CS480/680: Introduction to Machine Learning

Homework 3

Due: 11:59 pm, July 08, 2024, submit on LEARN.

NAME

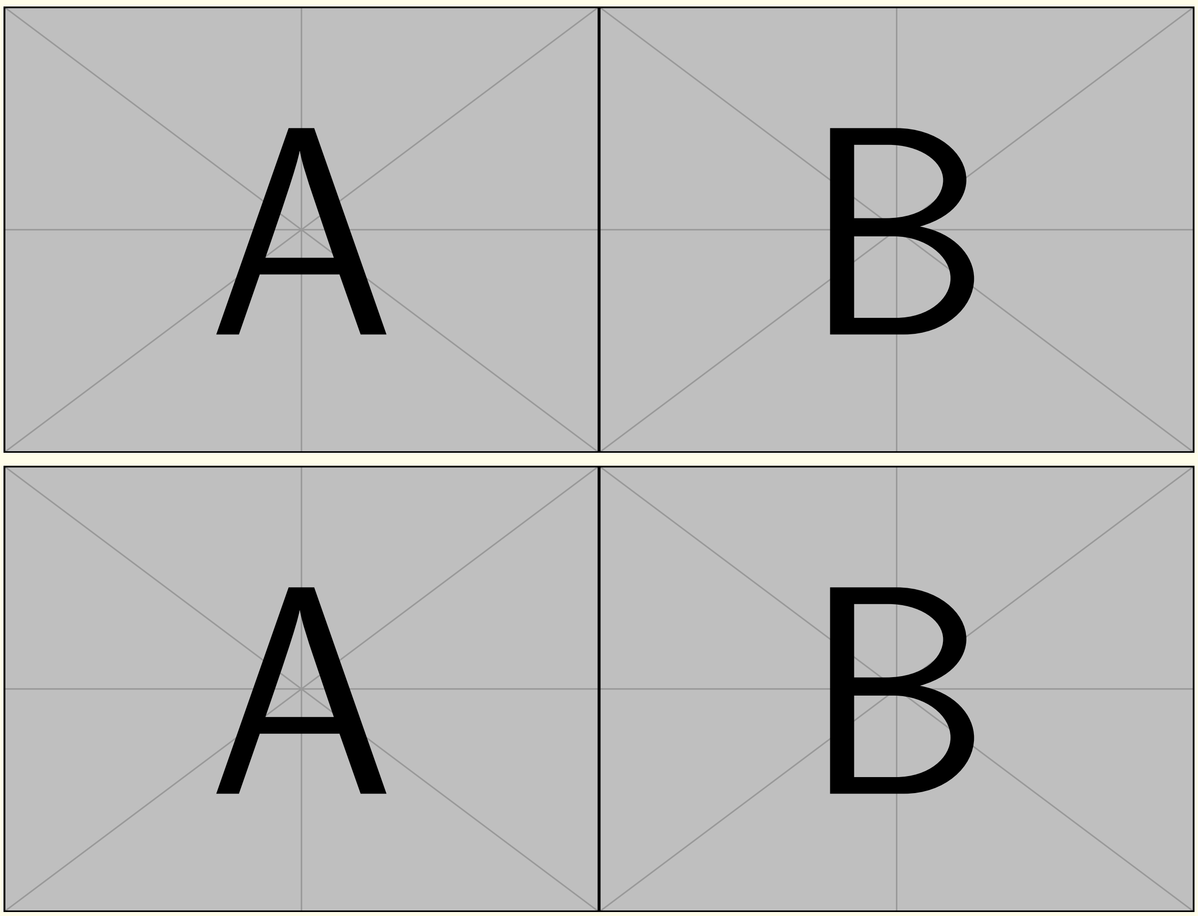
student number

Submit your writeup in pdf and all source code in a zip file (with proper documentation). Write a script for each programming exercise so that the TA can easily run and verify your results. Make sure your code runs!
[Text in square brackets are hints that can be ignored.]

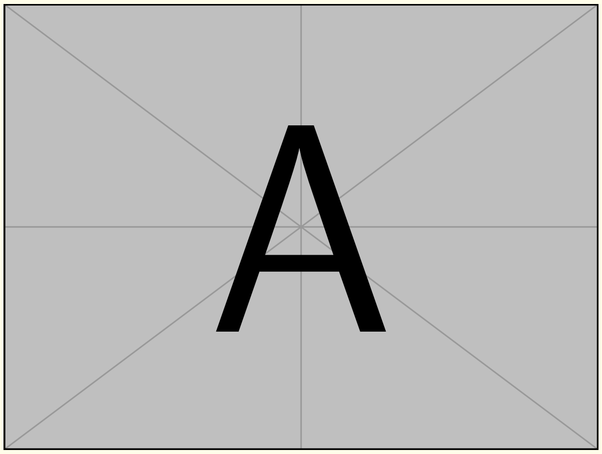
Exercise 1: Vision Transformers (10 pts)

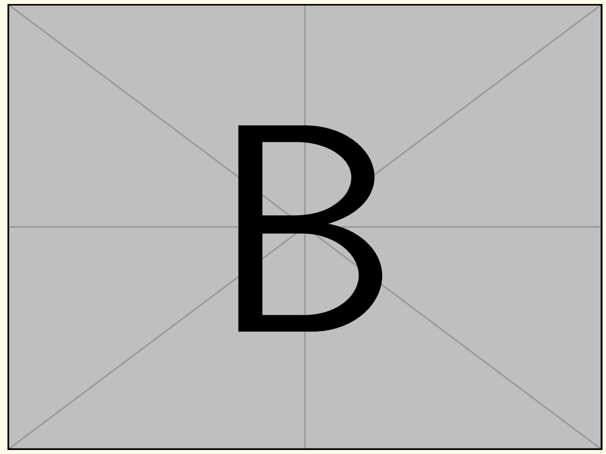
Please follow the instructions of this `ipynb` file.

1. (1+3+2 = 6 pts) Complete the missing coding parts in the provided `ipynb` file.
2. (1 pt) Visualization of patches:



3. (1 pt) The test accuracy I obtained on MNIST is: xxx%
4. (2 pts) Training / Validation accuracy vs. epoch:





Exercise 2: Adaboost (8 pts)

In this exercise we will implement Adaboost. Recall that Adaboost aims at minimizing the exponential loss:

$$\min_{\mathbf{w}} \sum_i \exp\left(-y_i \sum_j w_j h_j(\mathbf{x}_i)\right), \tag{1}$$

where h_j are the so-called weak learners, and the combined classifier

$$h_{\mathbf{w}}(\mathbf{x}) := \sum_j w_j h_j(\mathbf{x}). \tag{2}$$

Note that we assume $y_i \in \{\pm 1\}$ in this exercise, and we simply take $h_j(\mathbf{x}) = \mathrm{sign}(\pm x_j + b_j)$ for some $b_j \in \mathbb{R}$. Upon defining $M_{ij} = y_i h_j(\mathbf{x}_i)$, we may simplify our problem further as:

$$\min_{\mathbf{w} \in \mathbb{R}^d} \mathbf{1}^\top \exp(-M\mathbf{w}), \tag{3}$$

where exp is applied component-wise and $\mathbf{1}$ is the vector of all 1s.

Recall that $(s)_+ = \max\{s, 0\}$ is the positive part while $(s)_- = \max\{-s, 0\} = |s| - s_+$.

Algorithm 1: Adaboost.

```
Input: M ∈ ℝ^{n×d}, w_0 = 0_d, p_0 = 1_n, max_pass = 300
Output: w
for t = 0, 1, 2, ..., max_pass do
    p_t ← p_t / (1^⊤ p_t)                          // normalize
    ε_t ← (M)_-^⊤ p_t                              // (·)_- applied component-wise
    γ_t ← (M)_+^⊤ p_t                              // (·)_+ applied component-wise
    β_t ← 1/2 (ln γ_t − ln ε_t)                    // ln applied component-wise
    choose α_t ∈ ℝ^d                               // decided later
    w_{t+1} ← w_t + α_t ⊙ β_t                      // ⊙ component-wise multiplication
    p_{t+1} ← p_t ⊙ exp(−M(α_t ⊙ β_t))             // exp applied component-wise
```

1. (2 pts) We claim that Algorithm 1 is indeed the celebrated Adaboost algorithm if the following holds:

   - $\boldsymbol{\alpha}_t$ is one-hot (i.e., 1 at some entry and 0 everywhere else), namely, it indicates which weak classifier is chosen at iteration t.
   - $M \in \{\pm 1\}^{n \times d}$, i.e., if all weak classifiers are $\{\pm 1\}$-valued.

   With the above conditions, prove that (a) $\boldsymbol{\gamma}_t = \mathbf{1} - \boldsymbol{\epsilon}_t$, and (b) the equivalence between Algorithm 1 and the Adaboost algorithm in class. [Note that our labels here are $\{\pm 1\}$ and our $\mathbf{w}$ may have nothing to do with the one in class.]

Ans:

2. (2 pts) Let us derive each week learner h_j. Consider each feature in turn, we train d linear classifiers that each aims to minimize the weighted training error:

$$\min_{b_j \in \mathbb{R}, s_j \in \{\pm 1\}} \sum_{i=1}^{n} p_i [\![y_i(s_j x_{ij} + b_j) \leq 0]\!], \tag{4}$$

where the weights $p_i \geq 0$ and $\sum_i p_i = 1$. Find (with justification) an optimal value for each b_j and s_j. [If multiple solutions exist, you can use the middle value.] If it helps, you may assume p_i is uniform, i.e., $p_i \equiv \frac{1}{n}$.

Ans:

3. (2 pts) [Parallel Adaboost.] Implement Algorithm 1 with the following choices:
   - $\boldsymbol{\alpha}_t \equiv \mathbf{1}$
   - pre-process M by dividing a constant so that for all i (row), $\sum_j |M_{ij}| \leq 1$.

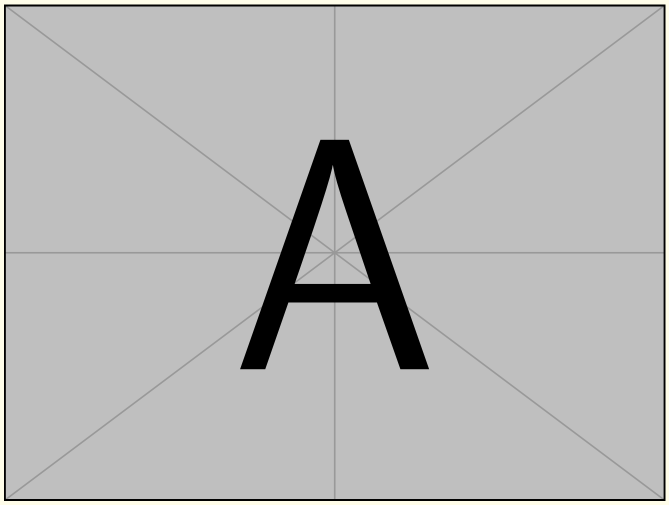
Run your implementation on the default dataset (available on course website), and report the training loss in (3), training error, and test error w.r.t. the iteration t, where

$$\mathrm{error}(\mathbf{w}; \mathcal{D}) := \frac{1}{|\mathcal{D}|} \sum_{(\mathbf{x},y)\in\mathcal{D}} [\![y h_{\mathbf{w}}(\mathbf{x}) \leq 0]\!]. \tag{5}$$

[Recall that $h_{\mathbf{w}}(\mathbf{x})$ is defined in (2) while each h_j is decided in Ex 2.2 with uniform weight $p_i \equiv \frac{1}{n}$. In case you fail to determine h_j, in Ex 2.3 and Ex 2.4 you may simply use $h_j(\mathbf{x}) = \mathrm{sign}(x_j - m_j)$ where m_j is the median value of the j-th feature in the training set.]

[Note that $\mathbf{w}_t$ is dense (i.e., using all weak classifiers) even after a single iteration.]

Ans: We report all 3 curves in one figure, with clear coloring and legend to indicate which curve is which.



4. (2 pts) [Sequential Adaboost.] Implement Algorithm 1 with the following choice:

   - $j_t = \mathrm{argmax}_j \, |\sqrt{\epsilon_{t,j}} - \sqrt{\gamma_{t,j}}|$ and $\boldsymbol{\alpha}_t$ has 1 on the j_t-th entry and 0 everywhere else.
   - pre-process M by dividing a constant so that for all i and j, $|M_{ij}| \leq 1$.

   Run your implementation on the default dataset (available on course website), and report the training loss in (3), training error, and test error in (5) w.r.t. the iteration t.

   [Note that $\mathbf{w}_t$ has at most t nonzeros (i.e., weak classifiers) after t iterations.]

   Ans: We report all 3 curves in one figure, with clear coloring and legend to indicate which curve is which.