COMBINATIONAL LOGIC DESIGN

Chapter 2

***Digital Design and Computer Architecture*, 2nd Edition**

David Money Harris and Sarah L. Harris

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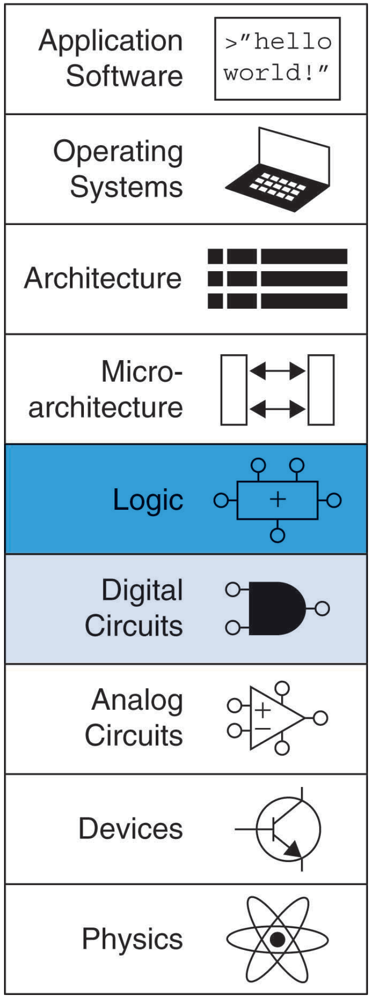
Chapter 2 :: Topics

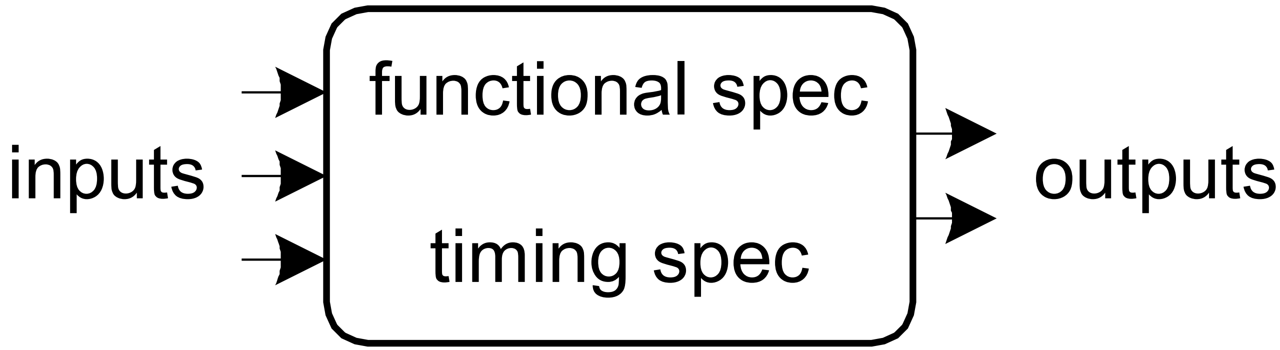
- Introduction
- Boolean Equations
- Boolean Algebra
- From Logic to Gates
- Multilevel Combinational Logic
- X's and Z's, Oh My
- Karnaugh Maps
- Combinational Building Blocks
- Timing



Introduction

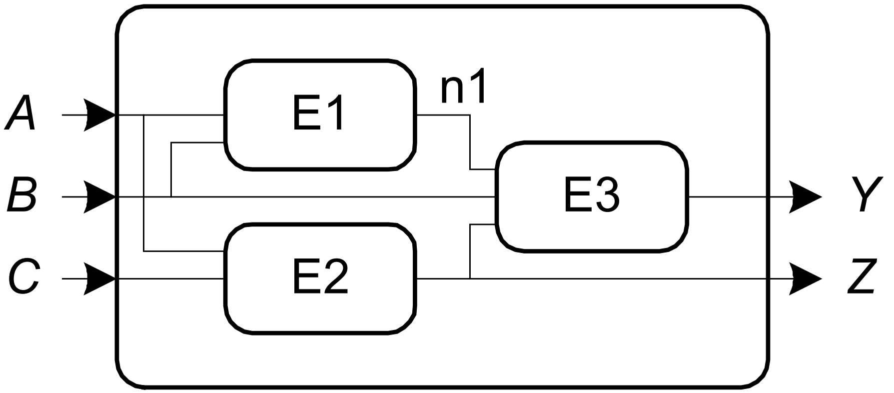
A logic circuit is composed of:

- Inputs
- Outputs
- Functional specification
- Timing specification



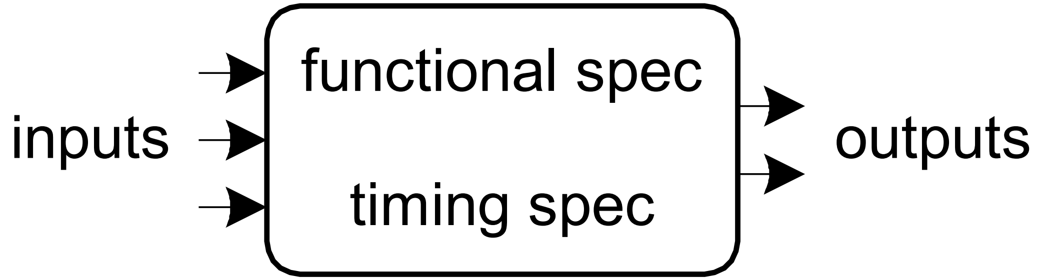
Circuits

- **Nodes**
  - Inputs: *A*, *B*, *C*
  - Outputs: *Y*, *Z*
  - Internal: n1
- **Circuit elements**
  - E1, E2, E3
  - Each a circuit



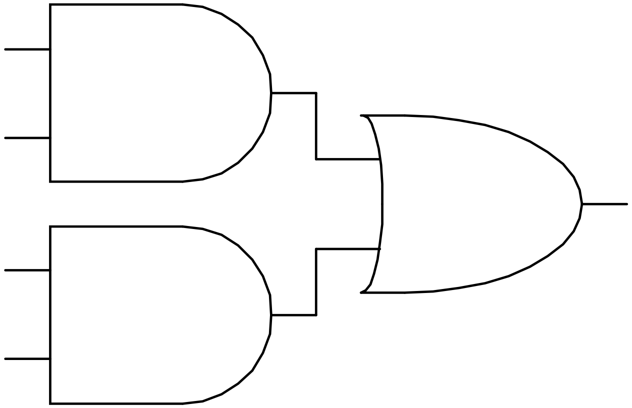
Types of Logic Circuits

- **Combinational Logic**
  - Memoryless
  - Outputs determined by current values of inputs
- **Sequential Logic**
  - Has memory
  - Outputs determined by previous and current values of inputs



Rules of Combinational Composition

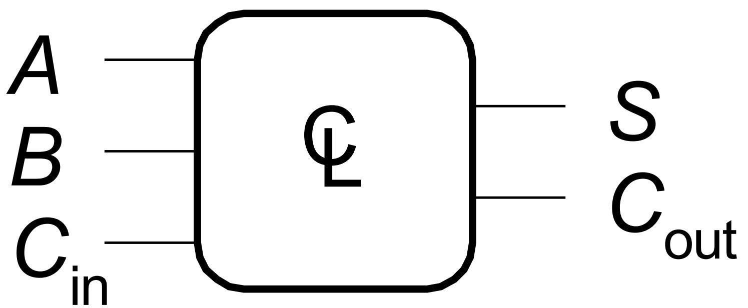
- Every element is combinational
- Every node is either an input or connects to *exactly one* output
- The circuit contains no cyclic paths
- **Example:**



Boolean Equations

- Functional specification of outputs in terms of inputs
- **Example:** $S = F(A, B, C_{in})$

  $C_{out} = F(A, B, C_{in})$



$$S = A \oplus B \oplus C_{in}$$
$$C_{out} = AB + AC_{in} + BC_{in}$$

Some Definitions

- **Complement:** variable with a bar over it
  $\bar{A}, \bar{B}, \bar{C}$
- **Literal:** variable or its complement
  $A, \bar{A}, B, \bar{B}, C, \bar{C}$
- **Implicant:** product of literals
  $AB\bar{C}, \bar{A}C, BC$
- **Minterm:** product that includes all input variables
  $AB\bar{C}, A\bar{B}\bar{C}, ABC$
- **Maxterm:** sum that includes all input variables
  $(A+\bar{B}+C), (\bar{A}+B+\bar{C}), (\bar{A}+\bar{B}+C)$

Sum-of-Products (SOP) Form

- All Boolean equations can be written in SOP form
- Each row has a **minterm**

A	B	Y	minterm	minterm name
0	0		$\overline{A}\ \overline{B}$	m_0
0	1		$\overline{A}\ B$	m_1
1	0		$A\ \overline{B}$	m_2
1	1		$A\ B$	m_3

Sum-of-Products (SOP) Form

- All Boolean equations can be written in SOP form
- Each row has a **minterm**
- A minterm is a **product** (AND) of literals
- Each minterm is TRUE for that row (and only that row)

A	*B*	*Y*	minterm	minterm name
0	0		$\overline{A}\ \overline{B}$	m_0
0	1		$\overline{A}\ B$	m_1
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1	1		$A\ B$	m_3

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- A minterm is a **product** (AND) of literals
- Each minterm is **TRUE** for that row (and only that row)
- Form function by **ORing minterms** where **output is 1**
- Thus, a **sum** (OR) of **products** (AND terms)

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\,\overline{B}$	m_0
0	1	**1**	$\overline{A}\,B$	m_1
1	0	0	$A\,\overline{B}$	m_2
1	1	**1**	$A\,B$	m_3

$$Y = \mathrm{F}(A, B) = \overline{A}B + AB$$

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1	1	**1**	$A\ B$	m_3

$$Y = \mathrm{F}(A, B) = \overline{A}B + AB = \Sigma(1, 3)$$

Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**

A	B	Y	maxterm	maxterm name
0	0		$A + B$	M_0
0	1		$A + \overline{B}$	M_1
1	0		$\overline{A} + B$	M_2
1	1		$\overline{A} + \overline{B}$	M_3

Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**
- A maxterm is a **sum** (OR) of literals
- Each maxterm is **FALSE** for that row (and only that row)

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1	0		$\overline{A} + B$	M_2
1	1		$\overline{A} + \overline{B}$	M_3

Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**
- A maxterm is a **sum** (OR) of literals
- Each maxterm is **FALSE** for that row (and only that row)
- Form function by **ANDing maxterms** where **output is 0**
- Thus, a **product** (AND) of **sums** (OR terms)

A	B	Y	maxterm	maxterm name
0	0	**0**	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	**0**	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

$$Y = \mathrm{F}(A, B) = (A + B) \bullet (A + \overline{B})$$

Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**
- A maxterm is a **sum** (OR) of literals
- Each maxterm is **FALSE** for that row (and only that row)
- Form function by **ANDing maxterms** where **output is 0**
- Thus, a **product** (AND) of **sums** (OR terms)

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

$$Y = \mathrm{F}(A, B) = (A + B)(A + \overline{B}) = \Pi(0, 2)$$

Boolean Equations Example

- You are going to the cafeteria for lunch
  - You won't eat lunch (E = 0)
  - If it's not open (O = 0) or
  - If they only serve corndogs (C = 1)
- Write a truth table for determining if you will eat lunch (E).

O	*C*	*E*
0	0	
0	1	
1	0	
1	1	

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O	*C*	*E*
0	0	0
0	1	0
1	0	1
1	1	0

SOP & POS Form

- **SOP** – sum-of-products

O	C	E	minterm
0	0		$\overline{O}\ \overline{C}$
0	1		$\overline{O}\ C$
1	0		$O\ \overline{C}$
1	1		$O\ C$

- **POS** – product-of-sums

O	C	E	maxterm
0	0		$O + C$
0	1		$O + \overline{C}$
1	0		$\overline{O} + C$
1	1		$\overline{O} + \overline{C}$

SOP & POS Form

- **SOP** – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(2)$$

- **POS** – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$E = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(0, 1, 3)$$

Boolean Algebra

- Axioms and theorems to **simplify** Boolean equations
- Like regular algebra, but simpler: variables have only two values (1 or 0)
- **Duality** in axioms and theorems:
  - ANDs and ORs, 0's and 1's interchanged

Boolean Axioms

Number	Axiom	Name
A1	$B = 0$ if $B \neq 1$	Binary Field
A2	$0 = \overline{1}$	NOT
A3	$0 \bullet 0 = 0$	AND/OR
A4	$1 \bullet 1 = 1$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	AND/OR

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A5	$0 \bullet 1 = 1 \bullet 0 = 0$	AND/OR

Dual: Replace: $\bullet$ with $+$
0 with 1

Boolean Axioms

Number	Axiom	Dual	Name
A1	B = 0 if B ≠ 1	B = 1 if B ≠ 0	Binary Field
A2	$0 = \overline{1}$	$1 = \overline{0}$	NOT
A3	0 • 0 = 0	1 + 1 = 1	AND/OR
A4	1 • 1 = 1	0 + 0 = 0	AND/OR
A5	0 • 1 = 1 • 0 = 0	1 + 0 = 0 + 1 = 1	AND/OR

Dual: Replace: • with +
0 with 1

Boolean Theorems of One Variable

Number	Theorem	Name
T1	$B \bullet 1 = B$	Identity
T2	$B \bullet 0 = 0$	Null Element
T3	$B \bullet B = B$	Idempotency
T4	$\overline{\overline{B}} = B$	Involution
T5	$B \bullet \overline{B} = 0$	Complements

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Dual: Replace: • with +
0 with 1

Boolean Theorems of One Variable

Number	Theorem	Dual	Name
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	$B + \overline{B} = 1$	Complements

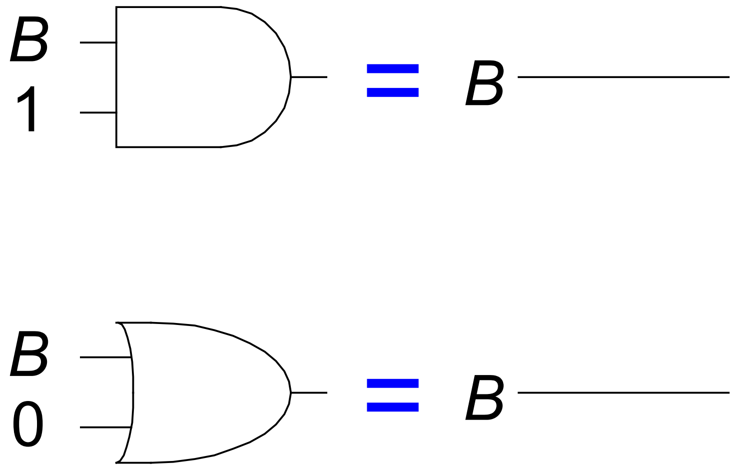
Dual: Replace: $\bullet$ with $+$
0 with 1

T1: Identity Theorem

- $B \bullet 1 = B$
- $B + 0 = B$

T1: Identity Theorem

- $B \cdot 1 = B$
- $B + 0 = B$

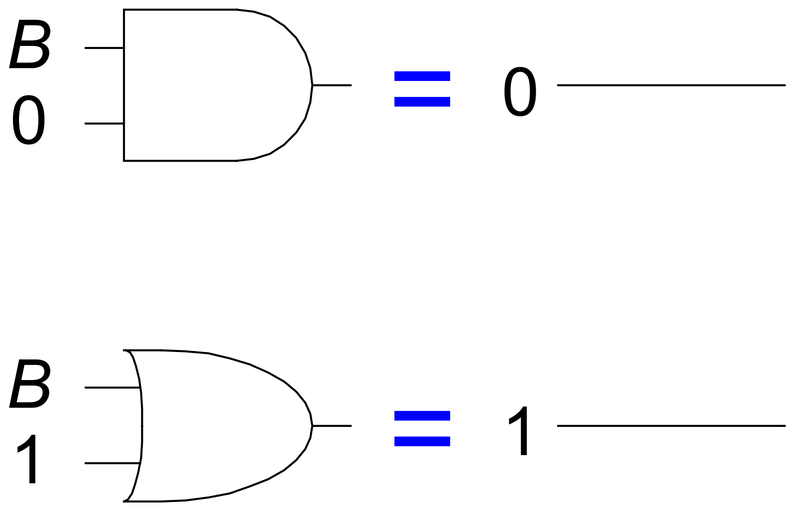


T2: Null Element Theorem

- $B \cdot 0 = 0$
- $B + 1 = 1$

T2: Null Element Theorem

- $B \cdot 0 = 0$
- $B + 1 = 1$

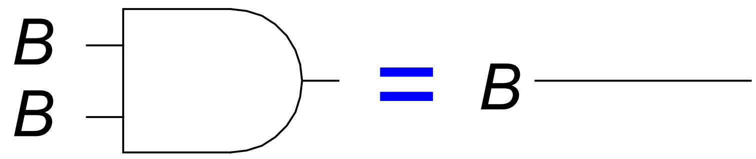


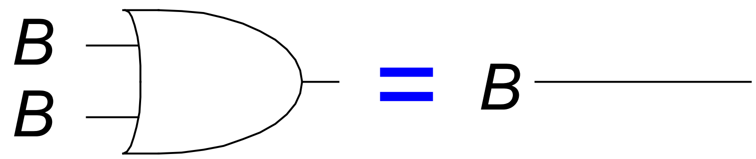
T3: Idempotency Theorem

- $B \cdot B = B$
- $B + B = B$

T3: Idempotency Theorem

- $B \cdot B = B$
- $B + B = B$



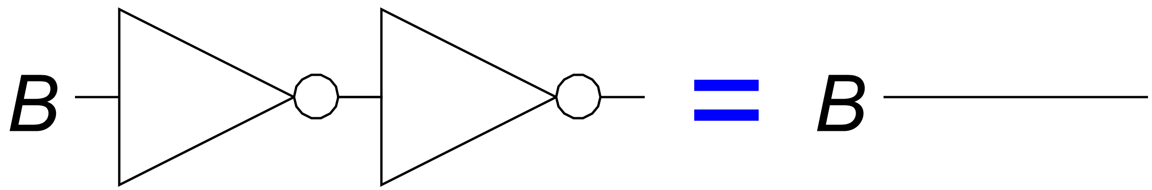


T4: Identity Theorem

- $\overline{\overline{B}} = B$

T4: Identity Theorem

- $\overline{\overline{B}} = B$

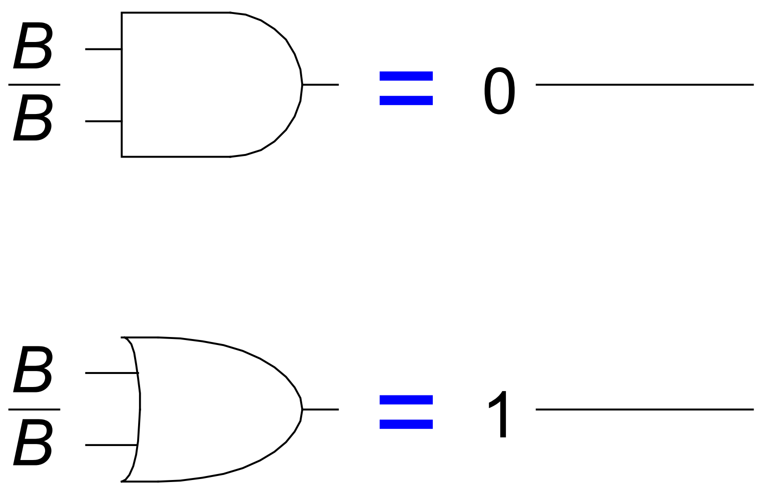


T5: Complement Theorem

- $B \bullet \overline{B} = 0$
- $B + \overline{B} = 1$

T5: Complement Theorem

- $B \cdot \overline{B} = 0$
- $B + \overline{B} = 1$



Recap: Basic Boolean Theorems

Number	Theorem	Dual	Name
T1	$B \bullet 1 = B$	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	$B + B = B$	Idempotency
T4	$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	$B + \overline{B} = 1$	Complements

Dual: Replace: • with +
0 with 1

Boolean Theorems of Several Vars

Number	Theorem	Name
T6	$B \bullet C = C \bullet B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	Distributivity
T9	$B \bullet (B+C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + (\overline{B} \bullet D)$	Consensus

Boolean Theorems of Several Vars

#	Theorem	Dual	Name
T6	$B \bullet C = C \bullet B$	$B+C = C+B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)\,(B+D)$	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

Dual: Replace: $\bullet$ with $+$

0 with 1

Boolean Theorems of Several Vars

#	Theorem	Dual	Name
T6	B•C = C•B	B+C = C+B	Commutativity
T7	(B•C) • D = B • (C•D)	(B + C) + D = B + (C + D)	Associativity
T8	B • (C + D) = (B•C) + (B•D)	$\boxed{\text{B + (C•D) = (B+C) (B+D)}}$	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	(B•C) + (B•$\overline{\text{C}}$) = B	(B+C) • (B+$\overline{\text{C}}$) = B	Combining
T11	(B•C) + ($\overline{\text{B}}$•D) + (C•D) = (B•C) + ($\overline{\text{B}}$•D)	(B+C) • ($\overline{\text{B}}$+D) • (C+D) = (B+C) • ($\overline{\text{B}}$+D)	Consensus

Warning: T8' differs from traditional algebra:
OR (+) distributes over AND (•)

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Boolean Theorems of Several Vars

#	Theorem	Dual	Name
T6	B•C = C•B	B+C = C+B	Commutativity
T7	(B•C) • D = B • (C•D)	(B + C) + D = B + (C + D)	Associativity
T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	(B•C) + (B•$\overline{C}$) = B	(B+C) • (B+$\overline{C}$) = B	Combining
T11	(B•C) + ($\overline{B}$•D) + (C•D) = (B•C) + ($\overline{B}$•D)	(B+C) • ($\overline{B}$+D) • (C+D) = (B+C) • ($\overline{B}$+D)	Consensus

How do we prove these are true?

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How to Prove

- **Method 1:** Perfect induction
- **Method 2:** Use other theorems and axioms to simplify the equation
  - Make one side of the equation look like the other

Proof by Perfect Induction

- Also called: **proof by exhaustion**
- Check every possible input value
- If the two expressions produce the same value for every possible input combination, the expressions are equal

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Example: Proof by Perfect Induction

Number	Theorem	Name
T6	B•C = C•B	Commutativity

B	***C***	***BC***	***CB***
0	0		
0	1		
1	0		
1	1		

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Example: Proof by Perfect Induction

Number	Theorem	Name
T6	B•C = C•B	Commutativity

B	*C*	*BC*	*CB*
0	0	0	0
0	1	0	0
1	0	0	0
1	1	1	1

Boolean Theorems of Several Vars

#	Theorem	Dual	Name
T6	B•C = C•B	B+C = C+B	Commutativity
T7	(B•C) • D = B • (C•D)	(B + C) + D = B + (C + D)	Associativity
T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
T9	B • (B+C) = B	B + (B•C) = B	Covering
T10	(B•C) + (B•$\overline{C}$) = B	(B+C) • (B+$\overline{C}$) = B	Combining
T11	(B•C) + ($\overline{B}$•D) + (C•D) = (B•C) + ($\overline{B}$•D)	(B+C) • ($\overline{B}$+D) • (C+D) = (B+C) • ($\overline{B}$+D)	Consensus

How do we prove these are true?

T7: Associativity

Number	Theorem	Name
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	Associativity

T8: Distributivity

Number	Theorem	Name
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	Distributivity

T9: Covering

Number	Theorem	Name
T9	$B \bullet (B+C) = B$	Covering

T9: Covering

Number	Theorem	Name
T9	$B \bullet (B+C) = B$	Covering

Prove true by:

- **Method 1:** Perfect induction
- **Method 2:** Using other theorems and axioms

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 1: Perfect Induction

B	*C*	*(B+C)*	*B(B+C)*
0	0		
0	1		
1	0		
1	1		

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 1: Perfect Induction

B	*C*	*(B+C)*	*B(B+C)*
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

T9: Covering

Number	Theorem	Name
T9	B• (B+C) = B	Covering

Method 1: Perfect Induction

B	*C*	*(B+C)*	*B(B+C)*
0	0	0	0
0	1	1	0
1	0	1	1
1	1	1	1

T9: Covering

Number	Theorem	Name
T9	$B \bullet (B+C) = B$	Covering

Method 2: Prove true using other axioms and theorems.

T9: Covering

Number	Theorem	Name
T9	$B\bullet(B+C) = B$	Covering

Method 2: Prove true using other axioms and theorems.

$B\bullet(B+C)$	$= B\bullet B + B\bullet C$	T8: Distributivity
	$= \mathbf{B} + B\bullet C$	T3: Idempotency
	$= B\bullet(1 + C)$	T8: Distributivity
	$= B\bullet(\mathbf{1})$	T2: Null element
	$= B$	T1: Identity

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T10: Combining

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet C) = B$	Combining

Prove true using other axioms and theorems:

T10: Combining

Number	Theorem	Name
T10	$(B \bullet C) + (B \bullet C) = B$	Combining

Prove true using other axioms and theorems:

$$B \bullet C + B \bullet \overline{C} \quad = B \bullet (C + \overline{C}) \quad \text{T8: Distributivity}$$

$$= B \bullet (\mathbf{1}) \quad \text{T5': Complements}$$

$$= B \quad \text{T1: Identity}$$

T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) = (B \bullet C) + (\overline{B} \bullet D)$	Consensus

Prove true using (1) perfect induction or (2) other axioms and theorems.

Boolean Theorems of Several Vars

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T8	B • (C + D) = (B•C) + (B•D)	B + (C•D) = (B+C) (B+D)	Distributivity
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T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

Axioms and theorems are useful for *simplifying* equations.

Simplifying an Equation

Reducing an equation to the **fewest number of implicants**, where each implicant has the **fewest literals**

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Recall:

- **Implicant:** product of literals

  $\overline{A}B\overline{C}$, $A\overline{C}$, BC

- **Literal:** variable or its complement

  $\overline{A}$, A, $\overline{B}$, B, $\overline{C}$, C

Simplifying an Equation

Reducing an equation to the **fewest number of implicants**, where each implicant has the **fewest literals**

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- **Literal:** variable or its complement

  $\bar{A}$, A, $\bar{B}$, B, $\bar{C}$, C

Simplifying the equation is also called ***minimizing*** *the equation*

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$

  $B + CD = (B+C)(B+D)$

- **Covering (T9')** $A + AP = A$

- **Combining (T10)** $P\overline{A} + PA = P$

Simplification methods

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- **Expansion** $P = P\overline{A} + PA$

  $A = A + AP$
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Simplification methods

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- **"Simplification" theorem** $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Proving the "Simplification" Theorem

"Simplification" theorem

$$PA + \overline{A} = P + \overline{A}$$

Method 1:

$PA + \overline{A}$	$= PA + (\mathbf{\overline{A} + \overline{A}P})$	**T9' Covering**
	$= PA + \mathbf{P\overline{A}} + \overline{A}$	**T6 Commutativity**
	$= \mathbf{P(A + \overline{A})} + \overline{A}$	**T8 Distributivity**
	$= P(\mathbf{1}) + \overline{A}$	**T5' Complements**
	$= P + \overline{A}$	**T1 Identity**

Proving the "Simplification" Theorem

"Simplification" theorem

$$PA + \overline{A} = P + \overline{A}$$

Method 2: $PA + \overline{A} = (\overline{A} + A)(\overline{A} + P)$ **T8' Distributivity**

$= 1(\overline{A} + P)$ **T5' Complements**

$= \overline{\mathbf{A}} + \mathbf{P}$ **T1 Identity**

T11: Consensus

Number	Theorem	Name
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	Consensus

Prove using other theorems and axioms:

T11: Consensus

Number	Theorem	Name
T11	$(B\bullet C) + (\overline{B}\bullet D) + (C\bullet D) = (B\bullet C) + (\overline{B}\bullet D)$	Consensus

Prove using other theorems and axioms:

$$\mathbf{B\bullet C + \overline{B}\bullet D + C\bullet D}$$

$= BC + \overline{B}D + (CDB+CD\overline{B})$	**T10: Combining**
$= BC + \overline{B}D + \overline{B}CD+BCD$	**T6: Commutativity**
$= BC + BCD + \overline{B}D + \overline{B}CD$	**T6: Commutativity**
$= (BC + BCD) + (\overline{B}D + \overline{B}CD)$	**T7: Associativity**
$= BC + \overline{B}D$	**T9': Covering**

Boolean Theorems of Several Vars

#	Theorem	Dual	Name
T6	$B \bullet C = C \bullet B$	$B+C = C+B$	Commutativity
T7	$(B \bullet C) \bullet D = B \bullet (C \bullet D)$	$(B + C) + D = B + (C + D)$	Associativity
T8	$B \bullet (C + D) = (B \bullet C) + (B \bullet D)$	$B + (C \bullet D) = (B+C)\,(B+D)$	Distributivity
T9	$B \bullet (B+C) = B$	$B + (B \bullet C) = B$	Covering
T10	$(B \bullet C) + (B \bullet \overline{C}) = B$	$(B+C) \bullet (B+\overline{C}) = B$	Combining
T11	$(B \bullet C) + (\overline{B} \bullet D) + (C \bullet D) =$ $(B \bullet C) + (\overline{B} \bullet D)$	$(B+C) \bullet (\overline{B}+D) \bullet (C+D) =$ $(B+C) \bullet (\overline{B}+D)$	Consensus

Simplification methods

- **Distributivity (T8, T8')** — $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** — $A + AP = A$
- **Combining (T10)** — $\mathbf{P\bar{A} + PA = P}$
- **Expansion** — $P = P\bar{A} + PA$
  $A = A + AP$
- **Duplication** — $A = A + A$
- **"Simplification" theorem** — $P\bar{A} + A = P + A$
  $PA + \bar{A} = P + \bar{A}$

Simplifying Boolean Equations

Example 1:

$$Y = A\overline{B} + AB$$

Simplifying Boolean Equations

Example 1:

$Y = A\overline{B} + AB$

Y = A   T10: Combining

or

$= A(\overline{B} + B)$ T8: Distributivity

$= A(1)$ T5': Complements

$= A$ T1: Identity

Simplification methods

- **Distributivity (T8, T8')** — **$B(C+D) = BC + BD$**

  $B + CD = (B+C)(B+D)$

- **Covering (T9')** — $A + AP = A$
- **Combining (T10)** — $P\overline{A} + PA = P$
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- **"Simplification" theorem** — $P\overline{A} + A = P + A$

  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

Simplifying Boolean Equations

Example 2:

$Y = A(AB + ABC)$

$= A(AB(1 + C))$	T8: Distributivity
$= A(AB(1))$	T2': Null Element
$= A(AB)$	T1: Identity
$= (AA)B$	T7: Associativity
$= AB$	T3: Idempotency

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\bar{A} + PA = P$
- **Expansion** $P = P\bar{A} + PA$
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- **Duplication** $A = A + A$
- **"Simplification" theorem** $P\bar{A} + A = P + A$
  $PA + \bar{A} = P + \bar{A}$

Simplifying Boolean Equations

Example 3:

$Y = A'BC + A'$

Recall: $A' = \overline{A}$

Simplifying Boolean Equations

Example 3:

$Y = A'BC + A'$

Recall: $A' = \overline{A}$

Note:

- A' is shorthand for $\overline{A}$.
- But use the tick symbol (') **only when typing.**
- It's easy to lose ticks (') when writing by hand!
- It is strongly recommended that you simplify equations by writing by hand.

Simplifying Boolean Equations

Example 3:

$Y = A'BC + A'$

Recall: $A' = \overline{A}$

$= A'$ T9' Covering: $X + XY = X$

or

$= A'(BC + 1)$ T8: Distributivity

$= A'(1)$ T2': Null Element

$= A'$ T1: Identity

Simplifying Boolean Equations

Example 4:

$Y = AB'C + ABC + A'BC$

Simplification methods

- **Distributivity (T8, T8')** $B(C+D) = BC + BD$
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** $A + AP = A$
- **Combining (T10)** $P\overline{A} + PA = P$
- **Expansion** $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** $A = A + A$
- **"Simplification" theorem** $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 4:

Y = AB'C + ABC + A'BC

= AB'C + **ABC + ABC** + A'BC	T3': Idempotency
= (AB'C+ABC) + (ABC+A'BC)	T7': Associativity
= AC + BC	T10: Combining

Simplification methods

- **Distributivity (T8, T8')** — **$B(C+D) = BC + BD$**
  $B + CD = (B+C)(B+D)$
- **Covering (T9')** — $A + AP = A$
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- **Expansion** — $P = P\overline{A} + PA$
  $A = A + AP$
- **Duplication** — $A = A + A$
- **"Simplification" theorem** — $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 5:

Y = AB + BC +B'D' + AC'D'

Method 1:

Y = AB + BC + B'D' + (ABC'D' + AB'C'D')	T10: Combining
= (AB + ABC'D') + BC + (B'D' + AB'C'D')	T6: Commutativity
	T7: Associativity
= AB + BC + B'D'	T9: Covering

Method 2:

Y = **AB** + BC + **B'D'** + AC'D' + **AD'**	T11: Consensus
= AB + BC + B'D' + AD'	T9: Covering
= AB + BC + B'D'	T11: Consensus

implification methods

- **Distributivity (T8, T8')** — $B(C+D) = BC + BD$
  $B + CD = (B + C)(B + D)$
- **Covering (T9')** — $A + AP = A$
- **Combining (T10)** — $P\overline{A} + PA = P$
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- **"Simplification" theorem** — $P\overline{A} + A = P + A$
  $PA + \overline{A} = P + \overline{A}$

Simplifying Boolean Equations

Example 6:

$Y = (A + BC)(A + DE)$

Apply T8' first when possible: $W+XZ = (W+X)(W+Z)$

Simplifying Boolean Equations

Example 6:

$Y = (A + BC)(A + DE)$

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: X = BC, Z = DE and rewrite equation

Y	= (A+X)(A+Z)	substitution (X=BC, Z=DE)
	= A + XZ	T8': Distributivity
	= A + BCDE	substitution

or

Y	= AA + ADE + ABC + BCDE	T8: Distributivity
	= A + ADE + ABC + BCDE	T3: Idempotency
	= **A + ADE** + ABC + BCDE	
	= **A** + ABC + BCDE	T9': Covering
	= A + BCDE	T9': Covering

Simplifying Boolean Equations

Example 6:

$Y = (A + BC)(A + DE)$

Apply T8' first when possible: $W+XZ = (W+X)(W+Z)$

Make: $X = BC$, $Z = DE$ and rewrite equation

Y	$= (A+X)(A+Z)$	substitution ($X=BC$, $Z=DE$)
	$= A + XZ$	T8': Distributivity
	$= A + BCDE$	substitution

or

Y	$= AA + ADE + ABC + BCDE$	T8: Distributivity
	$= A + ADE + ABC + BCDE$	T3: Idempotency
	$=$ **$A + ADE$** $+ ABC + BCDE$	
	$=$ **A** $+ ABC + BCDE$	T9': Covering
	$= A + BCDE$	T9': Covering

This is called ***multiplying out*** an expression to get sum-of-products (SOP) form.

Review: Canonical SOP & POS Forms

- **SOP – sum-of-products**

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$E = O\overline{C}$$
$$= \Sigma(2)$$

- **POS – product-of-sums**

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$E = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(0, 1, 3)$$

Multiplying Out: SOP Form

An expression is in **sum-of-products (SOP)** form when all products contain literals only.

- SOP form: Y = AB + BC' + DE
- **NOT** SOP form: Y = DF + E(A'+B)
- SOP form: Z = A + BC + DE'F

Multiplying Out: SOP Form

Example:

$Y = (A + C + D + E)(A + B)$

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: X = (C+D+E), Z = B and rewrite equation

Y	= (A+X)(A+Z)	substitution (X=(C+D+E), Z=B)
	= A + XZ	T8': Distributivity
	= A + (C+D+E)B	substitution
	= A + BC + BD + BE	T8: Distributivity

or

Y	= AA+AB+AC+BC+AD+BD+AE+BE	T8: Distributivity
	= **A**+AB+AC+AD+AE+BC+BD+BE	T3: Idempotency
	= A + BC + BD + BE	T9': Covering

Factoring: POS Form

An expression is in **product-of-sums (POS)** form when all sums contain literals only.

- POS form: Y = (A+B)(C+D)(E'+F)
- **NOT** POS form: Y = (D+E)(F'+GH)
- POS form: Z = A(B+C)(D+E')

Canonical POS form: each product contains 1 of each literal.

Factoring: POS Form

Example 1:

Y = (A + B'CDE)

Apply T8' first when possible: W+XZ = (W+X)(W+Z)

Make: X = B'C, Z = DE and rewrite equation

Y	= (A+XZ)	substitution (X=B'C, Z=DE)
	= (A+B'C)(A+DE)	T8': Distributivity
	= (A+B')(A+C)(A+D)(A+E)	T8': Distributivity

Factoring: POS Form

Example 2:

$Y = AB + C'DE + F$

Apply T8' first when possible: $W+XZ = (W+X)(W+Z)$

Make: $W = AB$, $X = C'$, $Z = DE$ and rewrite equation

Y	$= (W+XZ) + F$	substitution $W = AB$, $X = C'$, $Z = DE$
	$= (W+X)(W+Z) + F$	T8': Distributivity
	$= (AB+C')(AB+DE)+F$	substitution
	$= (A+C')(B+C')(AB+D)(AB+E)+F$	T8': Distributivity
	$= (A+C')(B+C')(A+D)(B+D)(A+E)(B+E)+F$	T8': Distributivity
	$= (A+C'+F)(B+C'+F)(A+D+F)(B+D+F)(A+E+F)(B+E+F)$	T8': Distributivity

DeMorgan's Theorem

Number	Theorem	Name
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} = \overline{B_0} + \overline{B_1} + \overline{B_2} \ldots$	DeMorgan's Theorem

DeMorgan's Theorem

Number	Theorem	Name
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} = \overline{B_0} + \overline{B_1} + \overline{B_2} \ldots$	DeMorgan's Theorem

The **complement** of the **product**
is the
sum of the **complements**

DeMorgan's Theorem: Dual

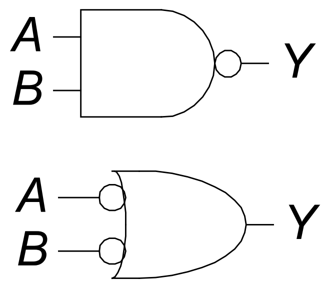
#	Theorem	Dual	Name
T12	$\overline{B_0 \bullet B_1 \bullet B_2 \ldots} =$ $\overline{B_0} + \overline{B_1} + \overline{B_2} \ldots$	$\overline{B_0 + B_1 + B_2 \ldots} =$ $\overline{B_0} \bullet \overline{B_1} \bullet \overline{B_2} \ldots$	DeMorgan's Theorem

The **complement** of the **product**
is the
sum of the **complements**.

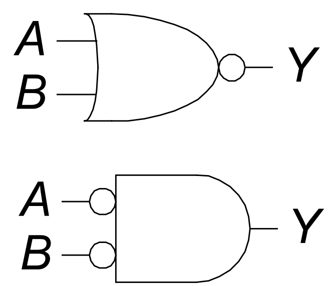
Dual: The **complement** of the **sum**
is the
product of the **complements**.

DeMorgan’s Theorem

- $Y = \overline{AB} = \overline{A} + \overline{B}$



- $Y = \overline{A + B} = \overline{A} \bullet \overline{B}$



DeMorgan's Theorem Example 1

$$Y = \overline{(A + \overline{BD})\overline{C}}$$

DeMorgan's Theorem Example 1

$$Y = \overline{(A+\overline{BD})\overline{C}}$$

$$= \overline{(A+\overline{BD})} + \overline{\overline{C}}$$

$$= (\overline{A}\bullet(\overline{\overline{BD}})) + C$$

$$= (\overline{A}\bullet(BD)) + C$$

$$= \overline{A}BD + C$$

DeMorgan's Theorem Example 2

$$Y = \overline{(\overline{ACE}+\overline{D}) + B}$$

DeMorgan's Theorem Example 2

$$Y = \overline{(\overline{AC}E+\overline{D}) + B}$$

$$= \overline{(\overline{AC}E+\overline{D})} \bullet \overline{B}$$

$$= (\overline{\overline{AC}E} \bullet \overline{\overline{D}}) \bullet \overline{B}$$

$$= ((\overline{\overline{AC}}+\overline{E}) \bullet D) \bullet \overline{B}$$

$$= ((AC+\overline{E}) \bullet D) \bullet \overline{B}$$

$$= (ACD + D\overline{E}) \bullet \overline{B}$$

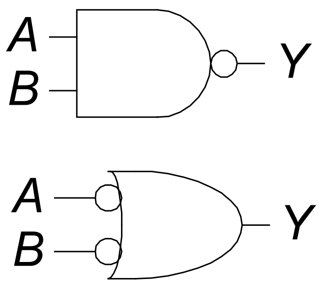
$$= A\overline{B}CD + \overline{B}D\overline{E}$$

COMBINATIONAL LOGIC DESIGN

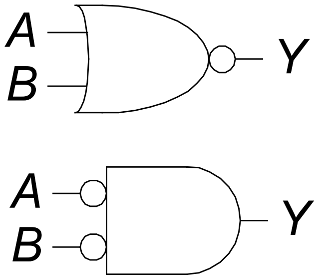
ELSEVIER

DeMorgan's Theorem

- $Y = \overline{AB} = \overline{A} + \overline{B}$

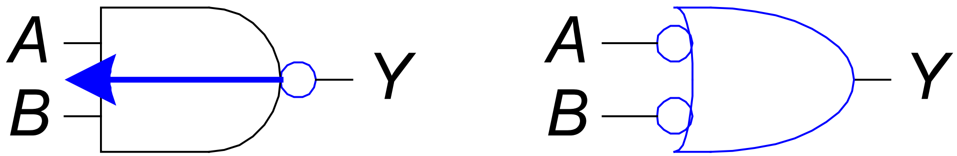


- $Y = \overline{A + B} = \overline{A} \bullet \overline{B}$



Bubble Pushing

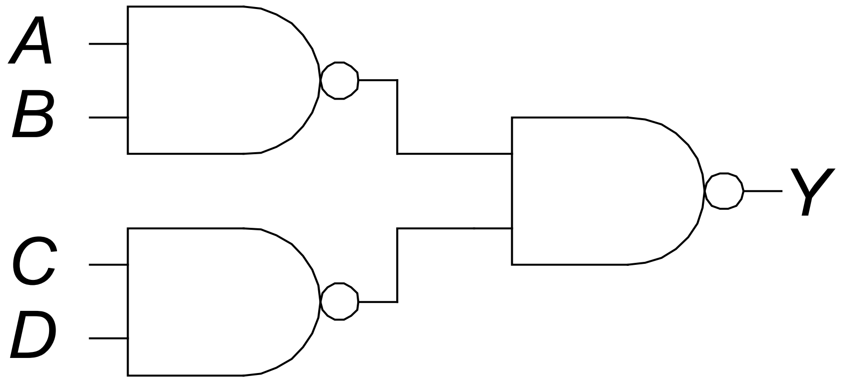
- **Backward:**
  - Body changes
  - Adds bubbles to inputs



- **Forward:**
  - Body changes
  - Adds bubble to output

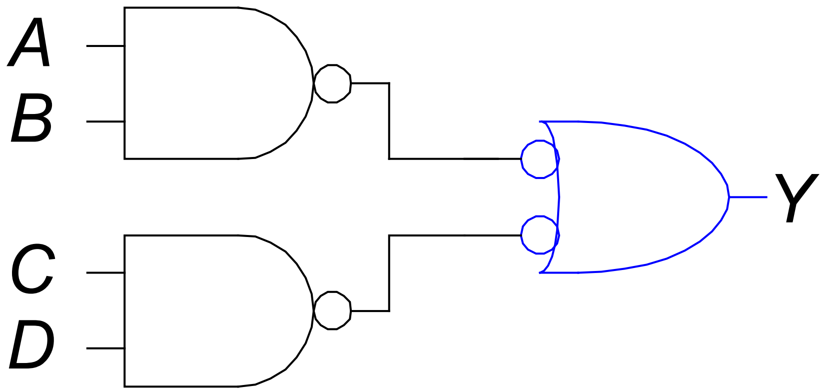
Bubble Pushing

- What is the Boolean expression for this circuit?



Bubble Pushing

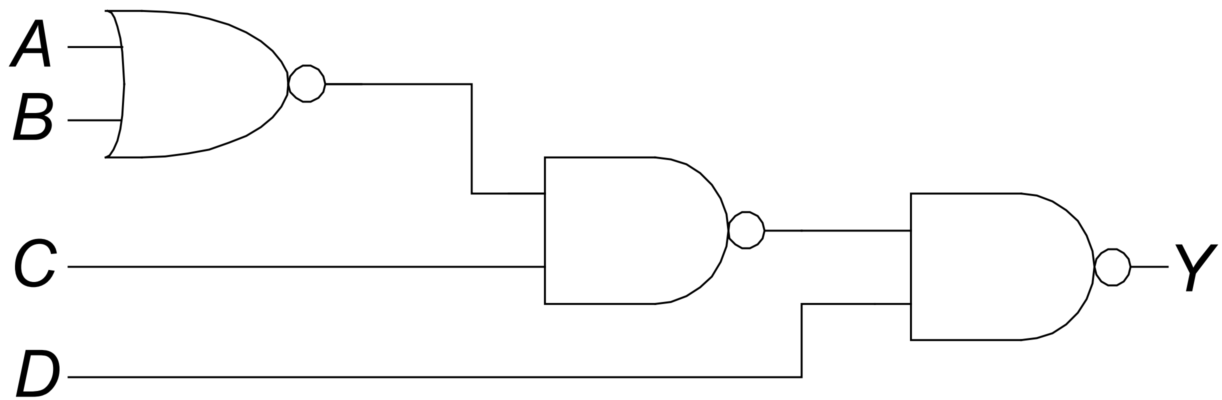
- What is the Boolean expression for this circuit?



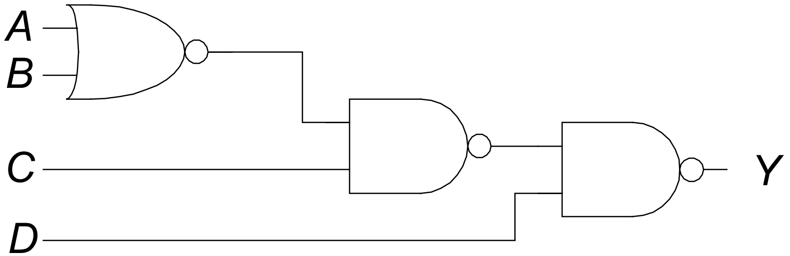
$$Y = AB + CD$$

Bubble Pushing Rules

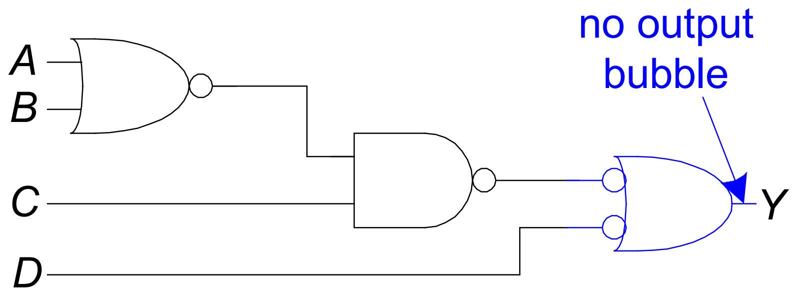
- Begin at output, then work toward inputs
- Push bubbles on final output back
- Draw gates in a form so bubbles cancel



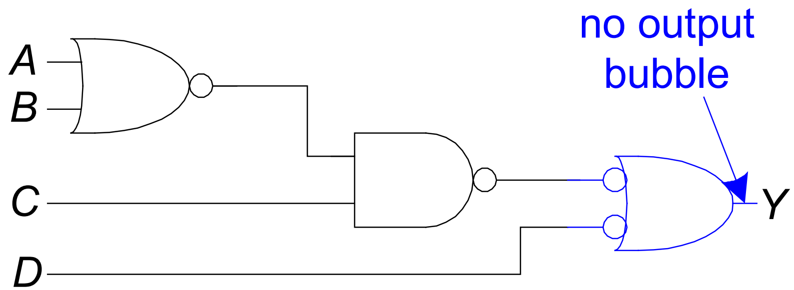
Bubble Pushing Example

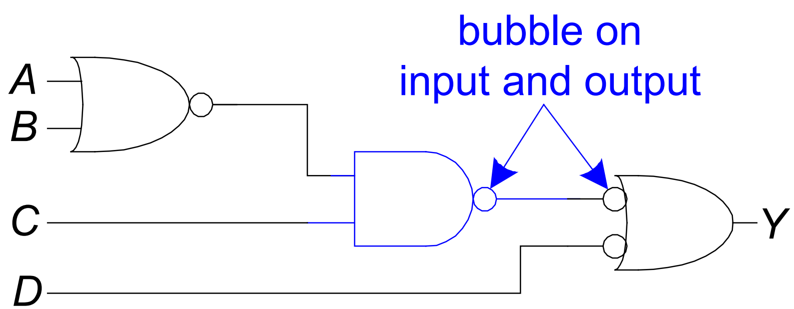


Bubble Pushing Example

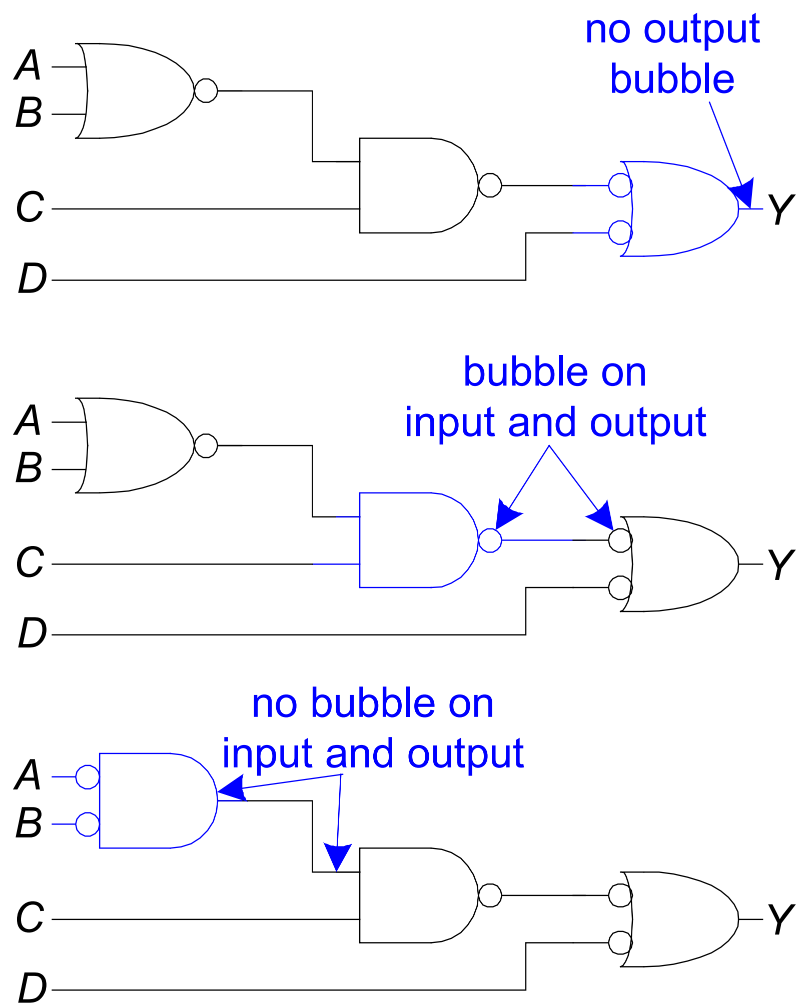


Bubble Pushing Example





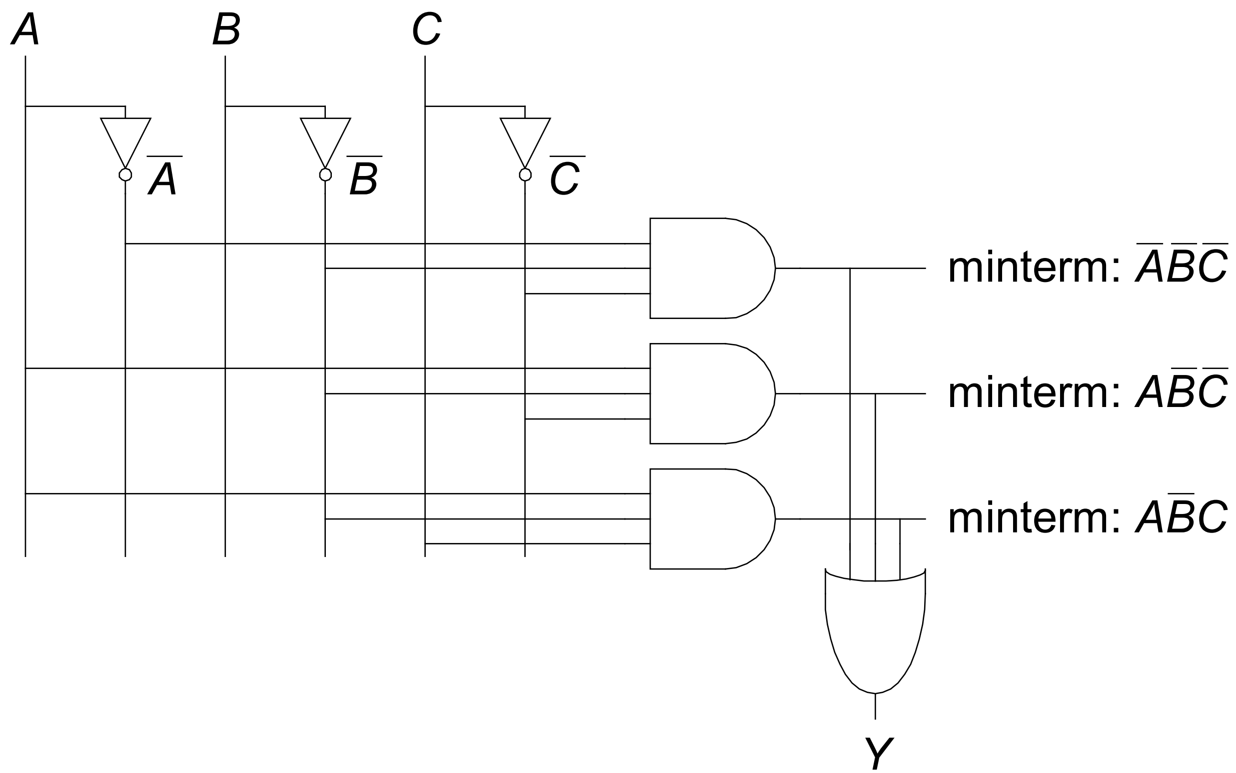
Bubble Pushing Example



$$Y = \overline{A}\,\overline{B}C + \overline{D}$$

From Logic to Gates

- Two-level logic: ANDs followed by ORs
- Example: $Y = \overline{A}\,\overline{B}\,\overline{C} + A\overline{B}\,\overline{C} + A\overline{B}C$



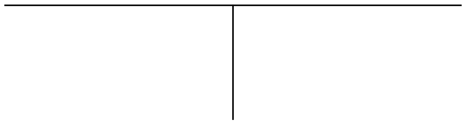
Circuit Schematics Rules

- Inputs on the left (or top)
- Outputs on right (or bottom)
- Gates flow from left to right
- Straight wires are best

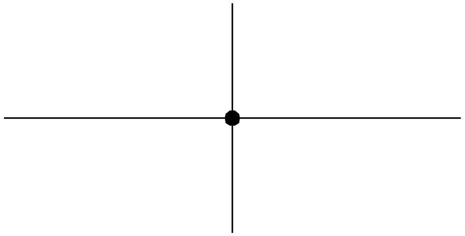
Circuit Schematic Rules (cont.)

- Wires always connect at a T junction
- A dot where wires cross indicates a connection between the wires
- Wires crossing *without* a dot make no connection

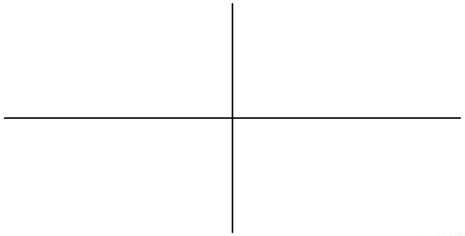
wires connect
at a T junction



wires connect
at a dot



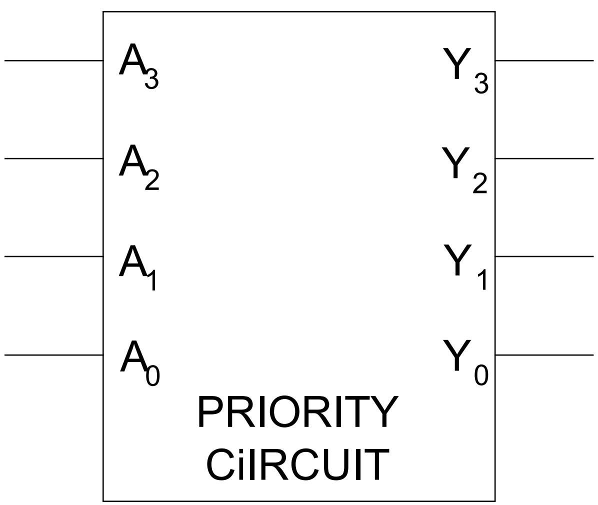
wires crossing
without a dot do
not connect



Multiple-Output Circuits

- **Example: Priority Circuit**

  Output asserted corresponding to most significant TRUE input

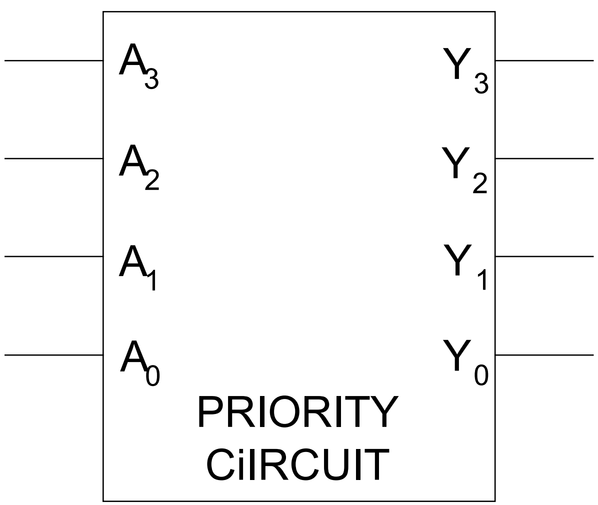


A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0				
0	0	0	1				
0	0	1	0				
0	0	1	1				
0	1	0	0				
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0				
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

Multiple-Output Circuits

- **Example: Priority Circuit**

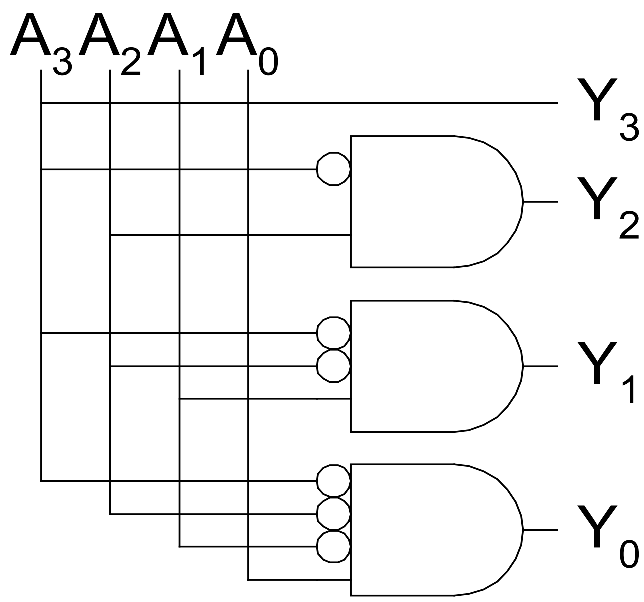
  Output asserted corresponding to most significant TRUE input



A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

Priority Circuit Hardware

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0



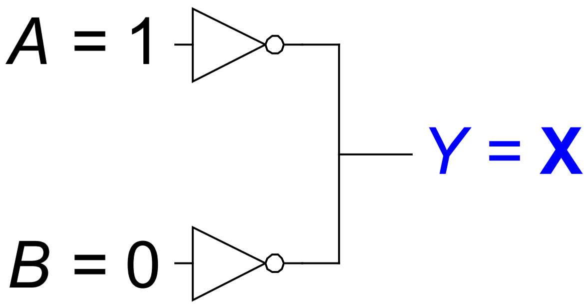
Don’t Cares

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

Contention: X

- Contention: circuit tries to drive output to 1 **and** 0
  - Actual value somewhere in between
  - Could be 0, 1, or in forbidden zone
  - Might change with voltage, temperature, time, noise
  - Often causes excessive power dissipation

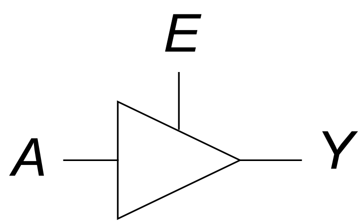


- **Warnings:**
  - Contention usually indicates a **bug**.
  - **X is used for "don't care" and contention** - look at the context to tell them apart

Floating: Z

- Floating, high impedance, open, high Z
- Floating output might be 0, 1, or somewhere in between
  - A voltmeter won't indicate whether a node is floating

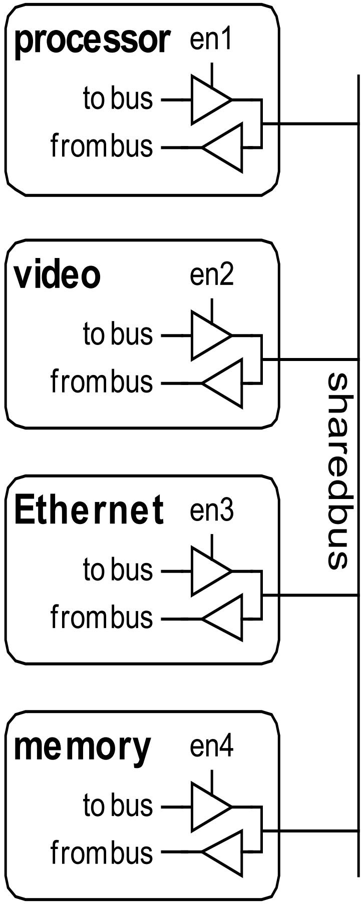
Tristate Buffer



E	A	Y
0	0	Z
0	1	Z
1	0	0
1	1	1

Tristate Busses

- Floating nodes are used in tristate busses
  - Many different drivers
  - Exactly one is active at once



Karnaugh Maps (K-Maps)

- Boolean expressions can be minimized by combining terms
- K-maps minimize equations graphically
- $PA + P\overline{A} = P$

A	B	C	Y
0	0	0	**1**
0	0	1	**1**
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y: C \ AB	00	01	11	10
0	**1**	0	0	0
1	**1**	0	0	0

Y: C \ AB	00	01	11	10
0	**$\bar{A}\bar{B}\bar{C}$**	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	**$\bar{A}\bar{B}C$**	$\bar{A}BC$	ABC	$A\bar{B}C$

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K-Map

- Circle 1's in adjacent squares
- In Boolean expression, include only literals whose true **and** complement form are ***not*** in the circle

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y: C \ AB	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$$Y = \overline{A}\,\overline{B}$$

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3-Input K-Map

Y: C \ AB	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

K-Map

Y: C \ AB	00	01	11	10
0				
1				

3-Input K-Map

$C \backslash AB$	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

K-Map

$C \backslash AB$	00	01	11	10
0	0	1	0	0
1	0	1	1	0

$$Y = \bar{A}B + BC$$

K-Map Definitions

- **Complement:** variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$

- **Literal:** variable or its complement

  $\bar{A}, A, \bar{B}, B, C, \bar{C}$

- **Implicant:** product of literals

  $A\bar{B}C, \bar{A}C, BC$

- **Prime implicant:** implicant corresponding to the largest circle in a K-map

K-Map Rules

- **Every 1 must be circled** at least once
- Each circle must span a **power of 2** (i.e. 1, 2, 4) squares in each direction
- Each circle must be as **large** as possible
- A circle may **wrap around the edges**
- Circle a “**don't care**” (X) **only if it helps** minimize the equation

4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ AB	00	01	11	10
00				
01				
11				
10				

4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ AB	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

$$Y = \overline{A}C + \overline{A}BD + A\overline{B}\,\overline{C} + \overline{B}\,\overline{D}$$

K-Maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ AB	00	01	11	10
00				
01				
11				
10				

K-Maps with Don’t Cares

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ *AB*	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

K-Maps with Don't Cares

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ *AB*	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

$$Y = A + \overline{B}\,\overline{D} + C$$

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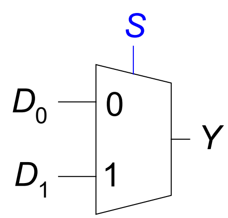
Combinational Building Blocks

- Multiplexers
- Decoders

Multiplexer (Mux)

- Selects between one of N inputs to connect to output
- $\log_2 N$-bit select input – control input
- **Example:** **2:1 Mux**

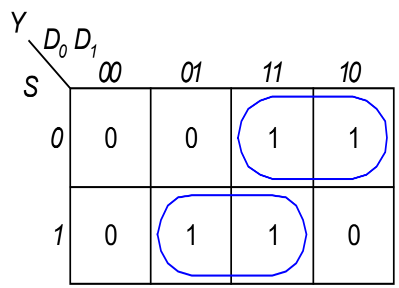


S	D_1	D_0	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

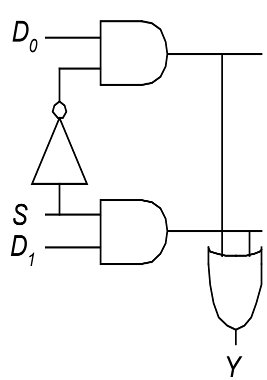
S	Y
0	D_0
1	D_1

Multiplexer Implementations

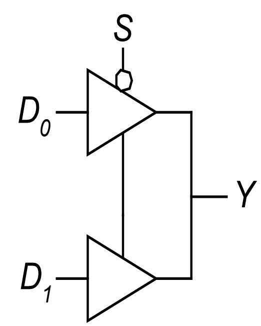
- **Logic gates**
  - Sum-of-products form



$Y = D_0\overline{S} + D_1S$



- **Tristates**
  - For an N-input mux, use N tristates
  - Turn on exactly one to select the appropriate input

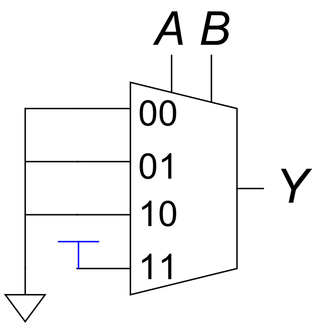


Logic using Multiplexers

Using the mux as a lookup table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$Y = AB$



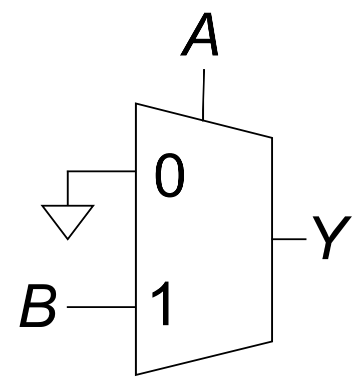
Logic using Multiplexers

Reducing the size of the mux

$Y = AB$

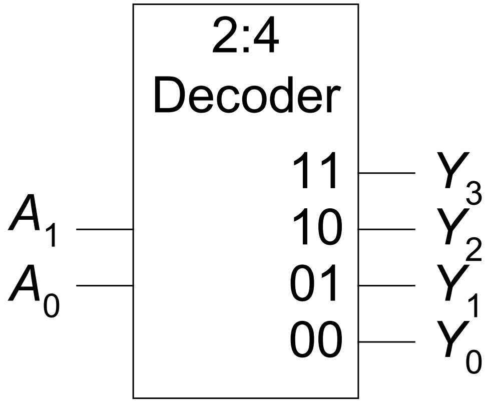
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

A	Y
0	0
1	B



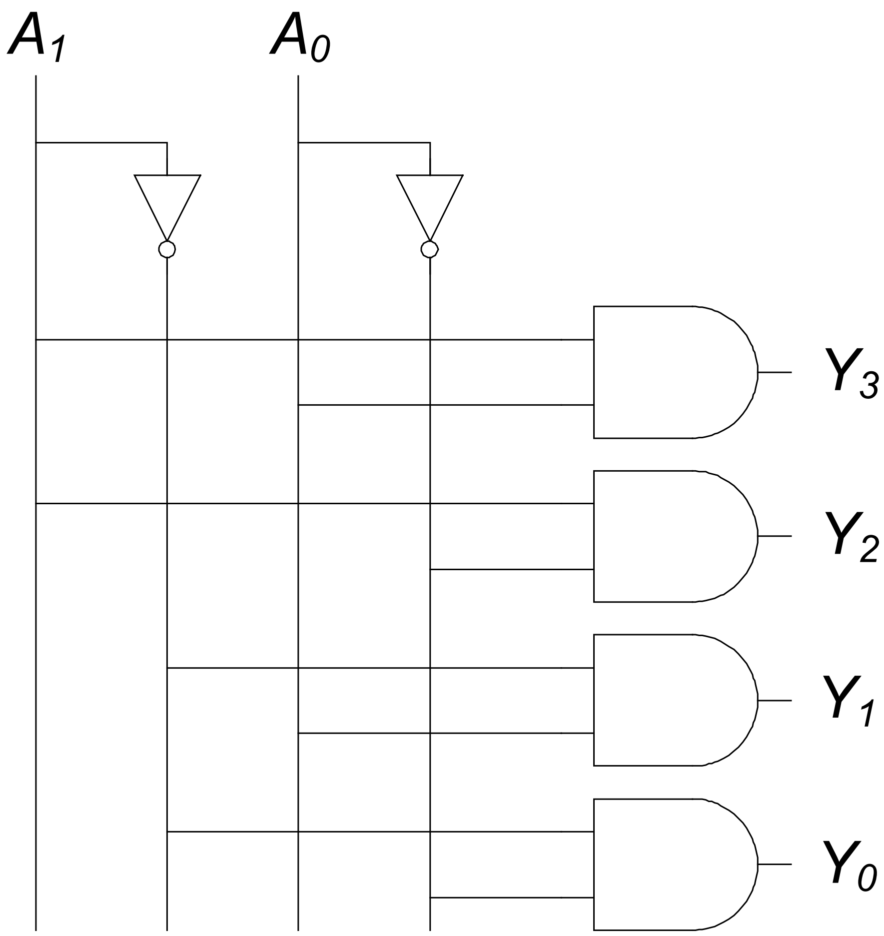
Decoders

- N inputs, 2^N outputs
- **One-hot** outputs: only **one output HIGH** at once



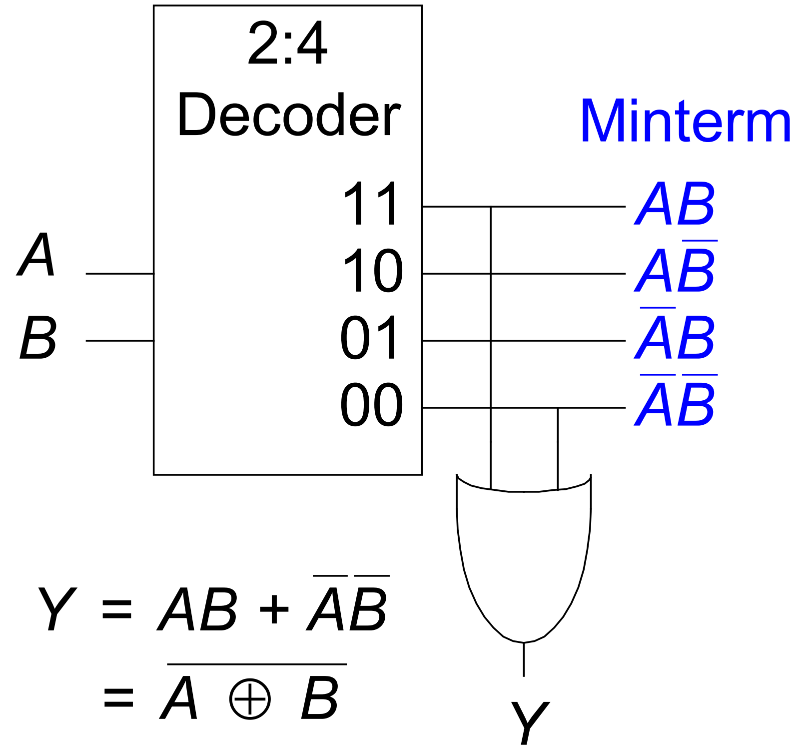
A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

Decoder Implementation



Logic Using Decoders

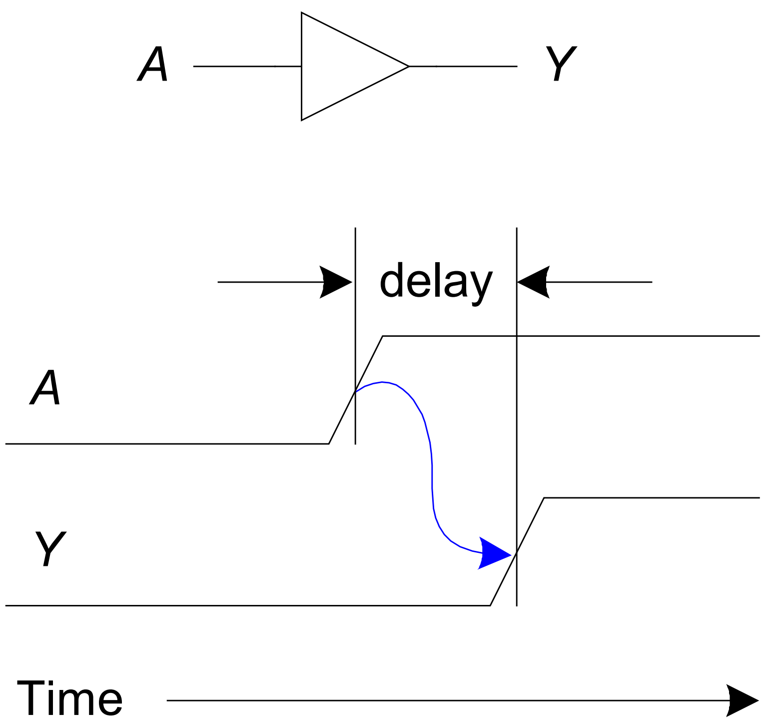
OR the minterms



$$Y = AB + \overline{A}\,\overline{B}$$
$$= \overline{A \oplus B}$$

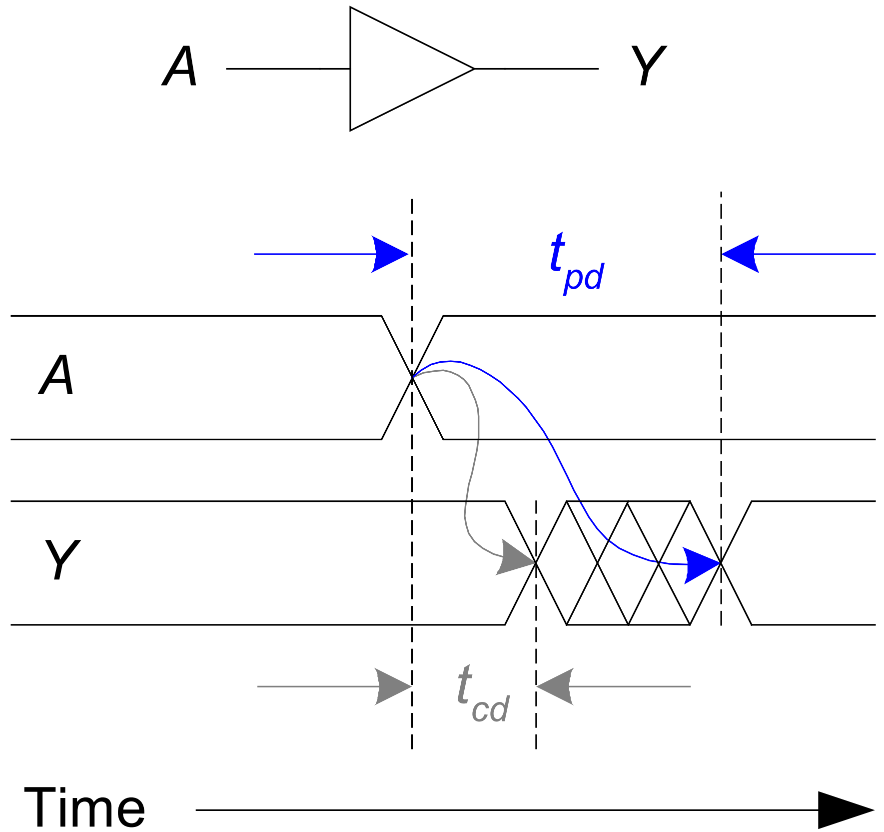
Timing

- **Delay** between input change and output changing
- How to build fast circuits?



Propagation & Contamination Delay

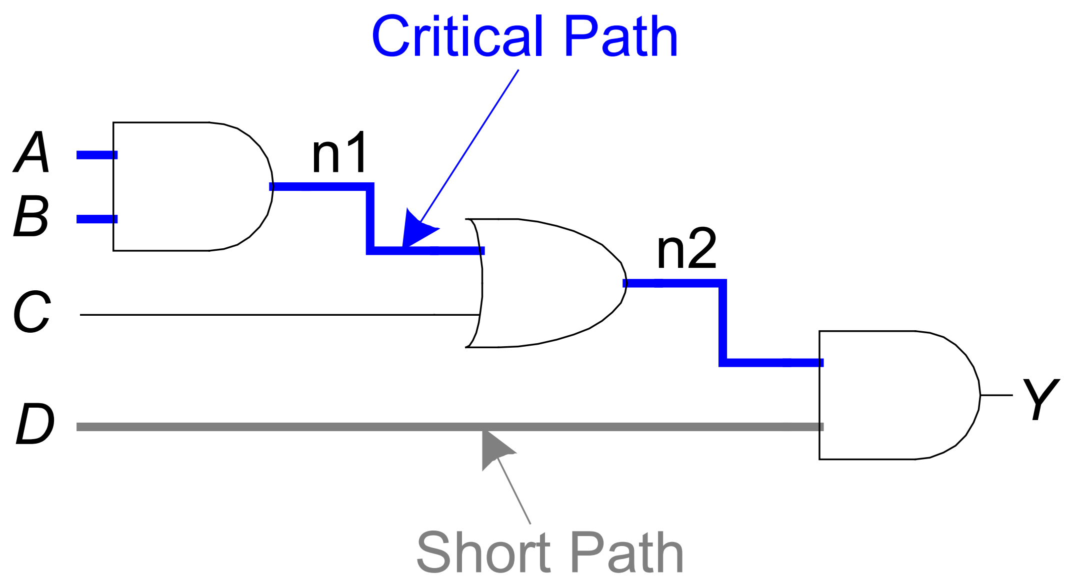
- **Propagation delay:** t_{pd} = max delay from input to output
- **Contamination delay:** t_{cd} = min delay from input to output



Propagation & Contamination Delay

- **Delay is caused by**
  - Capacitance and resistance in a circuit
  - Speed of light limitation
- **Reasons why t_{pd} and t_{cd} may be different:**
  - Different rising and falling delays
  - Multiple inputs and outputs, some of which are faster than others
  - Circuits slow down when hot and speed up when cold

Critical (Long) & Short Paths



Critical (Long) Path: $t_{pd} = 2t_{pd\_AND} + t_{pd\_OR}$

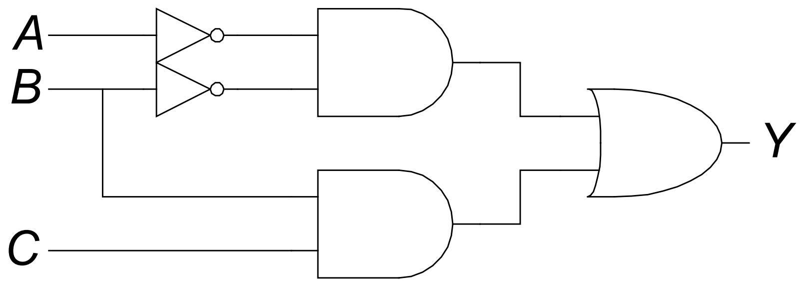
Short Path: $t_{cd} = t_{cd\_AND}$

Glitches

- When a single input change causes an output to change multiple times

Glitch Example

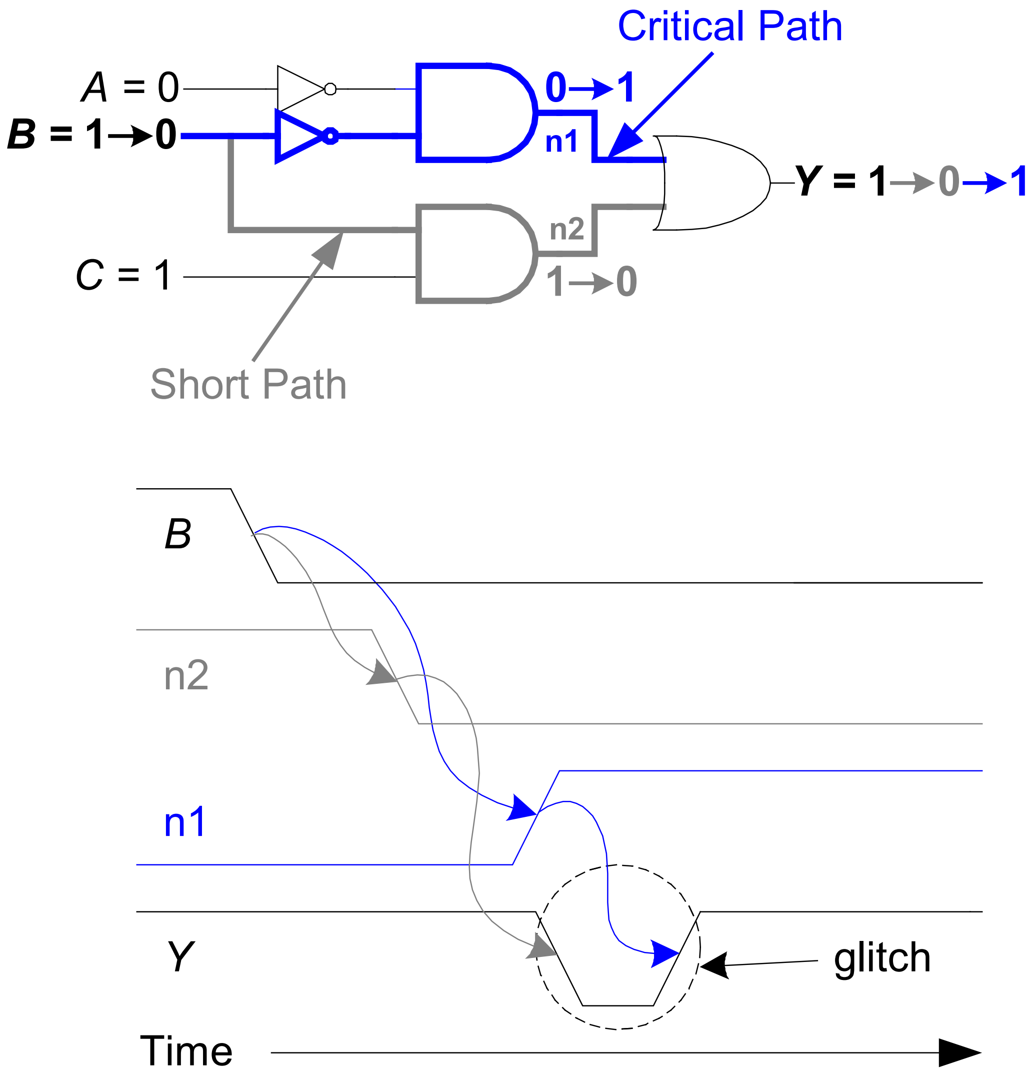
- What happens when A = 0, C = 1, B falls?



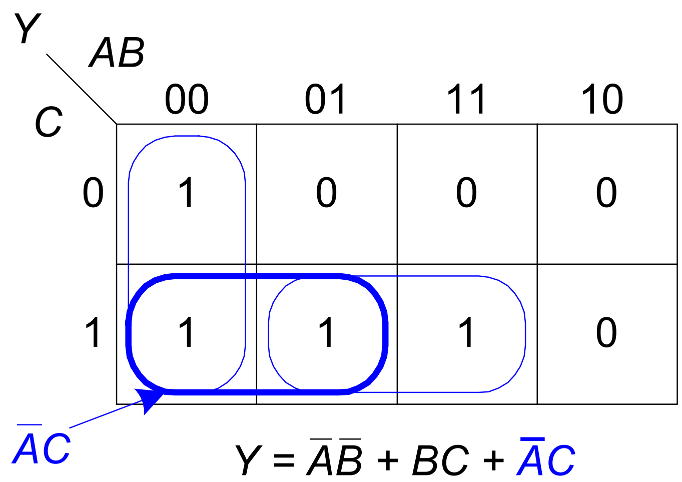
Y: C \ AB	00	01	11	10
0	1	0	0	0
1	1	1	1	0

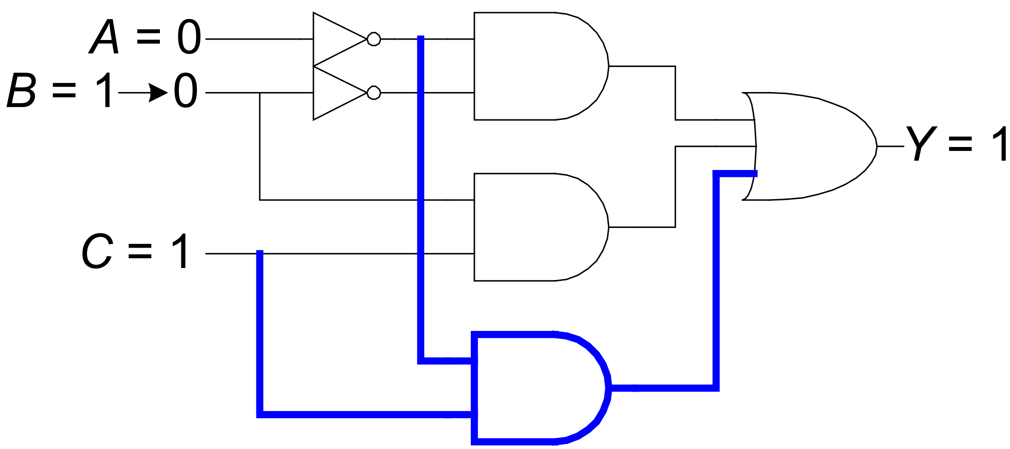
$$Y = \overline{A}\,\overline{B} + BC$$

Glitch Example (cont.)



Fixing the Glitch





Why Understand Glitches?

- Because of **synchronous design** conventions (see Chapter 3), glitches don't cause problems.
- It's important to **recognize** a glitch: in simulations or on oscilloscope.
- We **can't get rid of all glitches** – simultaneous transitions on multiple inputs can also cause glitches.