CS770 Machine Learning

Assignment1: Linear Regression, Ridge& lasso Regression

09/27/2024

Submitted by,

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Introduction

Linear regression is a type of supervised machine learning algorithm.

1. Simple linear regression: This involves predicting a dependent variable based on a single independent variable.

The equation of the regression line is represented as:

$$h(xi)=\beta 0+\beta 1xi$$

Here,

- h(x_i) represents the **predicted response value** for ith observation.
- b_0 and b_1 are regression coefficients and represent the **y-intercept** and **slope** of the regression line respectively.

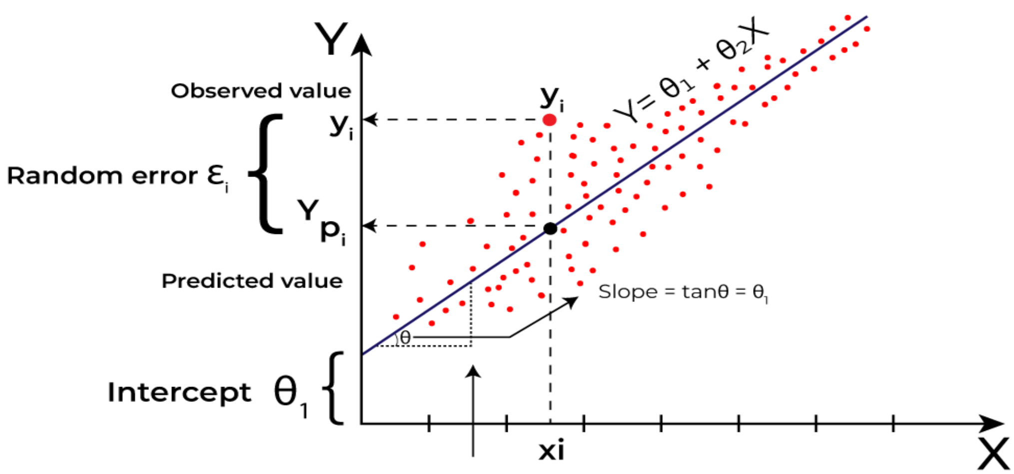
2. Multivariate linear regression: This involves predicting a dependent variable based on multiple independent variables.

$$y = \beta_0 + \beta_1 X1 + \beta_2 X2 + \ldots\ldots\ldots \beta_n Xn$$

where:

- Y is the dependent variable
- X1, X2, ..., Xn are the independent variables
- β0 is the intercept
- β1, β2, ..., βn are the slopes

The **Best Fit Line** equation provides a straight line that represents the relationship between the dependent and independent variables. The slope of the line indicates how much the dependent variable changes for a unit change in the independent variable(s).



Here Y is called the target variable and X is known as the predictor of Y(independent variable)

$\hat{Y} = \theta_1 + \theta_2 X$

OR

$\hat{y}_i = \theta_1 + \theta_2 x_i$

Here,

- $y_i \epsilon Y \quad (i = 1, 2, \cdots, n)$ are labels to data (Supervised learning)
- $x_i \epsilon X \quad (i = 1, 2, \cdots, n)$ are the input independent training data (univariate – one input variable(parameter))
- $\hat{y}_i \epsilon \hat{Y} \quad (i = 1, 2, \cdots, n)$ are the predicted values.

The model gets the best regression fit line by finding the best θ_1 and θ_2 values.

- **θ_1:** intercept
- **θ_2:** coefficient of x

To achieve the best-fit regression line, it is very important to update the θ1 and θ2 values, to reach the best value that minimizes the error between the predicted y value (pred) and the true y value (y).

$$minimize \frac{1}{n} \sum_{i=1}^{n} (\hat{y_i} - y_i)^2$$

Cost function for Linear Regression

The cost function or the loss function is nothing but the error or difference between the predicted value and the true value.

$$\text{Cost function}(J) = \frac{1}{n} \sum_{n}^{i} (\hat{y_i} - y_i)^2$$

where,

$$\hat{y}_i = \theta_1 + \theta_2 x_i.$$

Gradient Descent Algorithm

A linear regression model can be trained using the optimization algorithm gradient descent by iteratively modifying the model's parameters to reduce the mean squared error (MSE) of the model on a training dataset.

To update θ1 and θ2 values in order to reduce the Cost function (minimizing RMSE value) and achieve the best-fit line the model uses Gradient Descent.

After differentiating w.r.t θ1 and θ2, final values:

$$J'_{\theta_1} = \frac{\partial J(\theta_1, \theta_2)}{\partial \theta_1} \qquad J'_{\theta_2} = \frac{\partial J(\theta_1, \theta_2)}{\partial \theta_2}$$

$$\begin{aligned} \theta_1 &= \theta_1 - \alpha \left(J'_{\theta_1} \right) \\ &= \theta_1 - \alpha \left(\frac{2}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i) \right) \\ \theta_2 &= \theta_2 - \alpha \left(J'_{\theta_2} \right) \\ &= \theta_2 - \alpha \left(\frac{2}{n} \sum_{i=1}^{n} (\hat{y}_i - y_i) \cdot x_i \right) \end{aligned}$$

Evaluation Metrics for Linear Regression

Mean Square Error (MSE)

Mean Squared Error (MSE) calculates the average of the squared differences between the actual and predicted values.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(y_i - \widehat{y_i})^2$$

Here,

- n is the number of data points.
- y_i is the actual or observed value for the i^{th} data point.
- $\widehat{y_i}$ is the predicted value for the i^{th} data point.

Root Mean Squared Error (RMSE)

It describes how well the observed data points match the expected values, or the model's absolute fit to the data.

In mathematical notation:

$$RMSE = \sqrt{\frac{RSS}{n}} = \sqrt{\frac{\sum_{i=2}^{n}(y_i^{actual} - y_i^{predicted})^2}{(n-2)}}$$

R-Squared

It is a statistical measure which represents the goodness of fit of a regression model with value from 0 to 1. It is also called as coefficient of determination.

Mathematically,

$$R^2 = 1 - \left(\frac{RSS}{TSS}\right)$$

RSS - Residual sum of Squares

$$RSS = \sum_{i=2}^{n}(y_i - b_0 - b_1 x_i)^2$$

TSS- Total Sum of Squares

$$TSS = \sum(y - \overline{y_i})^2$$

Methods

Data Preparation

1. The load_boston dataset was **loaded** from sklearn.datasets.
2. **Exploratory Data Analysis on given Dataset**: Initial exploration was done which includes
   - Checking for missing values
   - Filling Missing values with mean, median, mode
   - Finding and handling outliers
   - Understanding the distribution of values of different independent feature and dependent target values with different plots
   - Visualizing relationships between feature and target variable with different plots
3. **Feature Selection**: With findings of correlation value features having more than 0.5 were selected:
   - RM

- LSTAT
- PTRATIO

4. **Target Variable**: Here, the MEDV is the target variable.

After data preparation, the following steps should be taken:

Regression Models Building Steps

1. **Split the Data into Training and Testing data**

   Divide randomly the dataset into two sets: training sets and testing sets. 25 percent of data will be used as test data or target value and 75 percent of data will be used as training data or input features value to predict the target value which mean test_size = 0.25

2. **Fit the dataset into the Linear Regression Model**
   - **Model Initialization**
   - **Model Training**

3. **Evaluation of Model**
   - **Predictions**
   - **Calculation of r-square value**
   - **Calculation of MSE value**

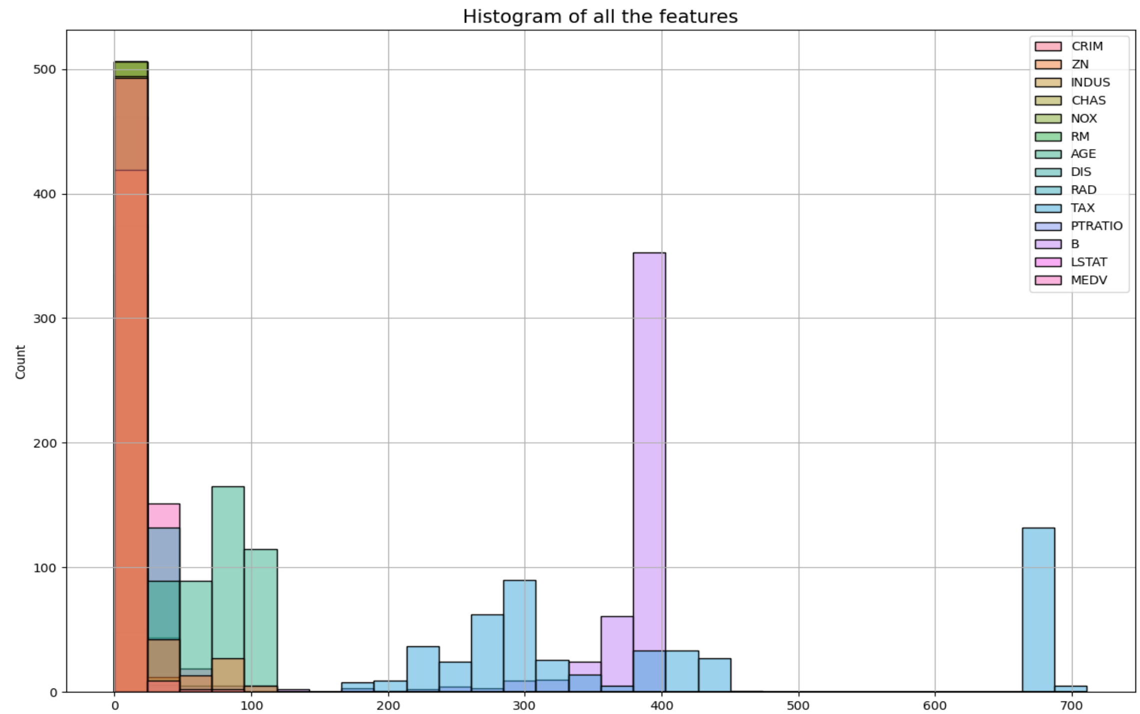
4. Visualization

A scatter plot was generated to compare the actual price of house against the predicted price from the different types of linear regression model.

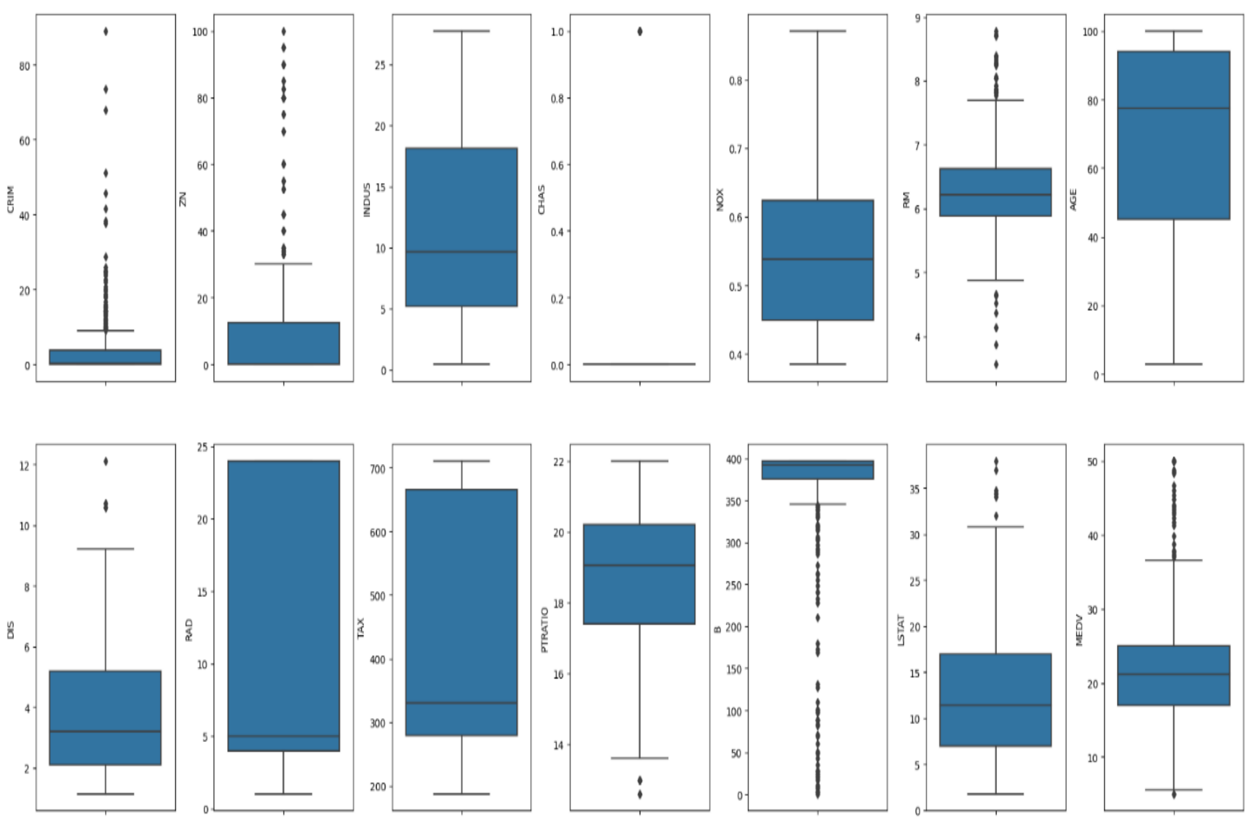
1. BOSTON HOUSING DATASET

DATA VISUALISATION

1. Histogram of all features

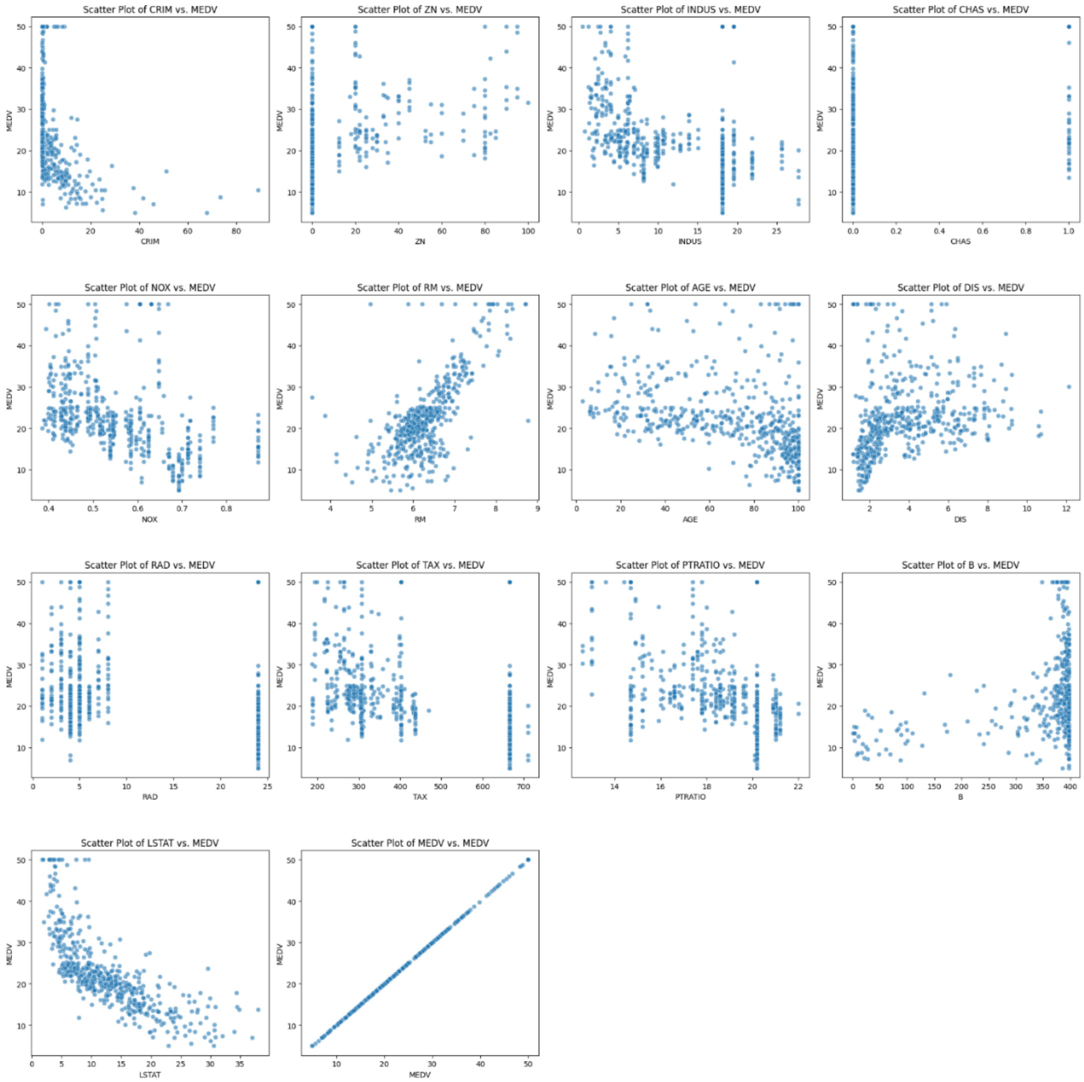

Histogram of all the features
Count
CRIM
ZN
INDUS
CHAS
NOX
RM
AGE
DIS
RAD
TAX
PTRATIO
B
LSTAT
MEDV

2. Boxplot

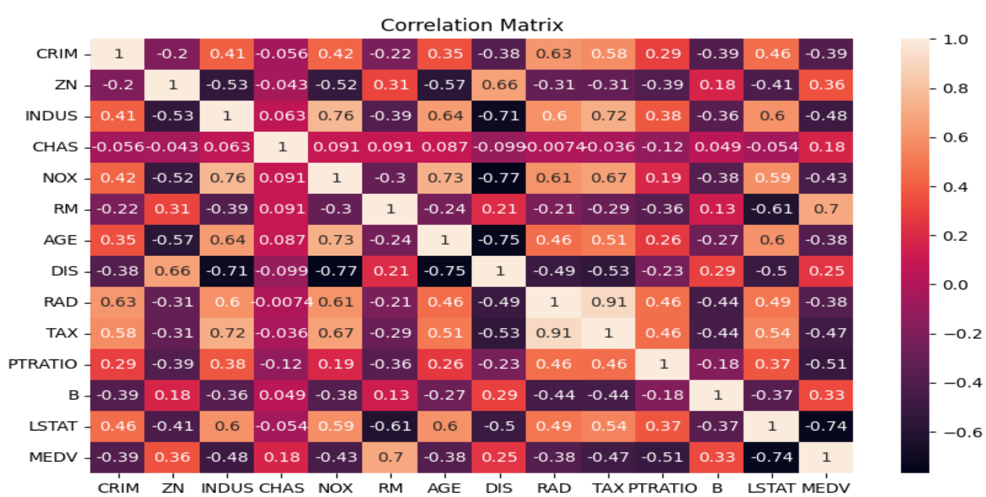

CRIM
ZN
INDUS
CHAS
NOX
RM
AGE
DIS
RAD
TAX
PTRATIO
B
LSTAT
MEDV

3. Scatter Plot

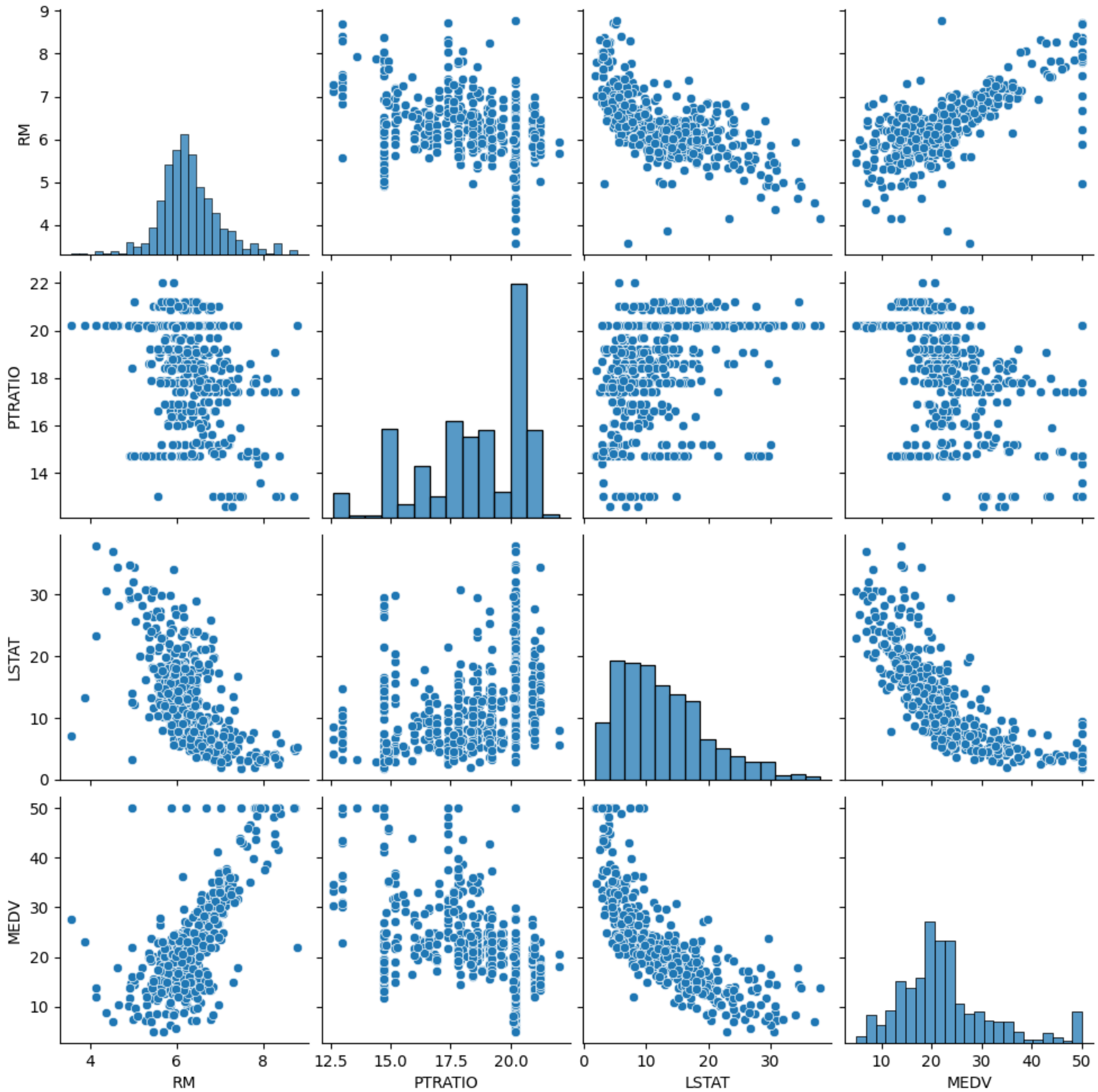
4. Scatter Plot against MEDV


Scatter Plot of CRIM vs. MEDV
Scatter Plot of ZN vs. MEDV
Scatter Plot of INDUS vs. MEDV
Scatter Plot of CHAS vs. MEDV
Scatter Plot of NOX vs. MEDV
Scatter Plot of RM vs. MEDV
Scatter Plot of AGE vs. MEDV
Scatter Plot of DIS vs. MEDV
Scatter Plot of RAD vs. MEDV
Scatter Plot of TAX vs. MEDV
Scatter Plot of PTRATIO vs. MEDV
Scatter Plot of B vs. MEDV
Scatter Plot of LSTAT vs. MEDV
Scatter Plot of MEDV vs. MEDV
MEDV

5. Heatmap


Correlation Matrix
CRIM 1 -0.2 0.41 -0.056 0.42 -0.22 0.35 -0.38 0.63 0.58 0.29 -0.39 0.46 -0.39
ZN -0.2 1 -0.53 -0.043 -0.52 0.31 -0.57 0.66 -0.31 -0.31 -0.39 0.18 -0.41 0.36
INDUS 0.41 -0.53 1 0.063 0.76 -0.39 0.64 -0.71 0.6 0.72 0.38 -0.36 0.6 -0.48
CHAS -0.056 -0.043 0.063 1 0.091 0.091 0.087 -0.099 -0.0074 -0.036 -0.12 0.049 -0.054 0.18
NOX 0.42 -0.52 0.76 0.091 1 -0.3 0.73 -0.77 0.61 0.67 0.19 -0.38 0.59 -0.43
RM -0.22 0.31 -0.39 0.091 -0.3 1 -0.24 0.21 -0.21 -0.29 -0.36 0.13 -0.61 0.7
AGE 0.35 -0.57 0.64 0.087 0.73 -0.24 1 -0.75 0.46 0.51 0.26 -0.27 0.6 -0.38
DIS -0.38 0.66 -0.71 -0.099 -0.77 0.21 -0.75 1 -0.49 -0.53 -0.23 0.29 -0.5 0.25
RAD 0.63 -0.31 0.6 -0.0074 0.61 -0.21 0.46 -0.49 1 0.91 0.46 -0.44 0.49 -0.38
TAX 0.58 -0.31 0.72 -0.036 0.67 -0.29 0.51 -0.53 0.91 1 0.46 -0.44 0.54 -0.47
PTRATIO 0.29 -0.39 0.38 -0.12 0.19 -0.36 0.26 -0.23 0.46 0.46 1 -0.18 0.37 -0.51
B -0.39 0.18 -0.36 0.049 -0.38 0.13 -0.27 0.29 -0.44 -0.44 -0.18 1 -0.37 0.33
LSTAT 0.46 -0.41 0.6 -0.054 0.59 -0.61 0.6 -0.5 0.49 0.54 0.37 -0.37 1 -0.74
MEDV -0.39 0.36 -0.48 0.18 -0.43 0.7 -0.38 0.25 -0.38 -0.47 -0.51 0.33 -0.74 1
CRIM ZN INDUS CHAS NOX RM AGE DIS RAD TAX PTRATIO B LSTAT MEDV

6. PAIRPLOT RM, PTRATIO, LSTAT, MEDV

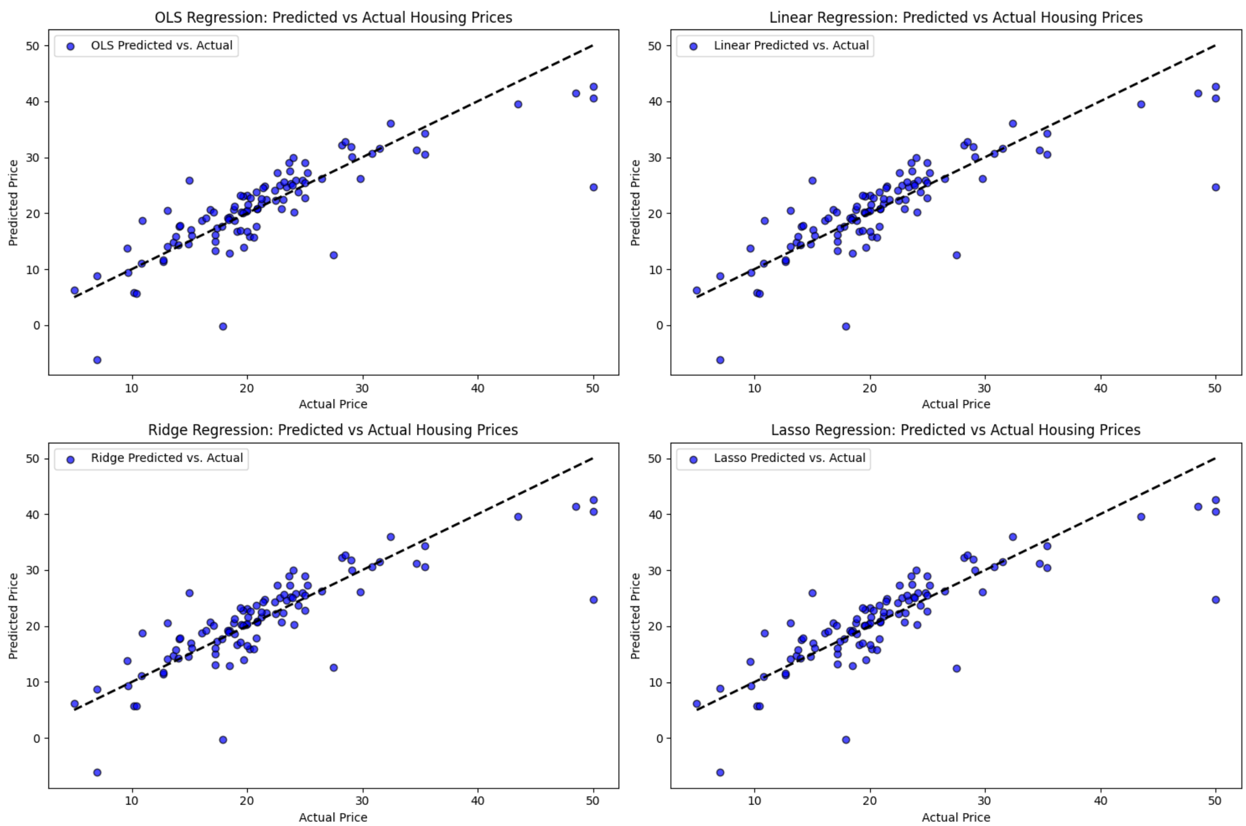


Results and Discussion

Case 1: Without outlier removal and without feature scaled

Error Calculation Table

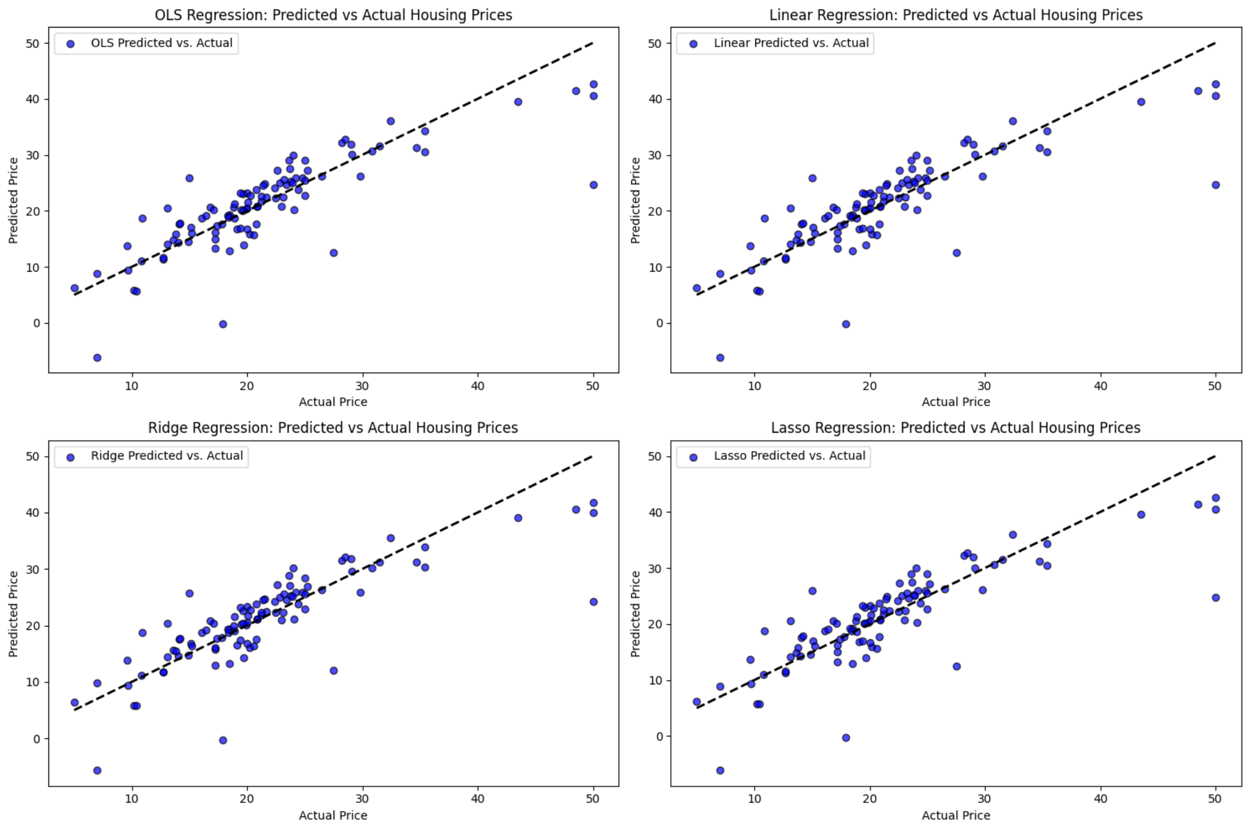
Regression	R^2 square	MSE
Ordinary Least Square (OLS) Regression	0.6688	24.2911
Linear Regression	0.6688	24.2911
Ridge Regression	0.6686	24.3010
Lasso Regression	0.6688	24.2888



Case 2: Without outlier removal and with feature scaling

Error Calculation Table

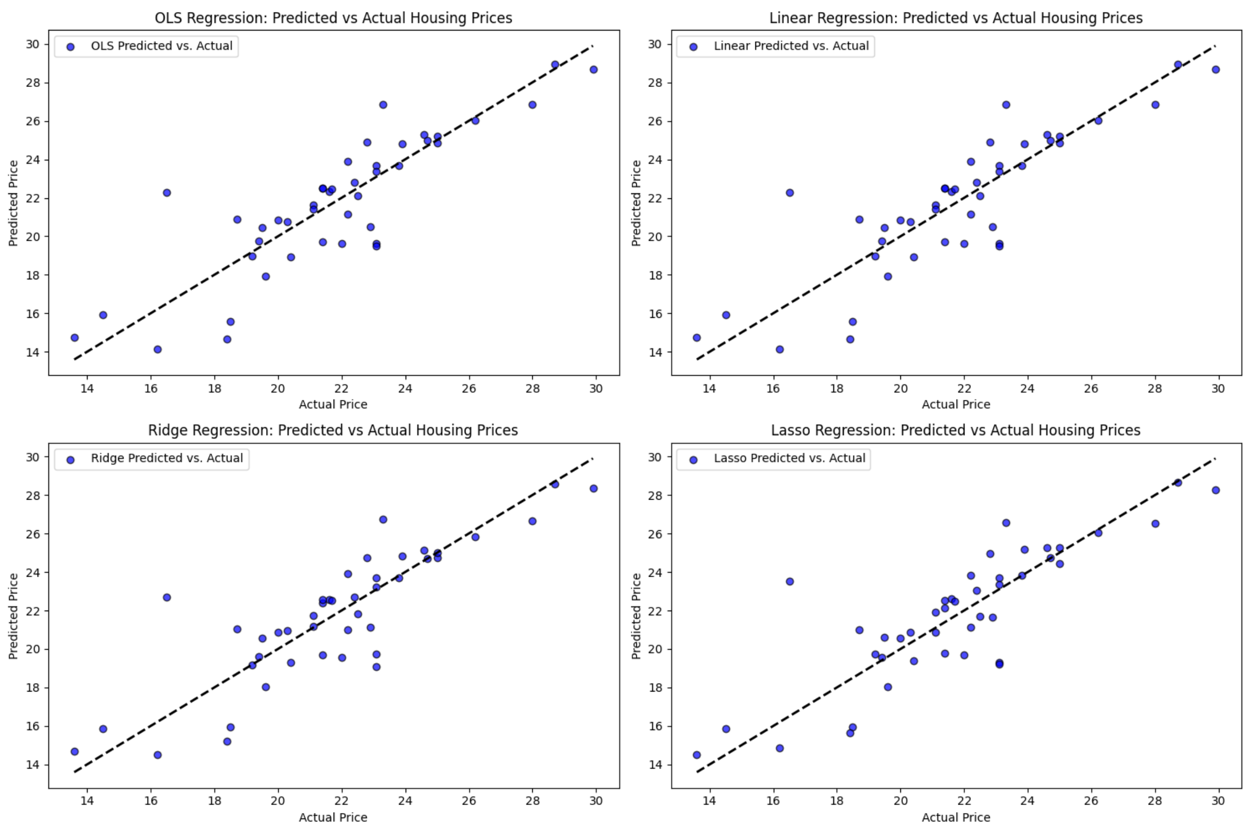
Regression	R^2 square	MSE
Ordinary Least Square (OLS) Regression	0.6688	24.2911
Linear Regression	0.6688	24.2911
Ridge Regression	0.6660	24.4958
Lasso Regression	0.6687	24.2945



Case 3: With outlier removal and feature scaled

Error Calculation Table

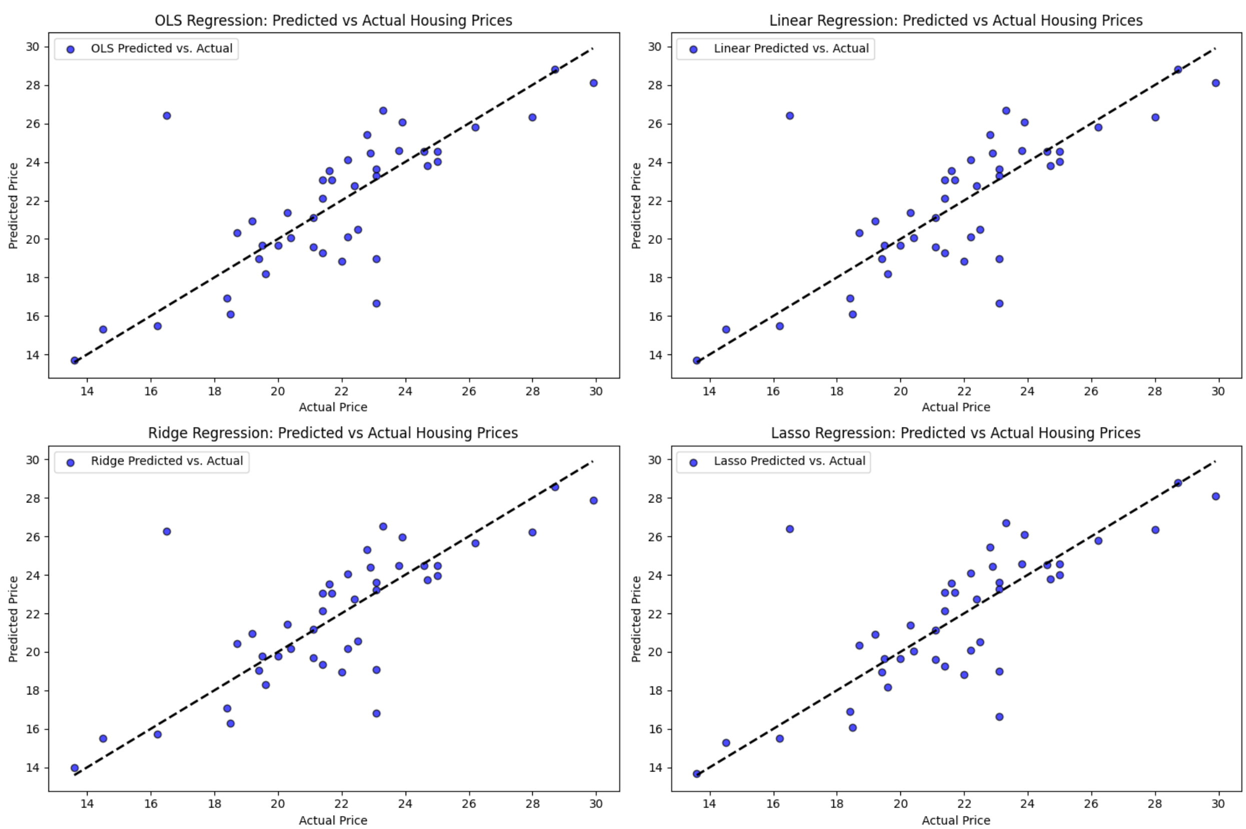
Regression	R^2 square	MSE
Ordinary Least Square(OLS) Regression	0.6938	3.3628
Linear Regression	0.6938	3.3628
Ridge Regression	0.6970	3.3275
Lasso Regression	0.6830	3.4815



Case 4: With feature having greater than 0.5, RM, PTRATIO, LSTAT and with feature scaled and outlier removed

Error Calculation Table

Regression	R^2 square	MSE
Ordinary Least Square (OLS) Regression	0.4758	5.7565
Linear Regression	0.4758	5.7565
Ridge Regression	0.4985	5.5075
Lasso Regression	0.4761	5.7537



Conclusion

From all the above cases, it can be concluded that

i. There is much improvement in MSE and r square value after outlier removal

ii. Feature scaling little bit decreased the MSE and r square value for each different types of regression.

iii. For each different types of Linear Regression the best line is plotted to fit the actual and predicted values.

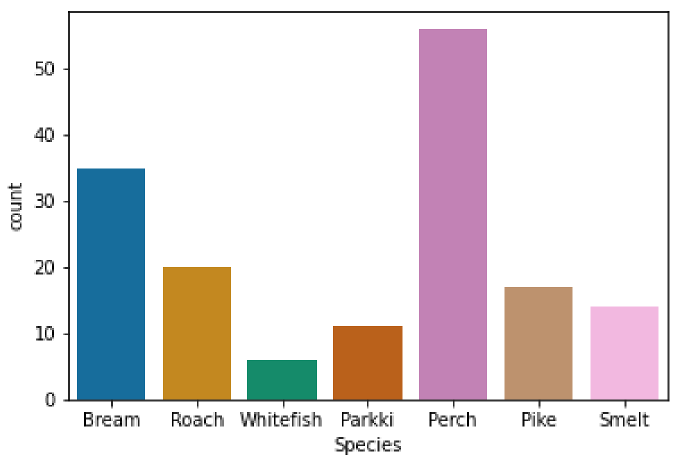
2. FISH.CV DATASET

This dataset consists of:

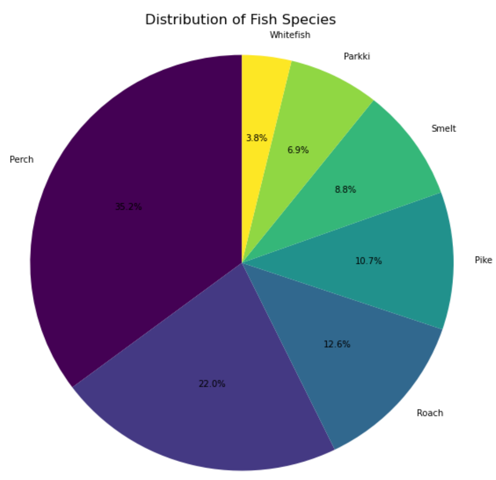
- Species = Species name of fish
- Weight = Weight of fish in Gram g
- Length1 = Vertical length in cm
- Length2 = Diagonal length in cm
- Length3 = Cross length in cm
- Height = Height in cm
- Width = Diagonal width in cm

VISUALIZATION:

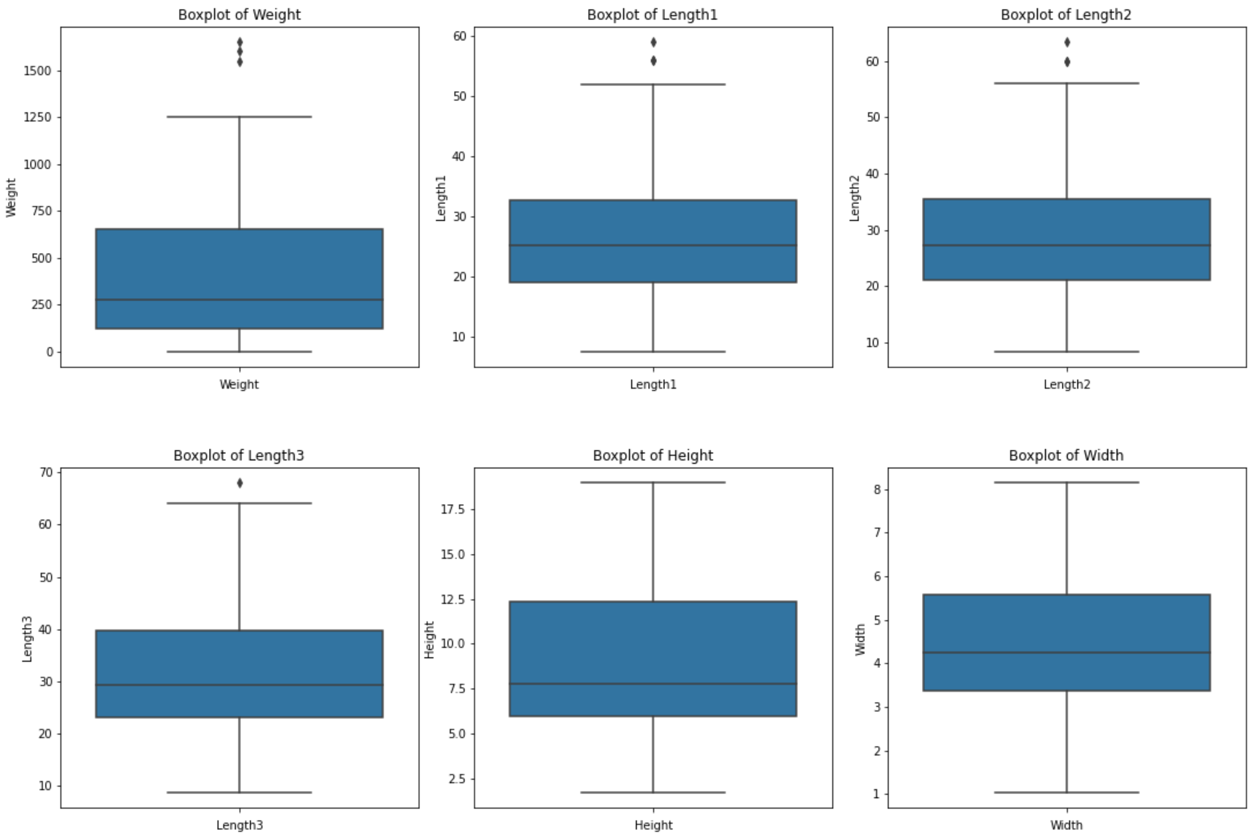
1. Histogram



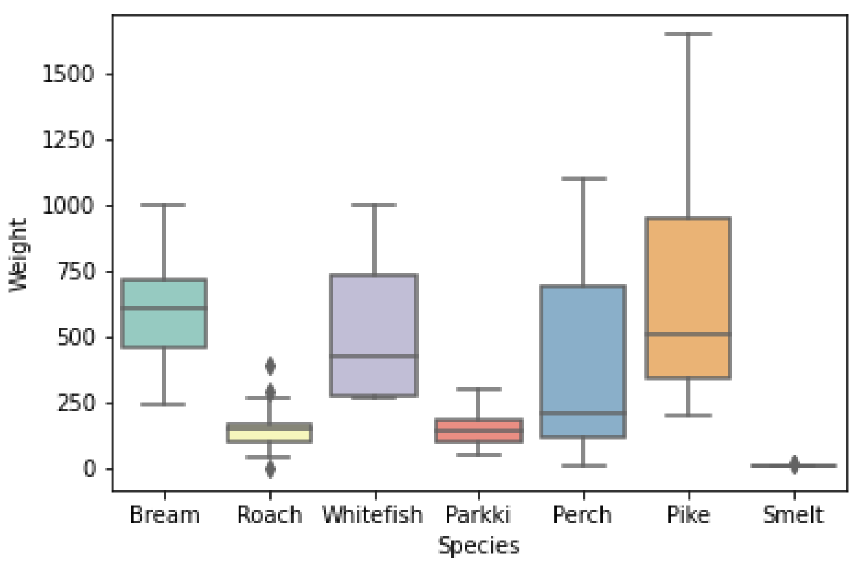
2. Pie chart



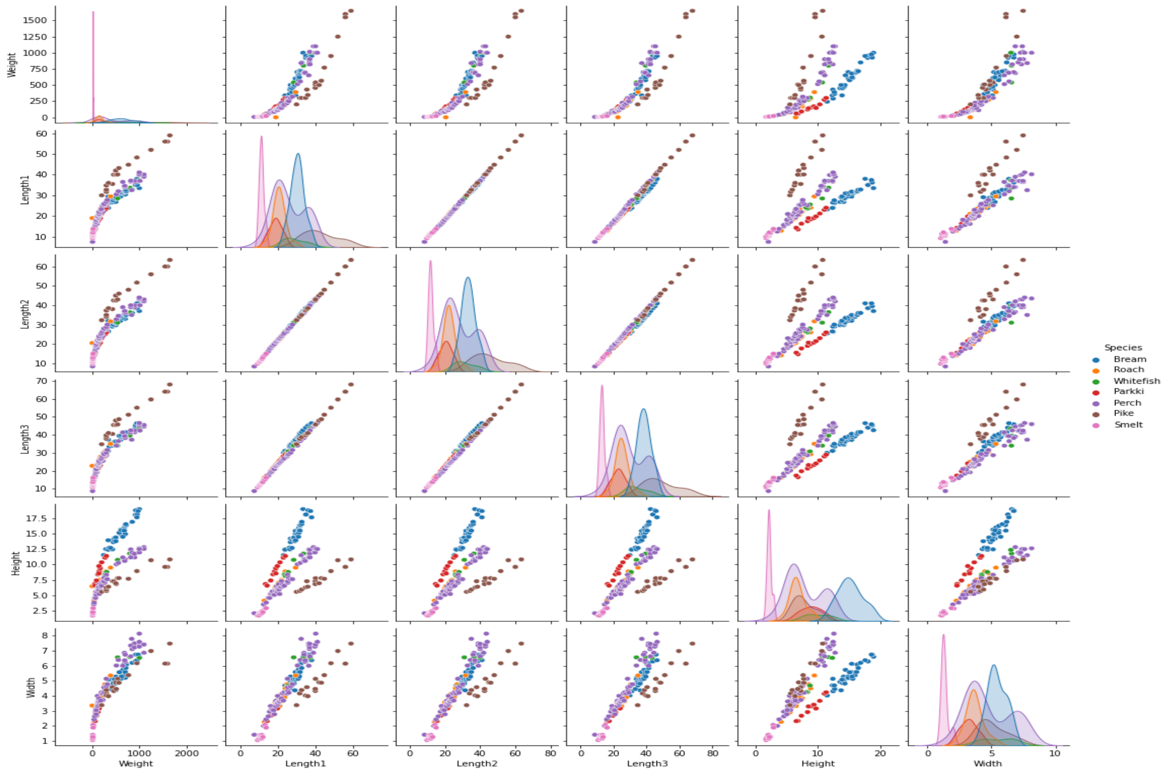
3. BOX PLOT



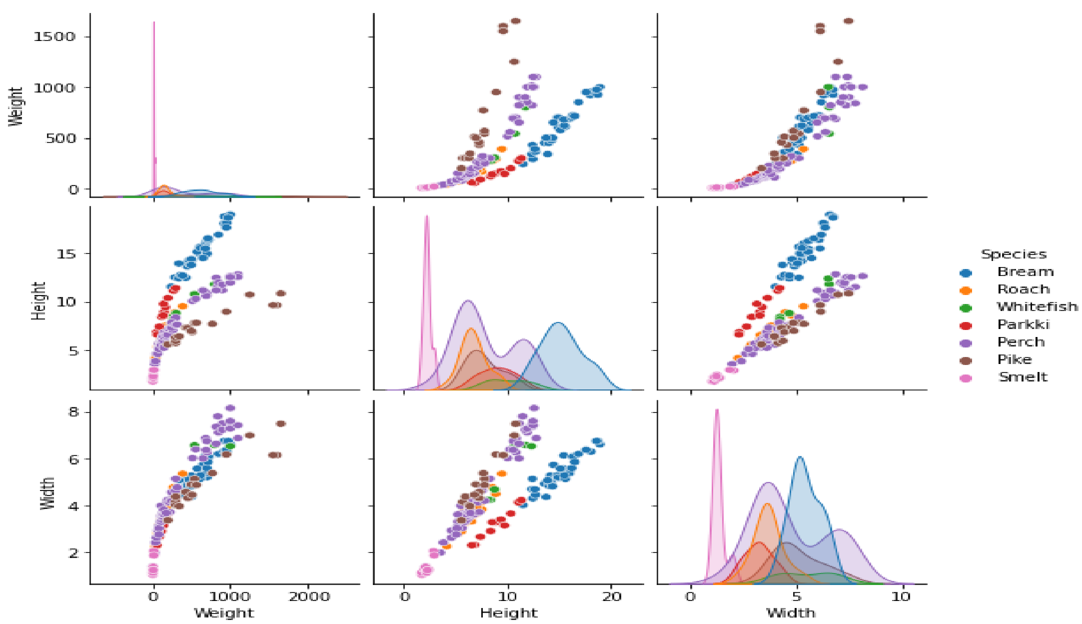
4. Species wise boxplot



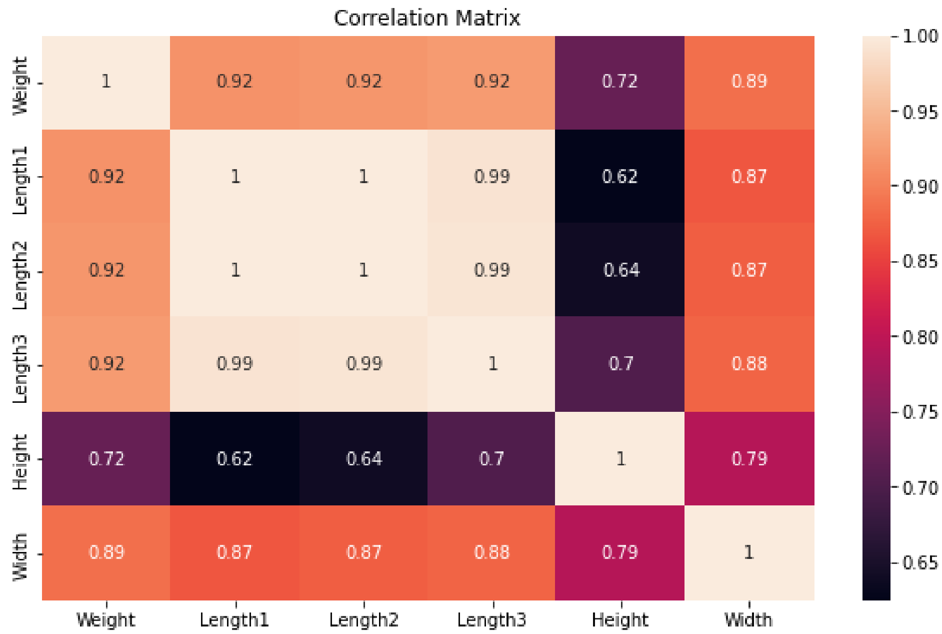
5. PAIR PLOT



5.1. PAIR PLOT FOR HEIGHT, WIDTH, WEIGHT



6. Heatmap



Results and Discussion

R^2 square and MSE value are listed below.
With this, coefficients are also calculated and scatter plots of actual vs predicted values are generated with the best fit line.

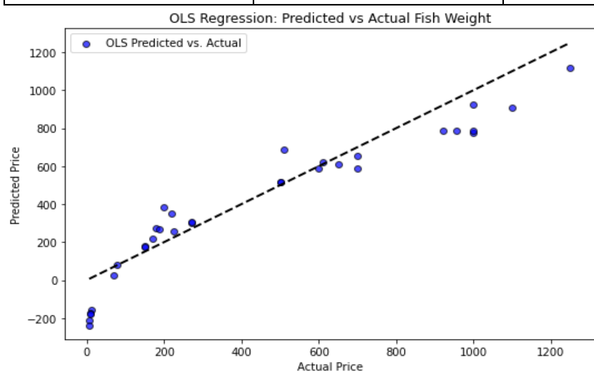
Case 1: Without outlier removal

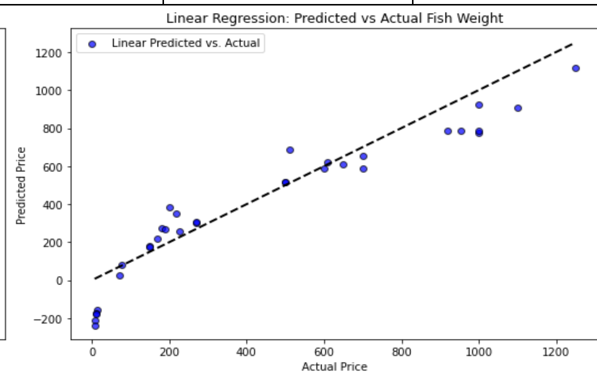
Error Calculation Table

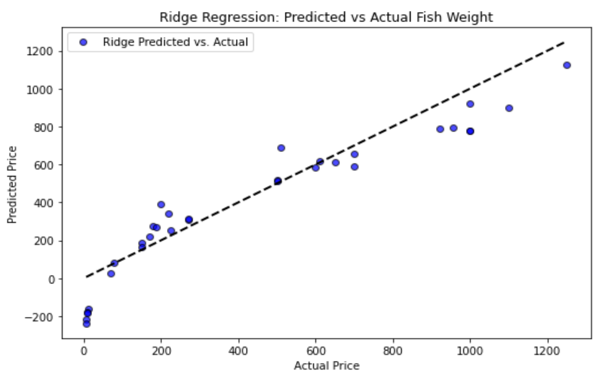
Regression	R^2 square	MSE
Ordinary Least Square (OLS) Regression	0.8821	16763.8872
Linear Regression	0.8821	16763.8872
Ridge Regression	0.8803	17022.0223
Lasso Regression	0.8774	17431.9637

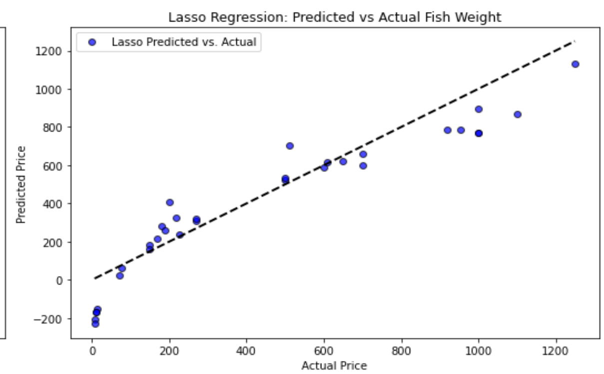
Coefficient Calculation

Features	OLS Regression Coefficients	Linear Regression Coefficients	Ridge Regression Coefficients	Lasso Regression Coefficients
Const	-515.305651			
Length1	43.535265	43.5352649	27.03858344	24.91762755
Length2	7.821796	7.82179624	17.69642626	0.1063255
Length3	-25.256701	-25.25670105	-19.51672787	-0.
Height	23.228912	23.2289123	20.3581969	13.44083754
Width	27.066493	27.06649294	27.51531904	29.06666741









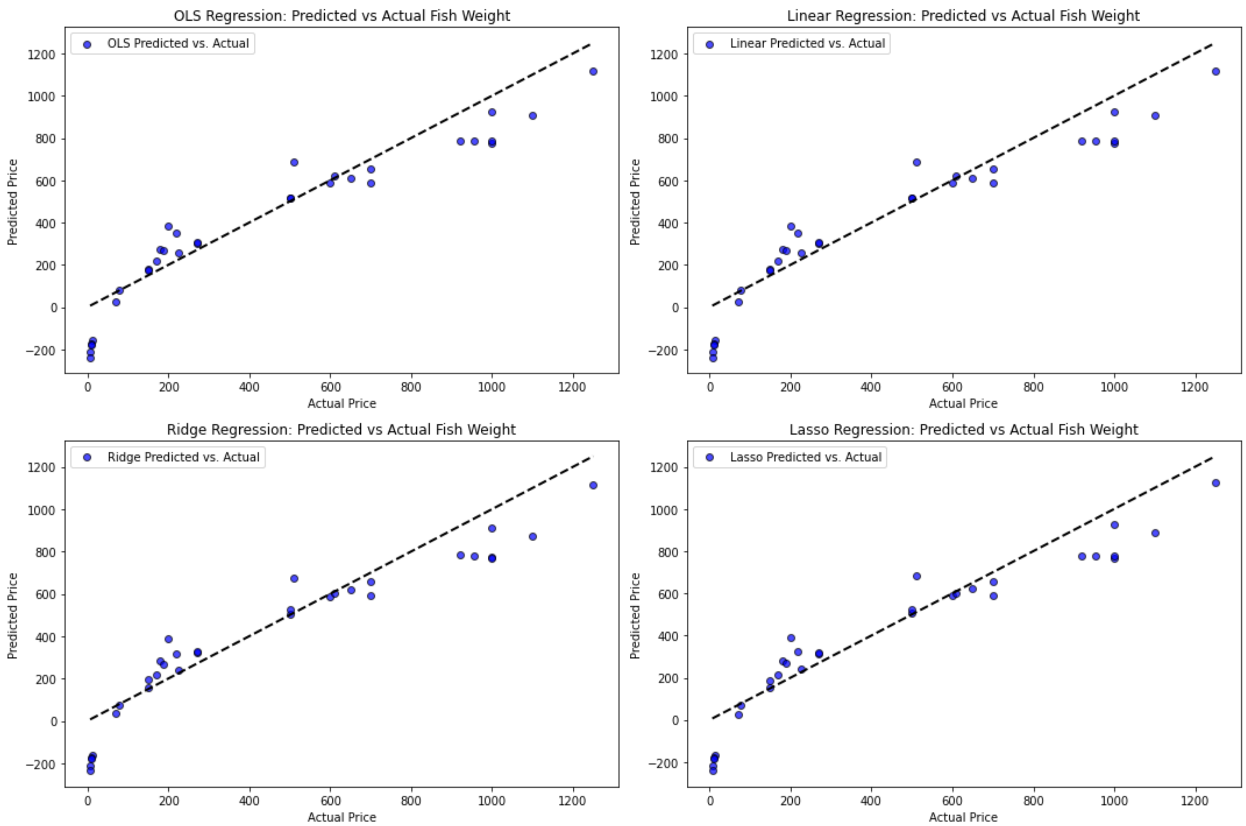
Case 2: Without outlier removal and with no feature scaling

Error Calculation Table

Regression	R^2 square	MSE
Ordinary Least Square(OLS) Regression	0.8821	16763.8872
Linear Regression	0.8821	16763.8872
Ridge Regression	0.8770	17488.5753
Lasso Regression	0.8773	17458.6532

Coefficient Calculation

Features	OLS Regression Coefficients	Linear Regression Coefficients	Ridge Regression Coefficients	Lasso Regression Coefficients
Const	-515.305651			
Length1	43.535265	432.27472554	83.29023683	229.96344143
Length2	7.821796	83.01304108	80.60503657	0.
Length3	-25.256701	-288.56797575	64.91737168	0.
Height	23.228912	92.52321581	35.7986499	44.01533366
Width	27.066493	44.06740907	77.01514167	75.78051359



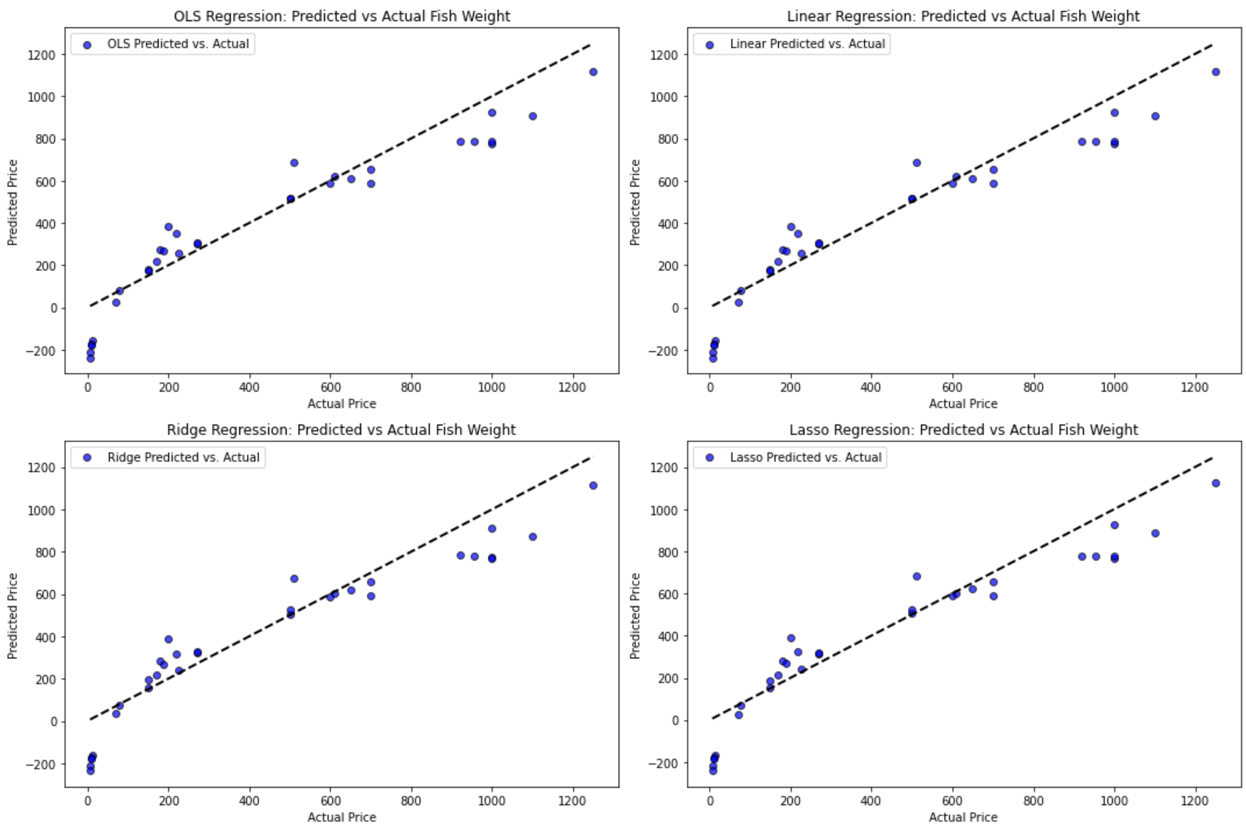
Case 3: With outlier removal and feature scaled

Error Calculation Table

Regression	R^2 square	MSE
Ordinary Least Square(OLS) Regression	0.8821	16763.8872
Linear Regression	0.8821	16763.8872
Ridge Regression	0.8770	17488.5753
Lasso Regression	0.8773	17458.6532

Coefficient Calculation

Features	OLS Regression Coefficients	Linear Regression Coefficients	Ridge Regression Coefficients	Lasso Regression Coefficients
Const	-515.305651			
Length1	43.535265	432.27472554	83.29023683	229.96344143
Length2	7.821796	83.01304108	80.60503657	0.
Length3	-25.256701	-288.56797575	64.91737168	0.
Height	23.228912	92.52321581	35.7986499	44.01533366
Width	27.066493	44.06740907	77.01514167	75.78051359



Conclusion

From all the above cases, it can be concluded that

i. There is not much difference in MSE and r square value even after outlier removal as the outlier count was very small.

ii. Feature scaling little bit increased the MSE and r square value for each different types of regression.

iii. Coefficient value increased very much after feature scaling for Linear, Lasso and Ridge Regression while in OLS coefficient, Feature scaling has no effect.