Intro to AI Assignment 1 - Fast Trajectory Planning

Rajeev Atla, Jasmin Badyal, Dhvani Patel

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§0 Part 0 - Setting Up the Environment

§1 Part 1 - Understanding Methods

§2 Part 2 - Repeated Forward A*

§3 Part 3 - Repeated Backward A*

§4 Part 4 - Heuristics in Adaptive A*

A heuristic function $h(s)$ is considered consistent if it satisfies the following properties:

1. $h_{\text{goal}} = 0$
2. $h(s) \leqslant c(s, a) + h(\text{succ}(s, a))$

§4.1 Part 4.1 - Manhattan Distance Consistency

The Manhattan distance in our grid world is defined as

$$h(s) = |x_s - x_{\text{goal}}| + |y_s - y_{\text{goal}}|$$

When $s = s_{\text{goal}}$,

$$\begin{aligned} h(s_{\text{goal}}) &= |x_{\text{goal}} - x_{\text{goal}}| + |y_{\text{goal}} - y_{\text{goal}}| \\ &= 0 \end{aligned}$$

We see that the first condition is satisfied, so we move to the next condition: the triangle inequality. Without loss of generality, assume the agent's current state is s, and the next state will be s', directly to the right of s. The y component of the Manhattan distance doesn't change, but the x component will become $|(x_s + 1) - x_{\text{goal}}|$. Therefore, $h(s') = h(s) \pm 1$. The maximum value attainable can be therefore described by the inequality $h(s') \leqslant h(s) + 1$. This assumption can be generalized to all 4 cardinal directions that are represented in the gridworld, with moving left changing the x component to $|(x_s - 1) - x_{\text{goal}}|$, and moving up or down changing the y component instead.

Recalling that $c(s, a) = 1$ in our gridworld, we see that $h(s') \leqslant h(s) + c(s, a)$, so the Manhattan distance is consistent.

§4.2 Part 4.2 - Heuristic Consistency in Adaptive A*

We need to show that Adaptive A*'s heuristic function is both admissible and consistent. We proceed to first show that it is admissible, essentially showing that it never overestimates the actual cost. Let $h^*(s)$ be the actual cost to reach the goal from state s Using the heuristic update formula and the definition of shortest path,

$$h_{\text{new}}(s) = g(s_{\text{goal}}) - g(s)$$
$$g(s_{\text{goal}}) = g(s) + h^*(s)$$

Solving this system of equations, we see that $h^*(s) = h_{\text{new}}(s)$. Therefore, the heuristic matches the cost exactly, and doesn't overestimate it, and is therefore admissible.

We next show that the heuristic is consistent. Suppose the agent is in state s and after updating, it is in state s'. We know that $h_{\text{new}}(s) = g(s_{\text{goal}}) - g(s)$, which becomes $h_{\text{new}}(s') = g(s_{\text{goal}}) - g(s')$. Substituting this into the triangle inequality,

$$g(s_{\text{goal}}) - g(s) \leqslant c(s, a) + g(s_{\text{goal}}) - g(s')$$
$$-g(s) \leqslant c(s, a) - g(s')$$
$$g(s') \geqslant g(s) + c(s, a)$$

The last inequality always holds true because of A*'s path expansion guarantee:

$$g(s') = \min(g(s'), g(s) + c(s, a))$$

Since these steps are all reversible, the trinagle inequality holds for $h_{\text{new}}(s)$, making it a consistent heuristic.

In the course of running Adaptive A*, action costs can increase. If we relax the assumption that the gridworld is unchanging, these action cost increases can be brought forth by a change in the maze that blocks a previously unblocked path. This means that $g(s)$ has to be recalculated for some states and may increase. However, this doesn't change the admissibility of $h_{\text{new}}(s)$ because it is exactly the true cost of moving from state s to the goal.

§5 Part 5 - Adaptive A*

§6 Part 6 - Statistical Analysis