

UMC 202
PROBLEM SET 5

- (1) Use the forward difference formula to approximate the derivative of $f(x) = \ln x$ at $x_0 = 1.8$ using $h = 0.1$, $h = 0.05$ and $h = 0.01$. Determine the bounds for the approximation errors.
- (2) Redo problem 1 for the backward difference formula and central difference formula.
- (3) Redo problem 1 for the three point endpoint formula and three point mid-point formula.

- (4) Consider the IVP

$$y' = t e^{3t} - 2y, \quad 0 \leq t \leq 1, \quad y(0) = 0 \text{ with } h = 0.5.$$

Use Euler's method to obtain the approximation to $y(1)$. The actual solution to the IVP is

$$y(t) = \frac{1}{5} t e^{3t} - \frac{1}{25} e^{3t} + \frac{1}{25} e^{-2t},$$

compare the actual error at each step to the error bound.

- (5) Consider the IVP

$$y' = 1 + \frac{y}{t}, \quad 1 \leq t \leq 2, \quad y(1) = 2.$$

Use Euler's method with $h = 0.25$ to obtain the approximation to $y(2)$. The actual solution to the IVP is

$$y(t) = t \ln t + 2t,$$

compare the actual error at each step to the error bound.