Brachistochrone Problem

Rajeev Atla

Administrivia

Definitions

Getting Started

Lagrangians

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Physics Club

November 20, 2020

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More advanced

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- More advanced
- Goals

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- More advanced
- Goals
 - ullet Get everyone to pass F=ma exam

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- More advanced
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 - ullet Get everyone to pass F=ma exam
 - USAPhO Qualifiers!!

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- More advanced
- Goals
 - Get everyone to pass F=ma exam
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- Prerequisites (recommended)

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- More advanced
- Goals
 - Get everyone to pass F=ma exam
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- Prerequisites (recommended)
 - Taken/currently taking a physics class

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- More advanced
- Goals
 - Get everyone to pass F=ma exam
 - USAPhO Qualifiers!!
- Prerequisites (recommended)
 - Taken/currently taking a physics class
 - Or...

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Getting Started

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- More advanced
- Goals
 - Get everyone to pass F=ma exam
 - USAPhO Qualifiers!!
- Prerequisites (recommended)
 - Taken/currently taking a physics class
 - Or...
 - Willingness to learn

PSA: Problems

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Definition:

Getting Started

On classroom

PSA: Problems

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On classroom

• Due date: next meeting

PSA: Problems

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- On classroom
- Due date: next meeting
- We hope to continue this pattern for the rest of this year

What Do I mean?

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Etymology

What Do I mean?

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Definitions

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Etymology

• Brachistos $(\beta \rho \alpha \chi \iota \sigma \tau \sigma)$ means "shortest"

What Do I mean?

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Definitions

Etymology

- Brachistos $(\beta \rho \alpha \chi \iota \sigma \tau \sigma)$ means "shortest"
- Chronos $(\chi \rho o \nu o \sigma)$ means "time"

What Do I mean?

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Getting Startet

- Etymology
 - Brachistos $(\beta \rho \alpha \chi \iota \sigma \tau \sigma)$ means "shortest"
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- A brachistochrone curve is the path such that a ball traveling along this path takes the least amount of time

What Do I mean?

Brachistochrone Problem

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- This is our problem

What Do I mean?

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Getting Started

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- This is our problem
- Formal problem statement

What Do I mean?

Brachistochrone Problem

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- Formal problem statement
 - Constraints: given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

What Do I mean?

Brachistochrone Problem

Definitions

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- A brachistochrone curve is the path such that a ball traveling along this path takes the least amount of time
- This is our problem
- Formal problem statement
 - Constraints: given two points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$
 - Find function y = f(x) such that the time it takes for a ball to travel under the influence of gravity from P_1 to P_2



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Getting Started

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• Let s be a position vector

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- Let s be a position vector
- Let v be the associated velocity vector

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- Let s be a position vector
- Let v be the associated velocity vector
- From last lecture, recall that

$$v = \frac{ds}{dt} \Rightarrow dt = \frac{ds}{v} \Rightarrow t_{12} = \int_{P_1}^{P_2} \frac{ds}{v}$$

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• Kinetic energy $K = \frac{1}{2}mv^2$

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- Kinetic energy $K = \frac{1}{2}mv^2$
- Gravitational potential energy U = mgy

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- Kinetic energy $K = \frac{1}{2}mv^2$
- Gravitational potential energy U = mgy
- Conservation of energy means that these two are equal

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- Kinetic energy $K = \frac{1}{2}mv^2$
- Gravitational potential energy U = mgy
- Conservation of energy means that these two are equal

$$\frac{1}{2}mv^2 = mgy \Rightarrow v = \sqrt{2gy}$$

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- Kinetic energy $K = \frac{1}{2}mv^2$
- Gravitational potential energy U = mgy
- Conservation of energy means that these two are equal

$$\frac{1}{2}mv^2 = mgy \Rightarrow v = \sqrt{2gy}$$

• We can substitute this into the last equation

Pythagorean Theorem

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$$ds^{2} = dx^{2} + dy^{2}$$

$$ds^{2} = dx^{2} \left(1 + \left(\frac{dy^{2}}{dx^{2}} \right) \right)$$

$$ds^{2} = dx^{2} \left(1 + \left(\frac{dy}{dx} \right)^{2} \right)$$

$$ds^{2} = dx^{2} \left(1 + y'^{2} \right)$$

$$ds = dx \sqrt{1 + y'^{2}}$$

Putting It All Together

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Original equation:

$$t_{12} = \int\limits_{P_1}^{P_2} \frac{ds}{v}$$

Putting It All Together

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Lagrangian

Original equation:

$$t_{12} = \int\limits_{P_1}^{P_2} \frac{ds}{v}$$

Conservation of energy:

$$v=\sqrt{2gy}$$

Putting It All Together

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Definitions

Getting Started

Lagrangian

Original equation:

$$t_{12} = \int\limits_{P_1}^{P_2} \frac{ds}{v}$$

• Conservation of energy:

$$v = \sqrt{2gy}$$

• Pythagorean theorem:

$$ds = dx \sqrt{1 + y'^2}$$

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$$t_{12} = \int\limits_{P_1}^{P_2} \sqrt{\frac{1 + y'^2}{2gy}} dx$$

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Lagrangians

$$t_{12} = \int\limits_{P_1}^{P_2} \sqrt{\frac{1 + y'^2}{2gy}} dx$$

• We want to minimize this by...

Lagrangians

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Getting Started

$$t_{12} = \int_{P_1}^{P_2} \sqrt{\frac{1 + y'^2}{2gy}} dx$$

- We want to minimize this by...
- picking a function y = f(x) to minimize integral

Lagrangians

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Getting Started

$$t_{12} = \int\limits_{P_1}^{P_2} \sqrt{\frac{1 + y'^2}{2gy}} dx$$

- We want to minimize this by...
- picking a function y = f(x) to minimize integral
- How do we do it???

Lagrangians

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Definitions

Getting Started

$$t_{12} = \int\limits_{P_1}^{P_2} \sqrt{\frac{1 + y'^2}{2gy}} dx$$

- We want to minimize this by...
- picking a function y = f(x) to minimize integral
- How do we do it???
- Lagrangians

More About Lagrangians

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Lagrangians

Let the Lagrangian be

$$\mathcal{L} = \sqrt{\frac{1 + y'^2}{2gy}}$$

More About Lagrangians

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Definitions

Getting Started

Lagrangians

• Let the Lagrangian be

$$\mathcal{L} = \sqrt{\frac{1 + y'^2}{2gy}}$$

• Remeber that y = f(x)

$$\mathcal{L}(x) = \sqrt{\frac{1 + f'(x)^2}{2gf(x)}}$$

• $f'(x) = \frac{df(x)}{dx}$ (Lagrangian notation)

Least Action Principle

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Lagrangians

• We need to minimize the time

$$t_{12} = \int\limits_{P_1}^{P_2} \mathcal{L}(x) dx$$

Least Action Principle

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Definitions

Getting Started

Lagrangians

• We need to minimize the time

$$t_{12} = \int\limits_{P_1}^{P_2} \mathcal{L}(x) dx$$

• Any ideas?

Least Action Principle

Brachistochrone Problem

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Lagrangians

• We need to minimize the time

$$t_{12} = \int\limits_{P_1}^{P_2} \mathcal{L}(x) dx$$

- Any ideas?
- Euler-Lagrange equation

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x}$$

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- ullet Symbol is ∂
- Hold all other variables constant while taking a derivative

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Getting Started

- Symbol is ∂
- Hold all other variables constant while taking a derivative
- Let f(x,y) = 2x + 3y, what are $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$?

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Getting Started

- ullet Symbol is ∂
- Hold all other variables constant while taking a derivative
- Let f(x,y) = 2x + 3y, what are $\frac{\partial f(x,y)}{\partial x}$ and $\frac{\partial f(x,y)}{\partial y}$?

$$\frac{\partial f(x,y)}{\partial x} = 2$$
$$\frac{\partial f(x,y)}{\partial y} = 3$$