ROB 498 Fall 2024: Assignment #2

Due on Oct 4th, at 11:59pm, on Gradescope.

Submission Instructions: Submit a PDF file with written answers to all questions, images of plots (included on the PDF). Paste your code to the end of the document. Label the document **[uniquename]_ROB498_Assignment2.pdf** and upload to Gradescope.

Problem 1 [30 pts] Consider now only the equations that govern the angle of the pendulum, i.e.,

$$(J + ml^2)\ddot{\theta} + \gamma\dot{\theta} - mgl\sin\theta = l\cos(\theta)\mathrm{F}.$$

(a) **[5 pts]** Write the state-space form of the nonlinear system above with state vector $\mathbf{x}_s = \begin{bmatrix}\theta & \dot{\theta}\end{bmatrix}$ and control input $u_s = \mathrm{F}$; linearize about the equilibrium $u_{s,eq} = 0$, $\mathbf{x}_{s,eq} = 0$, and write the linearized system in state-space form as

$$\dot{\mathbf{x}}_s = A_s\mathbf{x}_s + B_s u_s.$$

With the numerical values provided in Assignment 1, Problem 4 (i.e., $m = 0.2$ kg, $M = 0.5$ kg, $J = 0.006$ kg m^2, $l = 0.3$ m, $c = 0.1$ N sec/m, $\gamma = 0.1$ N m sec/rad, $g = 9.81$ m/sec^2), compute and write down the eigenvalues of the state matrix A_s. Is the system stable at the equilibrium $u_{s,eq} = 0$, $\mathbf{x}_{s,eq} = 0$?

Hint: We have done the exact same derivation in class!

(b) **[5 pts]** Consider a Proportional-Derivative (PD) controller for controlling the angle θ of the pendulum to the commanded angle $\theta_{cmd} = 0$:

$$u(t) = k_P(\theta_{cmd} - \theta) - k_D\dot{\theta}.$$

Determine the closed-loop state-space equations in the form

$$\dot{\mathbf{x}}_s = A_{s,cl}\ \mathbf{x}_s + B_{s,cl}\ \theta_{cmd}.$$

(c) **[10 pts]** Determine the range of the controller gains k_P and k_D such that the closed-loop system with PD controller is stable.

(d) **[10 pts]** Determine the values of k_P and k_D such that the 2nd-order system governing the response of the angle of the pendulum has a percent overshoot of P.O. = 12% and a 2% band settling time of $T_s = 5$ sec.

Problem 2 [15 pts] Consider the linearized model in Problem 1, given in the state-space form as $\dot{\mathbf{x}}_s = A_s\mathbf{x}_s + B_s u_s$, with the numerical values provided in Problem 1.

(a) **[5 pts]** Consider a Proportional-Integral-Derivative (PID) controller for controlling the angle θ of the pendulum to the commanded angle $\theta_{cmd} = 0$:

$$u(t) = k_P(\theta_{cmd} - \theta) + k_I \int_0^t (\theta_{cmd} - \theta(\tau))d\tau - k_D\dot{\theta}.$$

Determine the closed-loop state-space equations in the form

$$\dot{\mathbf{x}}_e = A_{e,cl}\, \mathbf{x}_e + B_{e,cl}\, \theta_{cmd},$$

where $\mathbf{x}_e$ is a properly defined state vector.

(b) **[10 pts]** Determine the range of the controller gains k_P, k_I and k_D such that the closed-loop system with PID controller is stable.

Problem 3 [15 pts] Consider again the linearized model in Problem 1, with the numerical values provided in Assignment 1, Problem 4, subject to constant commanded angle $\theta_{cmd} \neq 0$ and a constant additive disturbance $w \neq 0$ that perturbs the angular velocity $\dot{\theta}$ of the pendulum.

(a) **[5 pts]** Consider the Proportional-Derivative (PD) controller $u(t) = k_P(\theta_{cmd} - \theta) - k_D\dot{\theta}$ for controlling the angle θ of the pendulum to the commanded angle $\theta_{cmd} \neq 0$, for $k_P = 5, k_D = 0.1$. What is the steady-state value θ_{ss}? Simulate the response of $\theta(t)$ and $\dot{\theta}(t)$ for initial conditions $\theta(0) = \frac{\pi}{10}$ rad, $\dot{\theta}(0) = 0.3$ rad/s, commanded angle $\theta_{cmd} = \frac{\pi}{3}$ and disturbance $w = 3$ rad/sec over the time interval $[0, 10]$ sec.

(b) **[5 pts]** Consider the Proportional-Integral-Derivative (PID) controller $u(t) = k_P(\theta_{cmd} - \theta) + k_I \int_0^t(\theta_{cmd} - \theta(\tau))d\tau - k_D\dot{\theta}$ for controlling the angle θ of the pendulum to the commanded angle $\theta_{cmd} \neq 0$, with k_P and k_D equal to the gains you used in (a), and $k_I = 2.3562$. What is the steady-state value θ_{ss}? Simulate the response of $\theta(t)$ and $\dot{\theta}(t)$ for initial conditions $\theta(0) = \frac{\pi}{10}$ rad, $\dot{\theta}(0) = 0.3$ rad/s, commanded angle $\theta_{cmd} = \frac{\pi}{3}$ and disturbance $w = 3$ rad/sec over the time interval $[0, 10]$ sec.

(c) **[5 pts]** Draw the block diagrams of the pendulum orientation system under PD controller and PID controller, respectively.