

non-linear

# Tree

Hierarchy DS

classmate

Data

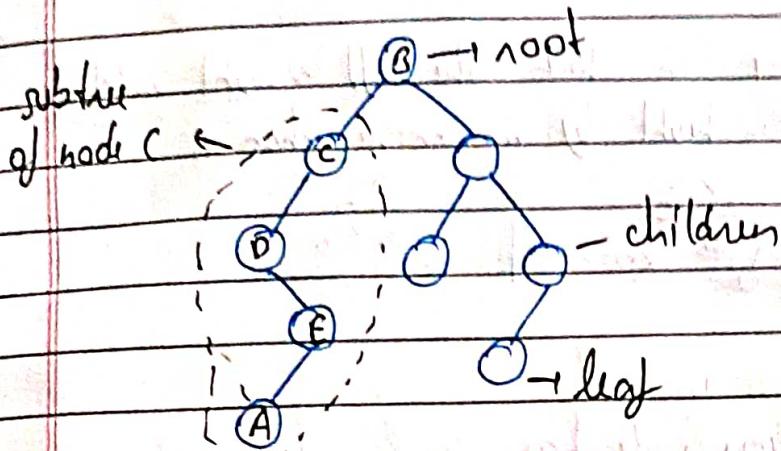
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Binary Tree :

- at most 2 children

descendants

- 3rd node in tree is subtree descendants to

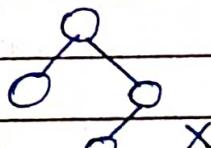
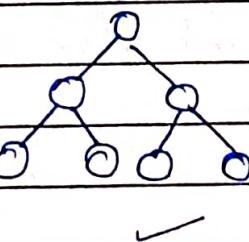
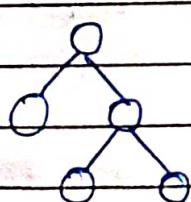


- for node A, (B, C, D, E) are ancestors.

Types of Binary Tree :

① Full Binary Tree :

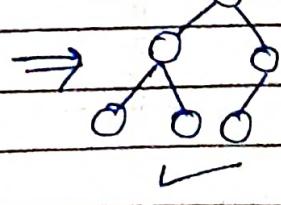
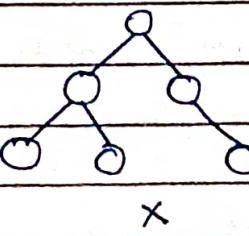
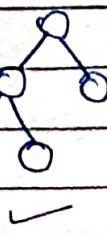
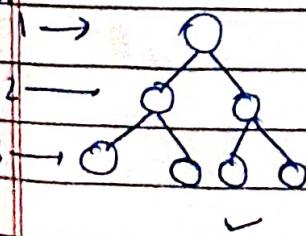
either 0 or 2 children



② Complete BT

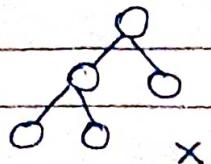
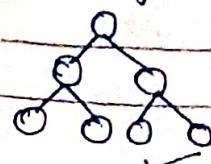
- all levels are completely filled except the last level
- the last level has all nodes on left as possible

level



③ Perfect BT

- all leaf nodes are at same level



## height of tree

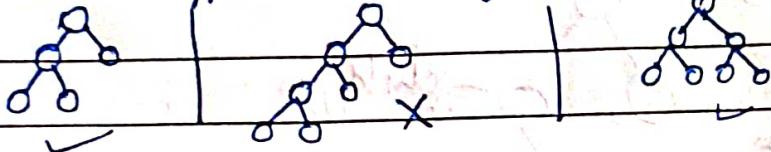
The no. of edges from the leaf node to the particular node of in the longest path.

## d) Balanced BT

height of tree can be maximum  $\log N$

e.g.  $n=8 \quad \log_2 8 = 3$

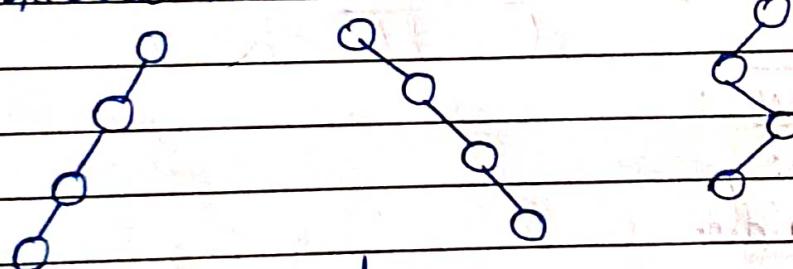
A BT is Balanced BT in which the left & right subtrees of every node differ in height by no more than 1



D

## e) Degenerate Tree (every node has single children)

- skewed tree



- such tree are performance wise same as linked list.

## Application of Tree

- ① Used to represent Hierarchical Data  
 , Organization Structure  
 , Folder " (Folder & subfolders)  
 , XML/HTML content (JSON object) (HTML tag tree & form  
ji hrd & )  
 ↳ In OOP (Inheritance)

## ② Binary Search Tree

- ③ Binary Heap (Used to represent Priority Queue)  
 Used in Dijkstra, Prim, Huffman

## ④ B and B+ Trees in DBMS

(B " " are in DBMS for indexing)

## ⑤ Root is shortest path, so it build sb tree

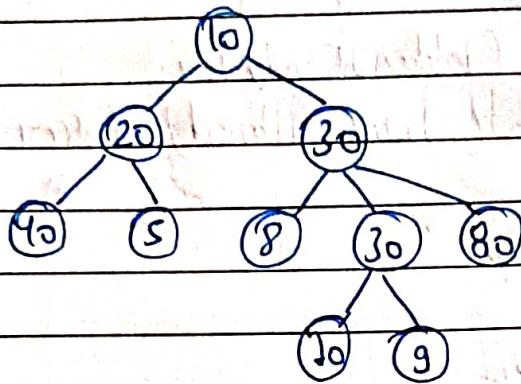
## ⑥ Spanning & Shortest path tree in Computer Network

## ⑦ Parse Tree, Expression Tree in Compiler

### Variation of Trees :

- ① Trie: used to represent dictionary  
 support operation like prefix search
- ② Suffix Tree: used to do fast search in String (Pattern & Text)
- ③ Binary Index Tree: used for range Query Searches with range
- ④ Segment Tree:
  - more beneficial
  - SST is faster for limited set of operations.

- Degree of a node: no. of children it has
- internal node: nodes which are not leaf are called IN



leaf node

LN: 40, 5, 8, 10, 9, 80

IN: 10, 20, 30, 30, 80

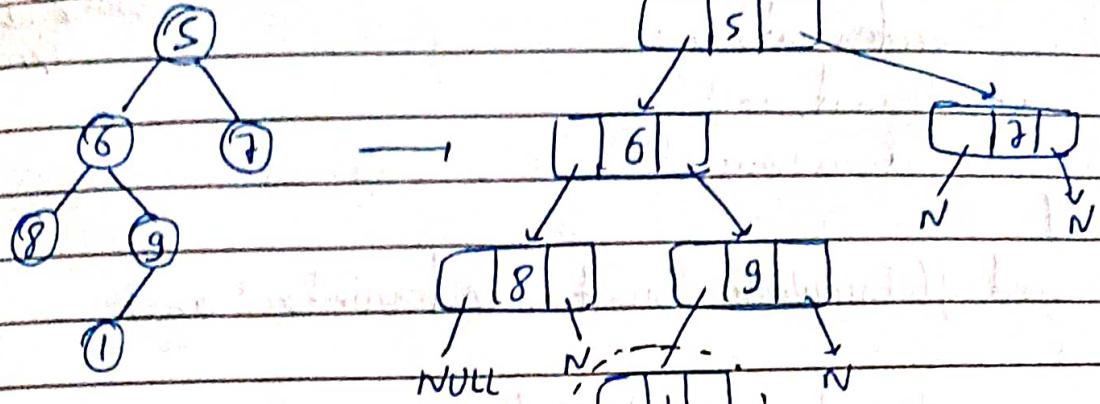
### Important

① Binary Search Tree is the most used Tree DS  
↓  
it is a Binary Tree Variation

② Binary Heap Tree is used to represent Priority Queue  
↓  
it is a Binary Tree Variation

③ Segment Tree DS is also a Binary Tree.

## Binary Tree Representation:



struct Node

```

{ int data;
  struct Node *left;
  struct Node *right;
}
  
```

Node(int val)

```

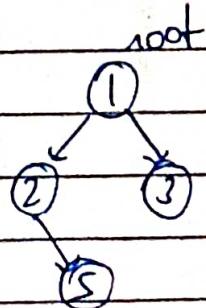
{ data = val;
  left = right = NULL;
}
  
```

};

ej: main()

```

{ struct Node *root = new Node(1);
  root->left = new Node(2);
  root->right = new Node(3);
  root->left->right = new Node(5);
}
  
```



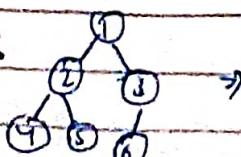
DFS

→ Inorder (left Root Right)

→ Preorder (Root Left Right)

→ Postorder (Left Right Root)

BFS



1 2 3 4 5 6 (level wise traversal)

## (1) Pre-Order Traversal $T \sim O(N)$ $SC \sim O(N)$

Recursive  $\text{vector<int>} \text{PT}(\text{TreeNode}^* \text{root})$

```
{
    vector<int> ans;
    f(root, ans);
    return ans;
}
```

↑  
Worst case when  
skewed tree

void f(TreeNode^\* root, vector<int> &ans)

```
{
    if (root == NULL) return;
    ans.push_back(root->val);
    f(root->left, ans);
    f(root->right, ans);
}
```

## Iterative $\text{vector<int>} \text{PT}(\text{TreeNode}^* \text{root})$

```
{
    vector<int> ans;
    if (root == NULL) return ans;
    stack<TreeNode^*> st;
    st.push(root);
    while (!st.empty())
```

```

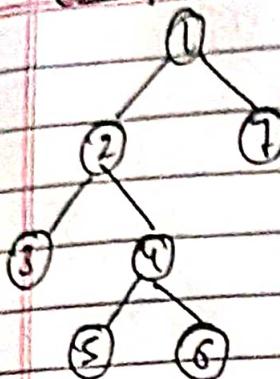
        TreeNode^ root = st.top(); // TreeNode^ node = st.top();
        st.pop();
        ans.push_back(node->val);
```

```

        if (node->right != NULL) st.push(node->right);
        if (node->left != NULL) st.push(node->left);
    }
```

return ans;

example



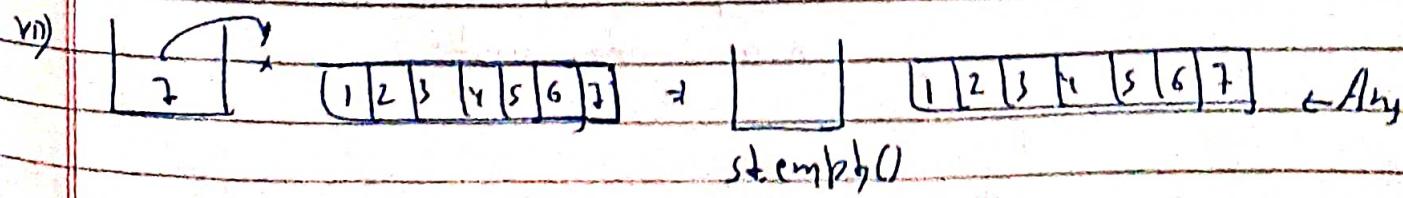
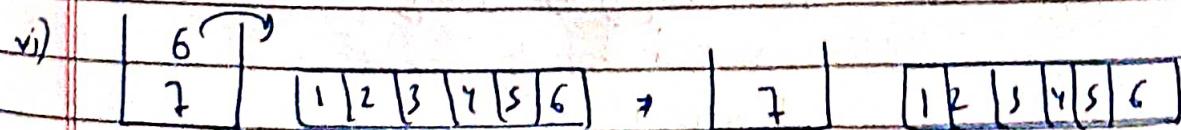
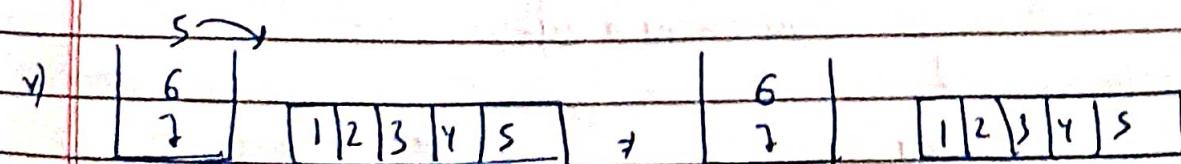
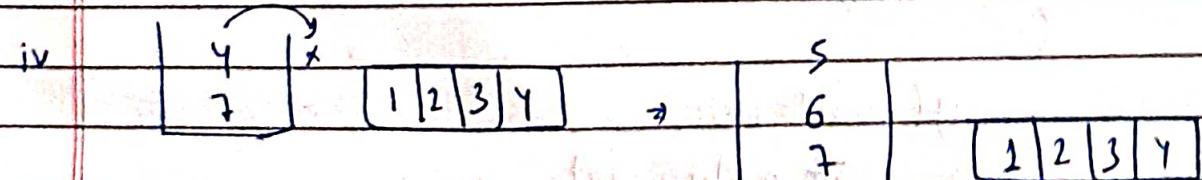
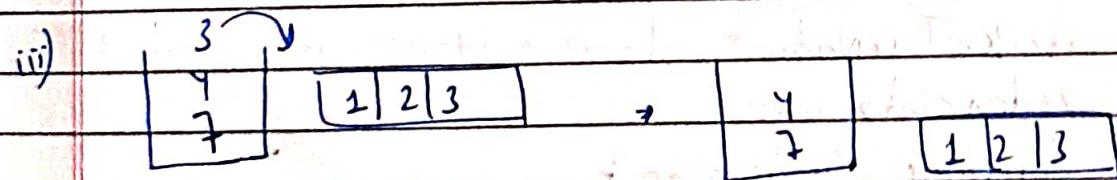
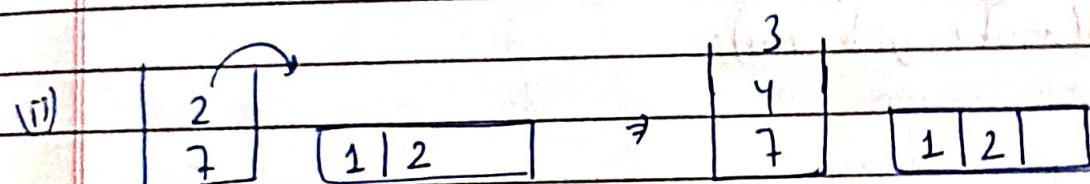
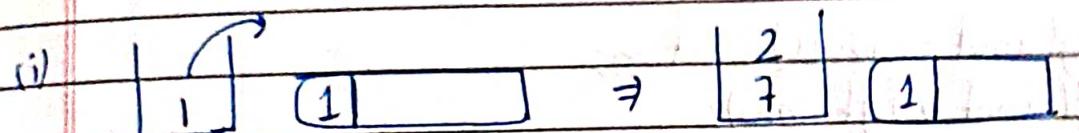
Ans - 1 2 3 4 5 6 7

Initially Stack



, is a node visited

visit right child stack if visit left  
visit parent at last visit (root LR)  
maintain stack



L Root R

## (II) InOrder

```

Recursive vector<int> II (TreeNode *root)
{
    vector<int> ans;
    f(root, ans);
    return ans;
}

void f(TreeNode *root, vector<int> &ans)
{
    if (root == NULL) return;
    f(root->left, ans);
    ans.push_back(root->val);
    f(root->right, ans);
}
  
```

## Iterative vector&lt;int&gt; II (TreeNode \*root)

```

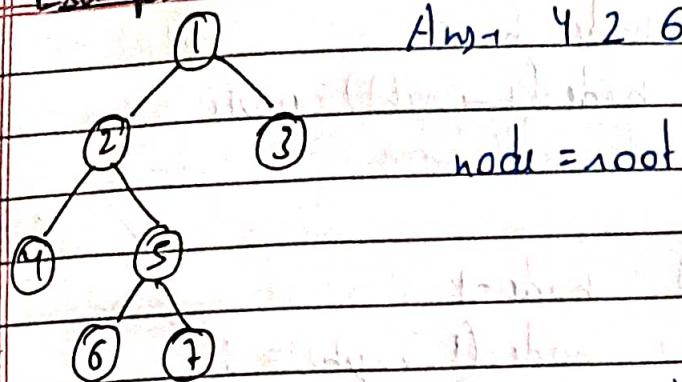
{
    stack<TreeNode *> st;
    vector<int> ans;
    TreeNode *node = root;
    while (true)
    {
        if (node == NULL)
            st.push(node);
        node = node->left;
    }
}
  
```

else

```

{
    if (st.empty()) break;
    node = st.top();
    st.pop();
    ans.push_back(node->val);
    node = node->right;
}
  
```

return ans;

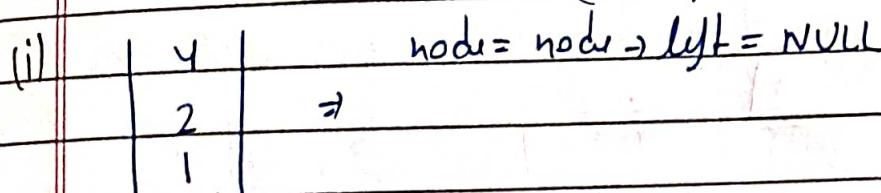
Example

Ans: 4 2 6 5 7 1 3

node = root

stack it wif node  
t |

(4 - lft)



stack it push node on lft (until (node!=NULL))

iii) node == NULL

node = 4

node = NULL

18

4

iii) node == NULL

node = 2

2

1

4

2

iv) node != NULL

6  
5  
1after 6 (6->lft = NULL)  
node == NULL6  
5  
14  
2  
6

node = 6

node = 6-&gt;right = NULL

v)

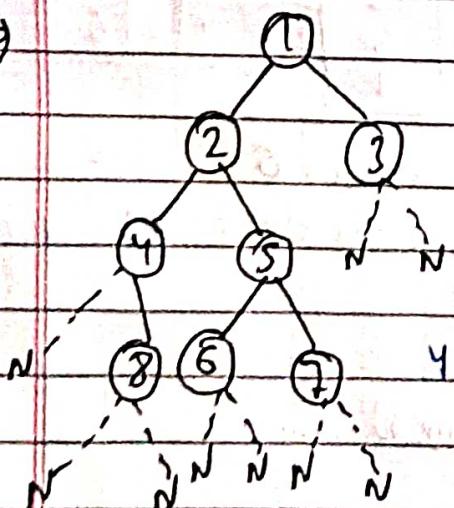
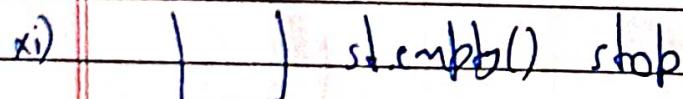
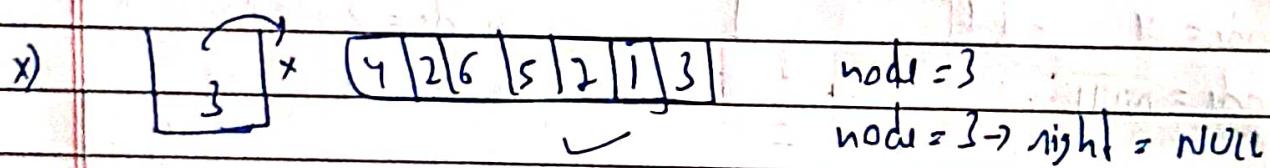
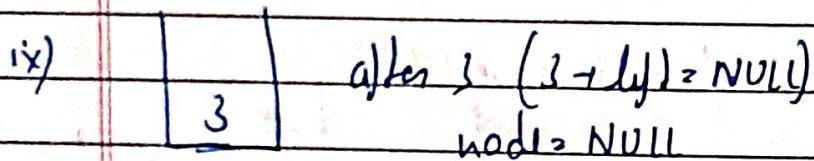
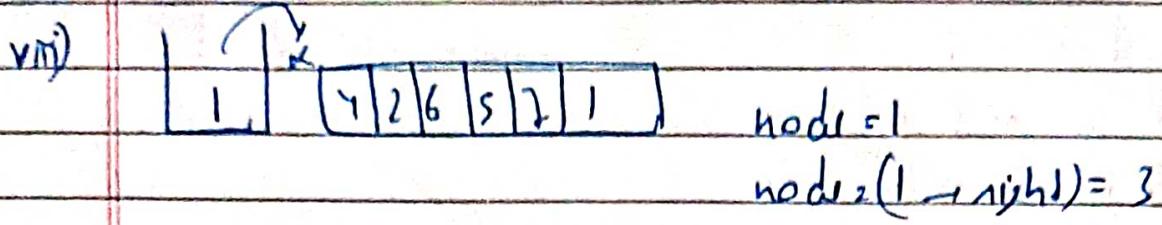
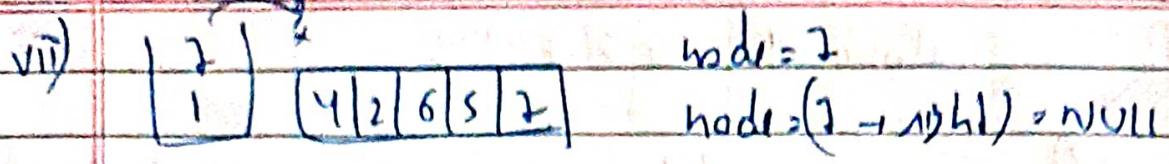
5  
1  
4  
2  
6  
5

node = 5

node = (5-&gt;right) = 7

vi)

7  
1  
after 7 (7->lft == NULL)  
node == NULL



- L Root R

  - यदि left = NULL (िनमें)
  - यह root ही point
  - यदि right नहीं था, तो वह किसी  
node->right को, दायरी, st, top() नहीं

L R Root

Post OrderRecursive

```
vector<int> PO(TreeNode *root)
{
    vector<int> ans;
    f(root, ans);
    return ans;
}
```

```
void f(TreeNode *root, vector<int> &ans)
{
    if (root == NULL) return;
    if (node->left)
        f(node->left);
    if (node->right)
        f(node->right);
    ans.push_back(node->val);
}
```

a) Iterative  $O(N)$  SC  $\rightarrow O(2N)$ 

in this sol<sup>n</sup> you can simply  
use ans vector in place of st2.  
(then reverse vector to get  
ans)

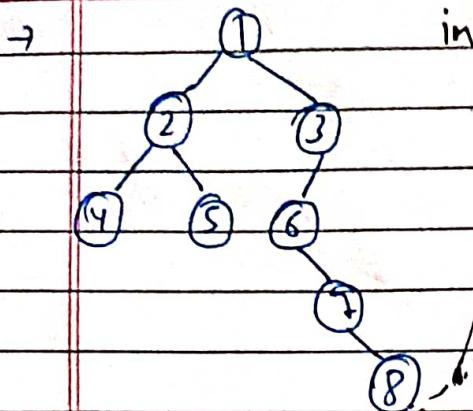
vector&lt;int&gt; PO(TreeNode \*root)

```
{
    vector<int> ans;
    if (root == NULL) return ans;
    stack<TreeNode *> st1, st2;
    st1.push(root);
    while (!st1.empty())
    {
        root = st1.top();
        st2.push(root);
        if (root->left != NULL) st1.push(root->left);
        if (root->right != NULL) st1.push(root->right);
    }
    while (!st2.empty())
    {
        ans.push_back(st2.top()->val);
        st2.pop();
    }
    return ans;
}
```

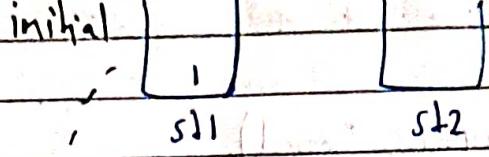
} not required

// reverse(ans);  
returns ans;

L R Root



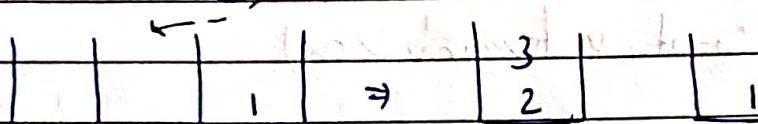
initial



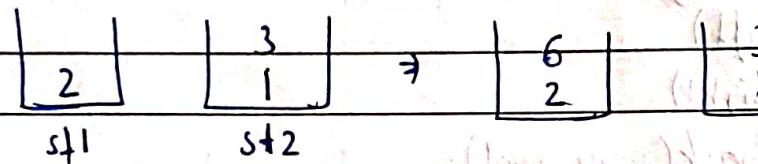
Any 4 5 2 8 7 6 3 |

basically it is to maintain  
in s1 s2 st

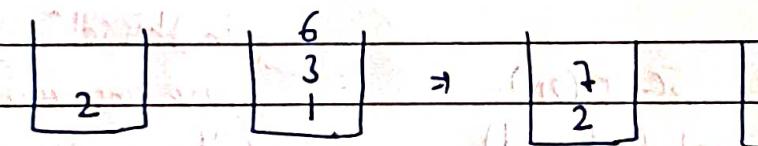
(i)



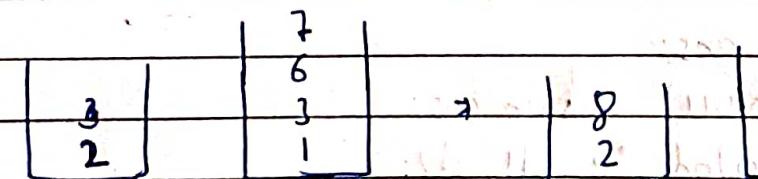
(ii)



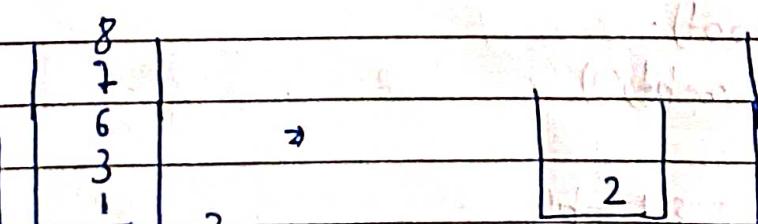
(iii)



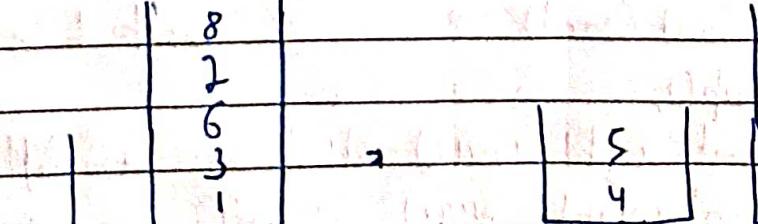
(iv)



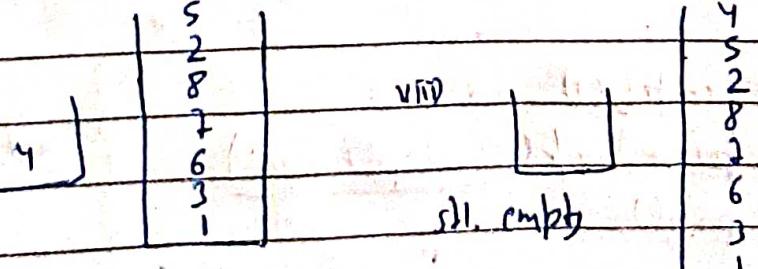
(v)



(vi)



(vii)

pop & push in  
new vertex

st2.

b) Iterative (with one Stack)  $T(-O(N)) \quad SC-1 \quad O(N)$

vector<int> po(TreeNode \*root)

{ TreeNode \*run, \*temp;

stack<TreeNode \*> st;

vector<int> ans;

if(root==NULL) return ans;

run=root;

while(run!=NULL || !st.empty())

{

if(run==NULL)

{ st.push(run);

run=run->left;

}

else

{

temp=st.top()->right;

if(temp==NULL)

{ temp=st.top();

st.pop();

ans.push\_back(temp->val);

while(!st.empty() && temp==st.top()->right)

{

temp=st.top();

st.pop();

ans.push\_back(temp->val);

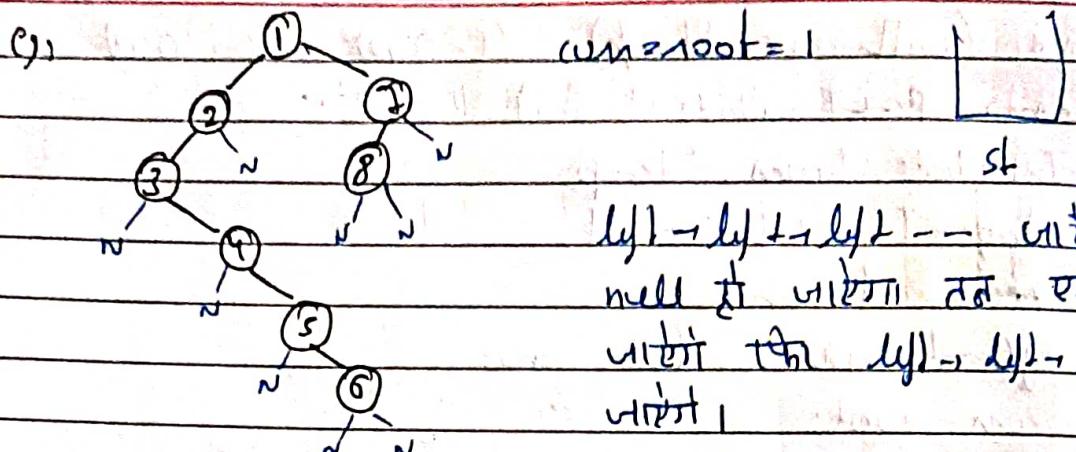
}

else

run=temp;

}

return ans;



ii)  $\text{cum} = x \geq s[N]$

3
2
1

iii)  $\text{cum} = N$

$\text{temp} = 3 \rightarrow R = (y != \text{null})$

$\text{cum} = 4$

iv)  $\text{cum} = N$

v)  $\text{cum} = 4 \leq N$

4
3
2
1

vi)  $\text{cum} = N$

$\text{temp} = 5 \rightarrow R = (6 != \text{null})$

$\text{cum} = 6$

vii)  $\text{cum} = 6 \leq N$

6
5
4
3
2
1

viii)  $\text{cum} = N$

$\text{temp} = 6 \rightarrow R = (\text{null} \geq N \vee II)$

$\text{temp} = 6$

6	X
5	
4	
3	
2	
1	

6

while loop - III (temp  $\geq s[1].top() \rightarrow \text{right}$ )

$$6 \geq (S[1].R) \Rightarrow 6 \geq 6$$

$\rightarrow \text{temp} = 5$

$S = 4 \rightarrow R \Rightarrow (S = S)$

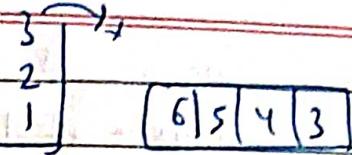
$\text{temp} = 4$

3
2
1
6   5   4

5	X
4	
3	
2	
1	

6	S
---	---

$$3 == 4 \Rightarrow (3 \rightarrow R) \Rightarrow 4 \Rightarrow 4 \\ \text{temp} = 3$$

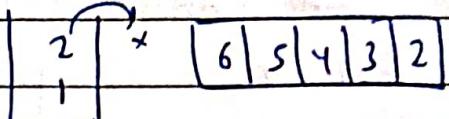


$(3 \neq 2 \rightarrow R)$  False while loop stops

ix)  $\text{num} = N$

$$\text{temp} = 2 \rightarrow R = N$$

$$\text{temp} = 2$$



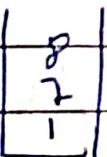
while loop start  $2 \neq (1 \rightarrow R)$  false stop

x)  $\text{num} = N$

$$\text{temp} = 1 \rightarrow R = 7$$

$$\text{temp} (\text{num} = 7)$$

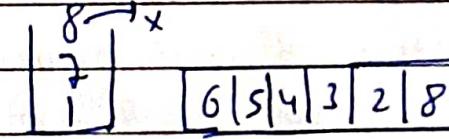
xi)  $\text{num} = 7 \neq N$



xii)  $\text{num} = N$

$$\text{temp} = 8 \rightarrow R = N$$

$$\text{temp} = 8$$

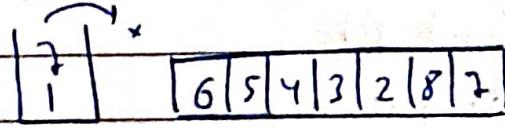


while loop start  $8 \neq 7 \rightarrow R$  false

xiii)  $\text{num} = N$

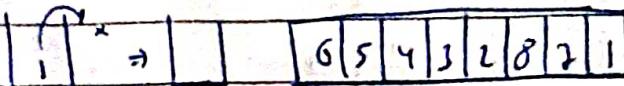
$$\text{temp} = 7 \rightarrow R = N$$

$$\text{num temp} = 7$$



while loop start  $7 \neq (7 \rightarrow R) \Rightarrow (7 == 2)$

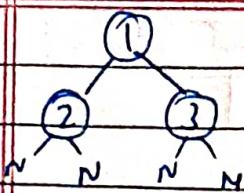
$$\text{temp} = 1$$



st.comph()

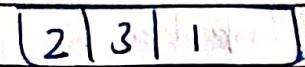
xiv) first while loop stops

eg)



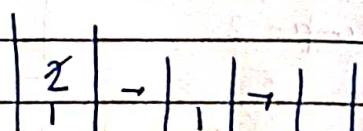
$$\text{sum} = x \times N \times N$$

$$\text{temp} = \cancel{x} + \cancel{x} \times \cancel{x} \times \cancel{x}$$



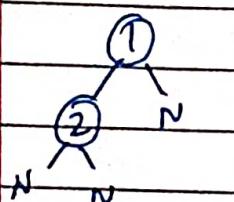
$$\text{sum} = x \times N \times N$$

$$\text{temp} = x \times N \times 1$$



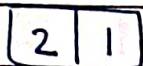
stop

eg)

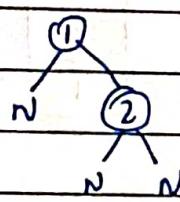


$$\text{sum} = x \times N \times N$$

$$\text{temp} = x \times N \times 1$$

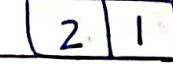


eg)



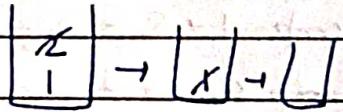
$$\text{sum} = x \times N \times N$$

$$\text{temp} = \cancel{x} \times \cancel{x} + 2$$



L-R

while loop



2 = L-R

→ Preorder, Inorder, Postorder in One Traversal

vector<int> PIP(TreeNode \*root)

{

TC → O(3N)

O(4N)

stack<pair<TreeNode \*, int>> st;

: st.push({root, 1});

vector<int> pre, in, post;

if(root == NULL) return;

while(!st.empty())

{

auto it = st.top();

st.pop();

$\text{if } (\text{it}.second == 1)$

{      $\text{bsr.push\_back(it.first \rightarrow \text{val});}$

$\text{it.second}++;$

$\text{st.push(it);}$

$\text{if } (\text{it}.first \rightarrow \text{left} == \text{NULL})$

$\text{st.push(it.first \rightarrow \text{left}, 1));}$

}

else if ( $\text{it}.second == 2$ )

{      $\text{in.push\_back(it.first \rightarrow \text{val});}$

$\text{it.second}++;$

$\text{st.push(it);}$

$\text{if } (\text{it}.first \rightarrow \text{right} == \text{NULL})$

$\text{st.push(it.first \rightarrow \text{right}, 1));}$

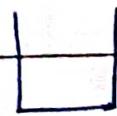
}

else

$\text{bst.push\_back(it.first \rightarrow \text{val});}$

}

Algo:



(node, num)

if num = 1

push in bsr

num++

left push in stack

if num = 2

push in in

num++

right

if num = 3

push in bst

Root push

Root L R

Root to arr left  
arr to left push

L Root R

L R Root

\* push bst root first

$\rightarrow$  out as root (leaf first root last)  
null at )

## → Level Order Traversal:

a) `vector<vector<int>> levelorder(TreeNode *root)`

```
queue<TreeNode*> q;
O(N)
```

```
vector<vector<int>> ans;
O(N)
```

```
if (root == NULL) return ans;
```

```
q.push(root);
```

```
while (!q.empty())
```

```
vector<int> v;
```

```
int n = q.size();
```

```
for (int i = 0; i < n; i++)
```

```
TreeNode *node = q.front();
```

```
q.pop();
```

```
if (node->left != NULL) q.push(node->left);
```

```
if (node->right != NULL) q.push(node->right);
```

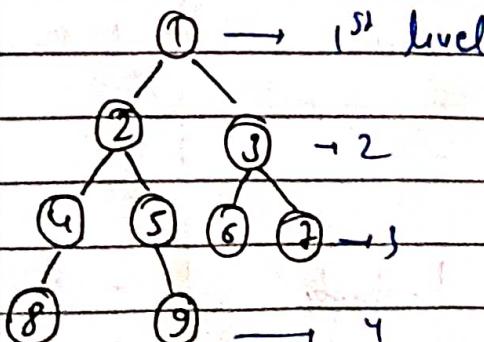
```
v.push_back(node->val);
```

```
ans.push_back(v);
```

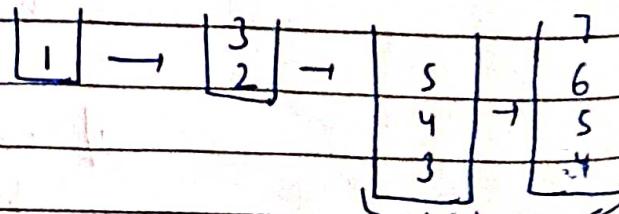
```
return ans;
```

```
}
```

Logic



level wise queue at pop time



9	8
7	2
6	5

b) void f(Node \*root)

{

    if (root == NULL) return NULL;

    queue<Node \*> q;

    q.push(root);

    while (!q.empty())

{

        Node \*cur = q.front();

        q.pop();

        cout << (cur->val) << " ";

        if (cur->left != NULL) q.push(cur->left);

        if (cur->right != NULL) q.push(cur->right);

    }

}

## ② Height of BT:

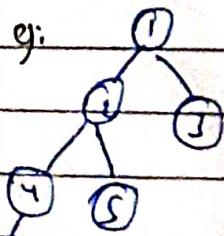
int height( TreeNode \*root)

{

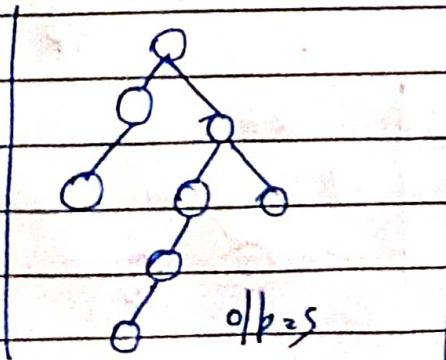
    if (root == NULL) return 0;

    return max( height(root->left), height(root->right)) + 1;

}



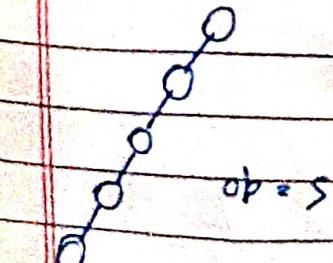
ob = 4



ob = 5

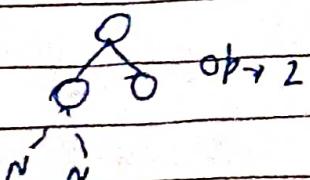
ob = 1

NULL ob = 0



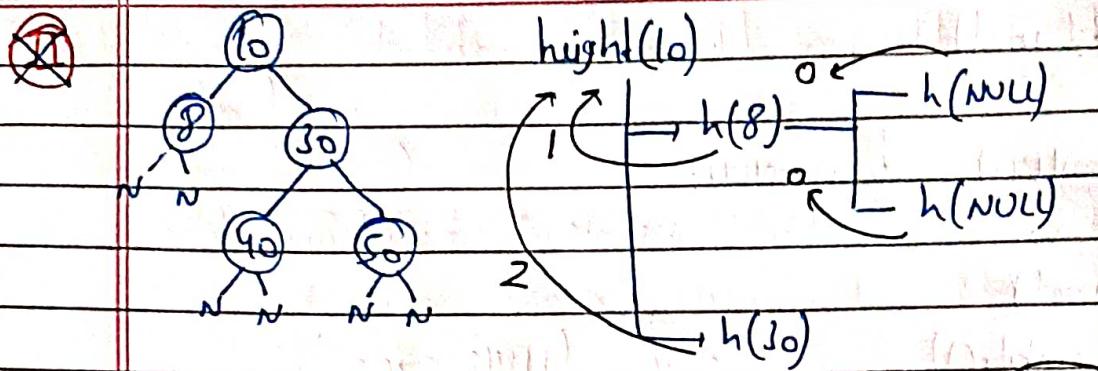
ob = 5

ob = 2



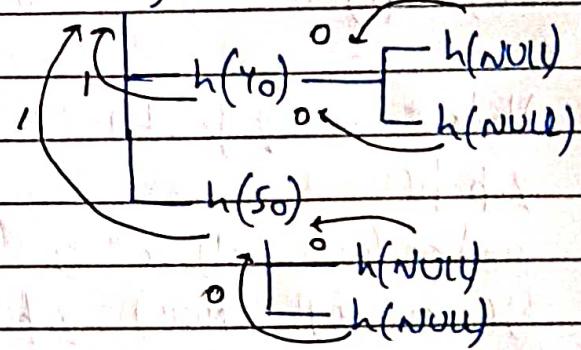
ob = 2

n n



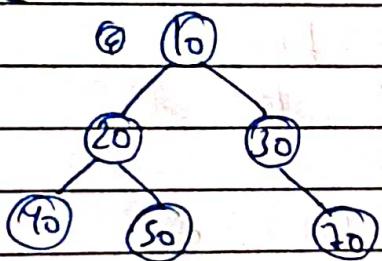
$$\max(1, 2) + 1$$

$$2 + 1 = 3$$



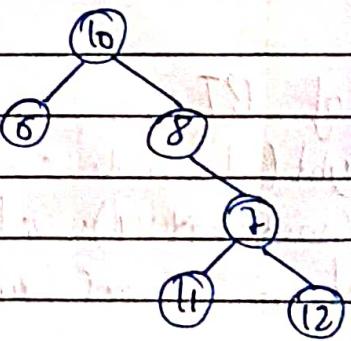
### Q1 Print Nodes at distance k :

K=2



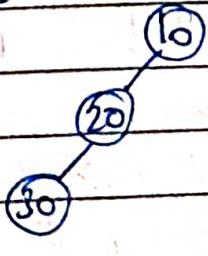
O/p - 80 50 70

K=3



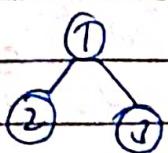
O/p 11 12

K=1



O/p - 20

K=0



O/p = 1

```
vector<int> Kdistance(Node *root, int k)
```

{

```
vector<int> ans;
```

```
f(root, ans, k, 0);
```

```
return ans;
```

}

```
void f(Node *root, vector<int> &ans, int k, int i)
```

{

```
if (root == NULL) return;
```

```
if (i == k)
```

```
{ ans.push_back(root->data);
```

```
return;
```

}

```
f(root->left, ans, k, i+1);
```

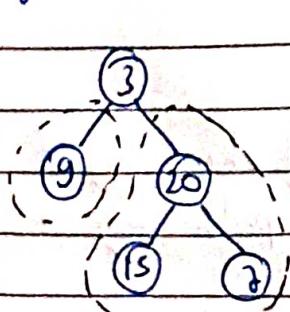
```
f(root->right, ans, k, i+1);
```

}

## II

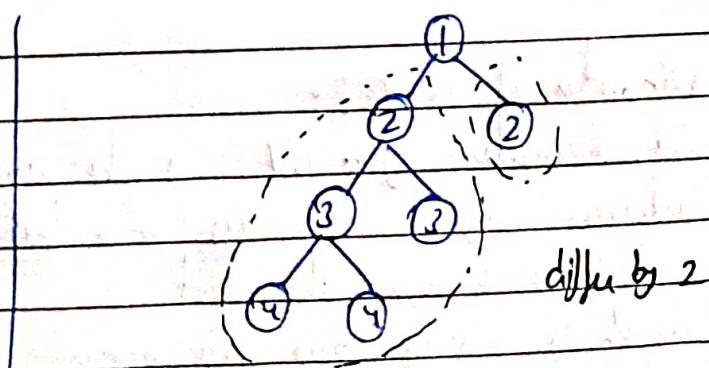
### Balanced Binary Tree:

A BT in which the left and right subtrees of every node differ in height by no more than 1.



differ by 1

$0 \leq 1$  (True)



differ by 2

Logic for Balanced BT for every node;

$$\text{height}(\text{left}) - \text{height}(\text{right}) \leq 1$$

a) Naive (calculate left height & right height of every node)  
 $O(n^2)$

b) efficient  $O(N)$   
 bool isBalanced (TruNode \*root)

```

    {
        if (root == NULL) return 1;
        int ans = f(root);
        return ans == -1 ? 0 : 1;
    }

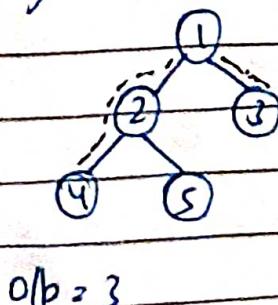
    int f(TruNode *root)
    {
        if (root == NULL) return 0;
        int h_left = f(root->left);
        if (h_left == -1) return -1;
        int h_right = f(root->right);
        if (h_right == -1) return -1;

        if (abs(h_left - h_right) > 1) return -1;
        return max(h_left, h_right) + 1;
    }
  
```

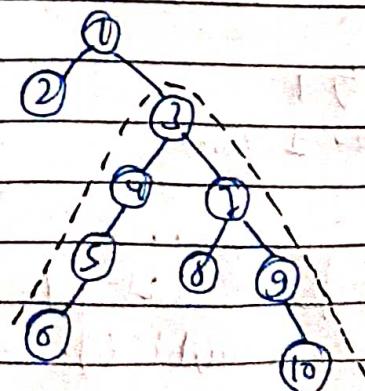
height of a tree function code

#### IV. Diameter of a BT:

- The diameter of a BT is the length of the longest path between any two nodes in a tree. This path may or may not pass through the root.
- length is no. of edges b/w nodes



$$O/p = 3$$



```
int diameter(TruNode *root)
```

```
{ int ans;
```

```
f(root, ans);
```

```
return ans;
```

```
}
```

```
int f(TruNode *root, int &ans)
```

```
{
```

```
if (root == NULL) return 0;
```

```
int h-left = f(root->left, ans);
```

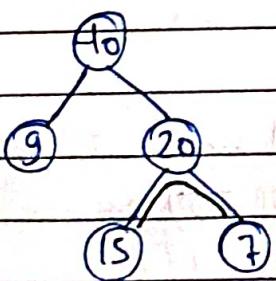
```
int h-right = f(root->right, ans);
```

```
ans = max(ans, h-left + h-right);
```

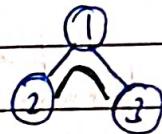
```
return max(h-left, h-right) + 1;
```

```
}
```

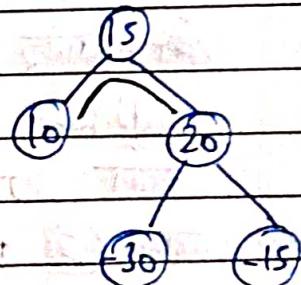
## ② BT Maximum Path Sum:



$$\text{Q/b } 15 + 20 + 7 = 42$$



$$\text{Q/b } 6$$



$$\text{Q/b } 45$$

Note: Never include a -ve path in ans. (here -10 & -15 path is not included b/z they don't part in answers)

```
int maxPathSum(TruNode *root)
```

```
{
```

```
int ans = INT_MIN;
```

```
f(root, ans);
```

```
return ans;
```

```
}
```

```
int J(TreeNode *root, int &ans)
```

{

~~If (root == NULL) return 0;~~

o b/c ve.

```
int left_sum = max(0, J(root->left, ans));
```

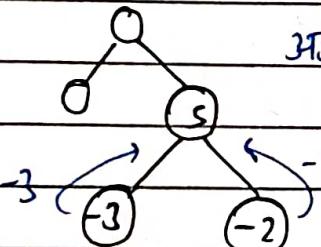
```
int right_sum = max(0, J(root->right, ans));
```

```
ans = max(ans, root->val + left_sum + right_sum);
```

```
return root->val + max(left_sum, right_sum);
```

}

★



प्रति left-sum = ~~max(0, J(root->left, ans))~~ दोनों

मात्र नहीं होता तो -3 नहीं

है जहां left-side पर 1 करे -2 नहीं

है — right — तो s के सभी

ve की देता।

- सभी रखिए

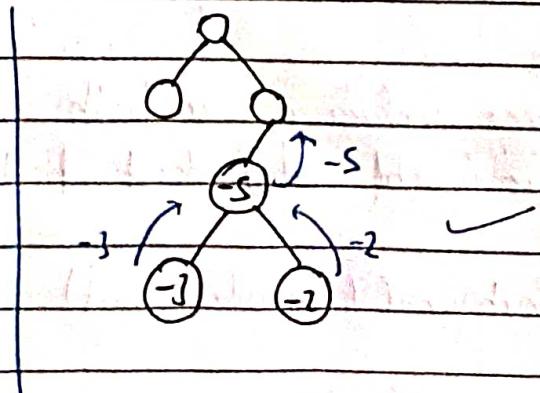
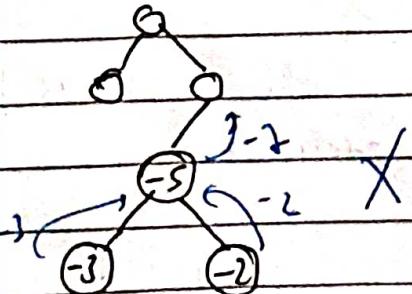
left-sum = max(0, J(root->left, ans));

तभी अपने ve के left-sum तक right-sum वे जान दे।

- मात्र (s) ने (-5) को अपने ve के कर्ता।

min कर

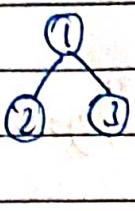
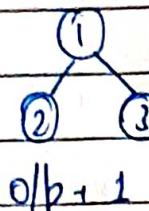
Q.



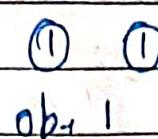
(VI)

Same Tree:

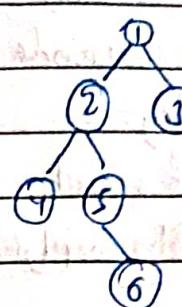
2 BT are identical if they are structurally identical and the nodes have the same value.



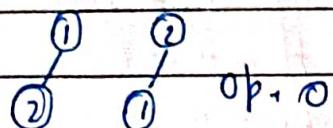
o/p - 1



o/p - 1



o/p - 1



o/p - 0

`bool isSameTree(TreeNode *p, TreeNode *q)`

{

\*  $\text{if } (p == \text{NULL} \text{ || } q == \text{NULL}) \text{ return } p == q;$

if ( $p->\text{val} == q->\text{val}$ )

return isSameTree(p->left, q->left) && isSameTree(p->right, q->right);

return 0;

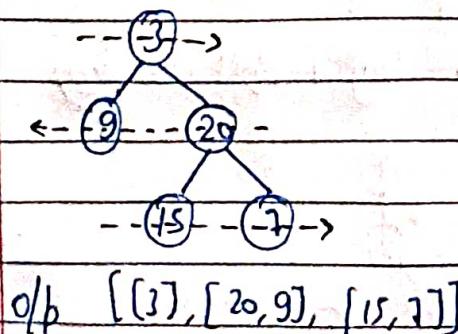
}

' if ( $p == \text{NULL} \text{ && } q == \text{NULL}$ ) return 1; }

if ( $p == \text{NULL} \text{ || } q == \text{NULL}$ ) return 0; }

} combining above line

(VII)

BT Zigzag Level Order Traversal:o/p  $[(3), [20, 9], [15, 7]]$ 

[1]

o/p  $[(1)]$ 

NULL

`vector<vector<int>> zigzag (Trinode *root)`

`<`  
`vector<vector<int>> ans;`

`if (root == NULL) return ans;`

`queue<Trinode *> q;`

`int flag = 0; // bool flag = 0;`

`q.push(root);`

`while (!q.empty())`

`{`

`int n = q.size();`

`vector<int> v(n);`

`for (int i = 0; i < n; i++)`

`{`

`Trinode *temp = q.front();`

`q.pop();`

`int ind = (flag == 0) ? i : n - i - 1;`

`v[ind] = temp->val;`

`if (temp->left) q.push(temp->left);`

`if (temp->right) q.push(temp->right);`

`}`

`ans.push_back(v);`

`flag = (flag == 0) ? 1 : 0; // flag = !flag;`

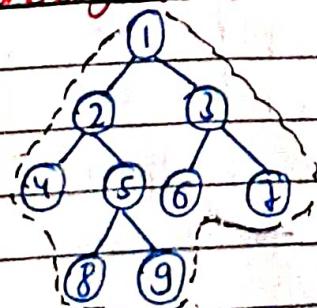
`}`

`} when ans;`

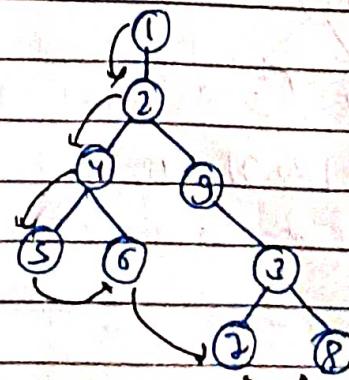
`left boundary + leaf + right boundary`

**VII**

**Boundary Traversal :**



`0/b 1 2 4 8 9 6 7 3`



$$T \rightarrow O(N) + O(N) + O(N) \rightarrow O(N)$$

$$S \rightarrow O(N)$$

classmate

Date \_\_\_\_\_

Page \_\_\_\_\_

vector<int> boundary(Node \*root)

{ vector<int> v;

if (root == NULL) return v;

if (root->left == NULL && root->right == NULL)

{ v.push\_back(root->data);

return v;

}

v.push\_back(root->data); // put the root node already in v

stack<int> st;

Node \*temp = root->left;

while (temp)

{

if (temp->left == NULL && temp->right == NULL) break;

v.push\_back(temp->data);

if (temp->left != NULL) temp = temp->left;

else temp = temp->right;

}

temp = root;

inorder(root, v);

temp = root->right;

while (temp)

{

if (temp->left == NULL && temp->right == NULL) break;

st.push(temp->data);

if (temp->right == NULL) temp = temp->left;

else temp = temp->right;

}

while (!st.empty())

{

v.push\_back(st.top());

st.pop();

}

return v;

} edge case

} left boundary

} boundary

} inorder traversal for leaf nodes occurs

} right boundary

void inorder(Nodi \*root, vct<int> &v)

{

    if (root->left == NULL && root->right == NULL)

        v.push\_back(root->data);  
        return;

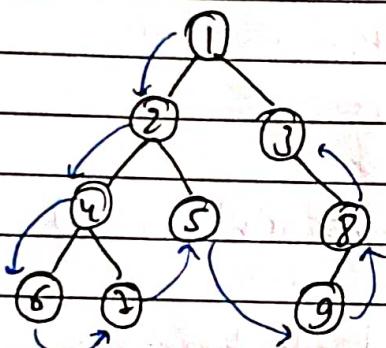
}

    if (root->left != NULL) inorder(root->left, v);

    if (root->right != NULL) inorder(root->right, v);

}

example

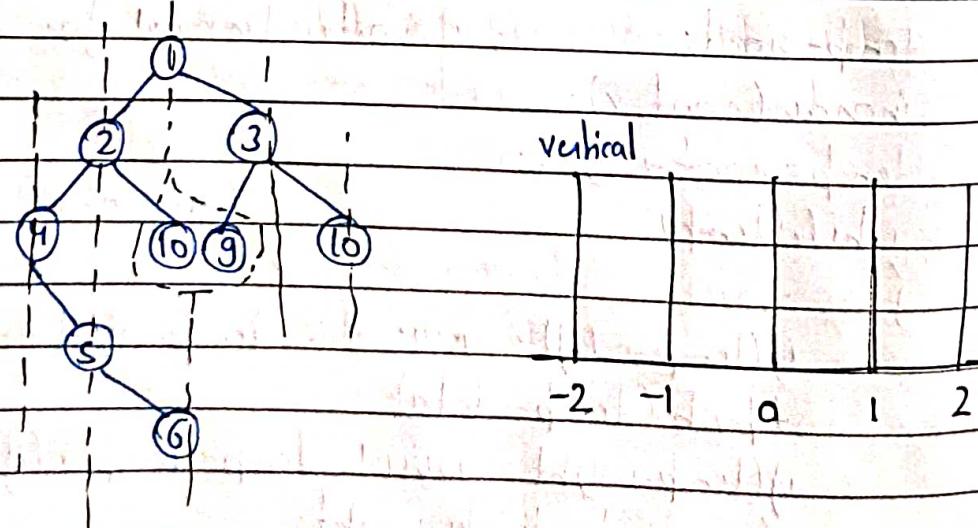


left boundary bush (2, 4) in vertical  
inorder bush (1, 2, 3, 8, 9)  
right boundary bush (8, 3)

\*\*\*

(ix)

Vertical Order Traversal:



O/p  $[(1), [2, 3], [4, 5, 6, 7], [8, 9, 10]]$

every vertical nodes in itself should be sorted

v.imp

\* एक level के नोड्स आपस में सारे same vertical लीफ़ (like in above (10, 9))

multiset uses .insert() function to insert

classmate

Date \_\_\_\_\_

Page \_\_\_\_\_

vector<vector<int>> vertical(TreeNode \*root)

{

queue<pair<TreeNode \*, pair<int, int>> q;

map<int, map<int, multiset<int>> m;

q.push({root, {0, 0}});

while(!q.empty())

{

auto it = q.front(); *beautiful line*

q.pop();

TreeNode \*node = it.first();

int x = it.second.first, y = it.second.second;

m[x][y].insert(node->val);

y(node->left) q.push({node->left, {x-1, y+1}});

y(node->right) q.push({node->right, {x+1, y+1}});

}

vector<vector<int>> ans;

for(auto x: m)

{

vector<int> v;

for(auto p: x.second)

v.insert(v.end(), p.second.begin(), p.second.end());

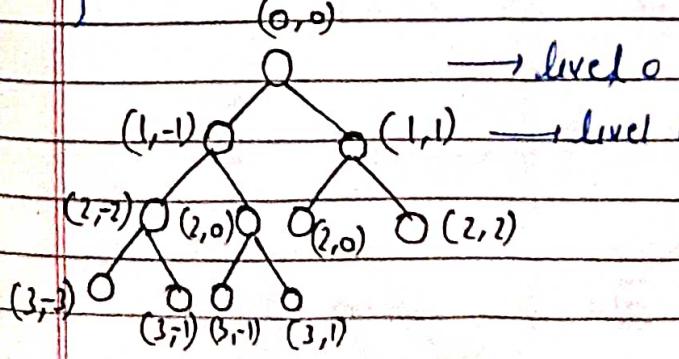
ans.push\_back(v);

}

return ans;

}

(level, vertical)



map<int, map<int, multiset<int>>

vertical

level

multiset b/z

left  
right  
(-1,+1)      (+1,+1)

we want sorted vertical

## → GFG Ques (Vertical Traversal)

- If there are multiple nodes passing through a vertical line, then they should be printed as they appear in level order traversal of the tree.

`vector<int> VerticalOrder(Node *root)`

```
{ queue<pair<Node *, int>> q;
```

```
map<int, vector<int>> m;
```

```
q.push({root, 0});
```

```
while(!q.empty())
```

```
{
```

```
auto it = q.front();
```

```
q.pop();
```

```
Node *node = it.first;
```

```
int x = it.second;
```

```
m[x].push_back(node->data);
```

```
if(node->left) q.push({node->left, x-1});
```

```
if(node->right) q.push({node->right, x+1});
```

```
}
```

```
vector<int> ans;
```

```
for(auto x : m)
```

```
{
```

```
for(auto b : x.second)
```

```
ans.push_back(b);
```

```
}
```

```
return ans;
```

```
}
```

```
1
```

```
2
```

```
3
```

```
4
```

```
5
```

```
6
```

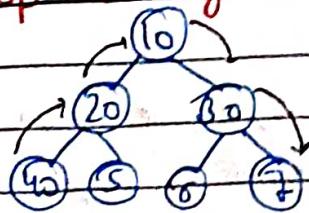
```
7
```

```
8
```

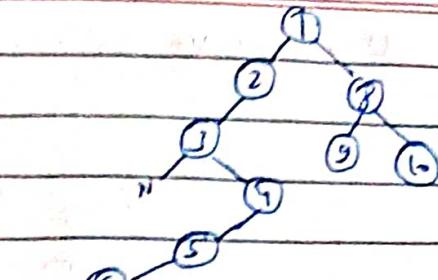
```
9
```

Output: 4 2 1 5 6 3 8 7 9.

## Q) Top View of BT



O/P [40, 20, 10, 30, 70]



O/P [2, 6, 3, 7, 8, 10]

vector<int> f(Node \*root)

{ vector<int> ans;

if(root == NULL) return ans;

queue<pair<Node \*, int>> q;

map<int, int> m;

q.push({root, 0});

while(!q.empty())

{

auto it = q.front();

q.pop();

Node \*node = it.first;

int x = it.second;

if(m.find(x) == m.end()) m[x] = node->data;

if(node->left) q.push({node->left, x-1});

if(node->right) q.push({node->right, x+1});

}

for(auto x : m)

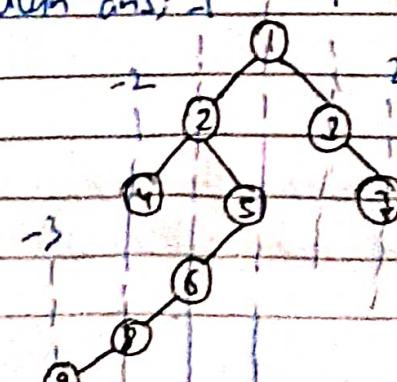
{

ans.push\_back(x.second);

}

return ans;

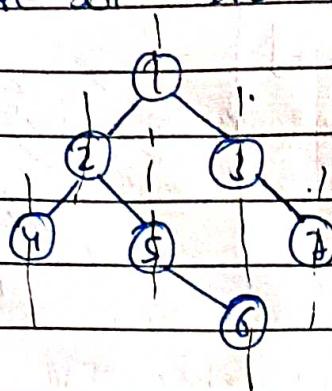
}



Queue  
→ (node, vertical)

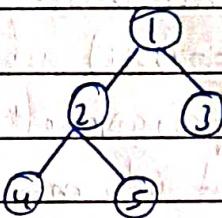
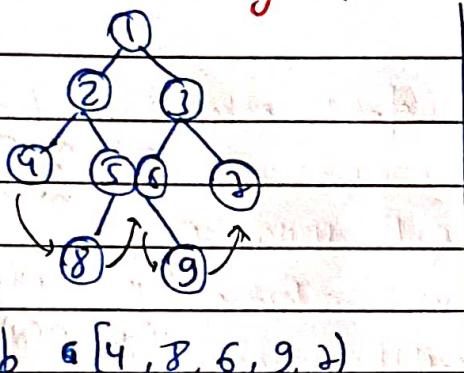
logic → simply vertical at top Node visited first

# Recursive soln DFS can't work b/c



when iteratively left we would have  
got 6 to inserted in map causing  
3 to not be inserted  
to

### (X) Bottom View of RT



O/p [4, 2, 5, 3]

O/p [4, 8, 6, 9, 2]

\* when nodes got overlapped o/p the right node (here in b/w 5, 6 we o/p 5)

```

vector<int> l(Node *root)
{
    vector<int> ans;
    if(root == NULL) return ans;
    queue<pair<Node *, int>> q;
    map<int, int> m;
    q.push({root, 0});
    while(!q.empty())
    {
        auto it = q.front();
        q.pop();
        Node *node = it.first;
        int x = it.second;
        m[x] = node->data;
        if(node->left) q.push({node->left, x-1});
        if(node->right) q.push({node->right, x+1});
    }
}
  
```

auto it = q.front();

q.pop();

Node \*node = it.first;

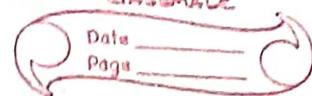
int x = it.second;

m[x] = node->data;

if(node->left) q.push({node->left, x-1});

if(node->right) q.push({node->right, x+1});

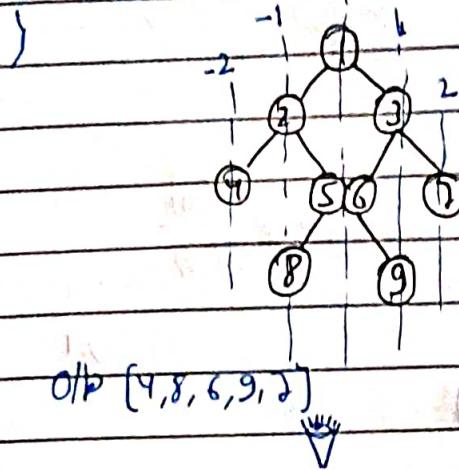
Recursive soln not possible b/c map updates values.  
(see by c):



for(auto x:m)

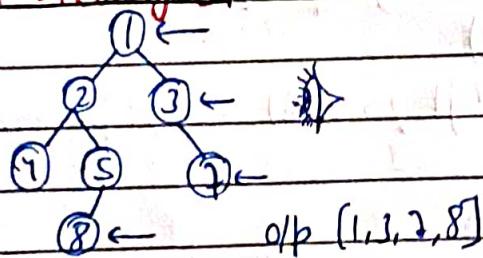
ans.push\_back(x.second);

return ans; }



logic simply bottom node 2nd vertical  
in ans #1

## XII Right Side View of BT



vector<int> l(TreeNode \*root)

{ vector<int> ans;

f(root, ans, 0);

return ans;

}

void f(TreeNode \*root, vector<int>& v, int l)

{ if(root == NULL) return;

if(root

if(v.size() == l)

v.push\_back(root->val);

f(root->right, v, l+1);

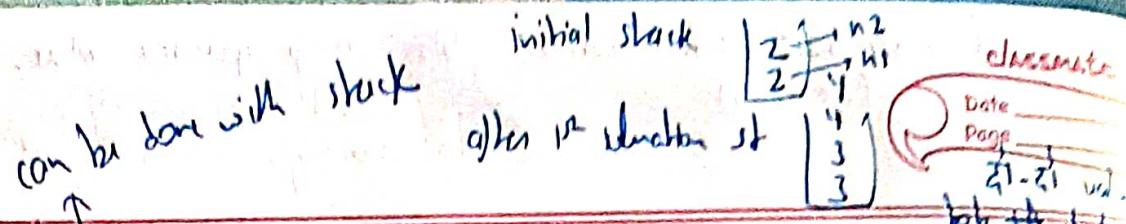
f(root->left, v, l+1);

}

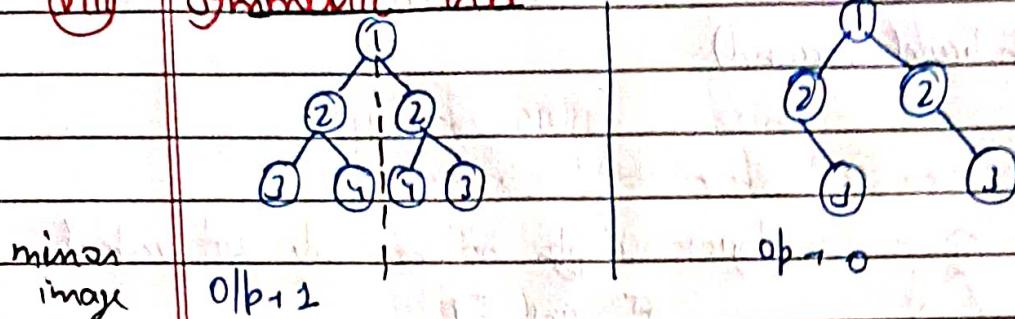
logic for level i right element ans it push by maintaining l variable.

for lth Side View of BT

swap these two line



## VIII Symmetric Tree



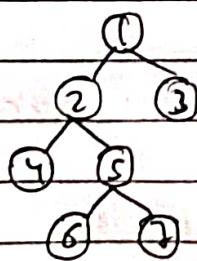
```
bool J(TreeNode *root)
{
    return J(root, root);
}
```

```
bool J(TreeNode *b, TreeNode *e)
```

```
{
    if (b == null || e == null) (b == e);
    if (b->val == e->val)
        return J(b->left, e->right) && J(b->right, e->left);
    return 0;
}
```

$O(N)$   
 $O(N) \rightarrow D$   
 $\log N$

## IX Root to Node Path



Given  $n = 7$

0/b [1, 2, 5, 2]

- No two nodes in tree are same

- Assume n to be parent in tree

vector<int> Path(TreeNode \*root, int n)

{

vector<int> ans;

$O(N)$

if (root == null) return ans;

$SC + O(N)$

bool a = J(<sup>root</sup>, ans, n)

return ans;

}

bool J(TreeNode \*root, vector<int> &ans, int n)

{

$\text{if } (\text{root} == \text{NULL}) \text{ return } \alpha;$

ans. bush-baile (root-ival);

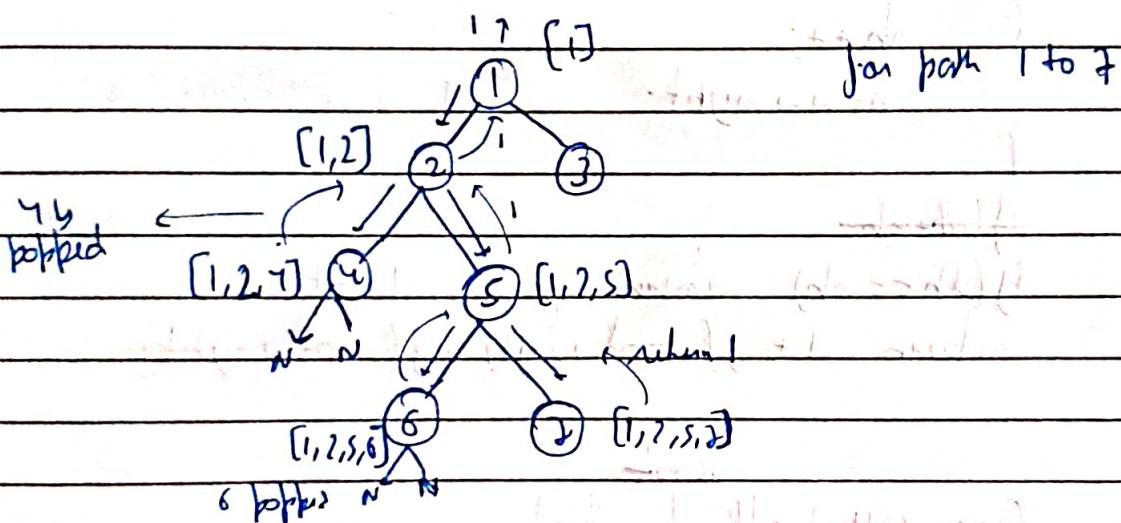
If ( $\text{root} \rightarrow \text{val} == n$ ) return 1;

~~$\text{if } (\text{f}(\text{root} \rightarrow \text{left}, \text{ans}, \text{n}) \text{ || } \text{f}(\text{root} \rightarrow \text{right}, \text{ans}, \text{n}))$~~

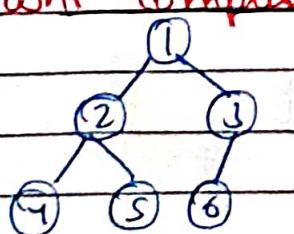
when 1:

ans.pop\_back(); // when backtracking pop-back return 0;

With  $f$  in  $\text{Polar}$ , when  $f$  calls, caller returns to caller so on.



## (xv) Count Complete Tree Nodes



Given variables BT

0/P-16

6

11

object

$\alpha/\beta - 1$

TC, O(N)

```
int countNodes(TreeNode *root)
{
    int c = 0;
    if (root, c);
    return c;
```

```

void f(TreeNode* root, int c)
{
    if (root == NULL) return;
    c++;
    f(root->left, c);
    f(root->right, c);
}

```

Crazy sol<sup>n</sup> see the implementation

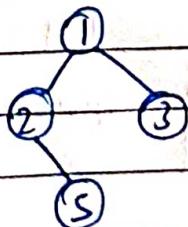
```
int f(TreeNode *root)
{
    if (root == NULL) return 0;
    TreeNode *l = root, *r = root;
    int lh = 0, rh = 0;
    while (l)
    {
        lh++;
        l = l->left;
    }
    while (r)
    {
        rh++;
        r = r->right;
    }
    if (lh > rh)
        return max(lh) - 1;
    return 1 + f(root->left) + f(root->right);
}
```

Root

(XVI)

Binary Tree All paths to leaf

: return all root to leaf path



dp [ "1->2->5", "1->3" ]

vector<string> f1(TreeNode \*root)

```
{
    vector<string> ans;
    string s;
    f1(root, ans, s);
    return ans;
}
```

```
void f(TreeNode *root, vector<string> &ans, string s)
{
    if (root == NULL) return;
```

```
    if (root->left == NULL && root->right == NULL)
        s += to_string(root->val);
        ans.push_back(s);
        return;
```

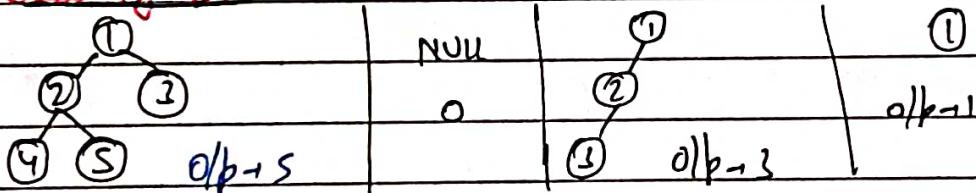
```
}
```

branch of  
Recursion

```
f(root->left, ans, s + to_string(root->val) + " -> ");
f(root->right, ans, s + to_string(root->val) + " -> ");
```

No. of nodes

~~(XVII)~~ Size of BT



```
int f(TreeNode *root)
```

```
{ if (root == NULL)
```

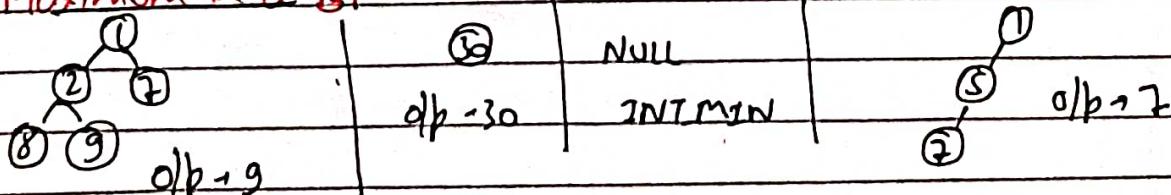
```
    return 0;
```

```
    return 1 + f(root->left) + f(root->right);
```

```
}
```

$O(h)$   
SC:  $O(h)$

~~(XVIII)~~ Maximum in a BT



```
int f(TreeNode *root)
```

```
{ if (root == NULL)
```

```
    return INTMIN;
```

```
    return max(root->val, max(f(root->left), f(root->right)));
```

```
}
```