Reference Materials for F=ma

1 Table of Constants

Constant	Value	SI Base Units
g	10	$\frac{\text{m}}{\text{s}^2}$
G	$6.67 \cdot 10^{-11}$	$\frac{\text{m}^3}{\text{kg} \cdot \text{s}^2}$
M_E	$6.0 \cdot 10^{24}$	kg
R_E	$6.4 \cdot 10^{6}$	m
AU	$1.50 \cdot 10^{11}$	m
c	$3.0 \cdot 10^{8}$	$\frac{\text{m}}{\text{s}}$
ρ_w	997	$\frac{\text{kg}}{\text{m}^3}$

2 Dimensional Analysis

2.1 Notation

$$[g] = \frac{\text{m}}{\text{s}^2}$$

2.2 Buckingham Pi Theorem

If you have N unique quantities with D independent dimensions, then you can form $N - D$ dimensionless quantities. Dimensional analysis has nothing to say about them.

3 Error Analysis

3.1 Definition

For a measurement x, we can often say that it has an error δx. For professional physicists (who are most of the writers of the F=ma), δx is taken to mean the standard deviation of a Gaussian with mean x. Go back to Gaussians if you ever get confused.

3.2 Error Propogation Formulae

$$
\begin{aligned}
Q = x \pm y \Rightarrow \delta Q &= \sqrt{(\delta x)^2 + (\delta x)^2} \\
Q = \frac{xy}{z} \Rightarrow \frac{\delta Q}{Q} &= \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2 + \left(\frac{\delta z}{z}\right)^2} \\
Q = x^n \Rightarrow \delta Q &= \left|\frac{n}{x}\right| \delta x \\
Q = f(x) \Rightarrow \delta Q &\approx \left|\frac{df}{dx}\right| \delta x \\
Q = f(x, y, z) \Rightarrow \delta Q &\approx \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 (\delta x)^2 + \left(\frac{\partial f}{\partial y}\right)^2 (\delta y)^2 + \left(\frac{\partial f}{\partial z}\right)^2 (\delta z)^2}
\end{aligned}
$$

Last 2 only hold true if you can make the tangent line approximation.

4 Approximations

Exact	Approximate	Error
e^x	$1 + x + \frac{1}{2}x^2$	$\mathcal{O}\left(x^3\right)$
$\sin x$	$x - \frac{x^3}{6}$	$\mathcal{O}\left(x^5\right)$
$\cos x$	$1 - \frac{1}{2}x^2$	$\mathcal{O}(x^4)$
$\tan x$	$x + \frac{1}{3}x^3$	$\mathcal{O}(x^5)$
$(1+x)^n$	$1 + nx$	$\mathcal{O}(x^2 n^2)$
$\ln(1+x)$	$x - \frac{1}{2}x^2$	$\mathcal{O}(x^3)$

All assume that $x \ll 1$.

5 1D Kinematics

$$
\begin{aligned}
x &= x_0 + v_0 (t - t_0) + \frac{1}{2} g (t - t_0)^2 \\
v &= v_0 + a (t - t_0) \\
v^2 &= v_0^2 + 2a (x - x_0) \\
x &= x_0 + \frac{1}{2} (v + v_0)(t - t_0)
\end{aligned}
$$

6 2D Kinematics

$$
\begin{aligned}
x(t) &= x_0 + v_0\,(t - t_0)\cos\theta \\
y(t) &= y_0 + v_0\,(t - t_0)\sin\theta - \frac{1}{2}g\,(t - t_0)^2 \\
\max y &= y_0 + \frac{v_0^2 \sin^2\theta}{2g} \\
\arg\max_\theta y &= \frac{\pi}{2} \\
\max x &= x_0 + \frac{v_0\cos\theta}{g}\left(v_0\sin\theta + \sqrt{v_0^2\sin^2\theta + 2gy_0}\right) \\
\arg\max_\theta x &= \cos^{-1}\sqrt{\frac{v_0^2 + 2gy_0}{2v_0^2 + 2gy_0}}
\end{aligned}
$$

7 Newton's Laws

7.1 Constant Mass

$$\sum F = \frac{dp}{dt} = ma$$

7.2 Variable Mass

$$\sum F = m\dot{v} + \dot{m}v$$

7.3 Polar Coordinates

$$
\begin{aligned}
v_r &= \dot{r} \\
v_\theta &= r\dot{\theta} \\
F_r &= m\left(\ddot{r} - r\dot{\theta}^2\right) \\
F_\theta &= m\left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)
\end{aligned}
$$

7.4 Spherical Coordinates

$$\begin{aligned}
v_r &= \dot{r} \\
v_\theta &= r\dot{\theta}\cos\phi \\
v_\phi &= r\dot{\phi} \\
F_r &= m\left(\ddot{r} - r\dot{\theta}^2\cos^2\phi - 2r\dot{\phi}^2\right) \\
F_\theta &= m\left(2\dot{r}\dot{\theta}\cos\phi + r\ddot{\theta}\cos\phi - 2r\dot{\theta}\dot{\phi}\sin\phi\right) \\
F_\phi &= m\left(2\dot{r}\dot{\phi} + r\dot{\phi}^2\sin\phi\cos\phi + r\ddot{\phi}\right)
\end{aligned}$$

7.5 Atwood Machines

$$m_{\text{eff}} = \frac{4m_1m_2}{m_1+m_2}$$

8 Oscillations

$$ma = -kx$$

$$\begin{aligned}
x(t) &= A\cos(\omega t + \phi) \\
\omega &= \sqrt{\frac{k}{m}} = \frac{2\pi}{T} = 2\pi f \\
\tan\phi &= \frac{x_0}{v_0}\sqrt{\frac{k}{m}}
\end{aligned}$$

8.1 General Oscillators

Given a potential $V(x)$

$$\begin{aligned}
V''(x_0) &> 0 \\
\omega &= \sqrt{\frac{V''(x_0)}{m}}
\end{aligned}$$

9 Energy

$$K = \frac{1}{2}mv^2$$
$$U_g = -mgh$$
$$U_s = \frac{1}{2}kx^2$$

10 Elastic Collisions

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}m_1v_1'^2 + \frac{1}{2}m_2v_2'^2$$
$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

$$\begin{pmatrix} v_1' \\ v_2' \end{pmatrix} = \frac{1}{m_1 + m_2}\begin{pmatrix} m_1 - m_2 & 2m_2 \\ 2m_1 & m_2 - m_1 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

11 Inelastic Collisions

$$m_1v_1 + m_2v_2 = (m_1 + m_2)\,v'$$
$$v' = v_1\frac{m_1}{m_1 + m_2} + v_2\frac{m_2}{m_1 + m_2}$$
$$\Delta K = -\frac{1}{2}\frac{m_1m_2}{m_1 + m_2}\left(v_1 - v_2\right)^2$$

12 Moments of Inertia

Object	Axis Location	Moment
Point Particle	Center	MR^2
2 Point Particles	CM	μR^2
Rod	CM	$\frac{1}{12}ML^2$
Rod	End	$\frac{1}{3}ML^2$
Cylindrical Shell	CM	MR^2
Cylinder	CM	$\frac{1}{2}MR^2$
Cone	CM	$\frac{3}{10}MR^2$
Conical Shell	CM	$\frac{1}{2}MR^2$
Spherical Shell	CM	$\frac{2}{3}MR^2$
Sphere	CM	$\frac{2}{5}MR^2$

$$I = I_{\text{CM}} + md^2$$

13 Pendula

$$\omega = \sqrt{\frac{g}{\ell}}$$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

13.1 Center of Percussion

$$p = \frac{I}{dM}$$

14 Celestial Mechanics

$$F_{\mathrm{ab}} = -\frac{Gm_{\mathrm{a}}m_{\mathrm{b}}}{r^2}$$
$$v_{\mathrm{orbit}} = \sqrt{\frac{GM}{R}}$$
$$v_{\mathrm{escape}} = \sqrt{\frac{2GM}{R}}$$
$$\mu = \frac{m_{\mathrm{a}}m_{\mathrm{b}}}{m_{\mathrm{a}} + m_{\mathrm{b}}}$$
$$v^2 = GM\left(\frac{2}{r} - \frac{1}{a}\right)$$
$$E = -\frac{Gm_{\mathrm{a}}m_{\mathrm{b}}}{2a}$$
$$U_{\mathrm{self}} = -\frac{Gm^2}{2R}$$
$$\min r = a\,(1-e)$$
$$\arg\min_{v} r = \sqrt{\frac{GM}{a}\frac{1+e}{1-e}}$$
$$\max r = a\,(1+e)$$
$$\arg\max_{v} r = \sqrt{\frac{GM}{a}\frac{1-e}{1+e}}$$
$$\frac{a^3}{T^2} = \frac{GM}{4\pi^2}$$
$$e = \sqrt{1 + \frac{2Ev_0^2r_0^2}{G^2mM^2}} = \frac{r_{\mathrm{max}} - r_{\mathrm{min}}}{r_{\mathrm{max}} + r_{\mathrm{min}}}$$
$$e = 0 \iff \text{Circle} \iff E = -\frac{Gm_am_b}{R}$$
$$0 < e < 1 \iff \text{Ellipse} \iff E < 0$$
$$e = 1 \iff \text{Parabola} \iff E = 0$$
$$e > 1 \iff \text{Hyperbola} \iff E > 0$$

15 Fluids

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$
$$A_1 v_1 = A_2 v_2$$
$$\frac{dp}{dy} = -\rho g$$
$$F = \eta A \frac{dv}{dy}$$