Temporal Models:

1) Transition model: $P(X_{t+1} = A \mid X_t = A) = 0.6$

$P(X_{t+1} = B \mid X_t = A) = 0.4$

$P(X_{t+1} = A \mid X_t = B) = 0.4$

$P(X_{t+1} = B \mid X_t = B) = 0.6$

Observation model: $P(E_t = 0 \mid X_t = A) = 0.9$

$P(E_t = 1 \mid X_t = A) = 0.1$

$P(E_t = 0 \mid X_t = B) = 0.2$

$P(E_t = 1 \mid X_t = B) = 0.1$

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2) Filtering:

$$P(X_1 = B \mid E_1 = 1) = \alpha\, P(E_1 = 1 \mid X_1 = B) \times \sum_{X_0} P(X_0)\, P(X_1 = B \mid X_0)$$

$$= \alpha \times P(E_1 = 1 \mid X_1 = B) \times \big(P(X_0 = A) P(X_1 = B \mid X_0 = A) + P(X_0 = B) P(X_1 = B \mid X_0 = B) \big)$$

$$= \alpha \times 0.8 \times 0.5 \times (0.4 + 0.6) = \alpha \times 0.8$$

$$P(X_1 = A \mid E_1 = 1) = \alpha \times 0.1$$

$$\Rightarrow \begin{cases} P(X_1 = A \mid E_1 = 1) = \dfrac{0.1}{0.8 + 0.1} \approx 0.11 \\ P(X_1 = B \mid E_1 = 1) = \dfrac{0.8}{0.8 + 0.1} \approx 0.89 \end{cases}$$

3,4) Prediction:

* $P(E_2=1|X_1=A) = \sum_{X_2} P(E_2=1|X_2)\, P(X_2|X_1=A)$

$= P(E_2=1|X_2=A)\, P(X_2=A|X_1=A) + P(E_2=1|X_2=B)\, P(X_2=B|X_1=A)$

$= 0.1 \times 0.6 + 0.8 \times 0.4 = 0.38$

* $P(E_2=1|X_1=B) = \sum_{X_2} P(E_2=1|X_2)\, P(X_2|X_1=A)$

$= 0.1 \times 0.4 + 0.8 \times 0.6 = 0.52$

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4) Smoothing:

$P(X_1=A|E_1=1, E_2=1) =$

$\alpha\, P(X_1=A|E_1=1)\, P(E_2=1|X_1=A)$

$= \alpha \cdot 0.11 \times 0.38 \quad = \alpha \times 0.04$

$P(X_1=B|E_1=1, E_2=1) =$

$\alpha \times P(X_1=B|E_1=1)\, P(E_2=1|X_1=B)$

$= \alpha \times 0.88 \times 0.52 \quad = \alpha \times 0.46$

$\Rightarrow \alpha = \frac{1}{0.5} \approx 2$

$\Rightarrow P(X_1=A|E_1=1, E_2=1) = 0.08$

$P(X_1=B|E_1=1, E_2=1) = 0.92$

5) Most likely Sequence:

E_t / X_t	$E_0 = ?$	$E_1 = 1$	$E_2 = 1$
A	0.5	$P(X_0=A)P(X_1=A \mid X_0=A)P(E_1=1 \mid X_1=A) = 0.5 \times 0.6 \times 0.1 = \boxed{0.03}$ $\cancel{P(X_0=B)P(X_1=A \mid X_0=B)P(E_1=1 \mid X_1=A) = 0.5 \times 0.4 \times 0.1 = 0.02}$	$\cancel{P(X_1=A)P(X_2=A \mid X_1=A)P(E_2=1 \mid X_2=A) = 0.03 \times 0.6 \times 0.1 = 0.0018}$ $P(X_1=B)P(X_2=A \mid X_1=B) \cdot P(E_2=1 \mid X_2=A)$ $0.24 \times 0.4 \times 0.1 = \boxed{0.0096}$
B	0.5	$\cancel{P(X_0=A)P(X_1=B \mid X_0=A)P(E_1=1 \mid X_1=B) = 0.5 \times 0.4 \times 0.8 = 0.16}$ $P(X_0=B)P(X_1=B \mid X_0=B)P(E_1=1 \mid X_1=B) = 0.5 \times 0.6 \times 0.8 = \boxed{0.24}$	$\cancel{P(X_1=A)P(X_2=B \mid X_1=A)P(E_2=1 \mid X_2=B) = 0.03 \times 0.4 \times 0.8 = 0.0096}$ $P(X_1=B)P(X_2=B \mid X_1=B)P(E_2=1 \mid X_2=B) = 0.24 \times 0.6 \times 0.8 = \boxed{0.12}$

Therefore, the most likely sequence is $X_0 = B$, $X_1 = B$, $X_2 = B$.

Bayesian Networks:

1) $P(c \mid r, \neg p, l, s, \neg e) =$

$$\alpha P(c, r, \neg p, l, s, \neg e) =$$

$$\alpha \sum_{A} P(c, r, \neg p, l, s, \neg e, A) =$$

$$\alpha \left[P(c, r, \neg p, l, s, \neg e, a) + P(c, r, \neg P, l, s, \neg e, \neg a) \right] =$$

$$\alpha \left[P(\neg e) P(r) P(c \mid r) P(\neg p \mid c) P(l \mid c) P(s \mid c, a) P(a \mid \neg e) + P(\neg e) P(r) P(c \mid r) P(\neg p \mid c) P(l \mid c) P(s \mid c, \neg a) P(\neg a \mid \neg e) \right]$$

$$= \alpha \left[0.8 \times 0.1 \times 0.1 \times 0.1 \times 0.3 \times 0.8 \times 0 + 0.8 \times 0.1 \times 0.1 \times 0.1 \times 0.3 \times 0.7 \times 1 \right]$$

$$= \alpha \times 0.000168$$

$$P(\neg c \mid r, \neg p, l, s, \neg e) =$$

$$\alpha\, P(\neg c, r, \neg p, l, s, \neg e) =$$

$$\alpha \sum_{A} P(\neg c, r, \neg p, l, s, \neg e, A) =$$

$$\alpha \big[P(\neg c, r, \neg p, l, s, \neg e, a) +$$
$$P(\neg c, r, \neg p, l, s, \neg e, \neg a) \big] =$$

$$\alpha \big[P(\neg e) P(r) P(\neg c \mid r) P(\neg p \mid \neg c) P(l \mid \neg c) P(s \mid \neg c, a) \times P(a \mid \neg e)$$
$$+$$
$$P(\neg e) P(r) P(\neg c \mid r) P(\neg p \mid \neg c) P(l \mid \neg c) P(s \mid \neg c, \neg a)\, P(\neg a \mid \neg e) \big]$$

$$= \alpha \big[0.8 \times 0.1 \times 0.9 \times 0.95 \times 0.01 \times 0.20 \times 0$$
$$+$$
$$0.8 \times 0.1 \times 0.9 \times 0.95 \times 0.01 \times 0.01 \times 1$$

$$= \alpha \times 0.00000684$$

Thus, $P(c \mid r, \neg p, l, s, \neg e) = \dfrac{0.000168}{0.00000684 + 0.000168}$

$$= 0.96$$

2) C is independent of A given R because any variable is independent of its non-descendant given its parents

3) R is indepentent of P given C because $\{C\}$ is the Markov Blanket of R.

Problem 3:

(a) True

(b) False

(c) False

(d) True

(e) False

(f) False

(g) False

(h) False

(i) False

(j) False

(k) False

(l) True

(m) False

(n) True

(o) False

(p) False