ROB 498/599 Fall 2024: Assignment #4

Due on Nov. 8th, at 11:59 pm, on Gradescope.

Submission Instructions: Submit a PDF file with written answers to all questions, images of plots (included on the PDF). Paste your code to the end of the document. Label the document **[uniquename]_ROB498_Assignment_4.pdf** and upload to Gradescope. **No late submissions will be accepted.**

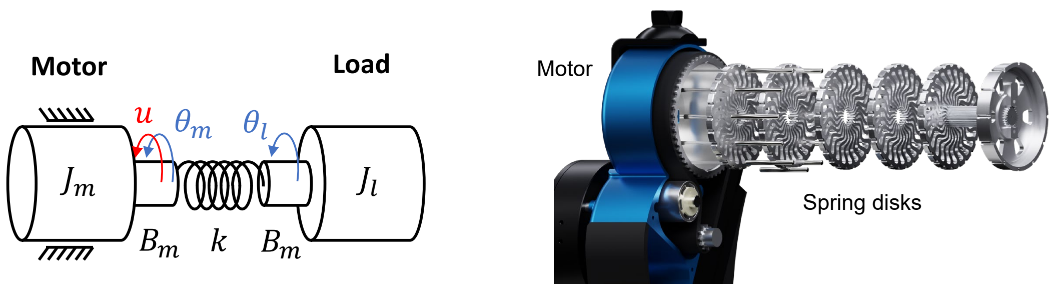


Figure 1: **Left:** Diagram of an SEA. Arrow represents positive directions. Right: exploded-view drawing of Open Source Leg v2 knee SEA. By changing the number of spring disk, the elastic element stiffness can be reconfigured. Figure adapted from Dr. Elliott Rouse with permission.

Series Elastic Actuator (SEA) refers to the serial connection between an electric motor, a spring, and a mechanical load. The series spring allows torque regulation by controlling the elongation of the spring, thus improving the torque tracking performance (though in this exercise we will only consider position control). SEAs can also reduce energy consumption by storing and releasing elastic energy. These benefits make SEAs suitable for many robotic applications like manipulators and wearable robots.

The equation of motion of the motor side is

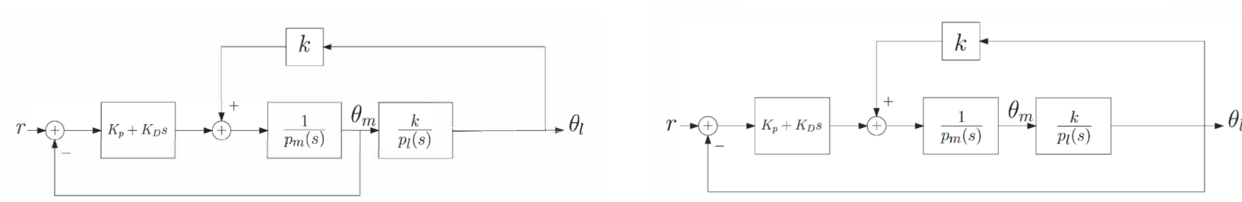
$$J_m\ddot{\theta}_m + B_m\dot{\theta}_m - k(\theta_l - \theta_m) = u \ .$$

The equation of motion of the load side is

$$J_l\ddot{\theta}_l + B_l\dot{\theta}_l + k(\theta_l - \theta_m) = 0 \ ,$$

where J_m, J_l are the motor and load inertia, B_m, B_l are the motor and load damping constants, θ_m, θ_l are the angle of the motor and load shafts, k is the stiffness of the spring, and u is the input torque applied to the motor shaft.

Problem 1 Suppose a proportional-derivative controller is used for position control with



(a) Motor angle feedback

(b) Load angle feedback

Figure 2: PD control with motor angle and load angle feedback. Refer to lecture notes for definitions of $p_m(s)$ and $p_l(s)$.

(a) Motor angle feedback:

$$u = K_p(\theta_{cmd} - \theta_m) - K_d\dot{\theta}_m$$

(b) Load angle feedback:

$$u = K_p(\theta_{cmd} - \theta_l) - K_d\dot{\theta}_l$$

The system diagrams of both closed-loop systems are shown in Fig. 2. Determine the closed-loop state-space equations in the form

$$\dot{\mathbf{x}} = A_{cl}\,\mathbf{x} + B_{cl}\,\theta_{cmd} \ .$$

Problem 2 Let $J_m = J_l = 0.0097\,\mathrm{kg\cdot m^2}, b_m = b_l = 0.04169\,\mathrm{Ns\cdot m^{-1}}, k = 100\,\mathrm{Nm\cdot rad^{-1}}$. Suppose $K_p = 10$, use the state-space representation you obtained from Problem1, and determine the range of $K_d > 0$ such that the closed-loop systems (a) and (b) in Fig. 2 are stable.

Hint: The `coeffs` function in MATLAB returns the coefficients of a symbolic polynomial.

Problem 3 Determine the transfer functions of the closed-loop systems (a) and (b) in Fig. 2,

$$H(s) = \frac{\Theta_l(s)}{R(s)} \ ,$$

where $\Theta_l(t) = \mathcal{L}\{\theta_l(t)\}$ and $R(s) = \mathcal{L}\{r(t)\}$ are the Laplace transforms of $\theta_l(t)$ and $r(t)$.

Hint: System A and System B in Fig. 3 have the same transfer functions.

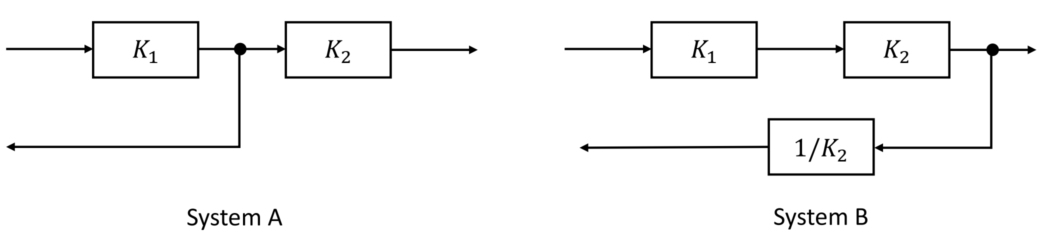


Figure 3: System A and system B are equivalent

Problem 4 Choose K_d as the gain to vary, and let $K_p/K_d = 10$. Create the root loci for system (a) and (b) in Fig. 2. Determine the range of K_d such that the closed-loop systems are stable, and compare with the result you obtained from Problem 1.

Problem 5 Create Simulink models for both systems (a) and (b) in Fig. 2. Use the same parameters as in Problem 2. Simulate the system responses to an unit step reference signal for 10 seconds with the following controller gains:

(a) $K_p = 10, K_d = 0.05$

(b) $K_p = 10, K_d = 0.1$

Attach screenshots of both Simulink models you created to the document. Additionally, create figures of the system output (load angle) along with the step reference signal.

Hint: For the second set of gains, you should obtain unstable system response for the load angle feedback case. This result should be in line with what you got in previous Problems.