Djordjevic-Sarkar as implemented in Ansys HFSS / SIwave

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Djordjevic-Sarkar Model

The model equation is

$$\varepsilon(\omega_1) = \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\omega_B + j\omega_1}{\omega_A + j\omega_1}\right) + \frac{\sigma_{DC}}{j\omega_1 \varepsilon_0}$$

- Requires measurements only at one frequency point

The required parameters be

1. Measurement frequency $f_1 = 1\,GHz$
2. Real part of relative permitivitty @ f_1 : $\varepsilon_r = 4.0$
3. Loss tangent @ f_1 $\tan\delta = 0.02$
4. Real part of relative permitivitty at DC $\varepsilon_r = 5.0$
5. Conductivity at DC $\sigma_{DC} = 1e-12 S/m$

Let the upper Corner frequency is assumed to be much higher than the frequencies of interest $f_B = 159.15\,GHz$, which is

The intermediate parameters can be computed using

$$\omega_A = \frac{\omega_B}{\exp\left(\frac{\Delta\varepsilon}{K}\right)} = \frac{\omega_B}{\exp\left(\frac{\varepsilon_{DC} - \varepsilon_\infty}{K}\right)}$$

$$K = \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} = \frac{\varepsilon_1 \tan\delta_1 - \frac{\sigma_{DC}}{\omega_1 \varepsilon_0}}{\tan^{-1}\left(\frac{\omega_B}{\omega_1}\right)}$$

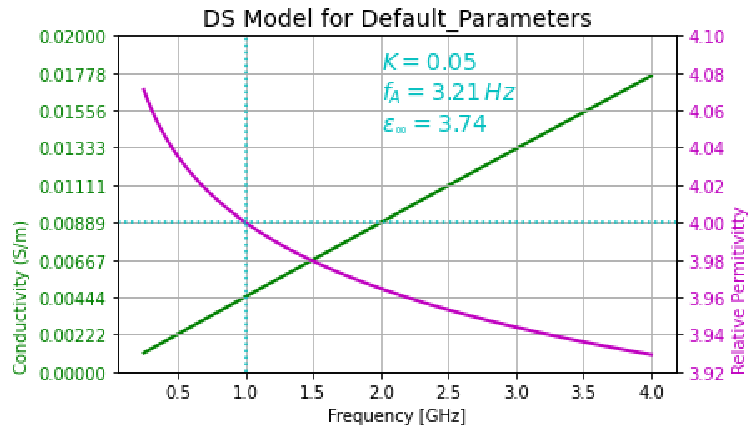
$$\varepsilon_\infty = \varepsilon_1 - K \ln\left(\frac{\sqrt{\omega_B^2 + \omega_1^2}}{\omega_1}\right)$$

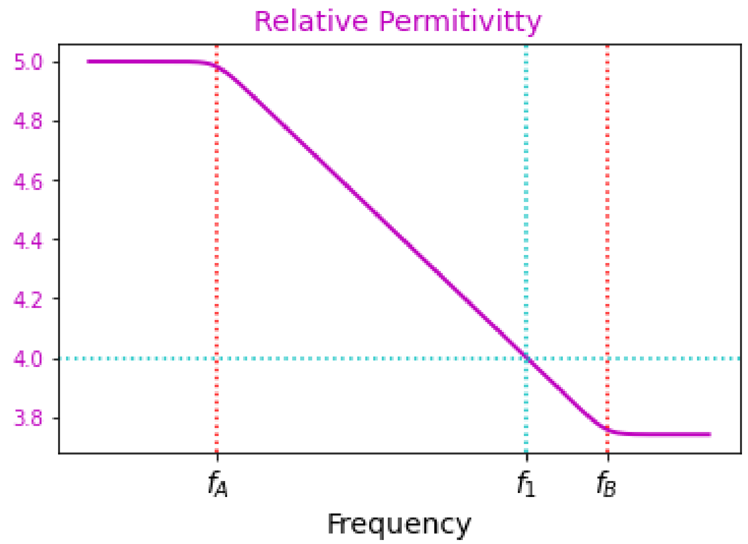
The equations for real part of permitivitty, conductivity and loss tangent are

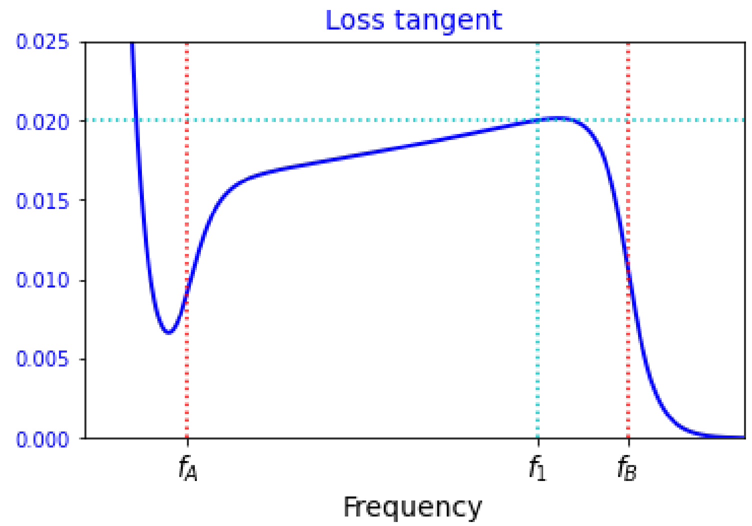
$$\varepsilon_1(f) = \varepsilon_\infty + K \ln\left(\frac{\sqrt{f_B^2 + f^2}}{\sqrt{f_A^2 + f^2}}\right) = \varepsilon_\infty + \frac{K}{2} \ln\left(\frac{f_B^2 + f^2}{f_A^2 + f^2}\right)$$

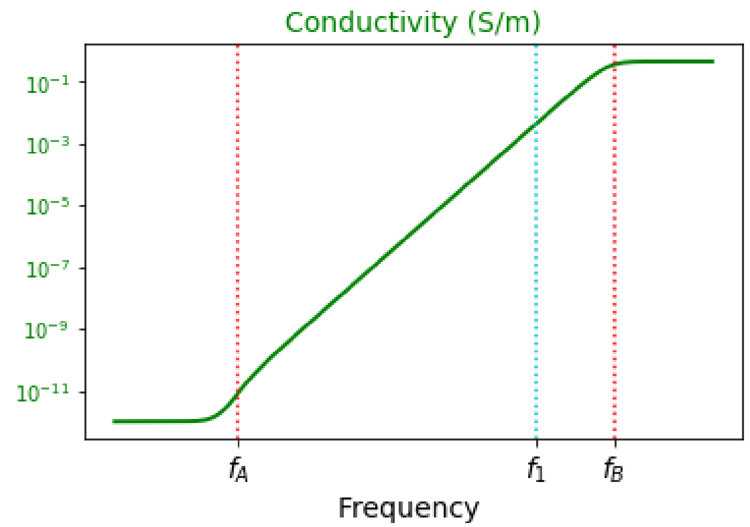
$$\sigma(f) = \sigma_{DC} + 2\pi f \varepsilon_0 K \left[\tan^{-1}\left(\frac{f}{f_A}\right) - \tan^{-1}\left(\frac{f}{f_B}\right)\right]$$

$$\tan\delta = \frac{\sigma_{DC} + 2\pi f \varepsilon_0 K \left[\tan^{-1}\left(\frac{f}{f_A}\right) - \tan^{-1}\left(\frac{f}{f_B}\right)\right]}{2\pi f \varepsilon_0 \left[\varepsilon_\infty + \frac{K}{2} \ln\left(\frac{f_B^2 + f^2}{f_A^2 + f^2}\right)\right]}$$









Djordjevic-Sarkar Model as implemented in Ansys HFSS

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May 13, 2022

- This model is causal
- Requires measurement DC and a RF frequency
- Only 5 input parameters
- Is characterized by two corner frequencies f_A and f_B
- f_B is usually assumed greater than the highest frequency of interest.
- The loss tangent will be almost a constant between the corner frequencies
- The conductivity will be linearly increasing with frequency between f_A and f_B

The model equation is

$$\varepsilon(\omega_1) = \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\omega_B + j\omega_1}{\omega_A + j\omega_1}\right) + \frac{\sigma_{DC}}{j\omega_1\varepsilon_0} \tag{1}$$

Let the following quantities of the material are known:

1. DC Conductivity : σ_{DC}
2. DC Permittivity : ε_{DC}
3. Measurement Freq : ω_1
4. Real Permittivity @ ω_1 : ε_1
5. Loss Tangent @ ω_1 : $\tan\delta_1$

and let the higher transition frequency be $\omega_B = 10^{12} rad/sec$

Now, in the model equation above, the unknown quantities are ε_∞ and ω_A with $\Delta\varepsilon = \varepsilon_{DC} - \varepsilon_\infty$

The following task is to express the unknown quantities in terms of the known quantities

For that, assuming $\omega_A << \omega_1$, Eq.(1) can be simplified as

$$
\begin{aligned}
\varepsilon(\omega_1) &\approxeq \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\omega_B + j\omega_1}{j\omega_1}\right) + \frac{\sigma_{DC}}{j\omega_1\varepsilon_0} \\
&= \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\omega_B + j\omega_1}{j\omega_1}\right) + \frac{\sigma_{DC}}{j\omega_1\varepsilon_0} \\
&= \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{-j\omega_B + \omega_1}{\omega_1}\right) + \frac{\sigma_{DC}}{j\omega_1\varepsilon_0} \\
&= \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\sqrt{\omega_B^2 + \omega_1^2}}{\omega_1} e^{-j\tan^{-1}(\omega_B/\omega_1)}\right) + \frac{\sigma_{DC}}{j\omega_1\varepsilon_0} \\
&= \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\sqrt{\omega_B^2 + \omega_1^2}}{\omega_1}\right) + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(e^{-j\tan^{-1}(\omega_B/\omega_1)}\right) + \frac{\sigma_{DC}}{j\omega_1\varepsilon_0} \\
&= \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\sqrt{\omega_B^2 + \omega_1^2}}{\omega_1}\right) - j\left[\frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \tan^{-1}\left(\frac{\omega_B}{\omega_1}\right) + \frac{\sigma_{DC}}{\omega_1\varepsilon_0}\right]
\end{aligned}
\tag{2}
$$

The permittivity expressed in terms of real part and loss tangent is

$$
\varepsilon(\omega_1) = \varepsilon_1 - j\varepsilon_{11} = \varepsilon_1 - j\varepsilon_1 \tan\delta_1 \tag{3}
$$

Comparing imaginary parts of Eq. (2) and Eq. (3)

$$
\varepsilon_1 \tan\delta_1 = \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \tan^{-1}\left(\frac{\omega_B}{\omega_1}\right) + \frac{\sigma_{DC}}{\omega_1\varepsilon_0} \tag{4}
$$

Expressing the term $\dfrac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)}$ as K, we have

$$
\begin{aligned}
\varepsilon_1 \tan\delta_1 &= \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \tan^{-1}\left(\frac{\omega_B}{\omega_1}\right) + \frac{\sigma_{DC}}{\omega_1\varepsilon_0} \\
\varepsilon_1 \tan\delta_1 &= K \tan^{-1}\left(\frac{\omega_B}{\omega_1}\right) + \frac{\sigma_{DC}}{\omega_1\varepsilon_0}
\end{aligned}
$$

$$
\implies K = \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} = \frac{\varepsilon_1 \tan\delta_1 - \dfrac{\sigma_{DC}}{\omega_1\varepsilon_0}}{\tan^{-1}\left(\dfrac{\omega_B}{\omega_1}\right)} \tag{5}
$$

i.e. K is expressed entirely in terms of known quantities.

Now, Comparing real parts of Eq. (2) and Eq. (3) and including Eq. (5)

$$\varepsilon_1 = \varepsilon_\infty + \frac{\Delta\varepsilon}{\ln(\omega_B/\omega_A)} \ln\left(\frac{\sqrt{\omega_B^2+\omega_1^2}}{\omega_1}\right)$$

$$\varepsilon_1 = \varepsilon_\infty + K \ln\left(\frac{\sqrt{\omega_B^2+\omega_1^2}}{\omega_1}\right)$$

$$\implies \varepsilon_\infty = \varepsilon_1 - K \ln\left(\frac{\sqrt{\omega_B^2+\omega_1^2}}{\omega_1}\right) \tag{6}$$

Using Eq. (5) and Eq. (6), the other unknown quantity, ω_A can be expressed as

$$\omega_A = \frac{\omega_B}{\exp\left(\frac{\Delta\varepsilon}{K}\right)} = \frac{\omega_B}{\exp\left(\frac{\varepsilon_{DC}-\varepsilon_\infty}{K}\right)} \tag{7}$$

From equations (6), (7) the unknown quantities are obtained.

Now, recalling the model equation in Eq.(1) in terms of frequency f and without making any assumption regarding f_A, it can be broken into real and imaginary parts as follows

$$
\begin{aligned}
\varepsilon(f) &= \varepsilon_\infty + K\ln\left(\frac{f_B + jf}{f_A + jf}\right) + \frac{\sigma_{DC}}{j2\pi f\varepsilon_0} \\
\varepsilon(f) &= \varepsilon_\infty + K\ln\left(\frac{\sqrt{f_B^2+f^2}\,\exp\left(j\tan^{-1}(f/f_B)\right)}{\sqrt{f_A^2+f^2}\,\exp\left(j\tan^{-1}(f/f_A)\right)}\right) + \frac{\sigma_{DC}}{j2\pi f\varepsilon_0} \\
\varepsilon(f) &= \varepsilon_\infty + K\ln\left(\frac{\sqrt{f_B^2+f^2}}{\sqrt{f_A^2+f^2}}\right) + K\ln\left(\frac{\exp\left(j\tan^{-1}(f/f_B)\right)}{\exp\left(j\tan^{-1}(f/f_A)\right)}\right) + \frac{\sigma_{DC}}{j2\pi f\varepsilon_0} \\
\varepsilon(f) &= \varepsilon_\infty + K\ln\left(\frac{\sqrt{f_B^2+f^2}}{\sqrt{f_A^2+f^2}}\right) + jK\tan^{-1}\left(\frac{f}{f_B}\right) - jK\tan^{-1}\left(\frac{f}{f_A}\right) + \frac{\sigma_{DC}}{j2\pi f\varepsilon_0} \\
\varepsilon(f) &= \varepsilon_\infty + K\ln\left(\frac{\sqrt{f_B^2+f^2}}{\sqrt{f_A^2+f^2}}\right) - j\left[K\tan^{-1}\left(\frac{f}{f_A}\right) - K\tan^{-1}\left(\frac{f}{f_B}\right) + \frac{\sigma_{DC}}{2\pi f\varepsilon_0}\right]
\end{aligned}
\tag{8}
$$

Hence, the real part of the permitivitty, as a function of frequency, corresponds to

$$
\varepsilon_1(f) = \varepsilon_\infty + K\ln\left(\frac{\sqrt{f_B^2+f^2}}{\sqrt{f_A^2+f^2}}\right) = \varepsilon_\infty + \frac{K}{2}\ln\left(\frac{f_B^2+f^2}{f_A^2+f^2}\right) \tag{9}
$$

For computational purposes, the imaginary part of the permittivity is expressed as equivalent conductivity i.e. $\sigma(f) = 2\pi f\varepsilon_0\varepsilon_{11}(f)$. Hence the conductivity, as a function of frequency, is

$$
\sigma(f) = \sigma_{DC} + 2\pi f\varepsilon_0 K\left[\tan^{-1}\left(\frac{f}{f_A}\right) - \tan^{-1}\left(\frac{f}{f_B}\right)\right] \tag{10}
$$

The loss tangent is $\tan\delta = \sigma(f)/(2\pi f\varepsilon_0\varepsilon_1(f))$

$$
\tan\delta = \frac{\sigma_{DC} + 2\pi f\varepsilon_0 K\left[\tan^{-1}\left(\frac{f}{f_A}\right) - \tan^{-1}\left(\frac{f}{f_B}\right)\right]}{2\pi f\varepsilon_0\left[\varepsilon_\infty + \frac{K}{2}\ln\left(\frac{f_B^2+f^2}{f_A^2+f^2}\right)\right]} \tag{11}
$$

DS Model for Default Parameters

For the default parameters of

1. Measurement frequency $f_1 = 1\,GHz$
2. Real part of relative permitivitty @ f_1 : $\varepsilon_r = 4.0$
3. Loss tangent @ f_1 $\tan\delta = 0.02$
4. Upper Corner frequency $f_B = 159.15\,GHz$
5. Real part of relative permitivitty at DC $\varepsilon_r = 5.0$
6. Conductivity at DC $\sigma_{DC} = 1e-12S/m$

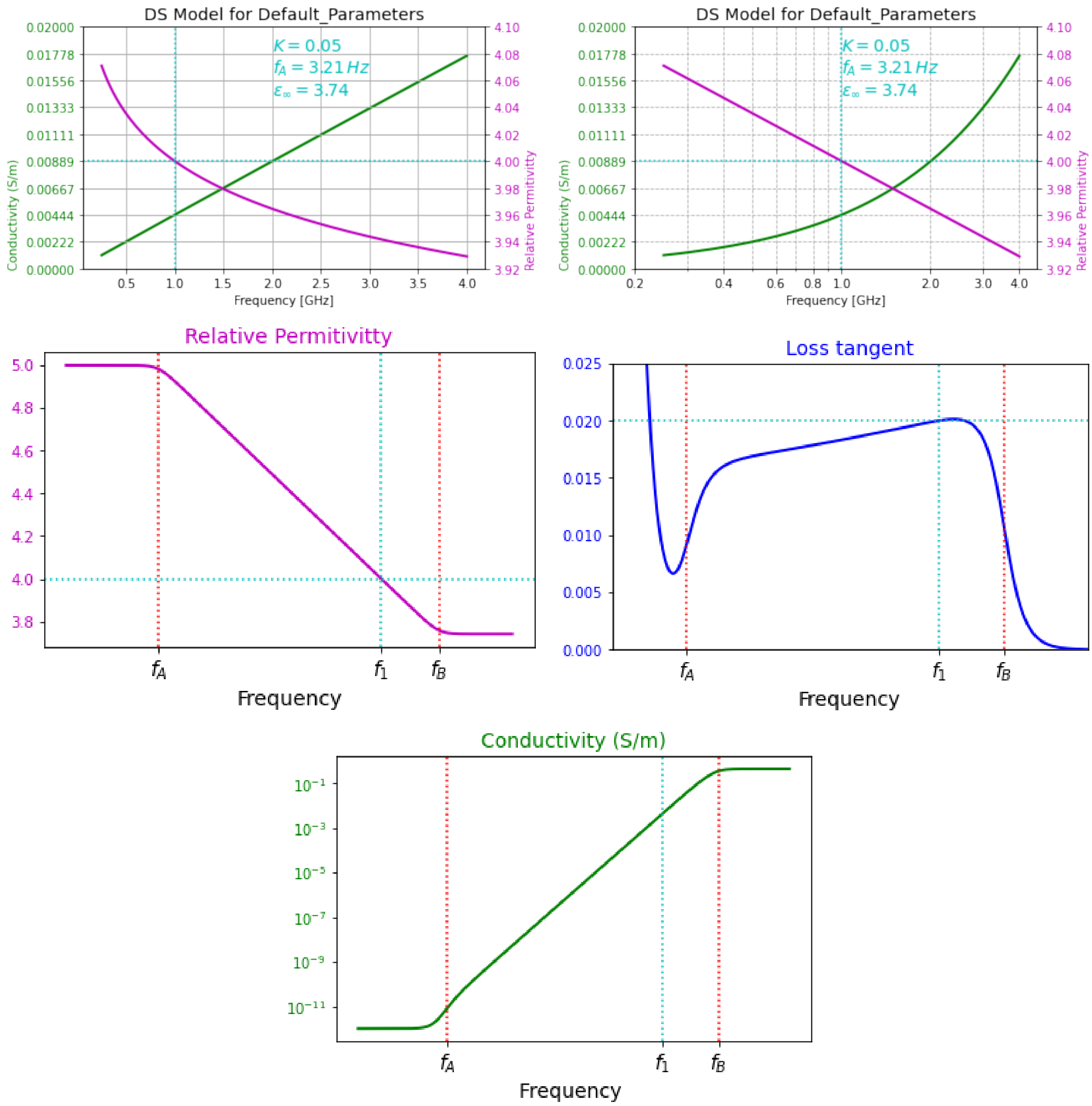


Figure 1: The DS Model for the default parameters in Ansys HFSS.
The first plot has frequency in linear and other have it in log scale.

DS_Model

April 13, 2022

[1]:
```
# Author : Rajavardhan Talashila
#          Application Engineer II
#          Ansys // India
#          rajavardhan.talashila@ansys.com
#          13th April 2022
```

[2]:
```
from pylab import *
%pylab inline
```

```
Populating the interactive namespace from numpy and matplotlib

/usr/local/lib/python3.8/dist-packages/IPython/core/magics/pylab.py:159:
UserWarning: pylab import has clobbered these variables: ['fft', 'random',
'power']
`%matplotlib` prevents importing * from pylab and numpy
  warn("pylab import has clobbered these variables: %s"  % clobbered +
```

1 The default inputs to the DS model are

[3]:
```
Material = "Default_Parameters"
f1 = 1e9 # Hz
er1 = 4
lt = 0.02
fB = 159.15494e9 # Hz
eDC = 5
sDC = 1e-12
e0 = 8.8541878128e-12
```

1.1 Calculte the unknown parameters

[4]:
```
K = ( (er1 * lt) - sDC / (2*pi*f1*e0) ) / arctan(fB/f1)
e8 = er1 - K * log(sqrt(fB**2 + f1**2)/f1) # Epsilon at infinity
fA = fB / e**((eDC - e8)/K)
```

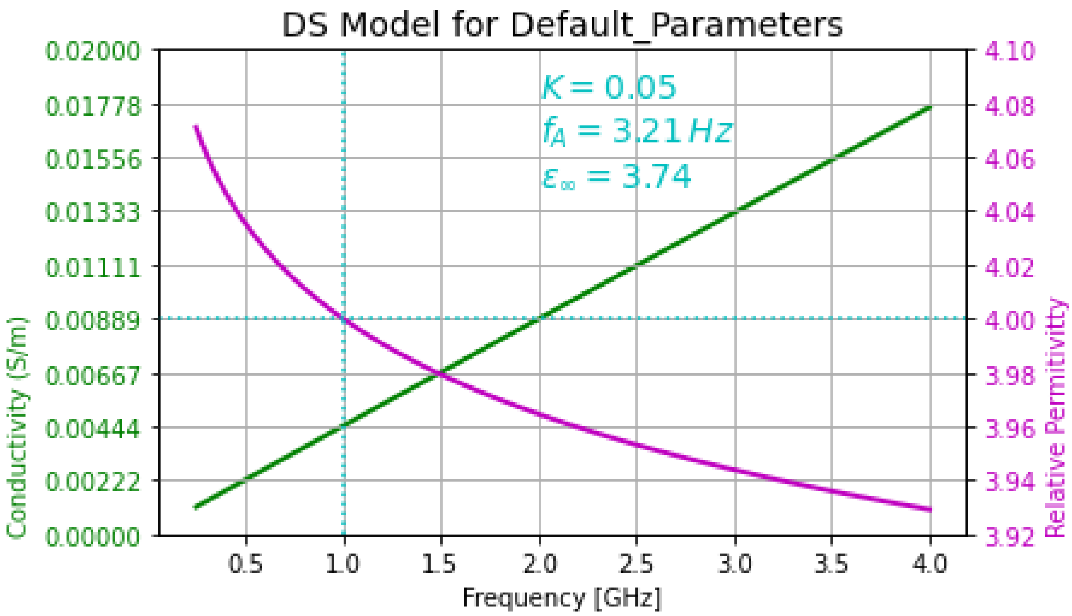
[5]:
```
K, e8, fA
```

[5]:
```
(0.05113411555261883, 3.7407552533068786, 3.2117976409283133)
```

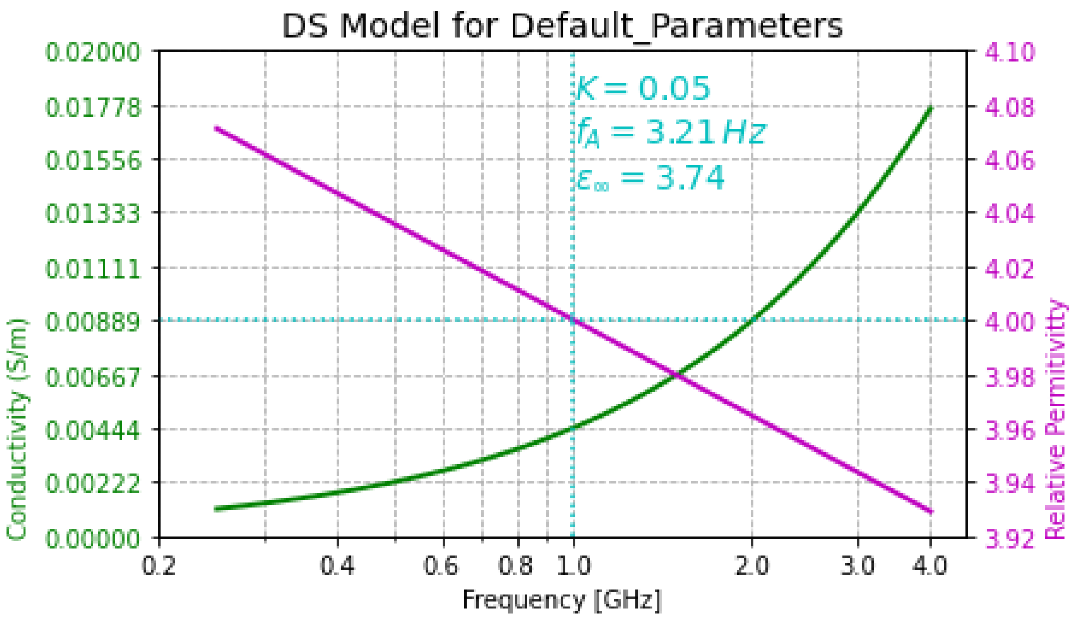
1.2 Calculte permitivitty and conductivity as a function of frequency

[6]:
```
f = linspace(0.25e9,4e9,128)
ef =  e8 + 0.5* K * log((fB**2+f**2)/(fA**2+f**2))
sf = sDC + 2*pi*f*e0 * K * ( arctan(f/fA) - arctan(f/fB)  )
```

[7]:
```
fig, ax1 = subplots(figsize=(6,6/1.618034))
ax1.plot(f/1e9,sf,c='g',lw=2)
ax2 = ax1.twinx()
ax1.tick_params(axis ='y', labelcolor='g')
ax1.set_ylabel("Conductivity (S/m)",c='g',loc='bottom')
ax1.set_yticks(linspace(0,0.02,10))
ax1.yaxis.set_major_formatter(FormatStrFormatter('%.5f'))
ax2.plot(f/1e9,ef,c='m',lw=2)
ax2.tick_params(axis ='y', labelcolor = 'm')
ax2.set_ylabel("Relative Permitivitty",c='m', loc='bottom')
ax2.set_yticks(linspace(3.92,4.1,10))
ax1.set_xlabel("Frequency [GHz]")
title("DS Model for %s"%Material,fontsize=14)
text(2,4.05,"$K = %0.2f$ \n$f_A=%0.2f\,Hz $ \n$\epsilon_{\infty}=%0.2f$␣
↪"%(K,fA,e8),fontsize=14,c='c')
ax2.grid(axis='both')
ax1.grid(axis='both')
axvline(x=1,c='c',ls=':')
axhline(y=4,c='c',ls=':')
savefig("%s_DS.png"%Material,bbox_inches='tight')
```



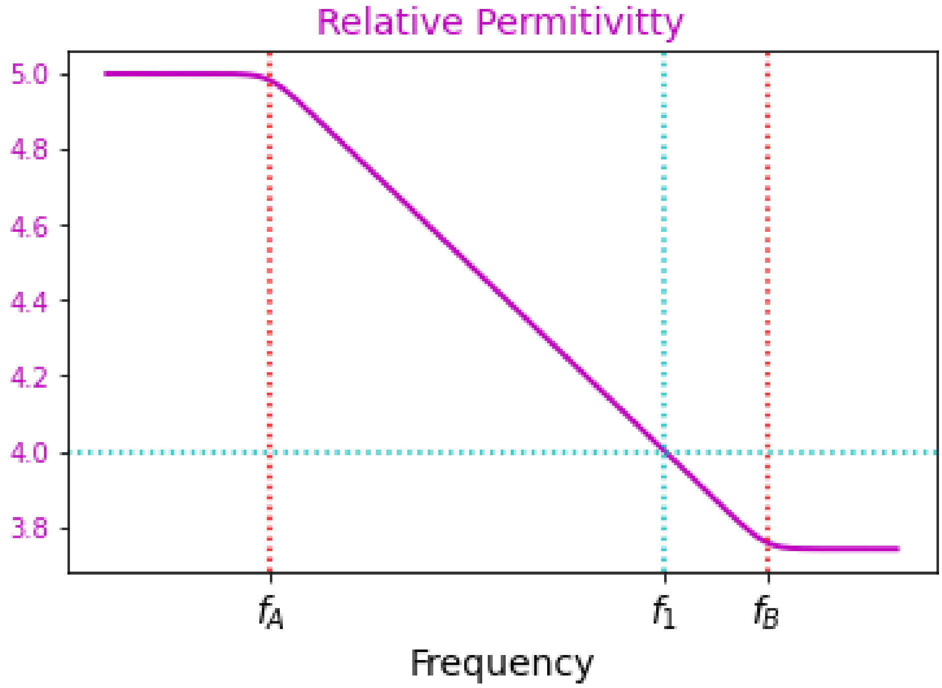
[8]:
```
fig, ax1 = subplots(figsize=(6,6/1.618034))
ax1.plot(f/1e9,sf,c='g',lw=2)
ax2 = ax1.twinx()
ax1.tick_params(axis ='y', labelcolor='g')
ax1.set_ylabel("Conductivity (S/m)",c='g',loc='bottom')
ax1.set_yticks(linspace(0,0.02,10))
ax1.yaxis.set_major_formatter(FormatStrFormatter('%.5f'))
ax2.plot(f/1e9,ef,c='m',lw=2)
ax2.tick_params(axis ='y', labelcolor = 'm')
ax2.set_ylabel("Relative Permitivitty",c='m', loc='bottom')
ax2.set_yticks(linspace(3.92,4.1,10))
ax1.set_xlabel("Frequency [GHz]")
title("DS Model for %s"%Material,fontsize=14)
text(1,4.05,"$K = %0.2f$ \n$f_A=%0.2f\,Hz $ \n$\epsilon_{\infty}=%0.
↪2f$"%(K,fA,e8),fontsize=14,c='c')
ax1.set_xscale('log')
ax1.xaxis.set_major_formatter(FormatStrFormatter("%.1f"))
ax1.grid(axis='both',which='both',ls='--')
ax1.set_xticks([0.2,0.4,0.6,0.8,1.0,2,3,4])
axvline(x=1,c='c',ls=':')
axhline(y=4,c='c',ls=':')
savefig("%s_DS_log.png"%Material,bbox_inches='tight')
```



1.2.1 Frequency range 2

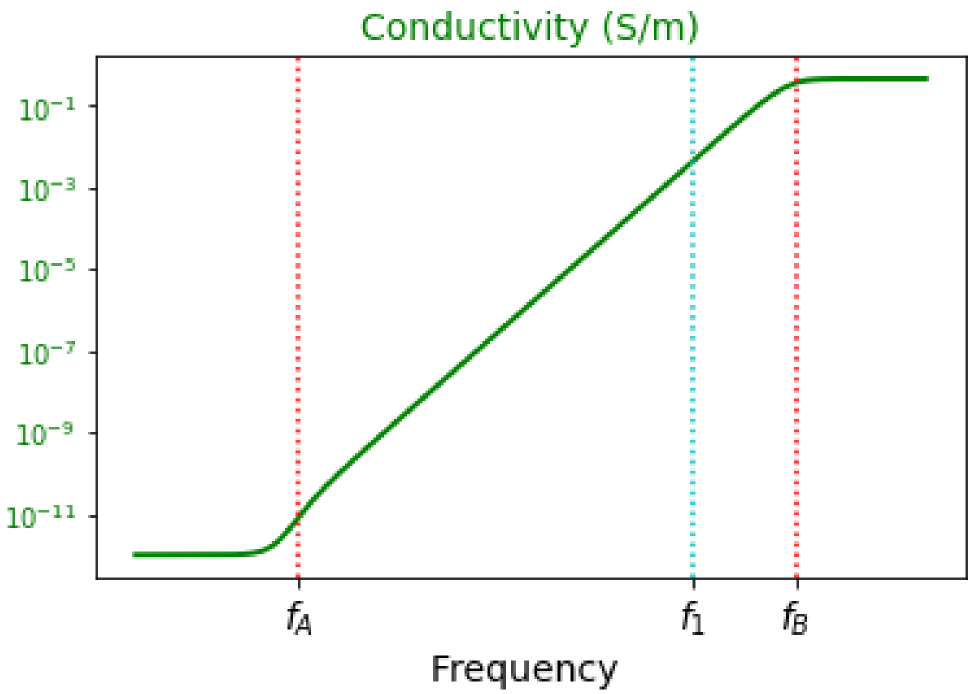
[9]:
```
fr = logspace(-3,14,1024)
ef =  e8 + 0.5* K * log((fB**2+fr**2)/(fA**2+fr**2))
sf = sDC + 2*pi*fr*e0 * K * ( arctan(fr/fA) - arctan(fr/fB)  )
```

[17]:
```
fig, ax1 = subplots(figsize=(6,6/1.618034))
ax1.semilogx(fr/1e9,ef,c='m',lw=2)
ax1.tick_params(axis ='y', labelcolor = 'm')
ax1.set_xlabel("Frequency",fontsize=14)
title("Relative Permitivitty",fontsize=14,c='m')
axvline(x=fB*1e-9,c='r',ls=':')
axvline(x=fA*1e-9,c='r',ls=':')
axhline(y=4,c='c',ls=':')
axvline(x=f1*1e-9,c='c',ls=':')
#ax1.set_xscale('log')
xticks([fA*1e-9,f1*1e-9,fB*1e-9],['$f_A$','$f_1$','$f_B$'],fontsize=14)
savefig("%s_DS_Permitivitty.png"%Material,bbox_inches='tight')
```



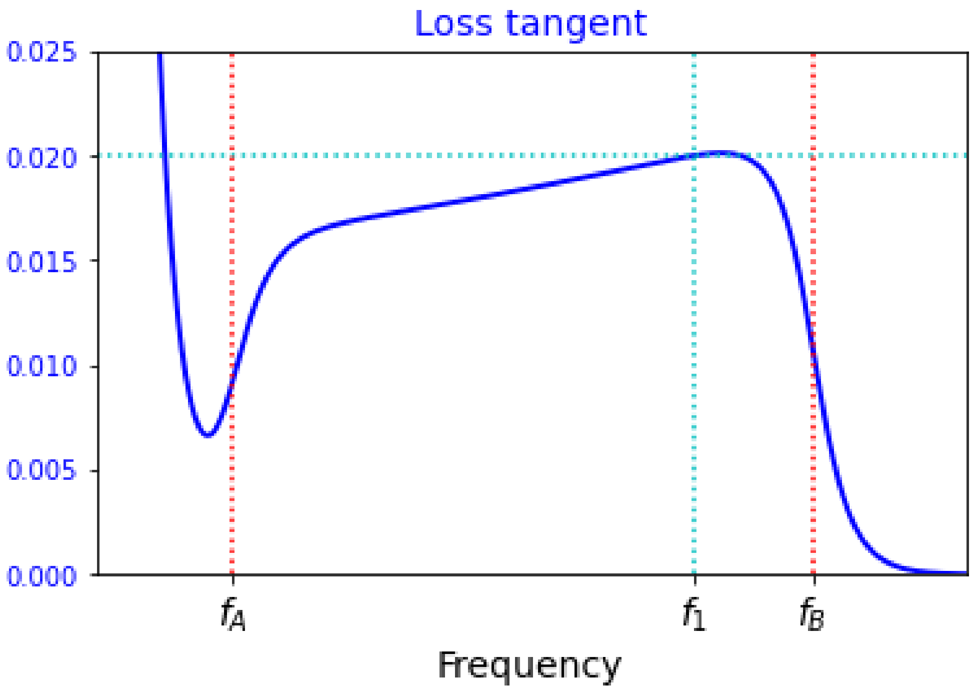
[16]:
```
fig, ax1 = subplots(figsize=(6,6/1.618034))
ax1.loglog(fr/1e9,sf,c='g',lw=2)
ax1.tick_params(axis ='y', labelcolor = 'g')
ax1.set_xlabel("Frequency ",fontsize=14)
title("Conductivity (S/m)",fontsize=14,c='g')
```

```
axvline(x=fB*1e-9,c='r',ls=':')
axvline(x=fA*1e-9,c='r',ls=':')
axvline(x=f1*1e-9,c='c',ls=':')
xticks([fA*1e-9,f1*1e-9,fB*1e-9],['$f_A$','$f_1$','$f_B$'],fontsize=14)
savefig("%s_DS_Conductivity.png"%Material,bbox_inches='tight')
```



[12]:
```
tanD = sf/(2*pi*fr*e0)/ef # Loss Tangent
```

[15]:
```
fig, ax1 = subplots(figsize=(6,6/1.618034))
ax1.tick_params(axis ='y', labelcolor = 'b')
ax1.set_xlabel("Frequency",fontsize=14)
semilogx(fr,tanD,c='b',lw=2)
axvline(x=f1,c='c',ls=':')
axhline(y=lt,c='c',ls=':')
axvline(x=fB,c='r',ls=':')
axvline(x=fA,c='r',ls=':')
title("Loss tangent",fontsize=14,c='b')
xticks([fA,f1,fB],['$f_A$','$f_1$','$f_B$'],fontsize=14)
xlim(0.01,fB+1e14)
ylim(0,0.025)
savefig("%s_DS_LossTangent.png"%Material,bbox_inches='tight')
show()
```


Loss tangent
0.025
0.020
0.015
0.010
0.005
0.000
f_A
f_1
f_B
Frequency

[ ]: