Problem 35

Let N be the event that it is nonrainy and let R be the event that it is rainy. Let E be the event that Joe is early and let L be the event that Joe is late.

Problem 35.a

$$\begin{aligned}
\mathbb{P}(E) &= \mathbb{P}(E|R)\,\mathbb{P}(R) + \mathbb{P}(E|N)\,\mathbb{P}(N) \\
&= (1 - \mathbb{P}(L|R))\,\frac{7}{10} + (1 - \mathbb{P}(L|N))\,\frac{3}{10} \\
&= \frac{7}{10}\cdot\frac{7}{10} + \frac{1}{10}\cdot\frac{3}{10} \\
&= \boxed{\frac{13}{25}}
\end{aligned}$$

Problem 35.b

$$\begin{aligned}
\mathbb{P}(R|E) &= \frac{\mathbb{P}(E|R)\,\mathbb{P}(R)}{\mathbb{P}(E)} \\
&= \frac{(1 - \mathbb{P}(L|R))\,\frac{7}{10}}{\frac{13}{25}} \\
&= \frac{\frac{21}{100}}{\frac{13}{25}} \\
&= \boxed{\frac{21}{52}}
\end{aligned}$$

Problem 40

Let W, B be the events of choosing a white ball and a black ball, respectively. Let H, T be the events of the coin landing heads and tails, respectively.

$$\begin{aligned}
\mathbb{P}(T|W) &= \frac{\mathbb{P}(W|T)\,\mathbb{P}(T)}{\mathbb{P}(W)} \\
&= \frac{\frac{1}{5}\cdot\frac{1}{2}}{\mathbb{P}(W|T)\,\mathbb{P}(T) + \mathbb{P}(W|H)\,\mathbb{P}(H)} \\
&= \frac{\frac{1}{10}}{\frac{1}{5}\cdot\frac{1}{2} + \frac{5}{12}\cdot\frac{1}{2}} \\
&= \frac{\frac{1}{10}}{\frac{1}{10} + \frac{5}{24}} \\
&= \boxed{\frac{12}{37}}
\end{aligned}$$

Problem 47

Problem 47.a

Since B is only given a chance to fix it if A fails, A and B are mutually exclusive. Therefore, $A \cap B = \varnothing$.

$$\begin{aligned}
\mathbb{P}(A \cup B) &= \mathbb{P}(A) + \mathbb{P}(B) \\
&= \frac{3}{10} + \mathbb{P}(B|A^c)\,\mathbb{P}(A^c) \\
&= \frac{3}{10} + \frac{2}{5} \cdot \frac{7}{10} \\
&= \boxed{\frac{29}{50}}
\end{aligned}$$

Problem 47.b

$$\begin{aligned}
\mathbb{P}((B|A^c) \,|\, (A \cup B)) &= \frac{\mathbb{P}((B|A^c) \cap (A \cup B))}{\mathbb{P}(A \cup B)} \\
&= \frac{\mathbb{P}(B|A^c)}{\mathbb{P}(A \cup B)} \\
&= \frac{\frac{2}{5}}{\frac{29}{50}} \\
&= \boxed{\frac{20}{29}}
\end{aligned}$$

Problem 49

Let W be the event that all the balls chosen are white. Let D be the dice value that is rolled.

$$\begin{aligned}
\mathbb{P}(W) &= \sum_{i=1}^{i=6} \mathbb{P}(W|D=i)\,\mathbb{P}(D=i) \\
&= \frac{1}{6} \sum_{i=1}^{i=6} \frac{\binom{5}{i}}{\binom{15}{i}} \\
&= \frac{1}{6}\left(\frac{1}{3} + \frac{2}{21} + \frac{2}{91} + \frac{1}{273} + \frac{1}{3003} + 0\right) \\
&= \boxed{\frac{5}{66}}
\end{aligned}$$

$$\begin{aligned}
\mathbb{P}\left(D = 3 | W\right) &= \frac{\mathbb{P}\left(W | D = 3\right) \mathbb{P}\left(D = 3\right)}{\mathbb{P}\left(W\right)} \\
&= \frac{\frac{1}{6} \cdot \frac{2}{91}}{\frac{5}{66}} \\
&= \boxed{\frac{22}{455}}
\end{aligned}$$