Partner: Kashish Gupta and Christopher Kao (both just for questions).

1 1.1
```
%% Clear the environment
clc
clear

%% Read an image
%Write code here to read in the image named 'Bikesgray.jpg' into the variable img1
img1 = imread('Bikesgray.jpg');

%% Display original image
figure; imagesc(img1); axis image; colormap(gray);

%% X gradient - Sobel Operator
f1 = [1 0 -1; 2 0 -2; 1 0 -1];

%% Convolve image with kernel f1 -> This highlights the vertical edges in the image
img2(:,:) = double(img1(:,:)); %converts to double
vertical_sobel = conv2(img2, f1);

%% Display the image
% Write code here to display the image 'vertical_sobel'
figure; imagesc(vertical_sobel); axis image; colormap(gray);

%% Y gradient - Sobel Operator
f2 = [1 2 1; 0 0 0; -1 -2 -1]; % Now if you want to highlight horizontal edges in the image, th

%% Convolve image with kernel f2 -> This should highlight the horizontal edges in the image
horz_sobel = conv2(img2, f2); %Write code here to convolve img1 with f2

%% Display the image
% Write code here to display the image 'horz_sobel'
figure; imagesc(horz_sobel); axis image; colormap(gray);
```

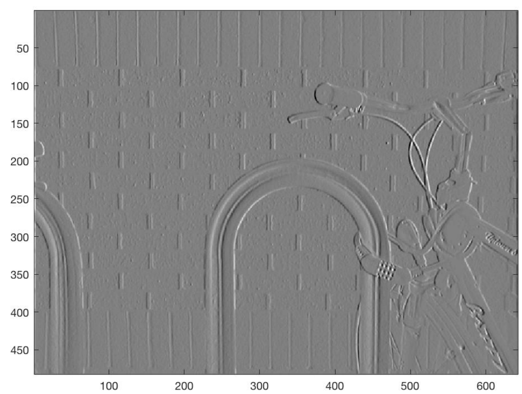
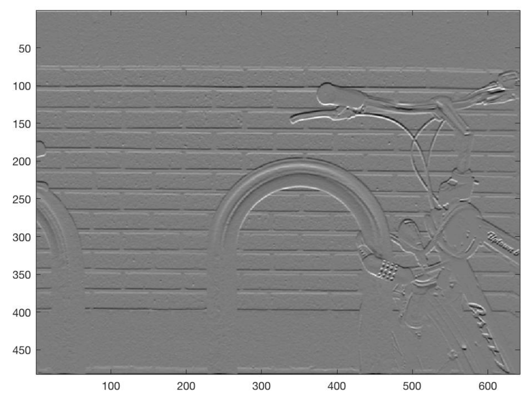

Figure on left: I_x (vertical lines showing); Figure on right: I_y (horizontal lines showing)

1.2 $I = \begin{bmatrix} 0.5 & 2 & 1.5 \\ 0.5 & 1.0 & 0.0 \\ 2.0 & 0.5 & 1.0 \end{bmatrix} \quad f = \begin{bmatrix} 0.5 & 1 & 0.0 \\ 0.0 & 1.0 & 0.5 \\ 0.5 & 0.0 & 0.5 \end{bmatrix}$

For example, the register for top left cell on image $(1,1)$: $= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0.5 & 2.0 \\ 0 & 0.5 & 1.0 \end{bmatrix}$. Thus when this register is multiplied with f, the result of the matrix multiplication is 2 so we insert it into g. So now $g = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

For example, the register for top right cell on image $(1,3) = \begin{bmatrix} 0 & 0 & 0 \\ 2 & 1.5 & 0 \\ 1 & 0.0 & 0 \end{bmatrix}$. Thus when this register is multiplied with f, the result of the matrix multiplication is 2 so we insert it into g. So now $g = \begin{bmatrix} 2 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

We can continue doing this for all 9 cells in the original image I (I didn't type every single calc out since they were the same format) and thus we will get the new image

$$I filteredwithf = \begin{bmatrix} 2 & 3 & 2 \\ 1.75 & 4.75 & 2.75 \\ 2.75 & 2.25 & 1.5 \end{bmatrix}$$

$$f filteredwithI = \begin{bmatrix} 1.5 & 2.25 & 2.75 \\ 2.75 & 4.75 & 1.75 \\ 2.0 & 3.0 & 2.0 \end{bmatrix}$$

Thus linear filtering is not communicative

2. 2.1 **First compute $I * f_x$:**
First since we're doing a convolution, we have to flip our kernel so our new kernel is $f_x = \begin{bmatrix} 1 & 0 & -1 \end{bmatrix}$. Now let's say the center our register around the top left cell $(1,1)$; then our register is $register = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}$. Thus, when we multiply the elements in the register with the convolved f_x we get -1 and so we put -1 into the $1,1$ position of our final matrix $I * f_x = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Now let's say we take the $register = \begin{bmatrix} 0 & 1 & -1 \end{bmatrix}$ (this is when we're looking at position $1,2$ in the original matrix I. Now, when we multiply the elements in the register with the convolved kernel we get 1. Thus we put 1 into position 12 of the $I * f_x$ matrix. Now, $I * f_x = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. If we continue doing this for all the positions in $I * f_x$, we get that

$$I * f_x = \begin{bmatrix} -1 & 1 & 1 \\ -1 & 2 & 1 \\ -3 & 1 & 3 \end{bmatrix}$$

Calculate $g_1 = I * f_x * f_y$: Once again, we need to flip our kernel for f_y for convolution so let's say our new kernel is $f_y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}'$. Now, if we take our register to be the top right cell of $I * f_x$ (position $1,3$), we get $register = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}'$. Thus, when we multiply the elements in this register with the convolved register, we get 2 so we put in 2 into element $1,3$ of the final g_1. Now $g_1 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Now let's say we take our register to be centered at point $2,2$. Thus our register becomes $register = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}'$. Thus when we multiply and add the elements in this register with the convolved kernel, we get 4 so we put 4 into element $2, 2$ of the final g_1. Thus, $g_1 = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 4 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. If we continue doing this for the rest of the registers and positions, we get that

$$g_1 = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

Now compute $f_{xy} = f_x * f_y$ First of all we need to remember that we need to the f_y kernel since we're doing a convolution. Thus $f_y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$. Now we need to remember that since we're essentially doing linear filtering on a 1x3 matrix by a 3x1 matrix, our output will be a 3x3 matrix and we'll be using the full operator.

So let's take the cell $1, 1$ on f_x and filter it over to cell $3, 1$ in the final matrix. So the register then is $register = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}'$. Thus, when we multiply and add the elements in this register, we get -1. Thus we put -1 into the final matrix for f_{xy}. Now $f_{xy} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$.

Now let's take the same cell $1, 1$ in f_x but instead filter it over to position $2, 1$ in the final matrix. Then, our register then is $register = \begin{bmatrix} 0 & -1 & 0 \end{bmatrix}'$. So when we now multiply and add by the kernel, we get -1 and put it into position $2, 1$ in f_{xy}. Thus $f_{xy} = \begin{bmatrix} 0 & 0 & 0 \\ -1 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}$. Now we can continue doing this for all 9 possible registers (since we're using full!) and we get that the final

$$f_{xy} = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

Calculate $g_2 = I * f_{xy}$ Now let's take $f_x y$ and then flip it since we're doing convolution. Thus our new $f_x y$ that we can essentially do traditional linear filtering on is $f_{xy} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$.

Now let's take the first cell in I (position $1, 1$) and do filtering on it. Thus, with 'same', our register becomes $register = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 2 & 1 \end{bmatrix}$. If we multiply all the elements and add them with the convolved f_{xy} matrix, we get -2 and so we put in -2 into position $1, 1$ of the final g_2 matrix. So $g_2 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

Now for the sake of example, let's say we want to convolve the element in position $3, 3$ of I. This means that our new register is $register = \begin{bmatrix} 1 & 0 & 0 \\ 3 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.Now when we multiply and add all these elements in the register with the convolved kernel, we get 4 and so we put 4 into element $3, 3$ of g_2. Thus our new g_2 matrix. So $g_2 = \begin{bmatrix} -2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 4 \end{bmatrix}$.

We can now continue doing this for all 9 elements in the original image I and since we're using the same size operator, our output matrix g_2 will also have 9 elements (3x3). The final

$$g_2 = \begin{bmatrix} -2 & 3 & 2 \\ -5 & 4 & 5 \\ -4 & 3 & 4 \end{bmatrix}$$

Verify associative property The associate property does indeed hold!

2.2 **For g_1:**
Let's calculate when convolving I with f_x. For each cell i, j in original matrix I, we need to multiply all the elements in the 3x1 register with all the elements in the 3x1 kernel. This leads to 3 multiplications per cell i, j. Then we add these three items up to come up with the final calculation that we output to the $I * f_x$ matrix. Since there are 9 cells (I is a 3x3), we thus have 3 multiplications * 9 cells = 27 multiplications and we have 2 additions * 9 cells = 18 additions. Note that this is just for the $I * f_x$ matrix.
When we now convolve the $I * f_x$ matrix with f_y, we also do the same. Each cell in the matrix $I * f_x$ will require 3 multiplications and 2 additions leading to a total of 27 multiplications and 18 additions for this part.
Thus in total to find g_x there are a total of $27+27 = 54$ multiplications and a total of $18+18 = 36$ additions.
For g_2:
Let's first calculate when convolving f_x with f_y. In this case, we will have a total of 9 registers of size 3x1 (since we're doing "full size") and so multiplying each register by the kernel will have 3 multiplications and 2 additions. Since we have 9 registers, we have a total of 27 multiplications and 18 additions.
Now let's calculate when convolving I with f_{xy}. In this case, we have once again 9 total registers but each register will be 3x3. Thus when multiplying each register with a kernel, we'll have 9 multiplications and 8 additions. Thus since we have 9 registers, we have a total of 81 multiplications and 72 additions.
In total, we thus have $27 + 81 = 108$ multiplications and $18 + 72 = 90$ additions.

2.3 It seems to be sharpening the left side and blurring the right side.

3. If we look at the bottom row of I and the right most column of I, we find that if we multiply those items by 5, we get the output in g. Thus, we put a 5 in the middle of f.

$$f = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now we notice that the 2x3 box at the top left of I (cells 11, 12, 21, and 22) when multiplied by 5 are off by a little from their true g values. In particular, they all seem to be a factor of 3 off when multiplied from the items in the bottom right box of I (cells 22, 23, 32, and 33). Thus, we notice that this is an impulse function and a scaled kernel; it has an impulse function and so we put a 1 in cell 11, and we scale that by 3, so we put in 3 at cell 11.

$$f = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4. 4.1 **Find f_1** Initially we notice that the kernel has to be 3x3 so that when we multiply all the numbers together and add them up, we can put them in the center of matrix from which we drew.
Now we notice that if we put a one in the lower-right hand of the kernel, we will essentially shift all the items in I_Q to the lower right. In other words, if $f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, then, $I_Q * f_x =$ $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$. Now we notice that essentially the entirety of I_Q can be composed of multiples

of 0.25. In particular, we notice that the center of I_Q (cells $2,2$; $2,3$; $3,2$; and $3,3$) are all $0.25 * 4$ and this tells us that we need to multiply the four ones in those positions by 0.25 to add up to get 1. Thus, in kernel f_1, we replace items $2,2$; $2,3$; $3,2$; and $3,3$ with 0.25, we get

$$f_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & .25 & .25 \\ 0 & .25 & .25 \end{bmatrix}$$

Note that in position $3,3$ of the kernel, the .25 has an impulse function of shifting the original image, and also scaling it down by 0.25. **Find** f_2 This item follows the same logic as finding f_1 except that we need to shift all the items to the top left corner. We can do this put putting a 1 in position $1,1$ of the kernel f_2. So now $f_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$. Now same as before, we just need to take that impulse function and scale it down by 0.25; and then we need to ensure that the middle four cells of the output image = 1. Thus we get the final matrix

$$f_2 = \begin{bmatrix} .25 & .25 & 0 \\ .25 & .25 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

4.2 Since $f = (f_1 + f_2)/2$, we get that

$$f = \begin{bmatrix} .125 & .125 & 0 \\ .125 & .25 & .125 \\ 0 & .125 & .125 \end{bmatrix}$$

$I'_Q = I_Q * f$ First we need to flip our kernel since we're doing convolution. However, this results in the same kernel.
Thus, let's take the 3x3 register of I_Q centered around cell $1,1$. This register is $register = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. When we multiply these elements with our kernel f and then add the items up, we get 0.625 and thus we put 0.625 into element $1,1$ of the final matrix I'_Q which is now $I'_Q \begin{bmatrix} 0.625 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

If we now take a 3x3 register of I_Q centered around cell $2,2$. This register is $register = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$. When we multiply these elements with our kernel f and then add the items up, we get 1 and thus we put 1 into element $2,2$ of the final matrix I'_Q which is now $I'_Q \begin{bmatrix} 0.625 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$. If we continue doing this for all 16 registers, we will get the final answer I'_Q which is now

$$I'_Q \begin{bmatrix} 0.625 & .75 & .50 & .125 \\ .75 & 1.0 & .75 & .25 \\ .50 & .75 & .625 & .25 \\ .125 & .25 & .25 & .125 \end{bmatrix}$$

$I'_T = I_T * f$ We can do the exact same thing and have the registers in the same positions as before (16 in total with 'same size') we get

$$I'_T = \begin{bmatrix} .125 & .25 & .25 & .125 \\ .25 & .625 & .75 & .50 \\ .25 & .75 & 1.00 & .75 \\ .125 & .50 & .75 & .625 \end{bmatrix}$$

Note that in this case, I didn't show calculations by hand because one would have the 3x3 registers in the 4x4 image the same way as was demonstrated in the convolution to find I'_Q.
Find $D(I'_Q, I'_T)$ The distance is simply the differences of the elements in I'_Q and I'_T, squared, and added up. So for example we'd subtract element $1, 1$ from $1, 1$ in both matrices, square the result, and then add to the total. In other words we have $(.625 - .125)^2 + (.75 - .25)^2 + ... + (.25 - .75)^2 + (.125 - .625)^2$. This is ultimately equal to 2.0312, which is significantly smaller than the distance of 10 we were originally getting for the distance of $D(I_Q, I_T)$.

5. 5.1 I have a iPhone 6s+ and there was not much info about size of sensor. However, I did find one source that said that the size for the rear camera (the camera through which the picture was taken) was 1/3.0-inch (4.89 x 3.67 mm). http://www.cameradebate.com/2015/sensor-size-comparison-iphone-6-plus-vs-samsung-galaxy-s6-edge-vs-note-4/. For the front camera, all i was able to find is that it's a 1/5" lens (http://www.anandtech.com/show/9686/the-apple-iphone-6s-and-iphone-6s-plus-review/9); however, the image was not taken from this lens anyways.
As the sensor size increases, the field of view also increases. This is because as the sensor size increases, the sensor can capture more photons from more angels. The size of the image is really limited by the pixel space decided by the side of the CCD.

   5.2 According to the same source the resolution is 3264x2448 pixels. Thus, since we have 3264 pixels in the x direction of a 4.89mm length sensor, we have that each pixel is 0.001489mm in length. And since we have 2448 pixels in the y direction of a 3.67mm height sensor, we have that each pixel is 0.001499mm in height. Thus, each pixel is around 0.001489 x 0.001499 mm.

   5.3 I was able to find that the reported focal length of the rear camera of iPhone 6s+ is around 4.15mm according to http://www.anandtech.com/show/9686/the-apple-iphone-6s-and-iphone-6s-plus-review/9. The focal length of the front camera is 2.65mm according to the same source.
If we calculate the focal length from scratch, we have $f = \frac{d}{2\tan(\varphi)}$. We know that $d = 4.89mm$.

$$f = \frac{4.89}{2 * 1.73}$$

$$f = 1.41$$

. This seems to be quite off from the 4.15mm reported.
As the focal length of the camera increases, the field of view decreases.

   5.4 The internal camera matrix is given by

$$\begin{bmatrix} f_x & 0 & p_x \\ 0 & f_y & p_y \\ 0 & 0 & 1 \end{bmatrix}$$

In our case,

$$f_x = f_m * \frac{W_{img}}{W_{ccd}}$$

so

$$f_x = 4.15 * \frac{3264pix}{4.89mm} = 2770.06$$

$$f_y = f_m * \frac{h_{img}}{h_{ccd}}$$

so

$$f_y = 4.15 * \frac{2448pix}{3.67mm} = 2768.17$$

Our p_x is simply our resolution width / 2 so $p_x = 1632$ and similarly our p_y is our resolution height /2 and so $p_y = 1224$. The internal camera matrix then is:

$$\begin{bmatrix} 2770.06 & 0 & 1632 \\ 0 & 2768.17 & 1224 \\ 0 & 0 & 1 \end{bmatrix}$$

5.5 

5.5.1 H_F is 5'10" or 1.778 meters. According to the image, the pixel height h_F is 1219 pixels. We can use the formula $y_{img} = f_m * \frac{h_{img}Y}{h_{ccd}Z}$. In other words,

$$Z = f_m * \frac{h_{img}Y}{h_{ccd} * y_{img}}$$

$$Z = 0.00415m\frac{4032pix * 1.778m}{0.00367m * 1219pix}$$

$$z = 6.65$$

5.5.2 Now H_B equals 5.4102m. We find h_B to be around 1399 pixels. Thus using a similar calculation as before

$$Z = f_m * \frac{h_{img}Y}{h_{ccd} * y_{img}}$$

$$Z = 0.00415m\frac{4032pix * 5.41m}{0.00367m * 1399pix}$$

$$z = 17.63$$

5.65.6.1 We need to find h''_B such that $h_F = h'_F$. In the first image, the height of statue is 1399 pixels. Thus in the new image,

$$Z = 0.00415m * \frac{3945pix * 5.41m}{0.00367m * 1219pix}$$

In other words, the Z of my friend in the new image is around 6.5m.
However, this is the distance from my camera to my friend in the new image. We want the distance from my camera to the statue in the old image. Well from part 5.5.1 and 5.5.2 we know that the difference between my friend and the statue is around $17.63m - 6.65 = 10.89$. So thus, our Z from the new further back position to the statue should be 6.5+10.89=17.39. Now to find h''_B we use

$$y_{img} = 0.00415m * \frac{3945 * 5.41m}{0.00367m * Z}$$

. Since we know that $Z = 6.5m$ we now have

$$y_{img} = 0.00415m * \frac{3945 * 5.41m}{0.00367m * 17.39}$$

. This leads to a h''_B of around 1387pixels.

5.6.2 The pixel height of B, h''_B is around 1738pixels in the actual picture. This seems to be 351 pixels off from the real value.

5.6.3 We want $h''_B = 3h'_B$ and we want $h''_F = h'_F$. So for the heights of the Statues:

$$h''_B = f_m * \frac{h_{img} * H_B}{h_{ccd} * Z}$$

$$3h'_B = f_m * \frac{h_{img} * H_B}{h_{ccd} * Z}$$

So

$$3 * 1399pix = f_m * \frac{4032pix * 5.41m}{0.00367m * Z}$$

$$4197 f_m = 5943629.42/Z$$

$$f_m = 1416.16/Z$$

For the heights of the person:

$$h''_F = f_m * \frac{h_{img} * H_F}{h_{ccd} * (Z - 10.89)}$$

$$h'_F = f_m * \frac{h_{img} * H_F}{h_{ccd} * (Z - 10.89)}$$

$$1219pix = f_m * \frac{4032pix * 1.778m}{0.00367m * (Z - 10.89)}$$

$$(Z - 10.89) = f_m * 1602.44$$

Now we just have a system of equations with two unknowns and two equations. So

$$f_m = 1416.16/(f_m * 1602.44)$$

$$f_m^2 = .8837$$

$$f_m = .940$$

$$Z = 1506.3$$