REVENUE MANAGEMENT

An Introduction to Linear Optimization

15.071x – The Analytics Edge

Airline Regulation (1938-1978)

- The Civil Aeronautics Board (CAB) set fares, routes, and schedules for all interstate air transport
- Most major airlines favored this system due to guaranteed profits
- Led to inefficiency and higher costs
  - Applications for new routes and fares often delayed or dismissed

Airline Deregulation (1978)

- The administration of President Jimmy Carter passed the Airline Deregulation Act in 1978
- The Act encouraged
  - **More competition**: 52 new airlines between 1980 and 2000
  - **New air routes**: saved passengers an estimated $10.3 billion each year in travel time
  - **Lower fares**: ticket prices are 40% lower today than they were in 1978
- This led to **more passengers**
  - The number of air passengers increased from 207.5 million in 1974 to 721.1 million in 2010

A Competitive Edge

- More competition led to heavy losses by air carriers
  - Need to lower fares while meeting operating costs
- 9 major carriers and more than 100 smaller airlines went bankrupt between 1978 and 2002
- How did airlines compete?

Discount Fares

- On January 17, 1985 American Airlines (AA) launched its Ultimate Super Saver fares to compete with PeopleExpress

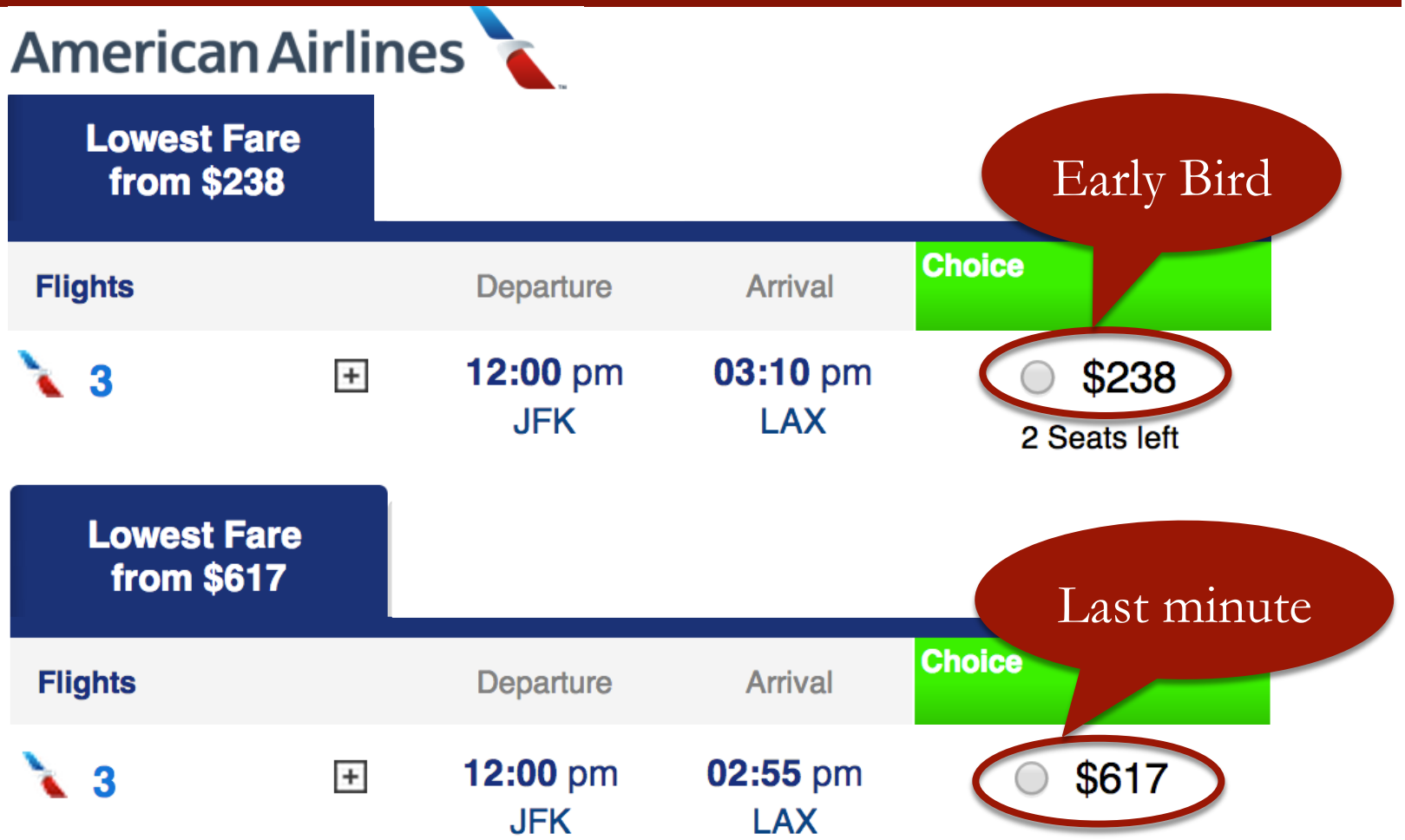
- Need to fill at least a minimum number of seats without selling every seat at discount prices
  - Sell enough seats to cover fixed operating costs
  - Sell remaining seats at higher rates to maximize revenues/profits

How Many Seats to Sell on Discount?

- Passengers have different valuations
  - Business people value flexibility (last-minute/refundable)
  - People seeking getaways value good deals (early birds)
- Sell too many discounted seats
  - Not enough seats for high-paying passengers
- Sell too few discounted seats
  - Empty seats at takeoff implying lost revenue
- How should AA allocate its seats among customers to maximize its revenue?

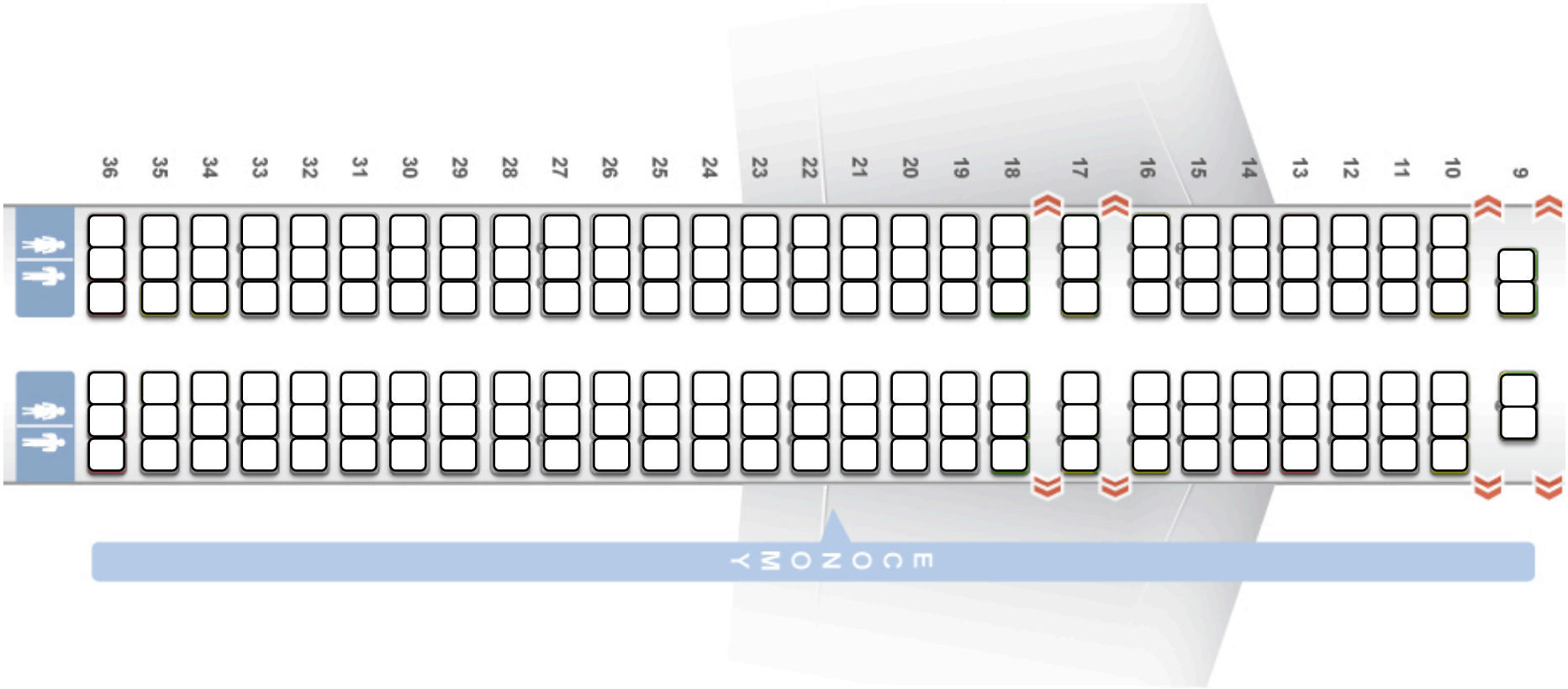
Let's Start Simple

Ticket Prices



Boeing 757-200 Seat Map

- 166 Economy seats



Demand Forecasting

- Demand for different prices can be forecasted using analytics tools, looking at historical data and incorporating models of human behavior
  - Time series methods
  - Linear regression
- Forecasts could be erroneous
  - Need to assess sensitivity to forecast errors
- We'll assume that demand has been forecasted

Myopic Solution

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	50	50
	Discount	238	150	116

Capacity 166

- How many discount seats to sell to maximize revenue?

Myopic Solution

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	100
	Discount	238	150	66

Capacity 166

- How many discount seats to sell to maximize revenue?

Myopic Solution

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	200	166
	Discount	238	150	0

Capacity 166

- How many discount seats to sell to maximize revenue?
- This seems simple, but what if we had 100 different flights?
- In the next video, we'll see how to formulate this mathematically

Single Route Example

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	

Capacity 166

- Problem: Find the optimal number of discounted seats and regular seats to sell to maximize revenue

- Let's formulate the problem mathematically

Step 1. Decisions

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	

Capacity 166

- What are our decisions?
  - Number of regular seats to sell – R
  - Number of discount seats to sell – D

Step 2. Objective

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	

Capacity 166

- What is our objective?
  - Maximizing total airline revenue
  - Revenue from each type of seat is equal to the number of that type of seat sold times the seat price

max 617*R + 238*D

Step 3. Constraints

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	

Capacity 166

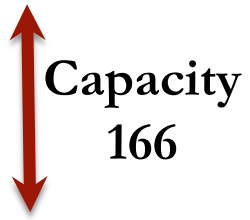
- AA cannot sell more seats than the aircraft capacity
  - Total number of seats sold cannot exceed capacity

$R + D \leq 166$

- AA cannot sell more seats than the demand
  - Regular seats sold cannot exceed 100 $R \leq 100$
  - Discount seats sold cannot exceed 150 $D \leq 150$

Step 4. Non-Negativity

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	



- AA cannot sell a negative number of seats

$R \geq 0$ $D \geq 0$

Problem Formulation

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	

Capacity 166

Maximize Total airline revenue

Subject to Seats sold cannot exceed capacity

Seats sold cannot exceed demand

Seats sold cannot be negative

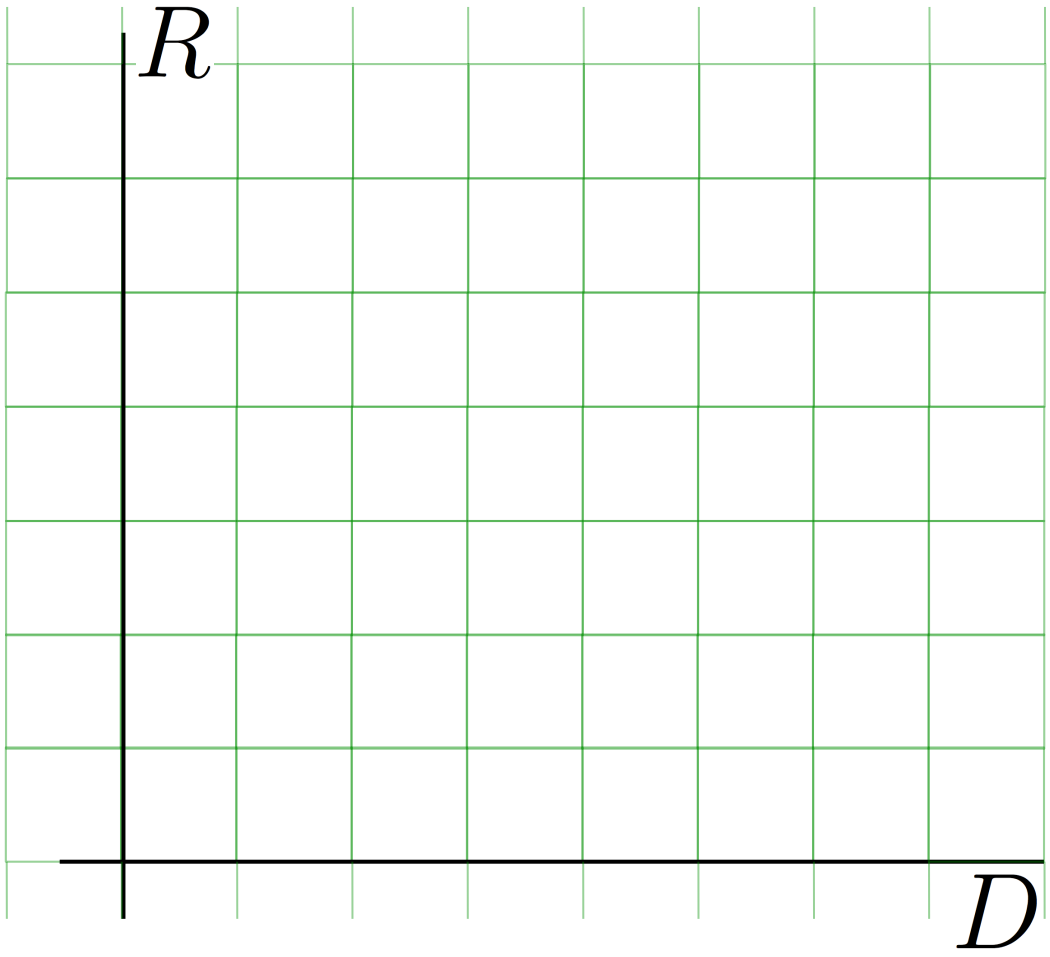
Problem Formulation

		Price	Demand	Seats to Sell
JFK - LAX	Regular	617	100	
	Discount	238	150	

Capacity 166

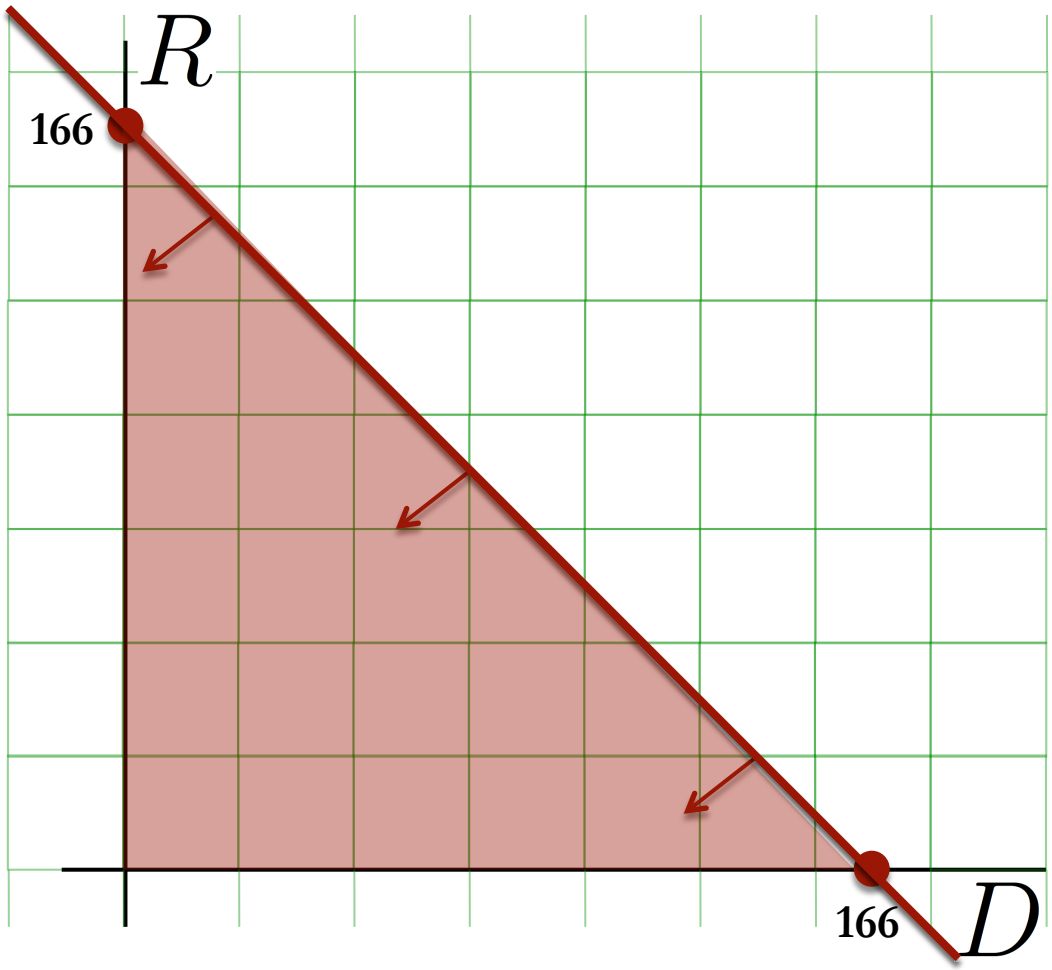
$$\begin{aligned} \text{Maximize} \quad & 617R + 238D \\ \text{Subject to} \quad & R + D \leq 166 \\ & R \leq 100, D \leq 150 \\ & R \geq 0, D \geq 0 \end{aligned}$$

Visualizing the Problem



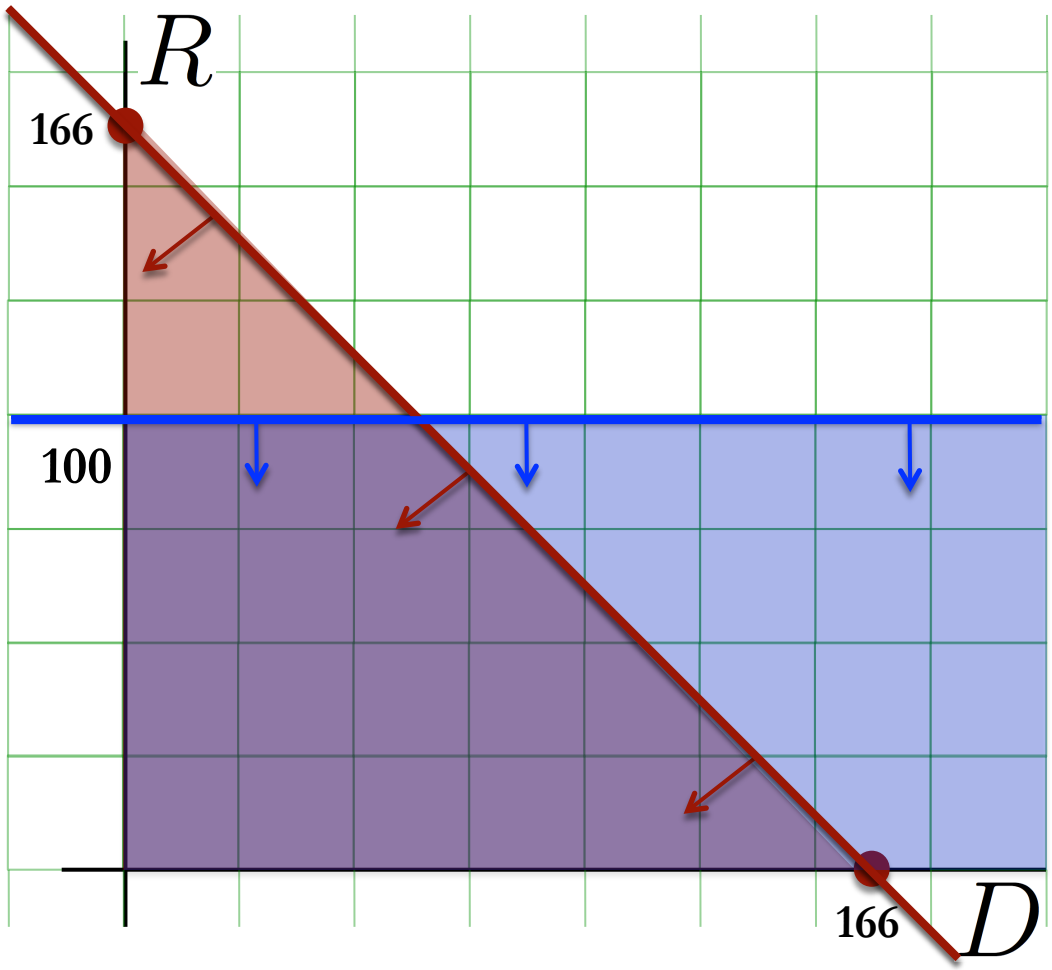
- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$

Visualizing the Problem



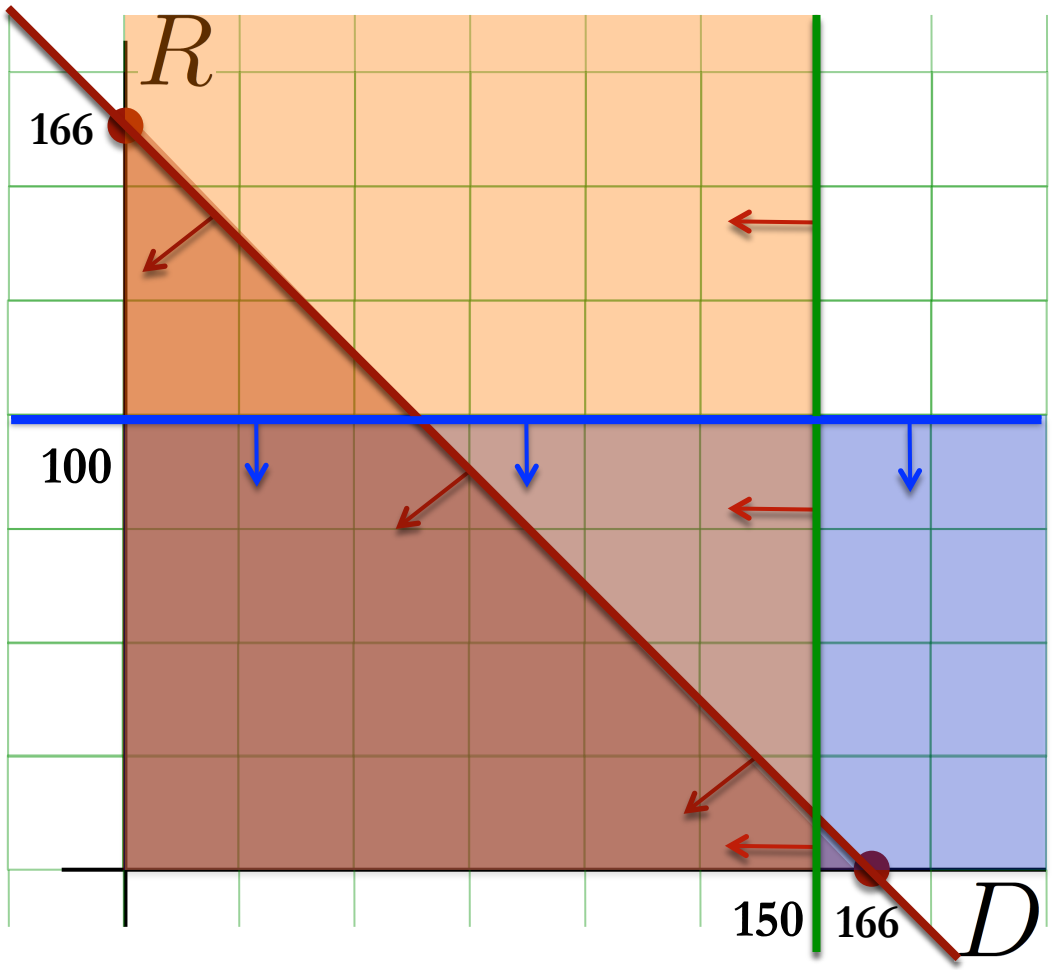
- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$
  - Capacity
    $R + D \leq 166$

Visualizing the Problem



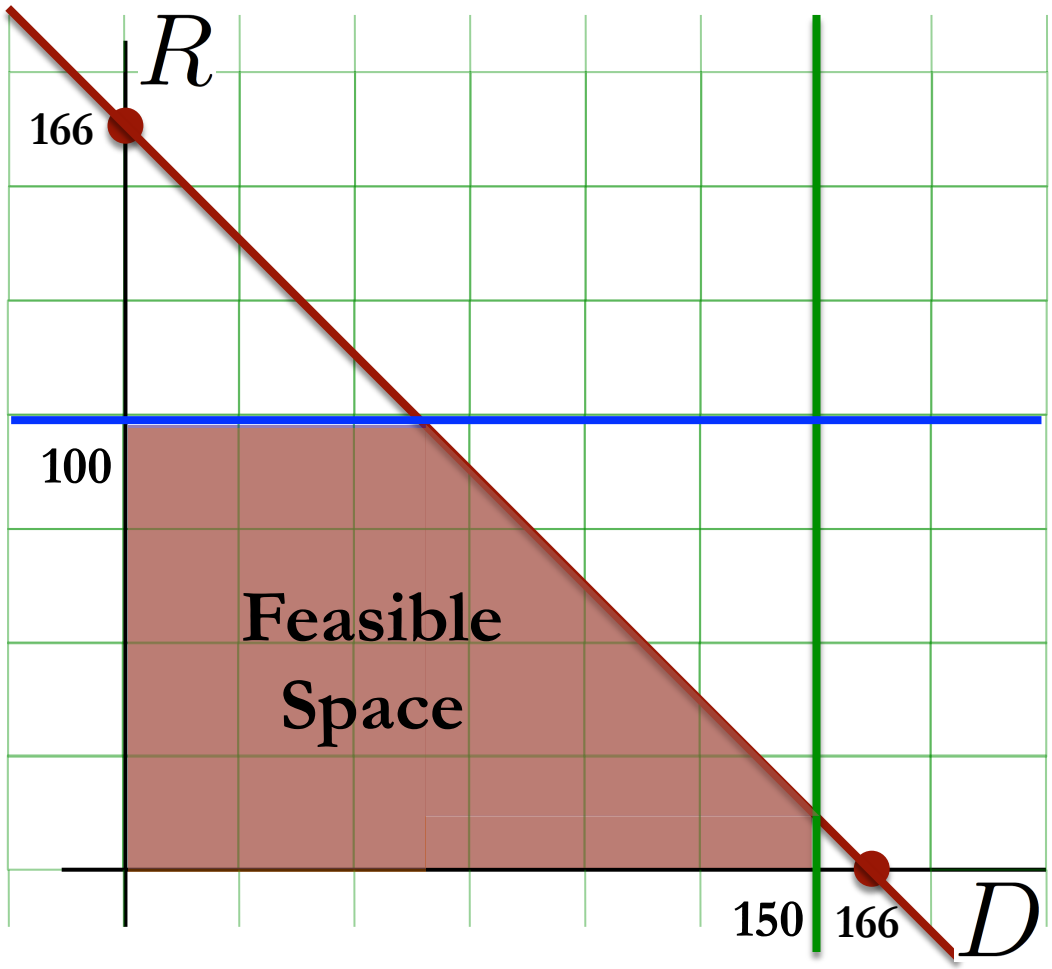
- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$
  - Capacity
    $R + D \leq 166$
  - Demand
    $R \leq 100, D \leq 150$

Visualizing the Problem

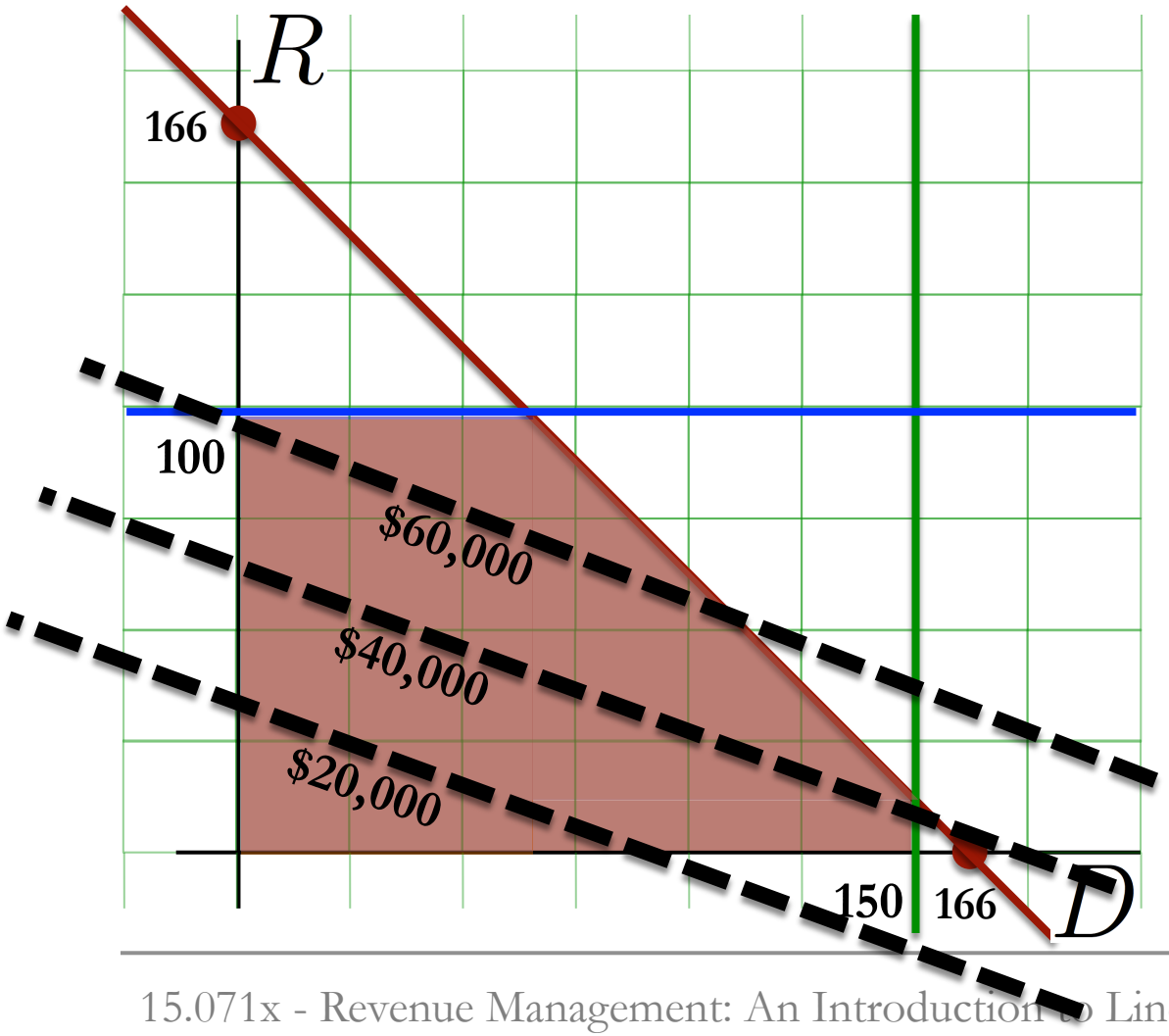


- 2D Representation
- Constraints
  - Non-negativity
    $R \geq 0, D \geq 0$
  - Capacity
    $R + D \leq 166$
  - Demand
    $R \leq 100, D \leq 150$

Feasible Space

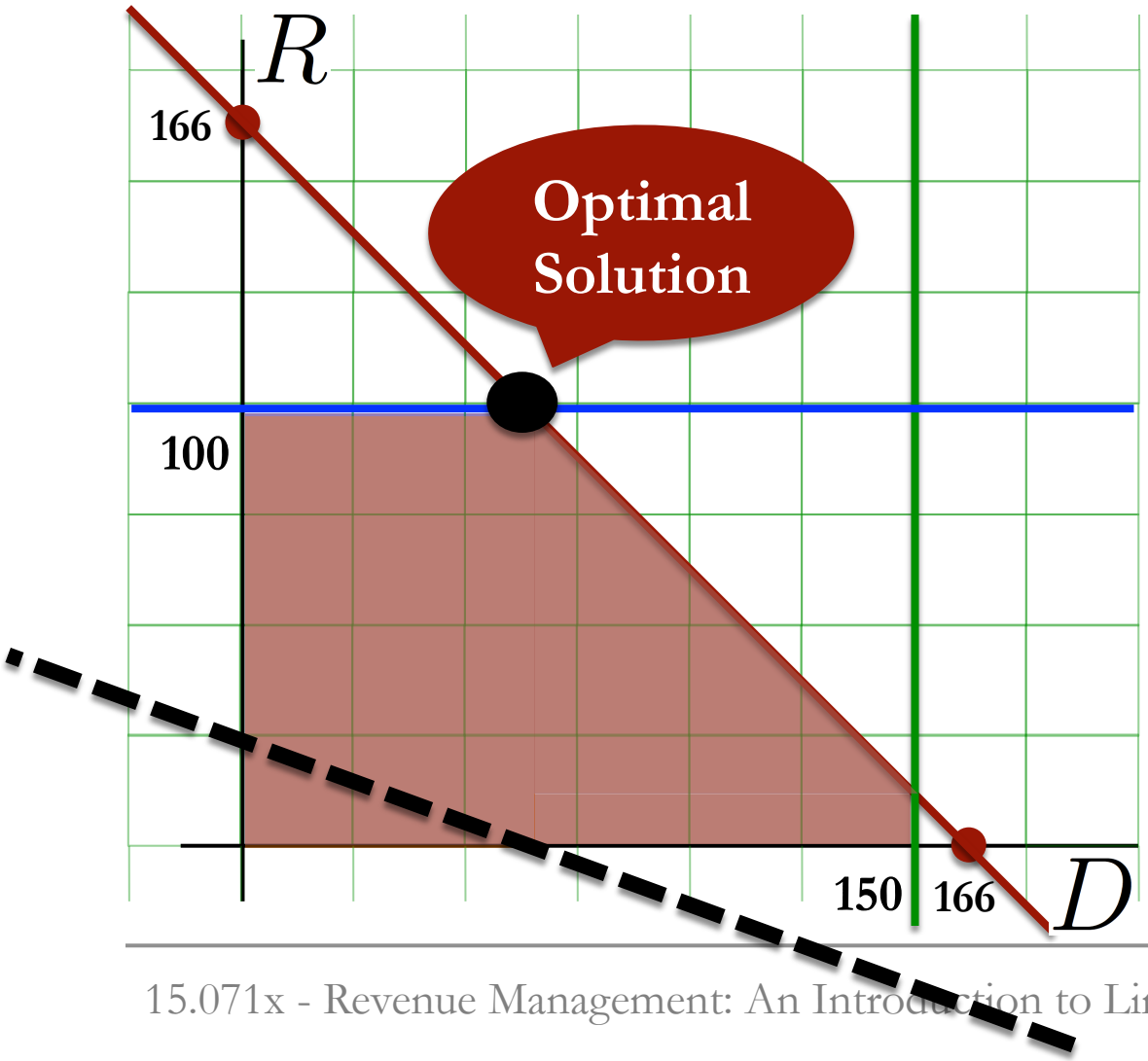


Possible Solutions



- Revenue $617R + 238D$
- How many seats to sell of each type to achieve a revenue of
  - $20,000?
  - $40,000?
  - $60,000?

Best Solution



- Revenue $617R + 238D$
- How many seats to sell of each type to achieve the highest revenue possible?

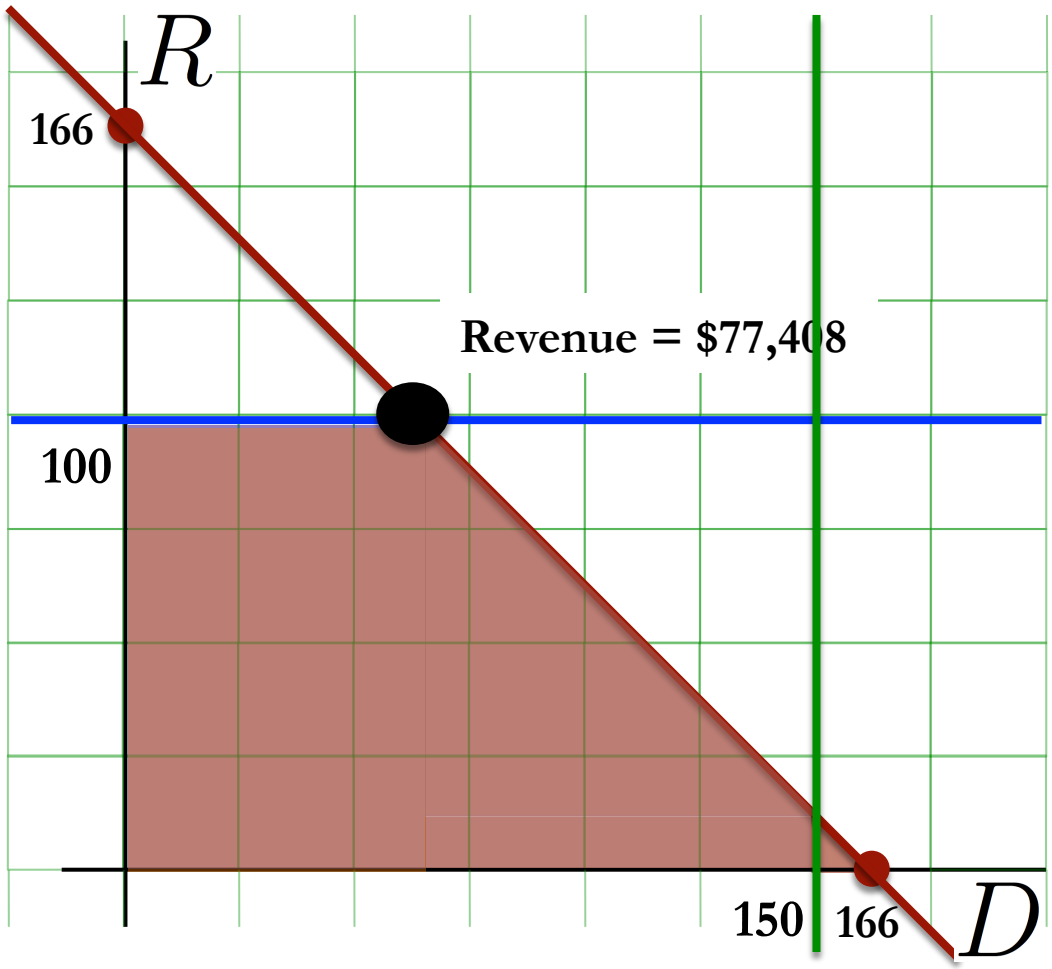
Marketing Decisions

- Management is trying to figure out whether it would be beneficial to invest in marketing its fares

- AA forecasts that its marketing effort is likely to attract one more unit of demand per **$200 spent**

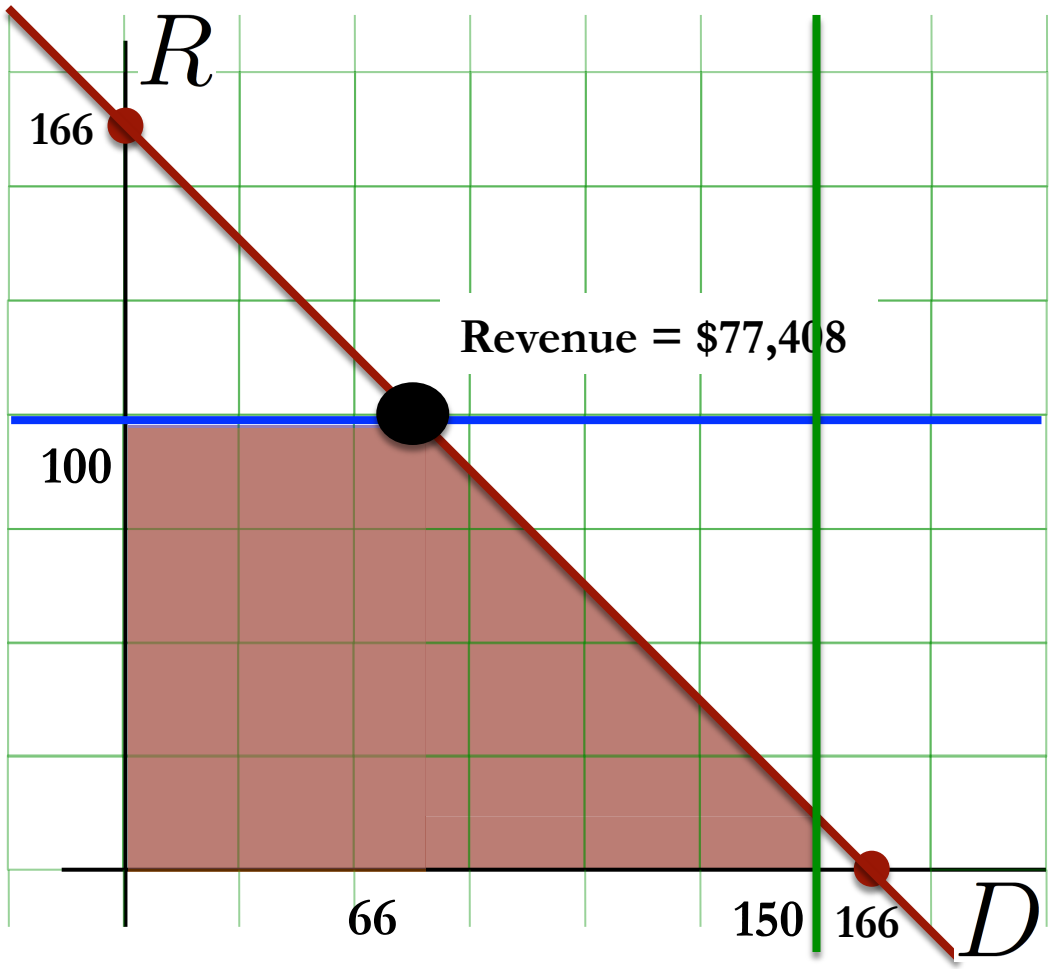
	Marketing Cost/unit	Marginal Revenue
Discount Fare	$200	
Regular Fare	$200	

Marketing Discount Fares



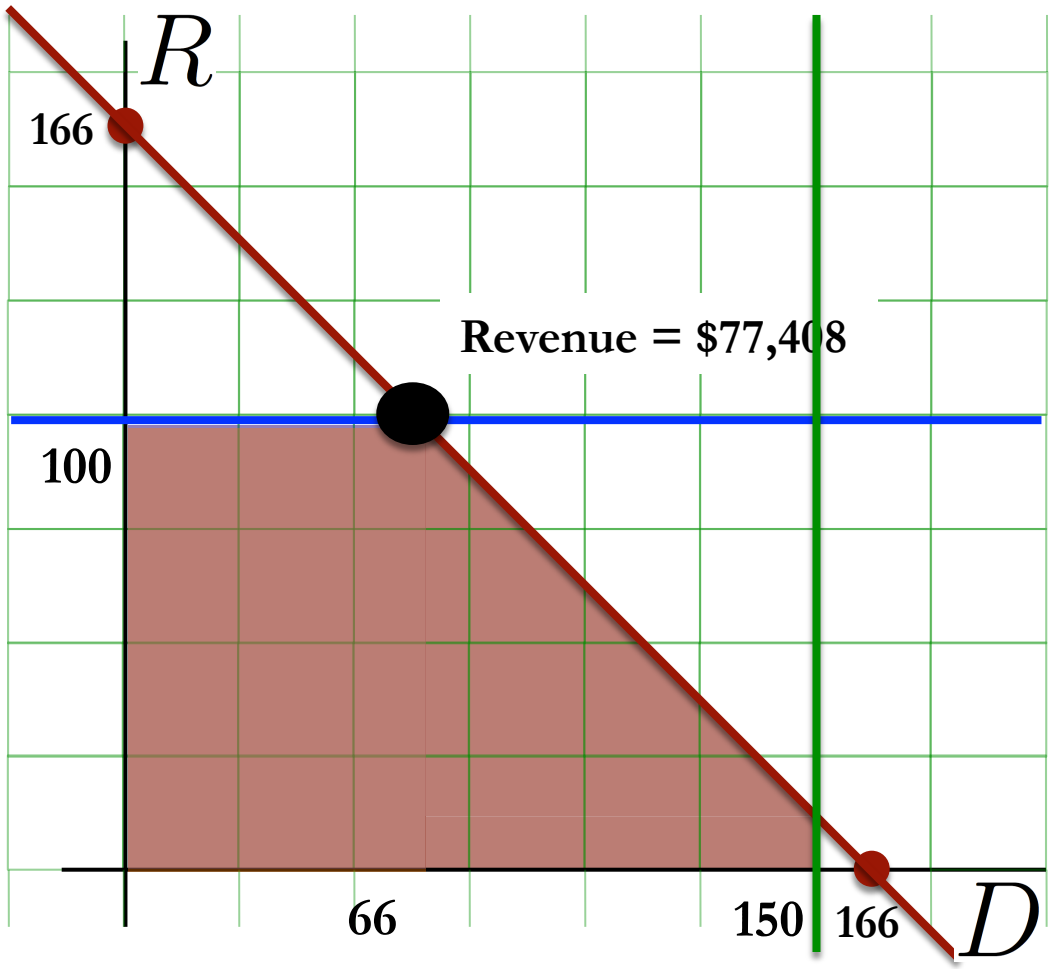
- What if AA increases its marketing budget for discount fares
- Higher demand for discount class
  - 150
  - 175
  - 200

Marketing Discount Fares



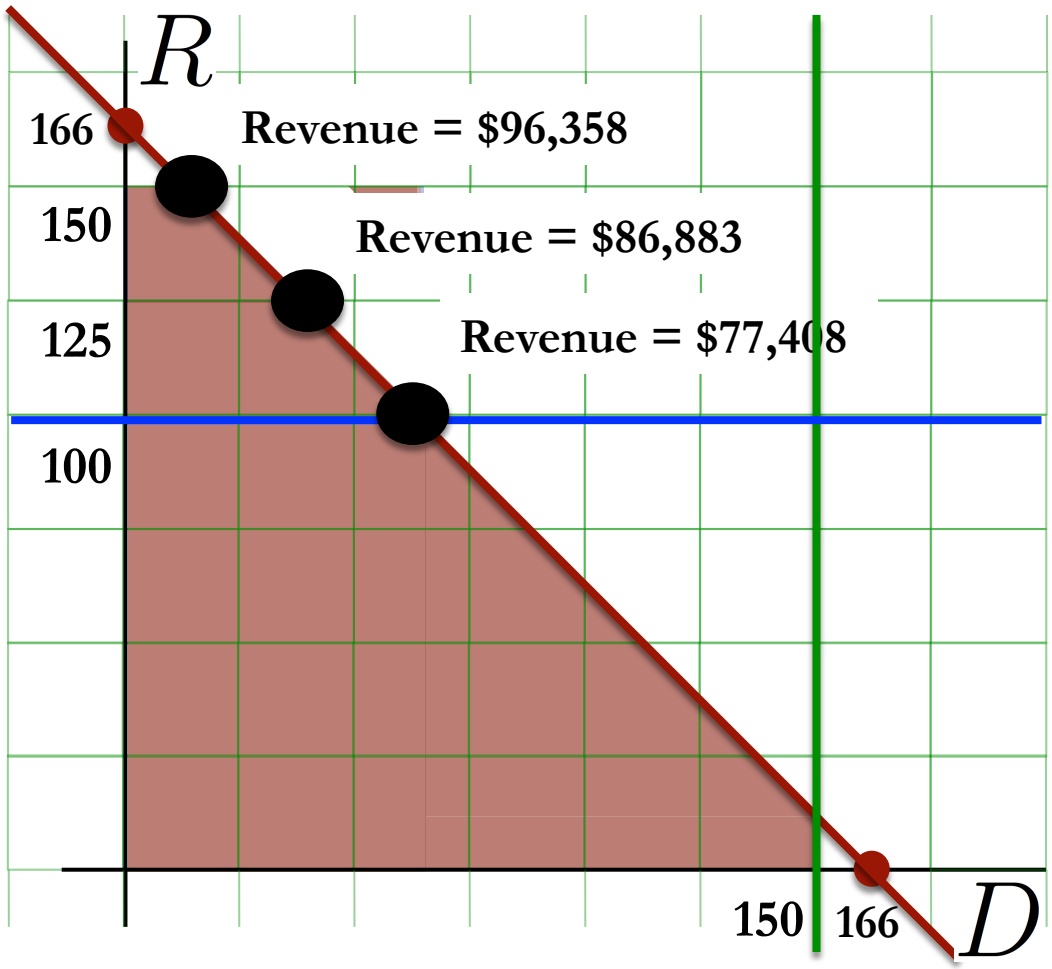
- What if AA decreases its budget to market discount fares?
- Lower demand for discount fare without affecting revenue

Marketing Discount Fares



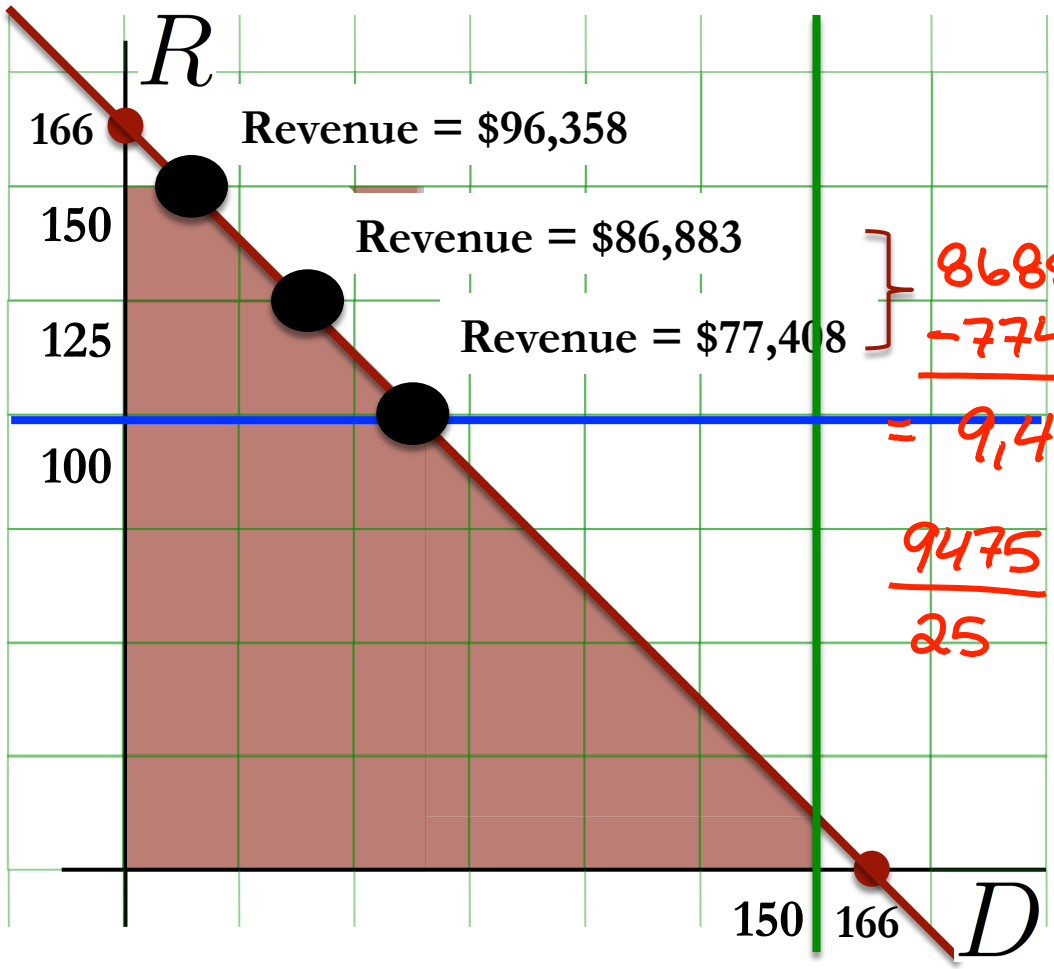
- "Shadow Price"
  - Marginal revenue of increasing discount demand by 1 unit
  - **ZERO** for discount demand greater than 66

Marketing Regular Fares



- AA is considering increasing its budget to market regular fares
- Higher demand for regular class
  - 100
  - 125
  - 150

Marketing Regular Fares



- "Shadow Price"
  - Marginal revenue for unit increase in demand of regular seats
  - **$379** for regular demand between 0 and 166

Marketing Decisions

- Management is trying to figure out whether it would be beneficial to invest in marketing its fares

- AA forecasts that its marketing effort is likely to attract one more unit of demand per **$200 spent**

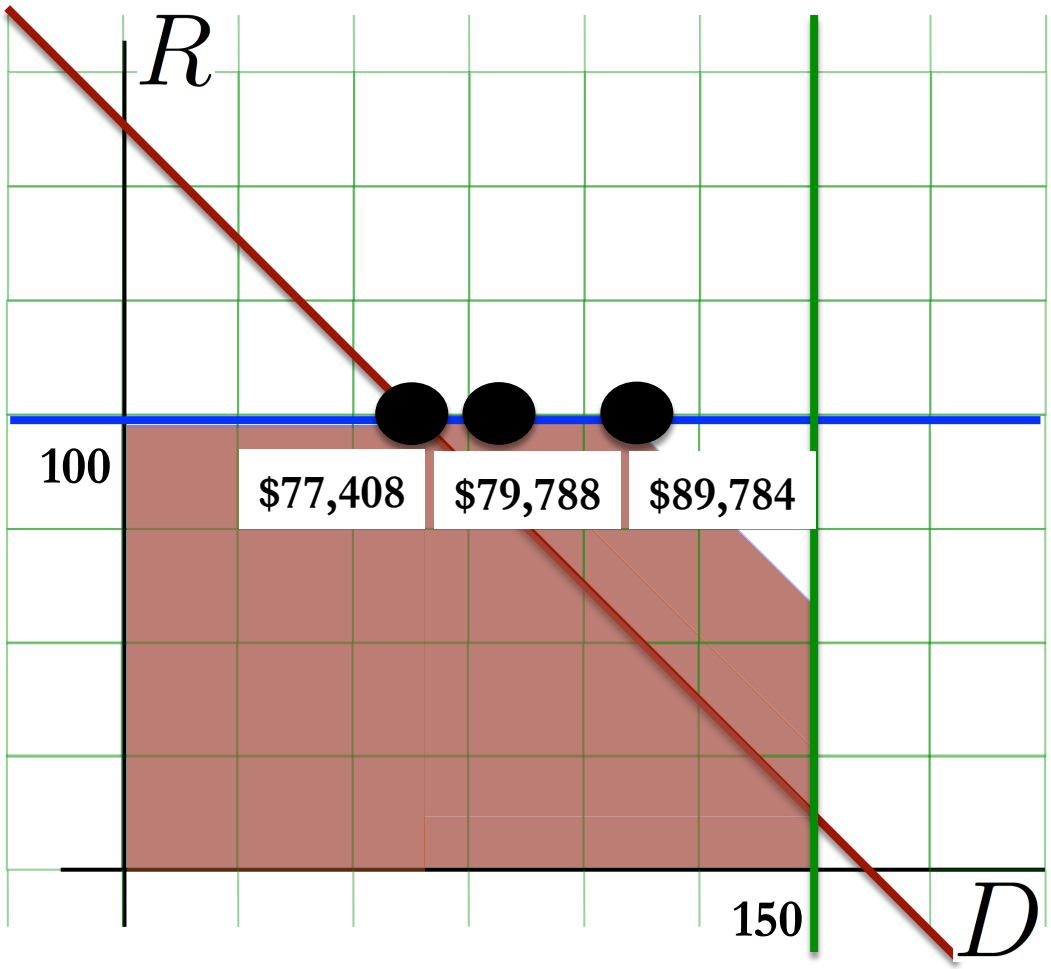
	Marketing Cost/unit	Marginal Revenue
Discount Fare	$200	0
Regular Fare	$200	$379

Capacity Allocation

- Management is trying to figure out whether it would be beneficial to allocate a bigger aircraft for the 6 hour JFK-LAX leg

	Cost/hr	Total Cost	Seats	Revenue
Original Aircraft	$12,067	$72,402	166	$77,408
Boeing 757-200	$12,765	$76,590	176	
Boeing 767-300	$14,557	$87,342	218	

Aircraft Capacity



- AA is considering increasing its aircraft capacity
  - 166
  - 176
  - 218

Capacity Allocation

- Management is trying to figure out whether it would be beneficial to allocate a bigger aircraft for the 6 hour JFK-LAX leg

	Total Cost	Revenue	Profit
Original Aircraft	$72,402	$77,408	$5,006
Boeing 757-200	$76,590	$79,788	$3,198
Boeing 767-300	$87,342	$89,784	$2,442

Connecting Flights

Step 1. Decisions

		Price	Demand	Seats to Sell	Flight Leg (capacity 166 on each)
JFK - LAX	Regular	428	80	?	1 & 2
	Discount	190	120	?	1 & 2
JFK - DFW	Regular	642	75	?	1
	Discount	224	100	?	1
DFW - LAX	Regular	512	60	?	2
	Discount	190	110	?	2

- Number of regular seats to sell

$$R_{\text{JFK-LAX}}, R_{\text{JFK-DFW}}, R_{\text{DFW-LAX}}$$

- Number of discount seats to sell

$$D_{\text{JFK-LAX}}, D_{\text{JFK-DFW}}, D_{\text{DFW-LAX}}$$

Step 2. Objective

		Price	Demand	Seats to Sell	Flight Leg (capacity 166 on each)
JFK - LAX	Regular	428	80	?	1 & 2
	Discount	190	120	?	1 & 2
JFK - DFW	Regular	642	75	?	1
	Discount	224	100	?	1
DFW - LAX	Regular	512	60	?	2
	Discount	190	110	?	2

- Maximize total revenue

$$\begin{aligned} & 428R_{\text{JFK-LAX}} + 190D_{\text{JFK-LAX}} \\ & + 642R_{\text{JFK-DFW}} + 224D_{\text{JFK-DFW}} \\ & + 512R_{\text{DFW-LAX}} + 190D_{\text{DFW-LAX}} \end{aligned}$$

Step 3. Constraints

		Price	Demand	Seats to Sell	Flight Leg (capacity 166 on each)
JFK - LAX	Regular	428	80	?	1 & 2
	Discount	190	120	?	1 & 2
JFK - DFW	Regular	642	75	?	1
	Discount	224	100	?	1
DFW - LAX	Regular	512	60	?	2
	Discount	190	110	?	2

- AA cannot sell more seats that the aircraft capacity
  - First leg - JFK-DFW

$$R_{\text{JFK-LAX}} + D_{\text{JFK-LAX}} + R_{\text{JFK-DFW}} + D_{\text{JFK-DFW}} \leq 166$$

  - Second leg - DFW-LAX

$$R_{\text{JFK-LAX}} + D_{\text{JFK-LAX}} + R_{\text{DFW-LAX}} + D_{\text{DFW-LAX}} \leq 166$$

Step 3. Constraints

		Price	Demand	Seats to Sell	Flight Leg (capacity 166 on each)
JFK - LAX	Regular	428	80	?	1 & 2
	Discount	190	120	?	1 & 2
JFK - DFW	Regular	642	75	?	1
	Discount	224	100	?	1
DFW - LAX	Regular	512	60	?	2
	Discount	190	110	?	2

- AA cannot sell more seats than the demand

$$\begin{cases} R_{\text{JFK-LAX}} \leq 80 & D_{\text{JFK-LAX}} \leq 120 \\ R_{\text{JFK-DFW}} \leq 75 & D_{\text{JFK-DFW}} \leq 100 \\ R_{\text{DFW-LAX}} \leq 60 & D_{\text{DFW-LAX}} \leq 110 \end{cases}$$

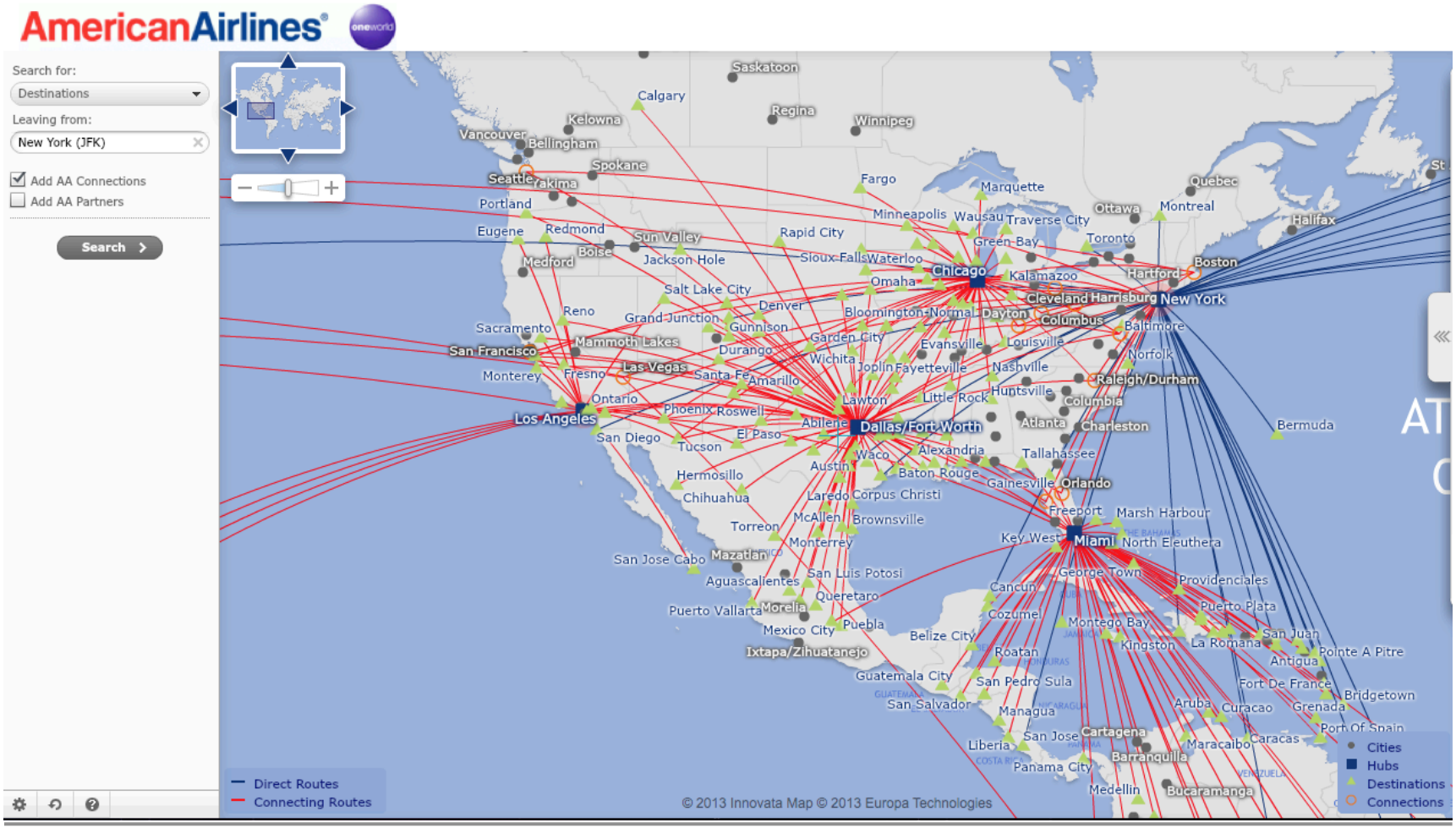
Step 4. Non-Negativity

		Price	Demand	Seats to Sell	Flight Leg (capacity 166 on each)
JFK - LAX	Regular	428	80	?	1 & 2
	Discount	190	120	?	1 & 2
JFK - DFW	Regular	642	75	?	1
	Discount	224	100	?	1
DFW - LAX	Regular	512	60	?	2
	Discount	190	110	?	2

- AA cannot sell a negative number of seats

$$\begin{cases} R_{\text{JFK-LAX}} \geq 0 & D_{\text{JFK-LAX}} \geq 0 \\ R_{\text{JFK-DFW}} \geq 0 & D_{\text{JFK-DFW}} \geq 0 \\ R_{\text{DFW-LAX}} \geq 0 & D_{\text{DFW-LAX}} \geq 0 \end{cases}$$

Complex Network



Multiple Fare Classes

Fare	Domestic Upg.	International Upg.	EQP	EQM	Mileage
A	First Class	First Class	1.5	1.0	150%
B	Yes	Yes	1.5	1.0	100%
C	NA	Business Upgrade	N/A	N/A	N/A
D	NA	Business Fare	1.5	1.0	125%
E	No	No	N/A	N/A	N/A
F	First Class Fare	First Class	1.5	1.0	150%
G	Government	Government	.5	1.0	100%
H	Yes*	Waitlist only	1.0	1.0	100%
I	NA	Business Class Fare	1.5	1.0	125%
J	NA	Business Class Fare	1.5	1.0	125%
K	Yes	No	1.0	1.0	100%
L	Yes	No	1.0	1.0	100%
M	Yes	No	1.0	1.0	100%

Fare	Domestic Upg.	International Upg.	EQP	EQM	Mileage
N	Yes	No	.5	1.0	100%
O	Yes*	No	.5	1.0	100%
P	First Class Fare	First Class Fare	1.5	1.0	150%
Q	Yes	No	.5	1.0	100%
R	NA	Business Class Upgrade or waitlist	N/A	N/A	N/A
S	Yes*	No	.5	1.0	100%
T	Coach Award	No	N/A	N/A	N/A
U	NA	Business Class Award	N/A	N/A	N/A
V	Yes*	No	1.0	1.0	100%
W	Yes*	No	1.0	1.0	100%
X	First Class Upgrade	Business Class Upgrade	N/A	N/A	N/A
Y	Yes	Yes	1.5	1.0	100%
Z	First Class Award	NA	N/A	N/A	N/A

EQP: Elite-Qualifying Points / EQM: Elite-Qualifying Miles

The Competitive Strategy of AA

- PEOPLExpress could not compete with AA's Ultimate Super Savers fares

"We were a vibrant, profitable company from 1981 to 1985, and then we tipped right over into **losing 50 million a month**."

"We had been profitable from the day we started until American came at us with Ultimate Super Savers."

Donald Burr, CEO of PEOPLExpress (1985)

The Competitive Strategy of AA

- Selling the right seats to the right customers at the right prices

"**Revenue management** is the single most important technical development in transportation management since we entered the era of airline deregulation."

"We estimate that revenue management has generated **$1.4 billion in incremental revenue** in the last three years."

Robert Crandall, former CEO of AA (~1985)

The Edge of Revenue Management

- Sabre Holdings
  - Built revenue management system for AA
  - As of November 2012, ranked 133 among America's largest private companies with $3.15 billion in sales
  - 400 airlines, 90,000 hotels, 30 car-rental companies
- Today, companies prosper from revenue management
  - Delta airlines increased annual revenue by $300 million
  - Marriott hotels increased annual revenue by $100 million