Lab 1

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ECE 4830 B03

January 27, 2025

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1 Problem 1

a. The provided matlab code generates a signal of 181 samples. The signal consists of a line, unit pulse, sine wave and a random segment. Once the main signal is created, a second signal is created by adding some noise to the main signal. The noise is generated with the `normrnd` function. The two signals, main and noise, are then plotted on the same graph.

b. Using ChatGPT to describe the code we get the following response:

```
This MATLAB script generates a synthetic signal composed of linear, constant,
sinusoidal, and noisy segments to simulate a real-world signal. It introduces
anomalies, such as spikes and plateaus, and adds Gaussian noise to mimic measurement
imperfections. The original and noisy signals are plotted side by side, showcasing
the impact of noise and modifications.
```

This output was the result of the following prompt:

```
Please describe the provided MATLAB code.
<<code>>
```

c. Asking an LLM to convert the matlab code to python we get the following response:

```python
import numpy as np
import matplotlib.pyplot as plt

# Initialize signal
sig = np.zeros(181)

# Signal construction
sig[:51] = np.arange(0, 51)              # Linearly increasing section
sig[51:100] = 50                         # Constant value section
sig[100:125] = 100                       # Higher plateau
sig[125:130] = 0                         # Zero section
sig[130:181] = 50 + 5 * np.random.randn(50)  # Random noisy section

# Append sinusoidal signals
n = np.arange(0, 51)
s1 = 50 + 5 * np.sin(2 * np.pi * 0.1 * n)  # Sine wave 1
s2 = 50 + 5 * np.sin(2 * np.pi * 0.2 * n)  # Sine wave 2
sig = np.concatenate([sig, s1, s2])

# Add baseline offset and anomalies
sig += 50
sig[54:57] = 150  # Spike
sig[58:62] = 125  # Small plateau

# Add noise
xx = np.arange(1, 284)
noisy_sig = sig + np.random.normal(0, 5, size=sig.shape)

# Plot the signals
plt.figure(figsize=(10, 5))
plt.plot(xx, noisy_sig.astype(np.uint8), ’--’, label=’Noisy’, linewidth=1.5)
plt.plot(xx, sig.astype(np.uint8), label=’Original’, linewidth=2)
plt.axis([0, 283, 30, 170])
plt.legend()
plt.xlabel('Index')
plt.ylabel('Signal Value')
plt.title('Original and Noisy Signal')
plt.grid(True)
plt.show()
```

Code Snippet 1: Python code generated by LLM

This output was the result of the following prompt:

```
Please convert the provided MATLAB code to Python.
<<code>>
```

d. To highlight the transitory parts, we can find the standard deviation of the 30 signals at each sample and plot the results. This will show the parts of the signal that are changing the most.

2 Problem 2

a. To compute the sampling frequency of the provided signal, we can use the following formula. The signal has 2500 samples, and it is know the signal is 60 Hz, and within the 2500 samples, 6 periods are captured. Therefore, the sampling frequency can be calculated as:

$$\begin{aligned} f_s &= \frac{N}{P} \times f \\ &= \frac{2500}{6} \times 60 \\ &= 2500 \times 10 = 25 \text{ kHz} \end{aligned}$$

Labeling the x-axis with the correct time scale instead of the sample number, we get the result shown in Figure 1.

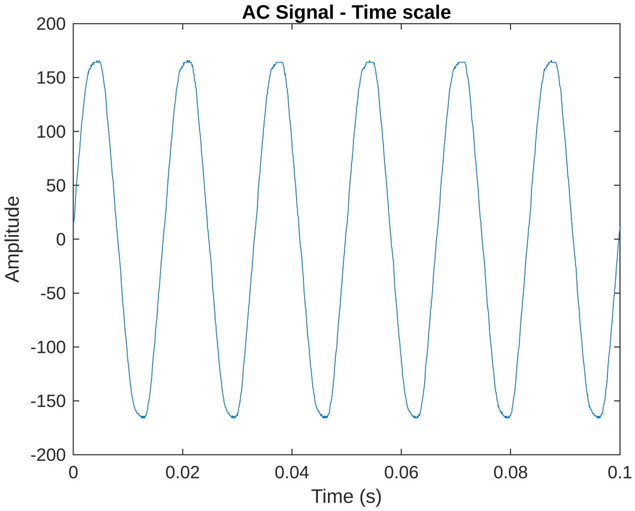


Figure 1: AC Signal with Correct Time Scale

b. Plotting the signal in the frequency domain using `fft`, we get the result shown in Figure 2.

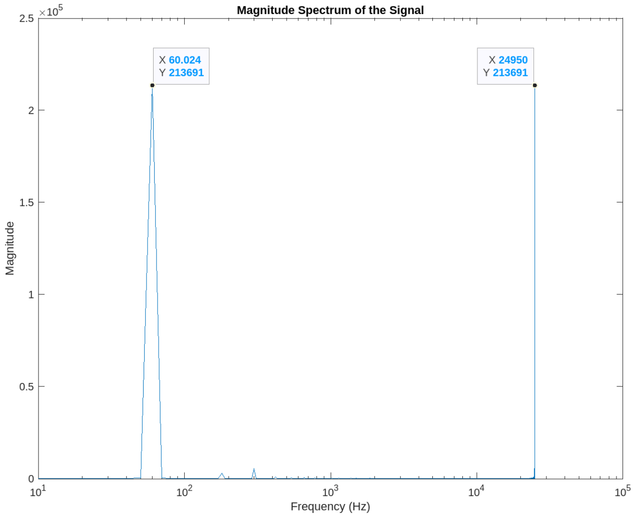


Figure 2: Frequency Domain of AC Signal

The result is as expected given that the signal is an approximate sine-wave with a frequency of 60 Hz. The frequency domain plot shows two peaks, one at 60 Hz, and the other at 24950 Hz. This is due to the symmetry of the FFT output, and if centered at 12.5 kHz, the two peaks would be shown at -12.5 kHz and 12.5 kHz. A log scale is used for the x-axis to better show the peaks.

c. To turn on a vacuum when the heavy load is on, we can detect the changes in frequencies of the signal before and after the heavy load is turned on. An RC circuit can be designed to detect the change and connect it to the relay that controls the vacuum.

Plotting the magnitude, in decibles, of the FFT of the two signals, we get the result shown in Figure 3.

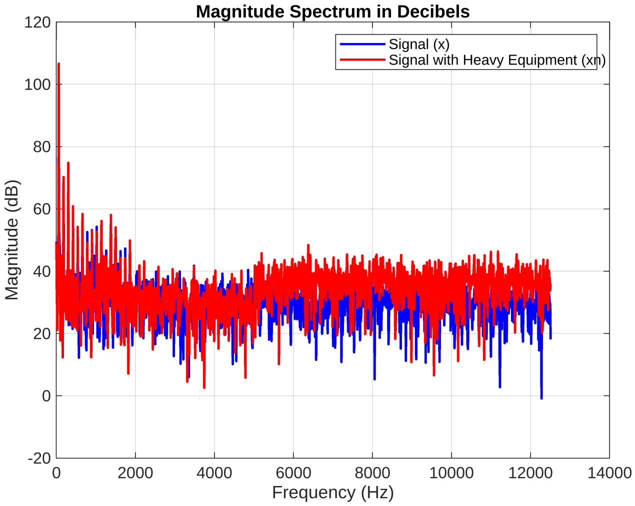


Figure 3: Magnitude of FFT of AC Signal

We can see that at higher frequencies, the signal with the machine on dominates and we can design an high-pass RC filter to detect these frequencies.

We can pick the cutoff frequency of the filter to be around 10 kHz. This will allow the signal with the machine on to pass through, while filtering out the signal with the machine off. The values of the resistor and capacitor can be calculated using the following formula:

$$\begin{aligned} f_c &= \frac{1}{2\pi RC} \\ \text{Let } f_c &= 10 \text{ kHz} \\ \text{Let } R &= 10 \text{ k}\Omega \\ \text{Then } C &= \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^3} = 1.59 \times 10^{-6} \text{ F} \\ \text{Choose } C &= 1.5\mu F \end{aligned}$$

With these values for the resistor and capacitor, we can read the output of the resistor, and then connect it to a amplifier to drive the relay that controls the vacuum.

3 Problem 3

3.1-6 We have:

$$x[n] = \begin{cases} \left(\frac{1}{3}\right)^n & \text{if } n \geq 0 \\ A^n & \text{if } n < 0 \end{cases}$$

Too determine the power and energy, we can use the following general equations:

For energy:

$$\begin{aligned} E_x &= \sum_{n=-\infty}^{\infty} |x[n]|^2 = \sum_{n=-\infty}^{-1} |A^n|^2 + \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^{2n} \\ &= \sum_{m=1}^{\infty} |A|^{-2m} + \sum_{n=0}^{\infty} \left(\frac{1}{9}\right)^n = \sum_{m=1}^{N} \frac{1}{A^{2m}} + \frac{1}{1-\frac{1}{9}} \\ &= \left(\sum_{m=0}^{N} \frac{1}{A^{2m}}\right) - 1 + \frac{9}{8} \\ &= \sum_{m=0}^{N} \left(\frac{1}{A^2}\right)^m + \frac{1}{8} \end{aligned}$$

If $|A| \leq 1$, then the sum will diverge to infinity as $1/A^2 \geq 1$. If $|A| > 1$, then the sum will converge to a finite value. Therefore:

$$E_x = \begin{cases} \infty & \text{if } |A| \leq 1 \\ \frac{1}{1-\frac{1}{A^2}} + \frac{1}{8} & \text{if } |A| > 1 \end{cases}$$

a. Determine the energy E_x and power P_x of $x[n]$ if A = 1/2.

Since $|A| < 1$, and using the results from above, the energy of the signal is infinite, that is:

$$E_x = \infty$$

The power can be found as follows:

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} \left(\frac{1}{2}\right)^{2n} + \sum_{n=0}^{N} \left(\frac{1}{3}\right)^{2n} \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} \left(\frac{1}{4}\right)^{n} + \sum_{n=0}^{N} \left(\frac{1}{9}\right)^{n} \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{m=1}^{N} 4^m + \sum_{n=0}^{N} \left(\frac{1}{9}\right)^{n} \right)
\end{aligned}$$

We can use the following two formulas to find the sum of the two series:

$$\sum_{m=1}^{N} 4^m = 4 + 4^2 + 4^3 + \ldots + 4^N = \frac{4(4^N-1)}{4-1} = \frac{4^{N+1}-4}{3}$$

$$\sum_{n=0}^{N} \frac{1}{9^n} = \frac{1-\frac{1}{9^{N+1}}}{1-\frac{1}{9}} = \frac{9}{8}\left(1-\frac{1}{9^{N+1}}\right)$$

We now have the following:

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \left(\frac{4^{N+1}-4}{3} + \frac{9}{8}\left(1-\frac{1}{9^{N+1}}\right) \right)$$

Now as $N \to \infty$, the term $\frac{1}{9^{N+1}}$ goes to 0, and the 4^{N+1} term grows very large. This means that the power of the signal is infinite, that is:

$$P_x \to \infty$$

b. Determine the energy E_x and power P_x of $x[n]$ if A = 1.

Since $|A| = 1$, and using the results from above, the energy of the signal is infinite, that is:

$$E_x = \infty$$

The power can be found as follows:

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 1 + \sum_{n=0}^{N} \left(\frac{1}{3}\right)^{2n} \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 1 \right) + \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=0}^{N} \left(\frac{1}{9}\right)^{n} \right)
\end{aligned}$$

The second term, as N grows large will approach 0, so we can disregard and focus on the first term.

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 1 \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=1}^{N} 1 \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} (N) \\
&= \lim_{N\to\infty} \frac{N}{2N+1} \\
&= \frac{1}{2}
\end{aligned}$$

c. Determine the energy E_x and power P_x of $x[n]$ if A = 2.

Since $|A| > 1$, and using the results from above, the energy of the signal is finite, that is:

$$E_x = \frac{1}{1-\frac{1}{A^2}} + \frac{1}{8} = \frac{1}{1-\frac{1}{4}} - \frac{1}{8} = \frac{4}{3} + \frac{1}{8} = \frac{35}{24}$$

The power can be found as follows:

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2 \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 2^{2n} + \sum_{n=0}^{N} \left(\frac{1}{3}\right)^{2n} \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 2^{2n} \right) + \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=0}^{N} \left(\frac{1}{9}\right)^{n} \right)
\end{aligned}$$

Similar to the previous part, the second term will approach 0 as N grows large, so we can disregard it.

$$\begin{aligned}
P_x &= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=-N}^{-1} 4^{n} \right) \\
&= \lim_{N\to\infty} \frac{1}{2N+1} \left(\sum_{n=1}^{N} 4^{-n} \right)
\end{aligned}$$

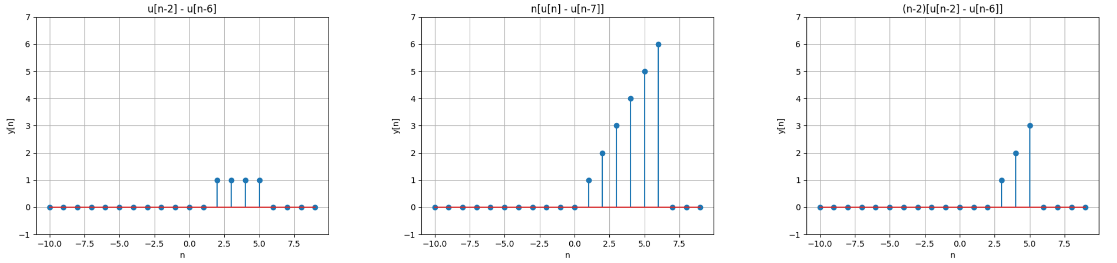
We can use the following formula to find the sum of the series:

$$\sum_{n=1}^{N} 4^{-n} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \ldots + \frac{1}{4^N} = \frac{1}{3}\left(1 - \frac{1}{4^N}\right)$$

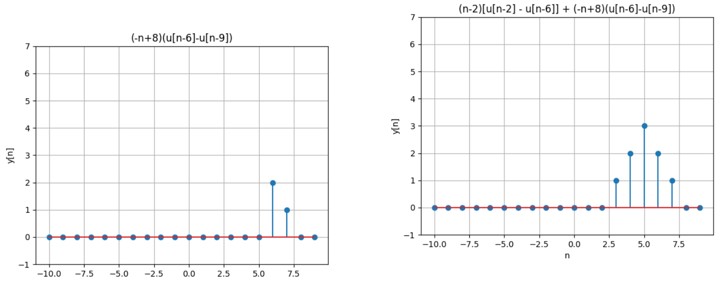
We can now substitute this back into the equation for P_x:

$$\begin{aligned} P_x &= \lim_{N\to\infty} \frac{1}{2N+1}\left(\frac{1}{3}\left(1-\frac{1}{4^N}\right)\right) \\ &= \lim_{N\to\infty} \frac{1}{2N+1}\left(\frac{1}{3}-\frac{1}{3\cdot 4^N}\right) \\ &= 0 \end{aligned}$$

3.3-3 Figure 4 shows the plots of the provided signals. The code to generate these signals is provided in the appendix at Code Snippet 3.



(a) $u[n-2]-u[n-6]$ (b) $n(u[n]-u[n-7])$ (c) $(n-2)(u[n-2]-u[n-6])$



(d) $(-n+8)(u[n-6]-u[n-9])$ (e) $(n-2)(u[n-2]-u[n-6])+$ $(-n+8)(u[n-6]-u[n-9])$

Figure 4: Plots of various signals

3.6-1

We have the following difference equation:

$$y[n]+\frac{1}{6}y[n-1]-\frac{1}{6}y[n-2]=\frac{1}{3}x[n]+\frac{2}{3}x[n-2]$$
$$y_0[-1]=3, \quad y_0[-2]=-1$$

The following code can be used to output the first 10 values of $y[n]$, assuming $x[n]=u[n]$:

```python
def unit_step(n):
    return 1 if n >= 0 else 0

y = {}
x = {}

# Initial conditions
y[-1] = 3
y[-2] = -1

for n in range(10):
    x[n] = unit_step(n)
    x[n-2] = unit_step(n-2)
    y[n] = (-1/6) * y[n-1] + (1/6) * y[n-2] + (1/3) * x[n] + (2/3) * x[n-2]
    print(f"y[{n}] = {y[n]:.5f}")
```

Code Snippet 2: Python code to solve difference equation

Plotting the first 10 values of $y[n]$ we get the result shown in Figure 5.

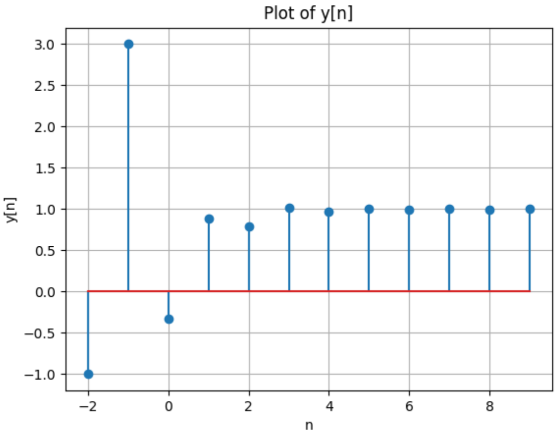


Figure 5: First 10 values of $y[n]$

A Code

```
import matplotlib.pyplot as plt
import numpy as np

def unit_step(n):
    return 1 if n >= 0 else 0

# plot 1
# u[n-2] - u[n-6]

n = np.arange(-10, 10)
y1 = np.zeros(20)
for i in range(20):
    y1[i] = unit_step(n[i] - 2) - unit_step(n[i] - 6)

plt.figure(1)
plt.stem(n, y1)
plt.title('u[n-2] - u[n-6]')
plt.xlabel('n')
plt.ylabel('y[n]')
plt.ylim(-1, 7)
plt.grid(True)
plt.savefig('/plot1.png')

# plot 2
# n[u[n] - u[n-7]]
y2 = np.zeros(20)
for i in range(20):
    y2[i] = n[i] * (unit_step(n[i]) - unit_step(n[i] - 7))

plt.figure(2)
plt.stem(n, y2)
plt.title('n[u[n] - u[n-7]]')
plt.xlabel('n')
plt.ylabel('y[n]')
plt.ylim(-1, 7)

plt.grid(True)
plt.savefig('/plot2.png')

# plot 3
# (n-2)[u[n-2] - u[n-6]]
y3 = np.zeros(20)
for i in range(20):
    y3[i] = (n[i] - 2) * (unit_step(n[i]-2) - unit_step(n[i] - 6))

plt.figure(3)
plt.stem(n, y3)
plt.title('(n-2)[u[n-2] - u[n-6]]')
plt.xlabel('n')
plt.ylabel('y[n]')
plt.ylim(-1, 7)

plt.grid(True)
plt.savefig('/plot3.png')

# plot 4
# (-n+8)(u[n-6]-u[n-9])
y4 = np.zeros(20)
for i in range(20):
    y4[i] = (-n[i] + 8) * (unit_step(n[i] - 6) - unit_step(n[i] - 9))

plt.figure(4)
plt.stem(n, y4)
plt.title('(-n+8)(u[n-6]-u[n-9])')
plt.xlabel('n')
plt.ylabel('y[n]')
plt.ylim(-1, 7)
plt.grid(True)
plt.savefig('/plot4.png')

# plot 5
# (n-2)[u[n-2] - u[n-6]] + (-n+8)(u[n-6]-u[n-9])
y5 = y3 + y4

plt.figure(5)
plt.stem(n, y5)
plt.title('(n-2)[u[n-2] - u[n-6]] + (-n+8)(u[n-6]-u[n-9])')
plt.xlabel('n')
plt.ylabel('y[n]')
plt.ylim(-1, 7)

plt.grid(True)
plt.savefig('/plot5.png')
```

Code Snippet 3: Python code to generate plots