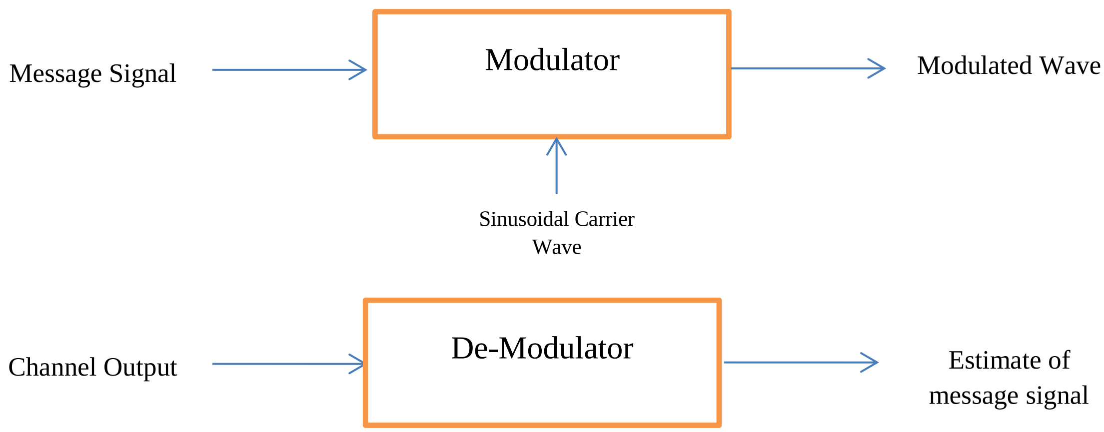
# Lecture 9

## Introduction

The purpose of communication system is to transmit information bearing signals through a communication signal separating the transmitter from the receiver. Information bearing signals also called baseband signals designate the band of frequencies representing the original signal as delivered by a source of information. The proper use of the communication channel requires a shift of the range of baseband frequencies into other frequency ranges suitable for transmission and corresponding shift back to original frequency range after reception. A shift of the range of frequencies in a signal is accomplished by using Modulation, which is defined as the process by which some characteristics of a carrier is varied in accordance with a modulating wave.
Define terms: Modulating Wave, Modulated Wave, De-modulation




Above Fig shows the process of Modulator and Demodulator. The signal received from the transmitter, the receiver input includes channel noise. The degradation in receiver performance due to channel noise is determined by the type of modulation used.

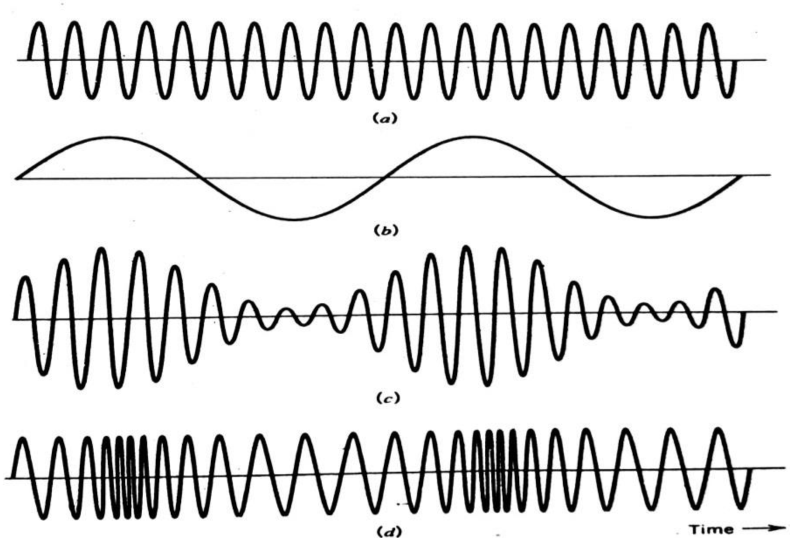



Fig 2.1 (a) Carrier (b) Sinusoidal Modulating Signal (c) Amplitude Modulated signal (d) Frequency Modulated signal

## Need for Modulation / Frequency Translation

Frequency Translation serves the following purposes

1. **Frequency Multiplexing**- Transmission of multiple signals along a single transmission channel demands signals recovery in a distinguishable manner. Such multiple transmission i.e. multiplexing may be achieved by translating each one of the original signals to a different frequency range. If one signal is translated to a frequency range $f_1'$ and $f_2'$ and the second signal is translated to a frequency range $f_1''$ and $f_2''$ and so on, . If these new frequency ranges do not overlap, then the signal may be separated at the receiver end by appropriate bandpass filters.

   *Give example via figure*

2. **Frequency Translation**
   With the help of this method, it is easy to adjust the low frequency signal into higher frequency range suitable to be transmitted over the allocated frequency range for a particular application. Like in AM, a 1 KHz signal is translated to 1 MHz using a carrier of high frequency.

3. **Practicability of Antennas**
   When free space is the communication channel, antenna radiate and receive the signal. It turns out that antennas operate effectively only when their dimensions are of the order of magnitude of the wavelength of the signal being transmitted. A signal of freq1KHz (audio tone length may be reduced to the point of practicability by translating the audio tone to a higher frequency
4. **Narrowbanding**
   Suppose a signal is of the range 50 to $10^4$ Hz. Ratio of highest to lowest audio frequency is 200. Therefore an antenna suitable for use at one end of the range would be too short and at other end would be too long. Suppose the signal is translated between $(10^6+50)$ to $(10^6+10^4)$. The ratio of highest to lowest frequency is now 1.01. This is called conversion of wideband signal to narrowband signal which may be well conveniently presented. Here wide/narrow band means fractional change in frequency from one band edge to the other
5. **Common Processing**
   It may happen that we may have to process, a number of signals similar in general character but occupying different spectral ranges. It will be necessary, as we go from signal to signal, to adjust the frequency range of our processing apparatus to correspond to the frequency range of the signal to be processed. If the processing apparatus is rather elaborate, it may well be wiser to leave the processing range of each signal in turn to correspond to this fixed frequency range.

**Method of Frequency Translation**

A method of Frequency Translation →

A signal may be translated to a new spectral range by multiplying the signal with an auxiliary signal.

Let the signal be

$$V_m(t) = A_m \cos \omega_m t = A_m \cos 2\pi f_m t$$

$$= \frac{A_m}{2}\left(e^{j\omega_m t} + e^{-j\omega_m t}\right)$$

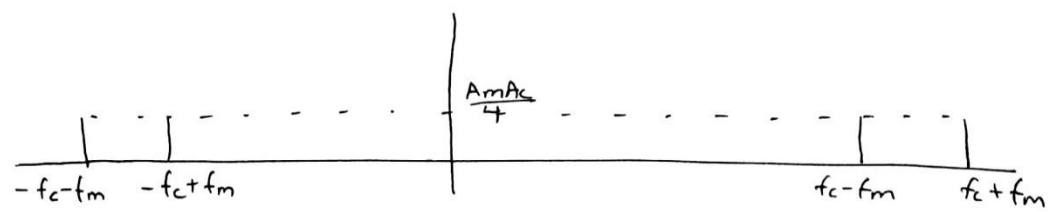
$$= \frac{A_m}{2}\left(e^{j2\pi f_m t} + e^{-j2\pi f_m t}\right)$$

Consider now the result of multiplication of $V_m(t)$ with an auxiliary signal

$$V_c(t) = A_c \cos \omega_c t = \frac{A_c}{2}\left(e^{j2\pi f_c t} + e^{-j2\pi f_c t}\right)$$

in which $A_c$ is the constant amplitude and $f_c$ is the frequency.

$$V_m(t)\, V_c(t) = \frac{A_m A_c}{4}\left[e^{j(\omega_c+\omega_m)t} + e^{-j(\omega_c+\omega_m)t} + e^{j(\omega_c-\omega_m)t} + e^{-j(\omega_c-\omega_m)t}\right]$$




A generalization of fig ① is shown in ② where a signal is chosen which consists of a superposition of four sinusoidal signals.

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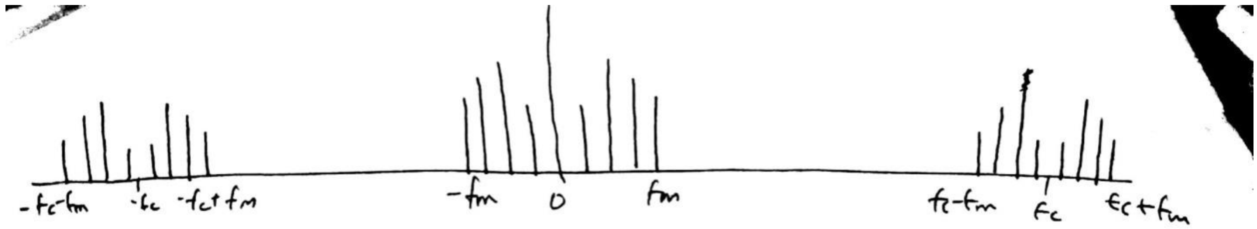
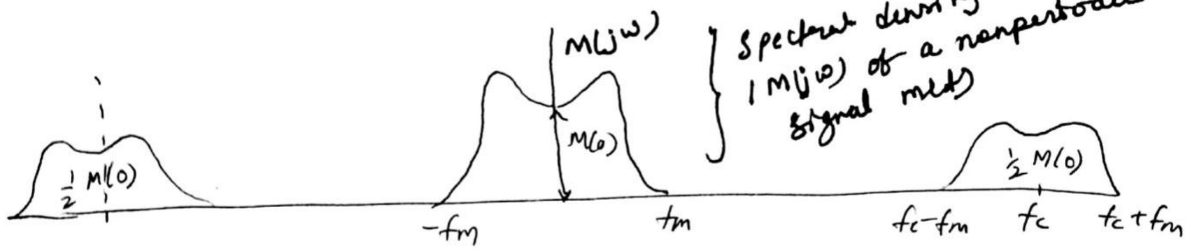



Spectral density of $m(t) \cos 2\pi f_c t$

The operation of multiplying a signal with an auxiliary sinusoidal signal is called <u>mixing or hetrodyning</u>. In the translated signal, the part of the signal which consists of spectral component above the auxiliary signal, in the range $f_c$ to $f_c+f_m$ is called upper-sideband signal. The part of the signal which consists of the spectral components below the auxiliary signal, in the range $f_c-f_m$ to $f_c$ is called lower-sideband signal.

# Lecture 10

### Amplitude Modulation

A frequency translated signal from which the baseband signal is easily recoverable is generated by adding to the product of baseband and carrier, the carrier signal itself and is named as Amplitude Modulation. The presence of carrier gives it another name Double Side band with carrier (DSB-C). Thus AM is defined as a process in which the amplitude of the carrier wave is varied about a mean value, linearly with the baseband signal $m(t)$.

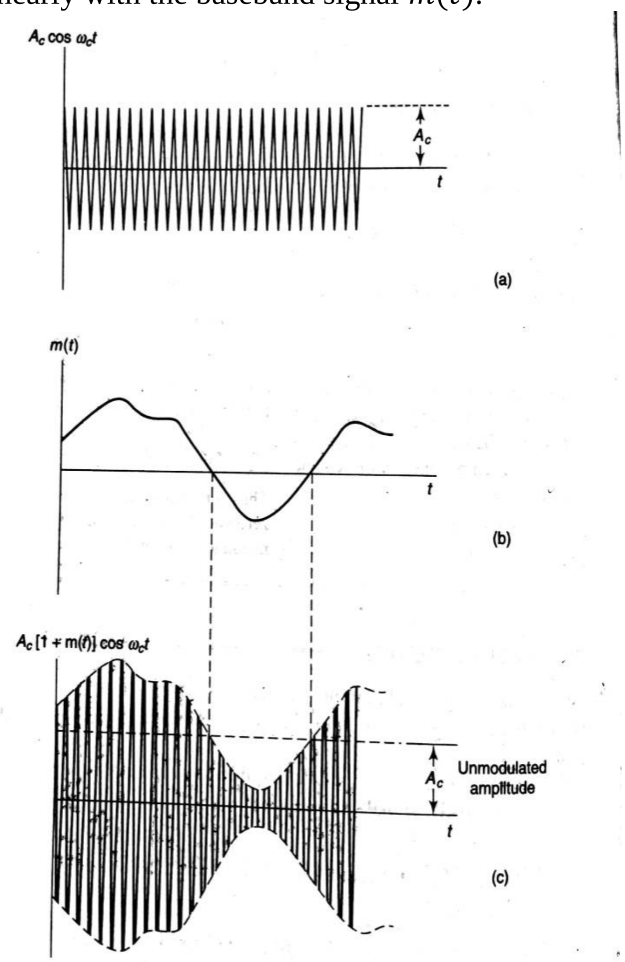



**Fig 2.2 AM process**

Let the baseband signal be

$$m(\mathrm{t}) = A_m \cos(w_m t)$$

And the carrier signal be

$$c(\mathrm{t}) = A_c \cos(w_c t)$$

The amplitude of the amplitude modulated waveis given by

$$A = A_c + A_m \cos(w_m t) = A_c + mA_c \cos(w_m t)$$
$$= A_c(1 + m\cos(w_m t))$$

Where $m = \frac{A_m}{A_c}$ called modulation index

The instantaneous voltage of the resulting amplitude modulated wave is

$$v(t) = A\cos(w_c t) = A_c[1 + m\cos(w_m t)]\cos(w_c t)$$

$$=[A_c + m(t)]\cos(w_c t) \qquad \text{(Ref:T\&S)}$$

$$v(t) = A_c\cos(w_c t) + mA_c\cos(w_m t)\cos(w_c t)$$

$$= A_c\cos(w_c t) + mA_c[\cos(w_c - w_m)t) + \cos(w_c + w_m)t)]$$

Thus, the equation of AM contains carrier and two sidebands of frequency $(f_c - f_m)$ (LSB) and $(f_c + f_m)$ (USB). Thus the Bandwidth requirement for AM is twice the frequency of the modulating signal. Thus in AM broadcasting, the bandwidth required is twice the highest modulating frequency

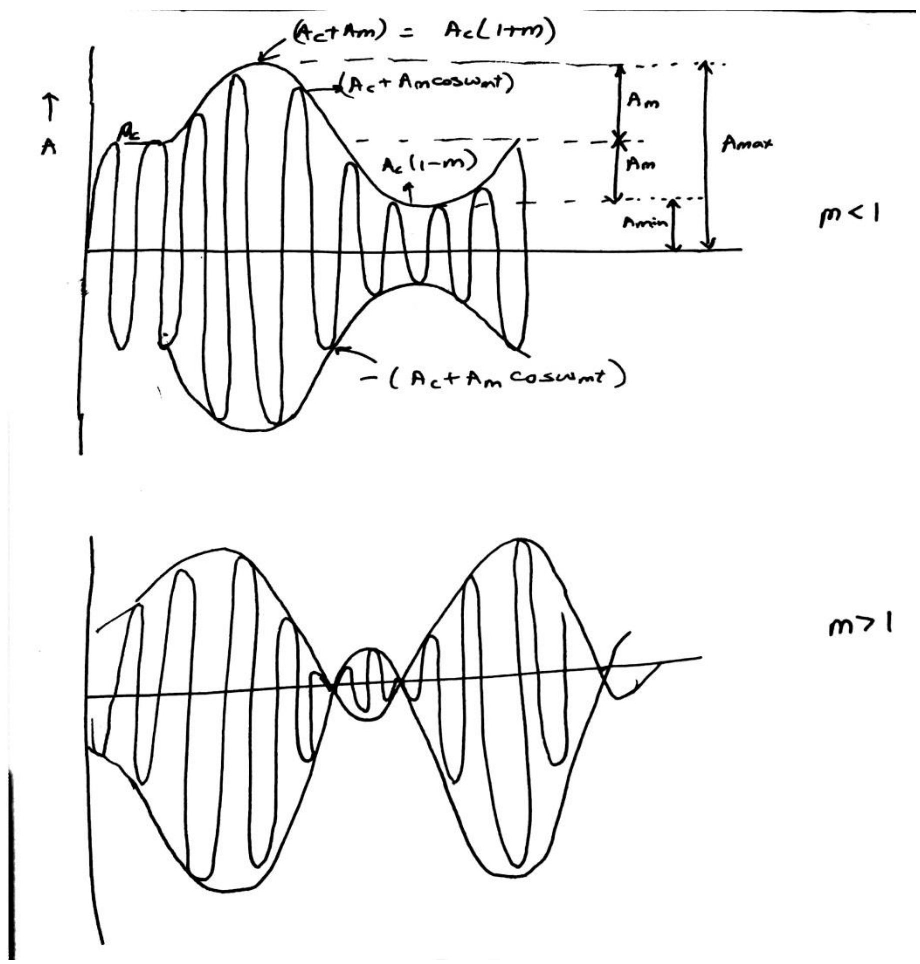



**Fig 2.3. Amplitude Modulated wave (a) m<1 (b) m>1**

For m>1, the carrier wave becomes overmodulated, resulting in carrier phase reversals whenever the factor $m$ crosses zero. In this case, amplitude of a baseband signal exceeds maximum carrier amplitude i.e.

$|m(t)| > A_c$ . Here m > 1

**Modulation Index**

$$A_m = \frac{A_{max} - A_{min}}{2}$$

and

$$A_c = A_{max} - A_{m=}A_{max} - \frac{A_{max} - A_{min}}{2} = \frac{A_{max} + A_{min}}{2}$$

From the above 2 equations,

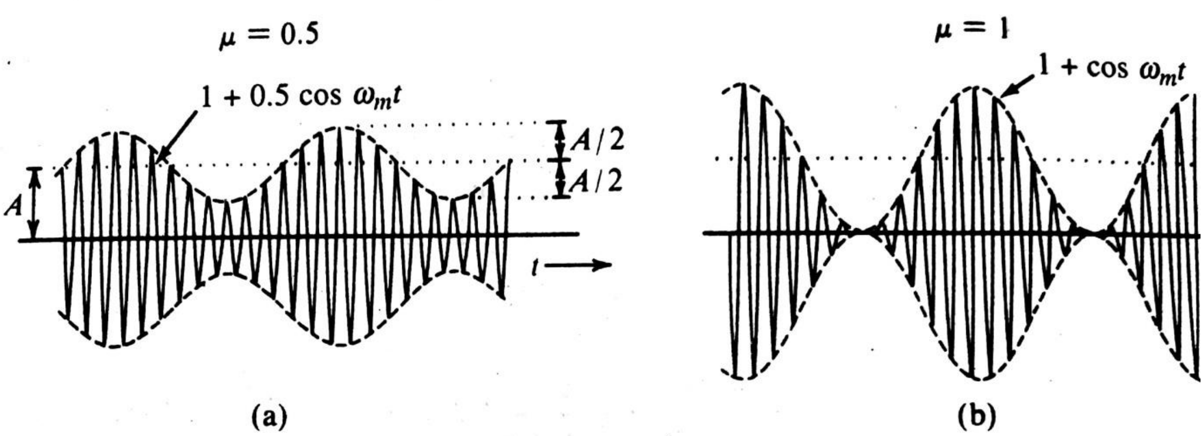
$$m = \frac{A_m}{A_c} = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

**Example 1-** The tuned circuit of the oscillator is an simple AM transmitter employs a 50 microhenry (50 $\mu H$) coil and a 1 nanofarad (nF) capacitor. If the oscillator output is modulated by audio frequencies upto 10 kHz, what is the frequency range occupied by the sidebands
**Sol:**

$$fc = \frac{1}{2\pi\sqrt{LC}} = 712\ kHz$$

Since the highest modulating frequency is 10 $kHz$, the frequency range occupied by the sidebands are at 722 $kHz$ and 702 $kHz$.

**Example 2-**, For AM wave for m=0.5 and m=1, sketch the AM Wave



## Power Content in an AM Wave

... of the AM wave may be drawn.

### 1.3 Power Relations in the AM Wave

It has been shown that the carrier component of the modulated wave has the same amplitude as the unmodulated carrier. That is, the amplitude of the carrier is unchanged; energy is either added or subtracted. The modulated wave contains extra energy in the two sideband components. Therefore, the modulated wave contains more power than the carrier had before modulation took place. Since the amplitude of the sidebands depends on the modulation index $V_m/V_c$, it is anticipated that the total power in the modulated wave will depend on the modulation index also. This relation may now be derived.

The total power in the modulated wave will be

$$P_t = \frac{V_{carr}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R} \text{ (rms)} \tag{3-11}$$

where all three voltages are (rms) values ($\sqrt{2}$ converted to peak), and $R$ is the resistance, (e.g., antenna resistance), in which the power is dissipated. The first term of Equation (3-11) is the unmodulated carrier power and is given by

$$P_c = \frac{V_{carr}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R}$$

$$= \frac{V_c^2}{2R} \tag{3-12}$$

Similarly,

$$P_{\text{LSB}} = P_{\text{USB}} = \frac{V_{\text{SB}}^2}{R} = \left(\frac{mV_c/2}{\sqrt{2}}\right)^2 \div R = \frac{m^2V_c^2}{8R}$$

$$= \frac{m^2}{4}\frac{V_c^2}{2R} \tag{3-13}$$

Substituting Equations (3-12) and (3-13) into (3-11), we have

$$P_t = \frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R} + \frac{m^2}{4}\frac{V_c^2}{2R} = P_c + \frac{m^2}{4}P_c + \frac{m^2}{4}P_c$$

$$\boxed{\frac{P_t}{P_c} = 1 + \frac{m^2}{2}} \tag{3-14}$$

Equation (3-14) relates the total power in the amplitude-modulated wave to the unmodulated carrier power. This is the equation which must be used to determine, among other quantities, the modulation index in instances not covered by Equation (3-10) of the preceding section. The methods of doing this, as well as solutions to other problems, will be shown in exercises to follow.

It is interesting to note from Equation (3-14) that the maximum power in the AM wave is $P_t = 1.5P_c$ when $m = 1$. This is important, because it is the maximum power that relevant amplifiers must be capable of handling without distortion.

**EXAMPLE 3-2** A 400-watt (400-W) carrier is modulated to a depth of 75 percent. Calculate the total power in the modulated wave.

**SOLUTION**

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) = 400\left(1 + \frac{0.75^2}{2}\right) = 400 \times 1.281$$

$$= 512.5 \text{ W}$$

**EXAMPLE 3-3** A broadcast radio transmitter radiates 10 kilowatts (10 kW) when the modulation percentage is 60. How much of this is carrier power?

**SOLUTION**

$$P_c = \frac{P_t}{1 + m^2/2} = \frac{10}{1 + 0.6^2/2} = \frac{10}{1.18} = 8.47 \text{ kW}$$

**Current calculations** The situation which very often arises in AM is that the modulated and unmodulated currents are easily measurable, and it is then necessary to calculate the modulation index from them. This occurs when the antenna current of the transmitter is metered, and the problem may be resolved as follows. Let $I_c$ be the unmodulated current and $I_t$ the total, or modulated, current of an AM transmitter, both being rms values. If $R$ is the resistance in which these currents flow, then

$$\frac{P_t}{P_c} = \frac{I_t^2 R}{I_c^2 R} = \left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{m^2}{2}$$

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ulation ⇒

$$\boxed{\frac{I_t}{I_c} = \sqrt{1 + \frac{m^2}{2}}} \quad \text{or} \quad I_t = I_c\sqrt{1 + \frac{m^2}{2}} \tag{3-15}$$

**EXAMPLE 3-4** The antenna current of an AM transmitter is 8 amperes (8 A) when only the carrier is sent, but it increases to 8.93 A when the carrier is modulated by a single sine wave. Find the percentage modulation. Determine the antenna current when the percent of modulation changes to 0.8.

**SOLUTION**

$$\left(\frac{I_t}{I_c}\right)^2 = 1 + \frac{m^2}{2}$$

$$\frac{m^2}{2} = \left(\frac{I_t}{I_c}\right)^2 - 1$$

$$m = \sqrt{2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]} \tag{3-16}$$

Here

$$m = \sqrt{2\left[\left(\frac{8.93}{8}\right)^2 - 1\right]} = \sqrt{2[(1.116)^2 - 1]}$$

$$= \sqrt{2(1.246 - 1)} = \sqrt{0.492} = 0.701 = 70.1\%$$

For the second part we have

$$I_t = I_c\sqrt{1 + \frac{m^2}{2}} = 8\sqrt{1 + \frac{0.8^2}{2}} = 8\sqrt{1 + \frac{0.64}{2}}$$

$$= 8\sqrt{1.32} = 8 \times 1.149 = 9.19\text{A}$$

Although Equation (3-16) is merely (3-15) in reverse, it will be found useful in other problems.

**Modulation by several sine waves** In practice, modulation of a carrier by several sine waves simultaneously is the rule rather than the exception. Accordingly, a way has to be found to calculate the resulting power conditions. The procedure consists of calculating the total modulation index and then substituting it into Equation (3-14), from which the total power may be calculated as before. There are two methods of calculating the total modulation index.

1. Let $V_1$, $V_2$, $V_3$, etc., be the simultaneous modulation voltages. Then the total modulating voltage $V_t$ will be equal to the square root of the sum of the squares of the individual voltages; that is,

   $$\boxed{V_t = \sqrt{V_1^2 + V_2^2 + V_3^2 + \cdots}}$$

   Dividing both sides by $V_c$, we get

$$\frac{V_t}{V_c} = \frac{\sqrt{V_1^2 + V_2^2 + V_3^2 + \cdots}}{V_c}$$

$$= \sqrt{\frac{V_1^2}{V_c^2} + \frac{V_2^2}{V_c^2} + \frac{V_3^2}{V_c^2} + \cdots}$$

that is,

$$m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \cdots} \tag{3-17}$$

2. Equation (3-14) may be rewritten to emphasize that the total power in an AM wave consists of carrier power and sideband power. This yields

$$P_t = P_c\left(1 + \frac{m^2}{2}\right) = P_c + \frac{P_c m^2}{2} = P_c + P_{SB}$$

where $P_{SB}$ is the total sideband power and is given by

$$P_{SB} = \frac{P_c m^2}{2} \tag{3-18}$$

If several sine waves simultaneously modulate the carrier, the carrier power will be unaffected, but the total sideband power will now be the sum of the individual sideband powers. We have

$$P_{SB_T} = P_{SB_1} + P_{SB_2} + P_{SB_3} + \cdots$$

Substitution gives

$$\frac{P_c m_t^2}{2} = \frac{P_c m_1^2}{2} + \frac{P_c m_2^2}{2} + \frac{P_c m_3^2}{2} + \cdots$$

$$m_t^2 = m_1^2 + m_2^2 + m_3^2 + \cdots$$

If the square root of both sides is now taken, Equation (3-17) will once again be the result.

It is seen that the two approaches both yield the same result. To calculate the total modulation index, *take the square root of the sum of the squares of the individual modulation indices*. Note also that this total modulation index must still not exceed unity, or distortion will result as with overmodulation by a single sine wave. Whether modulation is by one or many sine waves, the output of the modulated amplifier will be zero during part of the negative modulating voltage peak if overmodulation is taking place. This point is discussed further in Chapter 6, in conjunction with distortion in AM demodulators.

**EXAMPLE 3-5** A certain transmitter radiates 9 kW with the carrier unmodulated, and 10.125 kW when the carrier is sinusoidally modulated. Calculate the modulation index, percent of modulation. If another sine wave, corresponding to 40 percent modulation, is transmitted simultaneously, determine the total radiated power.

**SOLUTION**

$$\frac{m^2}{2} = \frac{P_t}{P_c} - 1 = \frac{10.125}{9} - 1 = 1.125 - 1 = 0.125$$

$$m^2 = 0.125 \times 2 = 0.250$$

$$m = \sqrt{0.25} = 0.50$$

For the second part, the total modulation index will be

$$m_t = \sqrt{m_1^2 + m_2^2} = \sqrt{0.5^2 + 0.4^2} = \sqrt{0.25 + 0.16} = \sqrt{0.41} = 0.64$$

$$P_t = P_c\left(1 + \frac{m_t^2}{2}\right) = 9\left(1 + \frac{0.64^2}{2}\right) = 9(1 + 0.205) = 10.84 \text{ kW}$$

**EXAMPLE 3-6** The antenna current of an AM broadcast transmitter, modulated to a depth of 40 percent by an audio sine wave, is 11 A. It increases to 12 A as a result of simultaneous modulation by another audio sine wave. What is the modulation index due to this second wave?

**SOLUTION**

From Equation (3-15) we have

$$I_c = \frac{I_t}{\sqrt{1 + m^2/2}} = \frac{11}{\sqrt{1 + 0.4^2/2}} = \frac{11}{\sqrt{1 + 0.08}} = 10.58 \text{ A}$$

Using Equation (3-16) and bearing in mind that here the modulation index is the total modulation index $m_t$, we obtain

$$m_t = \sqrt{2\left[\left(\frac{I_T}{I_c}\right)^2 - 1\right]} = \sqrt{2\left[\left(\frac{12}{10.58}\right)^2 - 1\right]} = \sqrt{2(1.286 - 1)}$$

$$= \sqrt{2 \times 0.286} = 0.757$$

From Equation (3-17), we obtain

$$m^2 = \sqrt{m_t^2 - m_1^2} = \sqrt{0.757^2 - 0.4^2} = \sqrt{0.573 - 0.16} = \sqrt{0.413}$$

$$= 0.643$$

## 3.7. TRANSMISSION EFFICIENCY

Transmission efficiency of an AM wave is the ratio of the transmitted power which contains the information (i.e., the total sideband power) to the total transmitted power.

or
$$\eta = \frac{P_{LSB} + P_{USB}}{P_t} = \frac{\left[\frac{m^2}{4}P_c + \frac{m^2}{4}P_c\right]}{\left[1 + \frac{m^2}{2}\right]P_c}$$

or
$$\eta = \frac{m^2/2}{1 + \frac{m^2}{2}} = \frac{m^2}{2 + m^2} \quad ...(3.28)$$

The percent transmission efficiency is given by,

$$\eta = \frac{m^2}{2 + m^2} \times 100\% \quad ...(3.29)$$

**EXAMPLE 3.10. The antenna current of an AM transmitter is 8 A if only the carrier is sent, but it increases to 8.93 A if the carrier is modulated by a single sinusoidal wave. Determine the percentage modulation. Also find the antenna current if the percent of modulation changes to 0.8.**

**(MDU, Rohtak, Sem., Exam., 2004-05)**

**Solution :** (i) The current relation for a single-tone amplitude modulation is expressed as

$$I_t = I_c\sqrt{1+\frac{m_a^2}{2}} \quad \text{(i)}$$

where $I_t$ = total or modulated current

$I_c$ = carrier or unmodulated current

$m_a$ = modulation index

Using equation (i), we get

$$\frac{I_t}{I_c} = \sqrt{1+\frac{m_a^2}{2}}$$

or $$\left(\frac{I_t}{I_c}\right)^2 = 1+\frac{m_a^2}{2}$$

or $$\frac{m_a^2}{2} = \left(\frac{I_t}{I_c}\right)^2 - 1$$

or $$m_a^2 = 2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]$$

or $$m_a = \sqrt{2\left[\left(\frac{I_t}{I_c}\right)^2 - 1\right]}$$

Putting all the given values, we have

$$m_a = \sqrt{2\left[\left(\frac{8.93}{8}\right)^2 - 1\right]} = \sqrt{2\left[(1.116)^2 - 1\right]} = \sqrt{2(1.246-1)} = \sqrt{0.492} = 0.701 = 70.1\%$$

(ii) Since $$I_t = I_c\sqrt{1+\frac{m_a^2}{2}}$$

Here, $I_c = 8A$

and $m_a = 0.8$

Therefore, $$I_t = 8\times\sqrt{1+\frac{0.8^2}{2}} = 8\sqrt{1+\frac{0.64}{2}} = 8\sqrt{1.32} = 8 \times 1.149 = 9.19 \text{ A}$$ **Ans.**

**EXAMPLE 3.15. The antenna current of AM broadcast transmitter modulated to the depth of 40% by an audio sine wave is 11 Amp. It increases to 12 Amp. as a result of simultaneous modulation by another audio sine wave. What is the modulation index due to this second wave?**

**Solution:** Given that $m_1 = 0.4$, $I_{t1} = 11$ Amp., $I_{t2} = 12$ Amp.

We know that $$\left[\frac{I_{t1}}{I_c}\right]^2 = 1 + \frac{m_1^2}{2}$$

Simplifying, we get $$I_c = \frac{I_{t1}}{\left[1+\frac{m_1^2}{2}\right]^{1/2}} = \frac{11}{\left[1+\frac{(0.4)^2}{2}\right]^{1/2}} = 10.58 \text{ Amp.} \quad \text{...(i}$$

After modulation with the second signal, we have

$$\left[\frac{I_{t2}}{I_c}\right]^2 = 1 + \frac{m_t^2}{2}$$

where $m_t$ is the total modulation index.

Therefore, we have $$I_{t2}^2 = I_c^2\left[1+\frac{m_t^2}{2}\right]$$

or $$m_t = \left[2\left\{\frac{I_{t2}^2}{I_c^2} - 1\right\}\right]^{1/2}$$

Substituting all the values, we obtain

$$m_t = \left[2\left\{\left(\frac{12}{10.58}\right)^2 - 1\right\}\right]^{1/2} = 0.7553 \quad \text{...(ii}$$

Total modulation index $m_t = \left[m_1^2 + m_2^2\right]^{1/2}$

or $m_2 = [m_t^2 - m_1^2]^{1/2} = [(0.7553)^2 - (0.4)^2]^{1/2}$

or $m_2 = 0.6407$ or 64.07% **Ans.**

## 3.14. LOW AND HIGH LEVEL MODULATION

In this section, let us study the different methods to generate AM. The generating circuits for AM wave are called as amplitude modulator circuits.

The modulator circuits are classified into following two categories:

**1. Low level modulation**

The generation of AM wave takes place at a low power level. The generated AM signal is then amplified using a chain of linear amplifiers. The linear amplifiers are required in order to avoid any waveform distortion.

**2. High level modulation**

In this method, the generation of AM wave takes place at high power levels. The carrier and the modulating signal both are amplified first to an adequate power level and the modulation takes place in the last RF amplifier stage of the transmitter. Highly efficient class C amplifiers are used in high level modulation. Therefore, the efficiency of high levels modulators is higher than that of low level modulation.

**EXAMPLE 3.7. A modulating signal 10 sin($2\pi \times 10^3t$) is used to modulate a carrier signal 20 sin ($2\pi \times 10^4t$). Determine the modulation index, percentage modulation, frequencies of the sideband components and their amplitudes. What will be the bandwidth of the modulated signal?**

**Solution:**

(i) The modulating signal $v_m = 10 \sin (2\pi \times 10^3 t)$.

Let us compare this with the following expression

$$v_m = V_m \sin(2\pi f_m t)$$

Then, we get $V_m = 10$ volt, $f_m = 1 \times 10^3$ Hz = 1 kHz

(ii) The carrier signal $v_c = 20 \sin (2\pi \times 10^4 t)$

Comparing this with the expression $v_c = V_c \sin(2\pi f_c t)$, we obtain

$V_c = 20$ volt, $f_c = 1 \times 10^4$ Hz = 10 kHz

(a) Modulation index and percentage modulation:

We know that $m = \frac{V_m}{V_c} = \frac{10}{20} = 0.5$ and % modulation = $0.5 \times 100 = 50$ % **Ans.**

(b) Frequencies of sideband components:

(i) Upper sideband $f_{USB} = f_c + f_m = (10 + 1) = 11$ kHz

(ii) Lower sideband $f_{LSB} = f_c - f_m = (10 - 1) = 9$ kHz

(c) Amplitudes of sidebands:

The amplitudes of upper as well as the lower sideband are given by,

Amplitude of each sideband = $\frac{mV_c}{2} = \frac{0.5 \times 20}{2} = 5$ Volt **Ans.**

(d) Bandwidth = $2 f_m = 2 \times 1$ kHz = 2 kHz **Ans.**

# Lecture 11

**AM Modulation**

AM Modulation

Square Law Diode Modulation

In fig., carrier and modulating signals
are applied across the diode. When two
diff. freq. are passed thru a non-linear
device, the process of AM takes place.
When carrier & mod. signal are applied
at the input of the diode, then
diff. freq. terms appears at the o/p of
the diode. These diff. freq. are applied
across a tuned ckt which is tuned to
carrier freq. & has a narrow BW just to
pass two SB & carrier & reject other freq.

Let $v_c = V_c \cos w_c t$

$v_m = V_m \cos w_m t$

total a.c. voltage across diode,

$v_s = v_c + v_m$

$= V_c \cos w_c t + V_m \cos w_m t$

The non-linear relationship betn voltage & current for a diode is

$i = a + b v_s + c v_s^2$

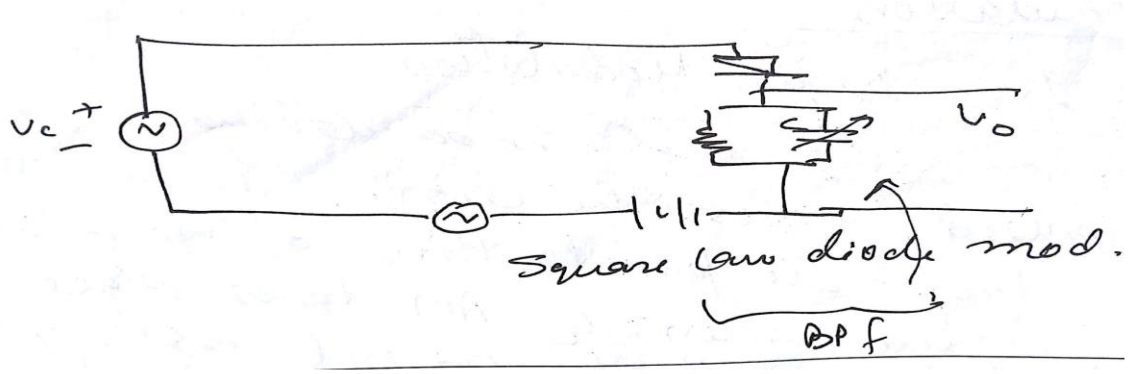
$= a + b(V_c \cos w_c t + V_m \cos w_m t)$
$+ c(V_c \cos w_c t + V_m \cos w_m t)^2$

Extract

$b V_c \cos w_c t$ — carrier

$c V_c V_m \cos(w_c + w_m)t$
$c V_c V_m \cos(w_c - w_m)t$ } SB

} select this term using tuned ckt




**AM Demodulation**

**Envelop Detector**

The output of an envelop detector follows the envelop of the modulated signal as shown in fig. On the positive cycle of the input signal, the input grows and may exceed the charged value on the capacity $v_c(t)$, turning on the diode and allowing the capacitor C to charge up to the peak voltage of the input signal cycle. As the input signal falls below this peak value, it falls quickly below the capacitor voltage (which is nearly the peak voltage), thus causing the diode to open. The capacitor now discharges through the resistor R at a slow rate (with a time constant RC). During the next positive cycle, the same procedure repeats. As the input signal rises above the capacitor voltage, the diode conducts again. The capacitor again charges to the peak value of this new cycle. The capacitor discharges slowly during the cutoff period.

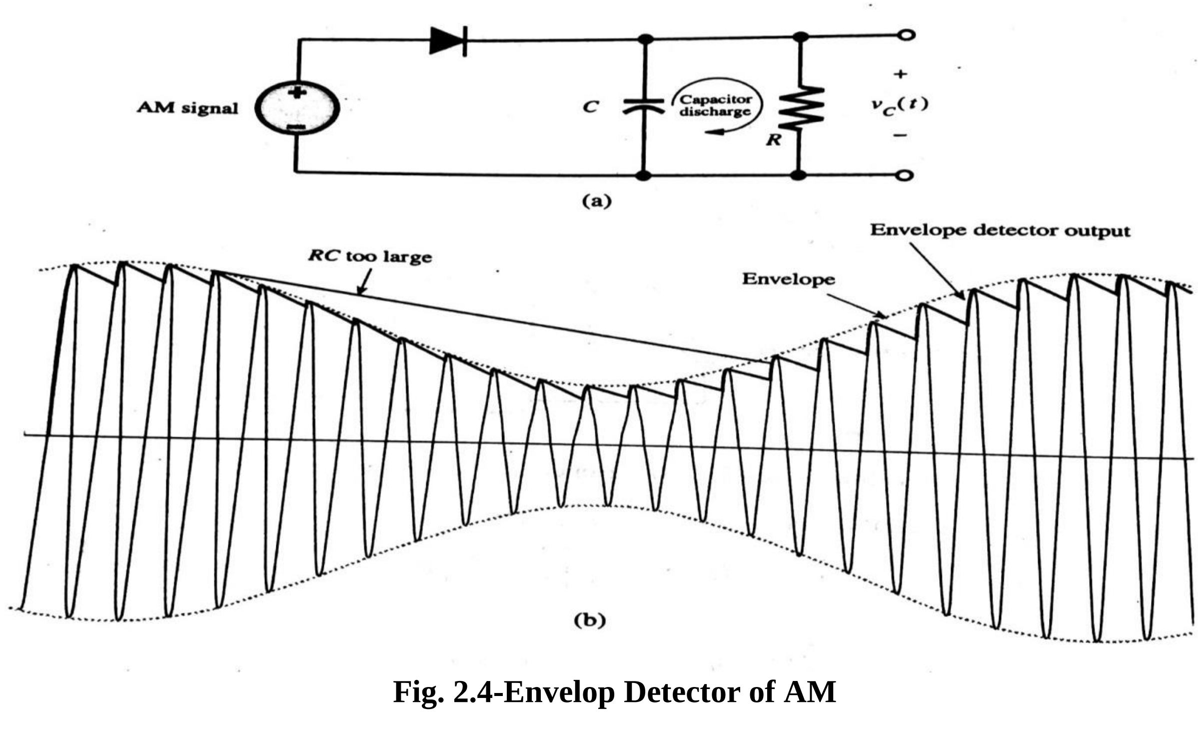



**Fig. 2.4-Envelop Detector of AM**

During each positive cycle, the capacitor charges up to the peak voltage of the input signal and then decays slowly until the next positive cycle as shown in Fig. 2.4. The output voltage $v_c(t)$ closely follows the (rising) envelop of the input AM cycle, Equally important, the slow capacity discharge via the resistor R allows the capacity voltage to follow a declining envelop. Capacitor discharge between positive peaks causes a ripple signal of frequency $w_c$in the output. This ripple can be reduced by choosing a larger time constant RC so the capacitor discharges very little between the positive peaks ($RC \gg \frac{1}{w_c}$). Picking RC to large would make it impossible for the capacitor voltage to follow a fast declining envelop (Fig 2.4b). Because the maximum rate of AM envelop decline is dominated by the bandwidth B of the message signal m(t), the design criterion of RC should be

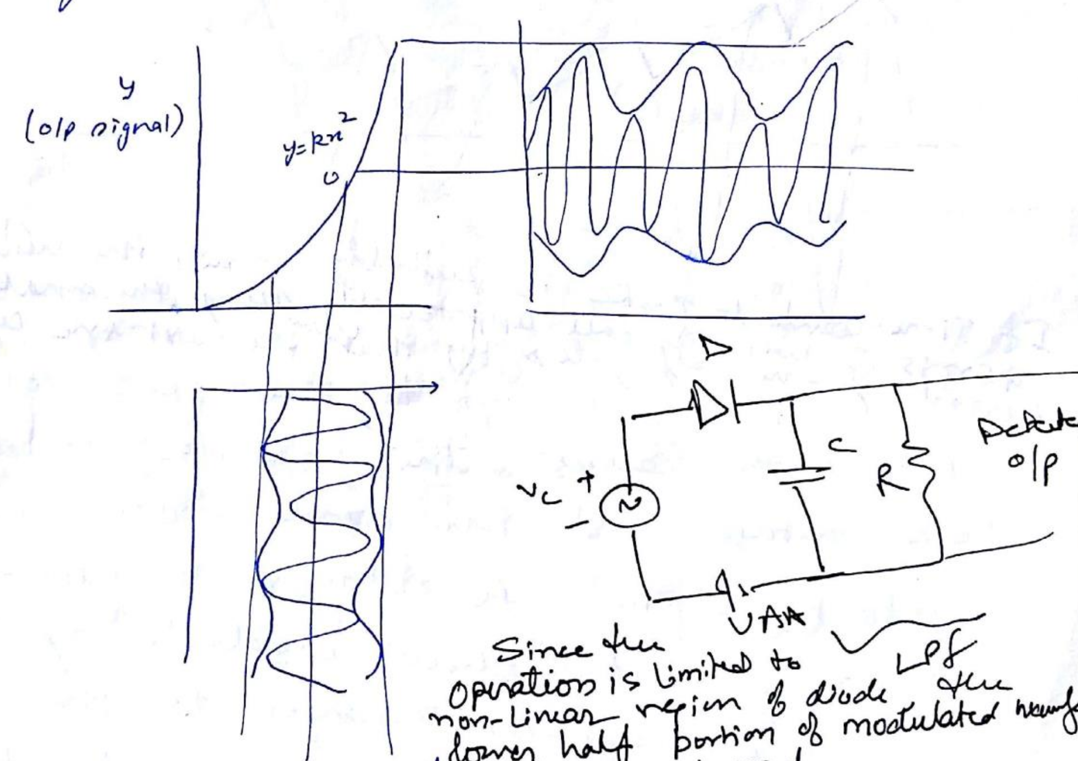
$$\frac{1}{w_c} \ll RC \ll \frac{1}{(2\pi B)} \qquad \text{(B is same as } f_m\text{)}$$

The envelop detector output is $v_c(t) = A + m(t)$. The dc term can be blocked by a capacitor or a high pass filter. The ripple (saw tooth waveform) may further be reduced by another RC filter.

## Square Law Demodulator –

A method to recover the baseband signal is to pass the AM signal thru a non-linear device. We assume that the device has a square-law relationship betn input signal x and output signal y.

Thus $y = kx^2$.




Since the operation is limited to non-linear region of diode the lower half portion of modulated waveform is compressed

The applied signal is

$$x = A_0 + A_c\,[\,1 + m(t)\,]\cos\omega_c t$$

Thus the o/p of the squaring ckt is

$$y = K\{A_0 + A_c[\,1 + m(t)\,]\cos\omega_c t\}^2$$

Squaring & dropping dc terms as well as terms whose spectral components are located near $\omega_c$ & $2\omega_c$

$$s_0(t) = \underbrace{KA_c^2\, m(t)}_{\text{reqd}} + \cdots$$

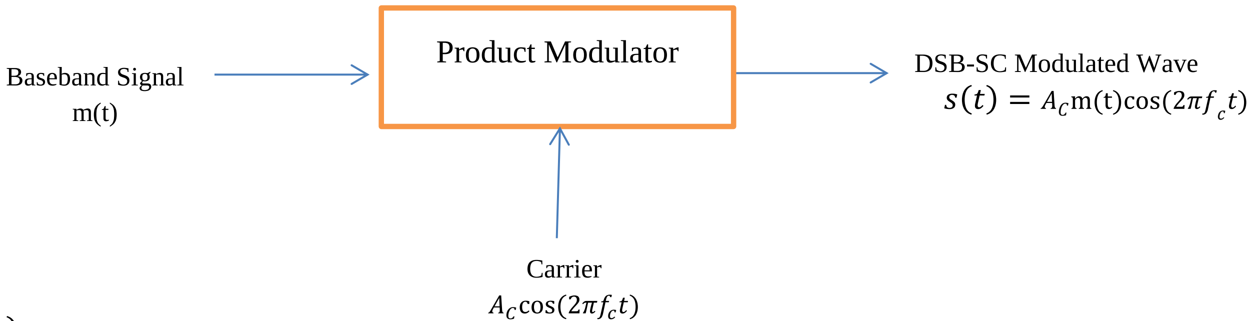
**Linear Modulation scheme is defined by**

$$s(t) = s_I(t)\cos(2\pi f_c t) - s_Q(t)\sin(2\pi f_c t)$$

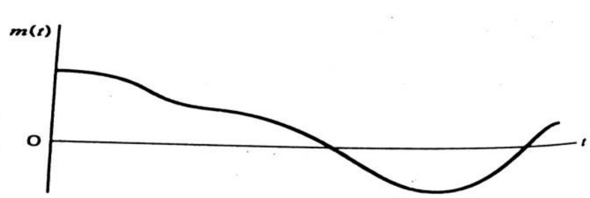
Where $s_I(t)$ is in-phase component of the modulated wave $s(t)$ and $s_Q(t)$is its quadrature component. The above equation is the canonical representation of a narrow band signal. There are 3 types of linear modulation involving a single message signal.

1) Double Side Band Suppressed Carrier (DSB-SC)- Only USB and LSB are transmitted
2) Single Side band (SSB) – Only One side band is transmitted (LSB or USB)
3) Vestigial Side Band (VSB)- Only a vestige (trace) of one of the side bands and a correspondingly modified version of the other side band are transmitted

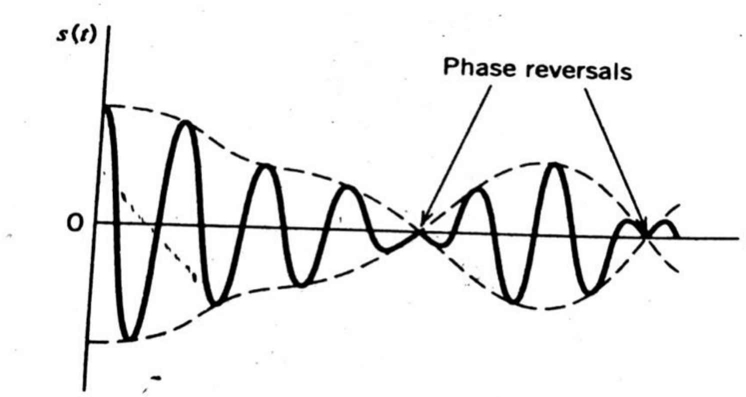
**DSB-SC**




$s(t) = A_C \mathrm{m(t)}\cos(2\pi f_c t)$




**Baseband Signal**




**DSB-SC modulated signal**

A DSB-SC signal $s(t)$is obtained is obtained by simply multiplying modulating signal $m(t)$ and carrier $\cos(w_c t)$. This is achieved by a product modulator as shown in the above figure. The modulated signal $m(t)$ undergoes a phase reversal whenever the message signal $m(t)$ crosses zero.

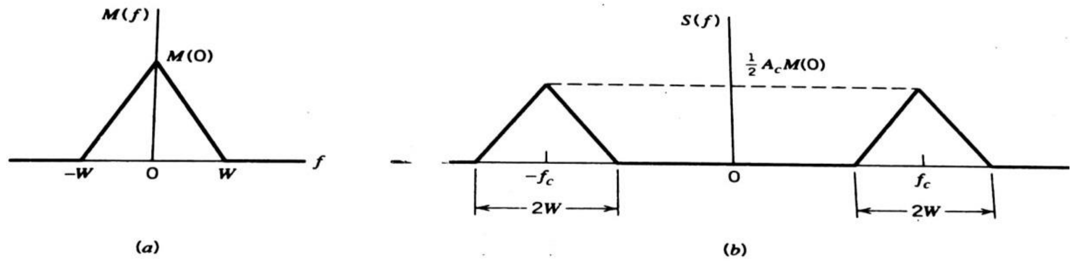


Fig. (a) Spectrum of Baseband Signal (b) Spectrum of DSB-SC modulated signal

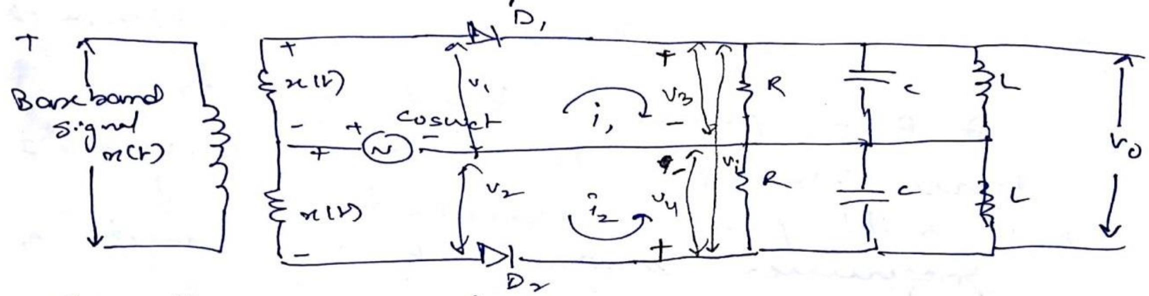
**Modulation using Non-Linear Device (Balanced Modulator)**

Generation of DSB-SC

M6

1) Balanced Modulator

If two non linear device such as diodes, transistors etc are connected in a balanced mode so as to supress the carriers of each other, then only side bands are left.




I/p voltage across diodes,

$v_1 = \cos w_c t + x(t)$ — (1)

$v_2 = \cos w_c t - x(t)$ — (2)

For $D_1$, the non-linear v-i relationship becomes

$i_1 = a v_1 + b v_1^2$ — (3)

for $D_2$, $i_2 = a v_2 + b v_2^2$ — (4)

Put (1) in (3),

$i_1 = a[\cos w_c t + x(t)] + b[\cos w_c t + x(t)]^2$

$= a\cos w_c t + a x(t) + b[\cos^2 w_c t + x^2(t) + 2x(t)\cos w_c t]$

or $= a\cos w_c t + a x(t) + b\cos^2 w_c t + b x^2(t) + 2b\, x(t)\cos w_c t$

Similarly $i_2 = a[\cos w_c t - x(t)] + b[\cos w_c t - x(t)]^2$

$= a\cos w_c t - a x(t) + b\cos^2 w_c t + b x^2(t) - 2b x(t)\cos w_c t$

the net i/p voltage at the i/p of BPF ~~across~~ BPF is

$$V_i = V_3 - V_4$$

Also $v_3 = i_1 R$

$V_4 = i_2 R$

and $v_i = R(i_1 - i_2)$

$= R[2ax(t) + 4bx(t)\cos w_c(t)]$

$= 2R[ax(t) + 2bx(t)\cos w_c(t)]$

A BPF passes a band of freq.
Since the BPF is centered around $\pm w_c$, it will pass a narrow band of frequencies centered at $\pm w_c$ with a BW of $2W_m$

$\therefore$ o/p of BPF

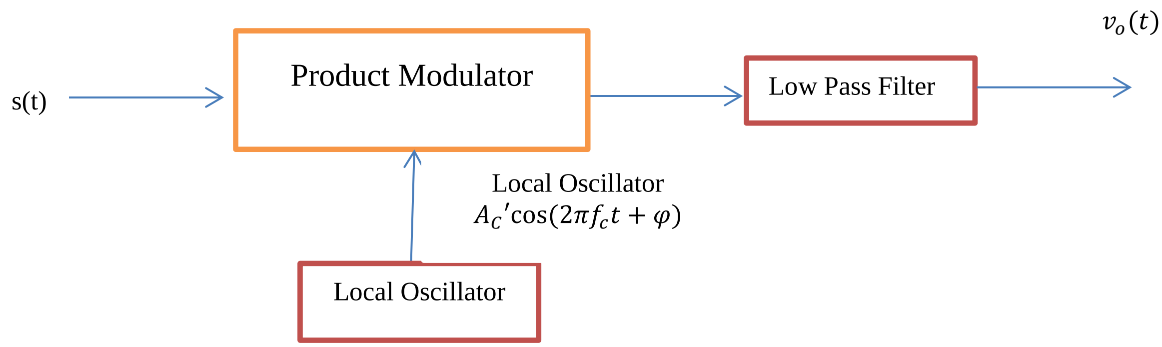
$v_0 = 4aRx(t)\cos w_c t$

$= Kx(t)\cos w_c t$

which is DSB-SC

## Coherent Detection of DSB-SC

The Baseband signal $m(t)$ can be uniquely recovered from a DSB-SC wave $s(t)$ with a locally generated sinusoidal wave and then low pass filtering the product as shown in figure below. It is assumed that the local oscillator signal is exactly coherent or synchronized, in both frequency and phase, with the carrier wave $c(t)$ used in the product modulator to generate $s(t)$. This method of generation is known as coherent detection or synchronous demodulation.



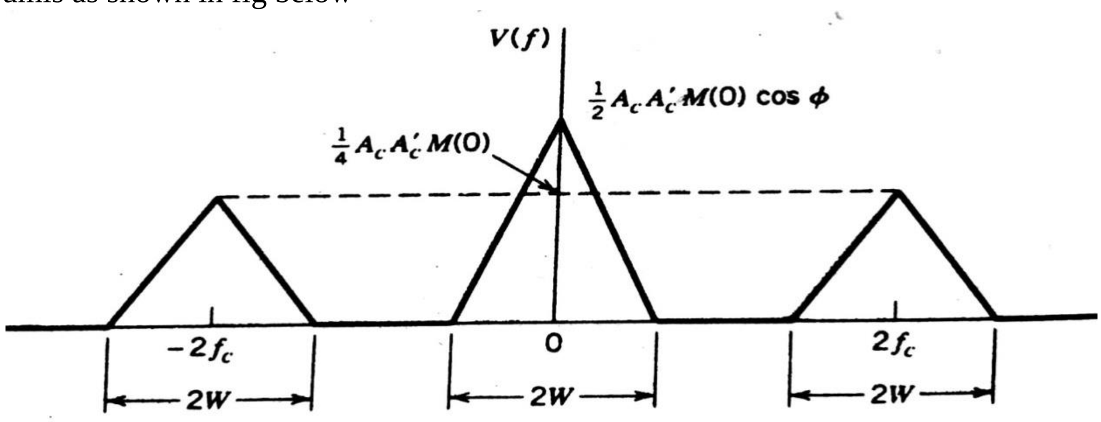

The output of the product modulator is

$$v(t) = A_C{}'\cos(2\pi f_c t + \varphi)s(t)$$

$$= A_C A_C' \cos(2\pi f_c t)\cos(2\pi f_c t + \varphi)\, m(t)$$

$$= \frac{1}{2} A_C A_C' \cos(4\pi f_c t + \varphi)\, m(t) + \frac{1}{2} A_C A_C' \cos(\varphi)\, m(t)$$

The first term in the above equation represents a DSB-SC modulated signal with a carrier frequency $2f_c$, whereas the second term is proportional to the baseband signal $m(t)$. The spectrumis as shown in fig below




Spectrum of product modulator output with a DSB-SC modulated wave as input

**Hilbert Transform**

## Hilbert Transform

We now introduce for later use a new tool known as the **Hilbert transform**. We use $x_h(t)$ and $\mathcal{H}\{x(t)\}$ to denote the Hilbert transform of signal $x(t)$

$$x_h(t) = \mathcal{H}\{x(t)\} = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\alpha)}{t-\alpha}\, d\alpha \tag{4.15}$$

Observe that the right-hand side of Eq. (4.15) has the form of a convolution

$$x(t) * \frac{1}{\pi t}$$

Now, application of the duality property to pair 12 of Table 3.1 yields $1/\pi t \Longleftrightarrow -j\,\mathrm{sgn}\,(f)$. Hence, application of the time convolution property to the convolution (of Eq. (4.15) yields

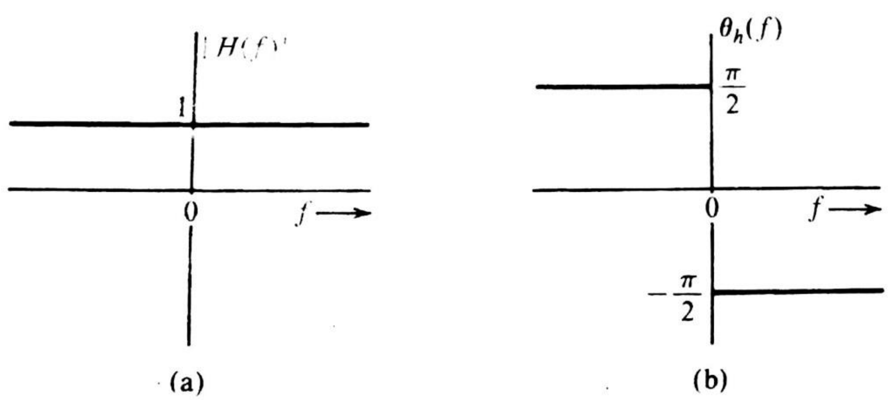
$$X_h(f) = -jX(f)\,\mathrm{sgn}\,(f) \tag{4.16}$$

From Eq. (4.16), it follows that if $m(t)$ passes through a transfer function $H(f) = -j\,\mathrm{sgn}\,(f)$, then the output is $m_h(t)$, the Hilbert transform of $m(t)$. Because

$$H(f) = -j\,\mathrm{sgn}\,(f) \tag{4.17}$$

$$= \begin{cases} -j = 1\cdot e^{-j\pi/2} & f > 0 \\ j = 1\cdot e^{j\pi/2} & f < 0 \end{cases} \tag{4.18}$$

it follows that $|H(f)| = 1$ and that $\theta_h(f) = -\pi/2$ for $f > 0$ and $\pi/2$ for $f < 0$, as shown in Fig. 4.14. Thus, if we change the phase of every component of $m(t)$ by $\pi/2$ (without changing its amplitude), the resulting signal is $m_h(t)$, the Hilbert transform of $m(t)$. Therefore, a Hilbert transformer is an ideal phase shifter that shifts the phase of every spectral component by $-\pi/2$.



# Lecture 11

## Single Side Band (SSB)

SSB has been quite possible the fastest spreading form of analog modulation, which offers many advantages such as ability to transmit good communication quality signals by using a very narrow bandwidth, with relatively low power for the distances involved.

**Evolution and description of SSB**

Review of basic transmission process –

1) Physical length of the antenna must equal the wavelength of the transmitted signal, usually in the RF range.
2) The audio signal is much too long to be transmitted directly by a conventional antenna
3) The intelligence (audio) must be processed by the electronic circuitry to meet the transmission requirements of the system. This process is known as mixing.

The SSB system incorporates the mixing process plus a signal multiplying filtering enhancement, to ensure that the signal meets the requirements of the antenna system. In AM, when a carrier is amplitude modulated, the resulting signal consists of three frequencies, carrier and two sidebands. This is automatic consequence of the AM mixing process and will always happen unless steps are taken to prevent it. In this chapter, we will see the factors involved in and the advantages and disadvantages of suppressing the carrier or any of the sidebands.

The AM power equation states that the ratio of total power to carrier power is given by $(1 + m^2/2)$. If the carrier is suppressed, only the sideband power remains. As this is only $P_c m^2/2$, a two-thirds saving is effected at 100 percent modulation and even more is saved as the depth of modulation is reduced. If one of the side band is now also suppressed, the remaining power is $P_c m^2/4$, a further saving of 50 percent over carrier suppressed AM.

Q- Calculate the percentage power saving when the carrier and one of the sidebands are suppressed in an AM wave in an AM wave modulated to a depth of

(a) 100 percent (b) 50 percent

(a) $P_t = P_c\left(1+\frac{m^2}{2}\right) = 1.5P_c$

$P_{SB=}P_c\left(\frac{m^4}{4}\right) = 0.25P_c$

Saving $=\frac{1.5-0.25}{1.5} = 83.3\%$ if one SB is suppressed

Saving $=\frac{1.5-1}{1.5}=\frac{1}{3} = 33.33\%$ if carrier is suppressed (shows carrier power is 2/3 of total power)

In practice, SSB is used to save power in applications, where such a power saving is warranted i.e. in mobile system, in which weight and power consumption must naturally be kept low. They are also used for applications in which bandwidth is at premium. For e.g. point to point communication, land, air, maritime mobile communication, television, telemetry, military communications, radio navigation and amateur radio.

The LSB and USB are uniquely related to each other by virtue of their symmetry about the carrier frequency i.e. if the amplitude and phase spectra of each sideband is given, we can uniquely determine the other. This means that as far as the transmission of information is concerned, only one side band is necessary. Thus if the carrier and one of the sideband are suppressed at the transmitter, no information is lost. Modulation of this type which provides a single side band with suppressed carrier is known as SSB-SC. Thus, SSB-SC reduces the transmission bandwidth by half. This means that in a given frequency, we can accommodate twice the number of channels by using a single sideband in place of both. SSB with LSB can be expressed as

SSB with LSB can be expressed as

$$\cos(w_{c-}w_m)\,t = \cos(w_m t)\cos(w_c t) + \sin(w_m t)\sin(w_c t) \quad (1)$$

SSB with USB can be expressed as

$$\cos(w_{c+}w_m)\,t = \cos(w_m t)\cos(w_c t) - \sin(w_m t)\sin(w_c t) \quad (2)$$

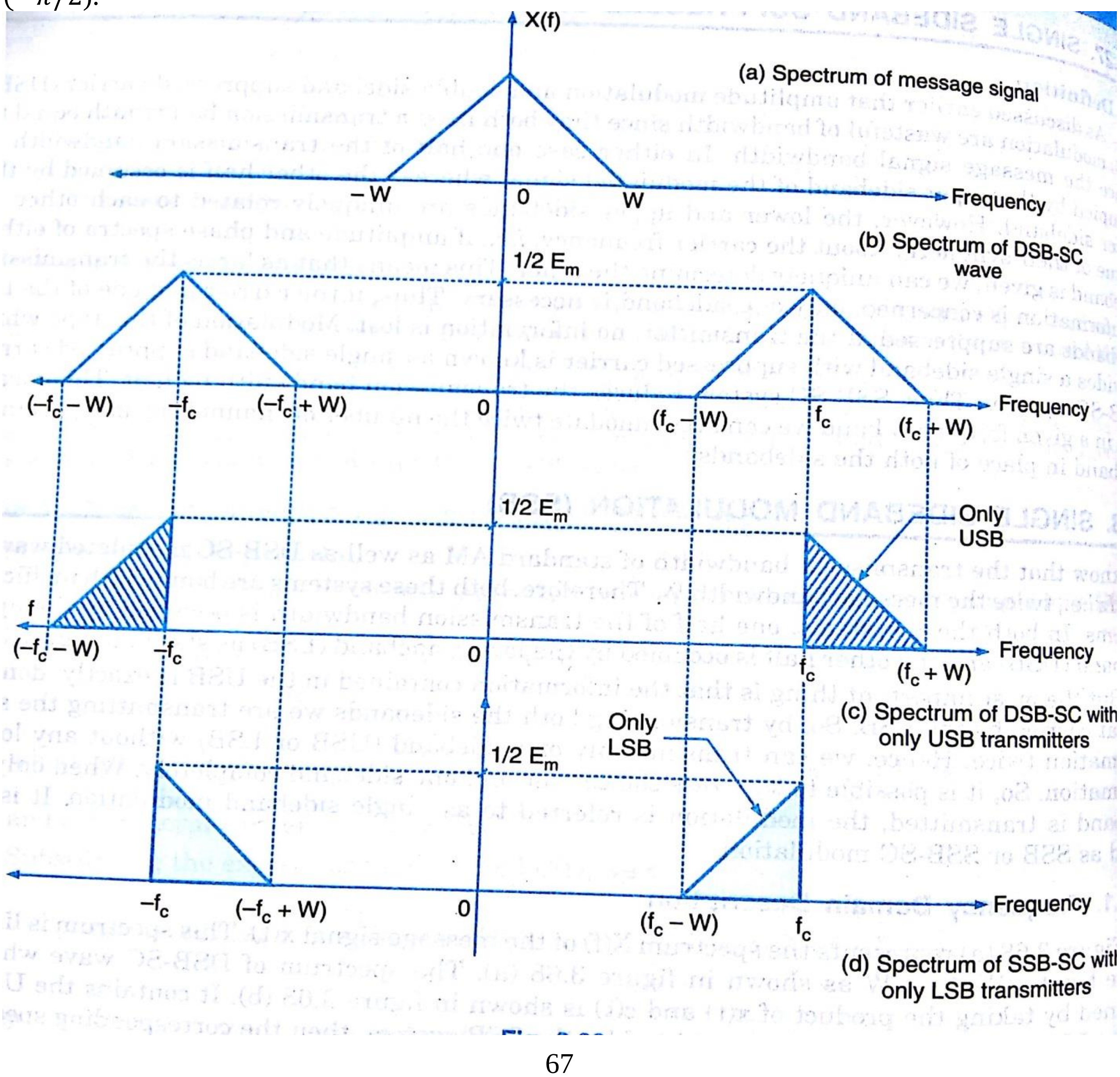
Eq. (1) & (2) can be combined as

$$s(t)_{SSB} = \cos(w_m t)\cos(w_c t) \pm \sin(w_m t)\sin(w_c t)$$

Or

$$s(t)_{SSB} = m(t)\cos(w_c t) \pm m_h(t)\sin(w_c t)$$

Where $m_h(t)$ is a signal obtained by shifting the phase of every component present in $m(t)$ by $(-\pi/2)$.



# Lecture 12

**Generation of SSB Signals**

## 2.4.1 SSB Modulator

### SSB Modulation by Multistage Filtering

Multistage filtering as a concept is used in SSB modulation to overcome the practical constraint associated with design of very sharp cutoff filter. Refer to Fig. 2.17. Here the baseband signal and a carrier are applied to a balanced modulator. The output of the balanced modulator bears both the upper- and lower-sideband signals. One or the other of these signals is then selected by a filter. The filter is a bandpass filter whose passband encompasses the frequency range of the sideband selected. The filter must have a cutoff sharp enough to separate the selected sideband from the other sideband. The frequency separation of the sidebands is twice the frequency of the lowest frequency spectral components of the baseband signal. Human speech contains spectral components as low as about 70 Hz. However, to alleviate the sideband filter selectivity requirements in an SSB system, it is common to limit the lower spectral limit of speech to about 300 Hz. It is found that such restriction does not materially affect the intelligibility of speech. Similarly, it is found that no serious distortion results if the upper limit of the speech spectrum is cut off at about 3000 Hz. Such restriction is advantageous for the purpose of conserving bandwidth. Altogether, then, a typical sideband filter has a passband which, measured from $f_c$, extends from about 300 to 3000 Hz and in which range its response is quite flat. Outside this passband the response falls off sharply, being down about 40 dB at 4000 Hz and rejecting the unwanted sideband also be at least 40 dB. The filter may also serve, further, to suppress the carrier itself. Of course, in principle, no carrier should appear at the

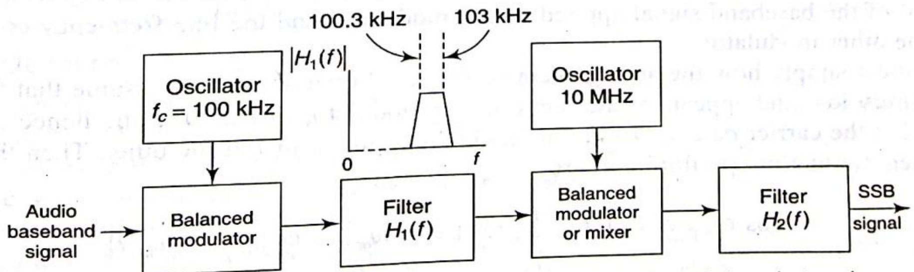



**Fig. 2.17** *Block diagram of the filter method of generating a single-sideband signal.*

output of a balanced modulator. In practice, however, the modulator may not balance exactly, and the precision of its balance may be subject to some variation with time. Therefore, even if a pilot carrier is to be transmitted, it is well to suppress it at the output of the modulator and to add it to the signal at a later point in a controllable manner.

Now consider that we desire to generate an SSB signal with a carrier of, say, 10 MHz. Then we require a passband filter with a selectivity that provides 40 dB of attenuation within 2 × 300 Hz = 600 Hz at a frequency of 10 MHz, a percentage frequency change of 0.006 percent. Filters with such sharp selectivity are very elaborate and difficult to construct. For this reason, it is customary to perform the translation of the baseband signal to the final carrier frequency in several stages. Two such stages of translation are shown in Fig. 2.17. Here we have selected the first carrier to be of frequency 100 kHz. The upper sideband, say, of the output of the balanced modulator ranges from 100.3 to 103 kHz. The filter following the balanced modulator which selects this upper sideband need now exhibit a selectivity of only a hundredth of the selectivity (40 dB in 0.6 percent frequency change) required in the case of a 10 MHz carrier. Now let the filter output be applied to a second balanced modulator, supplied this time with a 10 MHz carrier. Let us again select the upper sideband. Then the second filter must provide 40 dB of attenuation in a frequency range of 2 × 100.3 kHz = 200.6 kHz, which is nominally 2 percent of the carrier frequency.

If the second frequency-translating device in Fig. 2.17 were a mixer rather than a multiplier, then in addition to the upper and lower sidebands, the output would contain a component encompassing the range 100.3 to 103 kHz as well as the 10 MHz carrier. The range 100.3 to 103 kHz is well out of the range of the second filter intended to pass the range 10,100,300 to 10,103,000 Hz. And it is realistic to design a filter which will suppress the 10 MHz carrier, since the carrier frequency is separated from the lower edge of the upper sideband (10,100,300) by nominally a 1-percent frequency change.

Altogether, then, we note in summary that when a single-sideband signal is to be generated which has a carrier in the megahertz or tens-of-megahertz range, the frequency translation is to be done in more than one stage—frequently two but not uncommonly three. If the baseband signal has spectral components in the range of hundreds of hertz or lower (as in an audio signal), the first stage invariably employs a balanced modulator, while succeeding stages may use mixers.

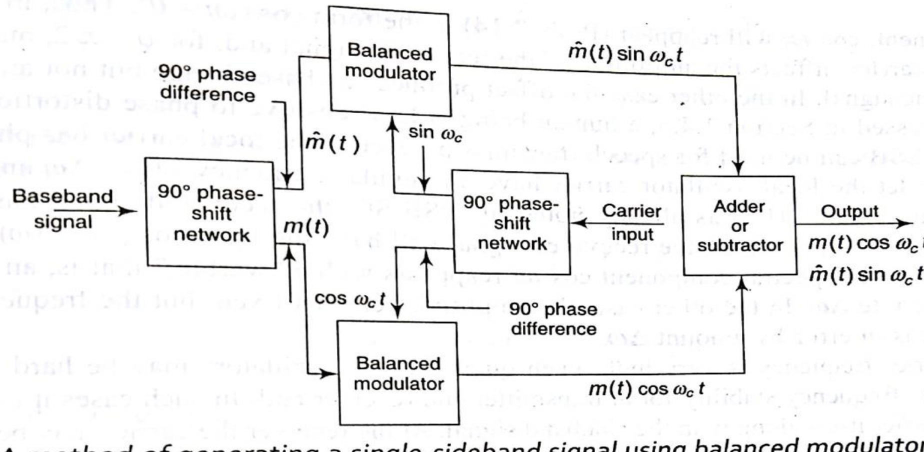
### SSB Modulation by Phase Shift

This alternative scheme comes from our mathematical presentation of SSB signal in Eq. (2.22). Refer to Fig. 2.18. The carrier signals of angular frequency $\omega_c$ which are applied to the modulators differ in phase by 90°. Similarly, the baseband signal, before application to the modulators, is passed through a 90° phase-shifting network so that there is a 90° phase shift between any spectral component of the baseband signal applied to one modulator and the like-frequency component applied to the other modulator.

To see most simply how the arrangement of Fig. 2.18 operates, let us assume that the baseband signal is sinusoidal and appears at the input to one modulator as $\cos \omega_m t$ and hence as $\sin \omega_m t$ at the other. Let the carrier be $\cos \omega_c t$ at one modulator and $\sin \omega_c t$ at the other. Then the outputs of the balanced modulators (multipliers) are

$$\cos \omega_m t \cos \omega_c t = \frac{1}{2}[\cos (\omega_c - \omega_m)t + \cos (\omega_c + \omega_m)t] \tag{2.23}$$

$$\sin \omega_m t \sin \omega_c t = \frac{1}{2}[\cos (\omega_c - \omega_m)t - \cos (\omega_c + \omega_m)t] \tag{2.24}$$




ig. 2.18 *A method of generating a single-sideband signal using balanced modulators and phase shifters.*

f these waveforms are added, the lower sideband results; if subtracted, the upper sideband appears t the output. In general, if the modulation $m(t)$ is given by

$$m(t) = \sum_{i=1}^{m} A_i \cos(\omega_i t + \theta_i) \tag{2.25}$$

hen, using Fig. 2.18, we see that the output of the SSB modulator is in general

$$m(t) \cos \omega_c t \pm \hat{m}(t) \sin \omega_c t \,. \tag{2.26}$$

where

$$\hat{m}(t) \equiv \sum_{i=1}^{m} A_i \sin(\omega_i t + \theta_i) \tag{2.27}$$

The single-sideband generating system of Fig. 2.18 generally enjoys less popularity than doe the filter method. The reason for this lack of favor is the need of precise 90 degree shift of phas shiter over a large frequency range, each modulator carefully balanced, etc.

### 3.31.3. Demodulation of SSB Waves

The SSB receivers are normally used for professional or commercial communications. The special requirements of SSB receivers are as follows :

1. Requirements of SSB receiver

(i) High reliability
(ii) Excellent suppression of adjacent signals
(iii) High signal to noise ratio
(iv) Ability to demodulate SSB

### 3.31.4. Coherent SSB Demodulation**

**(RTU, Kota, Sem. Exam; 2002-03) (10 marks)**

1. Definition and Block Diagram

The product modulator is a type of coherent SSB demodulator. To recover the modulating signal from the SSB-SC signal, we require a phase coherent or synchronous demodulator. The block

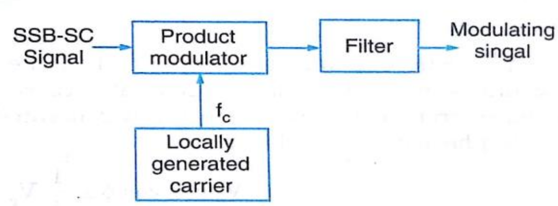



Fig. 3.78. Block diagram of coherent SSB demodulator

diagram of the coherent SSB-SC demodulator is as shown in figure 3.78. The received SSB signal is first multiplied with a locally generated carrier signal. The locally generated carrier should have exactly the same frequency as that of the suppressed carrier. The product modulator multiplies the two signals at its input and the product signal is passed through a low pass filter with a bandwidth equal to $f_m$. At the output of the filter, we get the modulating signal back.

### 2. Analysis of the Coherent Detector

Let the SSB wave at the input be given by,

$$s(t) = \frac{1}{2} V_c \left[x(t) \cos(2\pi f_c t) \pm \hat{x}(t) \sin(2\pi f_c t)\right]$$

The locally generated carrier is $\cos(2\pi f_c t)$

Therefore, output of the product modulator is given by,

$$v(t) = s(t) \cdot \cos(2\pi f_c t)$$

or $$v(t) = \frac{1}{2} V_c \cos(2\pi f_c t) \cdot \left[x(t) \cos(2\pi f_c t) \pm x(t) \sin(2\pi f_c t)\right]$$

$$v(t) = \frac{1}{2} V_c\, x(t) \cos(2\pi f_c t) \cos(2\pi f_c t) \pm \frac{V_c}{2} \hat{x}(t) \cos(2\pi f_c t) \sin(2\pi f_c t)] \quad \text{... (3.117)}$$

But, $$\cos A \cos B = \frac{1}{2} \{\cos(A+B) + \cos(A-B)\}$$

and $$\cos A \sin B = \frac{1}{2} \{\sin(A+B) + \sin(A-B)\}$$

$$\text{Therefore, } v(t) = \frac{1}{4} V_c\, x(t) \left[\cos(4\pi f_c t) + \cos(0)\right] \pm \frac{1}{4} V_c\, \hat{x}(t) \left[\sin(4\pi f_c t) - \sin(0)\right]$$

or $$v(t) = \underbrace{\frac{1}{4} V_c\, x(t)}_{\text{Scaled message signal}} + \underbrace{\frac{1}{4} V_c \left[x(t) \cos(4\pi f_c t) \pm \hat{x}(t) \sin(4\pi f_c t)\right]}_{\text{Unwanted terms}} \quad \text{... (3.118)}$$

When v(t) is passed through the filter, it will allow only the first term to pass through and will reject all other unwanted terms. Thus, at the output of the filter we get the scaled message signal and the coherent SSB demodulation is achieved.

Therefore, $$v_0(t) = \frac{1}{4} V_c\, x(t) \quad \text{... (3.119)}$$

### 3. Phase error and frequency error in coherent detection

**(BPTU, Orissa, Exam. Exam. 2002-03)**

The coherent detection explained in the preceding section, assumed the ideal operating conditions in which the locally generated carrier is in the perfect synchronization. But, in practice, a phase error $\phi$ may arise in the locally generated carrier wave. The detector output will get modified due to phase error as follows :

$$v_0(t) = \frac{1}{4} V_c\, x(t) \cos\phi \pm \frac{1}{4} V_c\, \hat{x}(t) \sin\phi \quad \text{... (3.120)}$$

In the above expression, the plus sign corresponds to the SSB input signal with only USB whereas the negative sign corresponds to SSB input with only LSB. Due to the presence of the Hilbert transform $\hat{x}(t)$ in the output, the detector output will suffer from the phase distortion. Such a phase distortion does not have serious effects with the voice communication. But in the transmission of music and video, it will have untolerable effects.

**Lecture 13**

# Vestigial Sideband Transmission

The exact frequency response requirements on the sideband filter in SSB-SC system can be relaxed by allowing a part of the unwanted sideband called vestige to appear in the output of the modulator. Due to this, the design of the sideband filter is simplified to a great extent.

But the bandwidth of the system is increased slightly.

To generate a VSB signal, we have to first generate a DSB-SC signal and then pass it through a sideband filter as shown in fig. 1. This filter will pass the wanted sideband as it is along with a part of unwanted sideband.

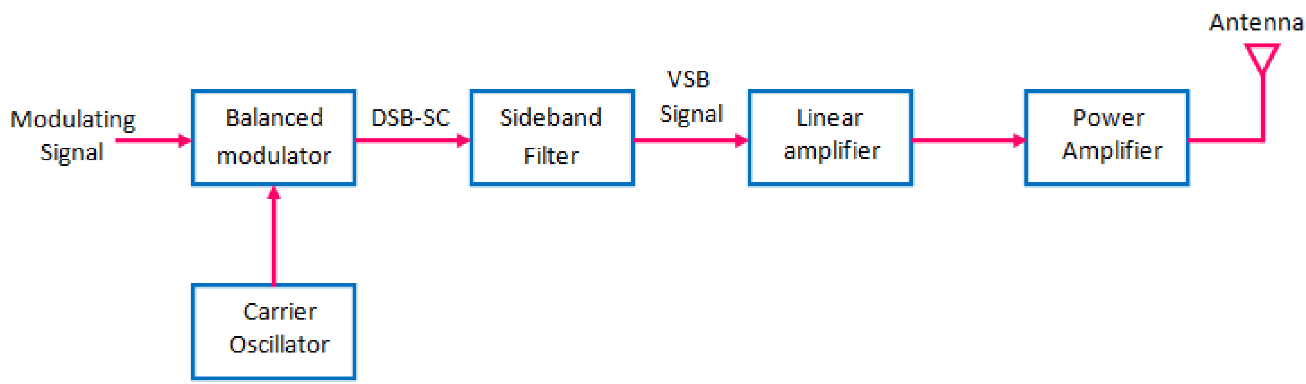
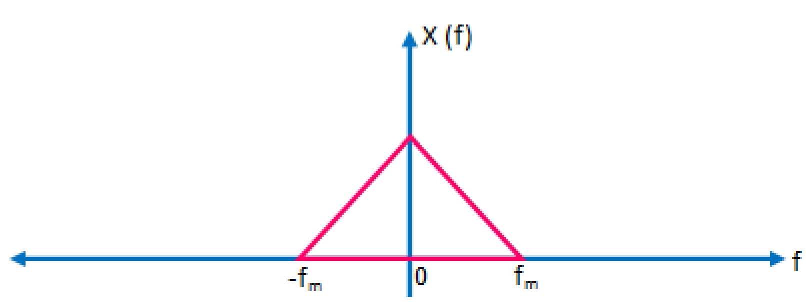



Fig.1 : VSB Transmitter

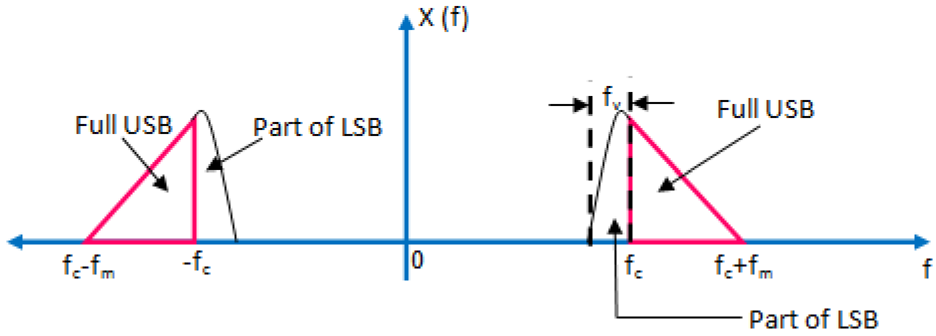
# Frequency Domain Description

## Frequency Spectrum

The spectrum of VSB is as shown in fig. 2.




(a) Spectrum of message signal




(b) Spectrum of VSB Signal

Fig. 2

The spectrum of message signal x(t) has also been shown. In the frequency spectrum, it is assumed that the upper sideband is transmitted as it is and the lower sideband is modified into vestigial sideband.

## Transmission Bandwidth

From fig. 2 (b), it is evident that the transmission bandwidth of the VSB modulated wave is given by:

$B = (f_m + f_v)Hz$ Where $f_m$ = Message bandwidth, f = Width of the vestigial sideband

## Advantages of VSB

1. The main advantage of VSB modulation is the reduction in bandwidth. It is almost as efficient as the SSB
2. Due to allowance of transmitting a part of lower sideband, the constraint on the filter have been relaxed. So practically, easy to design filters can be used .
3. It possesses good phase characteristics and makes the transmission of low frequency components possible.

## Application of VSB

VSB modulation has become standard for the transmission of television signal. Because the video signal need a large transmission bandwidth if transmitted using DSB-FC or DSB-SC techniques.

## Generation of VSB Modulated Wave

The block diagram of a VSB modulator is shown in fig.3 .

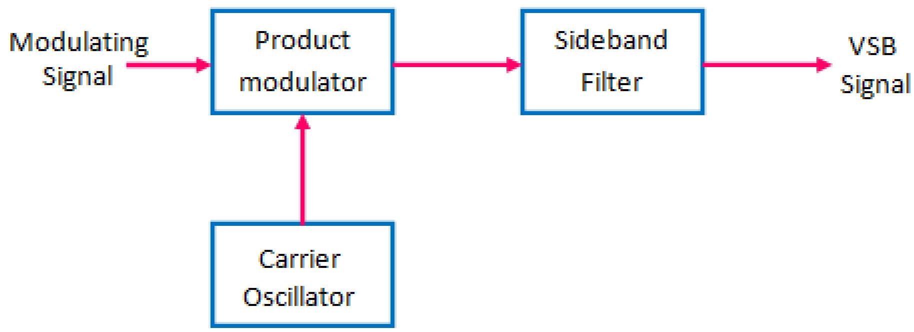



Fig.3 : Generation of VSB Signal

The modulating signal x(t) is applied to a product modulator . The output of the carrier oscillator is also applied to the other input of the product modulator . The output of the product modulator is then given by :

m(t) = x(t) . c(t)

= x(t) . $V_c$ cos(2π $f_c$t)
This represents a DSB-SC modulated wave .

This DSB-SC signal is then applied to a sideband shaping filter. The ddesign of this filter depends on the desired spectrum of the VSB modulated signal. This filter will pass the wanted sideband and the vestige of the unwanted sideband.

Let the transfer function of the filter be H(f).

Hence, the spectrum of the VSB modulated signal is given by :

$$S(f) = \frac{V_c}{2}[X(f - f_c) + X(f + f_c)]H(f)$$

# Demodulation of VSB Wave

The block diagram of the VSB demodulator is shown in fig.4 .

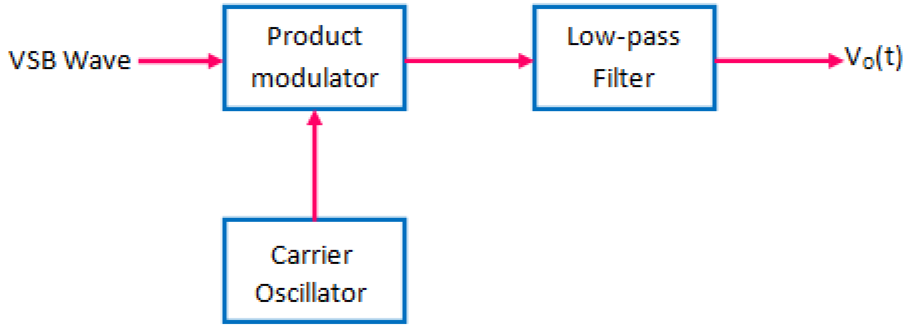



Fig.4 : VSB demodulator

## Working Operation

The VSB modulated wave is passed through a product modulator where it is multiplied with the locally generated synchronous carrier.

Hence, the output of the product modulator is given by:

$$m(t) = s(t) \times c(t) = s(t)V_c \cos(2\pi f_c t)$$

Taking the Fourier transform of both sides, we get

$$M(f) = S(f) \times \left[\frac{1}{2}\delta(f + f_c) + \frac{1}{2}\delta(f - f_c)\right] = \frac{1}{2}S(f + f_c) + \frac{1}{2}S(f - f_c)$$

But

$$S(f) = \frac{V_c}{2}[X(f - f_c) + X(f + f_c)]H(f)$$

Hence, we have

$$M(f) = \frac{V_c}{2}[X(f - 2f_c)H(f - f_c) + X\big((f + 2f_c)H(f + f_c)\big] + \frac{V_c}{4}[X(f)[H(f - f_c) + H(f + f_c)]$$

The first term in the above expression represents the VSB modulated wave, corresponding to a carrier frequency of $2f_c$ .This term will be eliminated by the filter to produce output $v_o(t)$. The second term in the above expression for M(f) represents the spectrum of demodulated VSB output. Therefore ,

$$V_O(f) = \frac{V_c}{4}[X(f)[H(f - f_c) + H(f + f_c)]$$

This spectrum is shown in fig.5.

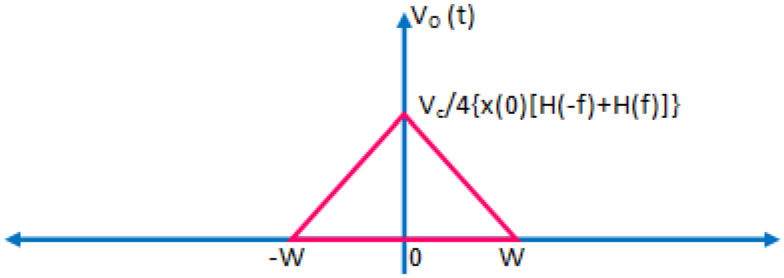



fig 5: Spectrum of VSB Demodulator

In order to obtain the undistorted message signal x(t) at the output of the demodulator, $V_o(f)$ should be a scaled version of X(f) .
For this the transfer function H(f) should satisfy the following conditions :

$$H(f - f_c) + H(f + f_c) = 2H(f + f_c)$$

Where H( $f_c$) is constant .

**EXAMPLE 3.34. The efficiency η of ordinary AM is defined as the percentage of the total power carried by the sidebands, that is,**

$$\eta = \frac{P_s}{P_t} \times 100\%$$

**where $P_s$ is the power carried by the sidebands and $P_t$ is the total power of the AM signal.**
**(i) Find η for $m_a = 0.5$ (50 percent modulation).**
**(ii) Show that for a single-tone AM, $\eta_{max}$ is 33.3 percent at $m_a = 1$.**

**(WBTU, Kolkata, Sem. Exam; 2005-06**

**Solution :** A single-tone AM signal can be expressed as under :

$$x_{AM}(t) = A \cos \omega_c t + m_a A \cos \omega_m t \cos \omega_c t$$

$$x_{AM}(t) = A \cos \omega_c t + \frac{1}{2} m_a A \cos(\omega_c - \omega_m)t + \frac{1}{2} m_a A \cos(\omega_c + \omega_m)t$$

$$P_c = \text{carrier power} = \frac{1}{2} A^2$$

$$P_s = \text{sideband power}$$

$$P_s = \frac{1}{2}\left[\left(\frac{1}{2} m_a A\right)^2 + \left(\frac{1}{2} m_a A\right)^2\right] = \frac{1}{2} m_a^2 A^2 \quad \text{...(i}$$

The total power $P_t$ is given by

$$P_t = P_c + P_s = \frac{1}{2} A^2 + \frac{1}{2} m_a^2 A^2$$

$$P_t = \frac{1}{2}\left(1 + \frac{1}{2} m_a^2\right) A^2 \quad \text{...(ii}$$

Thus $$\eta = \frac{P_s}{P_t} \times 100\% \quad \text{...(iii}$$

Substituting values of $P_s$ and $P_t$ in equation (iii), we get

or $$\eta = \frac{\frac{1}{4} m_a^2 A^2}{\left(\frac{1}{2} + \frac{1}{4} m_a^2\right) A^2} \times 100\% = \frac{m_a^2}{2 + m_a^2} 100\%$$

with the condition that $m_a \leq 1$.

(i) For $m_a = 0.5$, we have

$$\eta = \frac{(0.5)^2}{2 + (0.5)^2} \times 100\% = 11.1\%$$

(ii) Since $m_a \leq 1$, it can be observed that $\eta_{max}$ occurs at $m_a = 1$ and is given by

$$\eta = \frac{1}{3} \times 100\% = 33.3\% \quad \textbf{Ans.}$$

**EXAMPLE 3.70. An AM voltage is represented by the expression:**

$$V = 5\,[1 + 0.6 \cos (6280t)] \sin (2\pi \times 10^4 t) \text{ volts}$$

**Calculate: (i) Modulation Depth (ii) $f_m$ (iii) period of Carrier wave (iv) the peak instantaneous value of the modulated wave.**

**Expand the expression and calculate the rms voltage of lower side frequency component. The modulated wave is applied across the resistance of 1 k Ohms, what is the power dissipated?**

### Part I

**Solution:** Given that $V_{AM} = 5\,[1 + 0.6 \cos (6280\,t)] \sin (2\pi \times 10^4 t)$ volts

Let us compare this with the standard expression for AM which is as under:

$$e_{AM} = E_c\,[1 + m \cos (2\pi f_m t)] \sin (2\pi f_c t)$$

The comparison reveals the following values:

**(i) Modulation index** $m = 0.6$

Therfore, Modulation depth $= 0.6 \times 100 = 60\%$ **Ans.**

***(ii) Modulating frequency***

$$2\pi f_m = 6280$$

or $$f_m = \frac{6280}{2\pi} = 1000 \text{ Hz} \quad \textbf{Ans.}$$

**(iii) Period of carrier wave**

Carrier frequency $f_c = 10^4$ Hz

or Carrier wave period $= \frac{1}{f_c} = \frac{1}{10^4} = 100\ \mu\text{s}$ **Ans.**

**(iv) Peak instantaneous value of AM wave**

Peak carrier value $E_c = 5$V

Peak modulating amplitude $E_m = mE_c = 0.6 \times 5 = 3$ V

Therefore, Peak instantaneous value of AM wave is,

$$E_{max} = E_c + E_m = 5 + 3 = 8 \text{ volts} \quad \textbf{Ans.}$$

### Part II

**(i) RMS value of LSB**

Let us expand the given expression to get,

$$V_{AM} = 5 \sin (2\pi \times 10^4 t) + 3 \cos (6280\,t) \sin (2\pi \times 10^4 t)$$

or $$V_{AM} = 5 \sin (2\pi \times 10^4 t) + 1.5\,[\sin (2\pi \times 10^4 + 6280)\,t + \underbrace{\sin (2\pi \times 10^4 - 6280)\,t}_{\text{LSB Term}}]$$

The peak amplitude of LSB is 1.5 V.

Hence, its rms value $= \frac{1.5}{\sqrt{2}} = 1.06$ V **Ans.**

**(ii) Power dissipated in 1 kΩ resistance**

We have, $$P_t = \frac{E_c^2}{R_L} + \frac{E_{USB}^2}{R_L} + \frac{E_{LSB}^2}{R_L}$$

Where, $E_c$, $E_{USB}$ $E_{LSB}$ are the rms value.

Therefore, $$P_t = \frac{(5/\sqrt{2})^2}{1\times10^3} + \frac{(1.06)^2}{1\times10^3} + \frac{(1.06)^2}{1\times10^3}$$

or $$P_t = 14.75 \times 10^{-3} \text{ W or } 14.75 \text{ mW} \quad \textbf{Ans.}$$

**EXAMPLE 3.67. One output to an AM DSBFC modulator is an 800 kHz carrier with an amplitude of 40 $V_p$. The second input is a 25 kHz moduating signal whose amplitude is sufficient to produce a ± 10 V change in the amplitude if the envelope.**

**Calculate:**

**(a) Upper and lower side band frequencies**

**(b) Modulation coefficient and percentage modulation**

**(c) Maximum and minimum positive amplitude of the envelope**

**(d) Draw output frequency spectrum**

**(e) Draw the envelope and label it.**

**Solution:** Given that

$$f_c = 800 \text{ kHz}, E_c = 40 \text{ V},$$
$$f_m = 25 \text{ kHz}, E_m = 10\text{V}$$

**(a) $f_{USB}$ and $f_{LSB}$**

$$f_{USB} = (f_c + f_m) = (800 + 25) \text{ kHz} = 825 \text{ kHz}$$
$$f_{LSB} = (f_c - f_m) = (800 - 25) \text{ kHz} = 775 \text{ kHz} \quad \textbf{Ans.}$$

**(b) Modulation coefficient and percentage modulation**

$$m = \frac{E_m}{E_c} = \frac{10}{40} = 0.25$$

$$\%m = m \times 100 = 0.25 \times 100 = 25\% \quad \textbf{Ans.}$$

**(c) Maximum positive amplitude of envelope**

$$E_{max} = E_c + E_m = 40 + 10 = 50 \text{ volts}$$
$$E_{min} = E_c - E_m = 40 - 10 = 30 \text{ volts} \quad \textbf{Ans.}$$

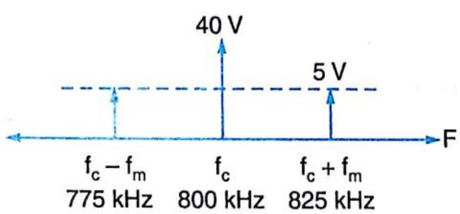



**Fig. 3.112.** Spectrum

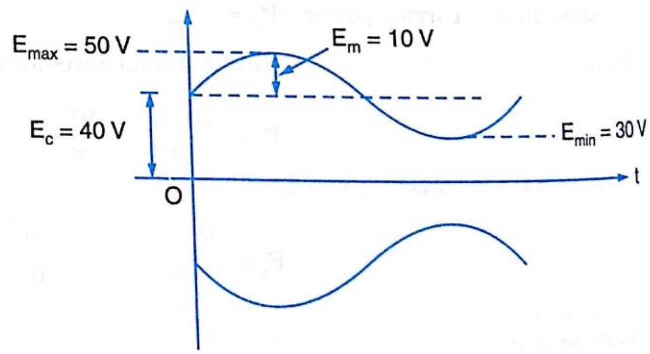



**Fig. 3.113.** Envelope of AM wave

**(d) Frequency spectrum**

$$\text{Carrier amplitude } E_c = 40 \text{ volts}$$

$$\text{Sideband amplitude} = m\frac{E_c}{2} = 0.25 \times \frac{40}{2} = 5 \text{ V}$$

Figure 3.112 shows the spectrum

**(e) Envelope:** Figure 3.113 shows the required envelope.

**EXAMPLE 3.68. An AM voltage is represented by the expression.**

$$\mathbf{V = 5\,[1 + 0.6 \cos (6280t)] \sin (2\pi \times 10^4\, t) \text{ volts}}$$

**What are the minimum and maximum amplitude of the AM wave? What frequency components are contained in the modulated wave and what is the amplitude of each component?**

**Solution:** Given Expression:

$$V_{AM} = \underbrace{5}_{E_c} [1 + \underbrace{0.6}_{m} \cos (\underbrace{6280}_{f_m = 1\text{ kHz}} t) \sin (2\pi \times \underbrace{10^4}_{f_c} t)$$

From the given expression, we have

$$E_c = 5V, m = 0.6, f_m = 1 \text{ kHz and } f_c = 10^4 \text{ Hz} = 10\text{kHz}$$

**(i) Minimum amplitude of AM wave ($E_{min}$)**

Peak amplitude of modulating signal is given by

$$E_m = m E_c = 0.6 \times 5 = 3 \text{ V}$$

Hence, $E_{min} = E_c - E_m = 5 - 3 = 2$ volts **Ans.**

**(ii) Maximum amplitude of AM wave ($E_{max}$)**

$E_{max} = E_c + f_m = 5 + 3 = 8$ volt **Ans.**

**(iii) Various frequency components and their amplitudes**

$$\text{USB: } f_{USB} = f_c + f_m = 10 + 1 = 11 \text{ kHz}$$
$$\text{LSB: } f_{LSB} = f_c + f_m = 10 - 1 = 9 \text{ kHz}$$
$$\text{Carrier: } f_c = 10 \text{ kHz}$$

Amplitude of carrier: $E_c = 5$ volt

Amplitude of each sideband $= \dfrac{mE_c}{2} = \dfrac{0.6 \times 5}{2} = 1.5$ volt **Ans.**

### 3.15.3. Distortions in the Envelope Detector Output

There are two types of distortions which can occur in the detector output. They are as under:

(i) Diagonal clipping, and

(ii) Negative peak clipping

#### 3.15.3.1. Diagonal clipping

This type of distortion occurs when the RC time constant of the load circuit is too long. Due to this, the RC circuit cannot follow the fast changes in the modulating envelope. The diagonal clipping has been shown in figure 3.35.

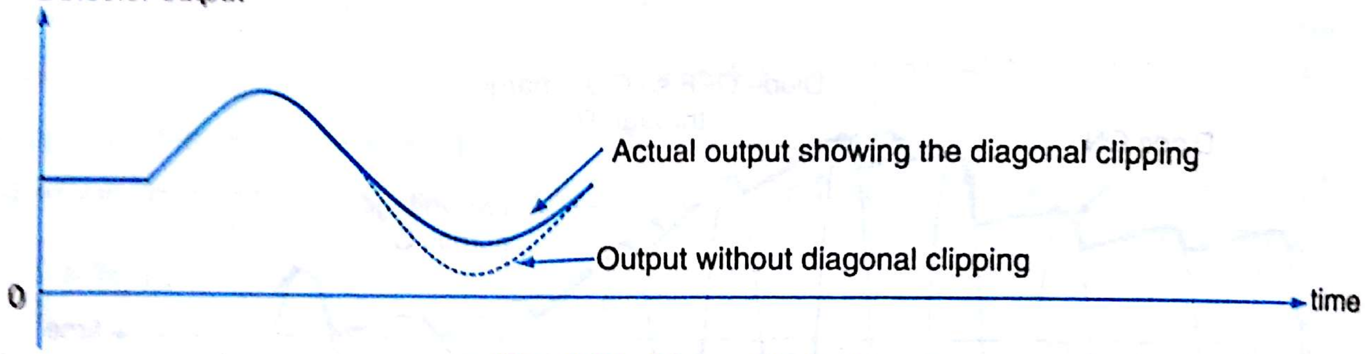



Fig. 3.35. Diagonal clipping

#### 3.15.3.2. Negative peak clipping

This distortion occurs due to a fact that the modulation index on the output side of the detector is higher than that on its input side. Hence, at higher depths of modulation of the transmitted signal, the overmodulation (more than 100% modulation) may take place at the output of the detector. The negative peak clipping will take place as a result of this overmodulation as shown in figure 3.36.

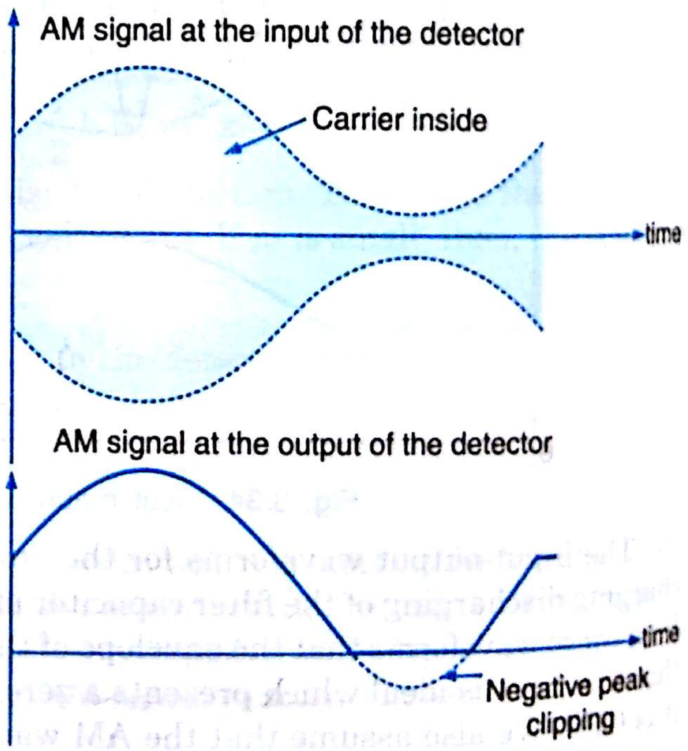



Fig. 3.36. Negative peak clipping

**Remedy:** The only way to reduce or eliminate the distortions is to choose the RC time constants properly as discussed earlier.

**EXAMPLE 3.22. Prove that the modulation index at the output of a diode detector is higher than that at its input and derive the maximum value of the transmitted modulation index to avoid the distortions in the output.**