Assignment 1
Probability And Random Processes

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I. Question 1.4.6

Verify that

$$\angle BOC = 2\angle BAC$$

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II. Solution

Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Also,we have a point $\mathbf{O} = \begin{pmatrix} \frac{-53}{12} \\ \\ \frac{5}{12} \end{pmatrix}$ which is intersection point of the perpendicular bisectors of AB and AC.

1) To find the value of $\angle BOC$:

$$\mathbf{B} - \mathbf{O} = \begin{pmatrix} \frac{5}{12} \\ \\ \frac{67}{12} \end{pmatrix} \tag{1}$$

$$\mathbf{C} - \mathbf{O} = \begin{pmatrix} \frac{17}{12} \\ \\ \frac{-65}{12} \end{pmatrix} \tag{2}$$

calculating the norm of $\mathbf{B} - \mathbf{O}$ and $\mathbf{C} - \mathbf{O}$,we get:

$$\|\mathbf{B} - \mathbf{O}\| = \frac{\sqrt{4514}}{12} \tag{3}$$

$$\|\mathbf{C} - \mathbf{O}\| = \frac{\sqrt{4514}}{12} \tag{4}$$

by doing matrix multiplication, we get:

$$(\mathbf{B} - \mathbf{O}) \cdot (\mathbf{C} - \mathbf{O}) = (\mathbf{B} - \mathbf{O})\,(\mathbf{C} - \mathbf{O})^{\top} \tag{5}$$

$$= \frac{-4270}{144} \tag{6}$$

to calcuate the $\angle BOC$:

$$\cos O = \frac{(\mathbf{B} - \mathbf{O}) \cdot (\mathbf{C} - \mathbf{O})}{\|\mathbf{B} - \mathbf{O}\|\,\|\mathbf{C} - \mathbf{O}\|} \tag{7}$$

$$= \frac{\frac{-4270}{144}}{\frac{\sqrt{4514}}{12} \times \frac{\sqrt{4514}}{12}} \tag{8}$$

$$= \frac{-4270}{4514} \tag{9}$$

$$\implies \angle O = \cos^{-1}\left(\frac{-4270}{4514}\right) \tag{10}$$

$$\implies \angle O = \angle BOC = 161.075355593 \tag{11}$$

Therefore $\angle BOC = 161.075355593$.

2) To find the value of $\angle BAC$:

$$\mathbf{B} - \mathbf{A} = \begin{pmatrix} -5 \\ 7 \end{pmatrix} \tag{12}$$

$$\mathbf{C} - \mathbf{A} = \begin{pmatrix} -4 \\ -4 \end{pmatrix} \tag{13}$$

calculating the norm of $\mathbf{B} - \mathbf{A}$ and $\mathbf{C} - \mathbf{A}$,we get:

$$\|\mathbf{B} - \mathbf{A}\| = \sqrt{74} \tag{14}$$

$$\|\mathbf{C} - \mathbf{A}\| = 4\sqrt{2} \tag{15}$$

by doing matrix multiplication, we get:

$$(\mathbf{B} - \mathbf{A}) \cdot (\mathbf{C} - \mathbf{A}) = (\mathbf{B} - \mathbf{A})\,(\mathbf{C} - \mathbf{A})^{\top} \tag{16}$$

$$= -8 \tag{17}$$

to calcuate the $\angle BAC$:

$$\cos A = \frac{(\mathbf{B}-\mathbf{A})\cdot(\mathbf{C}-\mathbf{A})}{\|\mathbf{B}-\mathbf{A}\|\,\|\mathbf{C}-\mathbf{A}\|} \tag{18}$$

$$= \frac{-8}{\sqrt{74}\times 4\sqrt{2}} \tag{19}$$

$$= \frac{-8}{4\sqrt{148}} \tag{20}$$

$$\implies \angle A = \cos^{-1}\left(\frac{-8}{4\sqrt{148}}\right) \tag{21}$$

$$\implies \angle A = \angle BAC = 99.4623222077 \tag{22}$$

Therefore $\angle BAC = 99.4623222077$.

As we can see,

$\angle BOC \neq 2\times\angle BAC$
Therefore given statement is wrong.