HW2 Report: Depth–Noise Tradeoff in QAOA

1. QAOA Implementation Details

I implemented a standard p-layer QAOA ansatz with alternating cost and mixer unitaries. The cost Hamiltonian was based on the Ising form:

$$f(z) = \sum_i h_i z_i + \sum_{i<j} J_{ij} z_i z_j, \qquad z_i \in \{-1, +1\}.$$

Linear terms h_i were implemented using RZ rotations, while quadratic terms J_{ij} were realized via CNOT–RZ–CNOT sequences. The mixer layer applied RX rotations to every qubit. For p layers, the circuit used $2p$ variational parameters $(\gamma_0, \beta_0, \ldots, \gamma_{p-1}, \beta_{p-1})$.

Measurement outcomes were converted to $\{\pm 1\}$ using the mapping $1 \to +1$ and $0 \to -1$. Expected energies were computed from the measurement distribution, and the global minimum was obtained via brute-force enumeration of all 2^6 bitstrings.

Optimization was performed with `COBYLA` using `scipy.minimize`. Random initialization in $[-\pi/2, \pi/2]$ and a limit of 100 iterations were used. Each evaluation required executing the circuit with 1024 shots.

2. Noise Model and Simulation

Ideal simulations were performed using the noiseless `AerSimulator`. Noisy simulations used IBM Torino's real hardware noise model. This includes gate errors, readout errors, thermal relaxation (T1, T2), and decoherence based on gate durations.

Circuits were transpiled to the backend's native gate set and connectivity. Since the problem uses only six qubits, all qubits mapped directly to physical qubits on Torino without additional routing overhead.

3. Ideal vs. Noisy Approximation Ratio Trends

Across all 12 instances (four graph topologies and three weight distributions), several consistent trends were observed:

- **Non-monotonic ideal performance:** Ideal QAOA often peaked at intermediate depths ($p = 2$–4) and declined afterward due to optimization challenges.
- **Noise-driven degradation:** Noisy QAOA substantially underperformed ideal QAOA, with degradation of 30–50% for 3-regular and ER graphs and up to 90% for fully connected graphs.
- **Topology dependence:** Dense graphs (SK model) accumulated two-qubit gate errors quickly, leading to severe performance collapse. Sparse graphs (Barabasi–Albert, 3-regular) showed greater resilience.

- **Weight distribution effects:** Random $\{-1, +1\}$ weights were the most erratic; uniform weights produced smoother ideal trends but degraded under noise; Gaussian weights showed surprising robustness at low depth.

4. Patterns and Depth–Noise Tradeoff

Several recurring patterns emerged across the experiments:

- **Noise wall at moderate depths ($p = 2$–4):** Ideal performance often continued improving, but noisy performance plateaued or declined as noise accumulation outpaced expressivity gains.
- **Sparse vs. dense behavior:** Sparse graphs accumulated noise slowly and often maintained 50–70% of ideal performance. Dense graphs degraded rapidly, with optimal depth often at $p = 1$ or 2.
- **Three depth regimes:**
  - Low depth ($p = 1$–2): limited expressivity, modest noise impact.
  - Middle depth ($p = 3$–4): best tradeoff.
  - High depth ($p = 5$–6): diminishing returns; noise dominates.
- **Rugged optimization landscape:** COBYLA often converged to poor local minima, producing AR values near zero even under ideal simulation. Noise occasionally acted as unintentional regularization, enabling slightly better performance in rare cases.
- **Instance-specific optimal depth:** No single depth performed best across all instances. For ideal circuits, optimal p ranged from 2 to 5; noisy circuits typically peaked at $p = 1$–3.

5. Conclusion

The results demonstrate the fundamental depth–noise tradeoff inherent in QAOA on near-term devices. While deeper circuits increase variational expressivity, noise accumulation particularly from two-qubit gates, quickly negates these benefits. For the IBM Torino noise model, optimal depths were surprisingly low ($p \approx 2$–3), and no instance achieved convergence to the global optimum by $p = 6$.