [1.5, 0.3]])  # Weights for incorrect prediction
b_incorrect = np.array([0.5, 0.6])  # Bias for incorrect prediction

# Compute cost for correct predictions
cost_correct = cost_softmax(X_correct, y_correct, W_correct, b_correct)

# Compute cost for incorrect predictions
cost_incorrect = cost_softmax(X_incorrect, y_incorrect, W_incorrect, b_incorrect)

# Check if the cost for incorrect predictions is greater than for correct predictions
assert cost_incorrect > cost_correct, f"Test failed: Incorrect cost {cost_incorrect} is not greater than correct cost {cost_correct}"

# Print the costs for verification
print("Cost for correct prediction:", cost_correct)
print("Cost for incorrect prediction:", cost_incorrect)

print("Test passed!")
```

```
Cost for correct prediction: 0.0006234354127112888
Cost for incorrect prediction: 0.2993086122417495
```

```
Test passed!
```

Computing Gradients:

```
def compute_gradient_softmax(X, y, W, b):

    logits = np.dot(X, W) + b
    probabilities = softmax(logits)
    error = probabilities - y
    grad_W = np.dot(X.T, error) / X.shape[0]
    grad_b = np.sum(error, axis=0) / X.shape[0]

    return grad_W, grad_b
```

Test case for compute_gradient function:

The test checks if the gradients from the function are close enough to the manually computed gradients using np.allclose, which accounts for potential floating-point discrepancies.

```
import numpy as np

# Define a simple feature matrix and true labels
X_test = np.array([[0.2, 0.8], [0.5, 0.5], [0.9, 0.1]])  # Feature matrix (3 samples, 2 features)
y_test = np.array([[1, 0, 0], [0, 1, 0], [0, 0, 1]])  # True labels (one-hot encoded, 3 classes)

# Define weight matrix and bias vector
W_test = np.array([[0.4, 0.2, 0.1], [0.3, 0.7, 0.5]])  # Weights (2 features, 3 classes)
b_test = np.array([0.1, 0.2, 0.3])  # Bias (3 classes)

# Compute the gradients using the function
grad_W, grad_b = compute_gradient_softmax(X_test, y_test, W_test, b_test)

# Manually compute the predicted probabilities (using softmax function)
z_test = np.dot(X_test, W_test) + b_test
y_pred_test = softmax(z_test)

# Compute the manually computed gradients
grad_W_manual = np.dot(X_test.T, (y_pred_test - y_test)) / X_test.shape[0]
grad_b_manual = np.sum(y_pred_test - y_test, axis=0) / X_test.shape[0]

# Assert that the gradients computed by the function match the manually computed gradients
assert np.allclose(grad_W, grad_W_manual), f"Test failed: Gradients w.r.t. W are not equal.\nExpected: {grad_W_manual}\nGot: {grad_W}"
assert np.allclose(grad_b, grad_b_manual), f"Test failed: Gradients w.r.t. b are not equal.\nExpected: {grad_b_manual}\nGot: {grad_b}"

# Print the gradients for verification
print("Gradient w.r.t. W:", grad_W)
print("Gradient w.r.t. b:", grad_b)

print("Test passed!")
```

```
Gradient w.r.t. W: [[ 0.1031051   0.01805685 -0.12116196]
 [-0.13600547  0.00679023  0.12921524]]
Gradient w.r.t. b: [-0.03290036  0.02484708  0.00805328]
Test passed!
```

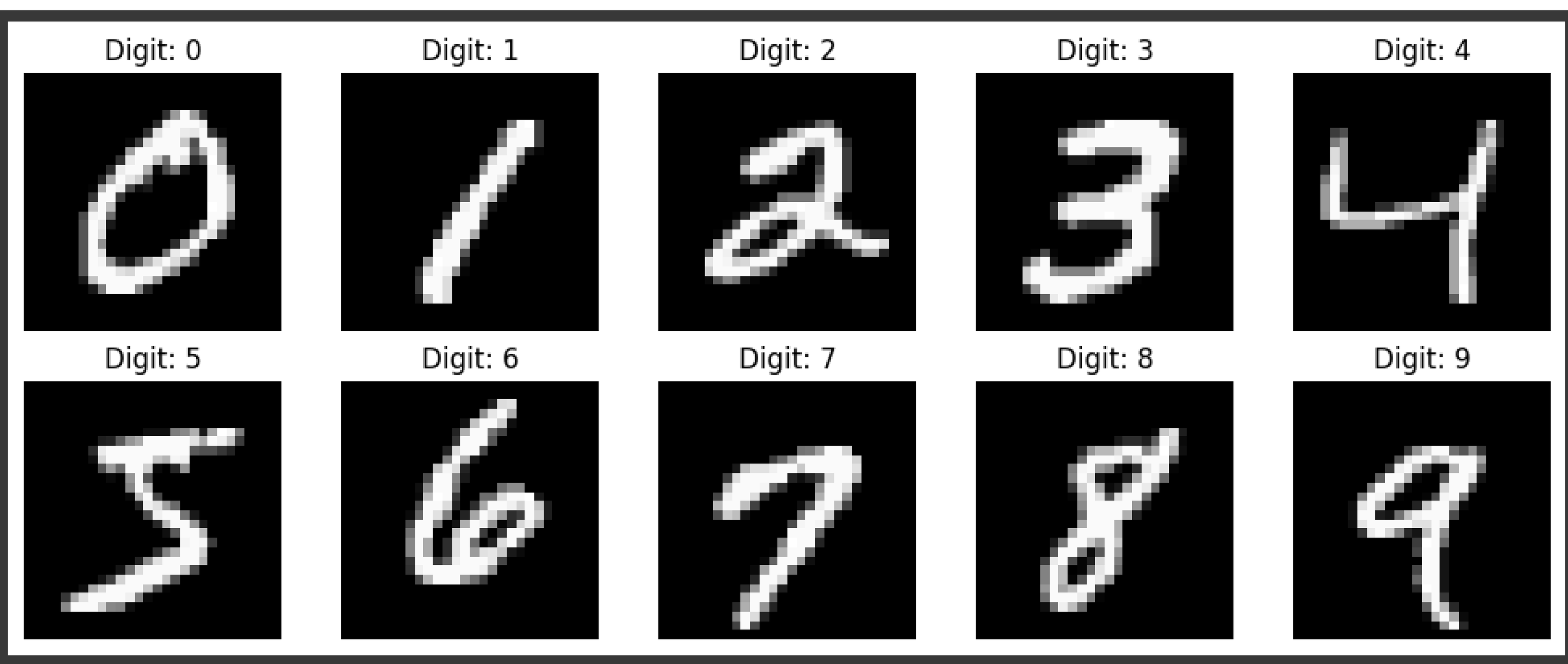
Implementing Gradient Descent:

```
def gradient_descent_softmax(X, y, W, b, alpha, n_iter, show_cost=False):

    cost_history = []

    for i in range(n_iter):
        grad_W, grad_b = compute_gradient_softmax(X, y, W, b)
        W -= alpha * grad_W
        b -= alpha * grad_b
        cost = cost_softmax(X, y, W, b)
        cost_history.append(cost)
        if show_cost and i % 100 == 0:
            print(f"Iteration {i}: Cost = {cost}")

    return W, b, cost_history
```

Preparing Dataset:

```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split

def load_and_prepare_mnist(csv_file, test_size=0.2, random_state=42):
    """
    Reads the MNIST CSV file, splits data into train/test sets, and plots one image per class.

    Arguments:
    csv_file (str)       : Path to the CSV file containing MNIST data.
    test_size (float)    : Proportion of the data to use as the test set (default: 0.2).
    random_state (int)   : Random seed for reproducibility (default: 42).

    Returns:
    X_train, X_test, y_train, y_test : Split dataset.
    """

    # Load dataset
    df = pd.read_csv(csv_file)

    # Separate labels and features
    y = df.iloc[:, 0].values  # First column is the label
    X = df.iloc[:, 1:].values  # Remaining columns are pixel values

    # Normalize pixel values (optional but recommended)
    X = X / 255.0  # Scale values between 0 and 1

    # Split data into train and test sets
    X_train, X_test, y_train, y_test = train_test_split(X, y, test_size=test_size, random_state=random_state)

    # Plot one sample image per class
    plot_sample_images(X, y)

    return X_train, X_test, y_train, y_test

def plot_sample_images(X, y):
    """
    Plots one sample image for each digit class (0-9).

    Arguments:
    X (np.ndarray): Feature matrix containing pixel values.
    y (np.ndarray): Labels corresponding to images.
    """

    plt.figure(figsize=(10, 4))
    unique_classes = np.unique(y)  # Get unique class labels

    for i, digit in enumerate(unique_classes):
        index = np.where(y == digit)[0][0]  # Find first occurrence of the class
        image = X[index].reshape(28, 28)  # Reshape 1D array to 28x28

        plt.subplot(2, 5, i + 1)
        plt.imshow(image, cmap='gray')
        plt.title(f"Digit: {digit}")
        plt.axis('off')

    plt.tight_layout()
    plt.show()
```

```
csv_file_path = "/content/mnist_dataset.csv"  # Path to saved dataset
X_train, X_test, y_train, y_test = load_and_prepare_mnist(csv_file_path)
```



A Quick debugging Step:

```
# Assert that X and y have matching lengths
assert len(X_train) == len(y_train), f"Error: X and y have different lengths! X={len(X_train)}, y={len(y_train)}"
print("Move forward: Dimension of Feture Matrix X and label vector y matched.")
```

```
Move forward: Dimension of Feture Matrix X and label vector y matched.
```

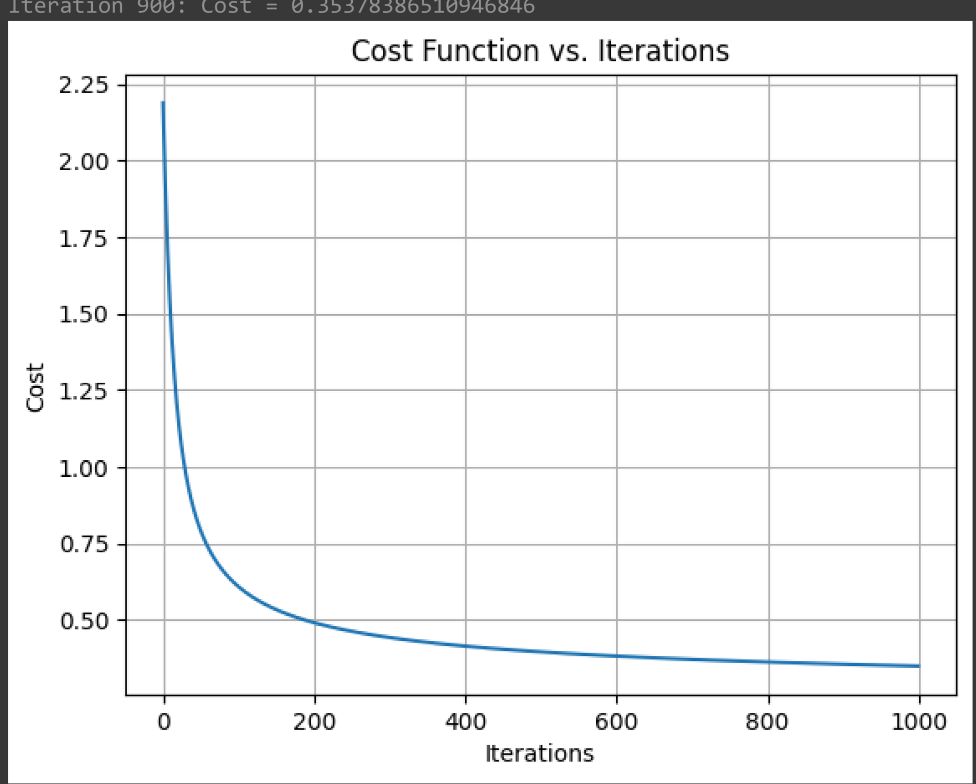
Train the Model:

```
print(f"Training data shape: {X_train.shape}")
print(f"Test data shape: {X_test.shape}")
```

```
Training data shape: (48000, 784)
Test data shape: (12000, 784)
```

```
from sklearn.preprocessing import OneHotEncoder

# Check if y_train is one-hot encoded
if len(y_train.shape) == 1:
    encoder = OneHotEncoder(sparse_output=False)  # Use sparse_output=False for newer versions of sklearn
    y_train = encoder.fit_transform(y_train.reshape(-1, 1))  # One-hot encode labels
    y_test = encoder.transform(y_test.reshape(-1, 1))  # One-hot encode test labels

# Now y_train is one-hot encoded, and we can proceed to use it
d = X_train.shape[1]  # Number of features (columns in X_train)
c = y_train.shape[1]  # Number of classes (columns in y_train after one-hot encoding)

# Initialize weights with small random values and biases with zeros
W = np.random.randn(d, c) * 0.01  # Small random weights initialized
b = np.zeros(c)  # Bias initialized to 0

# Set hyperparameters for gradient descent
alpha = 0.1  # Learning rate
n_iter = 1000  # Number of iterations to run gradient descent

# Train the model using gradient descent
W_opt, b_opt, cost_history = gradient_descent_softmax(X_train, y_train, W, b, alpha, n_iter, show_cost=True)

# Plot the cost history to visualize the convergence
plt.plot(cost_history)
plt.title('Cost Function vs. Iterations')
plt.xlabel('Iterations')
plt.ylabel('Cost')
plt.grid(True)
plt.show()
```

```
Iteration 0: Cost = 2.1886269327907515
Iteration 100: Cost = 0.6074026351640287
Iteration 200: Cost = 0.4896329763589784
Iteration 300: Cost = 0.44102197907282814
Iteration 400: Cost = 0.4129457870050173
Iteration 500: Cost = 0.3940817153178594
Iteration 600: Cost = 0.3802625511987804
Iteration 700: Cost = 0.3695570360281707
Iteration 800: Cost = 0.3609333539726234
Iteration 900: Cost = 0.35378386510946846
```



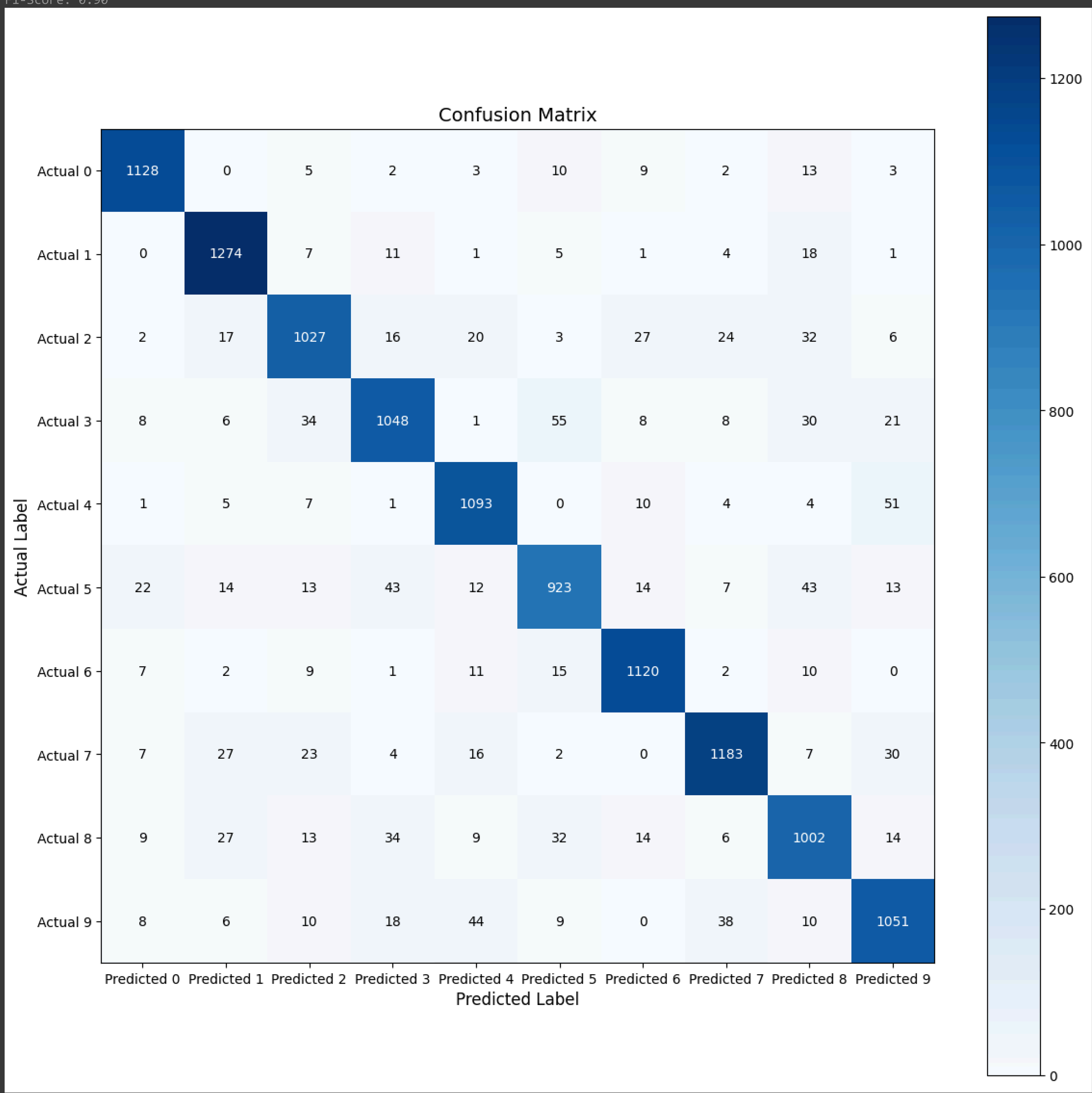
Evaluating the Model:

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.metrics import confusion_matrix, precision_score, recall_score, f1_score

# Evaluation Function
def evaluate_classification(y_true, y_pred):

    # Compute confusion matrix
    cm = confusion_matrix(y_true, y_pred)

    # Compute precision, recall, and F1-score
    precision = precision_score(y_true, y_pred, average='weighted')
    recall = recall_score(y_true, y_pred, average='weighted')
    f1 = f1_score(y_true, y_pred, average='weighted')

    return cm, precision, recall, f1
```

```
# Predict on the test set
y_pred_test = predict_softmax(X_test, W_opt, b_opt)

# Evaluate accuracy
y_test_labels = np.argmax(y_test, axis=1)  # True labels in numeric form

# Evaluate the model
cm, precision, recall, f1 = evaluate_classification(y_test_labels, y_pred_test)

# Print the evaluation metrics
print("\nConfusion Matrix:")
print(cm)
print(f"Precision: {precision:.2f}")
print(f"Recall: {recall:.2f}")
print(f"F1-Score: {f1:.2f}")

# Visualizing the Confusion Matrix
fig, ax = plt.subplots(figsize=(12, 12))
cax = ax.imshow(cm, cmap='Blues')  # Use a color map for better visualization

# Dynamic number of classes
num_classes = cm.shape[0]
ax.set_xticks(range(num_classes))
ax.set_yticks(range(num_classes))
ax.set_xticklabels([f'Predicted {i}' for i in range(num_classes)])
ax.set_yticklabels([f'Actual {i}' for i in range(num_classes)])

# Add labels to each cell in the confusion matrix
for i in range(cm.shape[0]):
    for j in range(cm.shape[1]):
        ax.text(j, i, cm[i, j], ha='center', va='center', color='white' if cm[i, j] > np.max(cm) / 2 else 'black')
```

```
# Add grid lines and axis labels
ax.grid(False)
plt.title('Confusion Matrix', fontsize=14)
plt.xlabel('Predicted Label', fontsize=12)
plt.ylabel('Actual Label', fontsize=12)

# Adjust layout
plt.tight_layout()
plt.colorbar(cax)
plt.show()
```

```
Confusion Matrix:
[[1128    0    5    2    3   10    9    2   13    3]
 [   0 1274    7   11    1    5    1    4   18    1]
 [   2   17 1027   16   20    3   27   24   32    6]
 [   8    6   34 1048    1   55    8    8   30   21]
 [   1    5    7    1 1093    0   10    4    4   51]
 [  22   14   13   43   12  923   14    7   43   13]
 [   7    2    9    1   11   15 1120    2   10    0]
 [   7   27   23    4   16    2    0 1183    7   30]
 [   9   27   13   34    9   32   14    6 1002   14]
 [   8    6   10   18   44    9    0   38   10 1051]]
Precision: 0.90
Recall: 0.90
F1-Score: 0.90
```

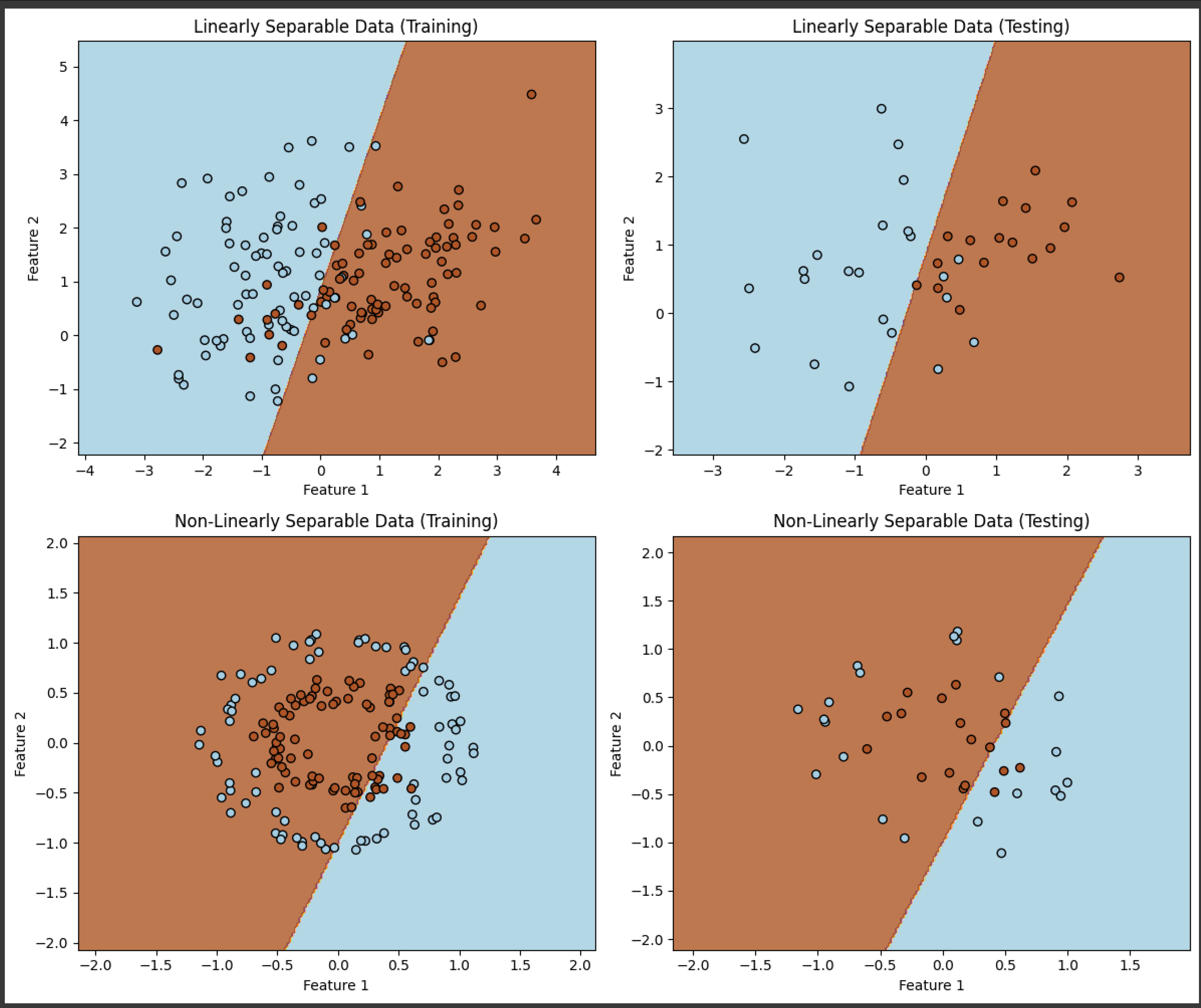


Linear Seperability and Logistic Regression:

Double-click (or enter) to edit

```
import numpy as np
import matplotlib.pyplot as plt
from sklearn.datasets import make_classification, make_circles
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LogisticRegression

# Set random seed for reproducibility
np.random.seed(42)

# Generate linearly separable dataset
X_linear_separable, y_linear_separable = make_classification(n_samples=200, n_features=2,
    n_informative=2, n_redundant=0, n_clusters_per_class=1, random_state=42)

# Split the data into training and testing sets
X_train_linear, X_test_linear, y_train_linear, y_test_linear = train_test_split(
    X_linear_separable, y_linear_separable, test_size=0.2, random_state=42
)

# Train logistic regression model on linearly separable data
logistic_model_linear_separable = LogisticRegression()
logistic_model_linear_separable.fit(X_train_linear, y_train_linear)

# Generate non-linearly separable dataset (circles)
X_non_linear_separable, y_non_linear_separable = make_circles(n_samples=200, noise=0.1, factor=0.5,
    random_state=42)

# Split the data into training and testing sets
X_train_non_linear, X_test_non_linear, y_train_non_linear, y_test_non_linear = train_test_split(
    X_non_linear_separable, y_non_linear_separable, test_size=0.2, random_state=42
)

# Train logistic regression model on non-linearly separable data
logistic_model_non_linear_separable = LogisticRegression()
logistic_model_non_linear_separable.fit(X_train_non_linear, y_train_non_linear)

# Plot decision boundaries for linearly and non-linearly separable data
def plot_decision_boundary(ax, model, X, y, title):
    h = .02  # step size in the mesh
    x_min, x_max = X[:, 0].min() - 1, X[:, 0].max() + 1
    y_min, y_max = X[:, 1].min() - 1, X[:, 1].max() + 1
    xx, yy = np.meshgrid(np.arange(x_min, x_max, h), np.arange(y_min, y_max, h))
    Z = model.predict(np.c_[xx.ravel(), yy.ravel()])
    Z = Z.reshape(xx.shape)
    ax.contourf(xx, yy, Z, alpha=0.8, cmap=plt.cm.Paired)
    ax.scatter(X[:, 0], X[:, 1], c=y, edgecolors='k', cmap=plt.cm.Paired)
    ax.set_title(title)
    ax.set_xlabel('Feature 1')
    ax.set_ylabel('Feature 2')

# Create subplots
fig, axes = plt.subplots(2, 2, figsize=(12, 10))

# Plot decision boundary for linearly separable data (Training)
plot_decision_boundary(axes[0, 0], logistic_model_linear_separable, X_train_linear, y_train_linear,
                       'Linearly Separable Data (Training)')

# Plot decision boundary for linearly separable data (Testing)
plot_decision_boundary(axes[0, 1], logistic_model_linear_separable, X_test_linear, y_test_linear,
                       'Linearly Separable Data (Testing)')

# Plot decision boundary for non-linearly separable data (Training)
plot_decision_boundary(axes[1, 0], logistic_model_non_linear_separable, X_train_non_linear,
                       y_train_non_linear, 'Non-Linearly Separable Data (Training)')

# Plot decision boundary for non-linearly separable data (Testing)
plot_decision_boundary(axes[1, 1], logistic_model_non_linear_separable, X_test_non_linear,
                       y_test_non_linear, 'Non-Linearly Separable Data (Testing)')
```

```
plt.tight_layout()

# Save the plots as PNG files
plt.savefig('decision_boundaries.png')
plt.show()
```



plots, answer the following questions:

• Question - 2: Provide an interpretation of the output based on your understanding.

Answer -

- "Logistic regression excels at classifying linearly separable data but struggles with non-linear data. For complex datasets, more advanced models are required."
- But when the data isn't linearly separable (like the circular data), logistic regression doesn't perform well because it can only draw straight lines. For more complex data, other models are needed.

Question - 3: Describe any challenges you faced while implementing the code above.

Answer -

- The main challenge is the inherent limitation of logistic regression when applied to non-linearly separable data, and recognizing that certain models are better suited for specific types of data.