Prof. Faouzi Bellili — ECE Department (University of Manitoba)

ECE–4260 Communication Systems

Assignment#1 (Winter 2025)

Remarks:

- This assignment is due on **January 28th, 2025**. Please scan your answers and upload them to UM Learn. Do not e-mail pdf documents.
- **Make sure to compile a single PDF document that contains all your answers, i.e., do not submit separate pages and make sure to number your pages.**
- **Presenting clean and well-justified answers helps you get full marks for questions.**
- Assignments submitted after the due date will not be marked.
- Clearly show the solution method. Marks are awarded for the method and not the final answer itself.

1. **Problem 1 (20 points)**
Let j be the pure imaginary number satisfying $j^2 = -1$, $n \in \mathbb{N}$ a nonzero natural number, and $0 < \varphi < \pi/n$. Consider the following complex numbers:

$$z_1 = \frac{5}{2} + \frac{5\sqrt{3}}{2}j \tag{1}$$
$$z_2 = \sqrt{3} - j \tag{2}$$
$$z_3 = 1 + e^{-jn\varphi} \tag{3}$$
$$z_4 = 1 - e^{j\varphi} \tag{4}$$

1.1 Find the Euler representation of the complex numbers z_5, z_6, and z_7 given by (**15 points**):

$$z_5 = \frac{z_1}{z_2^*}, \qquad z_6 = \frac{z_3^*}{z_4}, \qquad \text{and} \qquad z_7 = z_5 z_6^*. \tag{5}$$

Recall: The Euler representation of a given complex number z is $z = re^{j\theta}$ for some $r > 0$, called the magnitude of z and some $0 \leq \theta < 2\pi$, called the angle or argument of z. Also z^* stands for the conjugate of z.

1.2 Find the real and imaginary parts of z_7^*. (**5 points**).

2. **Problem 2 (20 points)**

In Fig. 1 the signal $g_1(t) = g(-t)$. Express the four other signals $g_2(t)$, $g_3(t)$, $g_4(t)$, and $g_5(t)$, in terms of the signal $g(t)$, and its *time-shifted* , *time-scaled*, or *time-inverted* versions. (**5 points** for each signal)

Hint: For instance, as a first step you may recognize that we have $g_2(t) = g(t-T) + g_1(t-T)$ for some suitable value of T that you need to find. For the signal $g_5(t)$, you might also recognize that it can be expressed as $g(t)$ time-scaled, time-shifted, and then multiplied by some constant.

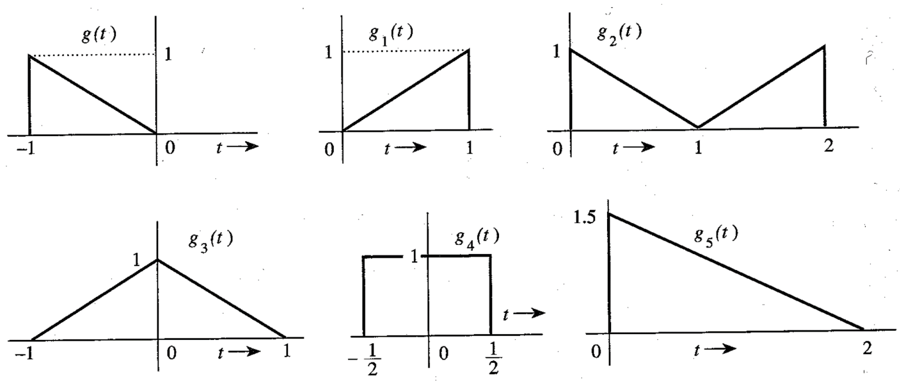

Figure 1: The six signals considered in Problem 2.

3. **Problem 3 (15 points)**

   For the signal $g(t)$ shown in Fig. 2, stech the following signals

   3.1) $g(3t)$. **(5 points)**

   3.2) $g(t+6)$. **(5 points)**

   3.3) $g(6-t)$. **(5 points)**

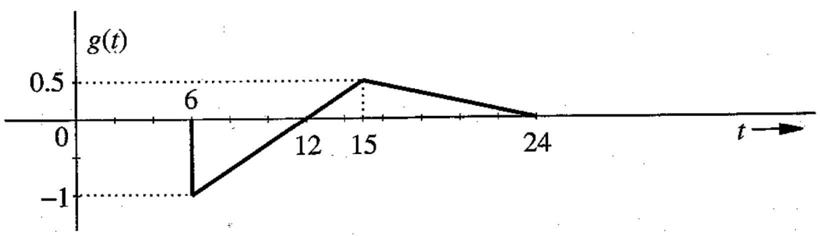

Figure 2: The signal $g(t)$ considered in Problem 3.

4. **Problem 4 (15 points)**

   Find the average power of the periodic signal $g(t)$ shown in Fig. 3.

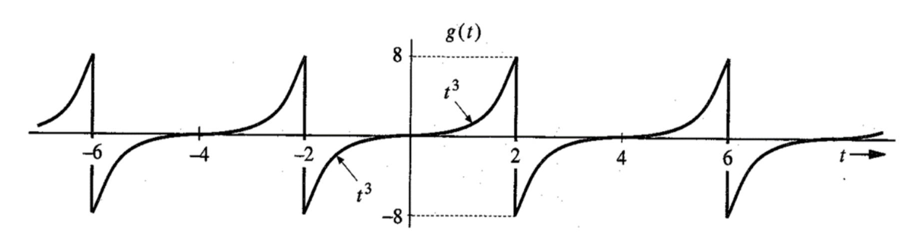

Figure 3: The signal $g(t)$ considered in Problem 4.

5. **Problem 5 (30 points)**

Consider the system $\mathcal{H}[.]$ whose input-output relationship is given by:

$$\mathcal{H}[x(t)] = h(t)x(t), \tag{6}$$

where $h(t)$ is a fixed time-dependent signal. Here $x(t)$ is the input of the system $\mathcal{H}[.]$ and the corresponding output is $\mathcal{H}[x(t)]$.

5.1) Show that $\mathcal{H}[.]$ is *linear*. (**6 points**)

5.2) Show that if $\mathcal{H}[.]$ is *time-invariant* then $h(t)$ must be constant (i.e., time-independent). In other words, we must have $h(t) = c$ for all t ($c \in \mathbb{R}$ is any constant). (**6 points**)

Switching on and off is a linear system. Now, suppose we have a system consisting of a "switch". When the switch is closed a signal goes through unchanged and when the switch is open the signal does not go through at all (so by convention what comes out the other end is the zero signal).

5.3 Suppose that the switch is closed (i.e., the switch is on) for $-1/2 \leq t \leq 1/2$. Using Question 5.1) show that this is a *linear* system. (**6 points**)

5.4 Suppose that the switch is turned on and off at various time intervals. Is the system still *linear* and why? Is it *time-invariant* and why? (**6 points**)

5.5 Suppose that the switch is turned on at $t = 0$ and stays on forever. Is the system now *linear* and why? Is it *time-invariant* and why? (**6 points**)