A primer on Generalized, Normalized, Power and Active S-Parameters

Rajavardhan Talashila, trvabc@gmail.com

Generalized S-Parameters

- They may have different reference impedances at each port.
- The travelling waves at the two ports are defined as

$$a_1 = \frac{V_1 + Z_{01}I_1}{2\sqrt{Z_{01}}} \qquad a_2 = \frac{V_2 + Z_{02}I_2}{2\sqrt{Z_{02}}}$$
$$b_1 = \frac{V_1 - Z_{01}I_1}{2\sqrt{Z_{01}}} \qquad b_2 = \frac{V_2 - Z_{02}I_2}{2\sqrt{Z_{02}}}$$

- Simulation solvers internally give out generalized S-parameters with the port impedances as the reference impedance at each port. They are then converted to Normalized S-parameters.
- Suppose a solver gives the generalized 2-port S-parameters at a particular frequency as

$$\mathbf{S}_G = \begin{bmatrix} \frac{b_1}{a_1}\Big|_{a_2=0} & \frac{b_1}{a_2}\Big|_{a_1=0} \\ \frac{b_2}{a_1}\Big|_{a_2=0} & \frac{b_2}{a_2}\Big|_{a_1=0} \end{bmatrix} = \begin{bmatrix} -0.234+0.2i & -0.628-0.672i \\ -0.628-0.672i & -0.328+0.213i \end{bmatrix}$$
$$|\mathbf{S}_G| = \begin{bmatrix} 0.308 & 0.92 \\ 0.92 & 0.391 \end{bmatrix} = \begin{bmatrix} -17.0 & -0.291 \\ -0.291 & -11.9 \end{bmatrix} dB$$

- The two device has port impedance of

$$\mathbf{Z}_P = \begin{bmatrix} Z_{01} & 0 \\ 0 & Z_{02} \end{bmatrix} = \begin{bmatrix} 50.93 & 0 \\ 0 & 88.07 \end{bmatrix}$$

i.e. The S-parameters S_G are calculated with the ports terminated with matched impedance according to Z_P

- A unique impedance matrix $\mathbf{Z}$ can be calculated as

$$\mathbf{Z} = \sqrt{Z_P}(I - S_G)^{-1}(I + S_G)\sqrt{Z_P}$$
$$= \begin{bmatrix} 3.94+32.4i & -3.39-57.2i \\ -3.39-57.2i & 0.213+52.0i \end{bmatrix}$$
$$|\mathbf{Z}_P| = \begin{bmatrix} 32.7 & 57.3 \\ 57.3 & 52.0 \end{bmatrix}$$

- From this impedance matrix, the S-parameters can be normalized to a common reference impedance, usually to 50Ω as

$$\mathbf{Z}_N = \begin{bmatrix} 50 & 0 \\ 0 & 50 \end{bmatrix}$$
$$\mathbf{S}_N = \sqrt{Y_N}(Z - Z_N)(Z + Z_N)^{-1}\sqrt{Z_N}$$
$$= \begin{bmatrix} -0.225-0.0569i & -0.708-0.667i \\ -0.708-0.667i & -0.0433+0.228i \end{bmatrix}$$
$$|\mathbf{S}_N| = \begin{bmatrix} 0.232 & 0.973 \\ 0.973 & 0.232 \end{bmatrix} = \begin{bmatrix} -12.7 & -0.24 \\ -0.24 & -12.7 \end{bmatrix} dB$$

Interpretation from $\mathbf{S}_G$

S_{11} Insertion Loss at Port 1 with Port 2 terminated in matched load ($a_2 = 0$)

S_{12} Reverse Voltage Gain (From port 2 to port 1)

S_{21} Forward Voltage Gain (From port 1 to port 2)

S_{22} Insertion Loss at Port 2 with Port 1 terminated in matched load ($a_1 = 0$)

Normalized S-Parameters

- The travelling waves at the two ports are defined with a common reference impedance of Z_0

$$a_1 = \frac{V_1 + Z_0 I_1}{2\sqrt{Z_0}} \qquad a_2 = \frac{V_2 + Z_0 I_2}{2\sqrt{Z_0}}$$
$$b_1 = \frac{V_1 - Z_0 I_1}{2\sqrt{Z_0}} \qquad b_2 = \frac{V_2 - Z_0 I_2}{2\sqrt{Z_0}}$$

- They usually refer to the measured S-parameters
- The nominal common reference impedance is $Z_0 = 50\Omega$.
- The Normalized S-parameters obtained from measurement or simulation with reference impedance of 50 Ω be

$$\mathbf{S}_N = \begin{bmatrix} -0.255-0.196i & 0.739+0.592i \\ 0.739+0.592i & 0.247+0.205i \end{bmatrix}$$

- These Normalized S-Parameters can be converted into Generalized S-parameters with any reference impedance at the ports. For that, the unique impedance matrix can be calculated as

$$\mathbf{Z} = \sqrt{Z_N}(I - S_N)^{-1}(I + S_N)\sqrt{Z_N}$$
$$\mathbf{Z} = \begin{bmatrix} 0.000122+36.4i & 0.000173+75.0i \\ 0.000173+75.0i & 0.000756+87.3i \end{bmatrix}$$

- Then the Generalized S-parameters can be obtained as

$$\mathbf{Z}_G = \begin{bmatrix} 50.93 & 0 \\ 0 & 88.10 \end{bmatrix}$$
$$\mathbf{S}_G = \sqrt{Y_G}(Z - Z_G)(Z + Z_G)^{-1}\sqrt{Z_G}$$
$$= \begin{bmatrix} -0.221+0.0653i & 0.721+0.653i \\ 0.721+0.653i & -0.0433+0.226i \end{bmatrix}$$

- This exercise simply says that the reference impedance of the S-parameters can be converted to the required reference impedances

Interpretation from $\mathbf{S}_N$

S_{11} Insertion Loss at Port 1 with Port 2 terminated in common reference load Z_0

S_{12} Reverse Gain (From port 2 to port 1)

S_{21} Forward Gain (From port 1 to port 2)

S_{22} Insertion Loss at Port 2 with Port 1 terminated in common reference load Z_0

The following definitions are valid w.r.t $\mathbf{S}_N$ when the source and load impedances are matched to the reference impedance Z_0

$$\text{Transducer Gain } G_T = \frac{\text{Power transferred to Load}}{\text{Available power from Generator}} = |S_{21}|^2$$

$$\text{Available Power Gain } G_a = \frac{\text{Available power from Network}}{\text{Available Power from Generator}} = \frac{|S_{21}|^2}{1 - |S_{22}|^2}$$

$$\text{Operating Power Gain } G_P = \frac{\text{Power Transferred to Load}}{\text{Power fed to the DUT}} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}$$

Power S-Parameters

- The ***power waves*** at the two ports can be defined as

$$a_1 = \frac{V_1 + Z_G I_1}{2\sqrt{R_G}} \qquad a_2 = \frac{V_2 + Z_L I_2}{2\sqrt{R_L}}$$
$$b_1 = \frac{V_1 - Z_G^* I_1}{2\sqrt{R_G}} \qquad b_2 = \frac{V_2 - Z_L^* I_2}{2\sqrt{R_L}}$$

- In the above definitions, it can be observed that the denominators have only real part of the impedance under the square root.
- These are defined by anticipating complex source and load of impedances of Z_G and Z_L respectively.
- The power waves can be interpreted in terms of power transferred to and from the DUT.
- Assume that the source and load impedance are same as the port impedance.i.e. $Z_G = Z_{01}$ and $Z_L = Z_{02}$.
  Then the Power S-parameters would be same as the Generalized S-Parameters

$$\mathbf{S}_P = \mathbf{S}_G = \begin{bmatrix} -0.234+0.2i & -0.628-0.672i \\ -0.628-0.672i & -0.328+0.213i \end{bmatrix}$$

- Consider a scenario where the reference impedance be complex. Say $Z_{01} = 45 + 90i\Omega$ and $Z_{02} = 60 - 80i\Omega$. The Generalized impedance in this case are

$$S_{GC} = \begin{bmatrix} -0.234+0.2i & -0.628-0.672i \\ -0.628-0.672i & -0.328+0.213i \end{bmatrix}$$
$$|S_{GC}| = \begin{bmatrix} 0.308 & 0.92 \\ 0.92 & 0.391 \end{bmatrix} = \begin{bmatrix} -10.2 & -0.721 \\ -0.721 & -8.15 \end{bmatrix} dB$$

The Power S-Parameters are

$$S_{PC} = \begin{bmatrix} 0.694+0.44i & -0.295-0.445i \\ -0.295-0.445i & -0.117-0.873i \end{bmatrix}$$
$$|S_{PC}| = \begin{bmatrix} 0.822 & 0.534 \\ 0.534 & 0.881 \end{bmatrix} = \begin{bmatrix} -1.71 & -5.45 \\ -5.45 & -1.1 \end{bmatrix} dB$$

- Consider a case where the source and load impedance are almost matched to the network i.e. $Z_{01} = 32\Omega$ and $Z_{02} = 49\Omega$, the Power S-Parameters are

$$S_{P2} = \begin{bmatrix} -0.00506-0.0699i & -0.718-0.694i \\ -0.718-0.694i & -0.0434+0.00954i \end{bmatrix}$$
$$|S_{P2}| = \begin{bmatrix} 0.0701 & 0.999 \\ 0.999 & 0.0445 \end{bmatrix}$$

- The corresponding gains are $G_T = 0.997$, $G_a = 0.999$ and $G_P = 0.999$

Interpretation from $\mathbf{S}_P$

$$\text{Transducer Gain } G_T = \frac{\text{Power transferred to Load}}{\text{Available power from Generator}} = |S_{21}|^2$$

$$\text{Available Power Gain } G_a = \frac{\text{Available power from Network}}{\text{Available Power from Generator}} = \frac{|S_{21}|^2}{1 - |S_{22}|^2}$$

$$\text{Operating Power Gain } G_P = \frac{\text{Power Transferred to Load}}{\text{Power fed to the DUT}} = \frac{|S_{21}|^2}{1 - |S_{11}|^2}$$

Reference : https://www.ece.rutgers.edu/ orfanidi/ewa/

Active S-Parameters

- Active S-Parameters are used to represent only the reflection coefficients when other ports are excited.
- One of the (possible) of definition of the Active S-parameter at port 1 (for a two port network is)

$$Active \quad S_1 = S11 + S12\frac{a_2}{a_1}$$

where a_1 and a_2 are related to the source powers at the ports 1 and 2.

- Similarly, the Active S-parameter at the port 2 is

$$Active \quad S_2 = S22 + S21\frac{a_1}{a_2}$$

- Note: When power supplied to port 2 is zero, the $Active \quad S_1$ is equivalent to S_{11} and vice-versa
- As evident from the definition, active S-parameters vary depend on the power with which the other ports are excited. For the same structure with the Normalized S-parameters S_N, the active S-parameters for the two ports as the power is varied in the other port is tabulated below:

P1 Power	P2 Power	$Active \quad S_1$
1W	0	$\lvert -0.225+0.0568i \rvert = 0.23 = -12.7dB$
1W	0.1W	$\lvert -0.449-0.154i \rvert = 0.474 = -6.4dB$
1W	0.5W	$\lvert -0.726-0.415i \rvert = 0.836 = -1.6dB$
1W	1W	$\lvert -0.933-0.610i \rvert = 1.114 = 0.94dB$

P1 Power	P2 Power	$Active \quad S_2$
0	1W	$\lvert -0.043+0.228i \rvert = 0.23 = -12.7dB$
0.1W	1W	$\lvert -0.267+0.017i \rvert = 0.267 = -11.4dB$
0.5W	1W	$\lvert -0.544-0.243i \rvert = 0.59 = -4.49dB$
1W	1W	$\lvert -0.751-0.439i \rvert = 0.870 = -1.21dB$

- It can be interpreted that the Active S-parameters vary with the power at the other port.
- These definitions can be extended for a network of higher number of ports.

Interpretation from $\mathbf{S}_A$

S_{11} Insertion Loss at Port 1 when the other port is excited

S_{22} Insertion Loss at Port 2 when the other port is excited