We define $P(L,n) := j$ such that

2^j is the n^{th} smallest value whose digit in base 10 starts with L.

We want $P(123, 678910)$. Let the answer be m, hence

$$2^m = 123\ldots \tag{1}$$

(some digits)
m is the 678910^{th} smallest number.
Taking log on both sides in (1), we get

$$m \log 2 = \log 123\ldots$$

$$= n + \log 1.23\ldots$$

n is the integer part of $\log 123\ldots$.

$$\lfloor m \log 2 \rfloor \text{ is also the integer part of } \log 123\ldots$$

Hence

$$n = \lfloor m \log 2 \rfloor$$

$$m \log 2 - \lfloor m \log 2 \rfloor = \log 1.23\ldots$$

$$\log 1.23 \leq \log 1.23\ldots < \log 1.24$$

Hence

$$\log 1.23 \leq m \log 2 - \lfloor m \log 2 \rfloor < \log 1.24$$

We can iterate over m finding the values for which the last condition satisfies we can use in built method for calculating the log values and double to store the fractional result there will be round of errors but are negligible for this case the C code implementing this is given in this file.