ROB-599 Assignment-5

11/18/2024

1(a). given first-order dynamics of the actuator

$$I\dot{v} + bv = T$$

where I - Inertia, b is the damping co-efficient.

The transfer function of this system is given as

$$G(s) = \frac{V(s)}{T(s)} = \frac{1}{Is+b}$$

From the magnitude plot it is given that the DC gain is 7.6dB & the -3dB frequency (cutoff frequency) is 6 rad/s with a magnitude 4.6 dB

The DC gain is 7.6dB

$$\Rightarrow 7.6 = 20\log_{10}(|G(j\omega)|)$$

$$\frac{7.6}{20} = \log_{10}(|G(j\omega)|)$$

$$10^{\frac{7.6}{20}} = |G(j\omega)|$$

$\omega = 0$ at DC.

$$\Rightarrow |G(0)| = 10^{\frac{7.6}{20}} = 2.398 \simeq 2.4$$

W.k.T

$$|G(j\omega)| = \frac{1}{\sqrt{(I\omega)^2 + b^2}} \quad (\omega = 0) \Rightarrow |G(0)| = \frac{1}{\sqrt{b^2}} = \frac{1}{b}$$

$$\Rightarrow |G(0)| = \frac{1}{b}$$

$$\Rightarrow 2.4 = \frac{1}{b} \quad \therefore \boxed{b = \frac{1}{2.4} = 0.4166}$$

Now we can use w_c to find I.

w.k.t $|G(jw)| = \dfrac{1}{\sqrt{(Ijw)^2 + b^2}}$

At $w_c = 6$ Rad/s, $b = \dfrac{1}{2.4}$ $\quad |G(j(6))| = 10^{4.6/20} = 1.698$

$$\Rightarrow 1.698 = \frac{1}{\sqrt{(I(6))^2 + \left(\frac{1}{2.4}\right)^2}}$$

$$1.698 = \frac{1}{\sqrt{36I^2 + \left(\frac{1}{2.4}\right)^2}}$$

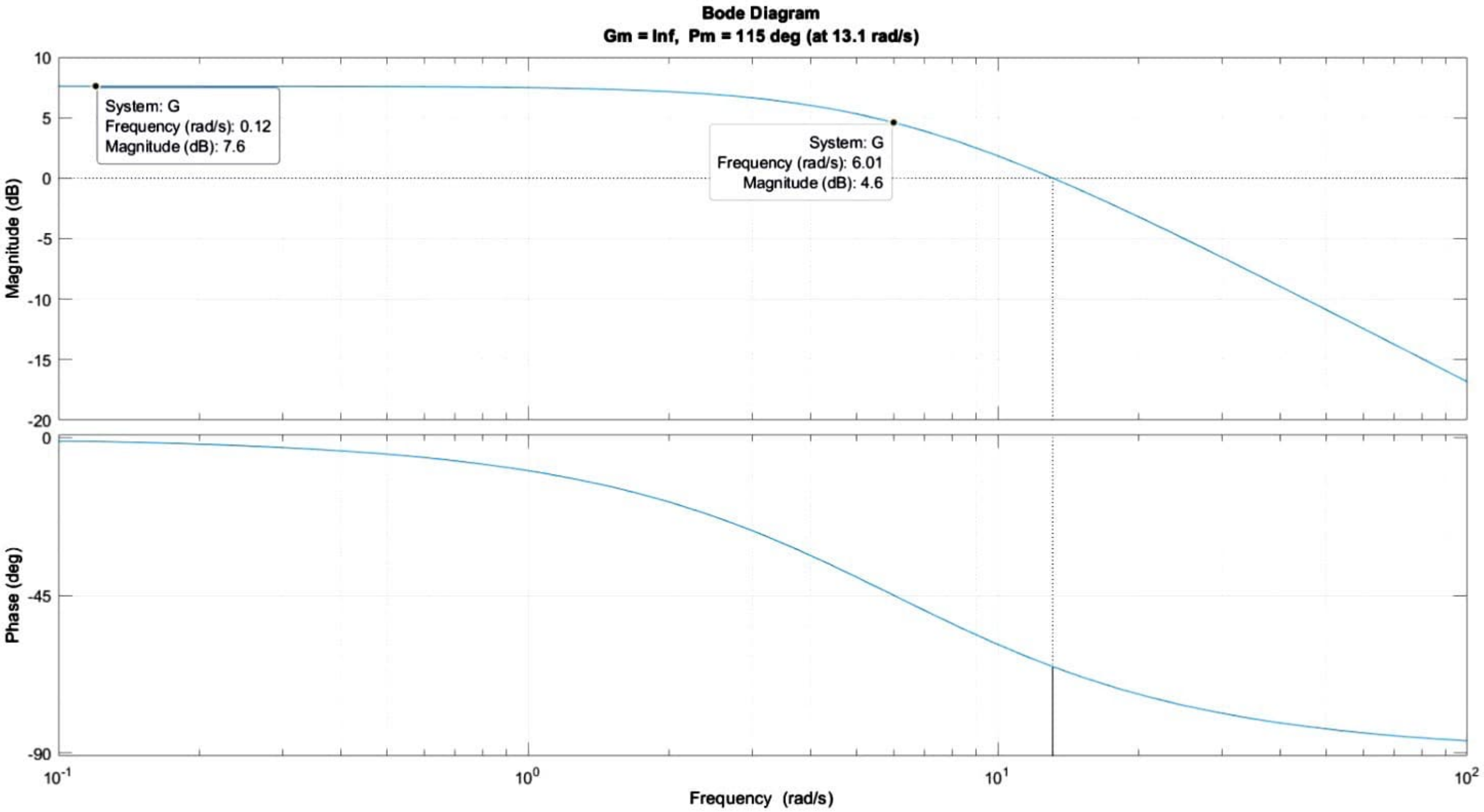
$$\Rightarrow \frac{1}{1.698} = \sqrt{36I^2 + \left(\frac{1}{2.4}\right)^2}$$

$$\left(\frac{1}{1.698}\right)^2 = 36I^2 + \left(\frac{1}{2.4}\right)^2$$

$$\sqrt{\frac{\left(\frac{1}{1.698}\right)^2 - \left(\frac{1}{2.4}\right)^2}{36}} = I \; = \; \frac{\sqrt{\left(\frac{1}{1.698}\right)^2 - \left(\frac{1}{2.4}\right)^2}}{6}$$

$$\Rightarrow \boxed{I = 0.069367.}$$

$$\boxed{\begin{aligned} \therefore \quad b &= 0.4166 \ \text{kg m}^2/\text{s} \\ I &= 0.069367 \ \text{kg m}^2 \end{aligned}}$$


Bode Diagram
Gm = Inf, Pm = 115 deg (at 13.1 rad/s)
System: G
Frequency (rad/s): 0.12
Magnitude (dB): 7.6
System: G
Frequency (rad/s): 6.01
Magnitude (dB): 4.6
Magnitude (dB)
Phase (deg)
Frequency (rad/s)
10
5
0
-5
-10
-15
-20
0
-45
-90
10^{-1}
10^{0}
10^{1}
10^{2}

Problem 2. given The first order dynamics of the actuator as $I\dot{v} + bv = T$
(a)
... ①

Eq ① is a first order system. A we were representing the eq^n of motion using v

~~w.r.t~~ w.r.t $v = \dot{\theta}$; $\dot{v} = \ddot{\theta}$ ... ②

lets sub^n ② in ① we get
$I\ddot{\theta} + b\dot{\theta} = T$ {Note - This becomes a 2nd order system}.

Taking the Laplace transform on both sides we get.

$$Is^2\theta(s) + bs\,\theta(s) = T(s)$$

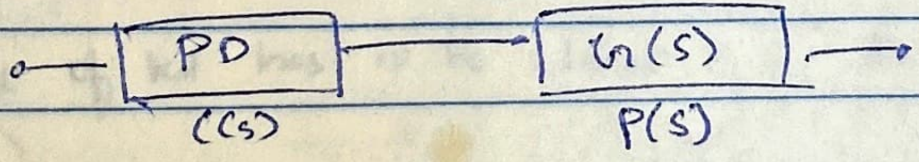
$$\theta(s) \ast [Is^2 + bs] = T(s)$$

$$\boxed{G(s) = \frac{\theta(s)}{T(s)} = \frac{1}{Is^2 + bs}}$$

(b) The PD controller can be written as

$$\tau = k_p(\theta_{cmd} - \theta) - K_d\dot{\theta} \quad \text{--} \quad (1)$$

The ~~b/o~~ open loop block diagram looks like



The open loop transfer function is given as

$$C(s) \cdot P(s)$$

~~C(S) = Laplase transform (t)~~

~~=> C(S) =~~

$$C(s) = K_p + K_d s$$

$$P(s) = G(s) = \frac{1}{Is^2 + bs}$$

The open loop transfer function is given as:-

$$L(s) = C(s) \cdot G(s) = \frac{k_P + k_D s}{I s^2 + bs}$$

w.k.t $\frac{K_P}{K_D} = 10$

$$\Rightarrow \frac{K_D(10 + s)}{I s^2 + bs}$$

To calculate the desired pole locations s for a second-order system based on the natural frequency (w_n) & damping ratio (ξ) the std. second order system formula can be used.

$$s = -\xi w_n \pm j w_n \sqrt{1 - \xi^2}$$

$\xi = 0.8$; $w_n = 10$ rad/s.

$$s = -8 \pm j\, 10 \sqrt{1 - (0.8)^2}$$

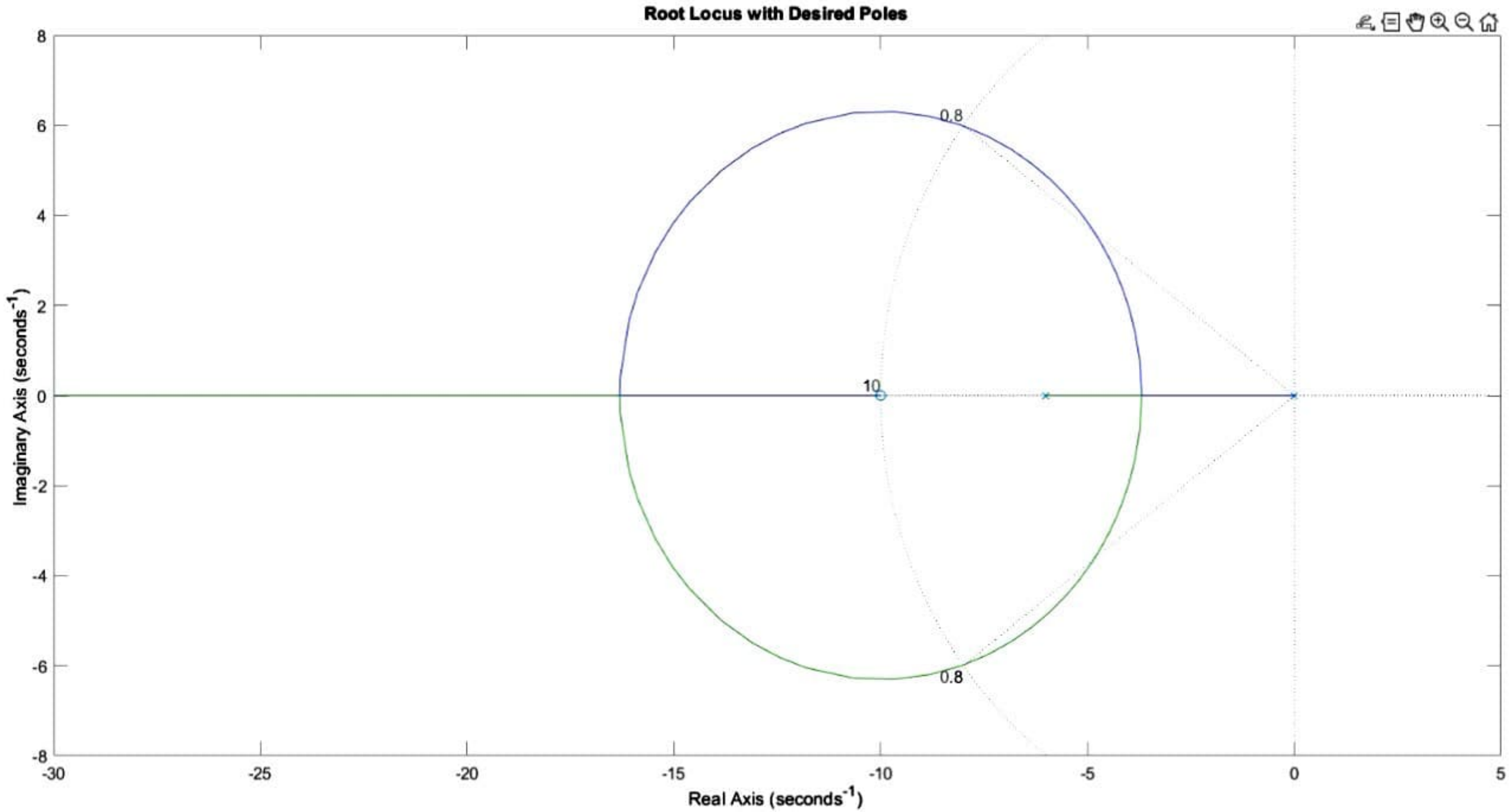
$$s = -8 \pm 6j$$

$\therefore$ The poles have to be on $(-8 + 6j)$ & $(-8 - 6j)$ for $\xi = 0.8$ & $w_n = 10$ rad/s.

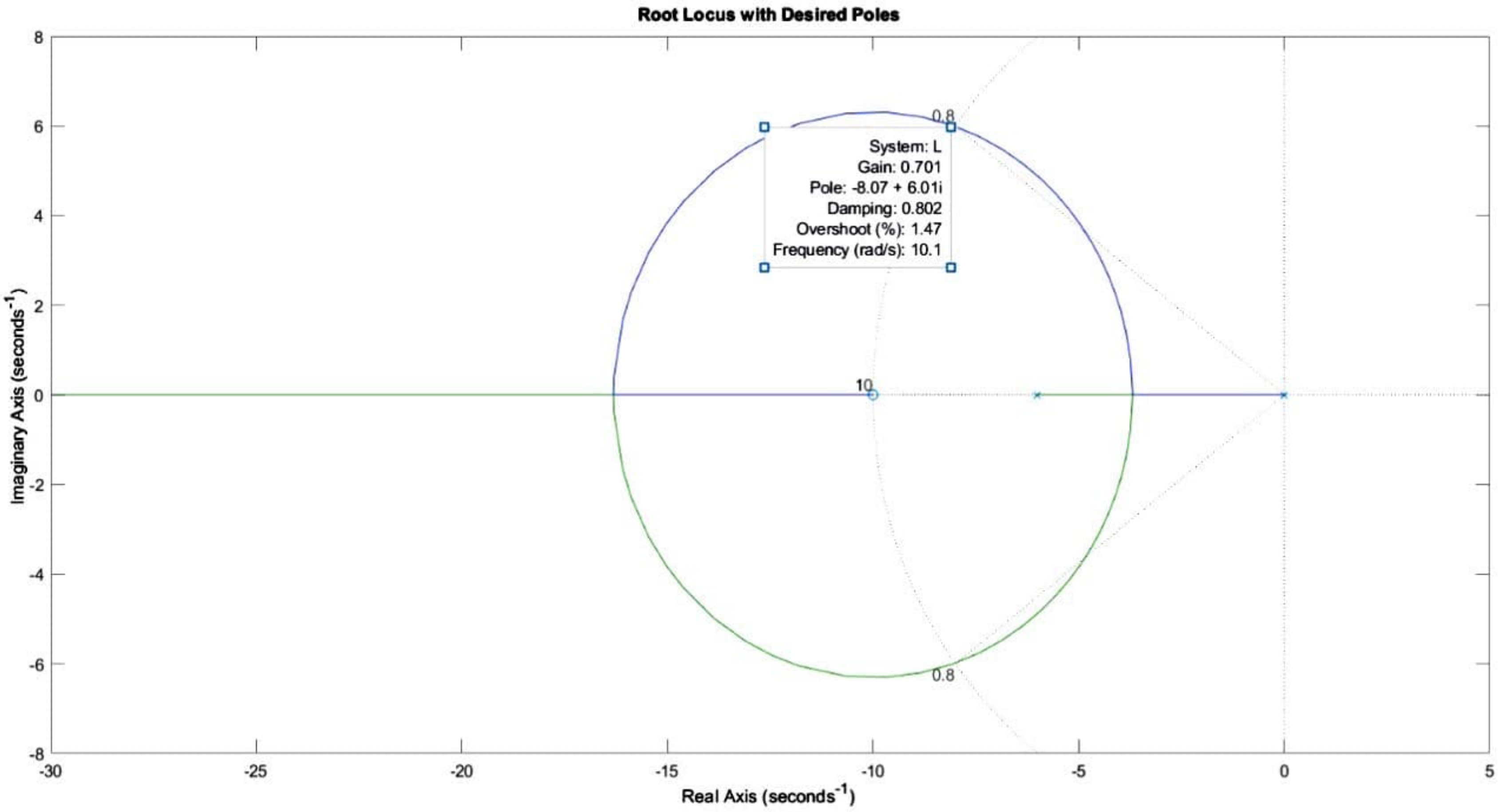
The value of kd has to be chosen s.t the poles lie on $(-8 + 6j)$ & $(-8 - 6j)$.

From the Root locus it is visible that $K_D \approx 0.701$ for the poles to lie on $(-8 + 6j)$ & $(-8 - 6j)$

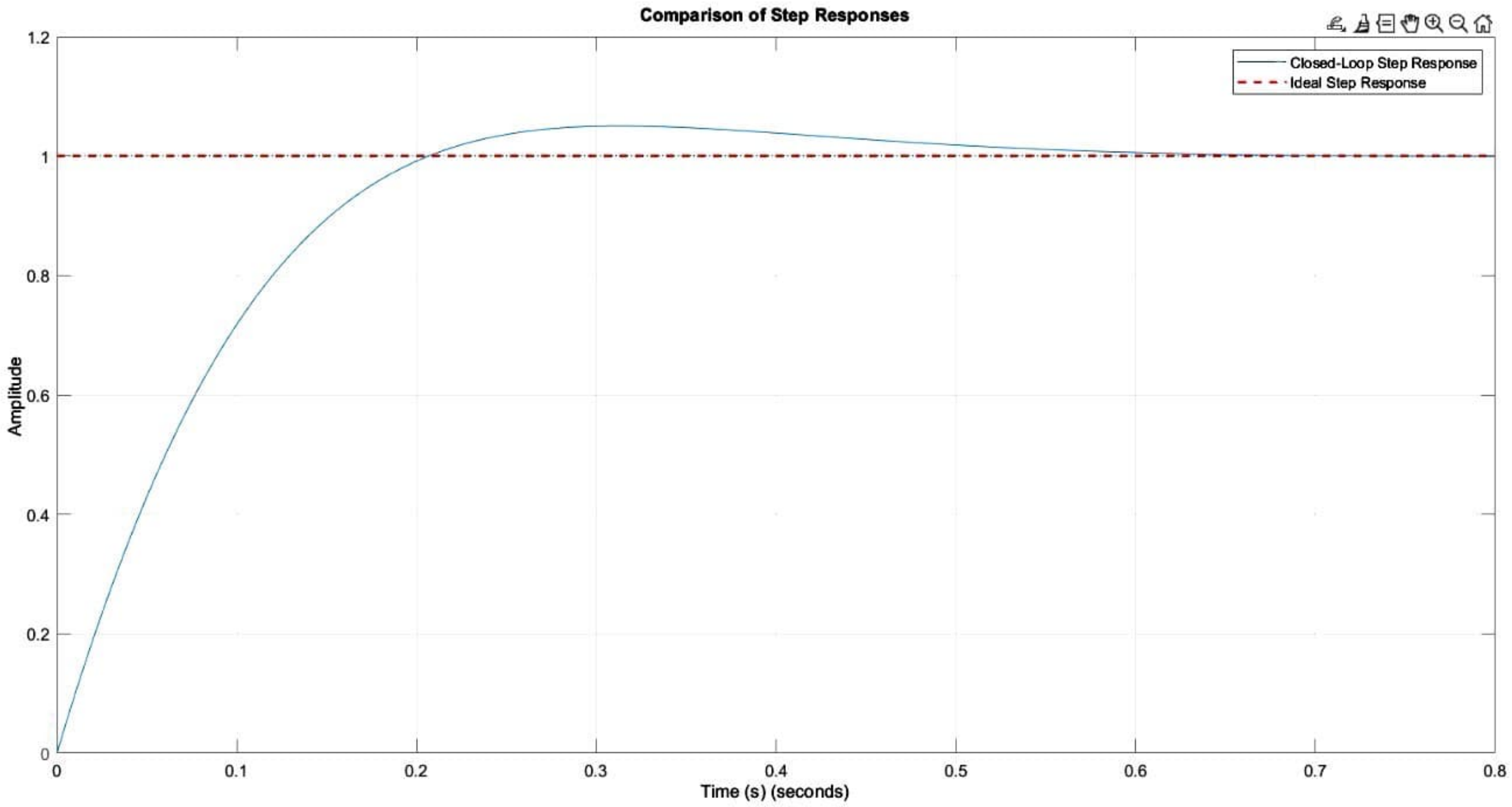
if $K_D = 0.701 \Rightarrow \frac{K_P}{K_D} = 10 \Rightarrow K_P = 10 \times 0.701 = 7.01$



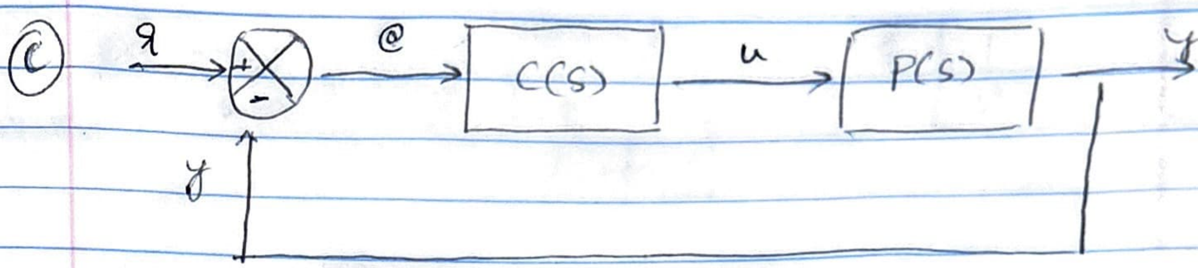
Root Locus with Desired Poles
Imaginary Axis (seconds^{-1})
Real Axis (seconds^{-1})
8
6
4
2
0
-2
-4
-6
-8
-30
-25
-20
-15
-10
-5
0
5
0.8
0.8
10



Root Locus with Desired Poles
System: L
Gain: 0.701
Pole: -8.07 + 6.01i
Damping: 0.802
Overshoot (%): 1.47
Frequency (rad/s): 10.1
0.8
0.8
10
Real Axis (seconds^{-1})
Imaginary Axis (seconds^{-1})
-30
-25
-20
-15
-10
-5
0
5
8
6
4
2
0
-2
-4
-6
-8


Comparison of Step Responses
Closed-Loop Step Response
Ideal Step Response
Amplitude
Time (s) (seconds)
0
0.2
0.4
0.6
0.8
1
1.2
0.1
0.3
0.5
0.7

(c)



$y = u\,P(s)$

$y = e\,C(s)\,P(s)$

$\Rightarrow r - y = e$

$\Rightarrow r - e\,C(s)\,P(s) = e$

$\Rightarrow r = e\,C(s)\,P(s) + e$

$\Rightarrow r = e\,(1 + C(s)\,P(s))$

$\Rightarrow \dfrac{E(s)}{R(s)} = \dfrac{1}{1 + C(s)\,P(s)} = S(s)$ → Sensitivity function.

~~Steady-state error~~ for $\frac{1}{s^2}$ From the final value theorem

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1}{s^2} \cdot \frac{1}{1 + C(s)P(s)} \right]$$

$(s)\left[R(s)\right]\left[S(s)\right]$ → Final value Theorm

$$e_{ss} = \lim_{s \to 0} s \left[\frac{1}{s^2} \cdot \frac{1}{1 + K\left(\frac{P_c}{z_c}\right)\left(\frac{s - z_c}{s - P_c}\right) \cdot \frac{1}{I s^2 + b s}} \right]$$

$$e_{ss} = \lim_{s \to 0} s\left[\frac{1}{s^2} \cdot \frac{1}{1+\frac{k}{Is^2+bs}}\right] = s\left[\frac{1}{s^2} \cdot \frac{1}{\frac{Is^2+bs+k}{Is^2+bs}}\right]$$

$$= \lim_{s \to 0} s\left[\frac{Is^2+bs}{s^2(Is^2+bs+k)}\right]$$

$$= \lim_{s \to 0} \frac{Is^3+bs^2}{s^2(Is^2+bs+k)}$$

$$e_{ss} = \lim_{s \to 0} \frac{I\cancel{s}^{\to 0}+b}{I\cancel{s^2}^{\to 0}+b\cancel{s}^{\to 0}+k} = \frac{b}{k}$$

$e_{ss} < 0.05$

$$\Rightarrow \frac{b}{k} < 0.05$$

$$\therefore \frac{k}{b} > \frac{1}{0.05}$$

$$k > \frac{1}{0.05}(b) \Rightarrow k > \frac{1}{0.05} \times 0.4166 = 8.332$$

let $k = 9$ for convience.

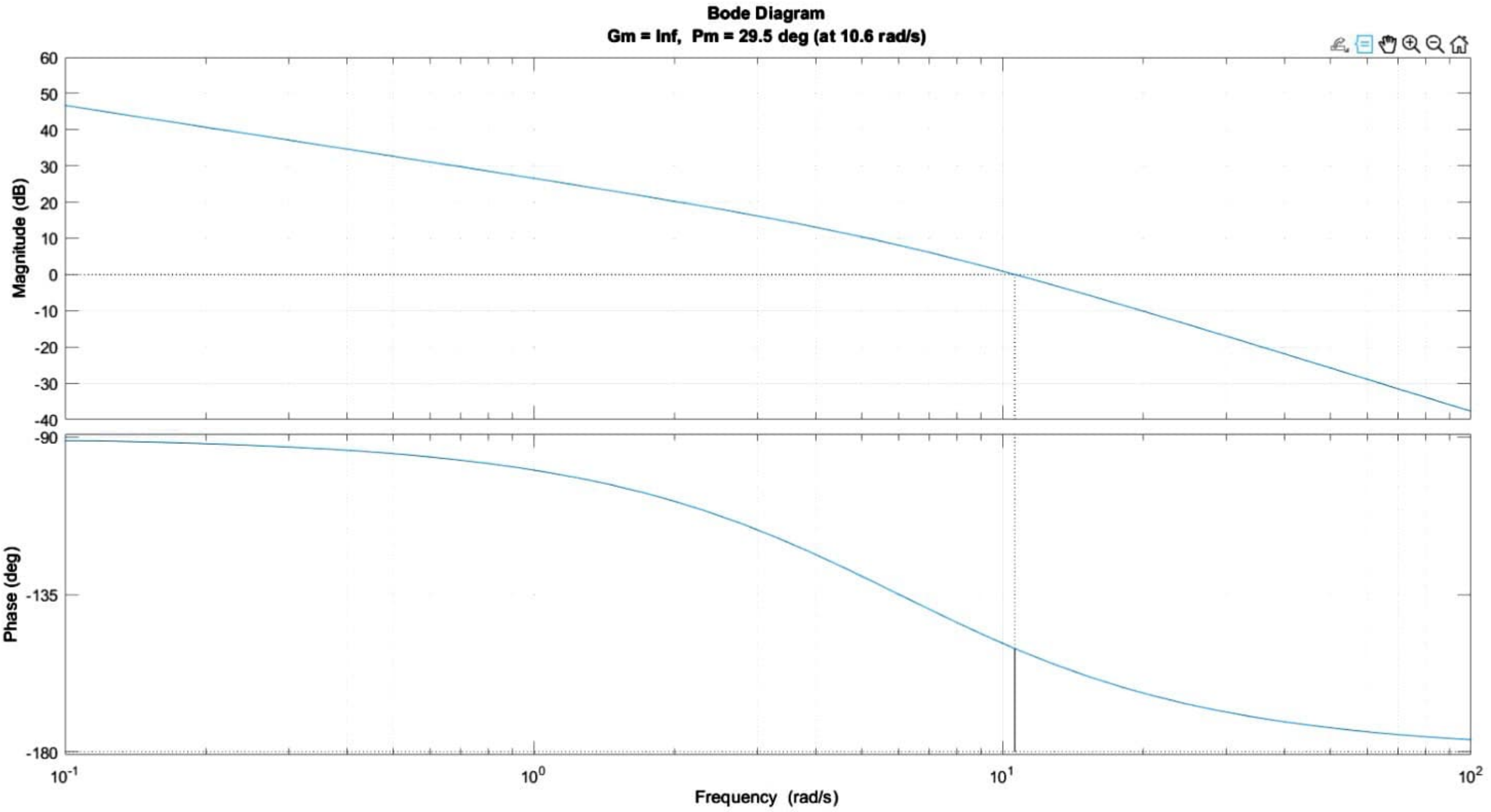
$$P'(s) = \frac{k}{Is^2+bs} = \frac{9}{Is^2+bs}$$

$I = 0.069367 \ kgm^2$

$b = 0.4166 \ kgm^2/s$

The Phase Margin of this plant is with gain k is -150^0

$(180 - 150^0) = 30^0$ (Approximate Value)



Bode Diagram
Gm = Inf, Pm = 29.5 deg (at 10.6 rad/s)
Magnitude (dB)
60
50
40
30
20
10
0
-10
-20
-30
-40
Phase (deg)
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
Frequency (rad/s)

$\phi_{max} = 45^\circ - (30) + 5 = 20^\circ$

$$a = \frac{1 - \sin\phi_{max}}{1 + \sin\phi_{max}}$$

$$a = \frac{1 - \sin(20^\circ)}{1 + \sin(20^\circ)} = 0.49$$

inorder to calculate $\omega\phi_{max}$

$$20 \log_{10} |P(j\omega m)| = 20 \log(\sqrt{a}) = 20 \log(\sqrt{0.49}) =$$

$$= -3.098 \text{ dB} < 0.$$

From the bode plot of Plant $\frac{k}{Is^2 + bs}$ the natural frequency

$\omega_m = 13.1$ rad/sec

From ω_m we can find z_c, P_c

$z_c = -\omega_m \sqrt{a}$

$P_c = z_c / a$

$z_c = -(13.1)(\sqrt{0.49}) = -9.17$

$P_c = -9.17/0.49 = -18.714$

$P_c < z_c < 0$

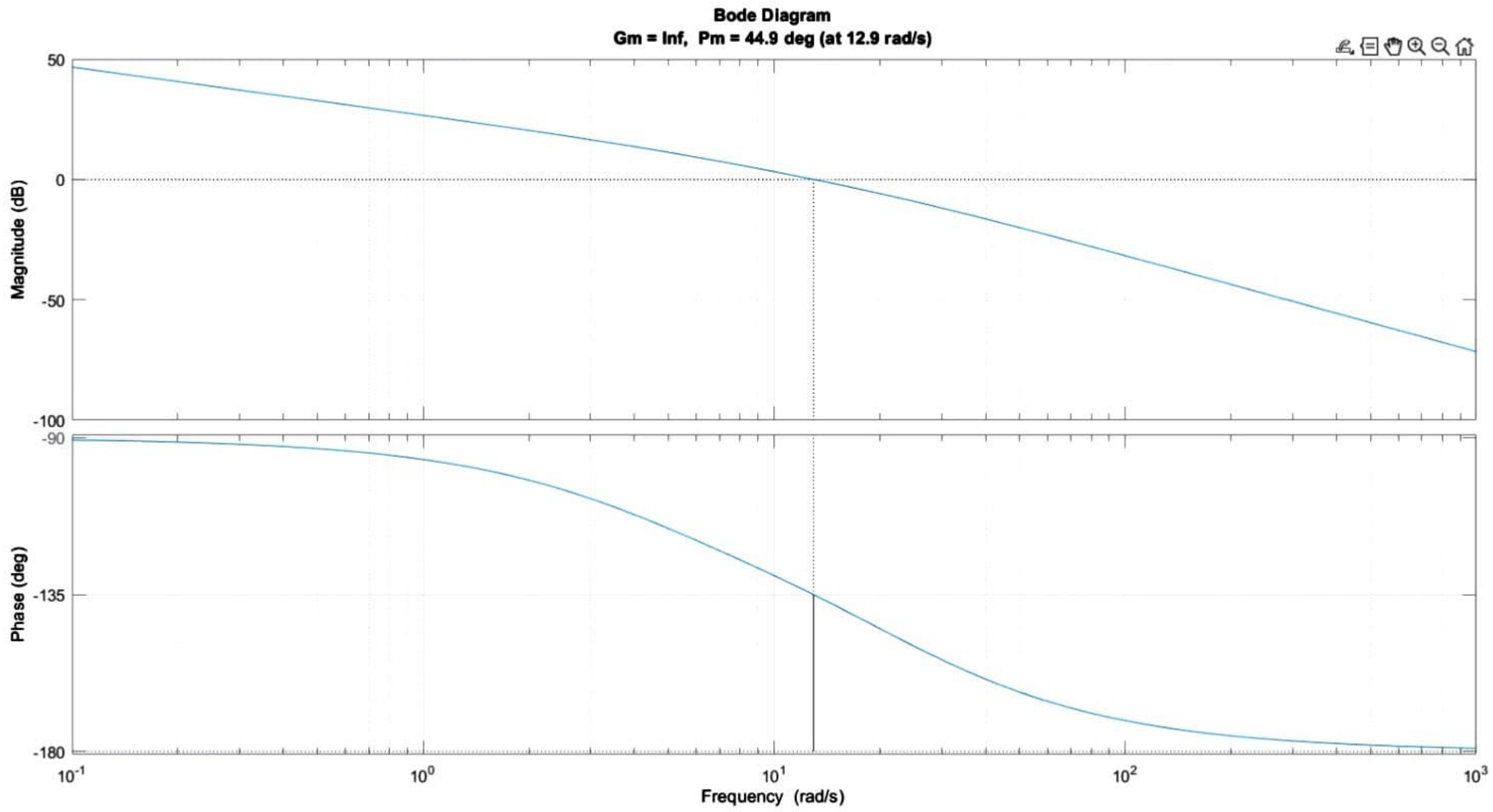
$$C'(s) = \frac{P_c}{z_c}\left[\frac{s + 9.17}{s + 18.714}\right] = \frac{\cancel{+9.17}\ 18.714}{9.17}\left[\frac{s + 9.17}{s + 18.714}\right]$$

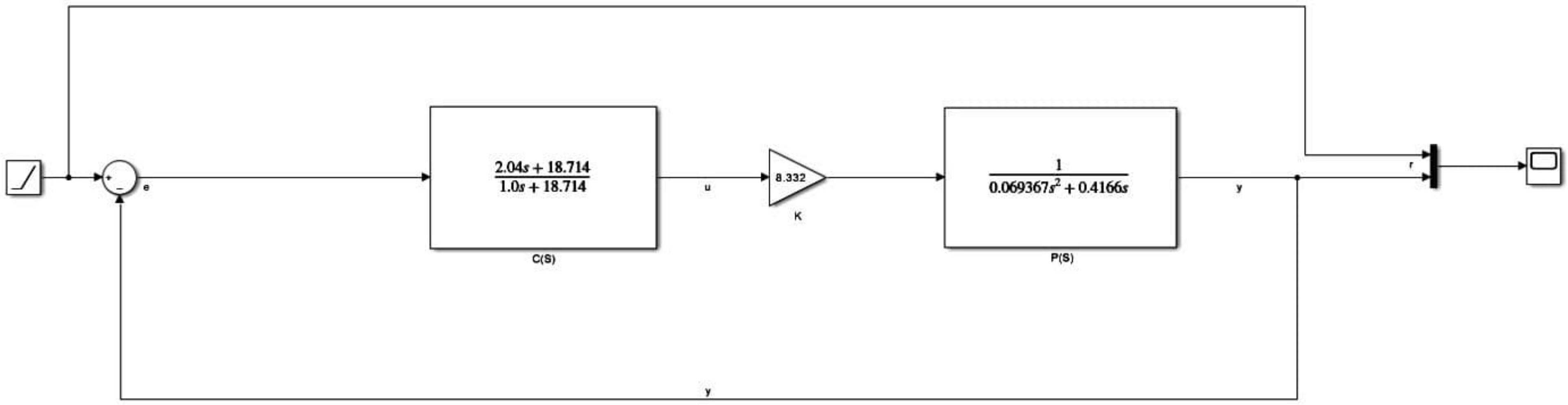
$$C'(s) = (9)\frac{2.04s + 18.714}{s + 18.714} \qquad P(s) = \frac{1}{Is^2 + bs}$$

$\boxed{\text{where } k = 9}$

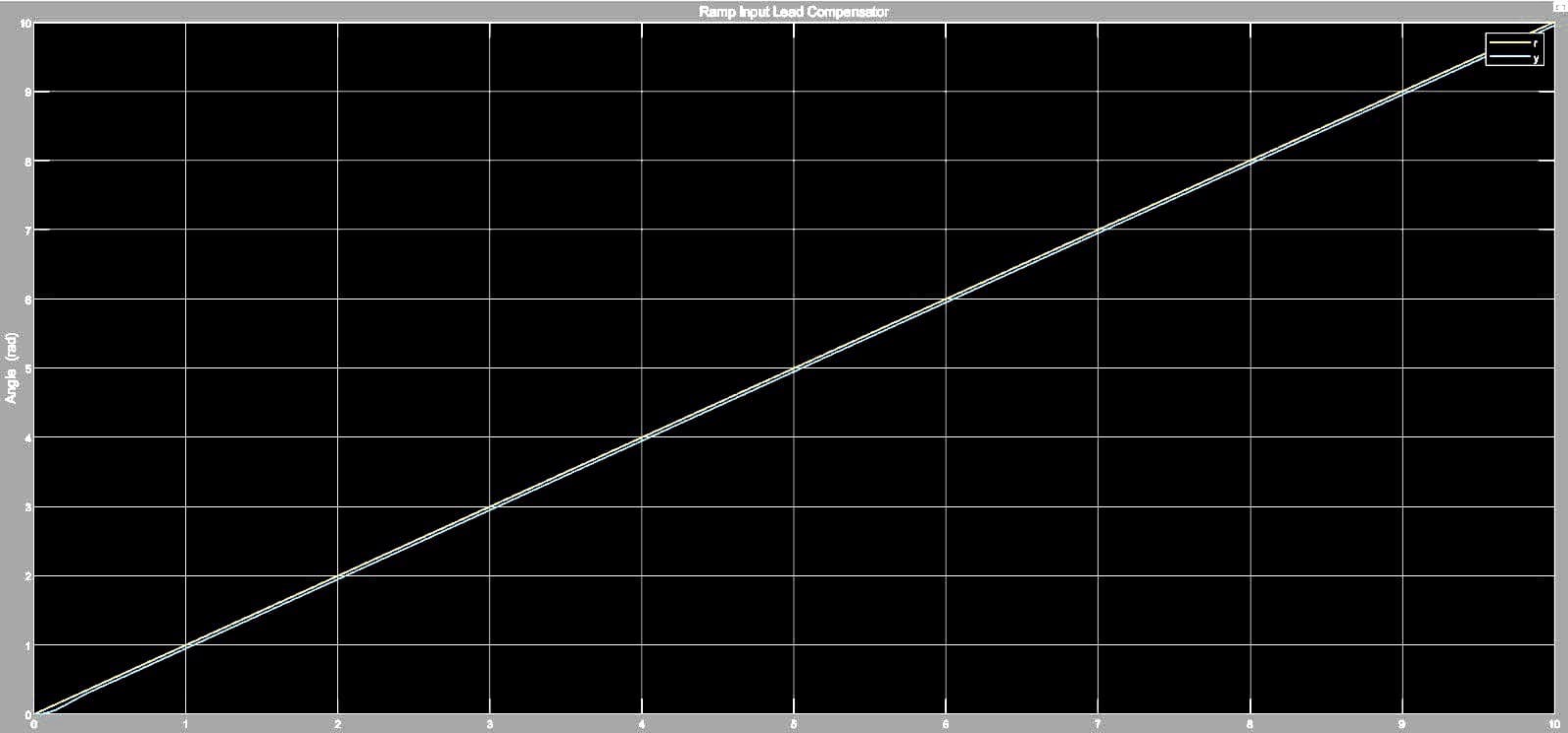
$L(s) = C(s) \cdot P(s)$

→ This will give Bode plot of Lead comp + plant


Bode Diagram
Gm = Inf, Pm = 44.9 deg (at 12.9 rad/s)
Magnitude (dB)
50
0
-50
-100
Phase (deg)
-90
-135
-180
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}
Frequency (rad/s)



e
2.04s + 18.714
1.0s + 18.714
C(S)
u
8.332
K
1
0.069367s^2 + 0.4166s
P(S)
y
r
y



Ramp Input Lead Compensator
r
y
Angle (rad)
0
1
2
3
4
5
6
7
8
9
10