# HW 6

## Problem 1

Let $X$ be a random variable denoting the winnings. Since doing otherwise would be cumbersome, we make a table illustrating what the possible values of $X$ are, as well as what ball combinations they correspond to.

| **Value of $X$** | **Ball 1** | **Ball 2** |
|---|---|---|
| 4 | Black | Black |
| 2 | Black | Orange |
| 1 | Black | White |
| 0 | Orange | Orange |
| -1 | White | Orange |
| -2 | White | White |

Note that **Ball 1** and **Ball 2** may be freely switched.

This means,

$$\begin{aligned}
\mathbb{P}(X = 4) &= \frac{4}{14} \cdot \frac{3}{13} \\
&= \boxed{\frac{6}{91}} \\
\mathbb{P}(X = 2) &= 2 \cdot \frac{4}{14} \cdot \frac{2}{13} \\
&= \boxed{\frac{8}{91}} \\
\mathbb{P}(X = 1) &= 2 \cdot \frac{4}{14} \cdot \frac{8}{13} \\
&= \boxed{\frac{32}{91}} \\
\mathbb{P}(X = 0) &= \frac{2}{14} \cdot \frac{1}{13} \\
&= \boxed{\frac{1}{91}} \\
\mathbb{P}(X = -1) &= 2 \cdot \frac{8}{14} \cdot \frac{2}{13} \\
&= \boxed{\frac{16}{91}} \\
\mathbb{P}(X = -2) &= \frac{8}{14}\frac{7}{13} \\
&= \boxed{\frac{4}{13}}
\end{aligned}$$

## Problem 6

There are 4 possible values of $X$: $\{3, 1, -1, -3\}$. The first and last values correspond to all heads and all tails, respectively. The second and third values correspond to 2 heads and 2 tails, respectively.

$$
\begin{aligned}
\mathbb{P}(X = 3) &= \left(\frac{1}{2}\right)^3 \\
&= \boxed{\frac{1}{8}} \\
\mathbb{P}(X = 1) &= \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) \\
&= \boxed{\frac{3}{8}} \\
\mathbb{P}(X = -1) &= \binom{3}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right) \\
&= \boxed{\frac{3}{8}} \\
\mathbb{P}(X = -3) &= \left(\frac{1}{2}\right)^3 \\
&= \boxed{\frac{1}{8}}
\end{aligned}
$$

# Problem 21

## Problem 21.a

$\mathbb{E}(X)$ will be greater than $\mathbb{E}(Y)$, as there are more buses than there are students.

## Problem 21.b

$$
\begin{aligned}
\mathbb{E}(X) &= 40 \cdot \frac{40}{148} + 33 \cdot \frac{33}{148} + 25 \cdot \frac{25}{148} + 50 \cdot \frac{50}{148} \\
&= \frac{1}{148}(1600 + 1089 + 625 + 2500) \\
&= \frac{5814}{148} \\
&= \boxed{\frac{2907}{74}} \\
&\approx 39.284 \\
\mathbb{E}(X) &= 40 \cdot \frac{1}{4} + 33 \cdot \frac{1}{4} + 25 \cdot \frac{1}{4} + 50 \cdot \frac{1}{4} \\
&= \frac{148}{4} \\
&= \boxed{37}
\end{aligned}
$$

# Problem 25

## Problem 25.a

When $X = 1$, only one of the coins may be heads. Therefore,

$$\begin{aligned}
\mathbb{P}(X = 1) &= \frac{3}{5} \cdot \frac{3}{10} + \frac{2}{5} \cdot \frac{7}{10} \\
&= \boxed{\frac{23}{50}}
\end{aligned}$$

## Problem 25.b

We must first calculate $\mathbb{P}(X = 0)$ and $\mathbb{P}(X = 2)$.

$$\begin{aligned}
\mathbb{P}(X = 0) &= \frac{2}{5} \cdot \frac{3}{10} \\
&= \frac{3}{25} \\
\mathbb{P}(X = 2) &= \frac{3}{5} \cdot \frac{7}{10} \\
&= \frac{21}{50}
\end{aligned}$$

Now, we can calculate $\mathbb{E}(X)$.

$$\begin{aligned}
\mathbb{E}(X) &= 0 \cdot \frac{3}{25} + 1 \cdot \frac{23}{50} + 2 \cdot \frac{21}{50} \\
&= \frac{23}{50} + \frac{42}{50} \\
&= \boxed{\frac{13}{10}} \\
&= 1.3
\end{aligned}$$