Assignment 3

Probabilistic Reasoning

Deadline: April 30^{th}, 11:55pm.
Perfect score: 100.

Assignment Instructions:

Teams: Assignments should be completed by teams of up to four students. You can work on this assignment individually if you prefer. No additional credit will be given for students that complete an assignment individually. Make sure to write the name and RUID of every member of your group on your submitted report.

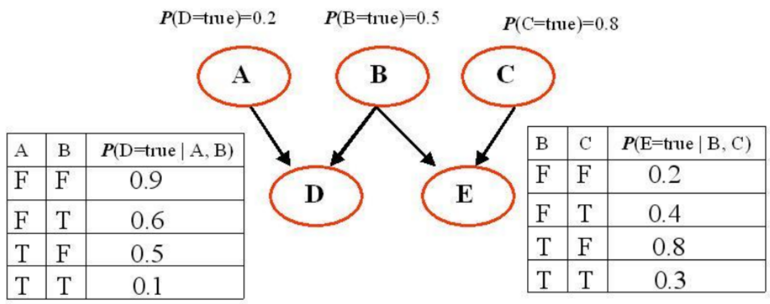
Submission Rules: Submit your reports electronically as a PDF document through Canvas (`canvas.rutgers.edu`). Do not submit Word documents, raw text, or hardcopies etc. Make sure to generate and submit a PDF instead. Each team of students should submit only a single copy of their solutions and indicate all team members on their submission. Failure to follow these rules will result in lower grade for this assignment.

Late Submissions: No late submission is allowed. 0 points for late assignments.

Precision: Try to be precise. Have in mind that you are trying to convince a very skeptical reader (and computer scientists are the worst kind...) that your answers are correct.

Collusion, Plagiarism, etc.: Each team must prepare its solutions independently from other teams, i.e., without using common notes, code or worksheets with other students or trying to solve problems in collaboration with other teams. You must indicate any external sources you have used in the preparation of your solution. **Do not plagiarize online sources** and in general make sure you do not violate any of the academic standards of the department or the university. Failure to follow these rules may result in failure in the course.

Problem 1 (*15 points):* Consider the following Bayesian network, where variables A through E are all Boolean valued. Note: there is a typo in the image, it should be $P(A = true) = 0.2$ instead of $P(D = true) = 0.2$.

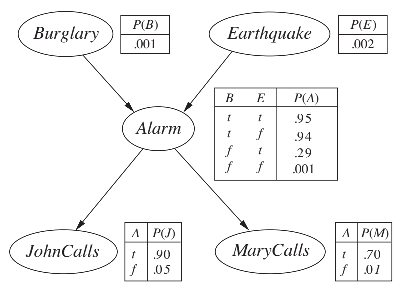


a) What is the probability that all five of these Boolean variables are simultaneously true?
[Hint: You have to compute the joint probability distribution. The structure of the Bayesian network suggests how the joint probability distribution is decomposed to the conditional probabilities available.]

b) What is the probability that all five of these Boolean variables are simultaneously false?
[Hint: Answer similarly to above.]

c) What is the probability that A is false given that the four other variables are all known to be true?

Problem 2 (*15 points):*



a) Calculate $P(Burglary|JohnsCalls = true, MaryCalls = true)$ and show in detail the calculations that take place. Use your book to confirm that your answer is correct.

b) Suppose a Bayesian network has the from of a *chain*: a sequence of Boolean variables $X_1, \ldots X_n$ where $Parents(X_i) = \{X_{i-1}\}$ for $i = 2, \ldots, n$. What is the complexity of computing $P(X_1|X_n = true)$ using enumeration? What is the complexity with variable elimination?

Problem 3 (*20 points):* Suppose you are working for a financial institution and you are asked to implement a fraud detection system. You plan to use the following information:

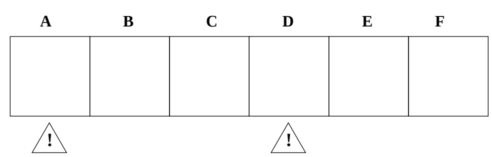
- When the card holder is travelling abroad, fraudulent transactions are more likely since tourists are prime targets for thieves. More precisely, 1% of transactions are fraudulent when the card holder is travelling, where as only 0.4% of the transactions are fraudulent when she is not travelling. On average, 5% of all transactions happen while the card holder is travelling. If a transaction is fraudulent, then the likelihood of a foreign purchase increases, unless the card holder happens to be travelling. More precisely, when the card holder is not travelling, 10% of the fraudulent transactions are foreign purchases where as only 1% of the legitimate transactions are foreign purchases. On the other hand, when the card holder is travelling, then 90% of the transactions are foreign purchases regardless of the legitimacy of the transactions.

- Purchases made over the internet are more likely to be fraudulent. This is especially true for card holders who don't own any computer. Currently, 75% of the population owns a computer or smart phone and for those card holders, 1% of their legitimate transactions are done over the internet, however this percentage increases to 2% for fraudulent transactions. For those who don't own any computer or smart phone, a mere 0.1% of their legitimate transactions is done over the internet, but that number increases to 1.1% for fraudulent transactions. Unfortunately, the credit card company doesn't know whether a card holder owns a computer or smart phone, however it can usually guess by verifying whether any of the recent transactions involve the purchase of computer related accessories. In any given week, 10% of those who own a computer or smart phone purchase (with their credit card) at least one computer related item as opposed to just 0.1% of those who don't own any computer or smart phone.

a) Construct a Bayes Network to identify fraudulent transactions.
What to hand in: Show the graph defining the network and the Conditional Probability Tables associated with each node in the graph. This network should encode the information stated above. Your network should contain exactly six nodes, corresponding to the following binary random variables:
OC : card holder owns a computer or smart phone.
Fraud : current transaction is fraudulent.
Trav : card holder is currently travelling.
FP : current transaction is a foreign purchase.
IP : current purchase is an internet purchase.
CRP : a computer related purchase was made in the past week.
The arcs defining your Network should accurately capture the probabilistic dependencies between these variables.

b) What is the prior probability (i.e., before we search for previous computer related purchases and before we verify whether it is a foreign and/or an internet purchase) that the current transaction is a fraud? What is the probability that the current transaction is a fraud once we have verified that it is a foreign transaction, but not an internet purchase and that the card holder purchased computer related accessories in the past week?
What to hand in: Indicate the two queries (i.e., $Pr(variables|evidence)$) you used to compute those two probabilities. Show each step of the calculation

Problem 4 (*40 points*): You are an interplanetary search and rescue expert who has just received an urgent message: a rover on Mercury has fallen and become trapped in Death Ravine, a deep, narrow gorge on the borders of enemy territory. You zoom over to Mercury to investigate the situation. Death Ravine is a narrow gorge 6 miles long, as shown below. There are volcanic vents at locations A and D, indicated by the triangular symbols at those locations.



The rover was heavily damaged in the fall, and as a result, most of its sensors are broken. The only ones still functioning are its thermometers, which register only two levels: *hot* and *cold*. The rover sends back evidence $E = \textit{hot}$ when it is at a volcanic vent (A and D), and $E = \textit{cold}$ otherwise. There is no chance of a mistaken reading. The rover fell into the gorge at position A on day 1, so $X_1 = A$. Let the rover's position on day t be $X_t \in \{A, B, C, D, E, F\}$. The rover is still executing its original programming, trying to move 1 mile east (i.e. right, towards F) every day. However, because of the damage, it only moves east with probability 0.80, and it stays in place with probability 0.20. Your job is to figure out where the rover is, so that you can dispatch your rescue-bot.

1. Filtering: Three days have passed since the rover fell into the ravine. The observations were $(E_1 = \textit{hot}, E_2 = \textit{cold}, E_3 = \textit{cold})$. What is $P(X_3 \mid \textit{hot}_1, \textit{cold}_2, \textit{cold}_3)$, the probability distribution over the rover's position on day 3, given the observations? (This is a probability distribution over the six possible positions).

2. Smoothing: What is $P(X_2 \mid \textit{hot}_1, \textit{cold}_2, \textit{cold}_3)$, the probability distribution over the rover's position on day 2, given the observations? (This is a probability distribution over the six possible positions).

3. Prediction: What is $P(\textit{hot}_4 \mid \textit{hot}_1, \textit{cold}_2, \textit{cold}_3)$, the probability of observing $\textit{hot}_4$ in day 4 given the previous observations in days 1,2, and 3? (This is a single value, not a distribution).

4. Prediction: You decide to attempt to rescue the rover on day 4. However, the transmission of E_4 seems to have been corrupted, and so it is not observed. What is the rover's position distribution for day 4 given the same evidence, $P(X_4 \mid \textit{hot}_1, \textit{cold}_2, \textit{cold}_3)$?

5. Bonus Question for Extra Credit (10 points): What is $P(hot_4, hot_5, cold_6 \mid hot_1, cold_2, cold_3)$, the probability of observing hot_4 and hot_5 and $cold_6$ in days 4,5,6 respectively, given the previous observations in days 1,2, and 3? (This is a single value, not a distribution). Note: The answer is a little bit too long.

You need to apply the formulas covered in the lecture to get to the answers. Do not just guess the answer using your logic. There are a lot of calculations involved, but most of them result in zeros. Save your time, if a product of probabilities contains a probability that is 0, then just write 0 and do not write down the entire product.

Problem 5 (*10 points*): Consider the Markov Decision Process (MDP) with transition probabilities and reward function as given in the tables below. Assume the discount factor $\gamma = 1$ (i.e., there is no actual discounting).

s	a	s'	$T(s,a,s')$
A	1	A	1
A	1	B	0
A	2	A	0.5
A	2	B	0.5

s	a	$R(s,a)$
A	1	0
A	2	-1

s	a	s'	$T(s,a,s')$
B	1	A	0
B	1	B	1
B	2	A	0
B	2	B	1

s	a	$R(s,a)$
B	1	5
B	2	0

We follow the steps of the Policy Iteration algorithm as explained in the class.

1. Write down the Bellman equation.

2. The initial policy is $\pi(A) = 1$ and $\pi(B) = 1$. That means that action 1 is taken when in state A, and the same action is taken when in state B as well. Calculate the values $V_2^\pi(A)$ and $V_2^\pi(B)$ from **two** iterations of policy evaluation (Bellman equation) after initializing both $V_0^\pi(A)$ and $V_0^\pi(B)$ to 0.

3. Find an improved policy π_{new} based on the calculated values $V_2^\pi(A)$ and $V_2^\pi(B)$.