Algorithmic Game Theory (**CS60025)**

PROBLEM TITLE: Strategy proofness in large

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Abstract:

We propose strategy-proofness in the large (SP-L), an approximate incentive compatibility criterion, and argue that it is a useful second-best to exact strategy-proofness (SP) for market design. From a theoretical point of view SP-L demands that an agent have a dominant strategy to report truthfully if she sees a mechanism's "prices" as exogenous to her report, whether they are traditional prices in an auction mechanism or price-like statistics in an assignment or matching mechanism. In big, two-sided matching markets, there is a strong approximation of strategy-proofness. Simulations show that as the market size expands, the fraction of agents with useful deviations diminishes. Furthermore, there appears to be a significant relationship between the length of preference order lists, the correlation of agent preferences, and strategy proofness approximation.

1 Introduction

In large matching markets with a small fraction of agents who may advantageously deviate, approximate strategy-proofness is seen. Roth and Elliot, 1990[1] wrote a study report on this topic. It demonstrates how the labor-market matching of newly graduated physicians and hospitals benefits from the discovery that "opportunities for strategic manipulations are surprisingly small," i.e. how approximate strategy proofness makes their matching algorithm more robust in a large market. Clearly, the consequences of the characteristic may be crucial in many circumstances when considering how to design big matching markets. For example, by allowing market designers to trust agents to report truthfully despite the fact that the DA mechanism is not technically strategy-proof.

We are conducting several simulations in this study to see how and when one may approximate strategy proofness. Furthermore, we intend to simulate various levels of correlation between the market preferences of the actors. We intend to gain fresh insights into when approximation strategy proofness may consistently minimize system gaming, and for what market size, length of preference lists, and preference correlation it looks effective to a lower extent.

Our idea is that for sufficiently big markets, we will see near strategy-proofness, and that the duration of the stated preference ordering and the amount of correlation

2 Theory

Our study is based on several pieces of existing research. The following is a list of theorems with a description of how they apply to our paper:

- *Simplicity, Dynamic Stability, and Robustness are some of many wanted properties of strategy-proofness* (Lubin and Parkes, 2012[2]) Thus, we want to have a strategy-proof mechanism design

- *Theorem 12.5 Truthful reporting is a dominant strategy for students in the student-proposing DA mechanism.* (Parkes and Seuken, 2019[3])

- *Theorem 12.7 No mechanism for two-sided matching is both stable and strategy-proof* (Parkes and Seuken, 2019[3])

  Knowing that deferred acceptance returns a stable matching, and that the proposing side has truthful reporting as its dominant strategy, we can deduce that in general there exists useful deviations, and that these will only exist for the agents that are being proposed to.

will have an effect on the approximation to strategy-proofness. Thus, we only have to look for deviations among agents being proposed to.

- *In simple markets, all successful manipulations can [also] be accomplished by truncations* (Roth and Peranson, 1999[1])

  This finding will be extremely helpful in the construction of our simulation. Instead of searching for k! possible permutations of preferences, it is sufficient for preference orders of length k to truncate the preference order once at each location for each participant on the receiving side. This makes a difficult task manageable. As a result, we simply need to look for truncation errors.

Strategy-proofness was described as desired in the theorems. Furthermore, we may infer that looking for deviations exclusively in the form of truncation on the receiving side of the market can be done without losing generality in the search for approximated strategy proofness. This greatly decreases computational complexity.

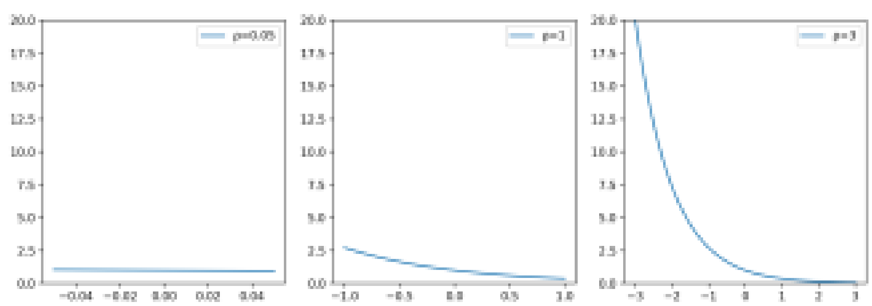

Figure 1: Plots of the three probability distributions we used for modeling different levels of correlation between agents' preferences. Left is the uncorrelated distribution, middle is the moderate, and right is the high correlation.

3 Method

In summary, our simulation simulates matching markets for a particular degree of preference correlation and duration of preference orders, progressively increasing market size, and counting the number of agents who may effectively deviate for each market size.

Our algorithm can be divided into three parts: *(i) Preference Generation*, *(ii) Deferred Acceptance*, *(iii) Counting Deviations*.

3.1 Preference Generation

To ensure the validity of our results, we must realistically model agent preferences in a way that resembles the distribution of preferences in a real market. We must remember that some market participants are popular among all agents on the other side, for example, some men are usually attractive in most women's eyes, making them more likely to appear high on women's hypothetical preference orderings. We found that a distribution defined by the inverse exponential function would give us a fitting representation. We model three specific market scenarios with differing levels of correlation between agents' preferences: (1) almost no correlation, (2) moderate correlation, and (3) high correlation. In figure 1, one can see a plot of the three different distributions used, respectively.

Based on these distributions, we created preference orderings for one side of the market, where they were given a k-length preference list drawn at random from the distributions described above. Because it makes no sense for a hospital, for example, to have preferences over students they have not interviewed, we decided to base the proposal-receiving side's preferences on the proposers' preferences. This will also ensure that we have the highest possible match ratio. We allowed the proposal receiving side to have preferences of varying lengths.

3.2 Deferred Acceptance

We used the postponed acceptance approach, which is known to create stable matches, to locate stable matches. This method reads incomplete preference orders as the agent prefers to be unmatched than being matched by agents not on their list. E.g. suppose *m*'s preference over f_1, f_2, f_3 given as

Algorithm 1 Deferred Acceptance

There are 5 males who have one specific female among their preferences, that female will have a preference ordering of length 5. With everyone's preferences in order we are ready to proceed to the actual matching.

$f_1 \;_m f_3$. Then this will be interpreted as $f_1 \;_m f_3 \;_m \varnothing \;_m f_2$. The following is the pseudo code of our deferred acceptance algorithm. We had to implement this algorithm from the bottom up, instead of using libraries, because most existing code did not handle truncations and incomplete preference orders.

```
Initialize p in proposers and r in recipients unmatched while exists p
that is unmatched and has someone yet to propose to do for p in
unmatched proposers do
    r = most preferred r which p has not proposed yet
    if p is not in r's preference list then
      break
    else if r is unmatched then
      (p, r) become matched
    else
      if r prefers p to current match then
        (p, r) become matched
        r's previous match p' becomes unmatched
      else
        no change
      end if
    end if
  end for
end while
```

3.3 Counting Deviations

This study attempts to establish a relationship between D(n), i.e. the number of agents with meaningful deviations, and n =, the number of agents on one side of the market. The computation of D is described in Section 3.3. (n).

According to Section 2 Theory, the only way for helpful deviations for stable matchings created by a given mechanism is incorrect reporting in the form of truncation deviation by the receiving side of the market. In this part, we refer to a receiver as a female for convenience (without intention of being hetero-normative). When our simulation returns a stable matching based truthful reports (given a preference generation dependent on ρ, k, and n) we analyze this matching in the following way:

1. While there are females not

analyzed,select a female.

2. Given a female, truncate her last preference order entry.
3. Based on this new preference order, run deferred acceptance and check if this female benefits.

4. Repeat step 2 and 3 until deviation is found (if so set $D(n) = D(n) + 1$) or preference list is of length one (in this case mark fe male as analyzed and go to step 1).

We are counting how many females have at least one useful mis report. The $D(n)$ returned will be logged with the corresponding n = market size, k = preference length and ρ = preference correlation. Giving us our final results.

4 Results

In this section we briefly present the results from our extensive simulations in the form of a series of plots. This required 300+ CPU-hours.

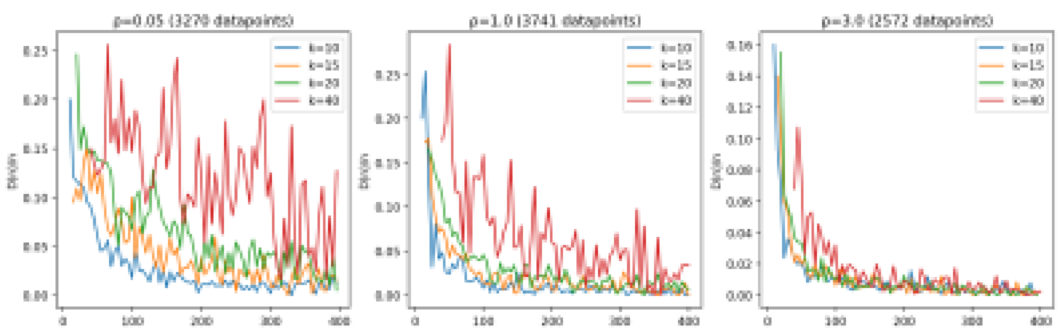

Figure 2: Simulations for ρ = 0.05, 1.0, 3.0 (from left to right) each with k = 10, 15, 20, 40.

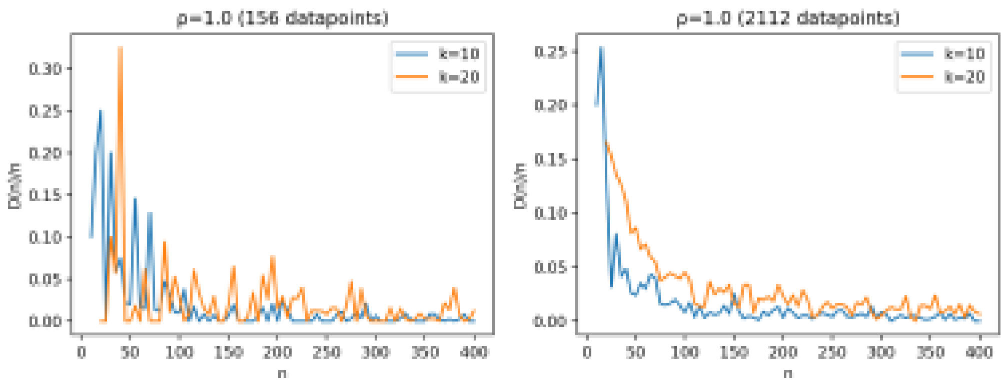

Figure 3: Plot for 156 data points to the left and 2112 data points to the right. With increasing data density, curves converge towards clearer separation.

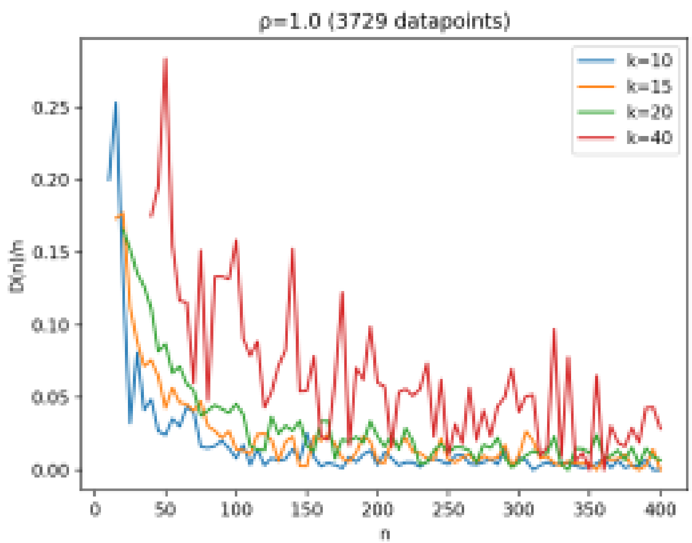

Figure 4: Plot for ρ = 1.0 and for multiple values of k. Observe that longer preference orderings tend to make the opportunities for strategic behavior larger.

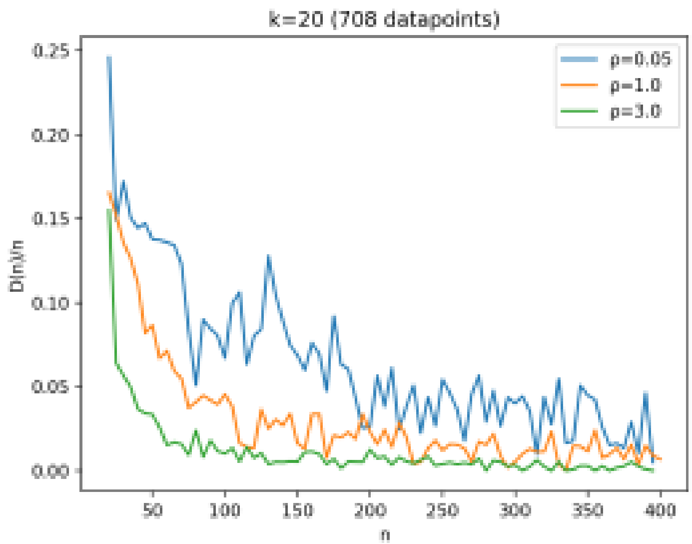

Figure 5: Plot for k = 20 for a range of different ρ. Observe that correlation between agents' preferences tend to increase the amount of opportunities for strategic behavior.

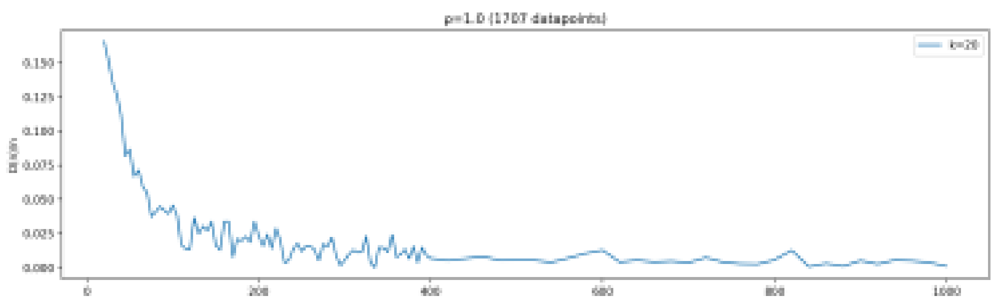

Figure 6: Plot for k = 20, ρ = 1, for markets of size up to n = 1000. We assume that other values for k and ρ will produce proportional results in the limit.

5 Discussion

5.1 The General Findings

There are two types of simulations carried out. The first is for four different preference lengths, k = 10, 15, 20, 40, and three preference correlation amounts, = 0.05, 1.0, 3.0, with a market size n ranging from 5 to 400. This shows how different k affects the approximate strategy proofness.The second type shows how, as the market grows even larger (up to n = 1000), $D(n)/n$ converges to 0 (figure 6). We show this while keeping k and ρ constant at 20 and 1.0, respectively. Because we lack the resources to run the same simulation for the other values of k and ρ, we chose k = 20 and ρ = 1.0 as representative values. We assume that the other curves would also tend towards 0.

In summary, our simulation simulates matching markets for a particular degree of preference correlation and duration of preference orders, progressively increasing market size, and counting the number of agents who may effectively deviate for each market size.

As a general conclusion, the results show that when markets grow from n = 10 to around n = 100, they rapidly approach strategy-proofness. In fact, given a large n, the percentage of agents with useful deviations drops from around 16% to 0.01%. As the market expands, the proportion of agents with useful deviations approaches zero, but the rate of convergence slows.

Our last image demonstrates how, as the number of simulation runs increases, the data plot gets smoother, with fewer random spikes in D(n)/n. Because the development of the preference ordering is largely reliant on randomness, having data from repeated simulations gives us model results that gradually approach an average value. Because our conclusions are based on a large number of various types of matching market circumstances, the random-based algorithm design makes them more easily adaptable to the actual world. The plot progressively approaches a monotone decreasing function. As a result, we argue that approximation strategy proofness occurs in big markets. We cannot, however, always rule out the prospect of some actor deviating from the norm.

5.2 Effect of Varying Preference-Order Length

In figure 2 we see the three plots: k = 10 in blue, k = 15 in orange, and k = 20 in green. The important thing to note is that in general the plots of $D(n)/n$ for a given n increases with k. Intuitively this makes sense as a larger k quite simply gives more preferences to truncate away, i.e. more possible deviations. For a market designer, the general advice would be that a smaller preference reporting allowed restricts the amount of beneficial mis reports available for all agents in total.

5.3 Effect of Varying the Preference Correlation

We note that an increasing correlation, ρ (ρ=0.05, ρ=1 and ρ=3), leads to a smaller ratio of agents who can fully deviate ($D(n)/n$). This allows market designers with a knowledge of low correlation to expand the market to secure approximate strategy proofness. As for the reason why stronger correlation leads to less possibility for useful deviation, having more agents interested in the same matches makes the rejection chain shorter, thus making it unlikely to return to the deviator.

6 Applications and Improvements

As discussed, a matching market designer has to choose between a stable and a strategy proof matching. Parkes and Seuken (2019[3]) argue that unstable markets have a tendency to un-ravel (possibly with catastrophic consequences) which makes stability an arguably more important property than strategy-proofness. However, as shown in this paper, large markets can gain approximate strategy-proofness while maintaining the stability of the outcome.

In DA, the proposal-receiving side gets their least preferred, achievable match. In some cases it might actually be more important for organizing parties to get the better matching. An example could be a market trying to match medical students to residencies at hospitals. It is potentially better if the hospital-optimal matching was achieved instead of the student optimal one. This is because the needs for students with specific talent and knowledge at the hospitals might be more important for the society at large the individual wants of the students.

The results in this paper are likely applicable to the real world. We have seen that in two-sided matching markets it quickly becomes difficult to find useful deviations for the receiving side. This effect is particularly evident when we have relatively short preference orderings and a high correlation between the preferences of the agents in the market. Both of these assumptions we think are reasonable in real-world applications.

As a familiar instance, Norway has a coordinated admissions system for almost every institution of higher education, *Samordna opptak*. There, students are ranked according to their high school grades, and students are allowed to create a strict preference order over at most 10 schools. Here we see that every agent on the proposing side has a preference ordering of length at most 10, while every recipient has a preference ordering over all students who have them on their preference ordering. Furthermore, we know from open statistics that a select few schools are extremely popular, while many schools are not as popular and struggle to fill up their classes. This effect is probably common, and is well captured by our medium or high correlation distribution. These observations, together with the fact that the market is very large (tens of thousands of agents), make us believe that this market would potentially reach strong approximate strategy-proofness.

As a last point, we want to argue that even though our simulation is implemented with an even-sized pool of proposers and proposed agents, a higher ρ models a matching market with different sized sides. When ρ is large almost all proposed agents are interested in a select few proposers. This is close to a real world situation where there are few proposers compared to the proposed side. As such, an increasing ρ could be used to model a situation with an increasingly large amount of agents on the proposed side of the market relative to the proposers.

7 Potential Issues and Next Steps

The following are some points of issues and possible future implementations:

- Our simulation assumes an equal number of proposers and recipients and does not account for situations in which there are more pros than recipients.
- We would benefit greatly from having a more highly optimized algorithm if we used our code to model more diverse markets.
- We have modeled one-to-one two sided matching market, but There are many other forms of matching that might require analysis. They include matching with couples, one-to-many matchings, and many-to-many matchings. Next step would be to expand our model to include these different matching markets.
- The existence of approximate strategy proofness in large two-sided matching markets has been empirically demonstrated in this paper. The next step would be to provide theoretical evidence for how and why this property occurs in large markets.

8 Conclusion

We discovered that near strategy-proofness is common in large marketplaces. According to our models, the fraction of agents with relevant deviations declines the most rapidly between market sizes of 10 and 100. The approximate strategy-proofness decreases with preference order length and increases with preference correlation. Finally, there are a number of real-world applications for these findings, such as the Norwegian university application process. There are still unresolved issues that could be addressed in future research.

References

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[2] Benjamin Lubin and David C. Parkes. *Approximate Strategyproofness*. https://dash.harvard.edu/bitstream/handle/1/11879945/Parkes_ApproximateStrategyproofness.pdf

[3] David C. Parkes and Sven Seuken *Algorithmic Economics: A Design Ap proach*. Harvard University/University of Zurich, 2019.

[4] Alvin E. Roth. *The economics of matching: Stability and incentives.* Math ematics of Operations Research, pages 617–628, 1982.