Problem 1

Let x be the value of the first roll and let y be the value of the second roll. The probability of them being the same, $\mathbb{P}(x = y)$, is $\frac{1}{6}$, so $\mathbb{P}(x \neq y) = \frac{5}{6}$.

Moreover, we next need to calculate the probability of at least one of x and y being 6 and $x \neq y$. Note that x and y cannot both be equal to 6, as they would be equal then. Therefore, we let $x = 6$, so $y \in \{1, 2, 3, 4, 5\}$, contributing 5 cases. We do the same with y, so we have a total probability of $\frac{10}{36} = \frac{5}{18}$.

The conditional probability is therefore $\frac{\frac{5}{18}}{\frac{5}{6}} = \boxed{\frac{1}{3}}$.

Problem 5

The probability of drawing 1 of 6 white balls out of 15 total balls is $\frac{6}{15}$. After we draw this ball, there are 5 white balls left and 14 total balls, making the probability of the second ball being white $\frac{5}{14}$. We can do the same for the black balls to get $\frac{9}{13}$ and $\frac{8}{12}$. Using the Law of Total Probability, we multiply these events, finding

$$\begin{aligned}
\left(\frac{6}{15}\right)\left(\frac{5}{14}\right)\left(\frac{9}{13}\right)\left(\frac{8}{12}\right) &= \left(\frac{2}{5}\right)\left(\frac{5}{14}\right)\left(\frac{9}{13}\right)\left(\frac{2}{3}\right) \\
&= \boxed{\frac{6}{91}}
\end{aligned}$$

Problem 8

For the sake of this problem, we assume that the probability of having a girl or boy is equal: $\frac{1}{2}$.

The probability that both are girls is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$. The probability that the older is a girl is $\frac{1}{2}$. Therefore, the conditional probability is $\frac{\frac{1}{4}}{\frac{1}{2}} = \boxed{\frac{1}{2}}$.

Problem 10

Let X_1, X_2, X_3 be the events that the first, second, and third card are spades, respectively. We are asked to find $\mathbb{P}(X_1 | X_2 X_3)$. Using the definition of conditional probability and the law of total probability,

$$\begin{aligned}
\mathbb{P}(X_1 | X_2 X_3) &= \frac{\mathbb{P}(X_1 X_2 X_3)}{\mathbb{P}(X_2 X_3)} \\
&= \frac{\mathbb{P}(X_1 X_2 X_3)}{\mathbb{P}(X_2 X_3 | X_1)\mathbb{P}(X_1) + \mathbb{P}(X_2 X_3 | X_1^c)\mathbb{P}(X_1^c)} \\
&= \frac{\mathbb{P}(X_1 X_2 X_3)}{\frac{\mathbb{P}(X_1 X_2 X_3)}{\mathbb{P}(X_1)}\mathbb{P}(X_1) + \frac{\mathbb{P}(X_1^c X_2 X_3)}{\mathbb{P}(X_1^c)}\mathbb{P}(X_1^c)} \\
&= \frac{\mathbb{P}(X_1 X_2 X_3)}{\mathbb{P}(X_1 X_2 X_3) + \mathbb{P}(X_1^c X_2 X_3)} \\
&= \frac{\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)}{\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right) + \left(\frac{39}{52}\right)\left(\frac{13}{51}\right)\left(\frac{12}{50}\right)} \\
&= \frac{\frac{11}{850}}{\frac{11}{850} + \frac{39}{850}} \\
&= \boxed{\frac{11}{40}}
\end{aligned}$$