Assignment 14: Simplex and TMR Reliability

Rajdeep Gill 7934493

ECE 3760 A01

April 9, 2025

1 Mean Time To Failure (MTTF)

1.1 MTTF Simulation

We first simulate the time to failure of a simplex system. If we have a system with a constant failure rate of $\lambda = 0.01$, we can generate the time it takes for a module to fail. The following code gives us the time it takes for a module to fail:

```
p = 0.01 # Probability / lambda
num_iterations = 10000

def simulate_simplex(p):
    time_steps = 0
    while True:
        time_steps += 1
        if np.random.rand() < p:
            break
    return time_steps
```

Code Snippet 1: Simplex Simulation

We loop until the system fails at each iteration, and then take the average of the time steps. The average time as a result of the simulation was found to be:

`Average time to failure (Simplex): 100.92 steps`

We can perform a similar simulation for a TMR system. The following code gives us the time it takes for a TMR system to fail:

```
def simulate_tmr(p):
    time_steps = 0
    working_modules = 3
    while working_modules >= 2:
        time_steps += 1
        failures = np.random.rand(working_modules) < p
        working_modules -= np.sum(failures)
    return time_steps
```

Code Snippet 2: TMR Simulation

At each time step, we find 3 random numbers and if any of them are less than the failure rate, we assume a module will fail. We then decrease the number of working modules by the number of failures and stop when we have less than 2 working modules. The average time as a result of the simulation was found to be:

`Average time to failure (TMR): 83.38 steps`

Lastly, a system which starts out as a TMR system and switches to a simplex system when one of the modules fails can also be simulated in a similar manner. The following code gives us the time it takes for a TMR system to fail:

```
def simulate_redundant_to_simplex(p):
    time_steps = 0
    active_modules = 3
    switched_to_simplex = False

    while True:
        time_steps += 1
        failures = np.random.rand(active_modules) < p
        active_modules -= np.sum(failures)

        if not switched_to_simplex and active_modules < 3:
            active_modules = 1  # Switch to simplex
            switched_to_simplex = True

        if active_modules <= 0:
            break
```

```
    return time_steps
```

Code Snippet 3: Redundant-to-Simplex Simulation

We loop until one of the systems fails, and then we switch to a simplex system. This was done by setting the active modules to 1, and then just simulating the random generation on a single module at each time step. The average time as a result of the simulation was found to be:

```
Average time to failure (Redundant-to-Simplex): 133.45 steps
```

1.2 MTTF Calculations

Given that we have a constant failure rate of $\lambda = 0.01$ failures per day, our reliability function is given by:

$$R(t) = e^{-\lambda t} = e^{-0.01t} \tag{1}$$

The mean time to failure (MTTF) is simply the integral of the reliability function over time, which can be expressed as:

$$\begin{aligned}\text{MTTF} = \int_0^\infty R(t)dt &= \int_0^\infty e^{-0.01t}dt \\ &= \left[-\frac{1}{0.01}e^{-0.01t}\right]_0^\infty \\ &= 1/0.01 = 100 \text{ days}\end{aligned}$$

This matches the result of the simulation, where we found the average time to failure of a simplex system to be 100.92 steps.

Assuming a TMR system with perfect voting with 3 modules. The system is reliable if 2 of the 3 agree. The system will fail if more than 1 module failes, this gives us the following reliability function:

$$R(t)_{\text{single}} = e^{-\lambda t}$$

We can find the reliability by first finding the probability of 2 out of 3 modules working and the probability of all 3 modules working.

$$\begin{aligned}P(2 \text{ out of } 3) &= \binom{3}{2}R(t)^2(1 - R(t)) \\ &= 3R(t)^2(1 - R(t)) \\ &= 3(e^{-\lambda t})^2(1 - e^{-\lambda t}) \\ &= 3e^{-2\lambda t} - 3e^{-3\lambda t}\end{aligned}$$

And when all 3 modules are working:

$$\begin{aligned}P(3 \text{ out of } 3) &= \binom{3}{3}R(t)^3 \\ &= e^{-3\lambda t}\end{aligned}$$

Therefore, the total reliability of the TMR system is given by:

$$\begin{aligned} R(t) &= P(2 \text{ out of } 3) + P(3 \text{ out of } 3) \\ &= 3e^{-2\lambda t} - 3e^{-3\lambda t} + e^{-3\lambda t} \\ &= 3e^{-2\lambda t} - 2e^{-3\lambda t} \end{aligned}$$

The MTTF of the TMR system can be calculated as:

$$\begin{aligned} \int_0^\infty R(t)dt &= \int_0^\infty (3e^{-2\lambda t} - 2e^{-3\lambda t})dt \\ &= \frac{3}{2\lambda} - \frac{2}{3\lambda} \\ &= \frac{3}{2(0.01)} - \frac{2}{3(0.01)} \\ &= 150 - 66.67 \\ &= 83.33 \text{ days} \end{aligned}$$

This closely matches the result of the simulation, where we found the average time to failure of a TMR system to be 83.38 steps.

If we are to switch to a simplex system when one of the modules fails, we can find the MTTF of the system by first finding the MTTF of all 3 modules working, and adding the MTTF of the simplex system. The MTTF of all 3 modules working is given by:

$$\begin{aligned} \int_0^\infty R(t)^3 dt &= \int_0^\infty e^{-3\lambda t} dt \\ &= \left[-\frac{1}{3\lambda} e^{-3\lambda t} \right]_0^\infty \\ &= \frac{1}{3(0.01)} = 33.33 \text{ days} \end{aligned}$$

Therefore, the MTTF of the switching system is given by:

$$\begin{aligned} \text{MTTF} &= \int_0^\infty R(t)^3 dt + \int_0^\infty R(t) dt \\ &= \int_0^\infty e^{-3\lambda t} dt + \int_0^\infty e^{-\lambda t} dt \\ &= 33.33 + 100 = 133.33 \text{ days} \end{aligned}$$

This also matches the result of the simulation, where we found the average time to failure of a redundant-to-simplex system to be 133.45 steps.