In[1]:=
```
Clear["Global`*"]
```

In[2]:=
```
num = 10; (* Number of terms in Ritz expansion *)
```

In[3]:=
```
h = Table[(x / l)^(i+1), {i, num}]; (* Defining the admissible functions *)
h // MatrixForm;
```

In[5]:=
```
m = Table[0, {i, num}, {j, num}]; (* Mass and stiffness matrix *)
k = Table[0, {i, num}, {j, num}];
```

In[7]:=
```
For[i = 1, i ≤ num, i++,
 For[j = 1, j ≤ num, j++,
  m[[i, j]] = Integrate[ρa * h[[i]] * h[[j]], {x, 0, l}];
  k[[i, j]] = Integrate[ei * D[h[[i]], {x, 2}] * D[h[[j]], {x, 2}], {x, 0, l}]
 ]
]
```

In[8]:=
```
FullSimplify[m] // MatrixForm;
```

In[9]:=
```
FullSimplify[k] // MatrixForm;
```

In[10]:=
```
ei = (200 * 10^9 * 0.05 * 0.025^3) / 12;
l = 1;
ρa = 2500 * (0.05 * 0.025); (* Values required for numerical solution *)
```

In[11]:=
```
ω = Sqrt[Eigenvalues[N@{k, m}]]
```

Out[11]=

{356130., 69598.5, 49933.5, 27748.8,
 19568.7, 12902.6, 7804.32, 3982.54, 1422.32, 226.958}

In[12]:=
```
Transpose[Eigensystem[N@{k, m}]] // MatrixForm;
```

In[13]:=
```
v = Eigenvectors[N@{k, m}];
v // MatrixForm
```

Out[14]//MatrixForm=

−0.0000434831	0.00116063	−0.0122425	0.0686141	−0.229049	0.478282	−0.63
−0.0000794515	0.00185629	−0.0174425	0.0882198	−0.268378	0.514564	−0.62
−0.000272595	0.00531178	−0.0412383	0.170082	−0.414235	0.619241	−0.56
0.0000198977	0.00035272	−0.00870294	0.0629604	−0.230886	0.492657	−0.63
0.000125864	0.000858301	−0.023566	0.140979	−0.403678	0.64004	−0.57
−0.000195395	0.00116315	−0.00423907	0.0333112	−0.171091	0.449208	−0.65
0.00169134	−0.00687463	0.00976835	−0.0637392	0.304118	−0.623154	0.63
−0.0237498	0.0619823	0.00252936	−0.0203183	−0.15452	−0.00936864	0.56
0.411556	−0.655781	−0.00100305	0.00692851	0.526959	−0.307764	−0.11
0.908433	−0.416821	-1.04243×10^{-7}	7.16468×10^{-7}	0.0311926	−0.00612728	−0.0000

In[15]:=
```
mode = Table[0, {i, num}]; newmode = Table[0, {i, num}];
For[i = num;
 p = 1, i > 0, i--;
 p++, mode[[p]] = Sum[(h[[j]] * v[[i, j]]), {j, 1, num}];
 newmode[[p]] = mode[[p]] / (mode[[p]] /. x → 1)]
Simplify[newmode] // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} x^2 \left(1.75801 - 0.806636\, x - 2.01732 \times 10^{-7} x^2 + 1.38652 \times 10^{-6} x^3 + 0.0603643\, x^4 - 0.0118576\, x^5 - 0.0\right. \\ x^2 \left(-11.0172 + 17.555\, x + 0.0268513\, x^2 - 0.185474\, x^3 - 14.1065\, x^4 + 8.23871\, x^5 + 3.\right. \\ x^2 \left(30.8436 - 80.4958\, x - 3.28486\, x^2 + 26.3872\, x^3 + 200.674\, x^4 + 12.167\, x^5 - 7\right. \\ x^2 \left(-61.2147 + 248.813\, x - 353.545\, x^2 + 2306.91\, x^3 - 11\,006.9\, x^4 + 22\,553.8\, x^5 - 2\right. \\ x^2 \left(103.3 - 614.927\, x + 2241.08\, x^2 - 17\,610.7\, x^3 + 90\,451.2\, x^4 - 237\,484.\, x^5 + 344\right. \\ x^2 \left(-105.524 - 719.593\, x + 19\,757.6\, x^2 - 118\,196.\, x^3 + 338\,440.\, x^4 - 536\,604.\, x^5 + 4\right. \\ x^2 \left(125.587 + 2226.24\, x - 54\,929.8\, x^2 + 397\,383.\, x^3 - 1.45727 \times 10^6 x^4 + 3.10947 \times 10^6 x^5 - 4.0\right. \\ x^2 \left(-706.913 + 13\,774.9\, x - 106\,942.\, x^2 + 441\,069.\, x^3 - 1.07422 \times 10^6 x^4 + 1.60586 \times 10^6 x^5\right. \\ x^2 \left(882.452 - 20\,617.4\, x + 193\,730.\, x^2 - 979\,840.\, x^3 + 2.98083 \times 10^6 x^4 - 5.71517 \times 10^6 x^5 + 6.9\right. \\ x^2 \left(-180.715 + 4823.58\, x - 50\,879.8\, x^2 + 285\,159.\, x^3 - 951\,925.\, x^4 + 1.98774 \times 10^6 x^5 - 2.6\right. \end{pmatrix}$$

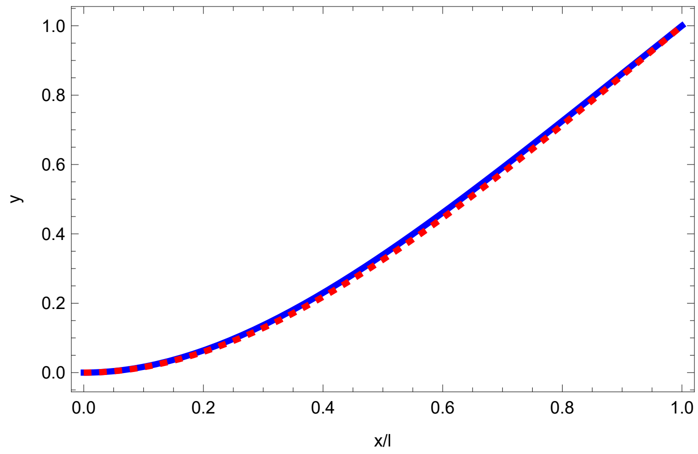
In[18]:=
```
(*Given values*) l = 1;
beta[n_] := - (2 * n - 1) * Pi / 2;

(*Define the expression for Wn(x)*)
Wn[x_, n_] := Sinh[beta[n] * x] - Sin[beta[n] * x] -
   ((Sinh[beta[n] * l] + Sin[beta[n] * l]) / (Cosh[beta[n] * l] + Cos[beta[n] * l])) *
    (Cosh[beta[n] * x] - Cos[beta[n] * x]);

(*Normalize Wn(x) for n=1*)
maxAbsWn1 = MaxValue[{Abs[Wn[x, 1]], 0 ≤ x ≤ l}, x];
NormalizedWn1[x_] := Wn[x, 1] / maxAbsWn1;

(*Plot*)
Plot[{newmode〚1〛, NormalizedWn1[x]}, {x, 0, l}, PlotStyle →
  {{Blue, Thick, Thickness[0.01]}, {Directive[Red, Dashed, Thickness[0.01]]}},
 Frame → True, FrameLabel → {"x/l", "y"}]
```

Out[23]=



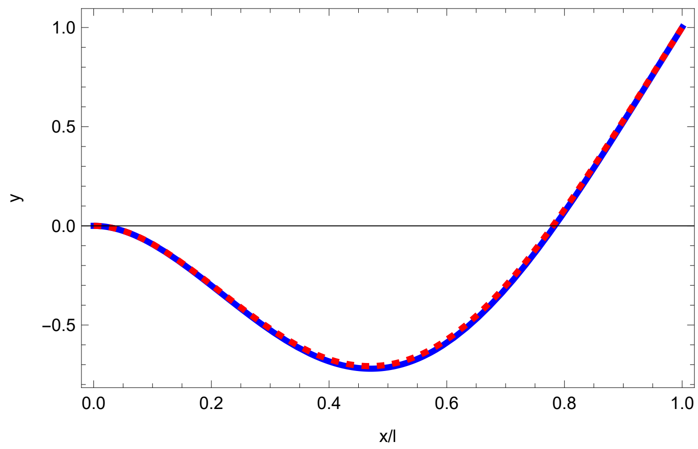
In[24]:=
```
(*Given values*) l = 1;
beta[n_] := (2 * n - 1) * Pi / 2;

(*Define the expression for Wn(x)*)
Wn[x_, n_] := Sinh[beta[n] * x] - Sin[beta[n] * x] -
   ((Sinh[beta[n] * l] + Sin[beta[n] * l]) / (Cosh[beta[n] * l] + Cos[beta[n] * l])) *
    (Cosh[beta[n] * x] - Cos[beta[n] * x]);

(*Normalize Wn(x) for n=2*)
maxAbsWn2 = MaxValue[{Abs[Wn[x, 2]], 0 ≤ x ≤ l}, x];
NormalizedWn2[x_] := Wn[x, 2] / maxAbsWn2;

(*Plot*)
Plot[{newmode[[2]], NormalizedWn2[x]}, {x, 0, l}, PlotStyle →
  {{Blue, Thick, Thickness[0.01]}, {Directive[Red, Dashed, Thickness[0.01]]}},
 Frame → True, FrameLabel → {"x/l", "y"}]
```

Out[29]=



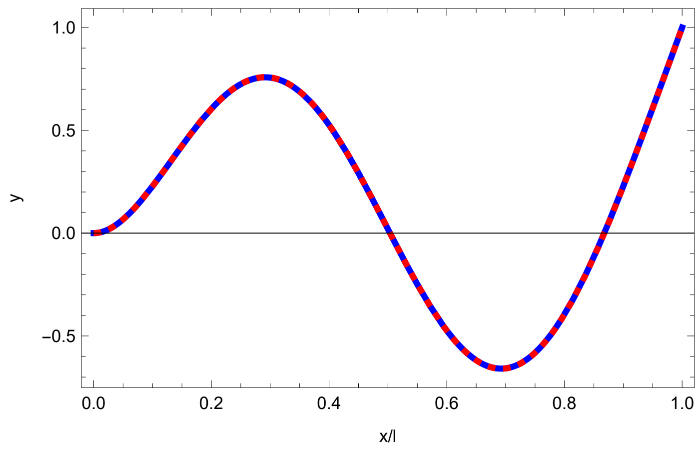
In[30]:=
```
(*Given values*) l = 1;
beta[n_] := -(2*n-1)*Pi/2;

(*Define the expression for Wn(x)*)
Wn[x_, n_] := Sinh[beta[n]*x] - Sin[beta[n]*x] -
   ((Sinh[beta[n]*l] + Sin[beta[n]*l]) / (Cosh[beta[n]*l] + Cos[beta[n]*l])) *
    (Cosh[beta[n]*x] - Cos[beta[n]*x]);

(*Normalize Wn(x) for n=3*)
maxAbsWn3 = MaxValue[{Abs[Wn[x, 3]], 0 ≤ x ≤ l}, x];
NormalizedWn3[x_] := Wn[x, 3] / maxAbsWn3;

(*Plot*)
Plot[{newmode〚3〛, NormalizedWn3[x]}, {x, 0, l}, PlotStyle →
  {{Blue, Thick, Thickness[0.01]}, {Directive[Red, Dashed, Thickness[0.01]]}},
 Frame → True, FrameLabel → {"x/l", "y"}]
```

Out[35]=



In[36]:=
```
(*Given values*) l = 1;
beta[n_] := (2 * n - 1) * Pi / 2;

(*Define the expression for Wn(x)*)
Wn[x_, n_] := Sinh[beta[n] * x] - Sin[beta[n] * x] -
   ((Sinh[beta[n] * l] + Sin[beta[n] * l]) / (Cosh[beta[n] * l] + Cos[beta[n] * l])) *
    (Cosh[beta[n] * x] - Cos[beta[n] * x]);

(*Normalize Wn(x) for n=4*)
maxAbsWn4 = MaxValue[{Abs[Wn[x, 4]], 0 ≤ x ≤ l}, x];
NormalizedWn4[x_] := Wn[x, 4] / maxAbsWn4;

(*Plot*)
Plot[{newmode[[4]], NormalizedWn4[x]}, {x, 0, l}, PlotStyle →
  {{Blue, Thick, Thickness[0.01]}, {Directive[Red, Dashed, Thickness[0.01]]}},
 Frame → True, FrameLabel → {"x/l", "y"}]
```

Out[41]=