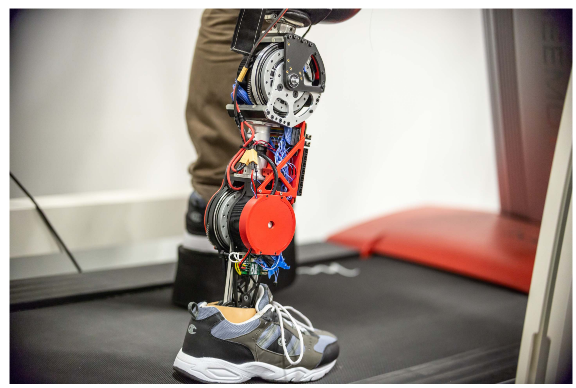
ROB 498/599 Fall 2024: Assignment #5

Due on Nov 22nd, at 11:59 pm, on Gradescope.

Submission Instructions: Submit a PDF file with written answers to all questions, images of plots (included on the PDF). Paste your code to the end of the document. Label the document **[uniquename]_ROB498_Assignment5.pdf** and upload to Gradescope. **No late submissions will be accepted.**

(a) CAD design of the knee actuator.



(b) Final assemblies of the Leg 2 prosthesis

Figure 1: **Left:** CAD design of the knee actuator. The exploded view on the left-hand side displays the components/sub-assemblies of the knee actuator, such as the upper/lower hinges, encoders, transmission, motor, and pylon. The image on the right-hand side presents the assembled knee actuator. The pyramid adapter on top connects to the user's socket, and the length-adjustable pylon on bottom connects to the ankle actuator module. **Right:** Prototype of Leg 2 knee-ankle prosthesis. Figures obtained from Dr. Robert Gregg with permission.

Problem 1 Consider the actuator shown in Fig. 1a. The first-order dynamics of the actuator can be described as

$$I\dot{v} + bv = \tau \ ,$$

where I is the inertia, b is the damping coefficient, v is the output velocity of the motor, and τ is the applied torque. The transfer function of this system from torque to velocity

$$G(s) = \frac{V(s)}{T(s)} = \frac{1}{Is + b} \ . \tag{1}$$

To identify the inertia I and the damping b, a benchtop test was performed. Sinusoidal torque signals at different frequencies were sent to the motor and the actuator's velocity was recorded to obtain the Bode magnitude plot of Fig. 2.

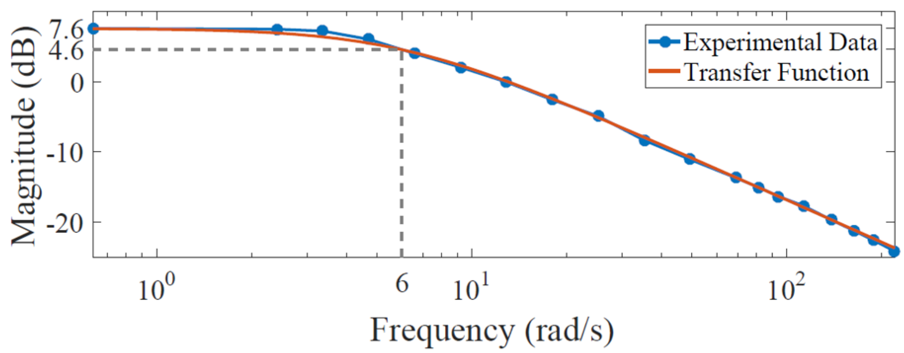


Figure 2: Magnitude plot for open-loop frequency response tests of the actuator. The DC gain is 7.6 dB, and the -3dB frequency (cutoff frequency) is 6 rad/s, with magnitude 4.6 dB.

(a) Compute the inertia I and damping b in (1), based on the frequency response shown in Fig. 2. All units are in the MKS-system.

(b) Create the Bode plot of the system (magnitude and phase) with the I and b you computed above. The magnitude plot you obtained should be the same as Fig. 2.

Problem 2 We now wish to control the angular **position** of the actuator in Problem 1, with the same parameters you found.

(a) Derive the transfer function from torque τ to position θ:

$$G(s) = \frac{\Theta(s)}{T(s)} \; .$$

(b) Now use root locus to design a PD-controller defined as

$$\tau = K_p(\theta_{cmd} - \theta) - K_d\dot{\theta} \; ,$$

where K_d is the parameter to vary with $K_p/K_d = 10$. Choose your K_p and K_d such that the closed loop system has a natural frequency $\omega_n = 10$ rad/s and damping ratio $\zeta = 0.8$. Simulate the step response of the closed-loop system.

Hint: `sgrid(zeta, wn)` generates s-plane grid line, with constant damping ratio `zeta` and natural frequency `wn`. `step(sys)` generates the step response plot of a system `sys`.

(c) The output signal magnitude of a PD controller increases with input frequency, which is undesirable since high-frequency noise is common in real systems. To alleviate the high-frequency amplification of PD controllers, lead compensators are used to limit the high-frequency amplification, while still increasing phase margin.

Now, design a lead compensator for the actuator, which allows **unit ramp position input** tracking with a steady-state error of less than 0.05. Additionally, the system should achieve 45° phase margin. Write out $C(s)$, the transfer function of the lead compensator, and create the Bode plot of $C(s)G(s)$, with amplitude and phase margin.

Hint: The lead compensator should have a transfer function in the following form:

$$C(s) = K \cdot \frac{p_c}{z_c} \cdot \frac{s - z_c}{s - p_c} \; ,$$

where K is a constant gain, p_c and z_c are the pole and zero of the lead compensator.

(d) Create a Simulink model of the position control system you designed in (c) and track a unit ramp position input signal. Simulate for 10 seconds, and create a figure of the reference signal and the output signal.