Assignment 8: CRC LFSR FSM Arithmetic in GF(2)

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1 Part 1

The polynomial representation for a 3-bit maximal length LFSR is $x^3 + x^2 + 1$. Since it is 3-bits, there are 7 non-zero states and 1 zero state.

We can verify this by doing the state transition and see if all 7 non-zero states are visited before returning to the initial state.

We will have the taps at bit's 0 and 1, so that the new bit that is shifted in is the XOR of bit 0 and bit 1.

$$s_2 s_1 s_0 \rightarrow s_{new} s_2 s_1, \quad s_{new} = s_1 \oplus s_0$$

$$\begin{aligned} \text{Start at: } s_2 s_1 s_0 = 100 &\rightarrow 010 \\ 010 &\rightarrow 101 \\ 101 &\rightarrow 110 \\ 110 &\rightarrow 111 \\ 111 &\rightarrow 011 \\ 011 &\rightarrow 001 \\ 001 &\rightarrow 100 \\ 100 &\rightarrow 010 \end{aligned}$$

We see that all 7 non-zero states are visited and will repeat the sequence. The zero-state will continously visit itself. The LSFR would look like the following:

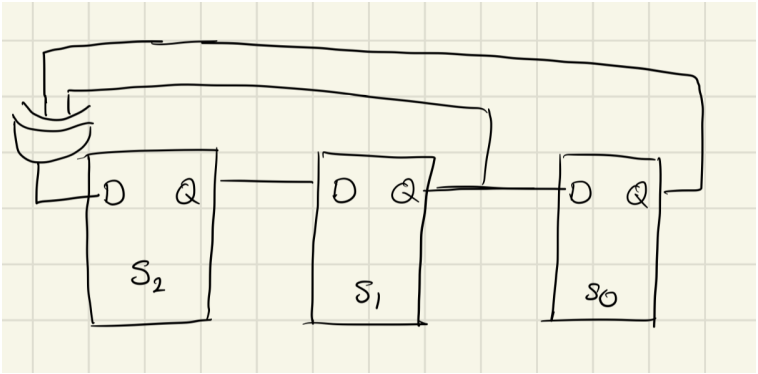

Figure 1: LFSR Diagram

Sender Side Operation:

We can divide 1011 into 1000 0000, and we get our remainder

$$\begin{aligned} 10000000 = 128, \quad 1011 = 11 \\ 128 \div 11 = 11 \text{ remainder } 7 \\ \text{Remainder: } 7 = 0111 \end{aligned}$$

We want to add to our original message to make the remainder 0, so we add (11-7) = 4 to the original message.

$$10000000 + 0100 = 10000100$$

Now we send this msg over to the receiver and if there is no remainder, then the message should be correct.

Receiver Side Operation:

On the receiver side, we divide the received message 1000 0100 by 1011 and determine if the remainder is 0. If the received message has a supposed error, and a bit-flip occurs, the remainder will not be 0.

Verifying this:

$$10000100 = 132, \quad 1011 = 11$$
$$132 \div 11 = 12 \text{ remainder } 0$$

If bitflip occurs, we can see that the remainder will not be 0.

$$10\mathbf{1}00100 = 164, \quad 1011 = 11$$
$$164 \div 11 = 14 \text{ remainder } 10$$

What does ethernet do on a CRC error? What does WiFi do? What does ethernet do on NO CRC error? What does WiFi do?

On a CRC error on both ethernet and WiFi, the frame is dropped and the receiver sends a NAK to the sender to resend the frame. Depending on the protocol, the sender may resend the frame after a timeout period, or resend on a NAK.

On no CRC error, the frame is accepted and the receiver sends an ACK to the sender to confirm that the frame was received.

Wi-Fi would have more error checking as compared to ethernet, as the wireless medium is more prone to errors.