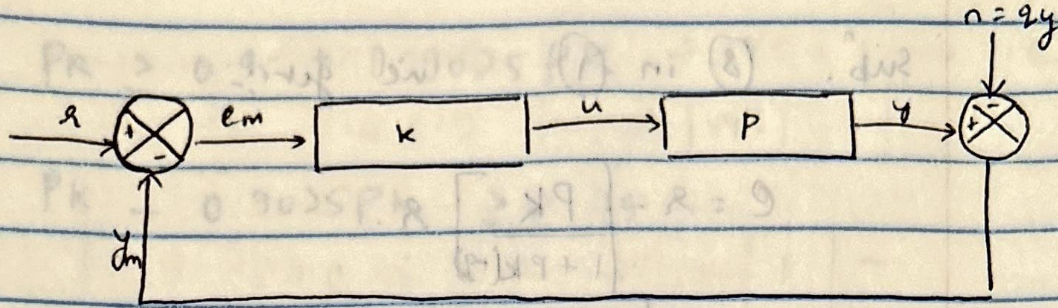
ROB-599 Robot-Control · Assignment-I.



$e = r - y$ ---- ①

Sol^n

$y = Pu$ --- ②

$u = Ke_m$ --- ③

$e_m = r - y_m$ --- ④

$y_m = y - qy = y(1-q)$ --- ⑤

~~fo~~ Sub ④ in ③ we get:-

$u = K[r - y_m]$ --- ⑥

Sub ⑥ in ② we get

$y = PK[r - y_m] = PK[r - y(1-q)]$

--- ⑦ {sub ⑤ in ⑦}

$\Rightarrow y = PK[r - y(1-q)]$

$y = PKr - PKy[1-q]$

$y + PKy[1-q] = PKr$

$y[1 + PK[1-q]] = PKr.$

$$y = \frac{PKr}{1+PK[1-q]} \longrightarrow ⑧$$

Sub^n. (8) in (1) we get :-

$$e = r - \left[\frac{PK}{1+PK(1-q)}\right] r.$$

$$e = r\left[1 - \frac{PK}{1+PK(1-q)}\right]$$

$$\frac{e}{r} = \left[1 - \frac{PK}{1+PK(1-q)}\right] \longrightarrow (9)$$

$$1 - \frac{PK}{1+PK(1-q)} \leq 0.05.$$

$$= \frac{-PK}{1+PK(1-q)} \leq -0.95$$

$$= \frac{PK}{(1+PK(1-q))} \geq 0.95$$ (Solve for k as the feasible range of k is required)

$$= \frac{PK}{1+PK(1-0.05)} \geq 0.95 \quad (q = 0.05)$$

$$= \frac{PK}{1+0.95PK} \geq 0.95$$

$$= PK \geq 0.95(1+0.95PK)$$

$$Pk \geq 0.95 + 0.9025\,Pk$$

$$Pk - 0.9025\,Pk \geq 0.95$$

$$Pk\left[1 - 0.9025\right] \geq 0.95$$

$$Pk\left[0.0975\right] \geq 0.95$$

$$K \geq \frac{0.95}{0.0975\,(P)}$$

$$\boxed{K \geq \frac{9.743}{P}} \quad \longrightarrow (+ve \text{ case})\ (10)$$

w.k.T $\left|\frac{e}{r}\right| \leq 0.05$

~~Case 1~~ +ve Case

$$+\left[\frac{e}{r}\right] \leq 0.05 \checkmark$$

−ve Case

$$-\left[\frac{e}{r}\right] \leq 0.05$$

$$= \frac{e}{r} \geq -0.05$$

Now lets consider the −ve case

$$\frac{e}{r} \geq -0.05$$

$$= \left[\frac{1 - Pk}{1 + Pk(1-q)}\right] \geq -0.05$$

$$= \left[1 - \frac{Pk}{1+Pk(1-q)} \right] \geq -0.05$$

$$- \left[1 - \frac{Pk}{1+Pk(1-q)} \right] \leq 0.05$$

$$= \frac{Pk}{1+Pk(1-q)} - 1 \leq 0.05$$

$$\frac{Pk}{1+Pk(1-q)} \leq 1.05 \qquad (\because q = 0.05)$$

$$= \frac{Pk}{1+Pk(0.95)} \leq 1.05$$

$$Pk \leq 1.05 \left[1 + 0.95Pk \right]$$

$$Pk \leq 1.05 + 0.9975 Pk$$

$$0.0025 Pk \leq 1.05$$

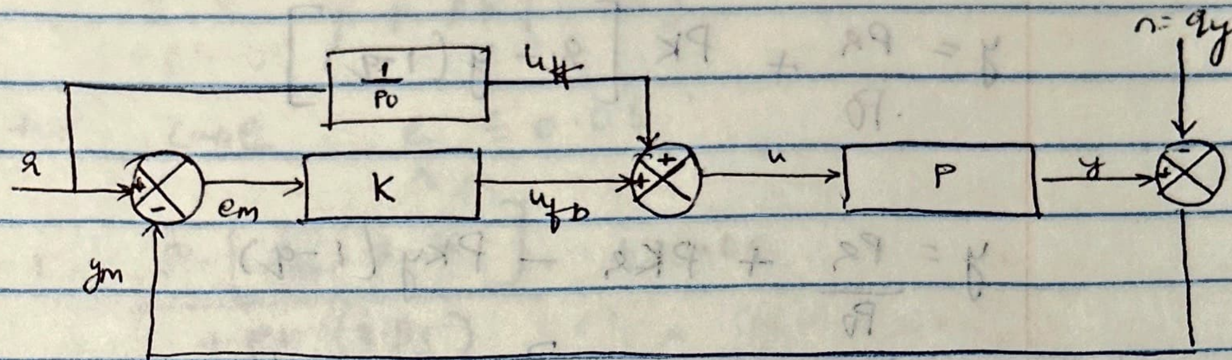
$$\cancel{P}k \leq \frac{1.05}{0.0025P}$$

$$\boxed{k \leq \frac{420}{P}} \text{ ------ (+ve case) (11)}$$

Combining both the +ve & -ve Cases i.e
Case (10) & Case (11) we get :-

$$\boxed{\frac{9.743}{P} \le K \le \frac{420}{P} \quad (P>0)}$$

Problem - 2



Sol^n $P_0 = 0.99P$, $P>0$, $e = r - y$ --- (0) $\left|\frac{e}{r}\right| \le 0.05$

$y = Pu$ ---- (1)

$u = u_{ff} + u_{fb}$ --- (2)

$u_{ff} = \frac{r}{P_0}$ --- (3)

$u_{fb} = Ke_m$ --- (4)

$e_m = r - y_m$ --- (5)

$y_m = y - n = y - qy = y(1-q)$ ---- (6)

$y = P[u_{ff} + u_{fb}]$ {sub (2) in (1)} --- (7)

$$y = P\left[\frac{r}{P_0} + k e_m\right] \quad \text{sub } (3) + (4) \text{ in } (7) \quad \text{---} (8)$$

$$y = P\left[\frac{r}{P_0} + k(r - y_m)\right] \quad \text{sub } (5) \text{ in } (8) \quad \text{---} (9)$$

$$y = P\left[\frac{r}{P_0} + k(r - y(1-q))\right] \quad \text{sub } (6) \text{ in } (9) \quad \text{---} (10)$$

$$y = \frac{Pr}{P_0} + Pk\left[r - y(1-q)\right]$$

$$y = \frac{Pr}{P_0} + Pkr - Pky(1-q)$$

$$y\left[1 + Pk(1-q)\right] = Pr\left[\frac{1}{P_0} + K\right]$$

$$y = \frac{Pr\left[\frac{1}{P_0} + K\right]}{\left[1 + Pk(1-q)\right]} \quad \text{---} (11)$$

$e = r - y.$ sub (11) in eq (1) we get

$$e = r - \frac{P\left[\frac{1}{P_0} + k\right]}{1 + Pk(1-q)} \cdot r$$

$$e = r\left[1 - \frac{P\left[\frac{1}{P_0} + k\right]}{1 + Pk(1-q)}\right]$$

$$\frac{e}{r} = \frac{1 - P\left[\frac{1}{P_0} + K\right]}{1 + PK(1-q)}$$

$q = 0.05$, $P_0 = 0.99P$, $\left|\frac{e}{r}\right| \leq 0.05$.

$$\frac{e}{r} = \frac{1 - P\left[\frac{1}{0.99P} + K\right]}{1 + PK(0.95)}$$

+ve case $\frac{e}{r} \leq 0.05$.

$$\frac{1 - P\left[\frac{1}{0.99P} + K\right]}{1 + PK(0.95)} \leq 0.05$$

$$\frac{-P\left[\frac{1}{0.99P} + K\right]}{1 + PK(0.95)} \leq -0.95$$

$$\frac{P\left[\frac{1}{0.99P} + K\right]}{1 + PK(0.95)} \geq 0.95$$

$$\frac{1.01 + PK}{1 + PK(0.95)} \geq 0.95$$

$$1.01 + PK \geq 0.95\left[1 + PK(0.95)\right]$$

$$\cancel{1 + PK \geq 0.95 + 0.9025PK}$$

$\cancel{Pk \geq \frac{0.95}{0.0975}}$ $\quad$ $1.01 + Pk \geq 0.95 + Pk(0.9025)$

$\cancel{Pk \geq 9.743}$ $\quad$ $(1 - 0.9025)Pk \geq (0.95 - 1.01)$

$\cancel{k \geq \frac{9.743}{P}}$ $\quad$ $0.0975\,Pk \geq -0.06$

$$Pk \geq \frac{-0.06}{0.0975}$$

$$Pk \geq -0.615 \Rightarrow K \geq \frac{-0.615}{P}$$

Since $K > 0$ & $P > 0$ $\quad$ $PK > 0$

$\Rightarrow \boxed{K > 0.} \rightarrow$ +ve case

(12)

-ve case.

$$\left|\frac{e}{R}\right| \leq 0.05$$

$$-\left(\frac{e}{R}\right) \leq 0.05$$

$$\frac{e}{R} \geq -0.05$$

$$1 - \frac{P\left[\frac{1}{P_0} + K\right]}{1 + PK(1-q)} \geq -0.05$$

$$\cancel{1 + 0.05 \geq \frac{P\left[\frac{1}{P_0} + K\right]}{1 + PK(1-q)}}$$

$$\frac{-P\left[\frac{1}{P_0}+K\right]}{1+PK(1-q)} \geq -0.05-1$$

$$\frac{-P\left[\frac{1}{P_0}+K\right]}{1+PK(1-q)} \geq -1.05$$

$$\frac{P\left[\frac{1}{P_0}+K\right]}{1+PK(1-q)} \leq 1.05$$

$$\frac{P\left[\frac{1}{0.99P}+K\right]}{1+PK(0.95)} \leq 1.05$$

$$\frac{1.01+PK}{1+(0.95)PK} \leq 1.05$$

$$1.01 + PK \leq 1.05 + 0.9975\,PK$$

$$0.0025\,PK \leq 0.04$$

$$PK \leq \frac{0.04}{0.0025}$$

$$K \leq \frac{16}{P}$$

$$\Rightarrow \boxed{0 < K \leq \frac{16}{P}}$$

Problem 3

Eq^n of the motion is given as:-

$$(M+m)\ddot{x} - ml\cos\theta\,\ddot{\theta} + c\dot{x} + ml\sin(\theta)(\dot{\theta})^2 = F \quad \text{--- (1)}$$

$$-ml\cos(\theta)\ddot{x} + (J + ml^2)\ddot{\theta} + \gamma\dot{\theta} - mgl\sin\theta = 0 \quad \text{--- (2)}$$

As eq of motion's from (1) & (2) have two second order differential eq^n terms in them ($\ddot{x}$ & $\ddot{\theta}$) we can consider that it is a 4th order differential eq^n, (i.e order of the system is 4)

hence the system can be converted to a 4th order state-space model of the form

$$\dot{x}_1 = f_1(x_1, x_2, x_3, x_4, u)$$
$$\dot{x}_2 = f_2(x_1, x_2, x_3, x_4, u)$$
$$\dot{x}_3 = f_3(x_1, x_2, x_3, x_4, u)$$
$$\dot{x}_4 = f_4(x_1, x_2, x_3, x_4, u)$$

$$x_1 = x \;\; (3); \quad x_2 = \dot{x} \;\; (4); \quad x_3 = \theta \;\; (5); \quad x_4 = \dot{\theta} \;\; (6); \quad u = F \;\; (7)$$

(1) & (2) can be written interms of x_1, x_2, x_3 & x_4.

$$\dot{x}_1 = \dot{x} \; ; \quad \dot{x}_2 = \ddot{x} \; ; \quad \dot{x}_3 = \dot{\theta} \; ; \quad \dot{x}_4 = \ddot{\theta}$$

$$\dot{x}_1 = x_2 \; ; \quad \dot{x}_3 = x_4 \quad \text{--- (8)}$$

So subtimbing the values of (3), (4), (5), (6), (7) & (8) to (1) & (2) we get

$$\cancel{(M+m)\dot{x}_2 \quad ml\cos\theta}$$

$$(M+m)\dot{x}_2 - ml\cos\theta\,\dot{x}_4 + Cx_2 + ml\sin\theta\,(x_4)^2 = u \quad \leftarrow (9)$$

$$-ml\cos(\theta)\,\dot{x}_2 + (J+ml^2)\,\dot{x}_4 + \gamma x_4 - mgl\sin\theta = 0 \quad \leftarrow (10)$$

$$\rightarrow (9)$$

$$\Rightarrow (M+m)\dot{x}_2 - ml\cos(x_3)\,\dot{x}_4 + Cx_2 + ml\sin(x_3)(x_4)^2 = u$$

$$-ml\cos(x_3)\cdot\dot{x}_2 + (J+ml^2)\dot{x}_4 + \gamma x_4 - mgl\sin(x_3) = 0$$

$$\rightarrow (10)$$

let $(M+m) = M_t$ & $(J+ml^2) = J_t$

$$\rightarrow (9a)$$

$$M_t\dot{x}_2 - ml\cos(x_3)\cdot\dot{x}_4 + Cx_2 + ml\sin(x_3)(x_4)^2 = u$$

$$-ml\cos(x_3)\cdot\dot{x}_2 + J_t\dot{x}_4 + \gamma x_4 - mgl\sin(x_3) = 0$$

$$\rightarrow (10a)$$

From (8) $\dot{x}_1 = x_2$ & $\dot{x}_3 = x_4$

||ly the above eqn can be solved using the Simpy library. & the soln is as follows.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{J_t c x_2 + J_t l m x_4^2 \sin(x_3) - J_t u - g l^2 m^2 \sin(x_3)\cos(x_3) + \gamma l m x_4 \cos(x_3)}{-J_t M_t + l^2 m^2 \cos^2(x_3)} \\ x_4 \\ \dfrac{-M_t g + M_t \gamma x_4 + c l m x_2 \cos(x_3) + l^2 m^2 g \sin(x_3) - l m u \cos(x_3) + l^2 m^2 x_4^2 \cdot \sin(x_3) \cdot \cos(x_3)}{-J_t M_t + l^2 m^2 \cos^2(x_3)} \end{bmatrix}$$

$$\downarrow$$

$$\dot{x} = \underbrace{f(x, u)}$$

$\Rightarrow$ The eq^n of the motion has been represented in the state space form.

$\dot{x}(t) = f(x(t), u(t))$

where $x_1 = x$, $x_2 = \dot{x}$, $x_3 = \theta$, $x_4 = \dot{\theta}$, $u = F$.

On further simplification the state space form can be written as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{\left[-ml \sin x_3\, x_4^2 + mg\left(ml^2/J_t\right)\cos x_3 \sin x_3 - c x_2 - (\gamma/J_t)\, ml(\cos x_3)(x_4) + u\right]}{\left[M_t - m\left(ml^2/J_t\right)(\cos x_3)^2\right]} \\ x_4 \\ \dfrac{-ml^2 \cos x_3 \sin x_3\, x_4^2 + M_t g l \sin x_3 - cl(\cos x_3)\, x_2 - \gamma(M_t/m)\, x_4 + l \cos x_3\, u}{\left(J_t (M_t/m) - m(l \cos x_3)^2\right)} \end{bmatrix}$$

Contents


```
clear
clc
close all
```

System Parameters

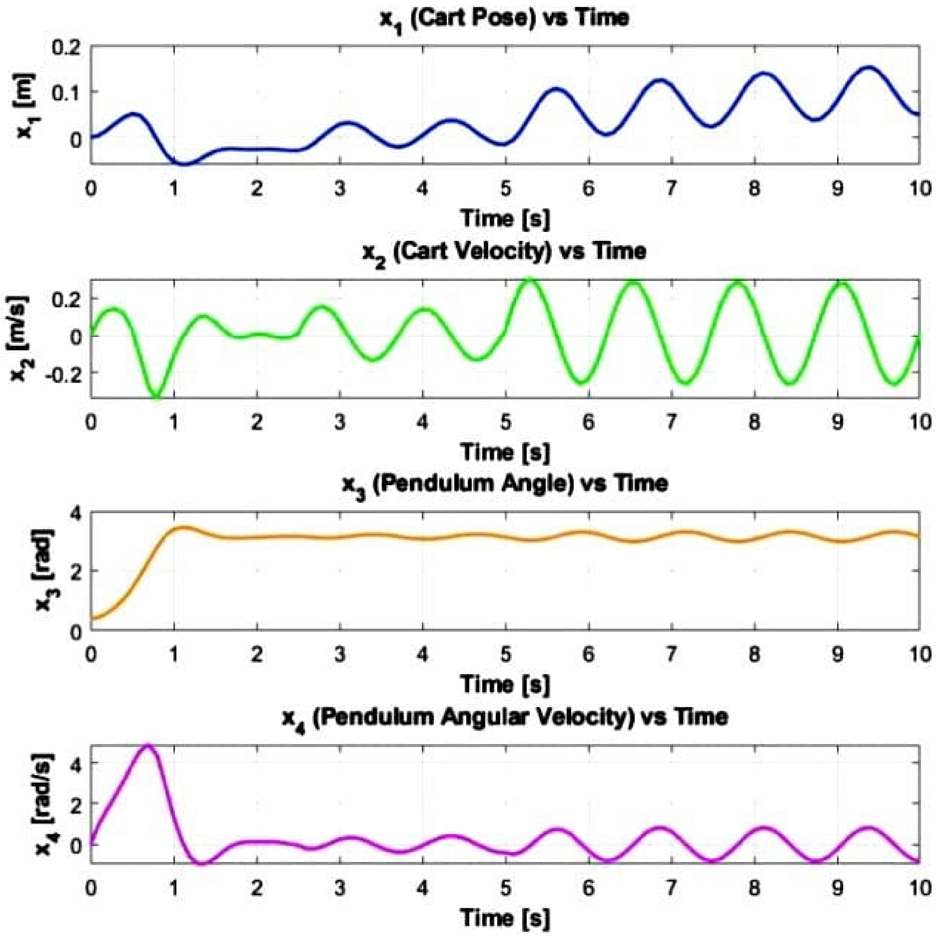
```
param.m = 0.2;
param.M = 0.5;
param.J = 0.006;
param.l = 0.3;
param.c = 0.1;
param.gamma = 0.1;
param.g = 9.81;
param.Jt= param.J + param.m*param.l^2;
param.Mt= param.M + param.m ;
```

Simulation Parameters

```
tSim = 0:0.1:10;
xo = [0; 0; pi/8; 0];

[tOut, xOut] = ode45(@(t,x) segway(t, x, param), tSim, xo);


% Calculate the applied force F(t) at each time step
F_t = zeros(length(tOut), 1);
for i = 1:length(tOut)
    F_t(i) = calculate_force(tOut(i));
end

% Create a new figure for the updated plots
figure('Position', [100, 100, 800, 900]); % Adjust figure size (width x height)

% Plot Cart Position (x1)
subplot(4,1,1);
plot(tOut, xOut(:,1), 'LineWidth', 2, 'Color', 'b');
xlabel('Time [s]', 'FontSize', 12, 'FontWeight', 'bold');
ylabel('x_1 [m]', 'FontSize', 12, 'FontWeight', 'bold');
title('x_1 (Cart Pose) vs Time', 'FontSize', 14, 'FontWeight', 'bold');
set(gca, 'FontSize', 12); % Increase font size of x and y ticks
grid on;

% Plot Cart Velocity (x2)
subplot(4,1,2);
plot(tOut, xOut(:,2), 'LineWidth', 2, 'Color', 'g');
xlabel('Time [s]', 'FontSize', 12, 'FontWeight', 'bold');
ylabel('x_2 [m/s]', 'FontSize', 12, 'FontWeight', 'bold');
title('x_2 (Cart Velocity) vs Time', 'FontSize', 14, 'FontWeight', 'bold');
set(gca, 'FontSize', 12); % Increase font size of x and y ticks
grid on;

% Plot Pendulum Angle (x3)
subplot(4,1,3);
plot(tOut, xOut(:,3), 'LineWidth', 2, 'Color', [1 0.5 0]);
xlabel('Time [s]', 'FontSize', 12, 'FontWeight', 'bold');
ylabel('x_3 [rad]', 'FontSize', 12, 'FontWeight', 'bold');
title('x_3 (Pendulum Angle) vs Time', 'FontSize', 14, 'FontWeight', 'bold');
set(gca, 'FontSize', 12); % Increase font size of x and y ticks
grid on;

% Plot Pendulum Angular Velocity (x4)
subplot(4,1,4);
plot(tOut, xOut(:,4), 'LineWidth', 2, 'Color', 'm');
xlabel('Time [s]', 'FontSize', 12, 'FontWeight', 'bold');
ylabel('x_4 [rad/s]', 'FontSize', 12, 'FontWeight', 'bold');
title('x_4 (Pendulum Angular Velocity) vs Time', 'FontSize', 14, 'FontWeight', 'bold');
set(gca, 'FontSize', 12); % Increase font size of x and y ticks
grid on;

% Adjust figure to have a clean layout and appropriate subplot spacing
set(gcf, 'Position', [100, 100, 800, 900]);
set(gcf, 'Color', 'w'); % Set white background for clarity
```

Calculate the Force Applied to the Cart

```
function F = calculate_force(t)
    if (t < 0)
        F = 0;
    elseif (t <= 2.5)
        F = 0.1 * cos(10 * t);
    elseif (t < 5)
        F = 0.5 * cos(5 * t);
    else
        F = 1.0 * cos(5 * t);
    end
end
```



State Space Of the System

```
function xdot = segway(t, x, param)

    % State variables
    x1 = x(1); % Cart position
    x2 = x(2); % Cart velocity
    x3 = x(3); % Pendulum angle
    x4 = x(4); % Pendulum angular velocity

    F = calculate_force(t);

    x1dot = x2;
    x2dot = (F*(param.J + param.m*param.l^2) - (param.J + param.m*param.l^2)*param.c*x2 - (param.J + param.m*param.l^2)*param.l*param.m*x4^2*sin(x3) + param.g*param
    x3dot = x4;
    x4dot = (F*param.l*param.m*cos(x3) + (param.M+param.m)*param.g*param.l*param.m*sin(x3) - (param.M+param.m)*param.gamma*x4 - param.c*param.l*param.m*x2*cos(x3) -

    xdot = [x1dot; x2dot; x3dot; x4dot];

end
```

Problem 5 :- From problem -3 the state space eq^n are as follows :-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{-ml\sin x_3\, x_4^2 + mg(ml^2/J_t)\cos x_3 \sin x_3 - c x_2 - (\gamma/J_t) ml(\cos x_3) x_4 + u}{M_t - m(ml^2/J_t)(\cos x_3)^2} \\ x_4 \\ \dfrac{-ml^2\cos x_3 \sin x_3\, x_4^2 + M_t g l \sin x_3 - cl(\cos x_3) x_2 - \gamma(M_t/m) x_4 + l\cos x_3 u}{J_t(M_t/m) - m(l\cos x_3)^2} \end{bmatrix} \quad \text{--- (1)}$$

input $u(t) = 0$ {i.e zero input} --- (2)

At equilibrium

$f(x_{eq}, u_{eq}) = 0$ & $u(t) = u_{eq} = 0$ --- (3)

$x = x_{eq}$ --- (4)

$\Rightarrow x_{2eq} = 0$ --- (5)

$$-ml\sin(x_{3eq})(x_{4eq})^2 + mg(ml^2/J_t)\cos(x_{3eq})\sin(x_{3eq}) - c\, x_{2eq} - \left(\frac{\gamma}{J_t}\right) ml\cos(x_{3eq})\, x_{4eq} + \cancel{u_{eq}} = 0$$

0 --- (6)

$x_{4eq} = 0$ --- (7)

--- (8)

$$-ml^2\cos(x_{3eq})\sin(x_{3eq})(x_{4eq})^2 + M_t g l \sin(x_{3eq}) - cl(\cos x_{3eq}) x_{2eq} - \gamma(M_t/m) x_{4eq} + l\cos x_{3eq}\, \cancel{u_{eq}} = 0$$

0

eq^n 5, 6, 7, 8 are obtained ~~from~~ by substituting 2, 3, 4, 5

From eq^n 5 + 7 w.k.t $x_{2eq} = 0$; $x_{4eq} = 0$.
substituting this in eq^n 6 + 8 we get.

$$-ml\sin x_{3eq} \cancel{x_{4eq}^2} + mg(ml^2/J_t)\cos x_{3eq} \sin x_{3eq} - \cancel{c x_{2eq}} - (\gamma/J_t)\, ml \cos x_{3eq} \cancel{x_{4eq}} = 0$$

$$= mg(ml^2/J_t)\cos(x_{3eq})\sin(x_{3eq}) = 0 \quad \text{--- (9)}$$

$$-ml^2\cos x_{3eq}\sin x_{3eq} \cancel{x_{4eq}^2} + M_t gl \sin x_{3eq} - cl(\cos x_{3eq}) \cancel{x_{2eq}} - \gamma(M_t/m) \cancel{x_{4eq}} \cancel{\cos} = 0$$

$$= M_t gl \sin x_{3eq} = 0 \quad \text{--- (10)}$$

From both eq^n 9 + 10 it is evident that the eq^n can become zero only if $\sin(x_{3eq}) = 0$

[Note - $\cos(x_{3eq}) \neq 0$; because eq^n 10 will not be equal to zero]

$\rightarrow$ if $\cos(x_{3eq}) = 0$
$x_{3eq} = \pm \frac{k\pi}{2}$ $(k = 2n+1)$ $n = 0, 1, 2, 3, \ldots$
for $k = 1$, $x_{3eq} = \frac{\pi}{2}$
$\sin(\frac{\pi}{2}) = 1$
i.e. eq 10 $\neq 0$.

$$\Rightarrow \sin(x_{3eq}) = 0$$
$$\Rightarrow x_{3eq} = \sin^{-1}(0)$$

$$\Rightarrow x_{3eq} = \pm k\pi; \quad \text{where } k = 0, 1, 2, 3 \ldots$$

$\therefore$ The equilibrium points are :-

$$\boxed{\begin{aligned} x_{2eq} &= 0 \\ x_{3eq} &= \pm k\pi ; \quad k = 0, 1, 2, 3 \ldots \\ x_{4eq} &= 0 \end{aligned}}$$

Observation :- The equilibrium corresponds to the straight - up or down positions of the pendulum ~~with~~ attached to the cart (i.e. Segway) with zero velocity & zero angular velocity ($x_{2eq} = \dot{x} = 0$; $x_{4eq} = \dot{\theta} = 0$). Since the velocities (i.e. linear velocity) is zero the displacement of the cart ($x_1 = x$) is also zero.

Problem-6 From problem-3 The state space form of the eq^n of motion of the Segway model is given as follows :-

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ \dfrac{-ml \sin x_3\, x_4^2 + mg(ml^2/J_t)\cos x_3 \sin x_3 - c x_2 - (\gamma/J_t)\, ml(\cos x_3)\, x_4 + u}{M_t - (m)(ml^2/J_t)(\cos x_3)^2} \\ x_4 \\ \dfrac{-ml^2 \cos x_3 \sin x_3\, x_4^2 + M_t g l \sin x_3 - cl(\cos x_3)x_2 - \gamma(M_t/m)x_4 + l\cos x_3\, u}{J_t(M_t/m) - m(l\cos x_3)^2} \end{bmatrix}$$

$\dot{x} = f(x, u)$

→ In order to obtain the linearized model $\mp$ about $x_{eq} = 0$; $u_{eq} = 0$ we have to find the Jacobians ~~about~~ of the state - space matrix.

$$\left.\frac{\partial F}{\partial x}\right|_{\substack{x = x_{eq} = 0 \\ U = U_{eq} = 0}} \qquad \left.\frac{\partial F}{\partial u}\right|_{\substack{x = x_{eq} = 0 \\ u = u_{eq} = 0}}$$

$$\frac{\partial F}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial F_1}{\partial x_2} & \frac{\partial F_1}{\partial x_3} & \frac{\partial F_1}{\partial x_4} \\ \partial f_2/\partial x_1 & \partial f_2/\partial x_2 & \partial f_2/\partial x_3 & \partial f_2/\partial x_4 \\ \partial F_3/\partial x_1 & \partial f_3/\partial x_2 & \partial F_3/\partial x_3 & \partial F_3/\partial x_4 \\ \partial f_4/\partial x_1 & \partial f_4/\partial x_2 & \partial f_4/\partial x_3 & \partial f_4/\partial x_4 \end{bmatrix}$$

$$\frac{\partial F}{\partial x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-c}{M_t - (m)\left(\frac{ml^2}{J_t}\right)(\cos(x_3))^2} & \dfrac{\partial F_2}{\partial x_3} & \dfrac{-2ml\,\cancel{\cos}\sin x_3\, x_4 - \left(\frac{\gamma}{J_t}\right) ml\cos x_3}{M_t - (m)\left(\frac{ml^2}{J_t}\right)(\cos(x_3))^2} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-cl(\cos x_3)}{J_t(M_t/m) - m(l\cos x_3)^2} & \dfrac{\partial F_4}{\partial x_3} & \dfrac{-2ml^2\cos x_3\sin x_3\, x_4 - \gamma(M_t/m)}{J_t(M_t/m) - m(l\cos x_3)^2} \end{bmatrix}$$

$$\frac{\partial F}{\partial u} = \begin{bmatrix} \dfrac{\partial f_1}{\partial u} \\ \dfrac{\partial f_2}{\partial u} \\ \dfrac{\partial f_3}{\partial u} \\ \dfrac{\partial f_4}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{1}{M_t - (m)(ml^2/J_t)(\cos x_3)^2} \\ 0 \\ \dfrac{(m)\, l\cos x_3}{J_t(M_t/m) - m(l\cos x_3)^2} \end{bmatrix}$$

$$\frac{\partial f_2}{\partial x_3} = \frac{\partial}{\partial x_3}\left[\frac{\left\{-ml\sin x_3\, x_4^2 + mg\left(\frac{ml^2}{J_t}\right)\cos x_3 \sin x_3 - c x_2 - \frac{\gamma}{J_t}(ml)(\cos x_3)\, x_4 + u\right\}}{M_t - (m)\left(\frac{ml^2}{J_t}\right)(\cos x_3)^2}\right]$$

using quotient rule we get.

$$\frac{\partial}{\partial x_3}\left(\frac{f_2(x_3)}{g(x_3)}\right) = \frac{f_2'(x_3)\, g(x_3) - f_2(x_3)\, g'(x_3)}{(g(x_3))^2}$$

$$\frac{\partial (f_2(x_3))}{\partial x_3} = \frac{\begin{aligned}&\left[-ml\cancelto{1}{\cos x_3}\, x_4^2 + mg\left(\frac{ml^2}{J_t}\right)\left[\cancelto{1}{\cos^2 x_3} - \cancelto{0}{\sin^2 x_3}\right] + \frac{\gamma}{J_t}(ml)\cancelto{0}{(\sin x_3)}(x_4)\right]\left[M_t - (m)\left(\frac{ml^2}{J_t}\right)\left(\cancelto{1}{\cos^2(x_3)}\right)\right] \\ &- \left\{\left[ml\cancelto{0}{\sin x_3}\, x_4^2 + mg\left(\frac{ml^2}{J_t}\right)\cancelto{0}{\cos x_3 \sin x_3} - c x_2 - \frac{\gamma}{J_t}(ml)\cancelto{1}{(\cos x_3)}\, x_4 + \cancelto{0}{u}\right]\right\} * \\ &\left\{+2(m)\left(\frac{ml^2}{J_t}\right)\cancelto{0}{(\cos x_3)(\sin x_3)}\right\}\end{aligned}}{\left[M_t - (m)\left(\frac{ml^2}{J_t}\right)\left(\cancelto{1}{\cos(x_3)}\right)^2\right]^2}$$

$$\left.\frac{\partial}{\partial x_3}\right|_{\substack{x_3 = x_{3eq} = 0 \\ u = u_{eq} = 0}} = \frac{\left[-ml x_4^2 + (mg)\left(\frac{ml^2}{J_t}\right)\right]\cancel{\frac{\gamma}{J_t}}\left[M_t - (M)\left(\frac{ml^2}{J_t}\right)\right] - \left\{\left[-c x_2 - \frac{\gamma}{J_t}(ml)(x_4)\right] * 0\right\}}{\left(M_t - (m)\left(ml^2/J_t\right)\right)^2}$$

$$\frac{\partial (f_4(x_3))}{\partial x_3} = \frac{f_4'(x_3)\cdot g(x_3) - f_2(x_3)\cdot g'(x_3)}{(g(x_3))^2}$$

$$= \Big[-ml^2 x_4^2 \big(\cancel{\cos^2(x_3)}^{1} - \cancel{\sin^2(x_3)}^{0}\big) + M_t g l \cancel{\cos x_3}^{1} + c\,l\,\cancel{\sin(x_3)}^{0}\cdot x_2 - l\,\cancel{\sin(x_3)}^{0}\cdot u\Big]\Big[J_t(M_t/m) - m(l\cancel{\cos x_3}^{1})^2\Big]$$

$$\frac{\partial (f_4(x_3))}{\partial x_3} = \frac{-\Big[-ml^2 \cancel{\cos x_3 \sin x_3}^{0}\, x_4^2 + \cancel{M_t g l \sin x_3}^{0} - c\cdot l \cos(x_3)\cdot x_2 - \gamma(M_t/m)x_4 + \cancel{l\cos x_3 u}^{0}\Big]\Big[+2ml^2\cancel{(\cos x_3)(\sin x_3)}^{0}\Big]}{\Big[J_t(M_t/m) - m(l\cancel{\cos x_3}^{1})^2\Big]^2}$$

$$\left.\frac{\partial (f_4(x_3))}{\partial x_3}\right|_{\substack{x_{3eq}=0\\ u_{eq}=0\\ x_{4eq}=0\\ x_{1eq}=0\\ x_{2eq}=0}} = \frac{\Big[\cancel{-ml^2 x_4^2}^{0} + M_t g l\Big]\Big[J_t(M_t/m) - ml^2\Big]}{\Big[J_t(M_t/m) - m(l)^2\Big]^2}$$

$$\left.\frac{\partial (f_2(x_3))}{\partial x_3}\right|_{\substack{x_{3eq}=0\\ u_{eq}=0\\ x_{4eq}=0\\ x_{1eq}=0\\ x_{2eq}=0}} = \frac{\Big[\cancel{-ml x_4^2}^{0} + mg\frac{(ml^2)}{J_t}\Big]\Big[M_t - (m)\frac{(ml^2)}{J_t}\Big]}{\Big[M_t - (m)\frac{(ml^2)}{J_t}\Big]^2}$$

$$
\left.\frac{\partial F_D}{\partial x}\right|_{\substack{x_1 = 0 = x_{1eq} \\ x_2 = 0 = x_{2eq} \\ x_3 = 0 = x_{3eq} \\ x_4 = 0 = x_{4eq}}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-c}{M_t - (m)\left(\frac{ml^2}{J_t}\right)} & \dfrac{\frac{Mg(ml^2)}{J_t}\left[M_t - \frac{(m)(ml^2)}{J_t}\right]}{\left[M_t - \frac{m(ml^2)}{J_t}\right]^2} & \dfrac{-\frac{\gamma}{J_t}(ml)}{\left(M_t - \frac{(m)(ml)}{J_t}\right)^2} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-cl}{J_t\left(M_t/m\right) - ml^2} & \dfrac{(M_t g l)\left[J_t(M_t/m) - ml^2\right]}{\left[J_t(M_t/m) - ml^2\right]^2} & \dfrac{-\gamma(M_t/m)}{J_t(M_t/m) - ml^2} \end{bmatrix}
$$

$$
\left.\frac{\partial F}{\partial u}\right|_{u_{eq} = u = 0} = \begin{bmatrix} 0 \\ \dfrac{1}{M_t - (m)\left(\frac{ml^2}{J_t}\right)(\cos x_3)^2} \\ 0 \\ \dfrac{l}{J_t(M_t/m) - ml^2} \end{bmatrix}
$$

So the linearized system of the segway model is given as follows:-

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \\ \dot{\tilde{x}}_3 \\ \dot{\tilde{x}}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-c}{M_t - \dfrac{m^2 l^2}{J_t}} & \dfrac{\dfrac{g m^2 l^2 \left[M_t - \dfrac{m^2 l^2}{J_t}\right]}{J_t}}{\left(M_t - \dfrac{m^2 l^2}{J_t}\right)^2} & \dfrac{\dfrac{-\gamma m l}{J_t}}{M_t - \dfrac{m^2 l^2}{J_t}} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-cl}{J_t\left(\dfrac{M_t}{m}\right) - ml^2} & \dfrac{M_t g l\left[J_t\left(\dfrac{M_t}{m}\right) - ml^2\right]}{\left[J_t\left(\dfrac{M_t}{m}\right) - ml^2\right]^2} & \dfrac{-\gamma\left(\dfrac{M_t}{m}\right)}{J_t\left(\dfrac{M_t}{m}\right) - ml^2} \end{bmatrix}}_{A} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \\ \tilde{x}_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \dfrac{1}{M_t - \dfrac{m^2 l^2}{J_t}} \\ 0 \\ \dfrac{l}{J_t\left(\dfrac{M_t}{m}\right) - ml^2} \end{bmatrix}}_{B} \tilde{u}$$

Q1(a) The key - observations in the curve obtained in the linear model & non-linear model are as follows.

1. Cart position :-

non-linear :- The cart ossilates around a small range, indicating that the cart is moving back & forth without any significant drift.

Linear - model :- The cart position shows a large exponential like increase, indicating that it moves away from the initial position without ossilating around an Equilibrium point.

Simulating pendulum on a cart using a linearised model (linearized only @ equillibrium x=0, u=0.

Contents

Cleanup

```
clear
clc
close all
```

System Parameters

```
param.m = 0.2;
param.M = 0.5;
param.J = 0.006;
param.l = 0.3;

param.c = 0.1;
param.gamma = 0.1;

param.g = 9.81;
```

Simulation Parameters

```
to = 0;
tFinal = 10;
tSim = 0:0.01:10;
xo = [0; 0; pi/8; 0];
```

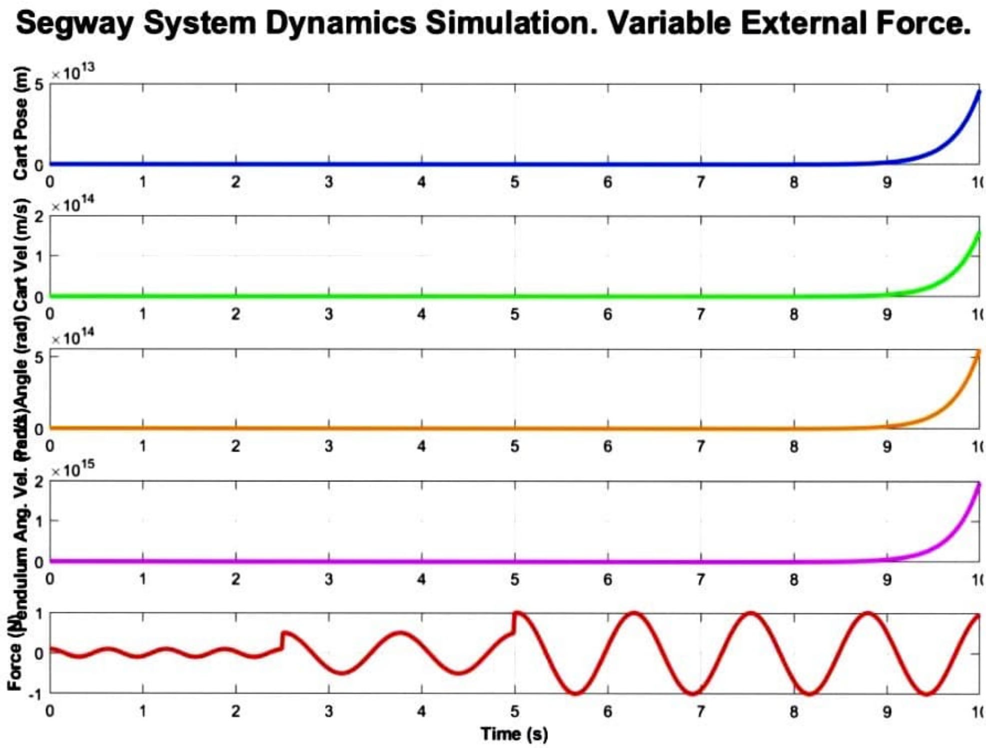
Simulation

```
[tOut, xOut] = ode45(@(t,x) segwayLinearized(t, x, param), tSim, xo);


% Calculate the applied force F(t) at each time step
F_t = zeros(length(tOut), 1);
for i = 1:length(tOut)
    F_t(i) = calculate_force(tOut(i));
end
```

Show the Plots with Improved Layout and Save to PNG

```
figure('Position', [100, 100, 1200, 900]);  % Set figure size (Width: 1200, Height: 900)

% Add a super-title for the whole figure
sgtitle('Segway System Dynamics Simulation. Variable External Force.', 'FontSize', 24, 'FontWeight', 'bold');

% Plot Cart Position
subplot(5,1,1);  % Create the first subplot in a 5-row grid
plot(tOut, xOut(:,1), 'LineWidth', 3, 'Color', 'b');
ylabel('Cart Pose (m)', 'FontSize', 12, 'FontWeight', 'bold');
set(gca, 'FontSize', 12);
grid on;

% Plot Cart Velocity
subplot(5,1,2);  % Create the second subplot
plot(tOut, xOut(:,2), 'LineWidth', 3, 'Color', 'g');
ylabel('Cart Vel (m/s)', 'FontSize', 12, 'FontWeight', 'bold');
set(gca, 'FontSize', 12);
grid on;

% Plot Pendulum Angle
subplot(5,1,3);  % Create the third subplot
plot(tOut, xOut(:,3), 'LineWidth', 3, 'Color', [1 0.5 0]);
ylabel('Pend. Angle (rad)', 'FontSize', 12, 'FontWeight', 'bold');
set(gca, 'FontSize', 12);
grid on;
```

```
% Plot Pendulum Angular Velocity
subplot(5,1,4);  % Create the fourth subplot
plot(tOut, xOut(:,4), 'LineWidth', 3, 'Color', 'm');
ylabel('Pendulum Ang. Vel. (rad/s)', 'FontSize', 12, 'FontWeight', 'bold');
set(gca, 'FontSize', 12);
grid on;

% Plot Applied Force
subplot(5,1,5);  % Create the fifth subplot
plot(tOut, F_t, 'LineWidth', 3, 'Color', 'r');
xlabel('Time (s)', 'FontSize', 14, 'FontWeight', 'bold');
ylabel('Force (N)', 'FontSize', 12, 'FontWeight', 'bold');
set(gca, 'FontSize', 12);
grid on;

% Save the figure as a high-resolution PNG image
print(gcf, 'SegwayPlot.png', '-dpng', '-r300');
```



Calculate the Force Applied to the Cart

```
function F = calculate_force(t)
    if (t < 0)
        F = 0;
    elseif (t <= 2.5)
        F = 0.1 * cos(10 * t);
    elseif (t < 5)
        F = 0.5 * cos(5 * t);
    else
        F = 1.0 * cos(5 * t);
    end
end
```

State Space Of the System (Linearized)

```
function xdot = segwayLinearized(t, x, param)

    % State variables
    x1 = x(1);
    x2 = x(2);
    x3 = x(3);
    x4 = x(4);

    F = calculate_force(t);


    x1dot = x2;
    x2dot = (1/((param.J + param.m*param.l^2)*(param.M+param.m) - param.l^2*param.m^2 )) * ( -((param.J + param.m*param.l^2)*param.c) * x2   +       (param.g*param
    x3dot = x4;
    x4dot = (1/((param.J + param.m*param.l^2)*(param.M+param.m) - param.l^2*param.m^2 )) * (       -(param.c*param.l*param.m)        * x2   +((param.M+param.m)*par

    xdot = [x1dot; x2dot; x3dot; x4dot];

end
```

2 Cart Velocity (Green Curve)

1. non-linear Model :- The velocity shows periodic oscillations with the system altering between moving forward & backward

Linear-Model :- The velocity increases steadily over time, suggesting that the cart is accelerating continuously in one direction.

3. Pendulum Angle (Orange Curve)

1. non-linear model :- The pendulum angle increases gradually with small oscillations stabilizing after a certain period.

2. Linear model :- The pendulum Angle grows rapidly & exponentially without oscillations, showing an unbounded increase.

4. Pendulum Angular Velocity (Magenta curve)

non-linear model :- The angular velocity oscillates with some delay, indicating a bounded motion

linear model :- The angular velocity increases steadily, similar to the other Variables, reflecting unbounded acceleration.

Key conclusions :-

i) non-linear model :- The system Exhibits stable, bounded Oscillations. with Natural damping or feedback effect present in Real - world non-linear systems

2) Linear - model :- The system exhibits unstable, unbounded behavior, which is typical in linear approximations. when the actual dynamics involve significant non - linearities. Linearization can miss the stabilizing effects present in the full non-linear system, leading to unrealistic prediction of exponen- - tial growth.

Problem -7

(b) The Jacobian matrix A is given as :-

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \dfrac{-c}{M_t - \dfrac{m^2 l^2}{J_t}} & \dfrac{\dfrac{g m^2 l^2}{J_t}\left[M_t - \dfrac{m^2 l^2}{J_t}\right]}{\left[M_t - \dfrac{m^2 l^2}{J_t}\right]^2} & \dfrac{\dfrac{-\gamma m l}{J_t}}{\left[M_t - \dfrac{m^2 l^2}{J_t}\right]} \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{-cl}{J_t\left(\dfrac{M_t}{m}\right) - ml^2} & \dfrac{M_t g l \left(J_t\left(\dfrac{M_t}{m}\right) - ml^2\right)}{J_t\left(\left(\dfrac{M_t}{m}\right) - ml^2\right)} & \dfrac{-\gamma\left(\dfrac{M_t}{m}\right)}{J_t\left(\dfrac{M_t}{m}\right) - ml^2} \end{bmatrix}$$

This can be simplified and the simplification is as follows:-

let $M_t J_t - m^2 l^2 = M$

So Row 2 of (A) is written as follows:-

$$\frac{-c J_t}{M_t J_t - m^2 l^2} = \frac{-c J_t}{M}$$

$$\frac{\dfrac{-\gamma m l}{\cancel{J_t}}}{\dfrac{M_t J_t - m^2 l^2}{\cancel{J_t}}} = \frac{-\gamma m l}{M}$$

$$\frac{\dfrac{g m^2 l^2}{\cancel{J_t}}}{\dfrac{M_t J_t - m^2 l^2}{\cancel{J_t}}} \Rightarrow \frac{g m^2 l^2}{M}$$

IIIly Row - 4 of [A] can be written as:

$\rightarrow -Clm/M$; $mM_t gl/M$; $-\gamma M_t/M$

$$\therefore A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-CJt}{M} & \frac{m^2 l^2 g}{M} & -\frac{\gamma m l}{M} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{Clm}{M} & \frac{mM_t gl}{M} & -\frac{\gamma M_t}{M} \end{bmatrix}$$

So the linerized system is as follows

$$\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-CJt}{M} & \frac{m^2 l^2 g}{M} & \frac{-\gamma ml}{M} \\ 0 & 0 & 0 & 1 \\ 0 & -\frac{Clm}{M} & \frac{mM_t gl}{M} & -\frac{\gamma M_t}{M} \end{bmatrix}}_{A} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ Jt/M \\ 0 \\ \frac{lm}{M} \end{bmatrix} w$$

The Eigen - values of the matrix (A) are as follows:-

$\lambda_1 = -8.8640$

$\lambda_2 = -0.1428$

$\lambda_3 = 3.5220$.

Conclusion:-

Since the system has one +ve eigenvalue, it is unstable at the equilibrium point (ie. $w_{eq} = 0$, $\theta_{eq} = 0$)
The unstable mode will dominate the systems behaviour overtime, causing it to diverge from the equilibrium point.