ROB-501 Assignment-10 18/11/2024.

① ⓐ $f(x_1, x_2, x_3) = 3x_1\left[2x_2 - (x_3)^3\right] + \frac{(x_2)^4}{3}$

$\frac{\partial f(x)}{\partial x} := \left[\frac{\partial f(x)}{\partial x_1}, \frac{\partial f(x)}{\partial x_2}, \frac{\partial f(x)}{\partial x_3}\right].$

$\frac{\partial f(x)}{\partial x_1} = 6x_2 - 3\cancel{x_1}(x_3)^3$

$\frac{\partial f}{\partial x_2} = 6x_1 + \frac{4}{3}x_2^3$

$\frac{\partial f}{\partial x_3} = -9x_1 x_3^2$

$\Rightarrow \frac{\partial f(x)}{\partial x} = \left[3(2x_2 - \cancel{x_1}(x_3)^3), 6x_1 + \frac{4}{3}x_2^3, -9x_1 x_3^2\right]$

$\frac{\partial f(x)}{\partial x}$ at $x^* = [1, 3, -1]^T = \left[3(6+1), 6 + \frac{4}{3}(+3)^3, -9\right]$

$= [21, 42, -9]$

(b) let $\delta_1 = \begin{bmatrix} 0.001 \\ 0 \\ 0 \end{bmatrix}$

$$\frac{\partial f(x^*)}{\partial x_1} = \frac{f(x^* + \delta_1) - f(x^* - \delta_1)}{2\delta_1} \quad \text{- - - - (1)}$$

$$x^* + \delta_1 = \begin{bmatrix} 1.001 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} ; \quad x^* - \delta_1 = \begin{bmatrix} 0.999 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

sub these values to the function

$$f(x) = f(x_1, x_2, x_3) = 3x_1 \left[2x_2 - (x_3)^3\right] + \frac{x_2^4}{3}$$

$$f(x^* + \delta_1) = 48.0210$$

$$f(x^* - \delta_1) = 49.9990$$

$\therefore$ Eq^n (1) becomes.

$$\frac{\partial f(x^*)}{\partial x_1} = \frac{48.0210 - 4\cancel{9}.\cancel{9}990}{2(0.001)} = 21.000$$

||ly let $\delta_2 = \begin{bmatrix} 0 \\ 0.001 \\ 0 \end{bmatrix}$

$$\frac{\partial f(x^*)}{\partial x_2} = \frac{f(x^* + \delta_2) - f(x^* - \delta_2)}{2\delta_2} = \frac{48.0420 - 47.958}{0.002}$$

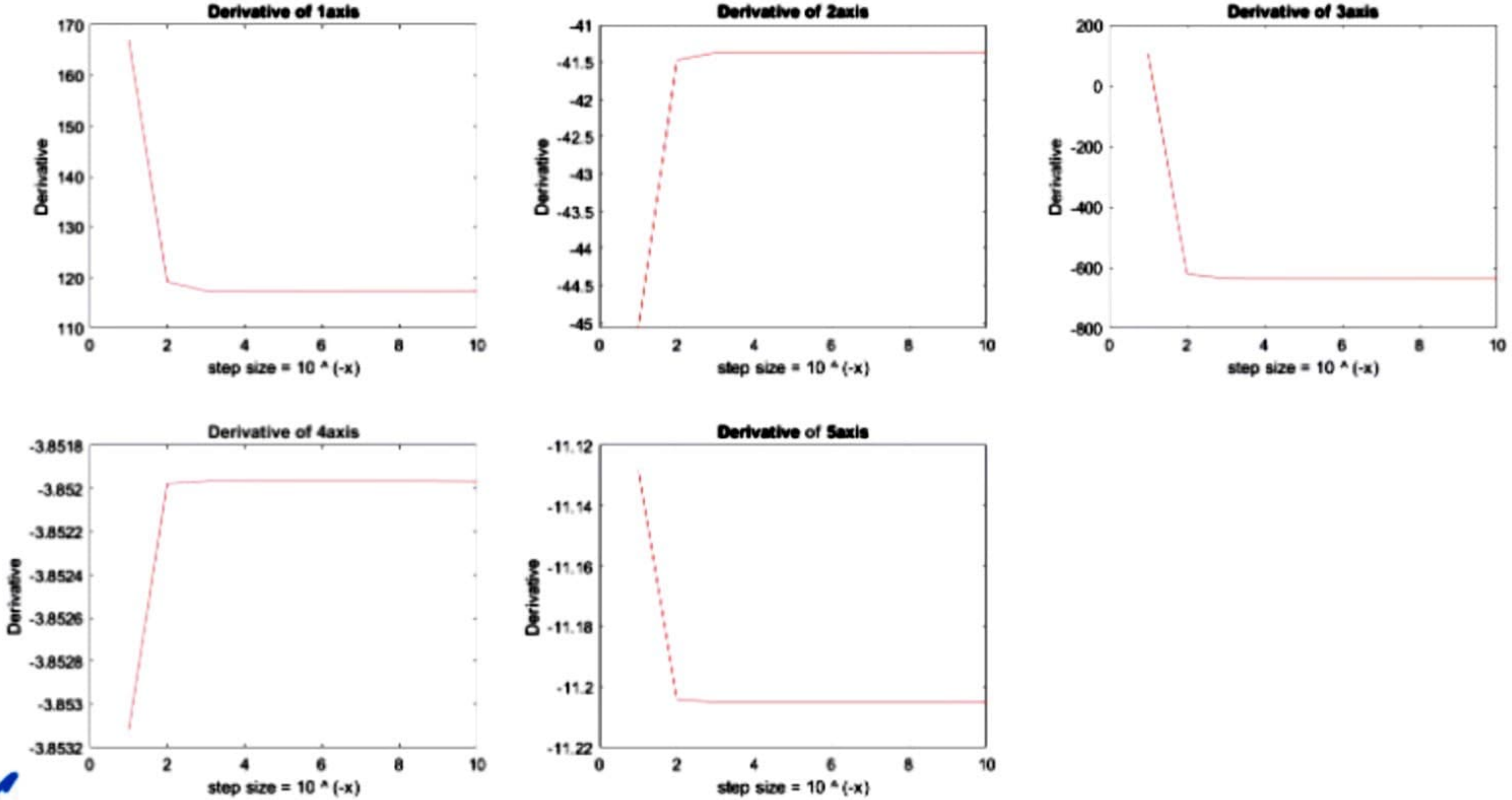
$$= 42.0000$$

let $\delta_3 = \begin{bmatrix} 0 \\ 0 \\ 0.001 \end{bmatrix}$

$$\frac{\partial f(x^*)}{\partial x_3} = \frac{f(x^* + \delta_3) - f(x^* - \delta_3)}{2\delta_3} = \frac{47.9910 - 48.009}{0.002}$$

$$= -9.0000$$

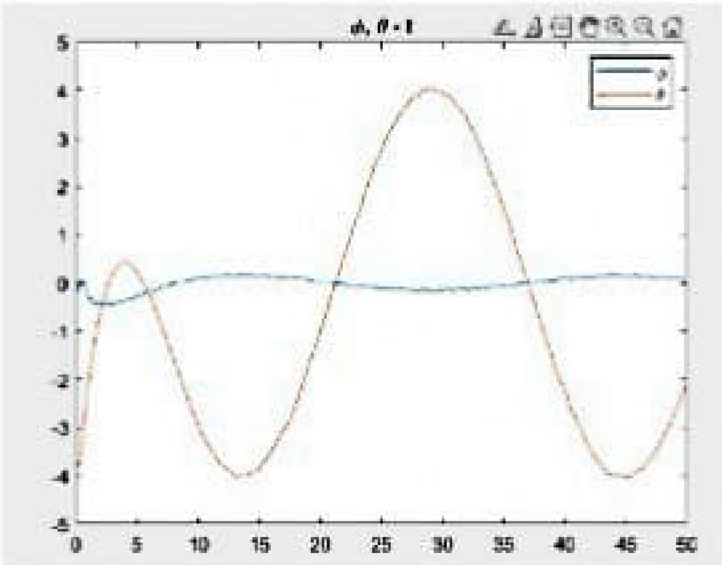
$\Rightarrow$ we get the same value as (a).

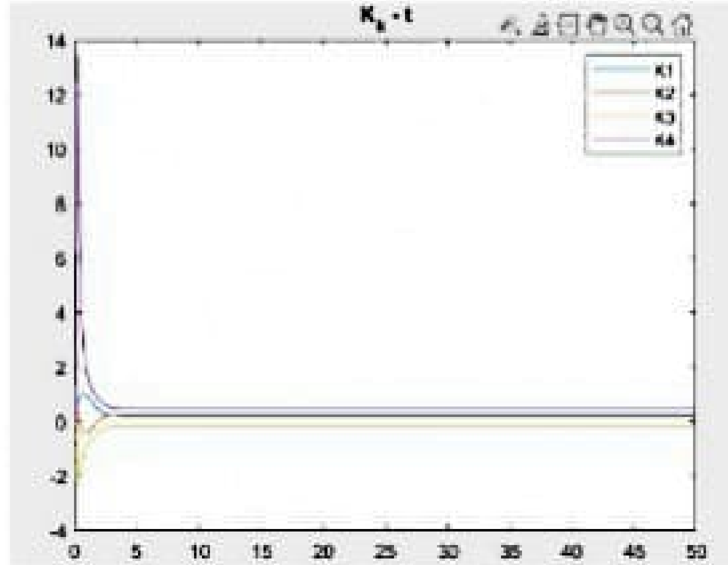

Derivative of 1axis
Derivative
step size = 10 ^ (-x)
Derivative of 2axis
Derivative
step size = 10 ^ (-x)
Derivative of 3axis
Derivative
step size = 10 ^ (-x)
Derivative of 4axis
Derivative
step size = 10 ^ (-x)
Derivative of 5axis
Derivative
step size = 10 ^ (-x)

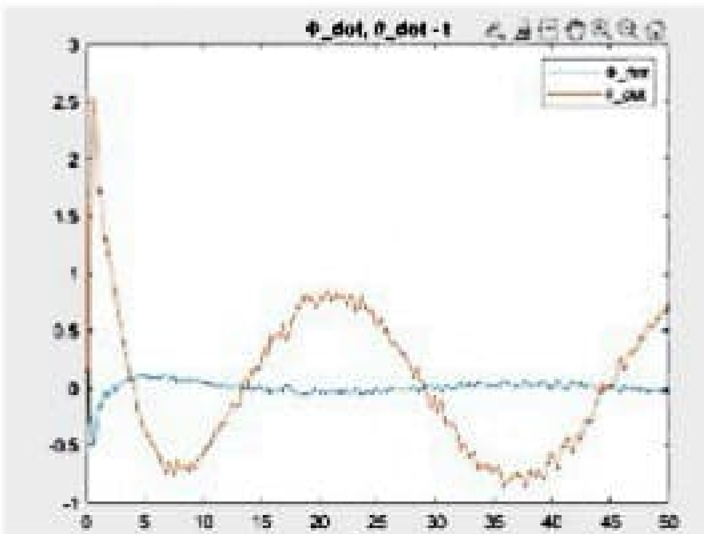
1(c) Jacobian at $x^{0} = [1, 1, 1, 1, 1]$

$$= [119.3523, \ -41.3685, \ -636.6021, \ -3.8520, \ -11.2049]$$

```
load SegwayData4KF.mat

phi = zeros(N, 1);
theta = zeros(N, 1);
phi_dot = zeros(N, 1);
theta_dot = zeros(N, 1);
K_1 = zeros(N, 1);
K_2 = zeros(N, 1);
K_3 = zeros(N, 1);
K_4 = zeros(N, 1);

x1 = x0;
P1 = P0;
tic
t=zeros(1,N);
for k =1:N
    uk = u(k);
    yk = y(k);

    K = P1 * C' / (C * P1 * C' + Q)
    x1 = A * x1 + B * uk + A * K * (yk - C * x1);
    P1 = A * (P1 - K * C * P1) * A' + G * R * G';

    x1_hat = x1;
    P1_hat = P1;


    phi(k)=[1 0 0 0] * x1_hat;
    theta(k)=[0 1 0 0] * x1_hat;
    phi_dot(k) = [0 0 1 0] * x1_hat;
    theta_dot(k) = [0 0 0 1] * x1_hat;
    K_1(k) = K(1);
    K_2(k) = K(2);
    K_3(k) = K(3);
    K_4(k) = K(4);

    t(k)=k*Ts;
    x1=x1_hat;
    P1 = P1_hat;
end
```




K1
K2
K3
K4



```
K =                    Kss =

    0.2113                  0.2113
    0.2559                  0.2559
   -0.1744                 -0.1744
    0.4816                  0.4816
```

(3) $x_1 \sim N(\mu_1, \Sigma_1) = N(1.7156, 0.0213)$

```
A = 1;
B = 0.1;
u = 10;
R = 16;
c = 3 * (10 ^ 8);
C = -2 / c;
Q = 10 ^ (-18);

z_1 = 2.2 * (10 ^ (-8));


X_0 = 1;
P_0 = 0.25;
X_hat = A * X_0 + B * u;

z_hat = 2 / c * (5 - X_hat);

P_hat = A * P_0 * A' + B * R * B';
K = P_hat * C' / (C * P_hat * C' + Q);
X_1_hat = X_hat + K * (z_1 - z_hat)
P_1_hat = P_hat -  K * C * P_hat
```

(4) $\hat{x} = \underset{x^T x = 1}{\operatorname{argmin}} \; x^T A^T A x$

let $\sigma = \underset{x^T x = 1}{\min} \; x^T A^T A x$

$\Rightarrow x\sigma = \sigma x = A^T A x$

σ = e-value, ~~and $A^T A$ = e-vectors of x~~

x is the e-vectors of $A^T A$.

Since we want e-values to be minimized, we pick the corresponding e-vectors to satisfy the condition.

$\because$ The smallest e-value is at the bottom right corner of Σ

$\Rightarrow$ The corresponding e-vectors is the last column of V

[$\because$ columns of V are evectors of $A^T A$]

(5). $\hat{A} = \begin{bmatrix} 4.0420 & 7.0450 & 3.0150 \\ 10.0426 & 17.0395 & 7.0245 \\ 16.0073 & 27.0037 & 11.0493 \end{bmatrix}$

$$\Delta A = \begin{bmatrix} 0.0010 & -0.0010 & 0.0010 \\ -0.0024 & 0.0025 & -0.0025 \\ 0.0013 & -0.0013 & 0.0013 \end{bmatrix}$$

$$\|\Delta A\|_2 = \sqrt{\lambda_{max}(\Delta A^T \Delta A)}.$$

$$= 0.0051$$