ECON 318 - Spring 2020: Practise Problems for Midterm1

Solutions

1 Question 1

A random variable X has a standard normal distribution with mean 0 and variance 1.

1. What is the probability that $X < 0$?

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535

- We use the standard normal distribution table to find that:

$$\begin{aligned} Pr(X < 0) &= 0.5 \\ &= 50.0\% \end{aligned}$$

2. What is the probability that $X > 2$?

2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574

- $$\begin{aligned} Pr(X > 2) &= 1 - Pr(X < 2) \\ &= 1 - 0.97725 \\ &= 0.02275 \\ &= 2.275\% \end{aligned}$$

3. What is the probability that $-2 < X < 2$?

-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831

- $$\begin{aligned} Pr(-2 < X < 2) &= Pr(X < 2) - Pr(X < -2) \\ &= 0.97725 - 0.02275 \\ &= 0.9545 \\ &= 95.45\% \end{aligned}$$

2 Question 2

In the past, a chemical plant has a stated goal of producing an average of 1100 pounds of chemical per day. The records for the past year, based on 260 operating days, show the following:
$\bar{Y} = 1060$ pounds per day.
$S_Y = 340$ pounds per day.
The managers of the firm wish to test whether or not the average daily production differs from its stated goal.

1. Give the appropriate null and alternative hypotheses.

- $H_0 : \bar{Y} = 1100$
  $H_1 : \bar{Y} \neq 1100$

2. Construct a 90% confidence interval for the sample mean.

- The confidence interval is given by: $\left[\hat{\theta} - C_{\alpha/2} \cdot SE(\hat{\theta}), \hat{\theta} + C_{\alpha/2} \cdot SE(\hat{\theta})\right]$, where $C_{\alpha/2}$ is the critical value taken from the appropriate distribution, and $SE(\hat{\theta})$ is an estimate of the standard deviation of $\hat{\theta}$.
- $Y_1, ..., Y_n$ randomly drawn from $N(\mu, \sigma^2)$, σ^2 not known. Specify the $(1-\alpha) * 100\%$ confidence for μ
- The estimator on which C.I. is based: $\bar{Y} \sim N(\mu, \sigma^2/n)$
- Use $\frac{\bar{Y}-\mu}{S_Y/\sqrt{n}} \sim t_{n-1}$, and student-t tables to specify $t_{\alpha/2}$ such that

$$Pr\left(-t_{\alpha/2} < \frac{\bar{Y}-\mu}{S_Y/\sqrt{n}} < t_{\alpha/2}\right) = 1-\alpha$$

- The confidence interval of μ is obtained by rewriting:

$$Pr\left(\bar{Y} - t_{\alpha/2}\frac{S_Y}{\sqrt{n}} < \mu < \bar{Y} + t_{\alpha/2}\frac{S_Y}{\sqrt{n}}\right) = 1-\alpha$$

- That is, with $1-\alpha$ confidence, $\mu \in \left[\bar{Y} - t_{\alpha/2}\frac{S_Y}{\sqrt{n}}, \bar{Y} + t_{\alpha/2}\frac{S_Y}{\sqrt{n}}\right]$
- In the above question, σ^2 (unknown) is estimated:

$$\begin{aligned}\left[\bar{Y} - t_{\alpha/2,n-1}\frac{S_Y}{\sqrt{n}}, \bar{Y} + t_{\alpha/2,n-1}\frac{S_Y}{\sqrt{n}}\right] &= \left[1060 - (1.65\frac{340}{\sqrt{260}}), 1060 + (1.65\frac{340}{\sqrt{260}})\right] \\ &= [1025.2, 1094.8]\end{aligned}$$

3 Question 3

Suppose you are told that an OLS regression analysis of average weekly earnings yields the following estimated model.

$$A\hat{W}E = 696.7 + 9.6AGE, R^2 = 0.023, ESS = 624.1 \tag{1}$$

1. Explain what the coefficient value means.
   - $\alpha = 696.7$: If age is 0, the average weekly earnings equal 696.7.
     $\beta = 9.6$: If age increases by 1 year, the average weekly earnings increase by \$9.6
2. What is the regression's predicted earnings for a 25 year-old worker and a 45 year-old worker?
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$$\begin{aligned}E[AWE|Age = 25] &= 696.7 + [9.6 * 25] \\ &= \$936.7\end{aligned}$$

$$\begin{aligned}E[AWE|Age = 45] &= 696.7 + [9.6 * 45] \\ &= \$1,128.7\end{aligned}$$

4 Question 4

Indicate whether each of the following statements about the simple regression model is true or false. Explain.

1. If the sample means of X and Y are zero, then the estimated Y-intercept is zero.
   - TRUE
     The estimated regression line passes through the mean of X and mean of Y. This implies that the equation $\bar{Y} = \hat{\alpha} + \hat{\beta}\bar{X}$ is always satisfied. Therefore, if $\bar{X}$ and $\bar{Y}$ are 0, the Y-intercept or $\hat{\alpha}$ is also 0

2. The slope of the simple regression model indicates how the actual value of Y changes as X changes.

   - TRUE

3. The residuals from a least squares regression are all zero.

   - FALSE
     The sum of the residuals from a least squares regression equal zero.

4. The sum of the residuals from a least squares regression is zero.

   - TRUE

5. The method of least squares minimizes the residuals.

   - FALSE
     The method of least squares minimizes the sum of the squared residuals

6. If the sample covariance between X and Y is zero, then the slope of the least squares regression line is zero.

   - TRUE

$$\begin{aligned}\hat{\beta} &= \frac{\sum\limits_i (X_i - \bar{X})(Y_i - \bar{Y})}{\sum\limits_i (X_i - \bar{X})^2} \\ &= \frac{Cov(X,Y)}{Var(X)}\end{aligned}$$

   Therefore, if the sample covariance between X and Y is 0, $\hat{\beta} = 0$