UMC 202
PROBLEM SET 9

(1) Use Gaussian elimination with backward substitution with tolerance 10^{-2} to solve the following linear system

$$\begin{aligned} 4x_1 - x_2 + x_3 &= 8, \\ 2x_1 + 5x_2 + 2x_3 &= 3, \\ x_1 + 2x_2 + 4x_3 &= 11. \end{aligned}$$

The exact solution of the system is $x_1 = 1$, $x_2 = -1$, $x_3 = 3$.

(2) The following linear system

$$\begin{aligned} 10x_1 - x_2 + 2x_3 &= 6, \\ -x_1 + 11x_2 - x_3 + 3x_4 &= 25, \\ 2x_1 - x_2 + 10x_3 - x_4 &= -11, \\ 3x_2 - x_3 + 8x_4 &= 15 \end{aligned}$$

has the unique solution $x = (1, 2, -1, 1)^T$. Use Gauss Jacobi's iterative technique to find the approximations $x^{(k)}$ to x with $x^0 = (0, 0, 0, 0)^T$ until

$$\frac{\|x^{(k)} - x^{(k-1)}\|_\infty}{\|x^{(k)}\|_\infty} < 10^{-3}$$

where $\|x\|_\infty = \max_{1 \le j \le 4} |x_j|$.

(3) Use the linear finite difference algorithm with $N = 9$ to approximate the solution to

$$y'' = \frac{-2}{x} y' + \frac{2}{x^2} y + \frac{sin(ln\, x)}{x^2}, \text{ for } 1 \le x \le 2,$$
$$\text{with } y(1) = 1 \text{ and } y(2) = 2,$$

Use Gauss-Jacobi and Gauss-Seidel methods to solve the resulting linear system and compare the results in both cases to the exact solution

$$y = c_1\, x + \frac{c_2}{x^2} - \frac{3}{10} sin(ln\, x) - \frac{1}{10} cos(ln\, x),$$

where $c_1 = 1.13921$ and $c_2 = -0.03921$.