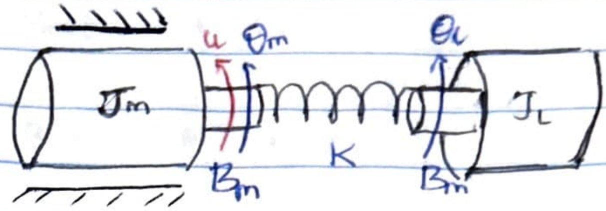
ROB-599 - ASSIGNMENT - 4.

02/11/2024



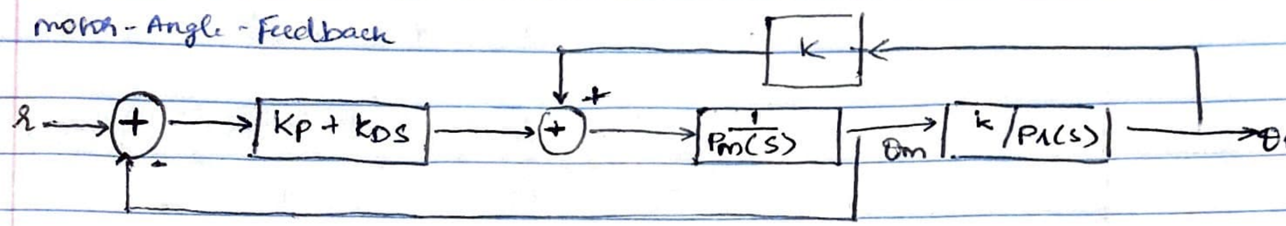
The eq^n of motion of the motor side is

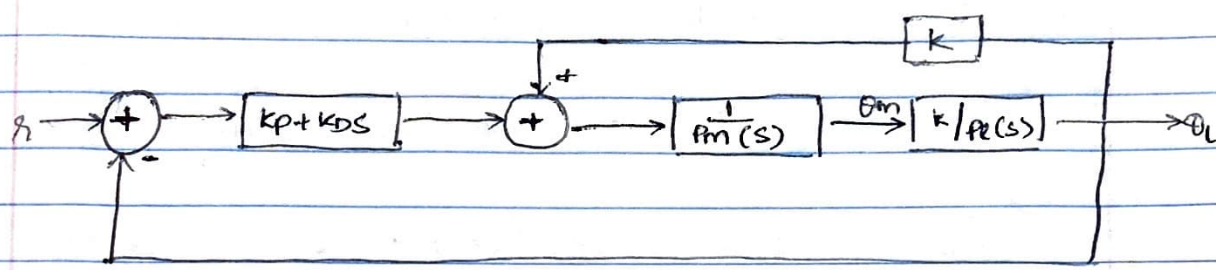
$$J_m \ddot{\theta}_m + B_m \dot{\theta}_m - k(\theta_L - \theta_m) = u$$

The eq^n of motion in the load side is

$$J_L \ddot{\theta}_L + B_L \dot{\theta}_L + k(\theta_L - \theta_m) = 0$$

motor - Angle - Feedback





Load - angle - Feedback

(a) Motor Angle - Feedback $\Rightarrow u = k_p(\theta_{cmd} - \theta_m) - k_d \dot{\theta}_m$

$$\cancel{u = k_p(\theta_{cmd} - \theta_L) - k_d \dot{\theta}_L}$$

$$P_m(s) = J_m s^2 + B_m s + k \; ; \quad P_L(s) = J_L s^2 + B_L s + k$$

(b) load Angle - Feed back

$$u = k_p(\theta_{cmd} - \theta_L) - k_d \dot{\theta}_L$$

The eq^n of motions are :-

$$J_m \ddot{\theta}_m + B_m \dot{\theta}_m - k(\theta_l - \theta_m) = u \quad ---- \ (1)$$

$$J_l \ddot{\theta}_l + B_l \dot{\theta}_l + k(\theta_l - \theta_m) = 0 \quad ---- \ (2)$$

$$x_1 = \theta_m \ ; \ x_2 = \dot{\theta}_m \ , \ x_3 = \theta_l \ ; \ x_4 = \dot{\theta}_l \quad --- \ (3)$$

$$\dot{x}_1 = \dot{\theta}_m = x_2 \ ; \ \dot{x}_2 = \ddot{\theta}_m \ ; \ \dot{x}_3 = \dot{\theta}_l = x_4 \ ; \ \dot{x}_4 = \ddot{\theta}_l \quad --- \ (4)$$

Sub^n (3) + (4) in (1) + (2)

$$J_m \dot{x}_2 + B_m x_2 - k(x_3 - x_1) = u \quad --- \ (5)$$

$$J_l \dot{x}_4 + B_l x_4 + k(x_3 - x_1) = 0 \quad ---- \ (6)$$

Case - 1 (Motor Angle Feed back),

$$u = k_p(\theta_{cmd} - \theta_m) - k_d \dot{\theta}_m$$

$$u = k_p(\theta_{cmd} - x_1) - k_d(x_2) \quad ----- \ (7)$$

Sub (7) in (5) we get.

$$J_m \dot{x}_2 + B_m x_2 - k(x_3 - x_1) = k_p(\theta_{cmd} - x_1) - k_d(x_2)$$

$$\dot{x}_2 = \frac{k_p(\theta_{cmd} - x_1) - k_d(x_2) - B_m x_2 + k(x_3 - x_1)}{J_m}$$

Rearranging the terms we get.

$$\dot{x}_2 = \frac{K_p \theta_{cmd} - (k_p + k) x_1 - (k_d + B_m) x_2 + k x_3}{J_m} \quad ---- \ (8)$$

from (6) we get $\dot{x}_4$

$$\dot{x}_4 = \frac{-B_l x_4 - k(x_3 - x_1)}{J_l} \quad ---- \ (9)$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{K_p\theta_{cmd} - (k_p + k)x_1 - (k_d + B_m)x_2 + k x_3}{J_m} \\ x_4 \\ -B_\ell x_4 - \dfrac{k(x_3 - x_1)}{J_\ell} \end{bmatrix}$$

$$A = \frac{\partial F}{\partial x} = \begin{bmatrix} \dfrac{\partial F_1}{\partial x_1} & \dfrac{\partial F_1}{\partial x_2} & \dfrac{\partial F_1}{\partial x_3} & \dfrac{\partial F_1}{\partial x_4} \\[2ex] \dfrac{\partial F_2}{\partial x_1} & \dfrac{\partial F_2}{\partial x_2} & \dfrac{\partial F_2}{\partial x_3} & \dfrac{\partial F_2}{\partial x_4} \\[2ex] \dfrac{\partial F_3}{\partial x_1} & \dfrac{\partial F_3}{\partial x_2} & \dfrac{\partial F_3}{\partial x_3} & \dfrac{\partial F_3}{\partial x_4} \\[2ex] \dfrac{\partial F_4}{\partial x_1} & \dfrac{\partial F_4}{\partial x_2} & \dfrac{\partial F_4}{\partial x_3} & \dfrac{\partial F_4}{\partial x_4} \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\[1ex] -\dfrac{(+k_p + k)}{J_m} & \dfrac{(-)(k_d + B_m)}{J_m} & \dfrac{k}{J_m} & 0 \\[2ex] 0 & 0 & 0 & 1 \\[1ex] \dfrac{k}{J_\ell} & 0 & \dfrac{-k}{J_\ell} & \dfrac{-B_\ell}{J_\ell} \end{bmatrix} \quad \text{---} \quad (10)$$

$$B = \frac{\partial F}{\partial \theta_{cmd}} = \begin{bmatrix} \dfrac{\partial F_1}{\partial \theta_{cmd}} \\[2ex] \dfrac{\partial F_2}{\partial \theta_{cmd}} \\[1ex] \partial F_3 / \partial \theta_{cmd} \\ \partial F_4 / \partial \theta_{cmd} \end{bmatrix} = \begin{bmatrix} 0 \\[1ex] \dfrac{k_p}{J_m} \\[1ex] 0 \\ 0 \end{bmatrix} \quad \text{---} \quad (11)$$

So for Case - 1, The State space Representation is given as :-
Sub (10) & (11)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(kp+k)}{Jm} & -\frac{(Bm+kd)}{Jm} & \frac{k}{Jm} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{Jl} & 0 & -\frac{k}{Jl} & -\frac{Bl}{Jl} \end{bmatrix}}_{Acl} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{kp}{Jm} \\ 0 \\ 0 \end{bmatrix}}_{Bcl} \theta_{cmd} \quad \text{---} (12)$$

Case 2 :- load angle feed back.

$$u = kp(\theta_{cmd} - \theta_l) - kd\,\dot{\theta}_l$$

Sub (3) we get.

$$u = kp(\theta_{cmd} - x_3) - kd\,x_4 \quad \text{-----} (13)$$

Sub (13) in (5) we get

$$Jm\dot{x}_2 + Bm\,x_2 - k(x_3 - x_1) = kp(\theta_{cmd} - x_3) - kd\,x_4$$

$$\dot{x}_2 = \frac{kp(\theta_{cmd} - x_3) - kd\,x_4 - Bm\,x_2 + k(x_3 - x_1)}{Jm}$$

Re arranging terms we get.

$$\dot{x}_2 = \frac{kp\,\theta_{cmd} + (k - kp)\,x_3 - kd\,x_4 - Bm\,x_2 - k\,x_1}{Jm} \quad \text{--} (14)$$

$$\dot{x}_4 = \frac{-Bl\,x_4 - k(x_3 - x_1)}{Jl}$$

$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{kp\,\theta_{cmd} + (k - kp)\,x_3 - kd\,x_4 - Bm\,x_2 - k\,x_1}{Jm} \\ x_4 \\ \dfrac{-Bl\,x_4 - k\,(x_3 - x_1)}{Jl} \end{bmatrix}
$$

$$
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k}{Jm} & -\dfrac{Bm}{Jm} & \dfrac{k-kp}{Jm} & \dfrac{-kd}{Jm} \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{Jl} & +\dfrac{Bl}{Jl} & \dfrac{-k}{Jl} & -\dfrac{Bl}{Jl} \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 \\ \dfrac{kp}{Jm} \\ 0 \\ 0 \end{bmatrix}
$$

So the State-Space eq^n for Case-2 is :-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{J_m} & -\frac{B_m}{J_m} & \frac{k-k_p}{J_m} & -\frac{k_d}{J_m} \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_l} & 0 & -\frac{k}{J_l} & -\frac{B_l}{J_l} \end{bmatrix}}_{A_{cl}} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ k_p/J_m \\ 0 \\ 0 \end{bmatrix}}_{B_{cl}} \theta_{cmd}$$

Problem-2

Case-1. $u = k_p(\theta_{cmd} - \theta_m) - k_d \dot{\theta}_m$ (motor Angle Feedback)

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{(k_p + k)}{J_m} & -\frac{(k_d + B_m)}{J_m} & \frac{k}{J_m} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{J_l} & 0 & -\frac{k}{J_l} & -\frac{B_l}{J_l} \end{bmatrix}$$

The characteristic polynomial is given as :-

$$|sI - A| = 0$$

$$|sI - A| = \begin{vmatrix} s & -1 & 0 & 0 \\ +\frac{(k_p + k)}{J_m} & s + \frac{(k_d + B_m)}{J_m} & \frac{-k}{J_m} & 0 \\ 0 & 0 & s & -1 \\ \frac{-k}{J_l} & 0 & +\frac{k}{J_l} & s + \frac{B_l}{J_l} \end{vmatrix}$$

$$|sI - A| = s \begin{vmatrix} s + \frac{(k_d + B_m)}{J_m} & -\frac{k}{J_m} & 0 \\ 0 & s & -1 \\ 0 & \frac{k}{J_l} & s + \frac{B_l}{J_l} \end{vmatrix}$$

$$-(-1) \begin{vmatrix} \frac{k_p + k}{J_m} & \frac{-k}{J_m} & 0 \\ 0 & s & -1 \\ \frac{-k}{J_l} & +\frac{k}{J_l} & s + \frac{B_l}{J_l} \end{vmatrix} + 0 + 0$$

$$= s\left[\left(s + \frac{(k_d + B_m)}{J_m}\right)\left\{(s)\left(s + \frac{B_l}{J_l}\right) - (-1)\left(\frac{k}{J_l}\right)\right\}\right]$$

$$+1\left[\frac{k_p + k}{J_m}\left((s)\left(s + \frac{B_l}{J_l}\right) + 1\left(\frac{k}{J_l}\right)\right) + \frac{k}{J_m}\left(\frac{-k}{J_l}\right)\right] = 0$$

$$s\left[\left(s+\frac{kd+Bm}{Jm}\right)\left\{(s)\left(s+\frac{Bl}{Jl}\right)+\frac{k}{Jl}\right\}\right]$$

$$+\frac{kp+k}{Jm}\left((s)\left(s+\frac{Bl}{Jl}\right)+\frac{k}{Jl}\right)+\frac{k}{Jm}\left(\frac{-k}{Jl}\right)\Big\}=0$$

$$s\left[(s)+\left(\frac{kd+Bm}{Jm}\right)\right]\left\{s^2+\frac{BL}{JL}s+\frac{k}{JL}\right\}\Big]$$

$$+\frac{kp+k}{Jm}\left(s^2+\frac{BL}{JL}s+\frac{k}{JL}\right)+\frac{k}{Jm}\left(\frac{-k}{JL}\right)=0$$

$$\cancel{s\left[s+\left(\frac{kd+Bm}{Jm}\right)s^2+\left(\frac{BL}{JL}\right)\left(\frac{kd+Bm}{Jm}\right)s+\frac{k(kd+Bm)}{Jm\,Jl}\right]}$$

$$s\left[s^3+\frac{Bl}{Jl}s^2+\frac{k}{Jl}s+\left(\frac{kd+Bm}{Jm}\right)s^2+\frac{Bl}{Jl}\left(\frac{kd+Bm}{Jm}\right)s+\frac{k}{JL}\left(\frac{kd+Bm}{Jm}\right)\right]\cancel{=0}$$

$$\cancel{+\frac{kp+k}{Jm}\left[s^2+\frac{Bl}{Jl}s+\frac{k}{Jl}+\right.}$$

$$+\frac{kp+k}{Jm}s^2+\left(\frac{Bl}{JL}\right)\left(\frac{kp+k}{Jm}\right)s+\frac{k}{JL}\left(\frac{kp+k}{Jm}\right)$$

$$-\frac{k^2}{Jm\,Jl}=0$$

$$|sI-A|=s^4+\frac{BL}{Jl}s^3+\frac{k}{Jl}s^2+\left(\frac{kd+Bm}{Jm}\right)s^3+\frac{BL}{JL}\left(\frac{kd+Bm}{Jm}\right)s^2$$

$$+\frac{k}{JL}\left(\frac{kd+Bm}{Jm}\right)s+\left(\frac{kp+k}{Jm}\right)s^2+\left(\frac{Bl}{JL}\right)\left(\frac{kp+k}{Jm}\right)s$$

$$+\left(\frac{k}{JL}\right)\left(\frac{kp+k}{Jm}\right)-\frac{k^2}{Jm\,Jl}=0$$

~~$|sI-A| = B_l B_m s^2 + B_l J_m s^3 + B_l k_d s^2 + B_l k_p s + B_l k s + B_m$~~

$$|sI-A| = \frac{B_l B_m s^2 + B_l J_m s^3 + B_l k_d s^2 + B_l k_p s + B_m J_l s^3 + B_m k s + J_l J_m s^4 + J_l k_d s^3 + J_l k_p s^2 + J_m k s^2 + k_d k s + k_p k}{J_l J_m} = 0$$

given. $J_m = J_l = 0.0097\ kg \cdot m^2$, $b_m = b_l = 0.04169\ Ns \cdot m^{-1}$

$k = 100\ Nm\ rad^{-1}$. $k_p = 10$

$$|sI-A| = \underbrace{s^4}_{(a_0)} + \underbrace{(103.093 k_d + 8.596)}_{(a_1)} s^3 + \underbrace{(443.086 k_d + 21667.957)}_{(a_2)} s^2$$

$$+ \underbrace{(1062812.201 k_d + 93048.145)}_{(a_3)} s + \underbrace{10628122.011}_{(a_4)} = 0$$

From this polynomial lets form the Herwitz matrix.

$$H = \begin{bmatrix} a_1 & a_3 & 0 & 0 \\ a_0 & a_2 & a_4 & 0 \\ 0 & a_1 & a_3 & 0 \\ 0 & a_0 & a_2 & a_4 \end{bmatrix}.$$

$$H = \begin{bmatrix} (103.093 k_d + 8.596) & (1062812.201 k_d + 93048.145) & 0 & 0 \\ 1 & (443.086 k_d + 21657.957) & 10628122.011 & 0 \\ 0 & (103.093 k_d + 8.596) & (1062812.201 k_d + 93048.145) & 0 \\ 0 & 1 & (443.086 k_d + 21657.957) & 10628122.011 \end{bmatrix}$$

$\Delta_1 > 0 \Rightarrow a_1 > 0$

$\Rightarrow 103.093\, kd + 8.596 > 0$

$\Rightarrow kd > -(0.083381)$ --- ① {given $kd > 0$}

$\Delta_2 > 0 \Rightarrow a_1 a_2 - a_3 a_0 > 0$

$\Rightarrow a_1 a_2 > a_3 a_0$

$(103.093kd + 8.596)(443.086kd + 21657.957)$

$> (1062812.201kd + 93048.145)$

$45,679.065\, kd^2 + 2,232,783.8\, kd + 3,808.76\, kd$
$+ 186,171.798 > 1062812.201kd + 93048.145$

$45,679.065\, kd^2 + 1,173,780.4\, kd + 93,123.653 > 0$

The roots of the eq^n are $kd > -0.0796$; $kd > -25.6167$

And given $kd > 0$ $\therefore \Delta_2 > 0$

~~$\Delta_3 > 0$~~

$\Delta_4 = a_4 \Delta_3 > 0 \Rightarrow a_4 > 0$ {And $a_4 > 0$}

$\Delta_3 > 0$

$\Rightarrow a_1(a_2 a_3 - a_1 a_4) - a_3(a_0 a_3) > 0$

$a_1 a_2 a_3 - (a_1)^2 a_4 - (a_3)^2 a_0 > 0$

$= 4854826 7610.1464\, kd^3 + 11388009 53467.1\, kd^2 +$
$18935402 0875.417\, kd + 787965 8342.69843 > 0$

The roots of this eq^n are

$kd > -23.289918$, $kd > -0.088$, $kd > -0.079$

given $kd > 0$; $\therefore \Delta_3 > 0$.

The lowest value ~~of kd~~ that kd can take is -0.079

$\therefore kd > -0.079$ for the system to be stable.

$\Rightarrow$ The system is stable for $kd > -0.079$

$\therefore$ Case 2:-

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{K}{Jm} & -\frac{B}{Jm} & \frac{K-Kp}{Jm} & -\frac{Kd}{Jm} \\ 0 & 0 & 0 & 1 \\ \frac{K}{Jl} & 0 & -\frac{K}{Jl} & -\frac{Bl}{Jl} \end{bmatrix}$$

$$[SI - A] = \begin{bmatrix} S & -1 & 0 & 0 \\ +\frac{K}{Jm} & S+\frac{B}{Jm} & -\frac{(K-Kp)}{Jm} & +\frac{Kd}{Jm} \\ 0 & 0 & S & -1 \\ -\frac{K}{Jl} & 0 & +\frac{K}{Jl} & S+\frac{Bl}{Jl} \end{bmatrix}$$

given $Jm = Jl = 0.0097$; $Bm = Bl = 0.04169$; $K = 100$ Nm Rad^{-1}

$Kp = 10$

Simplifying the determinant of $[SI - A]$ we get

$$\frac{B_l B_m s^2 + B_l J_m s^3 + B_l k s + B_m J_l s^3 + B_m k s + J_l J_m s^4 + J_l k s^2 + J_m k s^2 + k_d k s + k_p k}{J_l J_m}$$

$J_m = J_l = 0.0097$; $B_m = 0.04169$; $B\_l = 0.04169$

$k = 100$; $k\_p = 10$

$$\frac{\begin{gathered}(0.04169)(0.04169)s^2 + (0.04169)(0.0097)s^3 + (0.04169)(100)s + (0.04169)(0.0097)s^3 + (0.04169) \\ (100)s + (0.0097)(0.0097)s^4 + (0.0097)(100)s^2 + (0.0097)(100)s^2 + (k_d)(100)s + (10)(100)\end{gathered}}{(0.0097)(0.0097)}$$

$= 0$

$$\Rightarrow \underset{(a_0)}{s^4} + \underset{(a_1)}{8.5958762886598}\, s^3 + \underset{(a_2)}{20637.0289733234}\, s^2 + \left(1062812.2010840 7\, k_d + 88617.2813263896\right) s + \cancel{106}$$

$$\underset{a_4}{10628122.0108407} \quad (a_3)$$

The lower hurwitz Matrix H is

$$H = \begin{bmatrix} 8.596 & 1062812.201\, k_d + 88617.281 & 0 & 0 \\ 1 & 20637.029 & 10628122.01084 & 0 \\ 0 & 8.596 & (1062812.201\, k_d + 88617.281) & 0 \\ 0 & 1 & 20637.029 & 10628122.01084 \end{bmatrix}$$

$\Delta_1 > 0$

$8.591 > 0$

$\Delta_2 > 0$

$(8.596)(20637.029) - (1062812.201kd + 88617.281) > 0$

$= 0.0835318 > kd$

$\Rightarrow kd < 0.0835318 \quad \text{- - - -} \; (1)$

$\Delta_3 > 0$

$$\begin{vmatrix} 8.596 & 1062812.201kd + 88617.281 & 0 \\ 1 & 20637.029 & 10628122.01084 \\ 0 & 8.596 & 1062812.201kd + 88617.281 \end{vmatrix} > 0$$

This simplifies to

$-112956977459 4.46\, kd^2 + 171473359.535797kd + 7081995081.3222 > 0$

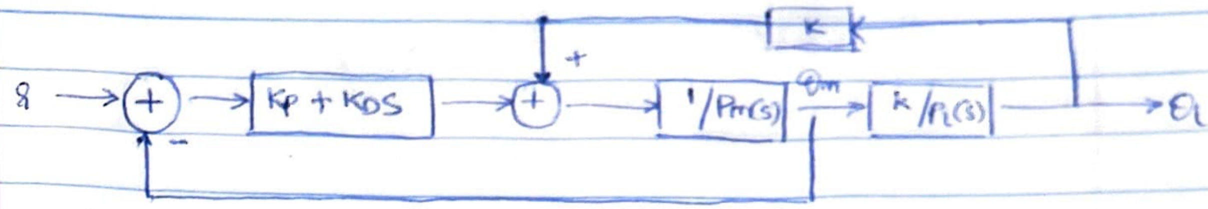
The value of kd which satisfies this are.

$$\boxed{-0.079105 < kd < 0.07925}$$

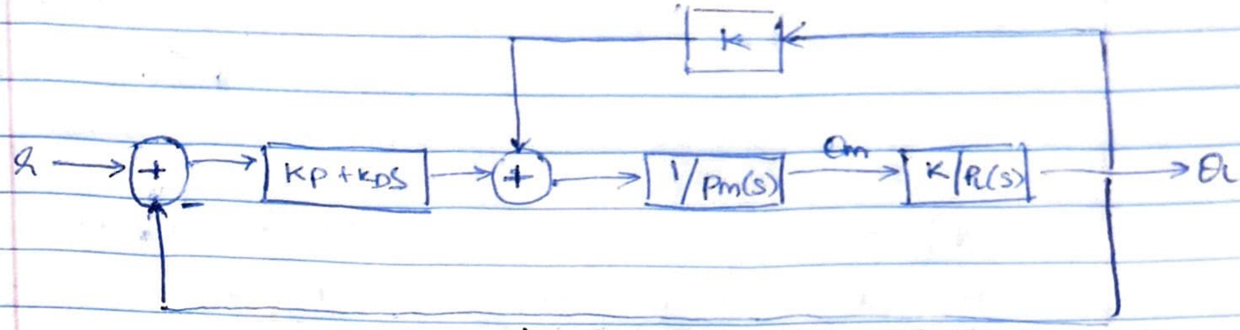
↳ This has to be the range of kd for the system to be stable.

Problem - 3

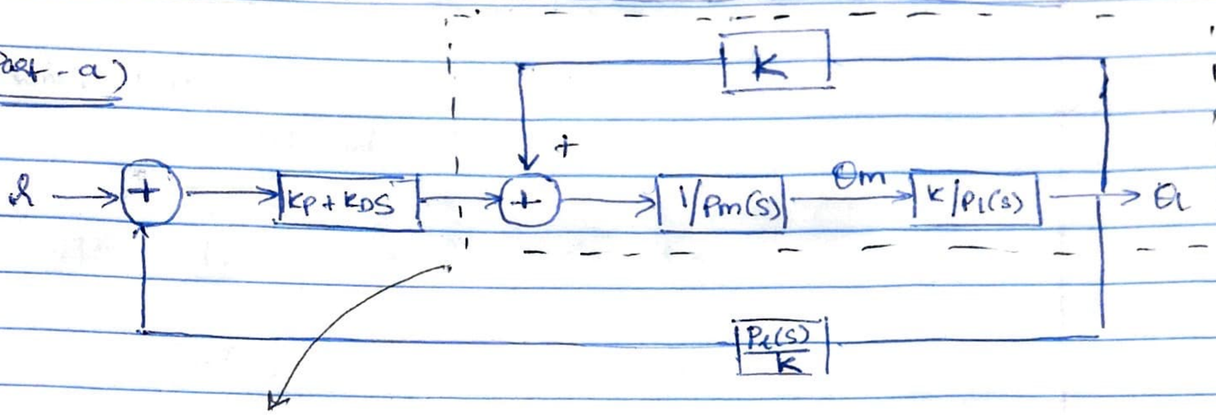
(a)



(b)



(Part - a)

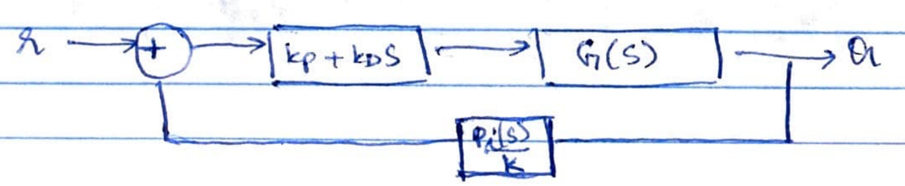


$$G'(s) \text{ (open loop gain)} = \frac{K}{P_l(s)\, P_m(s)}$$

$$\text{Closed loop gain } G(s) = \frac{G'(s)}{1 - K G'(s)} = \frac{\frac{K}{P_l(s)\, P_m(s)}}{1 - K\left[\frac{K}{P_l(s)\, P_m(s)}\right]}$$

$$\boxed{G(s) = \frac{K}{P_l(s)\, P_m(s) - K^2}}$$

now the loop simplifies To.



$H'(s)$ (open-loop-gain) $= (k_p + k_D s)(G(s))$

$$H(s) = \frac{H'(s)}{1 + \frac{P_l(s)}{k}\left[H'(s)\right]} = \frac{(k_p + k_D s)(G(s))}{1 + \frac{P_l(s)}{k}\left[(k_p + k_D s)(G(s))\right]}$$

[Closed loop gain with -ve feed back]

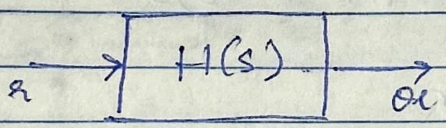
$$\boxed{H(s) = \frac{(k_p + k_D s)(G(s))}{\frac{k + P_l(s)(k_p + k_D s)(G(s))}{k}} = \frac{(k)(k_p + k_D s)(G(s))}{k + P_l(s)(k_p + k_D s)(G(s))}}$$

where $G(s) = \dfrac{k}{P_l(s)P_m(s) - k^2}$

~~s~~ $P_l(s) = J_l s^2 + B_l(s) + k$

$P_m(s) = J_m s^2 + B_m s + k$

The block diagram simplifies to:



~~Part~~ $$H(s) = \frac{(k_p + k_D s)\left(\frac{k}{P_l(s)P_m(s) - k^2}\right)(k)}{k + P_l(s)(k_p + k_D s)\left(\frac{k}{P_l(s)P_m(s) - k^2}\right)}$$

$$H(s) = \frac{(k_p + k_D s)\cancel{(k)}(k)}{\cancel{(k)}(P_l(s)P_m(s) - k^2) + P_l(s)(k_p + k_D s)\cancel{(k)}}$$

$$H(s) = \frac{(k_p + k_D s)(k)}{(J_l s^2 + B_l(s) + k)(J_m s^2 + B_m s + k) - k^2 + (J_l s^2 + B_l(s) + k)(k_p + k_D s)}$$

$$H(S) = \frac{k_p k + (K_D k)S}{J_l J_m S^4 + J_l B_m S^3 + J_l k S^2 + J_m B_l S^3 + B_m B_l S^2 + B_l k S + J_m k S^2 + B_m k S + \cancel{k} - \cancel{k} + J_l k_p S^2 + J_l K_D S^3 + B_l k_p S + B_l K_D S^2 + k k_p + (k K_D) S}$$

$$H(S) = \frac{k_D k S + k_p k}{(J_l J_m) S^4 + (J_l B_m + J_m B_l + J_l k_D) S^3 + (J_l k + J_m k + B_m B_l + J_l k_p + B_l k_D) S^2 + (B_l k + B_m k + B_l k_p + k k_D) S + k k_p}$$

W.K.T $J_m = J_l = 0.0097$; $B_m = B_l = 0.04169$; $k = 100$

(Sub^n problem 2 data)

~~H(S) = kkp~~

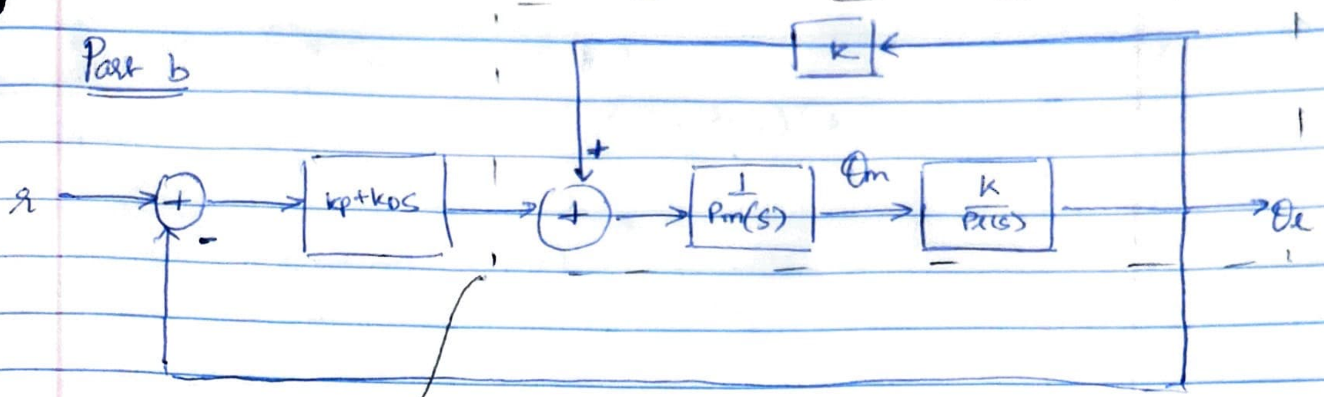
$$\boxed{H(S) = \frac{100 k_D S + 100 k_p}{(0.00009409) S^4 + (0.000808786 + 0.0097 k_D) S^3 + (1.94173806 + 0.0097 k_p + 0.04169 k_D) S^2 + (8.338 + 0.04169 k_p + 100 k_D) S + 100 k_p}}$$

Sub^n $k_p = 10$ we get.

$$\boxed{H(S) = \frac{100 k_D S + 1000}{(0.00009409) S^4 + (0.000808786 + 0.0097 k_D) S^3 + (\cancel{1.94173806 + 0.097} + 0.04169 k_D) S^2 + (\cancel{8.338 + 0.4169} + 100 k_D) S + 1000}}$$

2.038 8.7549

Part b

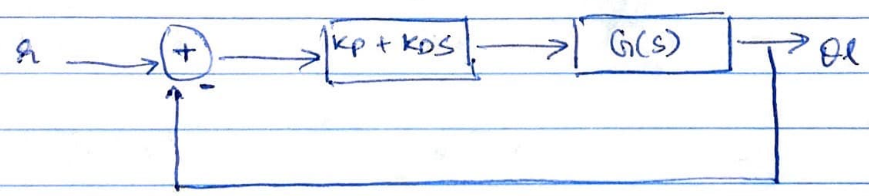


(open loop gain) $G'(s) = \frac{1}{P_m(s)} \cdot \frac{K}{P_l(s)}$

close loop gain. (+ve Feedback)

$$G(s) = \frac{G'(s)}{1 - K G'(s)} = \frac{\frac{K}{P_m(s) P_l(s)}}{1 - \frac{K^2}{P_m(s) P_l(s)}}$$

$$\boxed{G(s) = \frac{K}{P_m(s) P_l(s) - K^2}}$$

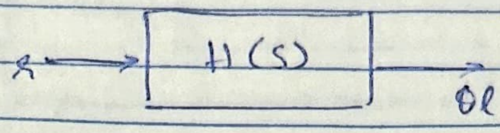


(open loop gain) $H'(s) = (K_p + K_D s)(G(s))$

closed loop gain $H(s) = \frac{H'(s)}{1 + H'(s)} = \frac{(K_p + K_D s)(G(s))}{1 + (K_p + K_D s)(G(s))}$

$$H(s) = \frac{(K_p + K_D s)\left(\frac{K}{P_m(s) P_l(s) - K^2}\right)}{1 + (K_p + K_D s)\left(\frac{K}{P_m(s) P_l(s) - K^2}\right)}$$

$$H(s) = \frac{(k_p + k_D s)(k)}{P_m(s)P_l(s) - k^2 + (k_p + k_D s)(k)}$$



where

$$P_m(s) = J_m s^2 + B_m s + k \; ; \qquad P_l(s) = J_l s^2 + B_l s + k.$$

$$H(s) = \frac{(k_p + k_D s)(k)}{(J_m s^2 + B_m s + k)(J_l s^2 + B_l s + k) - k^2 + (k_p + k_D s)k}$$

$$H(s) = \frac{K k_D s + K k_p}{J_m J_l s^4 + J_m B_l s^3 + J_m k s^2 + B_m J_l s^3 + B_m B_l s^2 + K B_m s + J_l k s^2 + B_l k s + \cancel{k^2} - \cancel{k^2} + k k_p + k k_D s}$$

$$H(s) = \frac{K k_D s + K k_p}{J_m J_l s^4 + (J_m B_l + B_m J_l)s^3 + (J_m k + B_m B_l + J_l k)s^2 + (K B_m + B_l k + K k_D)s + K k_p}$$

~~H(s) =~~ given $k = 100$; $J_m = J_l = 0.0097$;
$B_l = B_m = 0.04169$

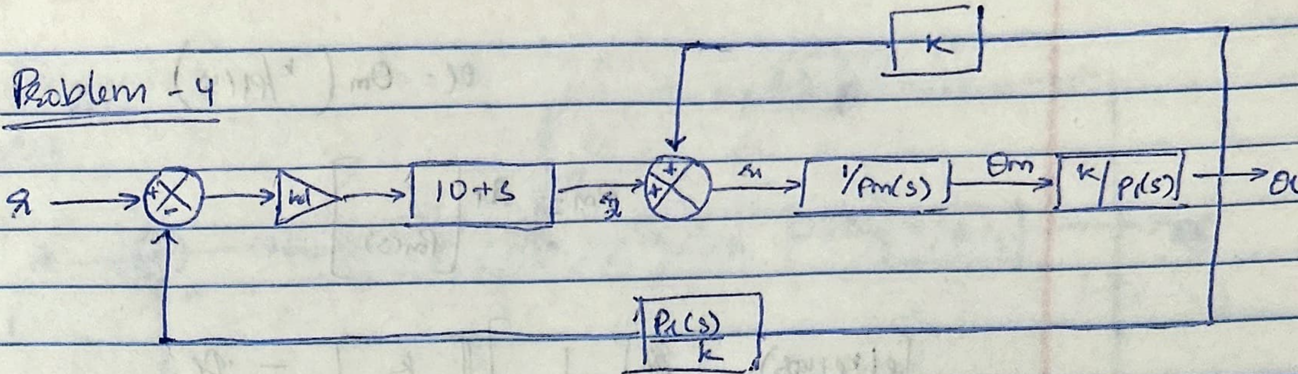
$$\boxed{H(s) = \frac{100 k_D s + 100 k_p}{(0.00009409)s^4 + (0.000808786)s^3 + (1.94173806)s^2 + (8.338 + 100 k_D)s + 100 k_p}}$$

sub $k_p = 10$ we get

$$H(s) = \frac{100 k_D s + 1000}{(0.00009409)s^4 + (0.000808786)s^3 + (1.94173806)s^2 + (8.338 + 100 k_D)s + 1000}$$

Problem - 4

(Part a)



$$\theta_l = \frac{k}{P_l(s)}\theta_m \; ; \; \theta_m = \frac{1}{P_m(s)} r_1 \quad \Rightarrow \theta_l = \frac{k \; r_1}{P_l(s)\, P_m(s)}$$

$$r_1 = r_2 + k\theta_l .$$

$$\Rightarrow \theta_l = \frac{k}{P_l(s)P_m(s)}\left(k\theta_l + r_2\right) \Rightarrow \theta_l = \frac{k}{P_l(s)\,P_m(s)}\left(k\theta_l + (k_p + k_d s) r\right)$$

Note → Taken $k_D = 1$; $k_P = 10$ (Maintaining the ratio $k_P/k_D = 10$)

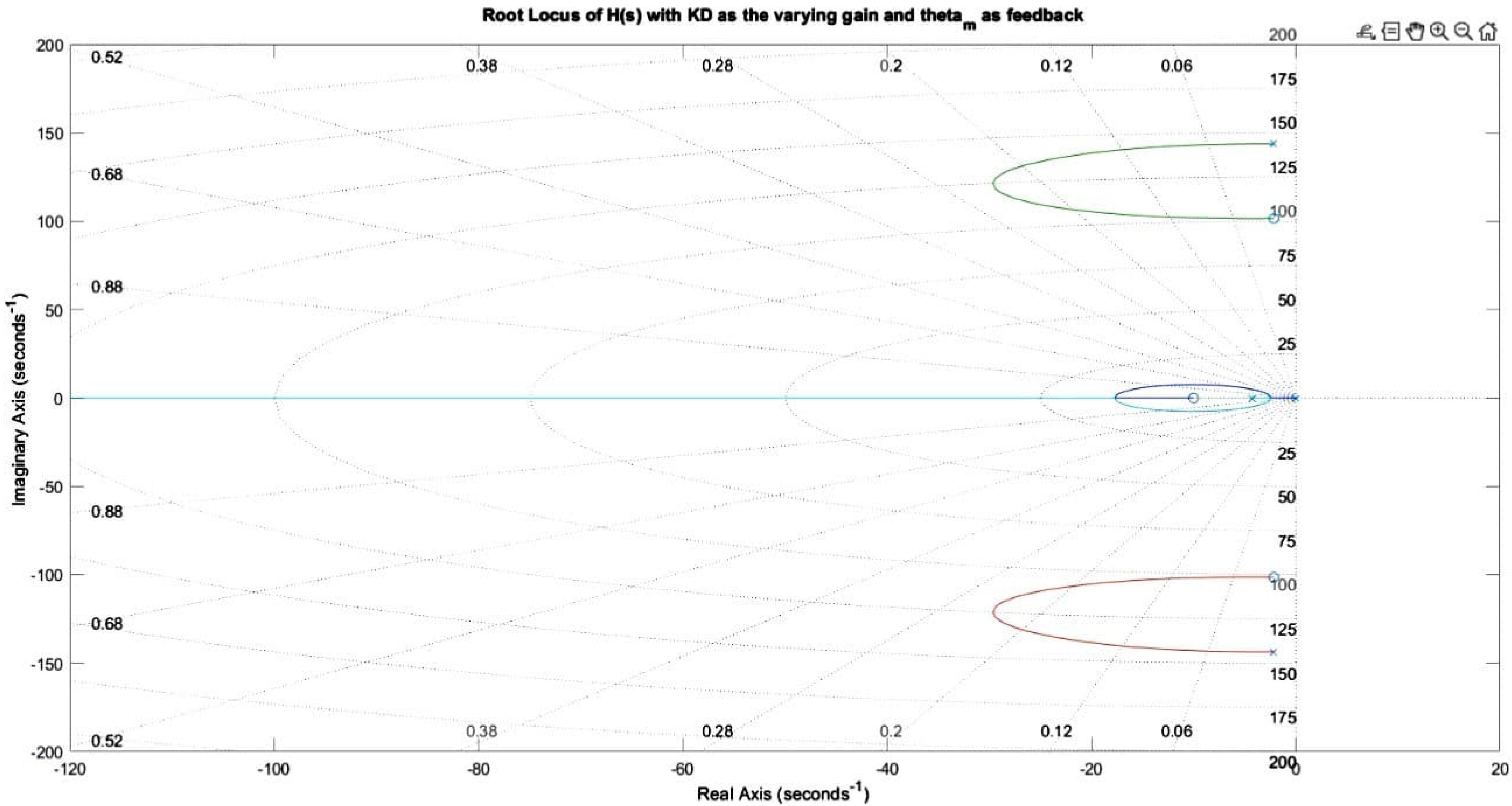
$$Q(s) = \frac{\theta_l(s)}{r_0(s)} = \frac{ks + 10k}{J_l J_m s^4 + (B_l J_m + B_m J_l)s^3 + (kJ_l + kJ_m + B_l B_m)s^2 + (kB_l + kB_m)s}$$

$$H(s) = \frac{Q(s) \times P_l(s)}{k} = \frac{(s+10)(J_l s^2 + B_l s + k)}{J_l J_m s^4 + (B_l J_m + B_m J_l)s^3 + (kJ_l + kJ_m + B_l B_m)s^2 + (kB_l + kB_m)s.}$$

Sub values $J_m = J_l = 0.0097$, $b_m = b_l = 0.04169$, $k = 100$, $k_p = 10$

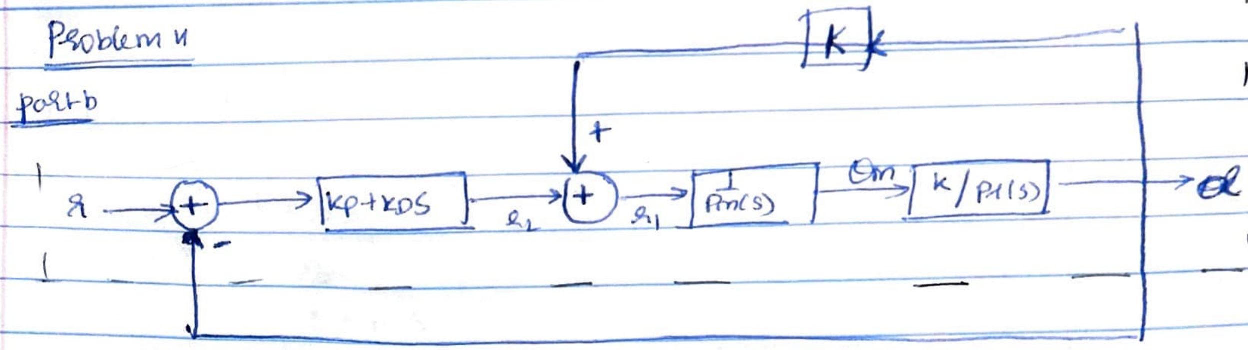
~~H(s) =~~

$$H(s) = \frac{0.0097s^3 + 0.13869s^2 + 100.4169s + 1000}{9.409 \times 10^{-5}s^4 + 0.00080878 6s^3 + \cancel{0.9217380561}\,1.94173\, s^2 + 8.3477s.}$$


Root Locus of H(s) with KD as the varying gain and theta$_m$ as feedback
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
0.52
0.38
0.28
0.2
0.12
0.06
0.68
0.88
200
175
150
125
100
75
50
25
-120
-100
-80
-60
-40
-20
0
20
-50
-100
-150
-200

Problem 4

part b



$$\theta l = \theta m \frac{k}{Pl(s)} \qquad \left\{ \theta m = \frac{e_1}{Pm(s)} \right\}$$

$$\Rightarrow \theta l = \frac{e_1 k}{Pm(s)\, Pl(s)} \qquad \left\{ e_1 = k\theta l + e_2 \right\}$$

$$\Rightarrow \theta l = \frac{(k\theta l + e_2)k}{Pm(s)\, Pl(s)} \qquad \left\{ e_2 = (kp + kDs)\, r \right\}$$

$$\Rightarrow \theta l = \frac{\left[k\theta l + (kp + kDs) r\right] k}{Pm(s)\, Pl(s)}$$

The open loop transfer function [Derived from Matlab]

$L(s) =$

$$\Rightarrow \theta l = \frac{k^2 \theta l}{Pm(s)\, Pl(s)} + \frac{k r (kp + kDs)}{Pm(s)\, Pl(s)}$$

$$\theta l \left[1 - \frac{k^2}{Pm(s)\, Pl(s)} \right] = \left[\frac{k(kp + kDs)}{Pm(s)\, Pl(s)} \right] r$$

$$Q(s) = \frac{\theta l}{r} = \frac{\left[\dfrac{k(kp + kDs)}{Pm(s)\, Pl(s)} \right]}{1 - \dfrac{k^2}{Pm(s)\, Pl(s)}} = \frac{k(kp + kDs)}{Pm(s)\, Pl(s) - k^2}$$

$$\frac{k(k_p + k_D S)}{(J_l s^2 + B_l s + k)(J_m s^2 + B_m s + k)} = Q(s)$$

$$Q(s) = \frac{K k_p + K K_D S}{J_l J_m s^4 + J_l B_m s^3 + J_l k s^2 + B_l J_m s^3 + B_m B_l s^2 + B_l k s + k J_m s^2 + K B_m s + \cancel{k^2} - \cancel{k^2}}$$

$\cancel{Q(s) =}$ Sub^n values i.e $k_p = 10$; $k_D = 1$ $\left\{ \Rightarrow \frac{k_p}{k_D} = 10 \right\}$.

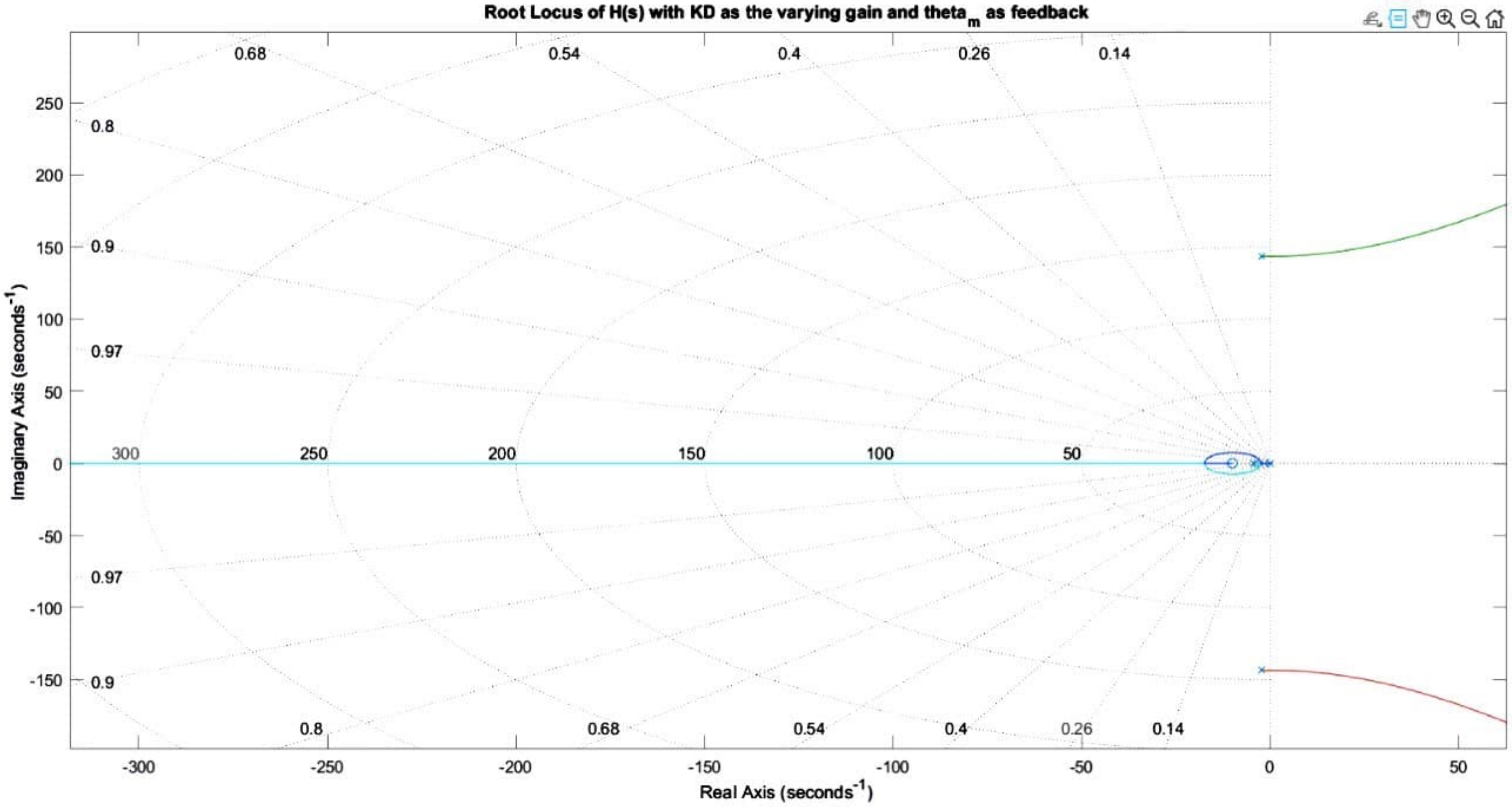
& $J_l = J_m = 0.0097$; $b_m = b_l = 0.04169$ $k = 100$

$$Q(s) = \frac{1000 + 100S}{0.00009409\, s^4 + 0.0008087865\, s^3 + \cancel{0.9717380561\, s^2}\ 1.941 s^2 + 8.338 s + \cancel{10{,}000}}$$

$H(s) = Q(s) \times 1$ $\Rightarrow$ $H(s) = Q(s)$

$\cancel{Q}$ ∴ $H(s) = Q(s) \times 1$

$$H(s) = \frac{1000 + 100s}{0.00009409\, s^4 + 0.0008087865\, s^3 + 1.941 s^2 + 8.338 s + 0}.$$


Root Locus of H(s) with KD as the varying gain and theta_m as feedback
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
0.68
0.54
0.4
0.26
0.14
0.8
0.9
0.97
300
250
200
150
100
50
0.97
0.9
0.8
0.68
0.54
0.4
0.26
0.14
250
200
150
100
50
0
-50
-100
-150
-300
-250
-200
-150
-100
-50
0
50

Problem 4

(a) When the motor Angle is fed back to the system & the root locus is obtained it can be observed that the gain kd does not cross the left hand side of the open plane (or) (OLHP). that means for any valued of $kd \geq 0$ the system is stable & there is no upper bound.

→ To give a better estimate of the lower bound $kd > -0.07$ & The Roth - horwitz criteria from which we got $kd > -0.079$.
{The graph below shows the min value of kd on the root locus} & ~~could~~ could point it to -0.0327 but on closer inspection it can be seen that kd is indeed greater than -0.079

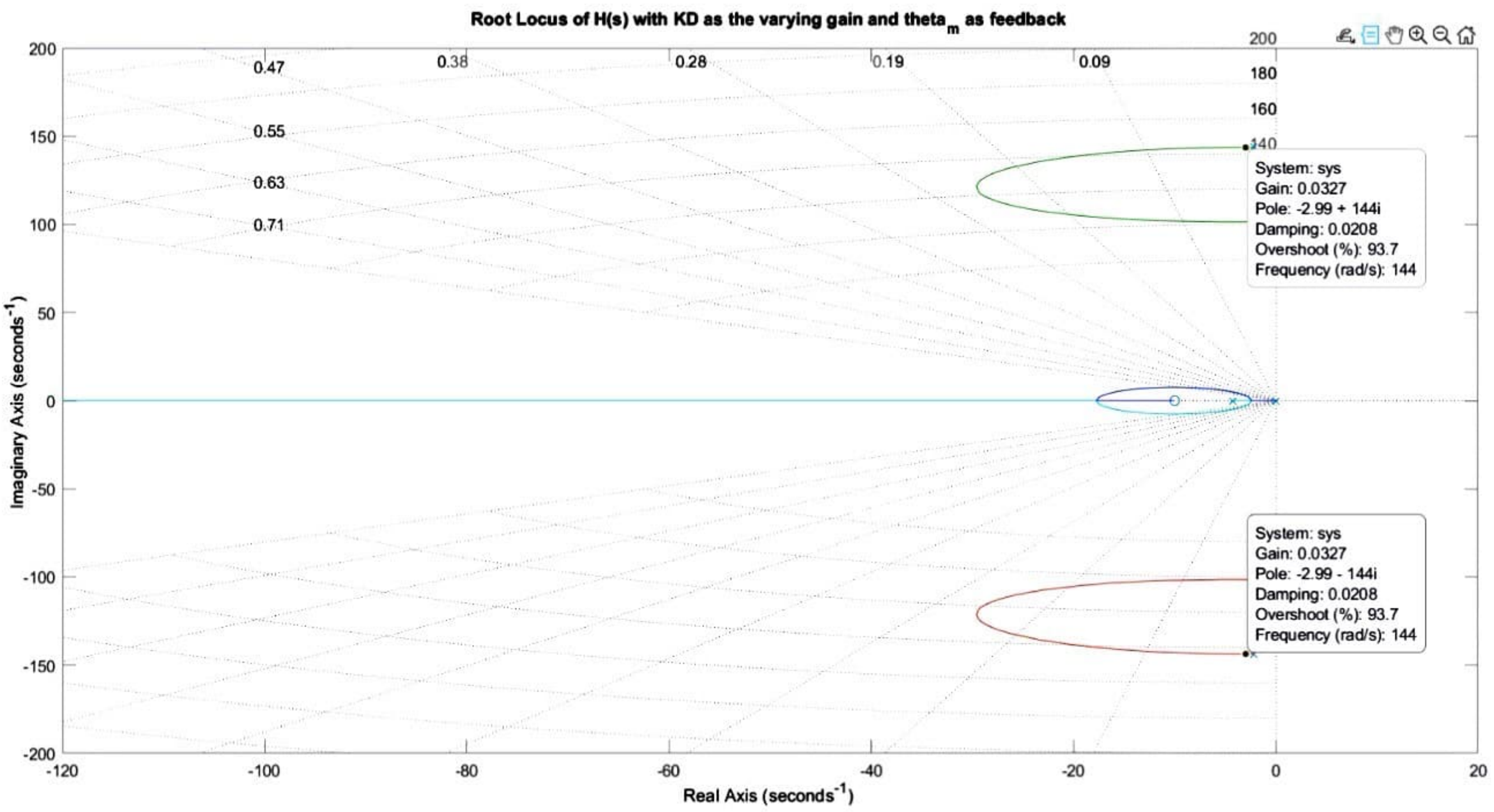
(b) When the load angle is fed back to the system & the root locus in obtained it can be observed that the poles cross the open left hand plane & hence there will be an upper bound & lower bound for k_d when we use the root - locus to find these bounds it can be observed that there is a slight discrepency with the Routh - Horwitz - Criteria.
The Routh horwitz criteria tells that the range of kd for the system to be stable should be

$$-0.079105 < kd < 0.07926$$

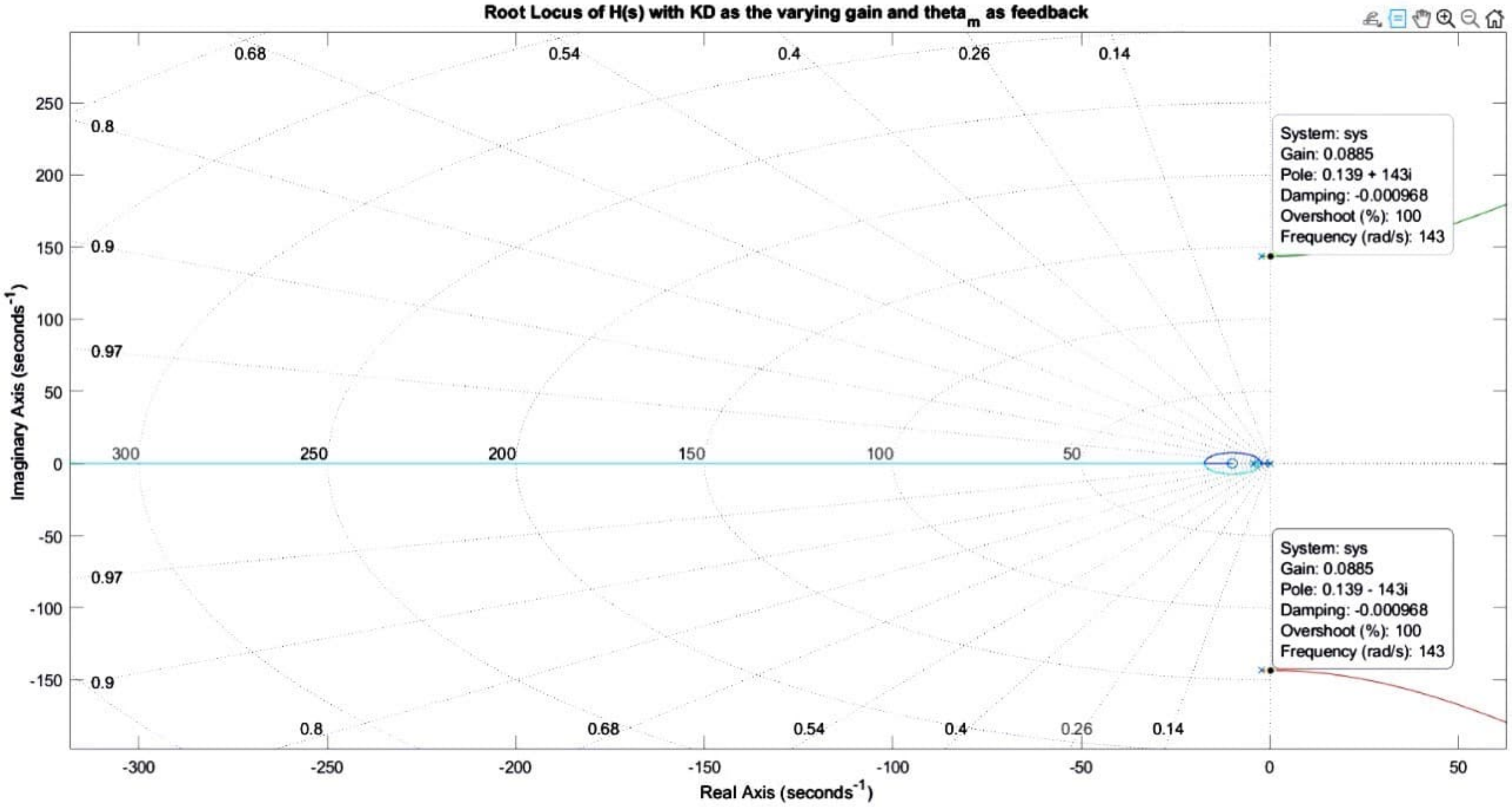
but thats not the case with the root - locus. the range of kd from the root - locus is :

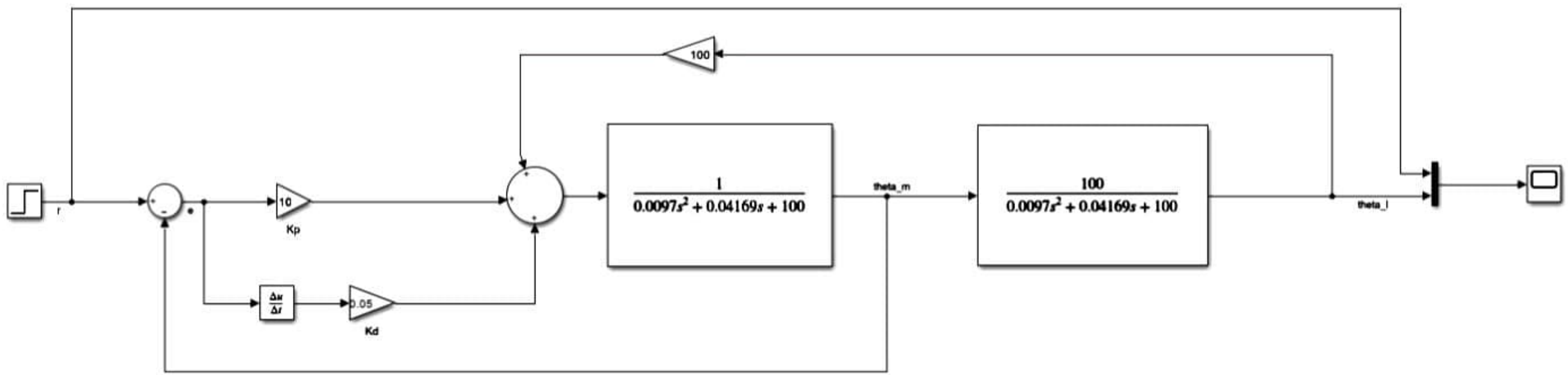
$$-0.083 < kd < +0.083$$

This discrepency is because in the Routh - horwitz criteria we fix kp & then then find the value of kd but in the root - locus we fix the ratio of kp/kd & then represent kp in terms of kd & then plot the root locus so due to this the discrepency occurs between the time domain (Routh-horwitz's) & frequency domain (Root - locus).



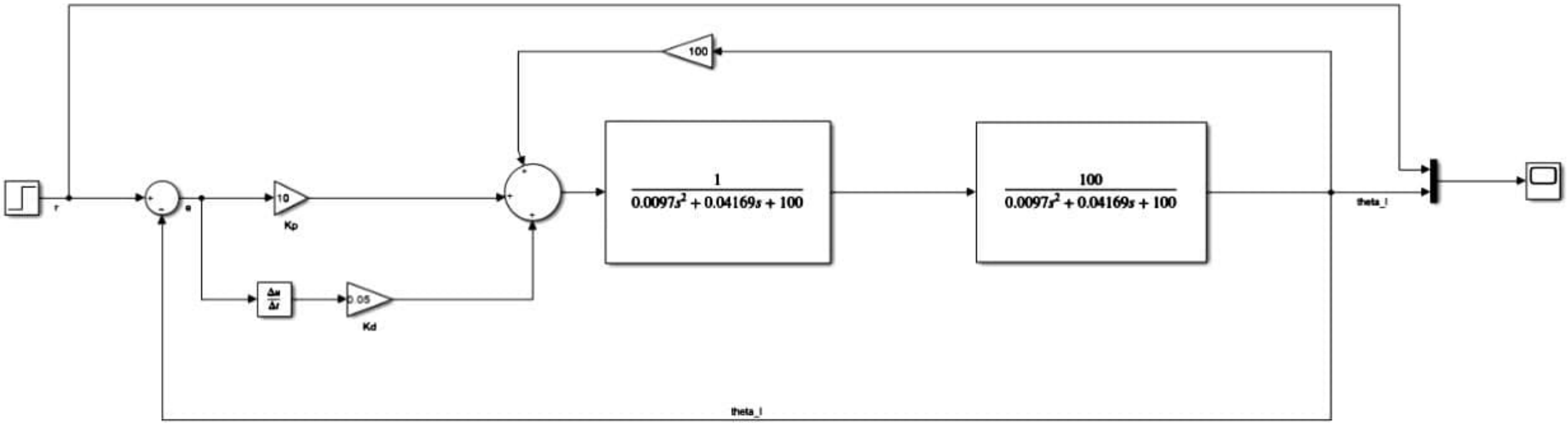
Root Locus of H(s) with KD as the varying gain and theta_m as feedback
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
200
150
100
50
0
-50
-100
-150
-200
-120
-100
-80
-60
-40
-20
0
20
0.47
0.38
0.28
0.19
0.09
0.55
0.63
0.71
200
180
160
140
System: sys
Gain: 0.0327
Pole: -2.99 + 144i
Damping: 0.0208
Overshoot (%): 93.7
Frequency (rad/s): 144
System: sys
Gain: 0.0327
Pole: -2.99 - 144i
Damping: 0.0208
Overshoot (%): 93.7
Frequency (rad/s): 144


Root Locus of H(s) with KD as the varying gain and theta_m as feedback
Imaginary Axis (seconds^-1)
Real Axis (seconds^-1)
System: sys
Gain: 0.0885
Pole: 0.139 + 143i
Damping: -0.000968
Overshoot (%): 100
Frequency (rad/s): 143
System: sys
Gain: 0.0885
Pole: 0.139 - 143i
Damping: -0.000968
Overshoot (%): 100
Frequency (rad/s): 143

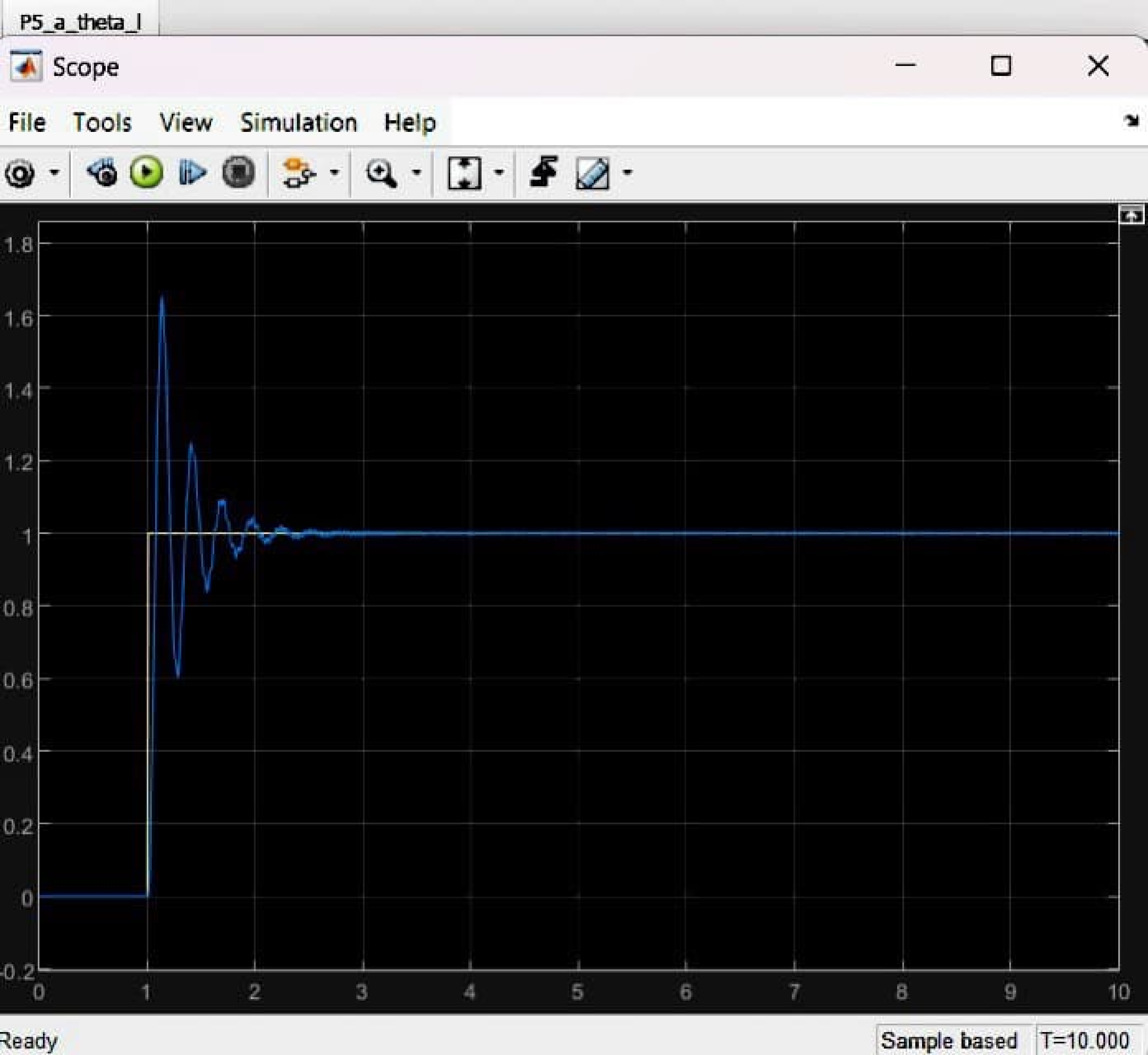


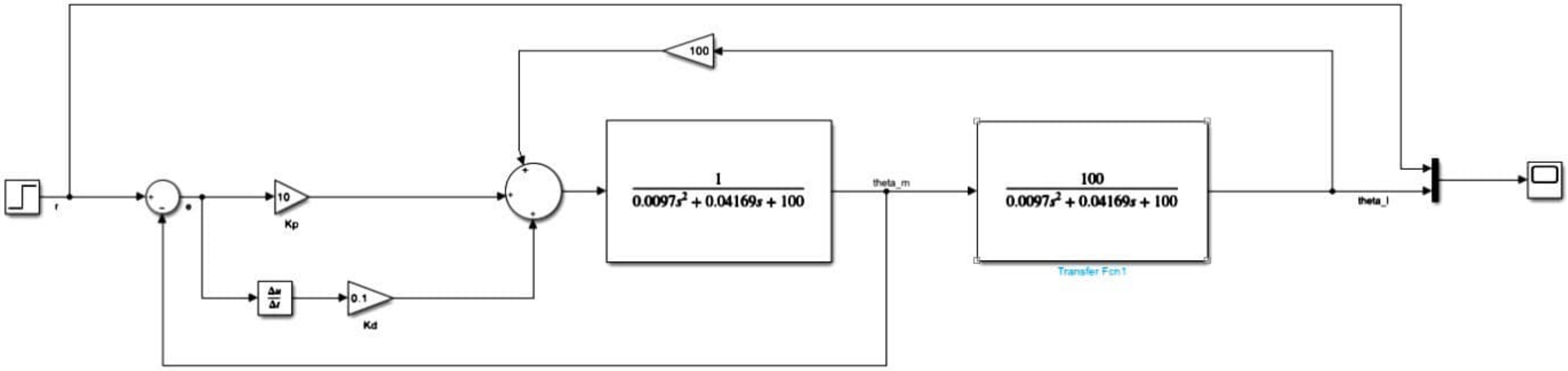
r
e
10
Kp
0.05
Kd
100
$\frac{1}{0.0097s^2 + 0.04169s + 100}$
theta_m
$\frac{100}{0.0097s^2 + 0.04169s + 100}$
theta_l


P5_a_theta_m
Scope
File Tools View Simulation Help
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0 1 2 3 4 5 6 7 8 9 10
Ready
Sample based
T=10.000

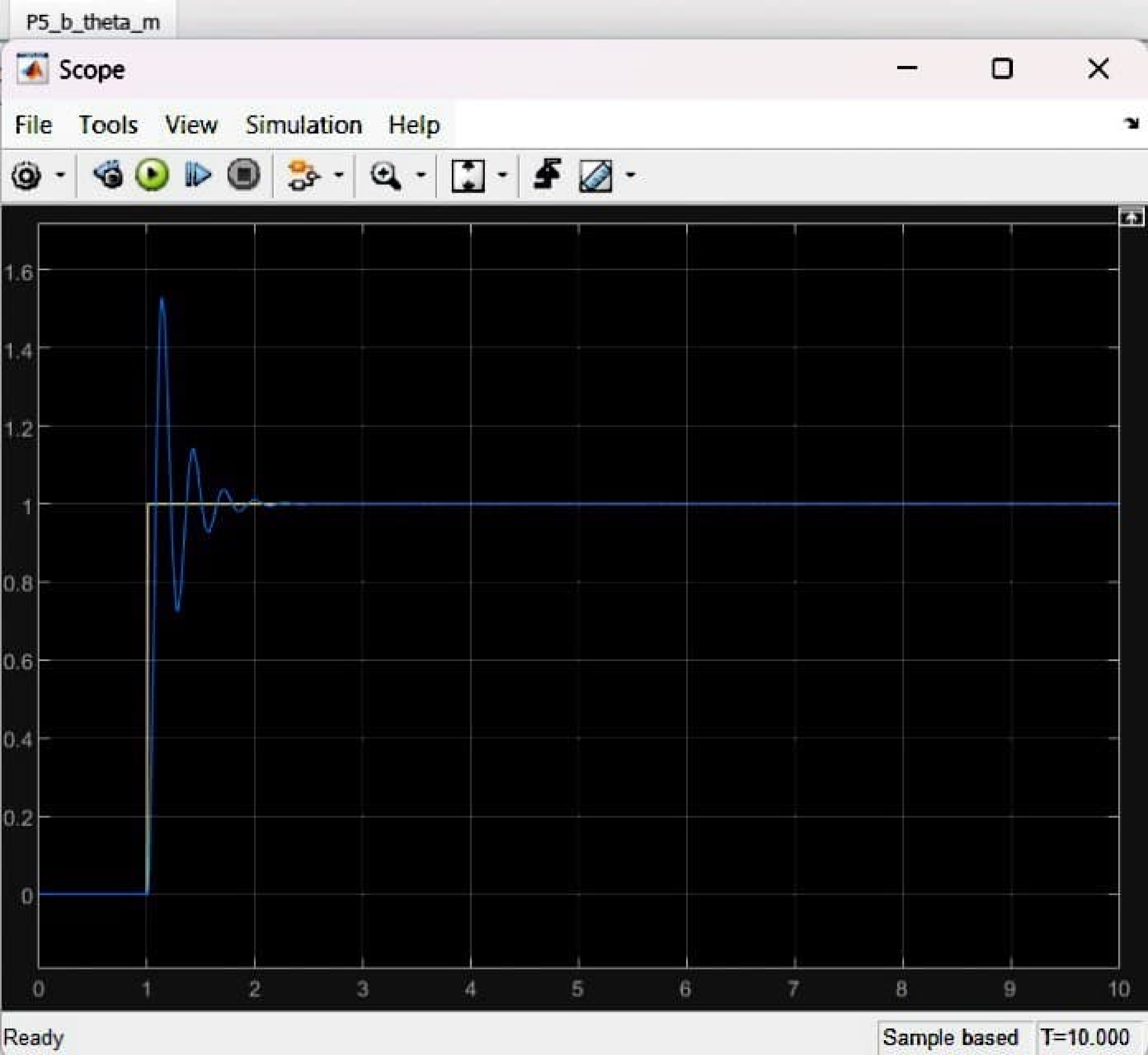


r
e
10
Kp
0.05
Kd
100
$\frac{1}{0.0097s^2 + 0.04169s + 100}$
$\frac{100}{0.0097s^2 + 0.04169s + 100}$
theta_l
theta_l

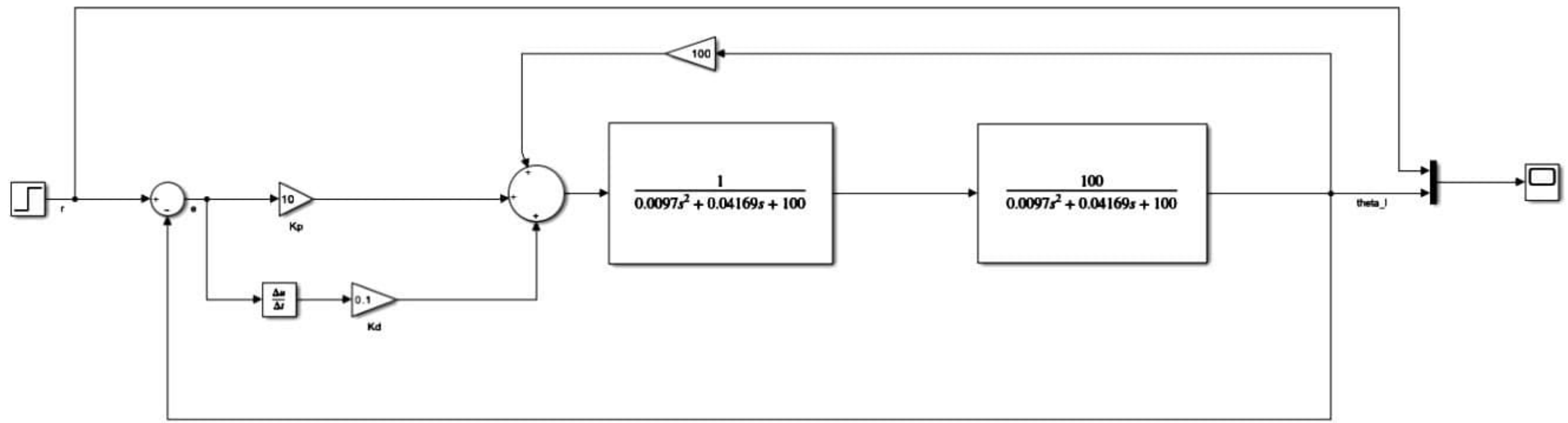

P5_a_theta_l
Scope
File Tools View Simulation Help
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
0 1 2 3 4 5 6 7 8 9 10
Ready
Sample based
T=10.000



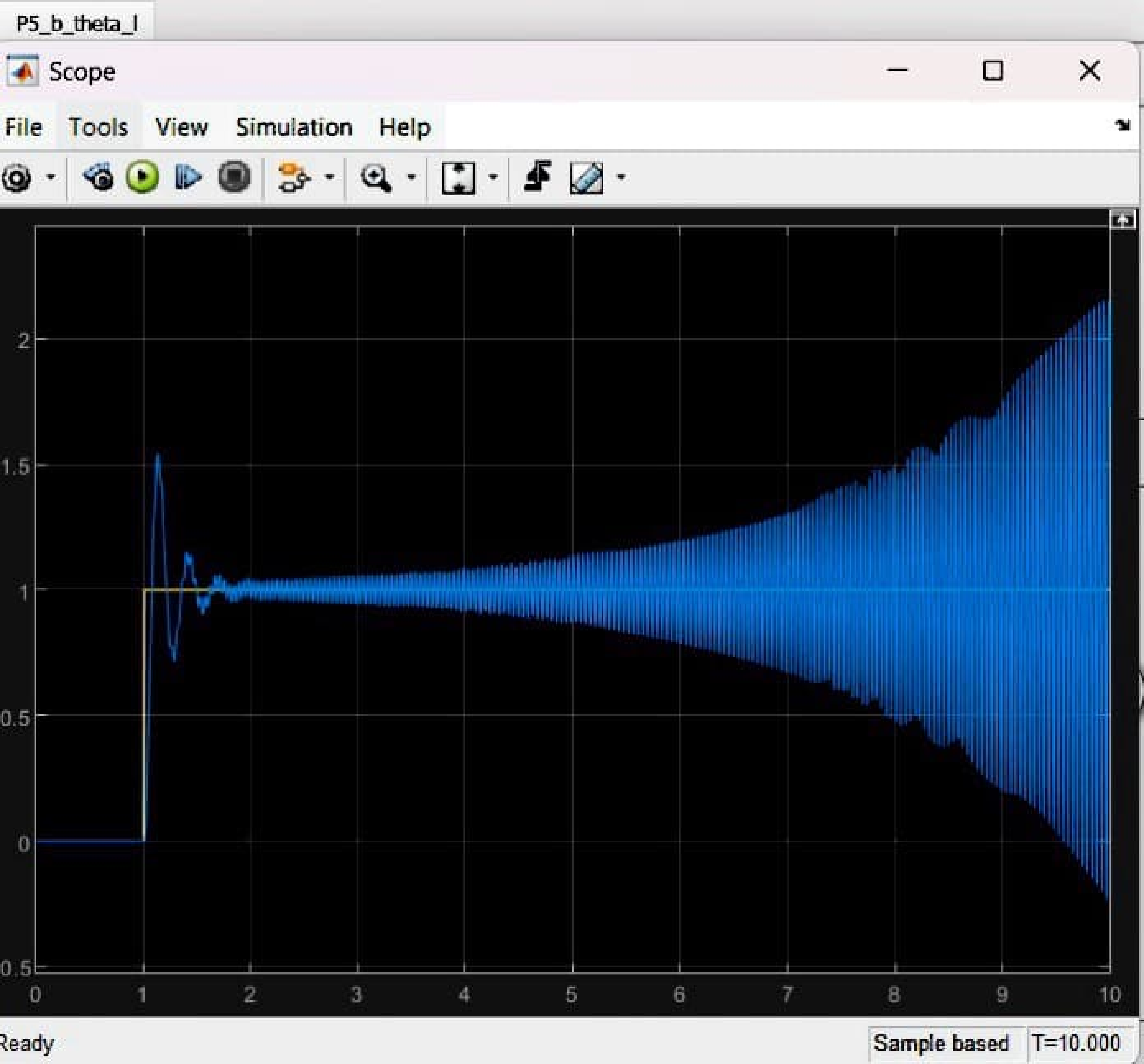
100
r
e
10
Kp
0.1
Kd
Δu/Δt
1 / (0.0097s^2 + 0.04169s + 100)
theta_m
100 / (0.0097s^2 + 0.04169s + 100)
Transfer Fcn1
theta_l



P5_b_theta_m
Scope
File
Tools
View
Simulation
Help
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000



r
e
10
Kp
0.1
Kd
100
$\frac{1}{0.0097s^2 + 0.04169s + 100}$
$\frac{100}{0.0097s^2 + 0.04169s + 100}$
theta_l


P5_b_theta_l
Scope
File
Tools
View
Simulation
Help
2
1.5
1
0.5
0
0.5
0
1
2
3
4
5
6
7
8
9
10
Ready
Sample based
T=10.000