ROB 501 Math for Robotics HW #6.

(1) Applying the Gram Schmidt process we have

$v^1 = y^1$

given $\langle x, y \rangle = x^T y$

$v^2 = y^2 - \dfrac{\langle y^2, v^1 \rangle}{\langle v^1, v^1 \rangle} v^1 = .$

$v^3 = y^3 - \dfrac{\langle y^3, v^1 \rangle}{\langle v^1, v^1 \rangle} v^1 - \dfrac{\langle y^3, v^2 \rangle}{\langle v^2, v^2 \rangle} v^2$

$y^1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \quad y^2 = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix}, \quad y^3 = \begin{bmatrix} 4 \\ -4 \\ 6 \end{bmatrix}$

$\Rightarrow v^1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$v^2 = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \dfrac{(y_2)^T v^1}{(v^1)^T v^1} \cdot v^1 = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \dfrac{\begin{bmatrix} 0 & 4 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}{\begin{bmatrix} -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}} \cdot \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \left[\dfrac{(0) + 4 - 1}{(2)^2 + (1)^2 + (1)^2} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix} \right] \Rightarrow \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \dfrac{3}{6} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow$

$$\Rightarrow v^2 = \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} -1 \\ 1/2 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{7}{2} \\ -\frac{3}{2} \end{bmatrix} = v^2$$

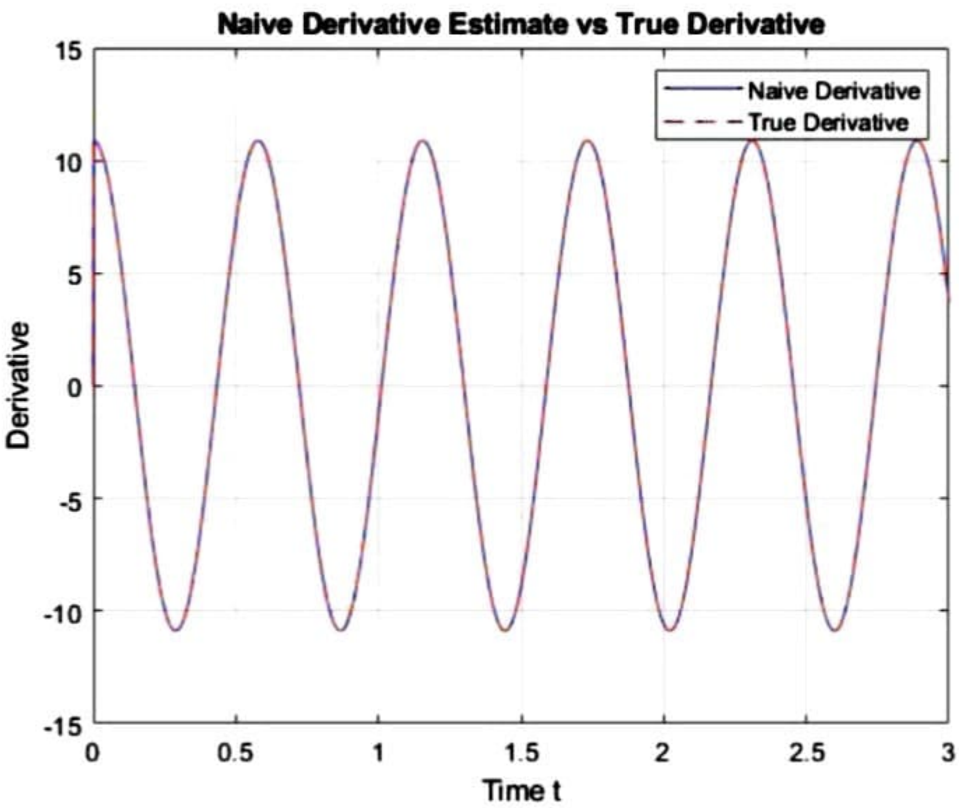
$$v^3 = y^3 - \left[\frac{(y^3)^T v^2}{\|v^2\|^2}\right] v^2 - \left[\frac{(y^3)^T v^1}{\|v^1\|^2}\right] v^1$$

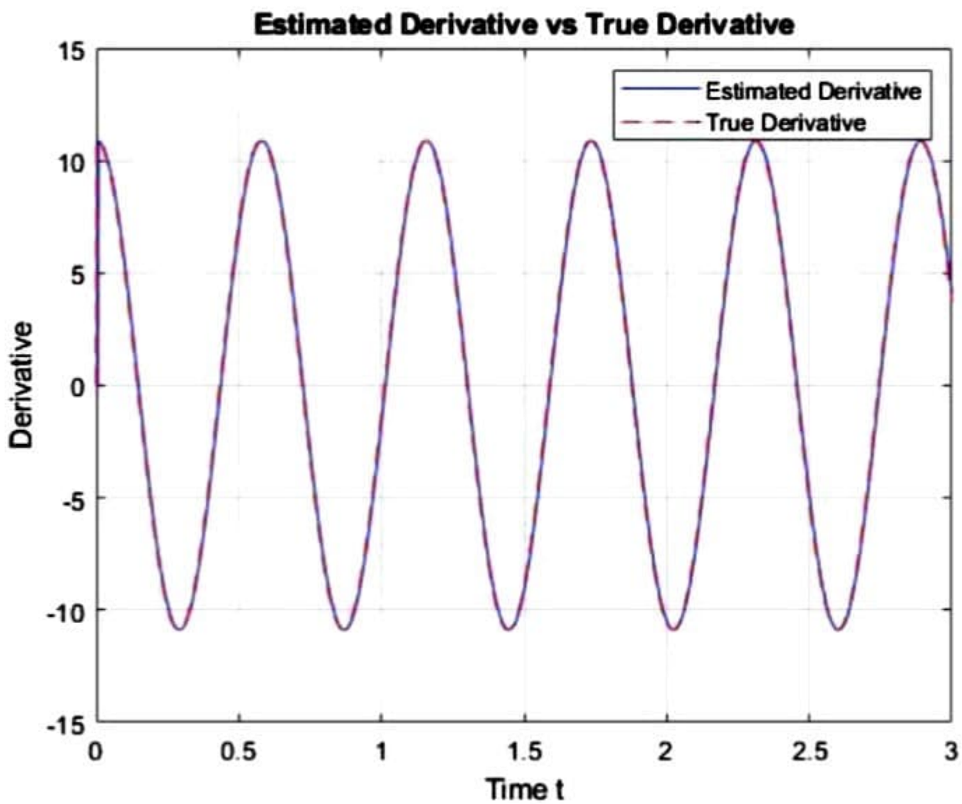
$$v^3 = \begin{bmatrix} 4 \\ -4 \\ 6 \end{bmatrix} - \left[\frac{\begin{bmatrix} 4 & -4 & 6 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{7}{2} \\ -\frac{3}{2} \end{bmatrix}}{(1)^2 + (3.5)^2 + (1.5)^2}\right]\begin{bmatrix} 1 \\ \frac{7}{2} \\ -\frac{3}{2} \end{bmatrix}$$

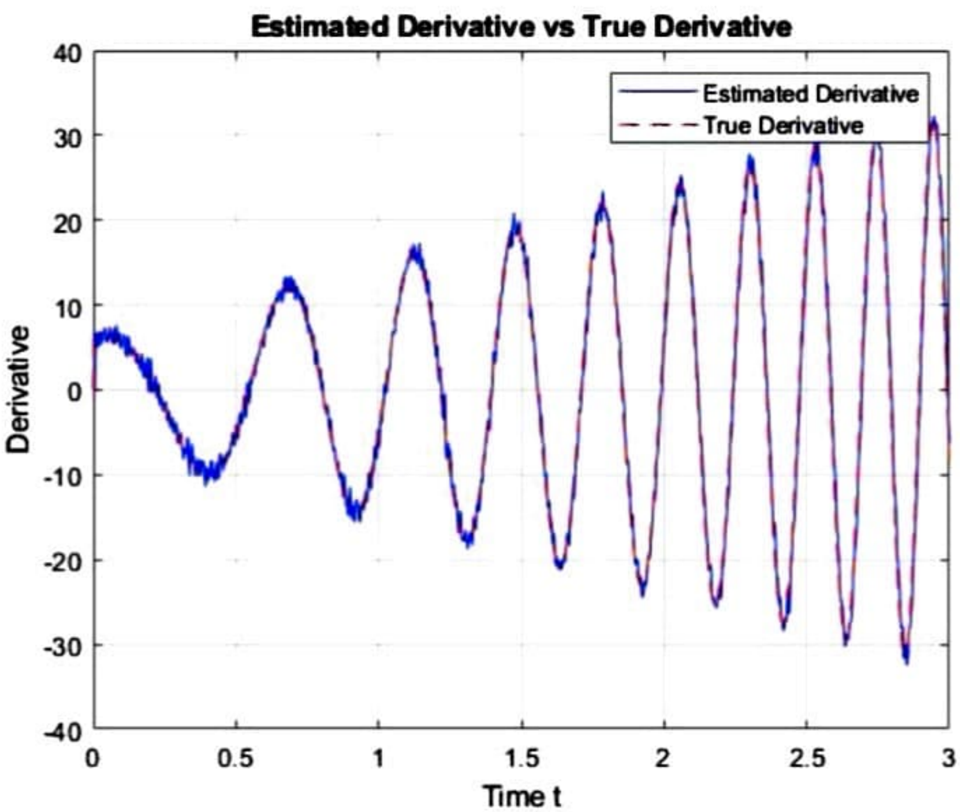
$$- \left[\frac{\begin{bmatrix} 4 & -4 & 6 \end{bmatrix}\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}}{4 + 1 + 1}\right]\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

$$v^3 = \begin{bmatrix} 4 \\ -4 \\ 6 \end{bmatrix} - \left(-1.2258\begin{bmatrix} 1 \\ 3.5 \\ 1.5 \end{bmatrix}\right) - (-1)\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

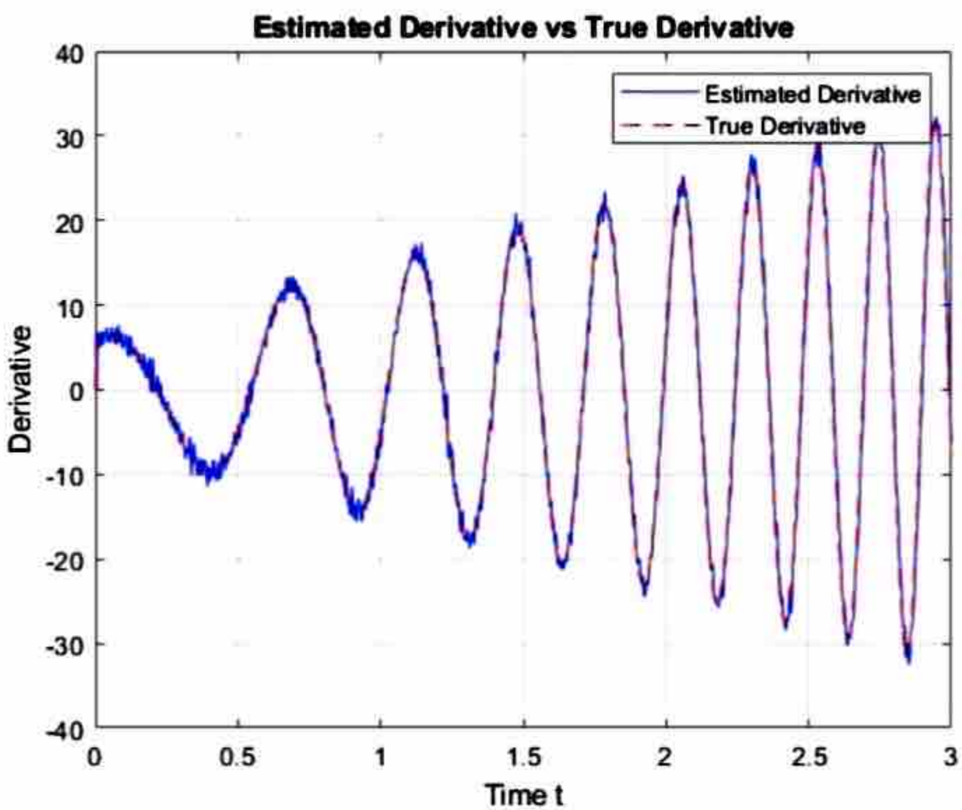
$$v^3 = \begin{bmatrix} 3.2258 \\ 1.2903 \\ 5.1613 \end{bmatrix} \qquad v^1, v^2, v^3 = \left\{\begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3.5 \\ -1.5 \end{bmatrix}, \begin{bmatrix} 3.2258 \\ 1.2903 \\ 5.1613 \end{bmatrix}\right\}$$


Naive Derivative Estimate vs True Derivative
Naive Derivative
True Derivative
15
10
5
0
-5
-10
-15
Derivative
0
0.5
1
1.5
2
2.5
3
Time t


Estimated Derivative vs True Derivative
Estimated Derivative
True Derivative
Derivative
Time t
15
10
5
0
-5
-10
-15
0
0.5
1
1.5
2
2.5
3


Estimated Derivative vs True Derivative
Estimated Derivative
True Derivative
Derivative
Time t
40
30
20
10
0
-10
-20
-30
-40
0
0.5
1
1.5
2
2.5
3

501_p3_a_contnued.pdf



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RMSE for Naive Estimate: 3.5854
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RMSE for Regression Model: 1.5676
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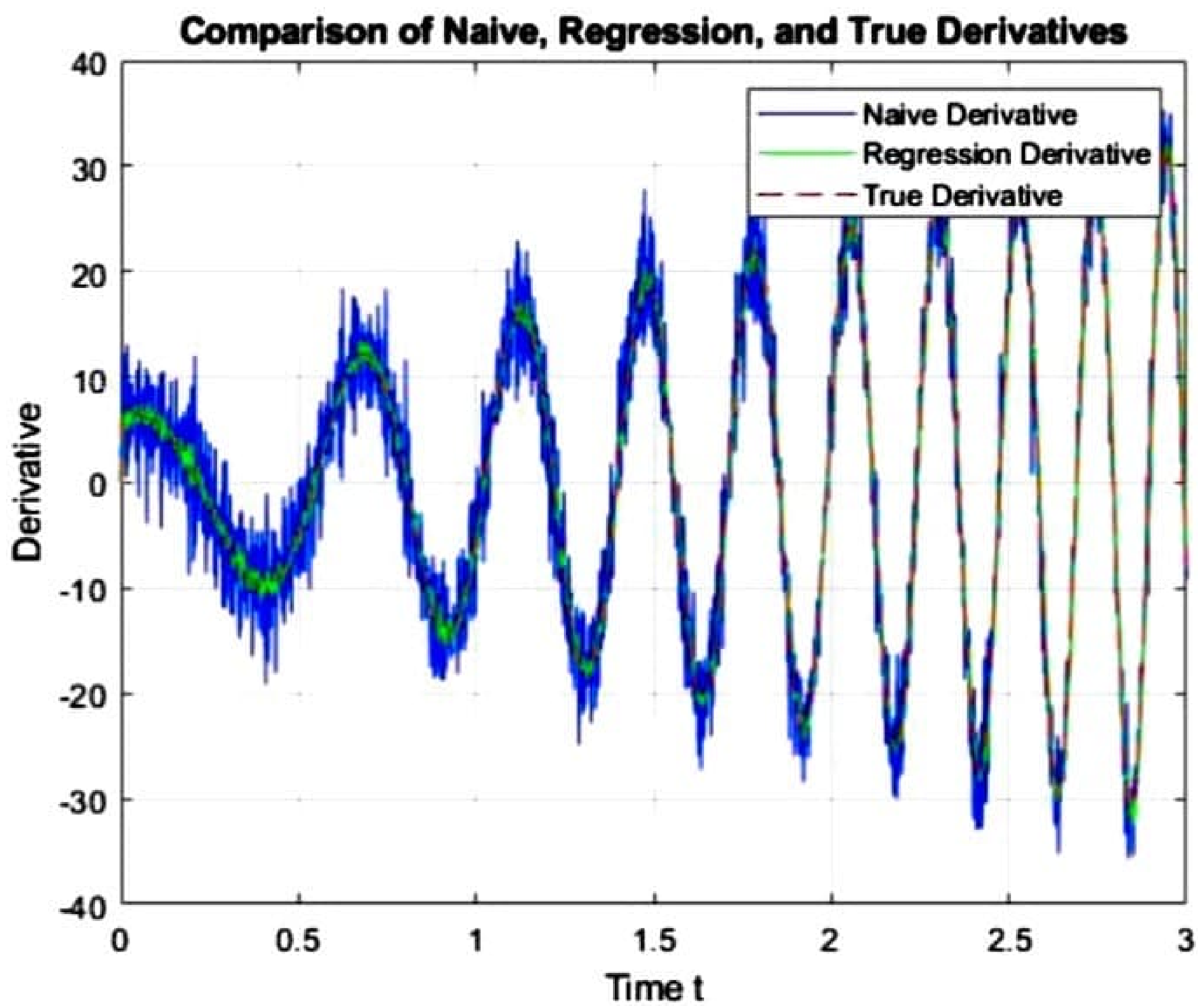

4. consider the vector space of real 2x2 matrices, with inner product $\langle A, B\rangle = \mathrm{tr}(A^T B)$

$$y^1 = \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}, \quad y^2 = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\hat{x} = \underset{y \in M}{\arg\min} \|x - y\| \text{ when } \quad x = \begin{bmatrix} 0 & -1 \\ 2 & 0 \end{bmatrix}$$

$$\cancel{G = \begin{bmatrix} \langle y^1, y^1\rangle & \langle y^2, y^3\rangle \\ \langle & \end{bmatrix}}$$

$$G = \begin{bmatrix} \langle y^1, y^1\rangle & \langle y^1, y^2\rangle \\ \langle y^2, y^1\rangle & \langle y^2, y^2\rangle \end{bmatrix}$$

$$\beta = \begin{bmatrix} \langle x, y^1\rangle \\ \langle x, y^2\rangle \end{bmatrix} \qquad \alpha = \begin{bmatrix} \alpha^1 \\ \alpha^2 \end{bmatrix}$$

$$\langle y^1, y^1\rangle = \mathrm{tr}\left\{\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}\right\} = \mathrm{tr}\left\{\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}\right\}$$

$$= \mathrm{tr}\left\{\begin{bmatrix} 1+4 & 0 \\ 0 & 0 \end{bmatrix}\right\} = 5$$

$$\langle y^1, y^2\rangle = \mathrm{tr}\left\{\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix}^T \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\} = \mathrm{tr}\left\{\begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right\}$$

$$= \mathrm{tr}\left\{\begin{bmatrix} 3 & 3 \\ 0 & 0 \end{bmatrix}\right\} = 3$$

$$\langle y^2, y^1\rangle = \text{TR}\left\{\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}^T \begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}\right\}$$

$$= \text{TR}\left\{\begin{bmatrix}3 & 0\\ 3 & 0\end{bmatrix}\right\} = 3$$

$$\langle y^2, y^2\rangle = \text{TR}\left\{\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}^T \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\} = \text{TR}\left\{\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}\right\} = 4$$

$$G = \begin{bmatrix}5 & 3\\ 3 & 4\end{bmatrix}$$

$$G^T = \begin{bmatrix}5 & 3\\ 3 & 4\end{bmatrix}^T = \begin{bmatrix}5 & 3\\ 3 & 4\end{bmatrix}$$

$$\langle x, y^1\rangle = \text{TR}\left\{\begin{bmatrix}0 & -1\\ 2 & 0\end{bmatrix}^T \begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}\right\} = \text{TR}\left\{\begin{bmatrix}0 & 2\\ -1 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\ 2 & 0\end{bmatrix}\right\}$$

$$= \text{TR}\left\{\begin{bmatrix}4 & 0\\ -1 & 0\end{bmatrix}\right\} = 4$$

$$\langle x, y^2\rangle = \text{TR}\left\{\begin{bmatrix}0 & -1\\ 2 & 0\end{bmatrix}^T \begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\} = \text{TR}\left\{\begin{bmatrix}0 & 2\\ -1 & 0\end{bmatrix}\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\right\}$$

$$= \text{TR}\left\{\begin{bmatrix}2 & 2\\ -1 & -1\end{bmatrix}\right\} = 1$$

$$\beta = \begin{bmatrix} \langle x, y^1 \rangle \\ \langle x, y^2 \rangle \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}.$$

$G^T \alpha = \beta$ find the values of α from this eq^n.

$$\begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{11} \begin{bmatrix} +(4) & -(3) \\ -(3) & +(5) \end{bmatrix}^T$$

$$\begin{bmatrix} 5 & 3 \\ 3 & 4 \end{bmatrix}^{-1} = \frac{1}{11} \begin{bmatrix} 4 & -3 \\ -3 & 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \frac{1}{11} \underbrace{\begin{bmatrix} 4 & -3 \\ -3 & 5 \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} 4 \\ 1 \end{bmatrix}}_{2\times 1}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 16 & -3 \\ -12 & +5 \end{bmatrix} = \frac{1}{11} \begin{bmatrix} 13 \\ -7 \end{bmatrix}$$

$\alpha_1 = \frac{13}{11}$; $\alpha_2 = \frac{-7}{11}$

The $\underset{\text{sol}^n}{\text{arg.}}$ which min. $\|x - y\|$ is . $\hat{x}$

$$\hat{x} = \alpha_1 y^1 + \alpha_2 y^2$$

$$= \frac{13}{11}\begin{bmatrix} 1 & 0 \\ 2 & 0 \end{bmatrix} + \frac{-7}{11}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{13}{11} & 0 \\ \frac{26}{11} & 0 \end{bmatrix} + \begin{bmatrix} \frac{-7}{11} & \frac{-7}{11} \\ \frac{-7}{11} & \frac{-7}{11} \end{bmatrix}.$$

$$\hat{x} = \begin{bmatrix} \frac{6}{11} & -\frac{7}{11} \\ \frac{19}{11} & -\frac{7}{11} \end{bmatrix}$$

(5) let $m_1 \in M$; & $m_2 \in M$. both satisfy the property

$$\inf \|x-y\| \le \left\|x - \frac{m_1+m_2}{2}\right\| \quad (y \in M)$$

$$= \left\|\frac{x-m_1}{2} + \frac{x-m_2}{2}\right\| \le \frac{1}{2}\|x-m_1\| + \frac{1}{2}\|x-m_2\|$$

Also by def^n of strict norm.

$$\left\|\frac{x-m_1}{2} + \frac{x-m_2}{2}\right\| = \frac{1}{2}\|x-m_1\| + \frac{1}{2}\|x-m_2\|$$

where $(x-m_1) = \alpha(x-m_2)$ $(\alpha \geq 0)$

Also, $\left\|x - \frac{m_1+m_2}{2}\right\| = \gamma$

$$\left\|x - m_1 + m_1 - \frac{m_1+m_2}{2}\right\| = \gamma$$

$$\|x-m_1\| + \left\|m_1 - \frac{m_1+m_2}{2}\right\| = \gamma \text{ (strict norm)}$$

Also IIIly

$$\left\|x - m_2 + m_2 - \frac{m_1+m_2}{2}\right\| = \gamma$$

$$\|x-m_2\| + \left\|m_2 - \frac{m_1+m_2}{2}\right\| = \gamma \quad \text{(strict norm)}$$

Since $\|x-m_1\| = \|x-m_2\| = \gamma$

we have $m_2 - \cancel{\left(\frac{m_1+m_2}{2}\right)} = m_1 - \cancel{\frac{m_1+m_2}{2}}$

$\Rightarrow m_2 = m_1.$

(6)

(b) $\|x\|_2 = \sqrt{x_1^2 + x_2^2}$

$\|x_1\|_2 + \|y_1\|_2 = \sqrt{a_1^2 + b_1^2} + \sqrt{a_2^2 + b_2^2} = m$

$\|x_1 + y_1\|_2 = \sqrt{(a_1 + a_2)^2 + (b_1 + b_2)^2} = n$

$a_1^2 + b_1^2 + a_2^2 + b_2^2 + 2\sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2)^2}$
$= a_1^2 + a_2^2 + 2a_1a_2 + b_1^2 + b_2^2 + 2b_1b_2$

$= a_1a_2 + b_1b_2 = \sqrt{(a_1^2 + b_1^2)(a_2^2 + b_2^2)}$

$a_1^2a_2^2 + b_1^2b_2^2 + 2a_1a_2b_1b_2 = (a_1a_2)^2 + (b_1b_2)^2 + a_1^2b_2^2 + b_1^2a_2^2$

$\Rightarrow a_1^2b_2^2 + b_1^2a_2^2 - 2a_1a_2b_1b_2 = 0$

$= (a_1b_2 - b_1a_2)^2 = 0$

$a_1b_2 = b_1a_2$

$\Rightarrow \frac{a_1}{b_1} = \frac{a_2}{b_2} \Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2}$

$\therefore \|x\|_2$ is a strict norm.

(c) $\|x\|_\infty = \max\{|x_1|, |x_2|\}$

$\|x_1\| + \|y_1\| = \max\{|x_1|\} + \max\{|x_2|\}$

$\|x_1 + y_1\| = \max\{|x_1 + y_1|\}$

$\|x_1\| + \|y_1\| = \max\{|a_1|, |b_1|\} + \max\{|a_2|, |b_2|\}$

$\|x_1 + y_1\| = \max\{|a_1 + a_2|, |b_1 + b_2|\}$

Consider the case $a_1 > b_1$; $a_2 > b_2$

$\|x_1\| + \|y_1\| = |a_1| + |a_2| = a_1 + a_2$

$\|x_1 + y_1\| = |a_1 + a_2| = a_1 + a_2.$

$\Rightarrow \|x_1 + y_1\| = \|x_1\| + \|y_1\|$

$\therefore$ $\|x\|_\infty$ is not a strict norm.

```matlab
function F = matrix_inversion_lemma(A_inv, B, C, D)
    % This function implements the Matrix Inversion Lemma:
    % (A + BCD)^(-1) = A^(-1) - A^(-1)B(C^(-1) + DA^(-1)B)^(-1)DA^(-1)

    % Inputs:
    % A_inv - Inverse of matrix A (A^(-1)) [Assumed given]
    % B, C, D - Matrices involved in the lemma

    % Output:
    % F - The result of the Matrix Inversion Lemma

    % Step 0: Check dimensions of A_inv, B, and D
    [m_A, n_A] = size(A_inv);
    [m_B, n_B] = size(B);
    [m_D, n_D] = size(D);

    if n_A ~= m_A
        error('A_inv must be a square matrix.');
    end

    if m_A ~= m_B
        error('The number of rows in B must match the number of rows in A_inv.');
    end

    if n_B ~= m_D
        error('The number of columns in B must match the number of rows in D.');
    end

    if n_D ~= m_A
        error('The number of columns in D must match the number of columns in A_inv.');
    end

    % Step 1: Compute the inverse of C
    if isscalar(C)
        C_inv = 1 / C;  % If C is a scalar, simply take the reciprocal
    else
        C_inv = inv(C);  % Otherwise, compute the matrix inverse
    end

    % Step 2: Compute the term (C^(-1) + DA^(-1)B)
    intermediate_term = C_inv + D * A_inv * B;

    % Step 3: Compute the inverse of the intermediate term
    intermediate_term_inv = inv(intermediate_term);

    % Step 4: Apply the matrix inversion lemma formula
    F = A_inv - A_inv * B * intermediate_term_inv * D * A_inv;
end

% Test the function with given matrices
A_inv = diag([2, 1, 1, 2, 1]);  % A_inv is the inverse of A
B = [3; 0; 2; 0; 1];
C = 0.25;  % C is a scalar
D = transpose(B);
```

```matlab
F = matrix_inversion_lemma(A_inv, B, C, D);
disp(F);
```

```
    0.6667         0   -0.4444         0   -0.2222
         0    1.0000         0         0         0
   -0.4444         0    0.8519         0   -0.0741
         0         0         0    2.0000         0
   -0.2222         0   -0.0741         0    0.9630
```

(8) $X = \{f \mid f : \mathbb{R} \to \mathbb{R}\}$, $\mathbb{F} = \mathbb{R}$, define inner product

$$\langle f, g \rangle = \int_{-1}^{1} f(t) \cdot g(t) \cdot dt$$

$$M = \operatorname{Span}\{\underset{y^1}{1}, \underset{y^2}{t}, \underset{y^3}{\tfrac{1}{2}(3t^2-1)}\}, \quad x = e^t$$

$$\hat{x} = \arg \min_{y \in M} \| x - y \|$$

$$G = \begin{bmatrix} \langle y^1, y^1 \rangle & \langle y^1, y^2 \rangle & \langle y^1, y^3 \rangle \\ \langle y^2, y^1 \rangle & \langle y^2, y^2 \rangle & \langle y^2, y^3 \rangle \\ \langle y^3, y^1 \rangle & \langle y^3, y^2 \rangle & \langle y^3, y^3 \rangle \end{bmatrix}$$

$$\Rightarrow \langle y^1, y^1 \rangle = \int_{-1}^{1} 1 \cdot 1 \cdot dt = t \Big]_{-1}^{1} = 2$$

$$\langle y^1, y^2 \rangle = \int_{-1}^{1} 1 \cdot t \cdot dt = \left[\frac{t^2}{2}\right]_{-1}^{1} = 0 = \langle y^2, y^1 \rangle$$

$$\langle y^1, y^3 \rangle = \int_{-1}^{1} 1 \cdot \frac{1}{2}(3t^2 - 1) \cdot dt = \cancel{\frac{3}{2}\left(\frac{t^3}{3}\right]_{-1}^{1}} - \frac{t}{2}\Big]_{-1}^{1} = 1 - 2/2 \Rightarrow 1 - 1 \Rightarrow 0$$

$$= \langle y^3, y^1 \rangle$$

$$\langle y^2, y^2 \rangle = \int_{-1}^{1} t \cdot t \cdot dt = \frac{t^3}{3}\Big]_{-1}^{1} = \frac{1}{3} - \frac{(-1)}{3} = \frac{2}{3}$$

$$\langle y^2, y^3 \rangle = \int_{-1}^{1} (t)\left(\frac{1}{2}(3t^2 - 1)\right) \cdot dt = \frac{t^4}{4} \cdot \frac{3}{2}\Big]_{-1}^{1} - \frac{t^2}{2}\Big]_{-1}^{1}$$

$$= 0 = \langle y^3, y^2 \rangle$$

$$\langle y^3, y^3\rangle = \int_{-1}^{1} \frac{(3t^2-1)(3t^2-1)}{4} \cdot dt.$$

$$= \int_{-1}^{1} \frac{9t^4 + 1 - 6t^2}{4} .. dt$$

$$= \frac{1}{4}\left(\frac{9t^5}{5} + t - \frac{6t^3}{3}\right)\Bigg]_{-1}^{1} = \frac{1}{2} \times \frac{4}{5} = 0.4.$$

$$G = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix} \qquad \alpha = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix}$$

$$\langle x, y_1\rangle = \int_{-1}^{1} e^t . 1 . dt = \left[e^t\right]_{-1}^{1} = e^1 - e^{-1}$$
$$= 2.3504.$$

$$\langle x, y_2\rangle = \int_{-1}^{1} e^t . t \, dt = \left[e^t . t - \int e^t . 1 . dt\right]_{-1}^{1}$$

$$= \left[t . e^t - e^t\right]_{-1}^{1} = e^t(t-1)\Big]_{-1}^{1} = 0 + 2e^{-1}$$
$$= +0.7357.$$

$$\langle x, y_3\rangle = \int_{-1}^{1} e^t . \frac{1}{2}(3t^2-1)$$

$$= \int_{-1}^{1} \left[e^t \frac{1}{2}(3t^2-1) - \int_{-1}^{1} e^t . 3t\right]_{-1}^{1}$$

$$= e^t \frac{1}{2}(3t^2 - 1) - \left[e^t 3t - 3\int e^t \right]$$

$$= e^t \frac{1}{2}(3t^2 - 1) - \left[\left[e^t 3t - 3e^t \right] \right]_{-1}^{1}$$

$$e^t\left(\frac{3t^2}{2} - \frac{1}{2}\right) - 3te^t + 3e^t$$

$$e^t\left[\frac{3t^2}{2} - 3t + 3 - \frac{1}{2}\right]_{-1}^{1}$$

$$e^t\left[\frac{3t^2}{2} - 3t + \frac{5}{2}\right]_{-1}^{+1}$$

$$= e^t\left[\frac{3t^2 - 6t + 5}{2}\right]_{-1}^{+1}$$

$$\cancel{e^t\left[\frac{-6(2)+5}{2}\right]}$$

$$\Rightarrow e^1\left[\frac{(3) - (6) + 5}{2}\right] - e^{-1}\left[\frac{3(-1)^2 - 6(-1) + 5}{2}\right]$$

$$e\left[\frac{2}{2}\right] - e^{-1}\left[\frac{3+6+5}{2}\right]$$

$$\langle x, y_3 \rangle = e^1 - e^{-1}[7]$$

$$\langle x, y_3 \rangle = 0.1431$$

$$G^T \alpha = \beta$$

$$G^T = G = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 2.3504 \\ 0.7357 \\ 0.1431 \end{bmatrix}$$

$$\begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2/3 & 0 \\ 0 & 0 & 0.4 \end{bmatrix}^{-1} \begin{bmatrix} 2.3504 \\ 0.7357 \\ 0.1431 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 1.5 & 0 \\ 0 & 0 & 2.5 \end{bmatrix} \begin{bmatrix} 2.3504 \\ 0.7357 \\ 0.1431 \end{bmatrix}$$

$$0.5 \times 2.3504 = \alpha_1 = 1.1752$$

$$1.5 \times 0.7357 = \alpha_2 = 1.10355$$

$$2.5 \times 0.1431 = \alpha_3 = 0.35775$$

$$\hat{x} = \alpha_1 + \alpha_2(t) + \alpha_3\left(\frac{1}{2}(3t^2 - 1)\right)$$

$$= 1.1752 + 1.10355(t) + 0.35775\left(\frac{1}{2}(3t^2 - 1)\right)$$

$$\hat{x} = 1.1752 + 1.10355(t) + 0.536625t^2 - 0.178875$$

$$\hat{x} = 0.536625t^2 + 1.10355(t) + 0.996325$$