PRACTISE SET

Context Free Grammar (CFG)

Q1 Write a CFG that

(a) generates strings of balanced paranthesis

(b) generates palindrome for binary string

(c) generates strings having equal numbers of a's & b's

Q2 write a CFG for the regular expression

(a) $r = 0^* | (0+1)^*$

(b) $r = (a+b)^* aa (a+b)^*$

Q3 Design a CFG for the language

(a) $L = \{ a^n b^m \mid n \neq m \}$

(b) $L = \{ ab (bbaa)^n bba (ba)^n \mid n \geq 0 \}$

(c) $L = \{ 0^n 1^n \mid n \geq 0 \} \cup \{ 1^n 0^n \mid n \geq 0 \}$

(d) $L = \{ a^n b^n c^m d^m \mid n \geq 1 \text{ & } m \geq 1 \}$

(e) $L = \{ a^n b^m c^m d^n \mid n \geq 1 \text{ & } m \geq 1 \}$

(f) $L = \{ a^{2n} b^m \mid n \geq 0, m \geq 0 \}$

(g) $L = \{ 0^i 1^j 2^k \mid k \leq i \text{ or } k \leq j \}$

(h) $L = \{ a^n b^{2n} c^m \mid n, m > 0 \}$

(i) $L = \{ a^n b^m c^{2m} \mid n, m > 0 \}$

Q4 Consider a CFG, "G" whose production are

$S \to aAS \mid a$

$A \to SbA \mid SS \mid ba$

show that $S \overset{*}{\Rightarrow} aabbaa$ and construct a derivation tree whose yield is aabbaa

Q5 Change the following grammar into a CNF

(a) $S \to 1A \mid 0B$

$S \to 1AA \mid 0S \mid 0$

$B \to 0BB \mid 1$

(b) $S \to abSb \mid a \mid aAb$

$A \to bS \mid aAAb$

(c) $S \to bA \mid aB$

$A \to bAA \mid aS \mid a$

$B \to aBB \mid bS \mid b$

Properties of CFG

Q6) Prove the language $L = \{a^n 5^n c^n \mid n \geq 0\}$ is not context free Language

Q7) Prove that $L = \{a^n 5^m \mid n = m^2\}$ is not context free

Pushdown Automata

Q8) Design a PDA for the following Language

(a) $L = \{w \in \{a,5\}^* \mid w$ has the equal number of a's & 5's$\}$

(b) $L = \{w w^R \mid w \in \{a,5\}^*, w^R =$ reverse of $w\}$

(c) $L = \{a^n 5^n \mid n \geq 0\}$

(d) $L = \{a^n 5^{2n} \mid n > 0\}$

(e) $L = \{a^n 5^{n+1} \mid n = 1, 2, 3, \ldots\}$

(f) $L = \{a^n 5^m \mid n > m > 0\}$

Q9) Construct a PDA for the RE

$R = 0^* 1^*$

(Q10) Design a PDA for the following CFG

(a) $S \to \epsilon$

$S \to SS$

$S \to (S)$

(b) $S \to aSa \mid bSb \mid c$

(Q11) Construct a PDA for the language

(a) $L = \{a^n b^m c^{n+m} \mid n \geq 0, m \geq 0\}$

(b) $L = \{a^3 b^n c^n \mid n \geq c\}$

Turing machine (TM)

(Q12) Design a TM for the function

(a) $f(m) = m+1$

(b) $f(n) = n+2$

(c) $f(m,n) = m+n$

(d) $f(m,n) = m*n$

(e) $f(m,n) = \begin{cases} m-n & \text{if } m \geq n \\ 0 & \text{otherwise} \end{cases}$

(f) $f(n) = 2n$

(g) $f(n) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{if } x \leq y \end{cases}$

(Q13) Design a TM that replaces every 0 with 1 and every 1 with 0 in a binary string

(Q14) Design a TM that recognises the set of all strings of 0's and 1's containing atleast one 1

(Q15) Design a TM for the function $f(w) = ww^R$ where w^R is the reverse of w

(Q16) Prove that the following function is computable

$$f(m) = \begin{cases} m-2 & \text{if } m > 2 \\ 1 & \text{if } m \leq 2 \end{cases}$$

(Q17) Design a TM for the function $f(x, y) = x$

(Q18) Design a TM which accept the language

(a) $L = \{ a^n b^n \mid n \geq 1 \}$

(b) $L = \{ w \in \{a,b\}^* \mid w$ have equal number of a's & b's $\}$

(c) $L = \{ ab^n \mid n > 0 \}$

Some theory Question

(Q14) State & prove pumping lemma for the context free grammar

~~(Q20) prove that CFL is closed under~~

(Q20) prove that CFLs are closed under union and not closed under

(a) Intersection

(b) Complement

(Q21) what is the difference between DFA (Deterministic finite automata) and PDA (push down automata)

(Q22) What is Turing machine (TM)? What is its structure & rules of operation?

(Q23) Define Halting problem? Prove that it is undecidable.

(Q24) Differentiate between TM & PDA

(Q25) What do you mean by Ambiguity in CFG, Explain with example

(Q26) What is multi-track TM and show that it is equivalent to single track TM

(Q27) Define Non-Deterministic TM

(Q28) Define —
(a) Recursive languages
(b) Recursively Enumerable lang.

(Q29) Prof that Recursive languages are closed under
(1) complementation
(2) Union
(3) Intersection

Q30. Define the Universal Turing machine

Q31 What is diagonalization language and prove that it is not Recursively enumerable language.

~~Q32 Define the term with~~

Q32 Define the following term with example

(1) Class P
(2) Class NP
(3) Class NP Complete
(4) Class NP-Hard

Show their relationship.

Q33 If $L_1 \leq_p L_2$ then prove that if $L_2 \in P$ then $L_1 \in P$

Q34 Prove that if any NP-complete problem is polynomial time solvable then P = NP

Q35 If $L_1 \in NPC$ and $L_1 \leq_p L_2$ then prove that $L_2 \in NP\text{-Hard}$

Q16 Prove the following problems are NP-complete problem

(a) 3 CNF

(b) CLIQUE problem

(c) VERTEX Cover problem

(d) TSP problem.