$\mathcal{P}$1

For this problem, we define H to be all members of the species *Homo sapiens* (humans) currently living, D as all members of the species *Canis familiaris* (domesticated dogs) currently living. Let $L(x, y) :=$ x *loves* y.

$\mathcal{P}$1.a

$$\forall x \in H, \ \forall y \in D, \ L(x, y)$$

$\mathcal{P}$1.b

$$\forall x \in H, \ \exists y \in D : L(x, y) \lor x \in D$$

$\mathcal{P}$1.c

Define $z(x) :=$ x *is the greatest integer*.

$$\forall x \in \mathbb{Z}, \ \neg z(x)$$

$\mathcal{P}$1.d

Define $R(x) :=$ x *needs rest to be healthy*, $E(x) :=$ x *needs excercise to be healthy*, and $D(x) :=$ x *needs a good diet to be healthy*.

$$\forall x \in H, \ R(x) \land E(x) \land D(x)$$

$\mathcal{P}$1.e

$$\forall x \in \mathbb{Z}, \ \exists y \in \mathbb{Z} : y > x$$

$\mathcal{P}$1.f

Define $D(x, y) :=$ x *divides* y. Define $S := \mathbb{Z}^+ \setminus \{1, x\}$ as the positive integers without 1 and x.

$$\nexists y \in S : D(x, y)$$

$\mathcal{P}$2

All the definitions from the previous problem carry over as appropriate.

$\mathcal{P}$2.a

$$\exists x \in H : \exists y \in D : \neg L(x, y)$$

$\mathcal{P}$2.b

$$\exists x \in H : \forall y \in D, \neg L(x, y) \wedge x \notin D$$

$\mathcal{P}$2.c

$$\exists x \in \mathbb{Z} : z(x)$$

$\mathcal{P}$2.d

$$\exists x \in H : \neg R(x) \vee \neg E(x) \vee \neg D(x)$$

$\mathcal{P}$2.e

$$\exists x \in \mathbb{Z} : \forall y \in \mathbb{Z},\ \ y \leqslant x$$

$\mathcal{P}$2.f

$$\exists y \in S : \neg D(x, y)$$

$\mathcal{P}$3

$\mathcal{P}$3.a Consistency

$$\begin{aligned} (p \vee \neg p) &\iff T \\ \neg(p \vee \neg p) &\iff \neg T \\ \neg p \wedge p &\iff F \end{aligned}$$

$\mathcal{P}$3.b Absorption

$$\begin{aligned} (p \wedge (p \vee q)) &\iff ((p \wedge p) \vee (p \wedge q)) \\ &\iff (p \vee (p \wedge q)) \end{aligned}$$

$\mathcal{P}$3.c Contrapositive

$$\begin{aligned} (p \Rightarrow q) &\iff (\neg p \vee q) \\ &\iff (q \vee \neg p) \\ &\iff (\neg(\neg q) \vee \neg p) \\ &\iff (\neg q \Rightarrow \neg p) \end{aligned}$$

$\mathcal{P}$4

$$P \veebar Q \equiv (P \vee Q) \wedge (\neg (P \wedge Q))$$

Intuitively, this makes sense, because it explicitly checks that one of P or Q are true, but also explicitly checks that both aren't true.

$\mathcal{P}$5

$\mathcal{P}$5.a

For all x in the set of real numbers, there exists a y in the set of real numbers such that x and y sum to 0.

This is true because for each $x \in \mathbb{R}$, there's a $-x$, which is its additive inverse, and by definition, the two sum to 0.

$\mathcal{P}$5.b

There exists an x in the set of real numbers such that for all y in the set of real numbers x and y sum to 0.

This is false, because not every real number is the additive inverse to x.

$\mathcal{P}$6

The key issue here is that we need to specify that **exactly one** element of X satisfies $P(x)$, so we say that there's exists an element in the set that satisfies $P(x)$ and if another value satisfies $P(x)$ then it is equal to the original value.

$$\exists! x \in X : P(x) \equiv \exists x \in X : (P(x) \wedge \forall y \in X, (P(y) \Rightarrow (y = x)))$$