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Assignment 2: Matrix - I

Q1.

a) If $A = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$, Prove that $A^2 = I$

Ans. $A^2 = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix} \begin{bmatrix} 0 & 1 & -1 \\ 4 & -3 & 4 \\ 3 & -3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 0+4-3 & 0-3+3 & 0+4-4 \\ 0-12+12 & 4+9-12 & -4-12+16 \\ 0-12+12 & 3+9-12 & -3-12+16 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (I)}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{--- (II)}$$

By equation (I) & (II), Hence prove that $A^2 = I$.

b) Find x & y if

$$\left\{ 4 \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 & -1 \\ 2 & -3 & 4 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

Ans. $\left\{ 4 \begin{bmatrix} 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix} - 2 \begin{bmatrix} 1 & 3 & -1 \\ 2 & -3 & 4 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$

$$= \left\{ \begin{bmatrix} 4 & 8 & 0 \\ 8 & -4 & 12 \end{bmatrix} - \begin{bmatrix} 2 & 6 & -2 \\ 4 & -6 & 8 \end{bmatrix} \right\} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 & 2 \\ 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$\therefore \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $\qquad \therefore x = 2$ & $y = 4$

c) If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$ verify $(AB)^T = B^T A^T$

Ans. $AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$

$$(AB)^T = \begin{bmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix} \text{ —— (I)}$$

$$A^T = \begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix}, \quad B^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix}$$

$$B^T A^T = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 3 & 4 \\ 2 & 0 & 5 \\ -1 & 2 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 3 & 14 \\ 1 & 2 & 5 \\ -3 & 6 & 0 \end{bmatrix} \text{ —— (II)}$$

By equation (I) & (II), Hence prove that

$$(AB)^T = B^T A^T$$

d) If, $A = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ -3 & 0 \end{bmatrix}$, find x such that

$$2X + 3A - 4B = I$$

Ans $3A = \begin{bmatrix} 9 & -3 \\ 6 & 12 \end{bmatrix}$, $4B = \begin{bmatrix} 4 & 8 \\ -12 & 0 \end{bmatrix}$

$3A - 4B = \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix}$, $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore 2x + \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore 2x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 5 & -11 \\ 18 & 12 \end{bmatrix}$

$\therefore 2x = \begin{bmatrix} \cancel{-2}\ -4 & 11 \\ -18 & -11 \end{bmatrix}$

$\therefore x = \begin{bmatrix} -2 & 11/2 \\ -9 & -11/2 \end{bmatrix}$

Q2. Find the inverse of the matrix using Adjoint method

a) $A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{bmatrix}$

Ans. $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 1 & 1 & 5 \\ 2 & 4 & 7 \end{vmatrix} = 1(7-20) - 2(7-10) + 3(4-2)$

$= -13 + 6 + 6$

$= -1$

$|A| \neq 0$, So, A^{-1} is Exist.

$C_{11} = (-1)^{1+1} \times \begin{vmatrix} 1 & 5 \\ 4 & 7 \end{vmatrix}$ $\quad C_{12} = (-1)^{1+2} \times \begin{vmatrix} 1 & 5 \\ 2 & 7 \end{vmatrix}$ $\quad C_{13} = (-1)^{1+3} \times \begin{vmatrix} 1 & 1 \\ 2 & 4 \end{vmatrix}$

$C_{11} = 7-20$ $\quad C_{12} = (-1)(7-10)$ $\quad C_{13} = (4-2)$

$C_{11} = -13$ $\quad C_{12} = 3$ $\quad C_{13} = 2$

$C_{21} = -2$ $\quad C_{22} = 1$ $\quad C_{23} = 0$

$C_{31} = 7$ $\quad C_{32} = -2$ $\quad C_{33} = -1$

Cofactor Matrix $= \begin{bmatrix} -13 & 3 & 2 \\ -2 & 1 & 0 \\ 7 & -2 & -1 \end{bmatrix}$

Adj A = Transpose of the Cofactor Marix

$= \begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$

Now inverse of A is $A^{-1} = \frac{1}{|A|} \times \text{Adj } A$

$= \frac{1}{-1} \begin{bmatrix} -13 & -2 & 7 \\ 3 & 1 & -2 \\ 2 & 0 & -1 \end{bmatrix}$

$\therefore A^{-1} = \begin{bmatrix} 13 & 2 & -7 \\ -3 & -1 & 2 \\ -2 & 0 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

Ans. $|A| = \begin{vmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{vmatrix} = 1(2-6) + 0 + 1(0-2)$

$= -4 - 2$

$= -6$

$\therefore |A| \neq 0 \quad \therefore A^{-1}$ is Exist.

$C_{11} = (-1)^{1+1} \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix}$ $C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix}$ $C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 2 \\ 1 & 2 \end{vmatrix}$

$C_{11} = -4$ $C_{12} = 3$ $C_{13} = -2$

$C_{21} = 2$ $C_{22} = 0$ $C_{23} = -2$

$C_{31} = -2$ $C_{32} = -3$ $C_{33} = 2$

$$\text{Cofactor Matrix} = \begin{bmatrix} -4 & 3 & -2 \\ 2 & 0 & -2 \\ -2 & -3 & 2 \end{bmatrix}$$

$$\text{Adj}\cdot A = \text{Transpose of Cofactor Matrix}$$
$$= \begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} \times \text{Adj}A$

$$\therefore A^{-1} = \frac{1}{-6} \times \begin{bmatrix} -4 & 2 & -2 \\ 3 & 0 & -3 \\ -2 & -2 & 2 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 2/3 & -1/3 & 1/3 \\ -1/2 & 0 & 1/2 \\ 1/3 & 1/3 & -1/3 \end{bmatrix}$$

Q3. Find the inverse of the following matrices using elementary row transformations.

a) $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Ans. $AA^{-1} = I$

$$\therefore \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 6 \\ 2 & 2 & 5 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 3R_1 \ , \ R_3 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -3 & 0 \\ 1 & -2 & 1 \end{bmatrix}$$

$R_1 + R_2 \ , \ (-1) R_2$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$R_1 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$\therefore \ A^{-1} = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

b) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix}$

Ans. $AA^{-1} = I$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 + R_1 \ , \ R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$3R_1 - 2R_2$

$$\begin{bmatrix} 3 & 0 & -1 \\ 0 & 3 & 5 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

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$R_1 \times \frac{1}{3}$, $R_2 \times \frac{1}{3}$

$$\begin{bmatrix} 1 & 0 & -1/3 \\ 0 & 1 & 5/3 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 1/3 & -2/3 & 0 \\ 1/3 & 1/3 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$R_1 + \frac{1}{3} R_3$, $R_2 - \frac{5}{3} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A^{-1} = \begin{bmatrix} 0 & -2/3 & 1/3 \\ 2 & 1/3 & -5/3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 0 & -2/3 & 1/3 \\ 2 & 1/3 & -5/3 \\ -1 & 0 & 1 \end{bmatrix}$$