ROB 498/599 Fall 2024: Assignment #1

Due on September 13th, at 11:59pm, on Gradescope.

Submission Instructions: Submit a PDF file with written answers to all questions, images of plots (included on the PDF). Label the folder **[uniquename]_ROB498_Assignment1.pdf** and upload to Gradescope.

Problem 1 [5 pts] For the system shown in Figure 1, suppose that $P > 0$ and that the output is measured by an imperfect sensor whose noise is proportional to the output, $n = qy$, where $q = 0.05$. Determine the feasible range of $K > 0$ to ensure that the steady-state tracking error, $e = r - y$, is within 5% of the command r, i.e., $|\frac{e}{r}| \leq 0.05$. Justify your answer.

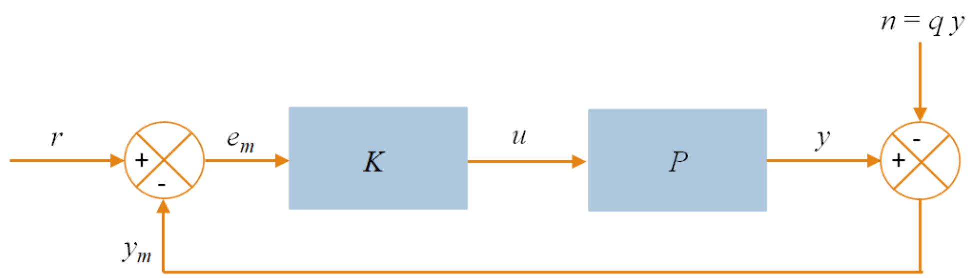

Figure 1: System of Problem 1.

Problem 2 [5 pts] For the system shown in Figure 2, suppose that $P_0 = 0.99P$, $P > 0$ and that the output is measured by an imperfect sensor whose noise is proportional to the output, $n = qy$, where $q = 0.05$. Determine the feasible range of $K > 0$ to ensure that the steady-state tracking error, $e = r - y$, is within 5% of the command r, i.e., $|\frac{e}{r}| \leq 0.05$. Justify your answer.

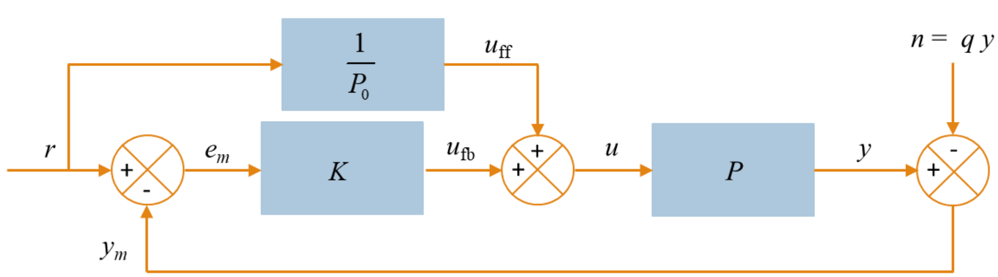

Figure 2: System of Problem 2.

Problem 3 **[5 pts]** Consider a Segway, modeled as an inverted pendulum on a base. The dynamics of the system are represented by the following equations of motion:

$$(M+m)\ddot{x} - ml\cos(\theta)\ddot{\theta} + c\dot{x} + ml\sin(\theta)(\dot{\theta})^2 = F,$$
$$-ml\cos(\theta)\ddot{x} + (J+ml^2)\ddot{\theta} + \gamma\dot{\theta} - mgl\sin\theta = 0,$$

where θ, $\dot{\theta}$ are the angle and angular rate (velocity) of the pendulum, x, $\dot{x}$ is the position and velocity of the base, F is the force applied to the base, M is the mass of the base, m, J are the mass and moment of inertia of the pendulum, l is the distance from the base to the center of mass of the pendulum, c, γ are coefficients of viscous friction, and g is the acceleration due to gravity. Obtain the equations of motion in the state-space form:

$$\dot{\mathbf{x}}(t) = f(\mathbf{x}(t), u(t)),$$

where $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \theta$, $x_4 = \dot{\theta}$, $u = F$.

Problem 4 **[5 pts]** Simulate the response of the model in Problem 3 to the initial conditions $x(0) = 0$ m, $\dot{x}(0) = 0$ m/sec, $\theta(0) = \frac{\pi}{8}$ rad, $\dot{\theta}(0) = 0$ rad/s and the input given as:

$$u(t) = \begin{cases} 0 & t < 0, \\ 0.1\cos(10t) & 0 \le t \le 2.5, \\ 0.5\cos(5t) & 2.5 < t < 5, \\ 1.0\cos(5t) & t \ge 5, \end{cases}$$

in MATLAB, and provide a MATLAB plot of the state variables x_1 [m], x_2 [m/s], x_3 [rad], x_4 [rad/s] versus time over the time interval $[0, 10]$ sec. Use the following values of numerical parameters: $m = 0.2$ kg, $M = 0.5$ kg, $J = 0.006$ kg m^2, $l = 0.3$ m, $c = 0.1$ N sec/m, $\gamma = 0.1$ N m sec/rad, $g = 9.81$ m/sec^2.

Problem 5 **[5 pts]** Given zero input $u(t) = 0$, $t \ge 0$, find all the equilibria of the Segway model in Problem 3.

Hint: Set $u = u_{eq} = 0$ in $f(\mathbf{x}, u)$, and solve for $\mathbf{x} = \mathbf{x}_{eq}$.

Problem 6 **[5 pts]** Linearize the equations of motion in state-space form in Problem 3 about the operating point

$$x_1 = x_{1,eq} = 0, \quad x_2 = x_{2,eq} = 0, \quad x_3 = x_{3,eq} = 0, \quad x_4 = x_{4,eq} = 0, \quad \text{and} \quad u(t) = u_{eq} = 0,$$

and give the linear model in the state-space form (without plugging in any numerical values for m, M, J, l, g, c, γ).

Problem 7 **[10 pts]** Consider the linearized model in Problem 6, with the numerical values provided in Problem 4.

(a) **[5 pts]** Simulate the response of the linearized model to the same initial conditions and input signal as in Problem 4. Provide a MATLAB plot of the state variables (x_1 in m, x_2 in m/sec, x_3 in rad, x_4 in rad/sec, versus time t in sec) of the linearized model over the time interval $[0, 10]$ sec. What is the difference of the response of the linearized model compared to the response of the nonlinear model in Problem 4?

Hint: If you are puzzled with the response of the system, see next question.

(b) **[5 pts]** Write the linearized model in the state-space form as $\dot{\mathbf{x}} = A\mathbf{x} + Bu$. Compute and write down the eigenvalues of the state matrix. Determine if the system is stable at the equilibrium $u_{eq} = 0, \mathbf{x}_{eq} = 0$.