$\begin{array}{c} \rm UMC\text{-}202 \\ \rm PROBLEM \ SET \ 4 \end{array}$

(1) Use rectangular rule and midpoint rule to evaluate the integral

$$\int_{1}^{5} \sqrt{1+x^2} \ dx$$

(2) Approximate the following integrals using the Trapezoidal rule and Simpson's rule.

(a)
$$\int_0^{0.5} \frac{2}{(x-4)} dx$$
,

(b)
$$\int_0^{\pi/4} e^{3x} \sin(2x) dx$$
.

(3) Use the Composite Trapezoidal rule and Composite Simpson's rule with the indicated values of n to approximate the following integrals

(a)
$$\int_{-2}^{2} x^3 e^x dx$$
, $n = 4$.

(b)
$$\int_0^{3\pi/8} \tan(x) dx \ n = 8.$$

(4) Evaluate the following integral by using one point Gauss quadrature and compute the true error.

$$\int_0^{\pi/4} x^2 \sin x \ dx$$

(5) Redo Problem 11 by using two point Gauss quadrature formula.

(6) Use Composite Simpson's rule with n=4 and m=2 to approximate

$$\int_{1.4}^{2.0} \int_{1.0}^{1.5} \ln(x+2y) \, dy \, dx.$$

(7) Redo Problem 6 using the Gaussian quadrature formula with $n=1,\,m=2$ in both dimensions.