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Assignment 1: Set Theory

Q1. If $U = \{1,2,3,4,5,6,7,8,9\}$, $A = \{1,2,3,4\}$, $B = \{2,4,6,8\}$, $C = \{3,4,5,6\}$. Find:

i) $(C-A)'$

$C-A = \{5,6\}$ $(C-A)' = \{1,2,3,4,7,8,9\}$

ii) $B \cap A$

$B \cap A = \cancel{\{4\}}\ \{2,4\}$

iii) $A \cup C$

$A \cup C = \{1,2,3,4,5,6\}$

iv) $(C \cap B) \cap (B \cap A)$

$C \cap B = \{4,6\}$ $B \cap A = \{4\}$

$(C \cap B) \cap (B \cap A) = \{4\}$

v) Show that $(B \cup C)' = B' \cap C'$

$B' = \{1,3,5,7,9\}$ $C' = \{1,2,7,8,9\}$ $B' \cap C' = \{1,7,9\}$ — I

$B \cup C = \{2,3,4,5,6,8\}$ $(B \cup C)' = \{1,7,9\}$ — II

By equation I & II hence Prove that

$(B \cup C)' = B' \cap C'$

Q2 If $U = \{1,2,3,4,5,6,7,8,9,10,11,12\}$, $A = \{2,4,7,8,10\}$, $B = \{1,2,4,6,7,9,11\}$, $C = \{3,4,5,6,7,9,12\}$. Find:

i) $A' \cap B$

$A' = \{1,3,5,6,9,11,12\}$, $A' \cap B = \{1,6,9,11\}$

ii) $A \cap C'$

$C' = \{1,2,8,10,11\}$, $A \cap C' = \{2,8,10\}$

iii) $(B \cup C) \cap A$

$B \cup C = \cancel{\{2,3,4,5,6,7,8,9,10,12\}}\ \{1,2,3,4,5,6,7,9,11,12\}$

$(B \cup C) \cap A = \{2,4,7\}$

iv) $B' \cap C'$

$B' = \{3, 5, 8, 10, 12\}$, $C' = \{1, 2, 8, 10, 11\}$

$B' \cap C' = \{8, 10\}$

v) Show that $(A \cap B)' = A' \cup B'$

$A' = \{1, 3, 5, 6, 9, 11, 12\}$, $B' = \{3, 5, 8, 10, 12\}$

$A \cap B = \{2, 4, 7, \}$, $(A \cap B)' = \{1, 3, 5, 6, 8, 9, 10, 11, 12\}$ — I

$A' \cup B' = \{1, 3, 5, 6, 8, 9, 10, 11, 12\}$ — II

By equation I & II, Hence prove that.

$(A \cap B)' = A' \cup B'$

Q3. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$

$B = \{2, 3, 4, 6, 8, 10\}$ $C = \{6, 7, 8, 9\}$. find:

i) $A - B$

$A - B = \{1, 5, 7, 9\}$

ii) $(B - C)'$

$B - C = \{2, 3, 4, 8, 10\}$, $(B - C)' = \{1, 5, 6, 7, 9\}$

iii) $(C \cap B) \cup (B \cap A)$

$C \cap B = \{6, 8\}$, $B \cap A = \{3, \}$

$(C \cap B) \cup (B \cap A) = \{3, 6, 8\}$

iv) $(C - A)'$

$C - A = \{6, 8\}$, $(C - A)' = \{1, 2, 3, 4, 5, 7, 9, 10\}$

v) Show that $(A \cup B)' = A' \cap B'$

$A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $(A \cup B)' = \{\}$ — I

$A' = \{2, 4, 6, 8, 10, \cancel{8}\}$, $B' = \{1, 5, 7, 9\}$

$A' \cap B' = \{\ \}$ — II

By equation I & II. Hence Prove That

$(A \cup B)' = A' \cap B'$

Q4. State which of the following sets are finite or infinite with appropriate reasons.

i) A = set of Natural Numbers

The set of natural numbers is an infinite set. A set is considered to be infinite if it has an infinite number of elements.

ii) $B = \{x \mid x \in Z, 3x-2=7\}$

The set B, defined as the set of all x such that x is an element of the set of integers Z & $3x-2=7$, is a finite set. Because 3 is the only integer that satisfies the eq^n $3x-2=7$

iii) $C = \{x \mid x^2-16=0\}$

The set C is finite set, Because ~~this~~ ~~ca~~ -4 & 4 are these two elements that satisfies the eq^n x^2-16.

iv) $D = \{x \mid x \in N, 2x-3=0\}$

The set of D contains only elements that are both natural numbers & solutions to the equation $2x-3=0$. However, the equation has a solution of ~~set~~ $x = 3/2$, which is not natural number. Therefore, the set D is empty. An empty set is considered as finite set, because it has a definite & limited numbers of elements, which is zero.

v) $E = \{y \mid y<-1, y \in Z\}$

The set E is an infinite set because it contains limitless number of elements which is all numbers are integers & less that -1.

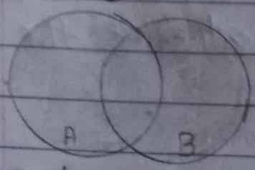
vi) $F = \{x \mid x \in N, x \text{ is even number}\}$

The set of F is an infinite set becaucause it contains natural even numbers & natural numbers ~~at~~ are limitless ~~&~~ or infinite.

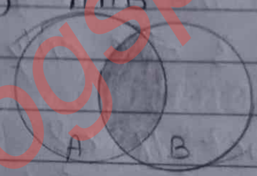
vi) $G = \{ y \mid y < 10, y \in N \}$
The~~s~~ set G is ~~an~~ a finite set because it contains numbers which is natural numbers & less that 10, also natural numbers are positive. So it contains only 9 elements.

Q5. Represents the following sets by venn diagram

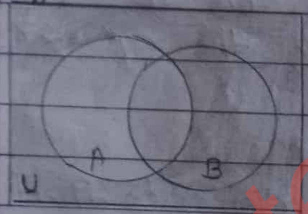
i) AUB



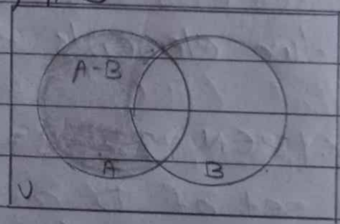
ii) A∩B



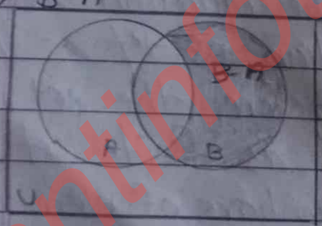
iii) A'



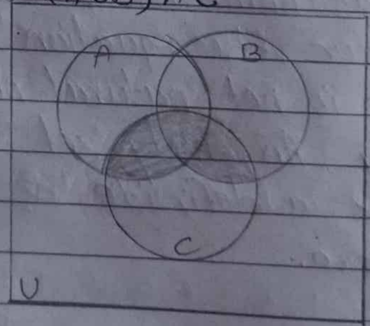
iv) A-B



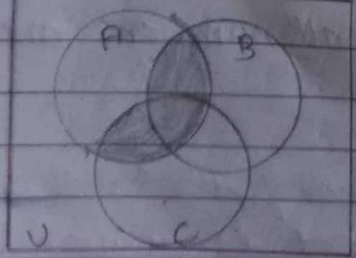
v) B-A



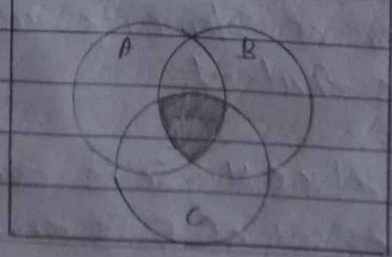
vi) (AUB)∩C



viii) (BUC)∩A



vii) A∩B∩C



Q6 Out of 100 persons in a group, 72 persons speak english & 43 persons speak French. Each one out of 100 persons speaks at least one language. Then how many speak only English? How many speak only french? How many of them speak English & French both?

Ans Let A be the set of persons speaking English & B be the set of persons speaking French.

So, $n(A) = 72$, $n(B) = 43$, $n(A \cup B) = 100$

Now,

$n(A) + n(B) = n(A \cup B) + n(A \cap B)$

$\therefore \quad n(A \cap B) = 72 + 43 - 100 = 15$

So, the number of persons who speak french & English both is 15.

Also,

$n(A) = n(A-B) + n(A \cap B)$

$\therefore \quad n(A-B) = 72 + 15 = 75$

So, the number of persons who speak only English is 57.

And,

$n(B) = n(B-A) + n(A \cap B)$

$\therefore \quad n(B-A) = 43 - 15 = 28$

So, the number of persons who speak only French is 28.

Q7. From amongst 2000 literate individuals of a town, 70% read Marathi newspaper, 50% read english newspaper & 32.5% read both Marathi & English newspaper. Find the number of individuals who read

i) at least one of the newspaper

ii) neither Marathi & English newspaper

iii) Only one of the newspaper

Ans Let M = set of individuals who read marathi newspaper

E = set of individuals who read English newspaper

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X = Set of all literate individuals

$\therefore n(X) = 2000,$

$n(M) = \frac{70}{100} \times 2000 = 1400$

$n(E) = \frac{50}{100} \times 2000 = 1000$

$n(M \cap E) = \frac{32.5}{100} \times 2000 = 650$

$n(M \cup E) = n(M) + n(E) - n(M \cap E)$

$\therefore n(M \cup E) = 1400 + 1000 - 650$

$\therefore n(M \cup E) = 1750$

i] No. of individuals who read at least one of the newspapers $= n(M \cup E) = 1750$

ii] No. of individuals who read neither Marathi & English newspaper

$= n(M' \cap E') = n(M \cup E)' = n(X) - n(M \cup E)$

$= 2000 - 1750$

$= 250.$

iii] No. of individuals who read only one of the newspapers $= n(M \cap E') + n(M' \cap E)$

$= n(M \cup E) - n(M \cap E)$

$= 1750 - 650$

$= 1100.$

Q8. In a board examination, 40 students failed in Physics, 40 in Chemistry & 35 in Maths, 20 failed in Maths & physics, 17 in Physics & chemistry, 15 in Maths & chemistry & 5 in all the three subjects. If 350 students appeared in the examination, how many of them did not fail in any subjects?

Ans. Given, $n(X) = 350$, $n(P) = 40$, $n(C) = 40$, $n(M) = 35$,
$n(P \cap M) = 20$, $n(P \cap C) = 17$, $n(M \cap C) = 15$, $n(P \cap M \cap C) = 5$

Let's find the number of students who failed in at least one of the subjects.

$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap M) - n(M \cap C) - n(P \cap C) + n(P \cap M \cap C)$

$\therefore n(P \cup C \cup M) = 40 + 40 + 25 - 17 - 20 - 15 + 5 = 68$

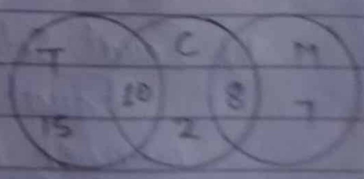
Let's find the number of students who didn't fail in any of the subjects $= n(X) - n(P \cup C \cup M)$
$= 350 - 68 = 282$

The number of students who didn't fail in any of the subjects is 282.

Q1. In a hostel, 25 students take tea, 20 students take cofee, 15 students take milk, 10 students take both tea & cofee, 8 students take both milk & coffee. None of them take both tea & milk & everyone takes atleast one beverage, find the total number of students in the hostel.

Ans. Let, T = set of students who take tea
C = set of students who take cofee
M = set of students who take milk.

$\therefore n(T) = 25$, $n(C) = 20$, $n(M) = 15$
$n(T \cap C) = 10$, $n(M \cap C) = 8$, $n(T \cap M) = 0$
$n(T \cap M \cap C) = 0$



$\therefore$ Number of students in the hostel
$= n(T \cup C \cup M)$
$= n(T) + n(C) + n(M) - n(T \cap C) - n(M \cap C) - n(T \cap M) + n(T \cap M \cap C)$
$= 25 + 20 + 15 - 10 - 8 - 0 + 0$
$= 42$

$\therefore$ 42 students in the hostel.

Q10. Define

i) Subset

A subset is a subgroup of any set. Consider two sets, A & B then A will be a subset of B ~~iffone~~ if & only if all the components of A are present in B. We can also say that A is contained in B.

ii) Union of two sets

Union of two or more sets is the set containing all the elements of the given sets. Union of sets can be written using the symbol "U".

iii) Intersection of two sets.

The intersection of sets for two given sets is the set that contains all the elements that are common to both sets. The symbol for the intersection of sets is "∩".

iv) Power set

A power set includes all the subsets of a given set including the empty set. The power set is denoted by the notation P(S) & the number of elements of the power set is given by 2^n.

v) Equal Sets

Equal sets are sets in set theory in which the number of elements is the same & all elements are equal. It is a concept of set equality.