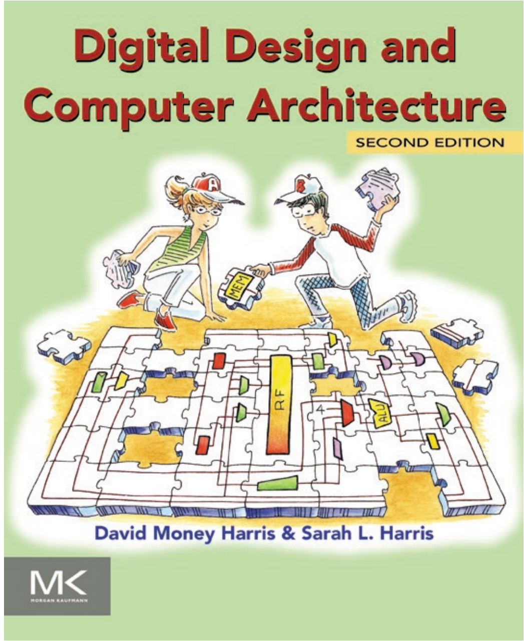
FROM ZERO TO ONE

Chapter 1

***Digital Design and Computer Architecture*, 2nd Edition**

David Money Harris and Sarah L. Harris



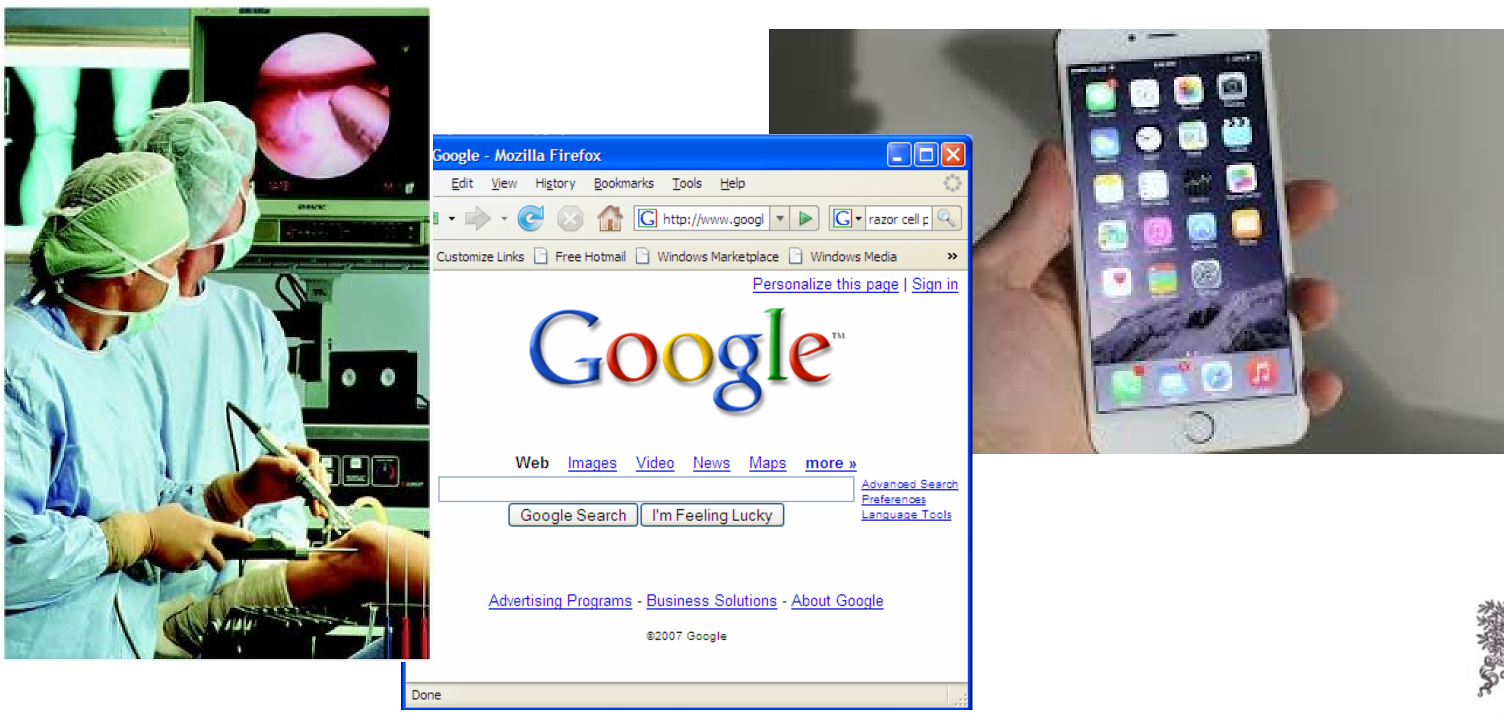
FROM ZERO TO ONE

Chapter 1 :: Topics

- Background
- The Game Plan
- The Art of Managing Complexity
- The Digital Abstraction
- Number Systems
- Logic Gates
- Logic Levels
- CMOS Transistors
- Power Consumption

Background

- Microprocessors have revolutionized our world
  - Cell phones, Internet, rapid advances in medicine, etc.
- The semiconductor industry has grown from $21 billion in 1985 to $306 billion in 2016



The Game Plan

- **Purpose of course:**
  - Understand what's under the hood of a computer
  - Learn the principles of digital design
  - Learn to systematically debug increasingly complex designs
  - Design and build a microprocessor

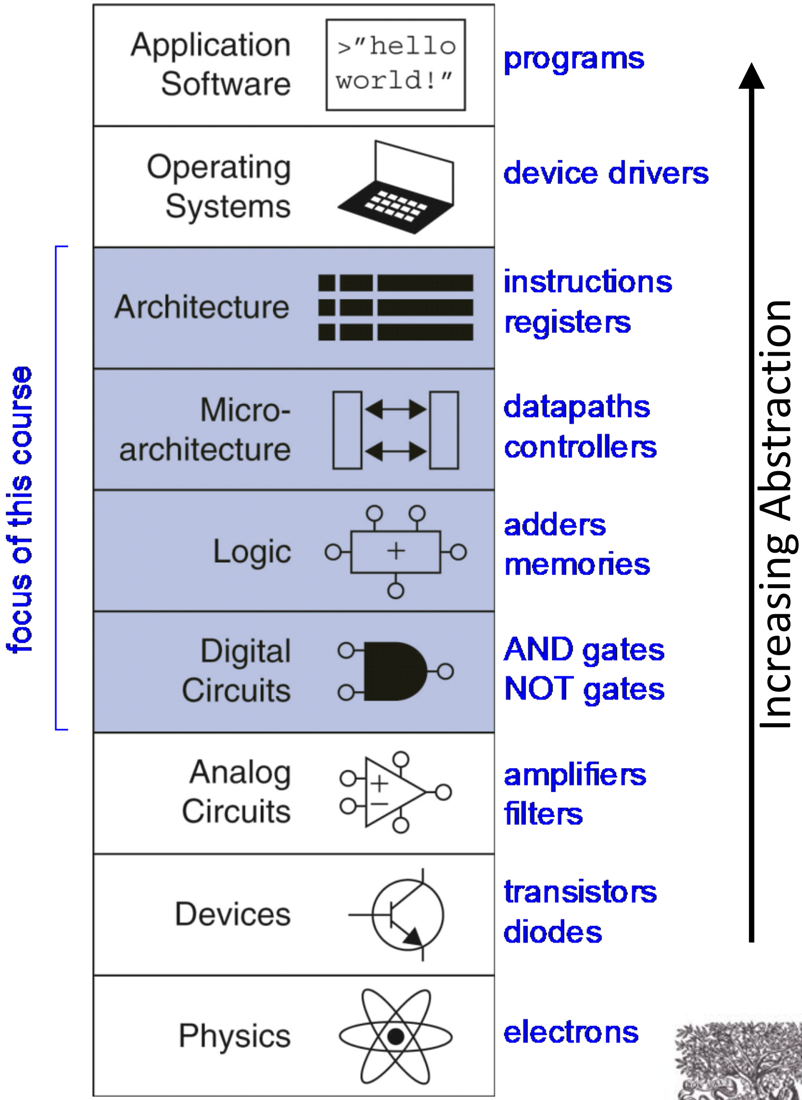
The Art of Managing Complexity

- Abstraction
- Discipline
- The Three –y's
  - Hierarchy
  - Modularity
  - Regularity

Abstraction

- What is **abstraction**?
  – Hiding details when they are not important

- Electronic computer abstraction



Discipline

- Intentionally restrict design choices
- Example: Digital discipline
  - **Discrete voltages** instead of continuous
  - **Simpler** to design than analog circuits – can build more sophisticated systems
  - Digital systems **replacing analog** predecessors:
    - i.e., digital cameras, digital television, cell phones, CDs

The Three -y's

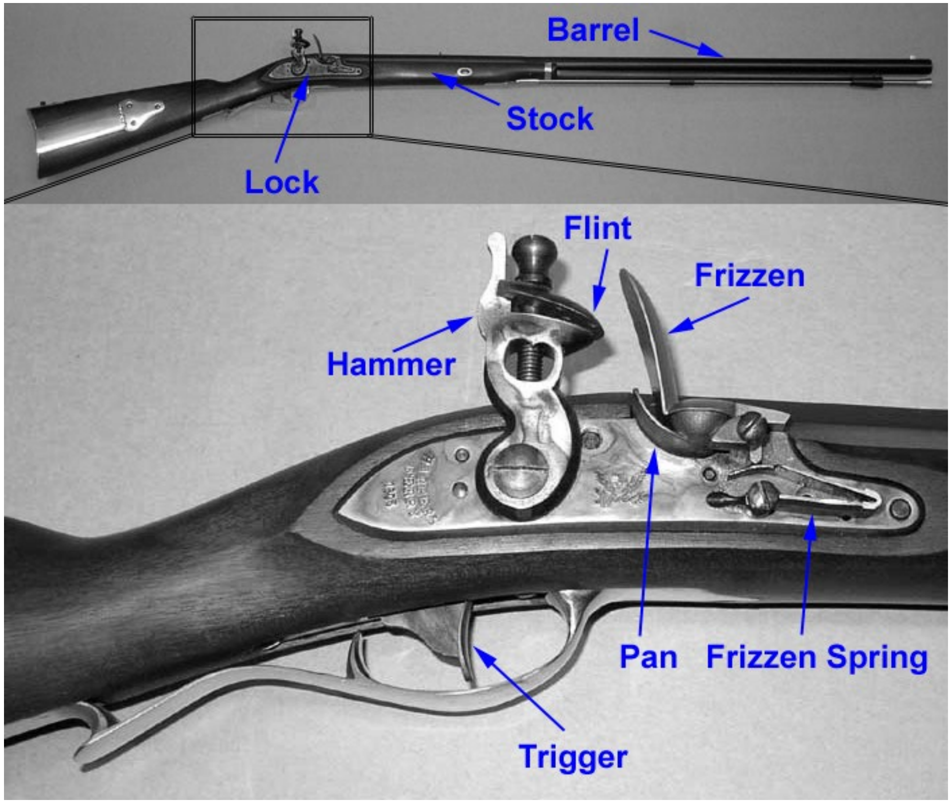
- **Hierarchy**
- **Modularity**
- **Regularity**

The Three -y's

- **Hierarchy**
  - A system divided into modules and submodules
- **Modularity**
  - Having well-defined functions and interfaces
- **Regularity**
  - Encouraging uniformity, so modules can be easily reused

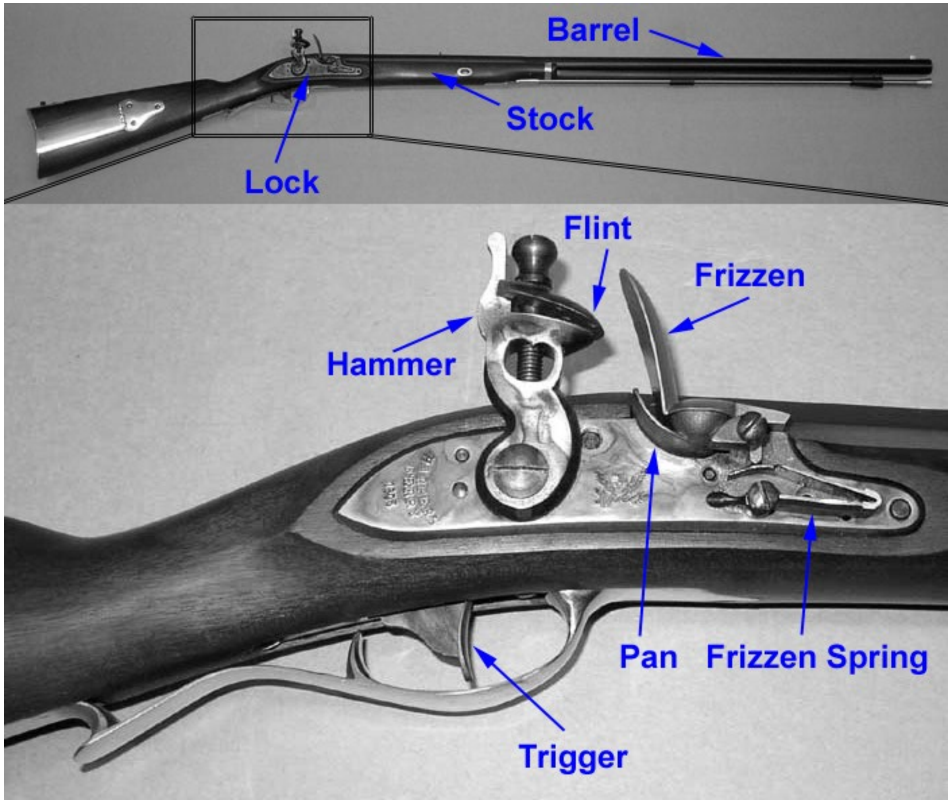
Example: The Flintlock Rifle

- **Hierarchy**
  - **Three main modules:** lock, stock, and barrel
  - **Submodules of lock:** hammer, flint, frizzen, etc.



Example: The Flintlock Rifle

- **Modularity**
  - **Function of stock:** mount barrel and lock
  - **Interface of stock:** length and location of mounting pins
- **Regularity**
  - Interchangeable parts

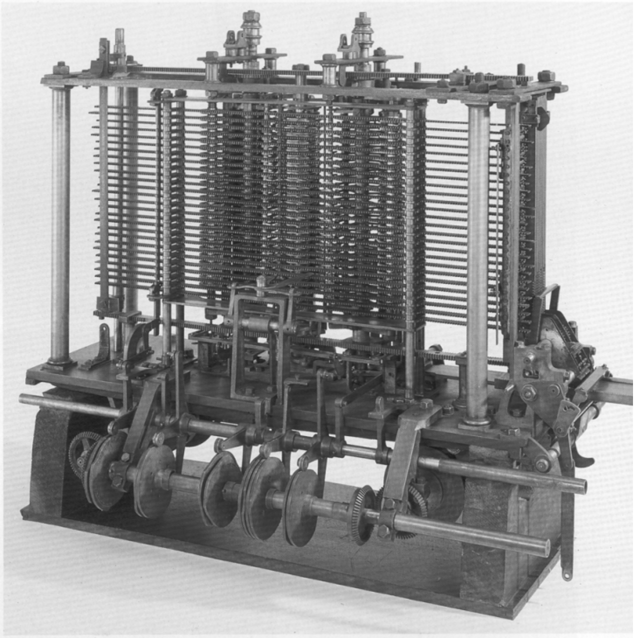


The Digital Abstraction

- Most physical variables are **continuous**
  - Voltage on a wire
  - Frequency of an oscillation
  - Position of a mass
- Digital abstraction considers **discrete subset** of values

The Analytical Engine

- Designed by Charles Babbage from 1834 – 1871
- Considered to be the first digital computer
- Built from mechanical gears, where each gear represented a discrete value (0-9)
- Babbage died before it was finished



Digital Discipline: Binary Values

- Digital abstraction considers **discrete subset** of values
- **Two discrete values:**
  - 1's and 0's
  - 1 = TRUE = HIGH
  - 0 = FALSE = LOW
- How to represent **1 and 0:**
  - voltage levels, rotating gears, fluid levels, etc.
- Digital circuits use **voltage levels** to represent 1 and 0
- ***Bit***: *B*inary dig*it*

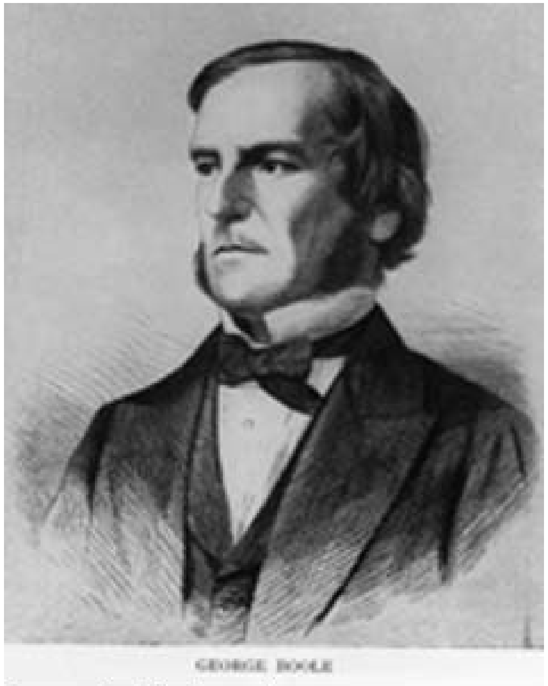
FROM ZERO TO ONE

Why Digital Systems?

- Easier to design
- Fast
- Can overcome noise
- Error detection/correction

George Boole, 1815-1864

- Born to working class parents
- **Taught himself mathematics** and joined the faculty of Queen's College in Ireland
- Wrote ***An Investigation of the Laws of Thought*** (1854)
- Introduced **binary variables**
- Introduced the **three fundamental logic operations**: AND, OR, and NOT



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FROM ZERO TO ONE

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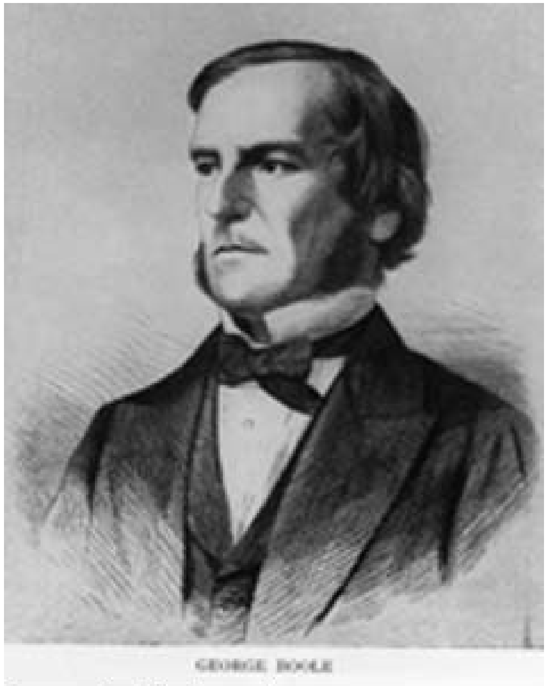

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Number Systems

- **Decimal**
  - Base 10
- **Binary**
  - Base 2
- **Hexadecimal**
  - Base 16

Review: Decimal Numbers

- **Base 10** (our everyday number system)

1000's column
100's column
10's column
1's column

$5374_{10} =$

Base 10

ELSEVIER

Review: Decimal Numbers

- **Base 10** (our everyday number system)

1000's column
100's column
10's column
1's column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

five thousands | three hundreds | seven tens | four ones

Base 10

Decimal and Binary Numbers

- **Base 10** (our everyday number system)

1000's column
100's column
10's column
1's column

$$5374_{10} = \underbrace{5 \times 10^3}_{\text{five thousands}} + \underbrace{3 \times 10^2}_{\text{three hundreds}} + \underbrace{7 \times 10^1}_{\text{seven tens}} + \underbrace{4 \times 10^0}_{\text{four ones}}$$

- **Base 2**: Binary numbers

8's column
4's column
2's column
1's column

$$1101_2 =$$

Base 2

Decimal and Binary Numbers

- **Base 10** (our everyday number system)

1000's column
100's column
10's column
1's column

$$5374_{10} = 5 \times 10^3 + 3 \times 10^2 + 7 \times 10^1 + 4 \times 10^0$$

five thousands · three hundreds · seven tens · four ones

- **Base 2**: Binary numbers

8's column
4's column
2's column
1's column

$$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 13_{10}$$

one eight · one four · no two · one one

Base 2

Powers of Two

- $2^0 =$
- $2^1 =$
- $2^2 =$
- $2^3 =$
- $2^4 =$
- $2^5 =$
- $2^6 =$
- $2^7 =$
- $2^8 =$
- $2^9 =$
- $2^{10} =$
- $2^{11} =$
- $2^{12} =$
- $2^{13} =$
- $2^{14} =$
- $2^{15} =$

Powers of Two

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
- $2^8 = 256$
- $2^9 = 512$
- $2^{10} = 1024$
- $2^{11} = 2048$
- $2^{12} = 4096$
- $2^{13} = 8192$
- $2^{14} = 16384$
- $2^{15} = 32768$
- Handy to memorize up to 2^9

Powers of Two

- $2^0 = 1$
- $2^1 = 2$
- $2^2 = 4$
- $2^3 = 8$
- $2^4 = 16$
- $2^5 = 32$
- $2^6 = 64$
- $2^7 = 128$
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- Handy to memorize up to 2^9

Binary to Decimal Conversion

- Binary to decimal conversion:
  - Convert 10011_2 to decimal

Binary to Decimal Conversion

- Binary to decimal conversion:
  - Convert 10011_2 to decimal
  - $16\times1 + 8\times0 + 4\times0 + 2\times1 + 1\times1 = 19_{10}$

Decimal to Binary Conversion

- **Two methods:**
  - **Method 1:** Find the **largest power of 2 that fits**, subtract and repeat. (***Recommended*** method)
  - **Method 2:** Repeatedly **divide by 2**, remainder goes in next most significant bit

Decimal to Binary Conversion

Method 1: Find the **largest power of 2 that fits**, subtract and repeat.

53_{10}

Method 2: **Repeatedly divide by 2**, remainder goes in next most significant bit.

Decimal to Binary Conversion

Method 1: Find the **largest power of 2 that fits**, subtract and repeat.

53_{10}	32×1
53-32 = 21	16×1
21-16 = 5	4×1
5-4 = 1	1×1

= 110101_2

Method 2: Repeatedly **divide by 2**, remainder goes in next most significant bit.

53_{10} =

53/2 = 26 R1
26/2 = 13 R0
13/2 = 6 R1
6/2 = 3 R0
3/2 = 1 R1
1/2 = 0 R1

= 110101_2

Number Conversion

- **Binary to decimal conversion:**
  – Convert 11101_2 to decimal

- **Decimal to binary conversion:**
  – Convert 47_{10} to binary

Number Conversion

- **Binary to decimal conversion:**
  - Convert 11101_2 to decimal
  - $16\times1 + 8\times1 + 4\times1 + 2\times0 + 1\times1 = 29_{10}$

- **Decimal to binary conversion:**
  - Convert 47_{10} to binary
  - $32\times1 + 16\times0 + 8\times1 + 4\times1 + 2\times1 + 1\times1 = 101111_2$

Binary Values and Range

- ***N*-digit decimal number**
  - How many values?
  - Range?
  - Example: 3-digit decimal number:
- ***N*-bit binary number**
  - How many values?
  - Range:
  - Example: 3-digit binary number:

Binary Values and Range

- ***N*-digit decimal number**
  - How many values? **10^N**
  - Range? **$[0, 10^N - 1]$**
  - Example: 3-digit decimal number:
    - **$10^3 = 1000$ possible values**
    - **Range: [0, 999]**
- ***N*-bit binary number**
  - How many values? **2^N**
  - Range: **$[0, 2^N - 1]$**
  - Example: 3-digit binary number:
    - **$2^3 = 8$ possible values**
    - **Range: $[0, 7] = [000_2$ to $111_2]$**

Binary Numbers

$$A: \{a_{N-1}, a_{N-2}, \dots a_1, a_0\}$$

$$A = \sum_{i=0}^{N-1} a_i 2^i$$

Example:

$1101_2 \quad = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 0 + 1$

$= 13$

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	
1	1	
2	2	
3	3	
4	4	
5	5	
6	6	
7	7	
8	8	
9	9	
A	10	
B	11	
C	12	
D	13	
E	14	
F	15	

Hexadecimal Numbers

Hex Digit	Decimal Equivalent	Binary Equivalent
0	0	0000
1	1	0001
2	2	0010
3	3	0011
4	4	0100
5	5	0101
6	6	0110
7	7	0111
8	8	1000
9	9	1001
A	10	1010
B	11	1011
C	12	1100
D	13	1101
E	14	1110
F	15	1111

Hexadecimal Numbers

- Base 16
- Shorthand for binary

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - **Convert $4AF_{16}$ (also written 0x4AF) to binary**

- Hexadecimal to decimal conversion:
  - **Convert $4AF_{16}$ to decimal**

Hexadecimal to Binary Conversion

- Hexadecimal to binary conversion:
  - **Convert $4AF_{16}$ (also written 0x4AF) to binary**
  - **$0100\ 1010\ 1111_2$**

- Hexadecimal to decimal conversion:
  - **Convert $4AF_{16}$ to decimal**
  - **$4 \times 16^2 + A \times 16^1 + F \times 16^0$**
  - **$4 \times 16^2 + 10 \times 16^1 + 15 \times 16^0 = 1199_{10}$**

FROM ZERO TO ONE

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Bits, Bytes, Nibbles…

- **Bits**
  - **msb:** most significant bit
  - **lsb:** least significant bit

$$\underbrace{1}_{\text{most significant bit}}001011\underbrace{0}_{\text{least significant bit}}$$

- **Bytes & Nibbles**

$$\overbrace{1001\underbrace{0110}_{\text{nibble}}}^{\text{byte}}$$

- **Bytes**
  - **MSB:** most significant byte
  - **LSB:** least significant byte

$$\underbrace{10100010}_{\text{most significant byte}}\underbrace{11100101}_{\text{least significant byte}}$$

Bits, Bytes, Nibbles…

- **Bits**
  - **msb:** most significant bit
  - **lsb:** least significant bit

10010110

most significant bit — least significant bit

- **Bytes & Nibbles**

byte

10010110

nibble

- **Bytes**
  - **MSB:** most significant byte
  - **LSB:** least significant byte
  - Each hex digit represents a nibble (4 bits)

CEBF9AD7

most significant byte — least significant byte

Large Powers of Two

- 2^{10} = 1 kilo ≈ thousand (1024)
- 2^{20} = 1 mega ≈ million (1,048,576)
- 2^{30} = 1 giga ≈ billion (1,073,741,824)
- 2^{40} = 1 tera ≈ trillion (1,099,511,627,776)
- 2^{50} = 1 peta ≈ 10^{15}
- 2^{60} = 1 exa ≈ 10^{18}

Large Powers of Two

- 2^{10} = 1 kilo (**kibi**) ≈ 10^3 (1024)
- 2^{20} = 1 mega (**mebi**) ≈ 10^6 (1,048,576)
- 2^{30} = 1 giga (**gibi**) ≈ 10^9 (1,073,741,824)
- 2^{40} = 1 tera (**tebi**) ≈ 10^{12}
- 2^{50} = 1 peta (**pebi**) ≈ 10^{15}
- 2^{60} = 1 exa (**exbi**) ≈ 10^{18}

Large Powers of Two: Abbreviations

- 2^{10} = 1 kilo ≈ 1000 (1024)

  kibibyte = **1 Ki**

  for example: 1 KiB = 1024 Bytes

  1 Kib = 1024 bits
- 2^{20} = 1 mega ≈ 1 million (1,048,576)

  mebibyte= **1 Mi**

  for example: 1 MiB, 1 Mib (1 megabit)
- 2^{30} = 1 giga ≈ 1 billion (1,073,741,824)

  gibibyte = **1 Gi**

  for example: 1 GiB, 1 Gib

Estimating Powers of Two

- What is the approximate value of 2^{24}?

- Approximately how many values can a 32-bit variable represent?

Estimating Powers of Two

- What is the approximate value of 2^{24}?

  $2^4 \times 2^{20} \approx$ 16 million

- Approximately how many values can a 32-bit variable represent?

  $2^2 \times 2^{30} \approx$ 4 billion

  First factor out the largest 2^{10x}. Then estimate.

Addition

- Decimal

$$\begin{array}{r} 3734 \\ +\ \ 5168 \\ \hline \end{array}$$

- Binary

$$\begin{array}{r} 1011 \\ +\ \ 0011 \\ \hline \end{array}$$

Addition

- Decimal

$$
\begin{array}{r l}
 & \color{blue}{11} \leftarrow \color{blue}{\text{carries}} \\
 & 3734 \\
+ & 5168 \\
\hline
 & 8902
\end{array}
$$

- Binary

$$
\begin{array}{r l}
 & \color{blue}{11} \leftarrow \color{blue}{\text{carries}} \\
 & 1011 \\
+ & 0011 \\
\hline
 & 1110
\end{array}
$$

Binary Addition Examples

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1001 \\ +\ 0101 \\ \hline \end{array}$$

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1011 \\ +\ 0110 \\ \hline \end{array}$$

Binary Addition Examples

- Add the following 4-bit binary numbers

$$\begin{array}{r} 1 \\ 1001 \\ +\ \ 0101 \\ \hline 1110 \end{array}$$

- Add the following 4-bit binary numbers

$$\begin{array}{r} 111 \\ 1011 \\ +\ \ 0110 \\ \hline 10001 \end{array}$$

Overflow!

Binary Addition: Number of Bits

- The addition of two 4-bit values (inputs) gives a 4-bit result (output).
  - Any additional bits on the left are ignored (**overflow!**)
- Generally, addition of two **n-bit** numbers gives an **n-bit** result.

$$
\begin{array}{r}
1 \\
1001 \\
+\ \ 0101 \\
\hline
1110
\end{array}
$$

$$
\begin{array}{r}
111 \\
1011 \\
+\ \ 0110 \\
\hline
10001
\end{array}
$$

Overflow!

Overflow

- Digital systems operate on a **fixed number of bits**
- **Overflow:** when result is **too big to fit** in the the available number of bits
- See previous example of 11 + 6

```
 111
  1011
+ 0110
 -----
 10001
```

Overflow!

Signed Binary Numbers

- Sign/Magnitude Numbers
- Two's Complement Numbers

Sign/Magnitude Numbers

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1

- Example, 4-bit sign/mag representations of ± 6:

  +6 =

  - 6 =

- Range of an *N*-bit sign/magnitude number:

Sign/Magnitude Numbers

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1

- Example, 4-bit sign/mag representations of ± 6:

  +6 = **0110**

  - 6 = **1110**

- Range of an *N*-bit sign/magnitude number:

  $[-(2^{N-1}-1), 2^{N-1}-1]$

FROM ZERO TO ONE

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Sign/Magnitude Numbers

- 1 sign bit, *N*-1 magnitude bits
- Sign bit is the most significant (left-most) bit
  - Positive number: sign bit = 0
  - Negative number: sign bit = 1

$$A: \{a_{N-1}, a_{N-2}, \ldots a_1, a_0\}$$

$$A = (-1)^{a_{N-1}} \sum_{i=0}^{N-2} a_i 2^i$$

- Example, 4-bit sign/mag representations of ± 6:

  +6 = **0110**

  - 6 = **1110**

- Range of an *N*-bit sign/magnitude number:

  [-(2^{N-1}-1), 2^{N-1}-1]

Unsigned Binary Numbers

$$A: \{a_{N-1}, a_{N-2}, \ldots a_1, a_0\}$$

$$A = \sum_{i=0}^{N-1} a_i 2^i$$

Example:

1101_2 $= 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 0 + 1$

$= 13$

FROM ZERO TO ONE

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Sign/Magnitude Numbers

$$A: \{a_{N-1}, a_{N-2}, \dots a_1, a_0\}$$

$$A = (-1)^{a_{N-1}} \sum_{i=0}^{N-2} a_i 2^i$$

Example:

$\mathbf{1101}_2$ $= (-1)^{\mathbf{1}} \times (\mathbf{1} \times 2^2 + \mathbf{0} \times 2^1 + \mathbf{1} \times 2^0)$

$= -1 \times (\mathbf{4} + \mathbf{0} + \mathbf{1})$

$= -5$

Sign/Magnitude Numbers

- Problems:
  - Addition doesn't work, for example -6 + 6:

```
   1110
 + 0110
 ------
  10100  (wrong!)
```

  - Two representations of 0 (± 0):

```
   1000
   0000
```

Two's Complement Numbers

- Don't have same problems as sign/magnitude numbers:
  - **Addition works**
  - **Single representation for 0**

Two's Complement Numbers

- Most significant bit (msb) has value of -2^{N-1}
- For example, a 4-bit 2's complement number:

___	___	___	___
-2^3	2^2	2^1	2^0
___	___	___	___
-8	4	2	1

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Two’s Complement Numbers

- Most significant bit (msb) has value of -2^{N-1}
- For example, a 4-bit 2’s complement number:

1	0	1	1
-2^3	2^2	2^1	2^0

1	0	1	1
-8	4	2	1

Value = -8 + 2 + 1 = -5

(We’ll show another way to find this value in a moment.)

Two’s Complement Numbers

- msb has value of -2^{N-1}
- Most positive 4-bit number:
- Most negative 4-bit number:
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an N-bit two’s complement number:

Two's Complement Numbers

- msb has value of -2^{N-1}
- Most positive 4-bit number: **0111**
- Most negative 4-bit number: **1000**
- The most significant bit still indicates the sign (1 = negative, 0 = positive)
- Range of an *N*-bit two's complement number:

$$[-(2^{N-1}), 2^{N-1}-1]$$

Unsigned Binary Numbers

$$A: \{a_{N-1}, a_{N-2}, \ldots a_1, a_0\}$$

$$A = \sum_{i=0}^{N-1} a_i 2^i$$

Example:

$1101_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= 8 + 4 + 0 + 1$

$= 13$

FROM ZERO TO ONE

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Two's Complement Numbers

$$A: \{a_{N-1}, a_{N-2}, \dots a_1, a_0\}$$

$$A = a_{N-1}\left(-2^{N-1}\right) + \sum_{i=0}^{N-2} a_i 2^i$$

Example:

1101_2 $= 1 \times (-2^3) + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$

$= -8 + 4 + 0 + 1$

$= -3$

“Taking the Two’s Complement”

- **Flips the sign** of a two’s complement number.
  - It makes a positive number negative.
  - It makes a negative number positive.
- **Method:**
  1. Invert the bits
  2. Add 1

"Taking the Two's Complement"

- **Flips the sign** of a two's complement number.
- **Method:**
  1. Invert the bits
  2. Add 1
- **Example:** Flip the sign of $3_{10} = 0011_2$
  1. **1100**
  2. **+ 1**

     1101 = -3_{10}

Two's Complement Examples

Take the two's complement of $6_{10} = 0110_2$

Two’s Complement Examples

Take the two’s complement of $6_{10} = 0110_2$

1. 1001
2. \+ 1

$1010_2 = -6_{10}$

Two's Complement Examples

What is the decimal value of the two's complement number 1001_2?

Two's Complement Examples

What is the decimal value of the two's complement number 1001_2?

- We know it's negative (msb = 1)
- Figure out magnitude by flipping the sign (i.e., "taking the two's complement")

1. **0110**
2. **~~+ 1~~**

$0111_2 = 7_{10}$

- So, we know it's a negative number with magnitude 7.
- Thus, $1001_2 = -7_{10}$

Taking the two's complement is the second (and **recommended**) way of figuring out the value of a negative two's complement number.

FROM ZERO TO ONE

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

$$\begin{array}{r} 0110 \\ +\ \ 1010 \\ \hline \end{array}$$

- Add -2 + 3 using two's complement numbers

$$\begin{array}{r} 1110 \\ +\ \ 0011 \\ \hline \end{array}$$

Two's Complement Addition

- Add 6 + (-6) using two's complement numbers

```
    111
    0110
 +  1010
 -------
   10000
```

- Add -2 + 3 using two's complement numbers

```
   111
   1110
 + 0011
 ------
  10001
```

Increasing Bit Width

- **Extend number from *N* to *M* bits (*M* > *N*) :**
  - Sign-extension
  - Zero-extension

Sign-Extension

- Sign bit copied to msb's
- Number value is same

- **Example 1:**
  - 4-bit representation of 3 = **0**011
  - 8-bit sign-extended value: **0000**0011
- **Example 2:**
  - 4-bit representation of -5 = **1**011
  - 8-bit sign-extended value: **1111**1011

Zero-Extension

- Zeros copied to msb's
- Value changes for negative numbers

- **Example 1:**
  - 4-bit value = $0011_2 = 3_{10}$
  - 8-bit zero-extended value: **0000**0011 = 3_{10}
- **Example 2:**
  - 4-bit value = 1011 = -5_{10}
  - 8-bit zero-extended value: **0000**1011 = 11_{10}

Number System Comparison

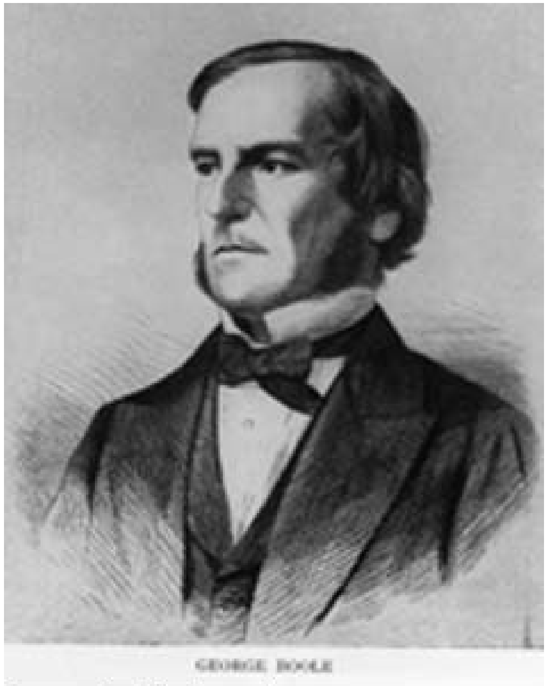
Number System	Range
Unsigned	$[0, 2^N-1]$
Sign/Magnitude	$[-(2^{N-1}-1), 2^{N-1}-1]$
Two's Complement	$[-2^{N-1}, 2^{N-1}-1]$

For example, 4-bit representation:

	-8	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Unsigned									0000	0001	0010	0011	0100	0101	0110	0111	1000	1001	1010	1011	1100	1101	1110	1111
Two's Complement	1000	1001	1010	1011	1100	1101	1110	1111	0000	0001	0010	0011	0100	0101	0110	0111								
Sign/Magnitude		1111	1110	1101	1100	1011	1010	1001	0000 1000	0001	0010	0011	0100	0101	0110	0111								

George Boole, 1815-1864

- Born to working class parents
- **Taught himself mathematics** and joined the faculty of Queen's College in Ireland
- Wrote ***An Investigation of the Laws of Thought*** (1854)
- Introduced **binary variables**
- Introduced the **three fundamental logic operations**: AND, OR, and NOT



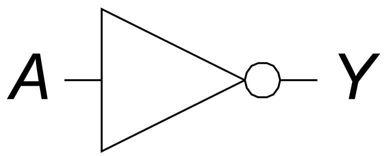
Scanned at the American Institute of Physics

Logic Gates

- **Perform logic functions:**
  – inversion (NOT), AND, OR, NAND, NOR, etc.
- **Single-input:**
  – NOT gate, buffer
- **Two-input:**
  – AND, OR, XOR, NAND, NOR, XNOR
- **Multiple-input**

Single-Input Logic Gates

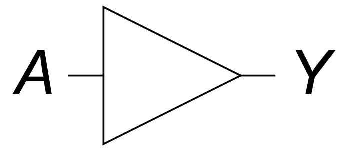
NOT



$$Y = \overline{A}$$

A	Y
0	
1	

BUF

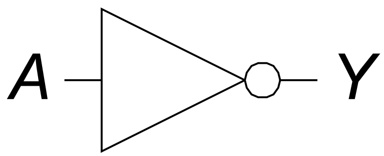


$$Y = A$$

A	Y
0	
1	

Single-Input Logic Gates

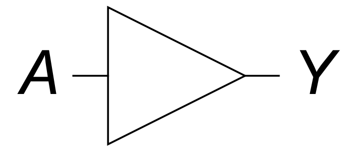
NOT



$$Y = \overline{A}$$

A	Y
0	1
1	0

BUF

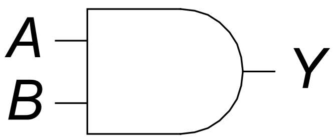


$$Y = A$$

A	Y
0	0
1	1

Two-Input Logic Gates

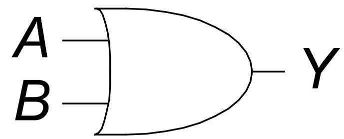
AND



$Y = AB$

A	B	Y
0	0	
0	1	
1	0	
1	1	

OR

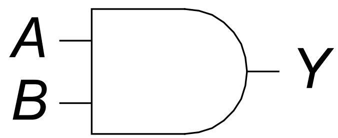


$Y = A + B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

Two-Input Logic Gates

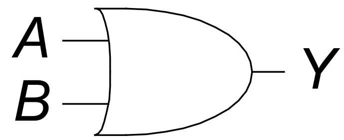
AND



$$Y = AB$$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR

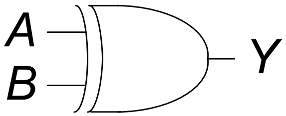


$$Y = A + B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

More Two-Input Logic Gates

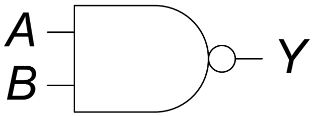
XOR



$Y = A \oplus B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

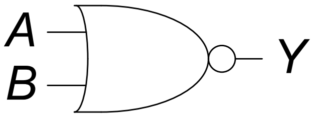
NAND



$Y = \overline{AB}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

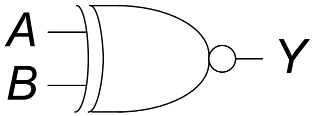
NOR



$Y = \overline{A + B}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

XNOR

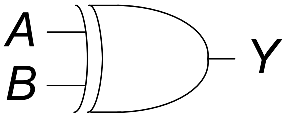


$Y = \overline{A \oplus B}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

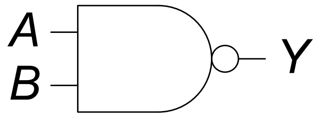
XOR



$Y = A \oplus B$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

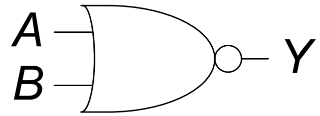
NAND



$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

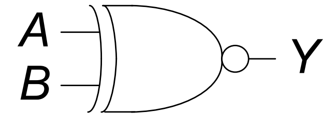
NOR



$Y = \overline{A + B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR

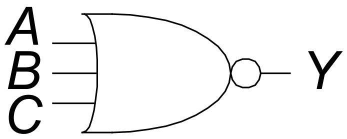


$Y = \overline{A \oplus B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Multiple-Input Logic Gates

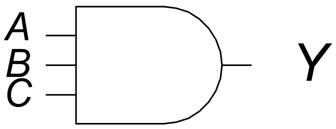
NOR3



$$Y = \overline{A+B+C}$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND3

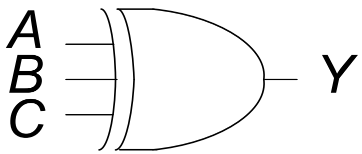


$$Y = ABC$$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Multiple-Input XOR

XOR3



$$Y = A \oplus B \oplus C$$

A	B	C	Y
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

- **Multi-input XOR:** Odd parity – the output is 1 when an **odd** number of inputs is 1.

Logic Levels

- Discrete voltages represent 1 and 0
- For example:
  - 0 = *ground* (GND) or 0 volts
  - 1 = V_{DD} or 5 volts
- What about 4.99 volts? Is that a 0 or a 1?
- What about 3.2 volts?

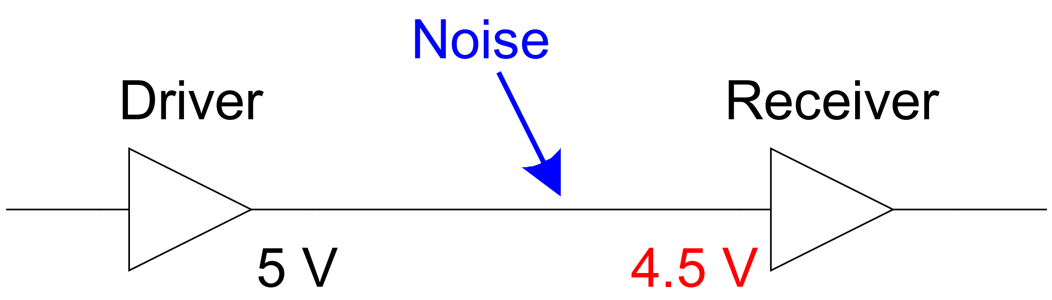
Logic Levels

- *Range* of voltages for 1 and 0
- Different ranges for inputs and outputs to allow for *noise*

What is Noise?

What is Noise?

- **Anything that degrades the signal**
  - E.g., resistance, power supply noise, coupling to neighboring wires, etc.
- **Example:** a gate (driver) outputs 5 V but, because of resistance in a long wire, receiver gets 4.5 V

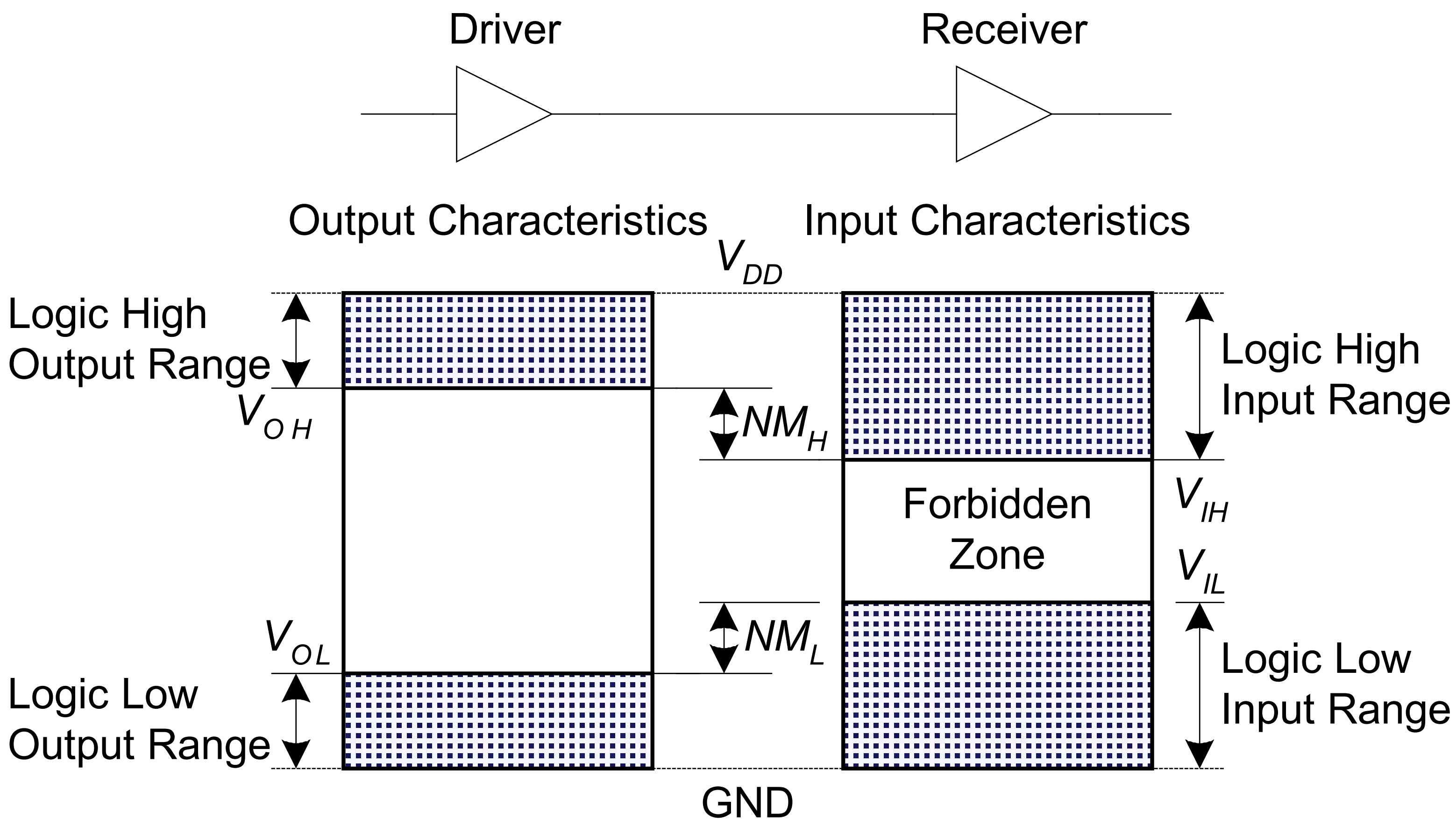


The Static Discipline

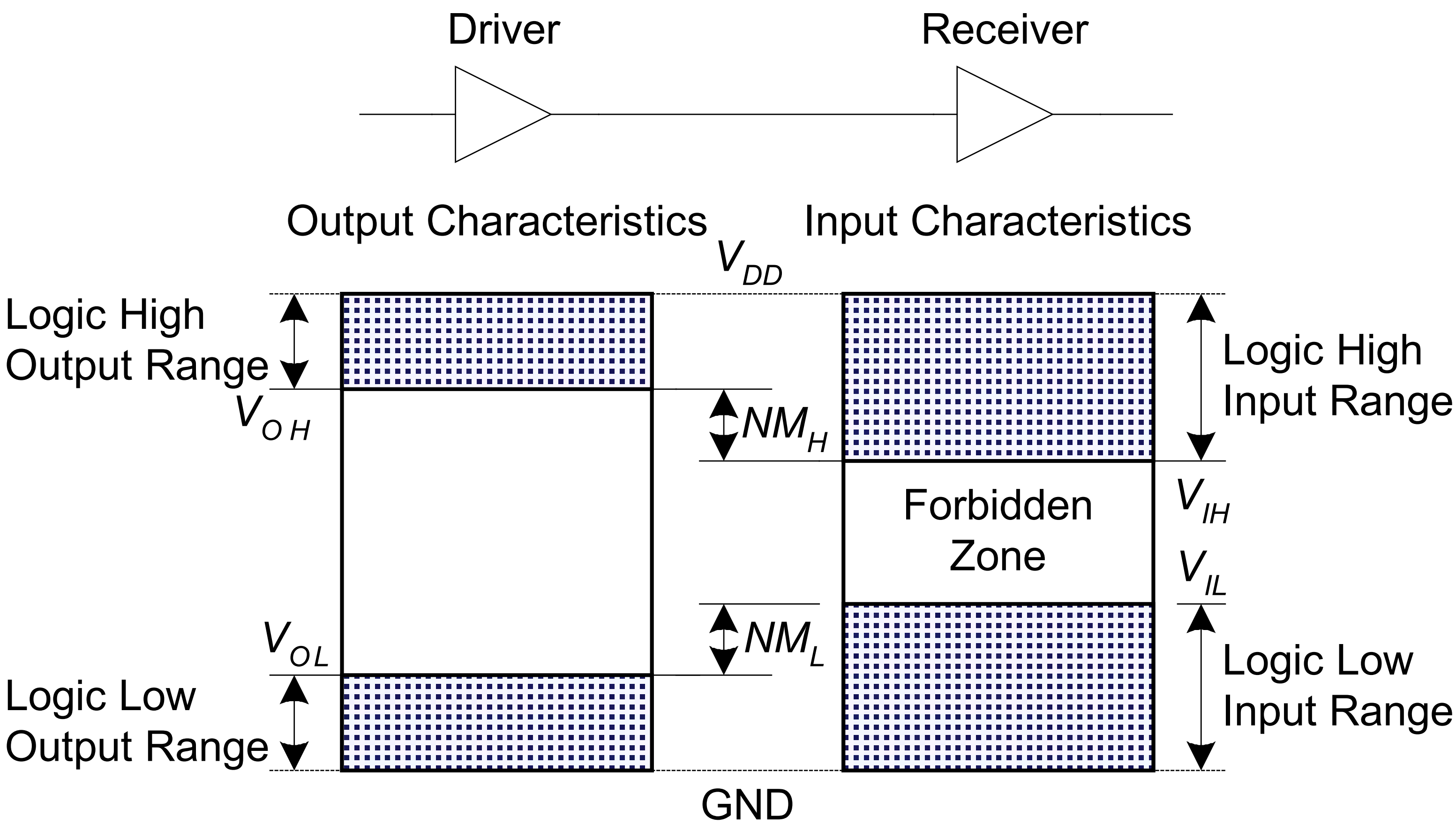
- With logically valid inputs, every circuit element must produce logically valid outputs

- Use limited ranges of voltages to represent discrete values

Logic Levels



Noise Margins

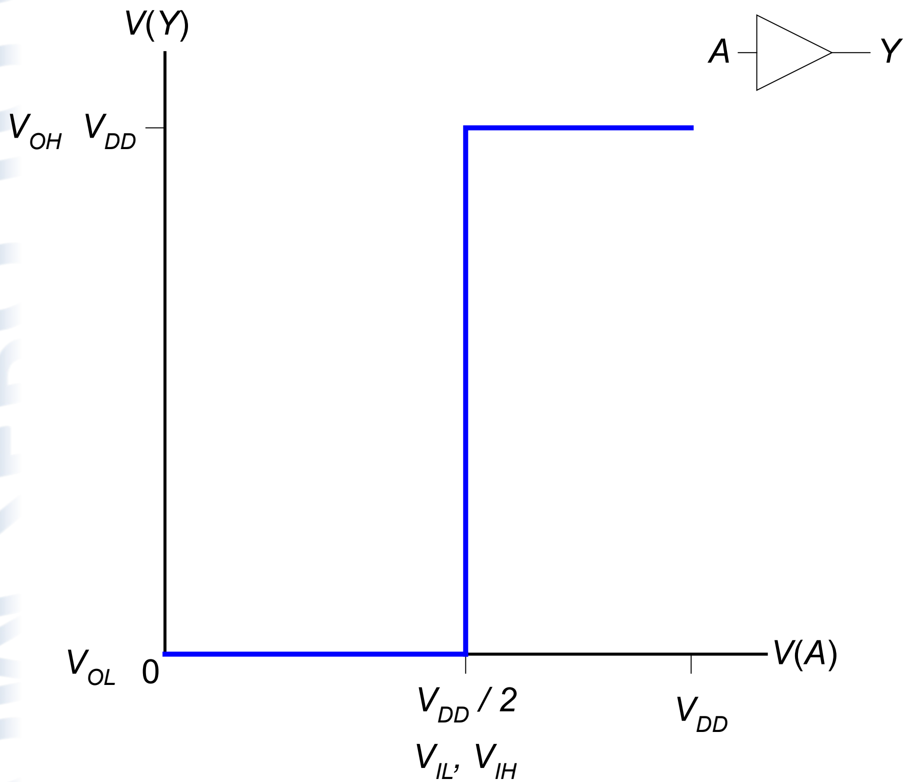


$$NM_H = V_{OH} - V_{IH}$$

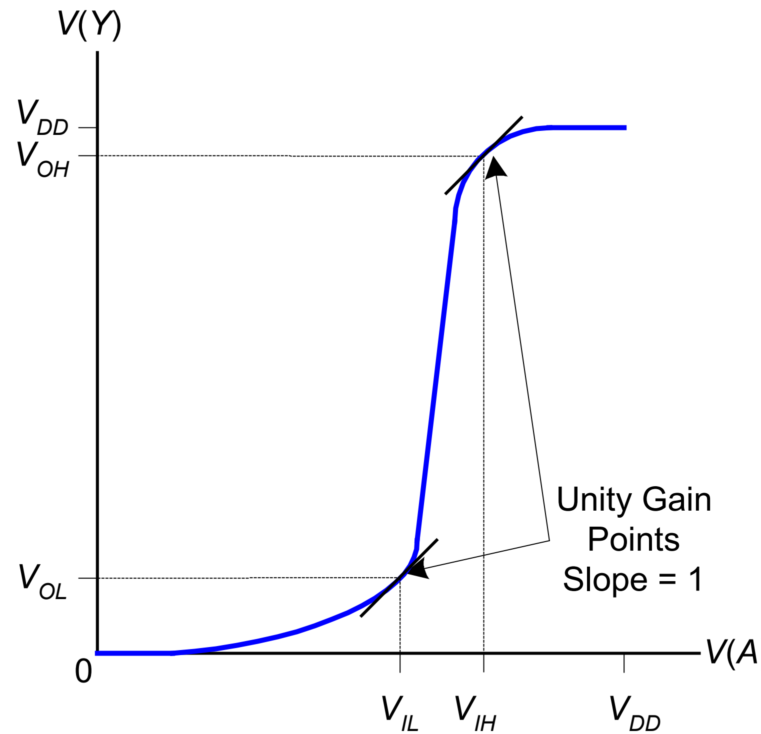
$$NM_L = V_{IL} - V_{OL}$$

DC Transfer Characteristics

Ideal Buffer:



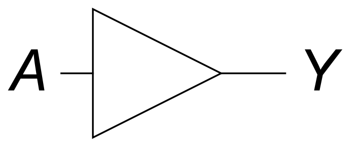
Real Buffer:

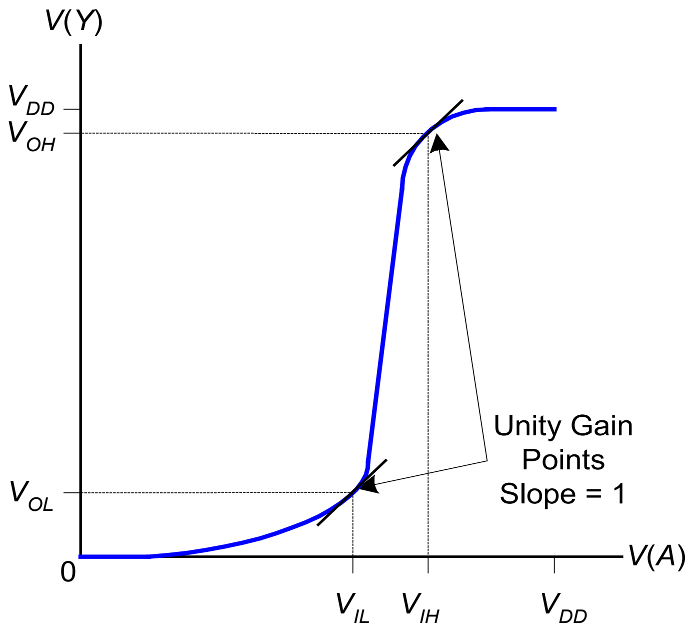


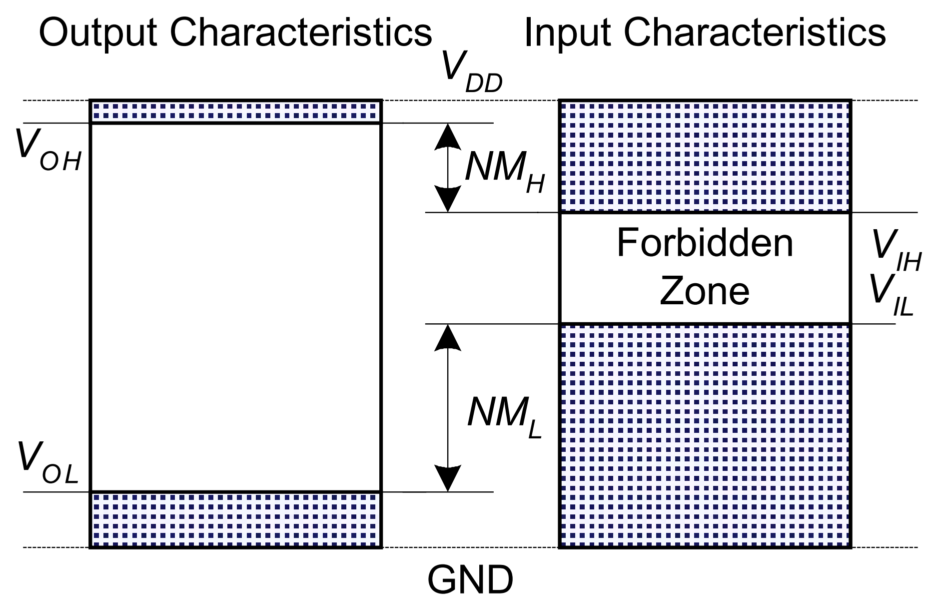
$$NM_H = NM_L = V_{DD}/2$$

$$NM_H\ , NM_L < V_{DD}/2$$

DC Transfer Characteristics

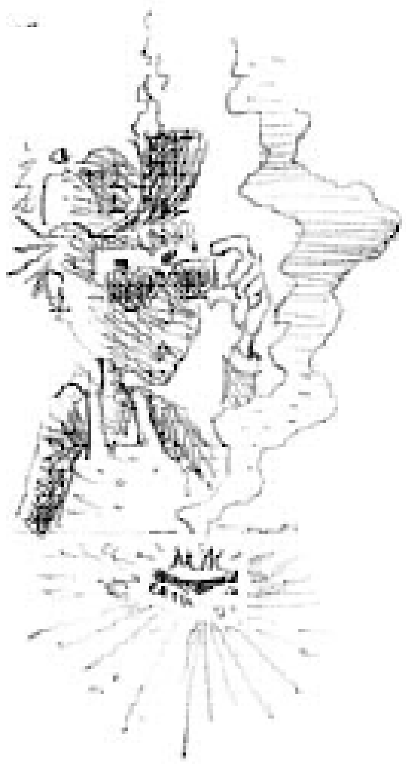






V_{DD} Scaling

- In 1970's and 1980's, V_{DD} = 5 V
- V_{DD} has dropped
  - Avoid frying tiny transistors
  - Save power
- 3.3 V, 2.5 V, 1.8 V, 1.5 V, 1.2 V, 1.0 V, …
- Be careful connecting chips with different supply voltages



Chips operate because they contain magic smoke.

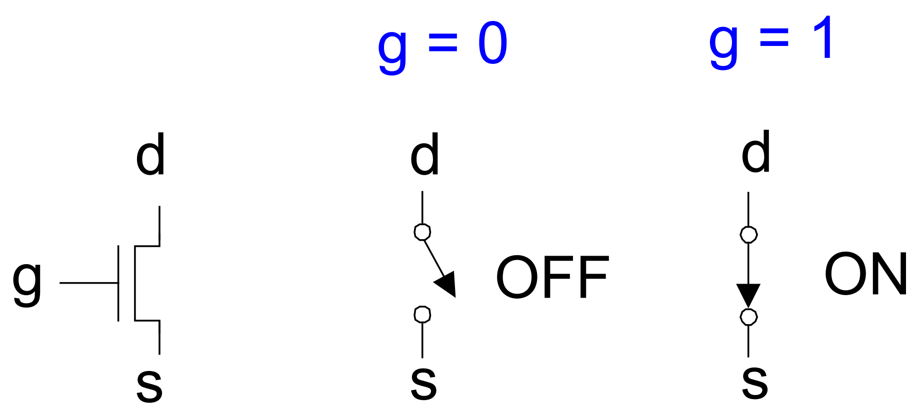
Proof: if the magic smoke is let out, the chip stops working

Logic Family Examples

Logic Family	V_{DD}	V_{IL}	V_{IH}	V_{OL}	V_{OH}
TTL	5 (4.75 - 5.25)	0.8	2.0	0.4	2.4
CMOS	5 (4.5 - 6)	1.35	3.15	0.33	3.84
LVTTL	3.3 (3 - 3.6)	0.8	2.0	0.4	2.4
LVCMOS	3.3 (3 - 3.6)	0.9	1.8	0.36	2.7

Transistors

- Logic gates built from transistors
- 3-ported voltage-controlled switch
  - 2 ports connected depending on voltage of 3rd
  - d and s are connected (ON) when g is 1

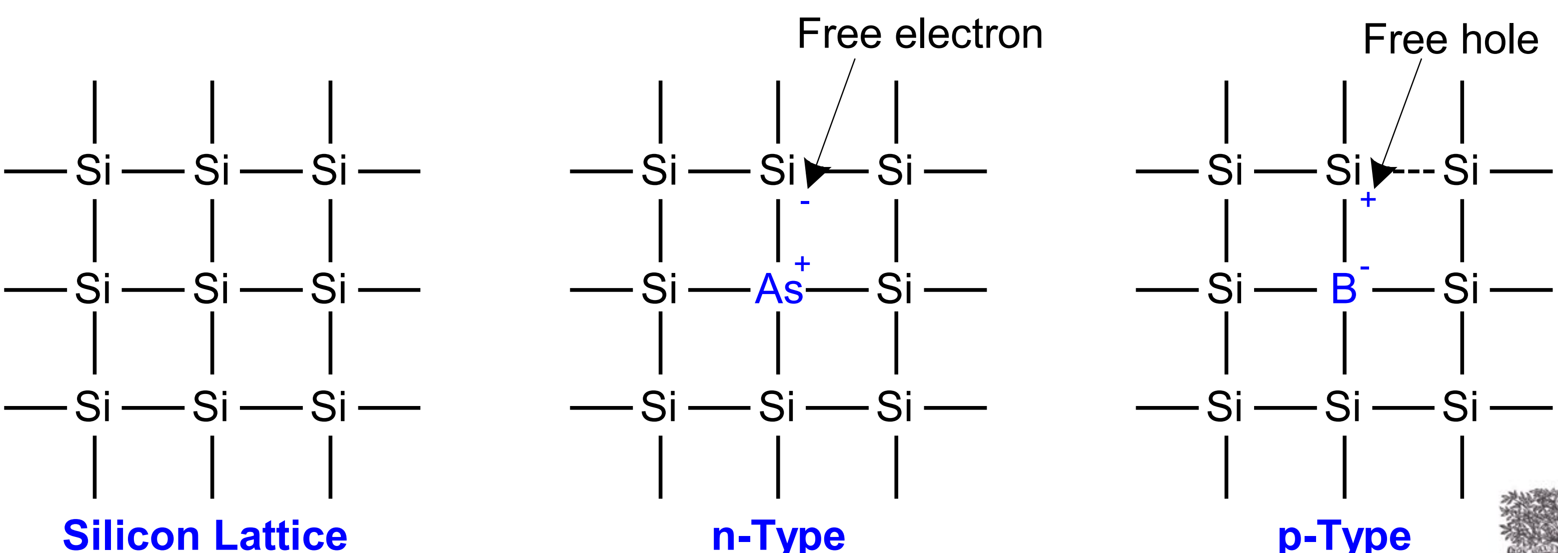


Robert Noyce, 1927-1990

- Nicknamed “Mayor of Silicon Valley”
- Cofounded Fairchild Semiconductor in 1957
- Cofounded Intel in 1968
- Co-invented the integrated circuit

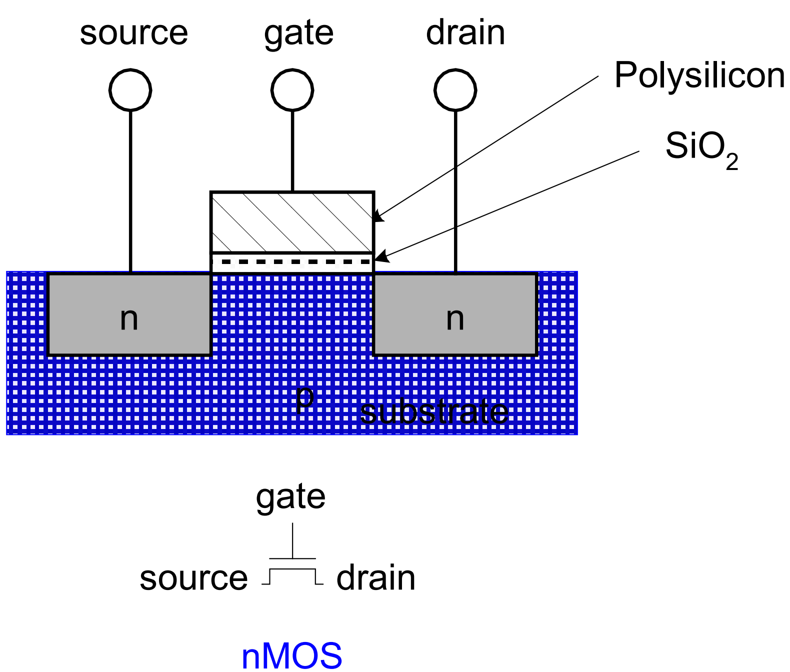
Silicon

- Transistors built from silicon, a semiconductor
- Pure silicon is a poor conductor (no free charges)
- Doped silicon is a good conductor (free charges)
  - **n-type** (free *n*egative charges, electrons)
  - **p-type** (free *p*ositive charges, holes)



MOS Transistors

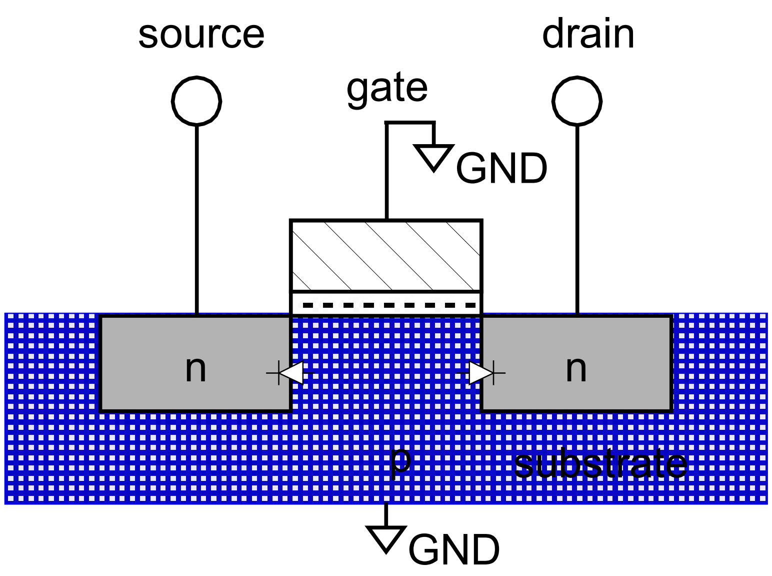
- **Metal oxide silicon (MOS) transistors:**
  - Polysilicon (used to be **metal**) gate
  - **Oxide** (silicon dioxide) insulator
  - Doped **silicon**



Transistors: nMOS

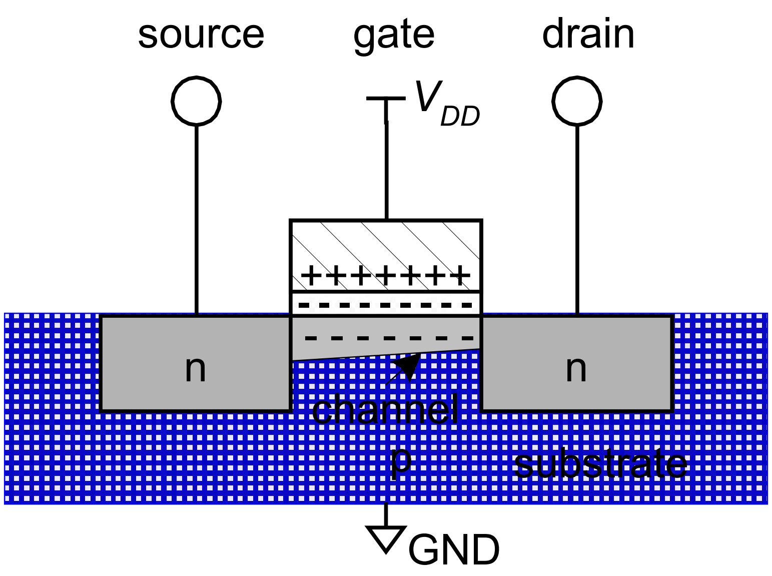
Gate = 0

OFF (no connection between source and drain)



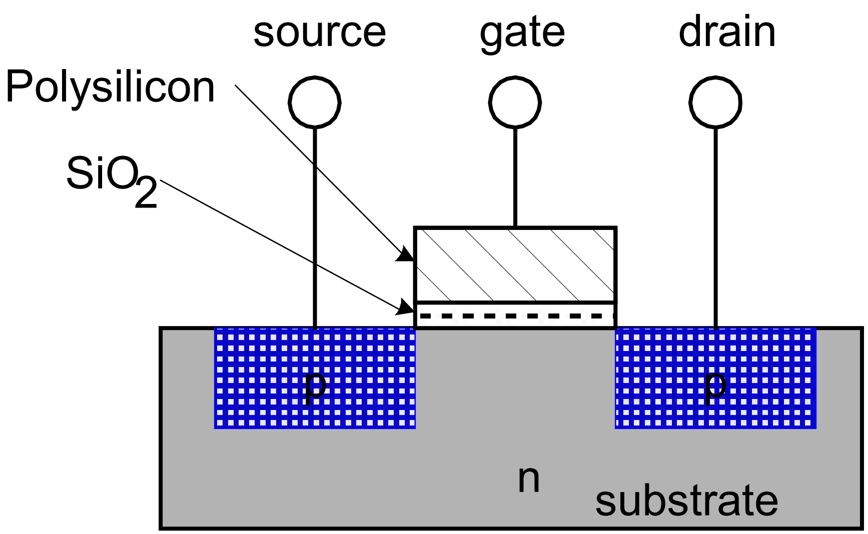
Gate = 1

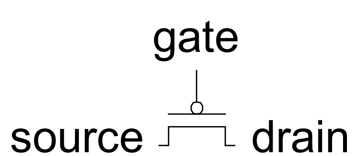
ON (channel between source and drain)



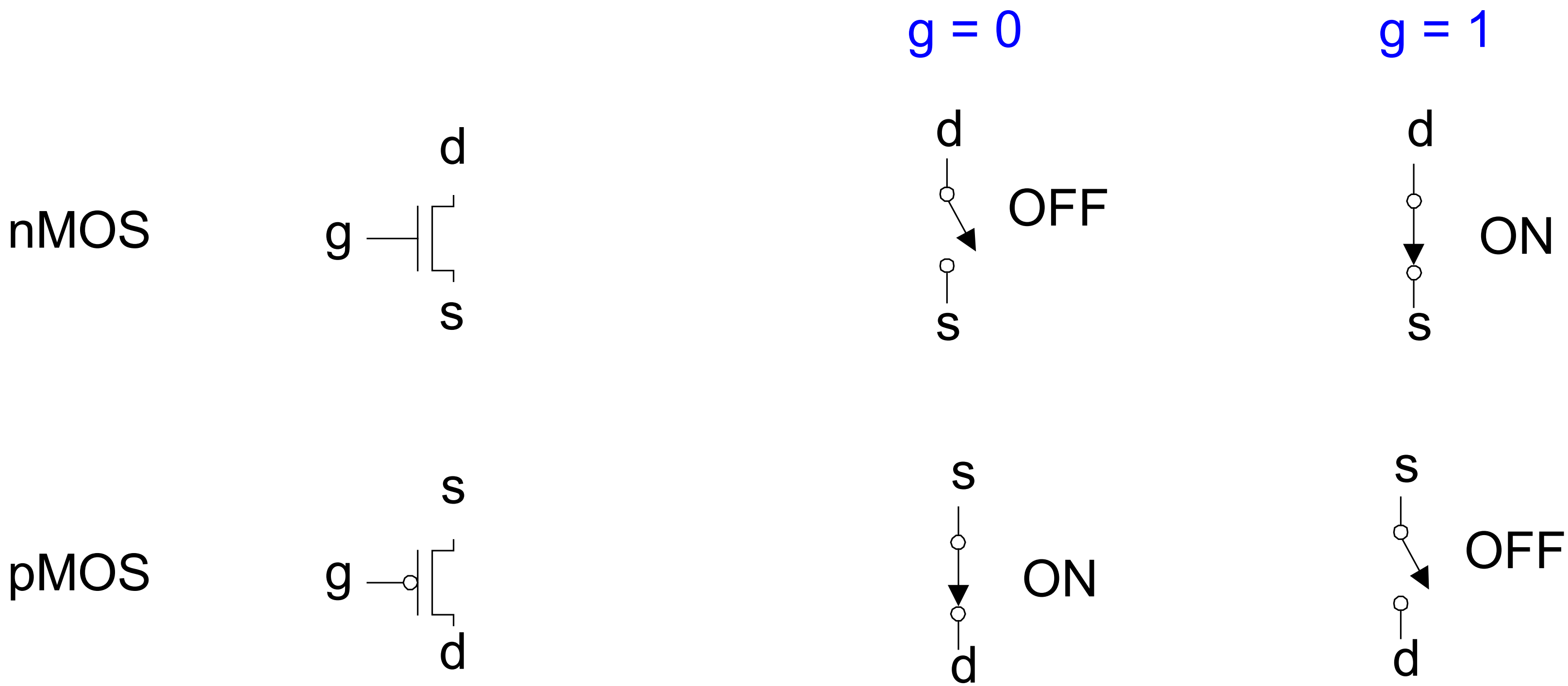
Transistors: pMOS

- pMOS transistor is opposite
  - ON when Gate = 0
  - OFF when Gate = 1



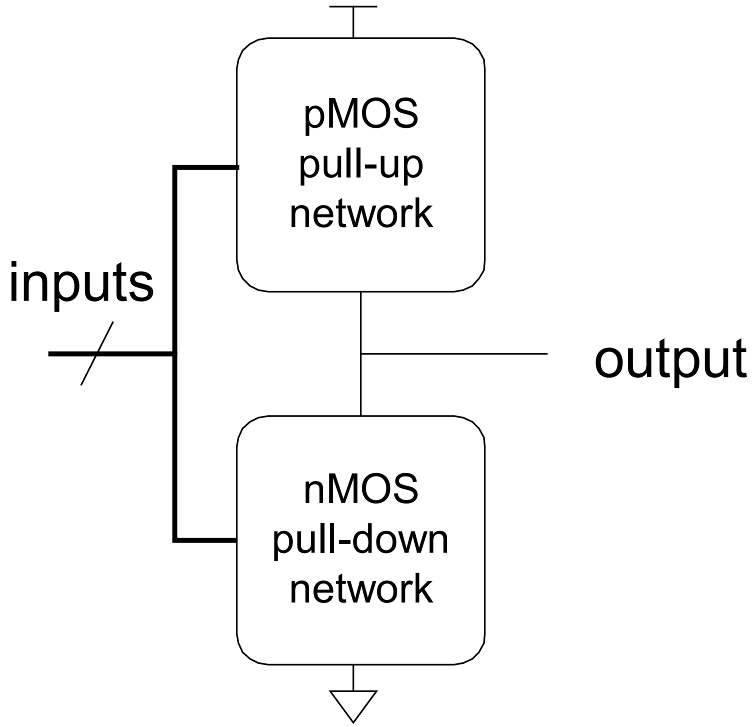


Transistor Function

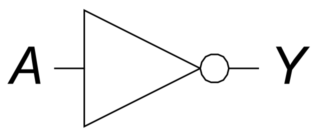


Transistor Function

- **nMOS:** pass good 0's, so connect source to GND
- **pMOS:** pass good 1's, so connect source to V_{DD}

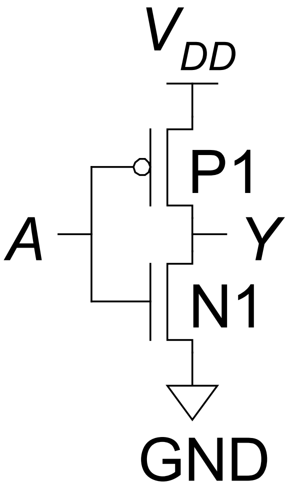


CMOS Gates: NOT Gate



$Y = \overline{A}$

A	Y
0	1
1	0

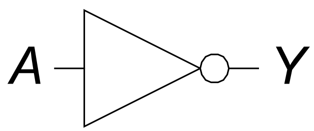


A	P1	N1	*Y*
0			
1			

FROM ZERO TO ONE

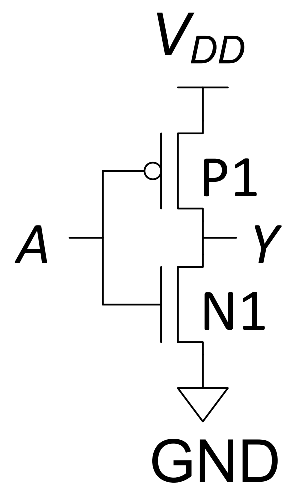
CMOS Gates: NOT Gate

NOT



$Y = \overline{A}$

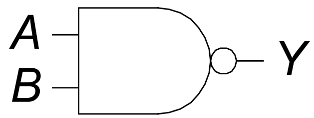
A	Y
0	1
1	0



A	P1	N1	*Y*
0	ON	OFF	1
1	OFF	ON	0

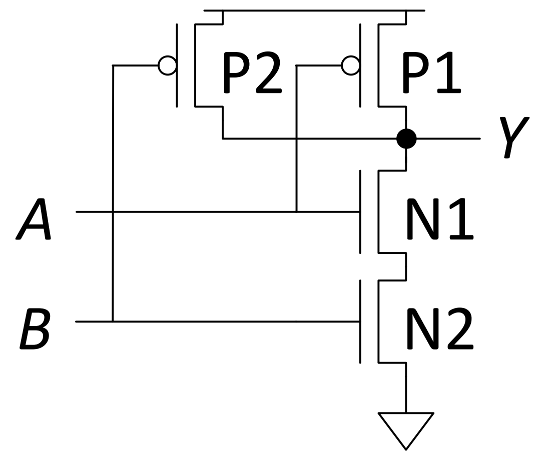
CMOS Gates: NAND Gate

NAND



$$Y = \overline{AB}$$

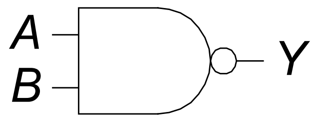
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



A	*B*	P1	P2	N1	N2	*Y*
0	0					
0	1					
1	0					
1	1					

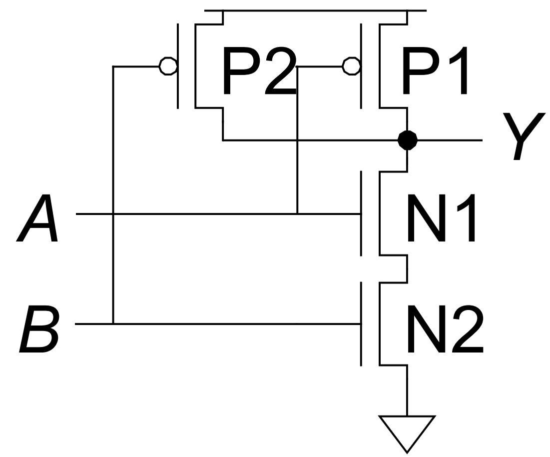
CMOS Gates: NAND Gate

NAND



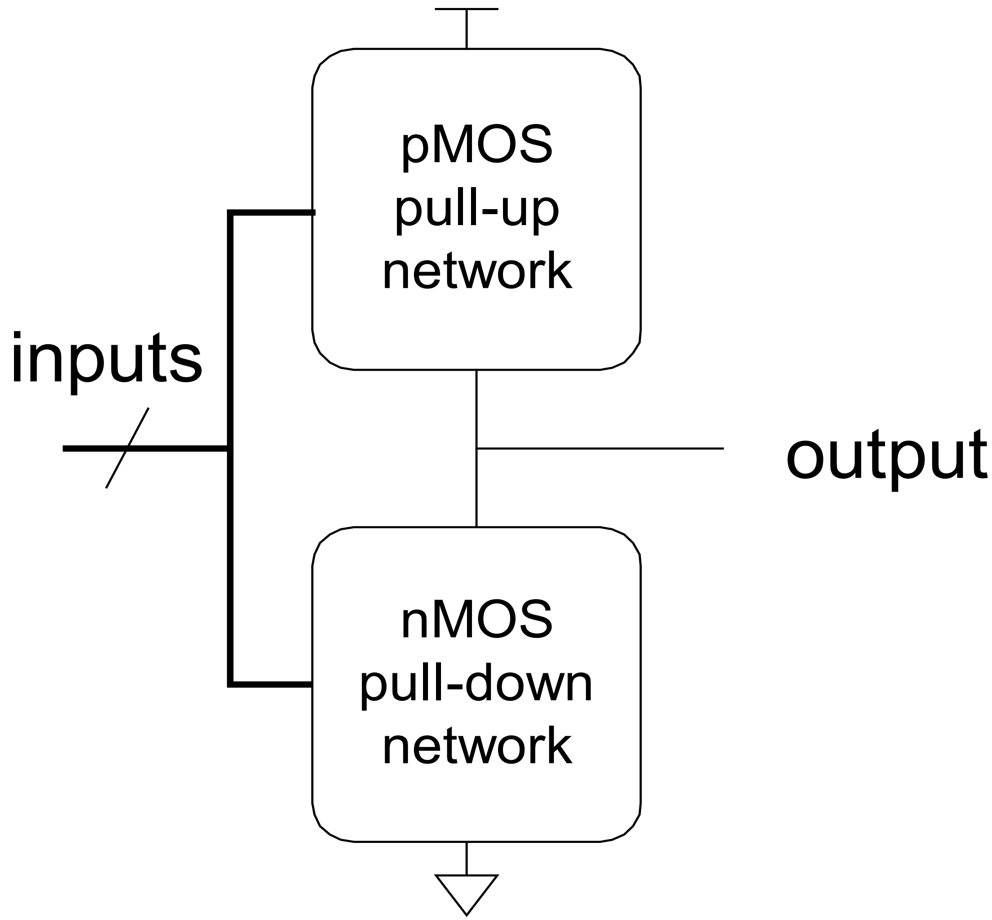
$$Y = \overline{AB}$$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0



A	*B*	P1	P2	N1	N2	*Y*
0	0	ON	ON	OFF	OFF	1
0	1	ON	OFF	OFF	ON	1
1	0	OFF	ON	ON	OFF	1
1	1	OFF	OFF	ON	ON	0

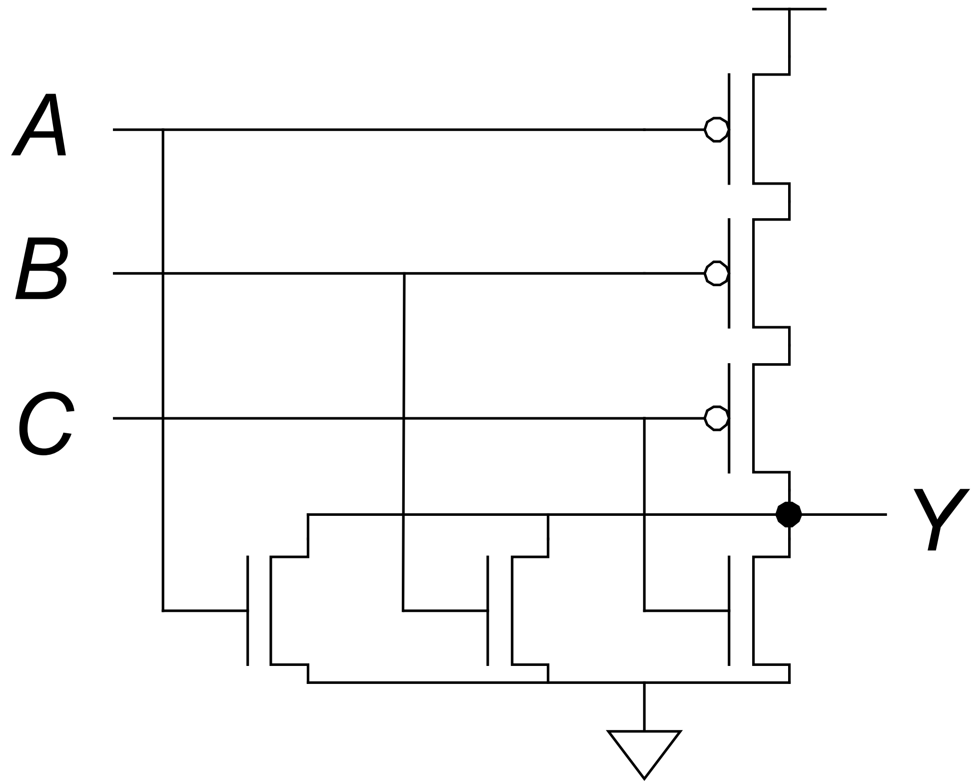
CMOS Gate Structure



NOR Gate

How do you build a three-input NOR gate?

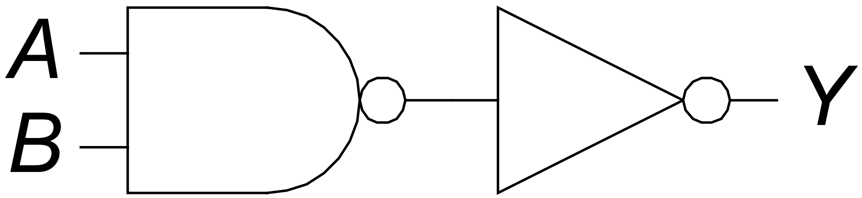
NOR3 Gate

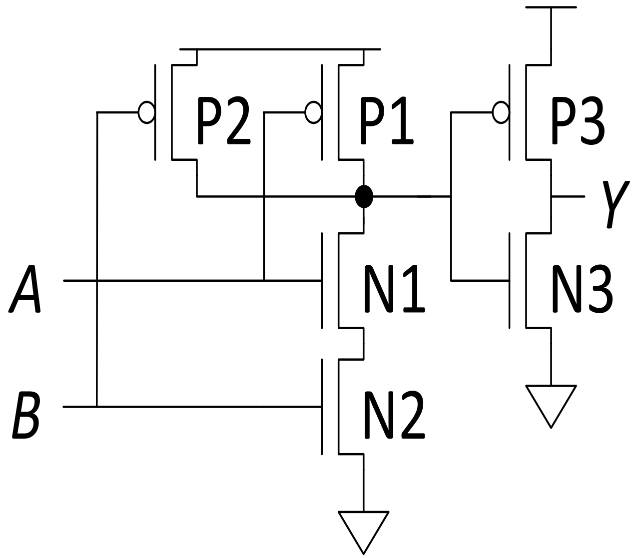


Other CMOS Gates

How do you build a two-input AND gate?

AND2 Gate

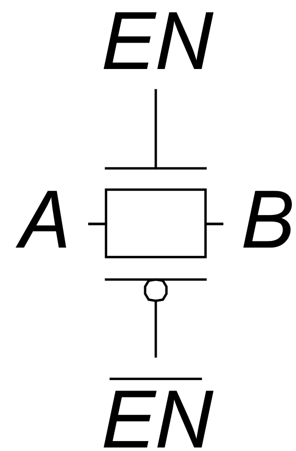




- CMOS is better at building **inverting gates** (i.e., NAND, NOR, etc.)
- They require **fewer transistors**

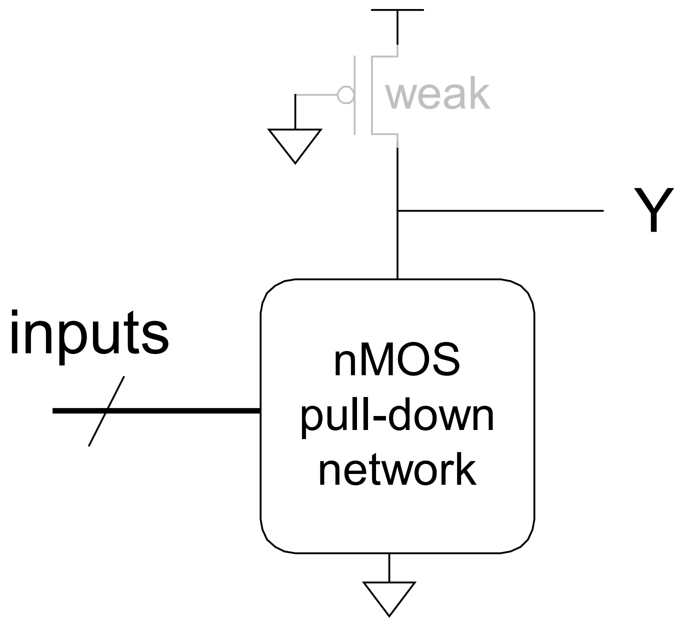
Transmission Gates

- nMOS pass 1's poorly
- pMOS pass 0's poorly
- Transmission gate is a better switch
  - passes both 0 and 1 well
- When *EN* = 1, the switch is ON:
  - $\overline{EN}$ = 0 and *A* is connected to *B*
- When *EN* = 0, the switch is OFF:
  - *A* is not connected to *B*



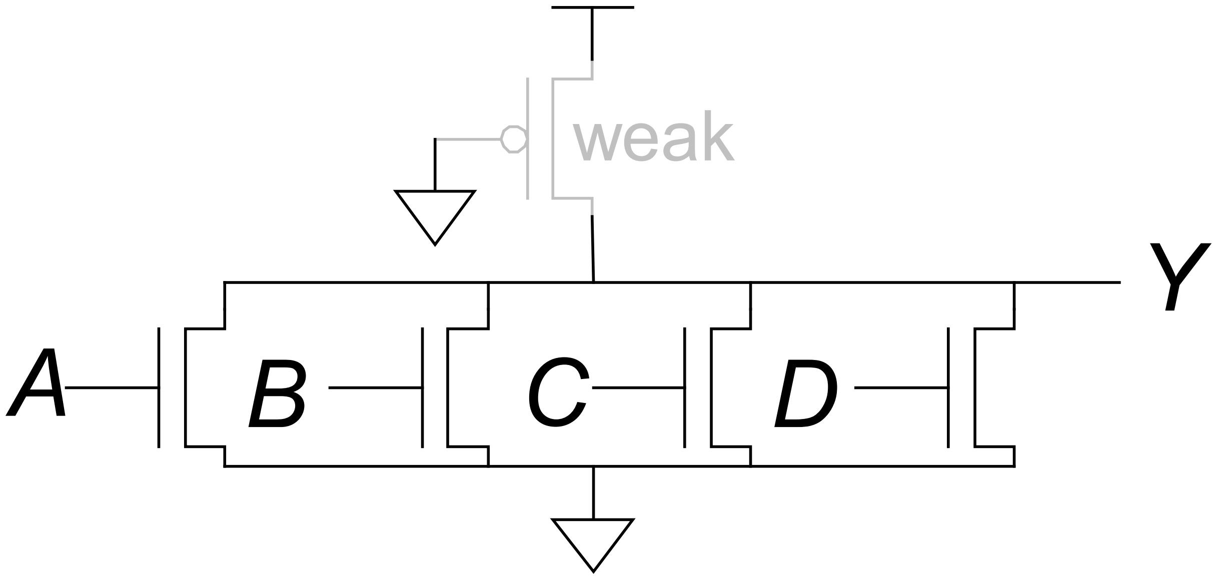
Pseudo-nMOS Gates

- Replace pull-up network with *weak* pMOS transistor that is always on
- pMOS transistor: pulls output HIGH *only* when nMOS network not pulling it LOW



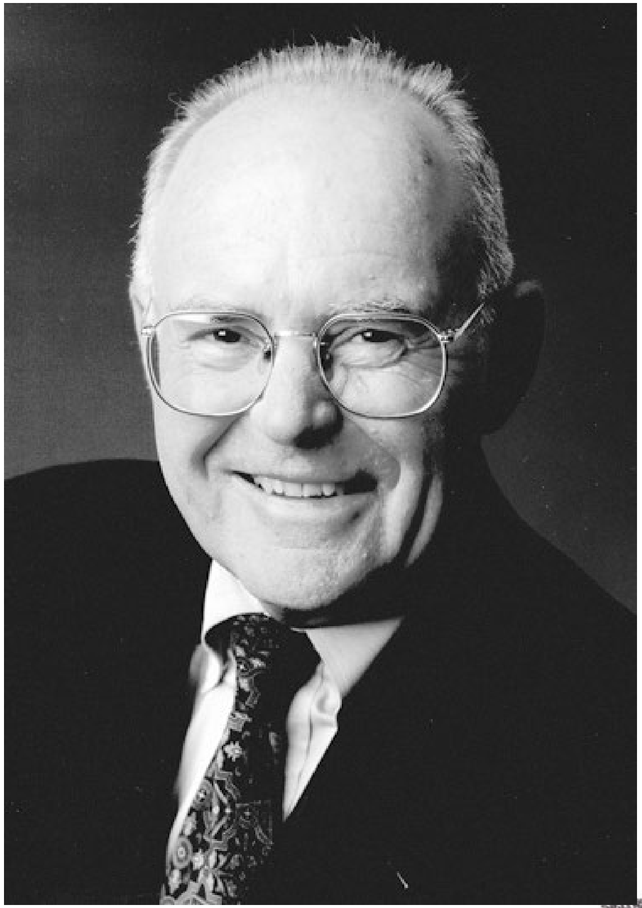
Pseudo-nMOS Example

Pseudo-nMOS **NOR4**

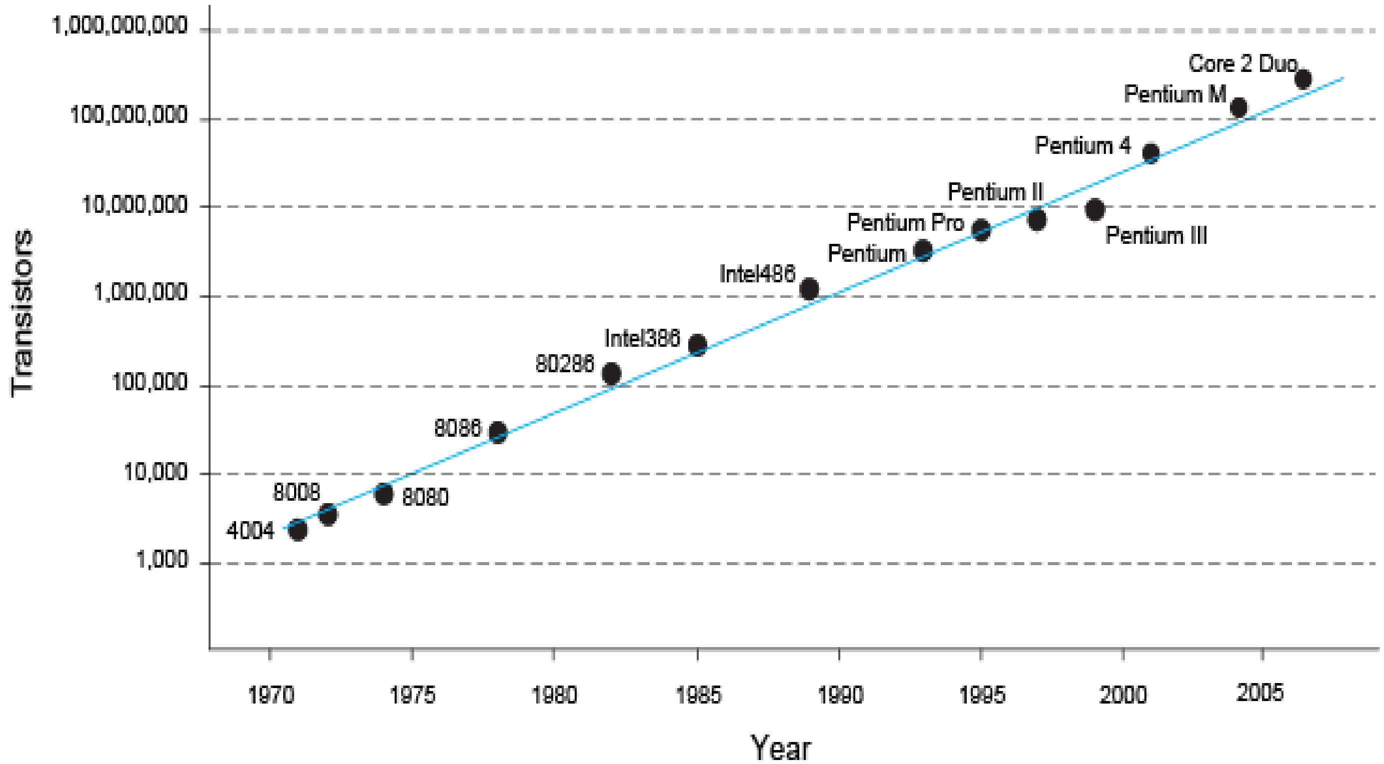


Gordon Moore, 1929-

- Cofounded Intel in 1968 with Robert Noyce.
- **Moore's Law:** number of transistors on a computer chip doubles every year (observed in 1965)
- Since 1975, transistor counts have doubled every two years.



Moore's Law



- *"If the automobile had followed the same development cycle as the computer, a Rolls-Royce would today cost $100, get one million miles to the gallon, and explode once a year . . ."*

  – Robert Cringley

Power Consumption

- Power = Energy consumed per unit time
  - Dynamic power consumption
  - Static power consumption

Dynamic Power Consumption

- **Power to charge transistor gate capacitances**
  - Energy required to charge a capacitance, C, to V_{DD} is CV_{DD}^2
  - Circuit running at frequency f: transistors switch (from 1 to 0 or vice versa) at that frequency
  - Capacitor is charged $f/2$ times per second (discharging from 1 to 0 is free)
- Dynamic power consumption:

$$\boxed{P_{dynamic} = \frac{1}{2} C V_{DD}^2 f}$$

Static Power Consumption

- Power consumed when no gates are switching
- Caused by the *quiescent supply current*, I_{DD} (also called the *leakage current*)
- Static power consumption:

$$\boxed{P_{static} = I_{DD}V_{DD}}$$

Total Power Consumption

- Dynamic power + static power

$$\boxed{P_{total} = P_{static} + P_{dynamic}}$$

Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - V_{DD} = 1.2 V
  - C = 20 nF
  - f = 1 GHz
  - I_{DD} = 20 mA

Power Consumption Example

- Estimate the power consumption of a wireless handheld computer
  - V_{DD} = 1.2 V
  - C = 20 nF
  - f = 1 GHz
  - I_{DD} = 20 mA

$$\mathbf{P = \tfrac{1}{2}CV_{DD}^2 f + I_{DD}V_{DD}}$$

$$= \tfrac{1}{2}(20\text{ nF})(1.2\text{ V})^2(1\text{ GHz}) + (20\text{ mA})(1.2\text{ V})$$

$$\mathbf{= (14.4 + 0.024)\ W \approx 14.4\ W}$$