Assignment 13: Kalman Filter Estimation

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1 Kalman Filter Estimation

We are given that the stride is 1 meter with a standard deviation of 0.2 meters, and variance of 0.04 meters. The PDF of one single stride is given by the following normal distribution:

$$\begin{aligned} & x \sim \mathcal{N}(\mu, \sigma^2) \\ & \mu = 1 \text{ m}, \quad \sigma^2 = 0.04 \text{ m}^2 \\ \implies & x \sim \mathcal{N}(1, 0.04) \end{aligned}$$

Extending this to 100 steps, assuming one step is independent of the other, we can find the new mean and variance. The mean and variance of the sum of independent random variables is given by:

$$\mu_{100} = \sum_{i=1}^{100} \mu_i = 100$$
$$\sigma_{100}^2 = \sum_{i=1}^{100} \sigma_i^2 = 100 \cdot 0.04 = 4$$

Therefore, the PDF of 100 steps is given by:

$$x_{100} \sim \mathcal{N}(\mu_{100}, \sigma_{100}^2) = \mathcal{N}(100, 4)$$

The two PDFs are shown in Figure 1.

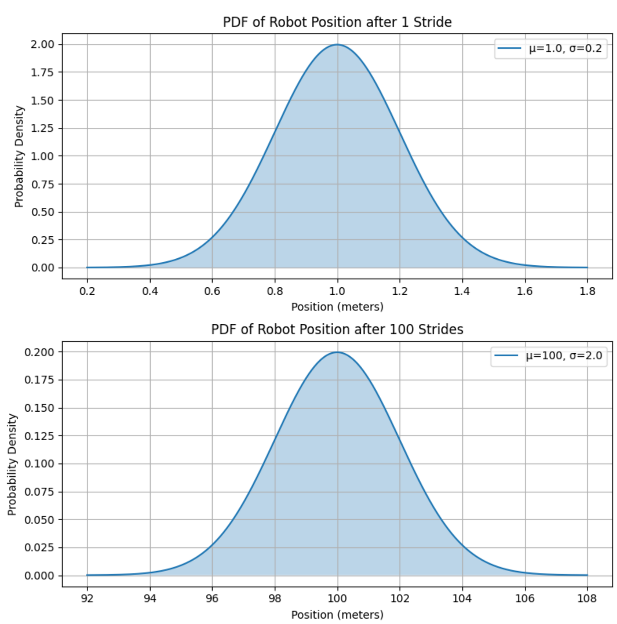

Figure 1: PDF of 1 step and 100 steps

Utilizing a second input, which estimates a position of 105 meters with variance 1, the resultant fused PDF is as seen in Figure 2. The mean and variance of the fused PDF is given by the following equations:

$$\mu_{fused} = \frac{\mu_{100} \cdot \sigma^2_{\text{sensor}} + \mu_{\text{sensor}} \cdot \sigma^2_{100}}{\sigma^2_{100} + \sigma^2_{\text{sensor}}} = \frac{100 \cdot 1 + 105 \cdot 4}{4 + 1} = \frac{520}{5} = 104$$
$$\sigma^2_{fused} = \frac{\sigma^2_{100} \cdot \sigma^2_{\text{sensor}}}{\sigma^2_{100} + \sigma^2_{\text{sensor}}} = \frac{4 \cdot 1}{4 + 1} = \frac{4}{5} = 0.8$$

Plotting this distribution over the previous two distributions, we can see that the position estimate is now more closer to the sensor input, and the variance is smaller than before.

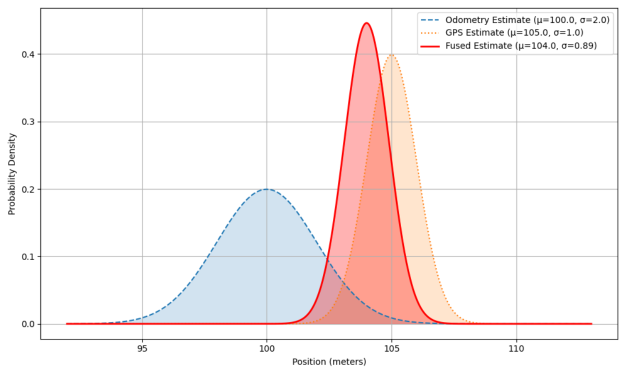

Figure 2: Fused PDF of 100 steps and sensor input