

Assignment 1

Probability And Random Processes

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I. QUESTION 1.4.6

Verify that

$$\angle BOC = 2\angle BAC$$

II. SOLUTION

Given,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix}$$

Also, we have a point $\mathbf{O} = \begin{pmatrix} \frac{-53}{12} \\ \frac{5}{12} \end{pmatrix}$ which is intersection point of the perpendicular bisectors of AB and AC.

1) To find the value of $\angle BOC$:

$$\mathbf{OB} = \mathbf{B} - \mathbf{O} \quad (1)$$

$$= \begin{pmatrix} \frac{5}{12} \\ \frac{67}{12} \end{pmatrix} \quad (2)$$

$$\mathbf{OC} = \mathbf{C} - \mathbf{O} \quad (3)$$

$$= \begin{pmatrix} \frac{17}{12} \\ \frac{-65}{12} \end{pmatrix} \quad (4)$$

calculating the norm of \mathbf{OB} and \mathbf{OC} , we get:

$$\|\mathbf{OB}\| = \frac{\sqrt{4514}}{12} \quad (5)$$

$$\|\mathbf{OC}\| = \frac{\sqrt{4514}}{12} \quad (6)$$

by doing matrix multiplication, we get:

$$\mathbf{OB} \cdot \mathbf{OC} = (\mathbf{OB})(\mathbf{OC})^T \quad (7)$$

$$= \frac{-4270}{144} \quad (8)$$

to calculate the $\angle BOC$:

$$\cos O = \frac{\mathbf{OB} \cdot \mathbf{OC}}{\|\mathbf{OB}\| \|\mathbf{OC}\|} \quad (9)$$

$$= \frac{\frac{-4270}{144}}{\frac{\sqrt{4514}}{12} \times \frac{\sqrt{4514}}{12}} \quad (10)$$

$$= \frac{-4270}{4514} \quad (11)$$

$$\Rightarrow \angle O = \cos^{-1}\left(\frac{-4270}{4514}\right) \quad (12)$$

$$\Rightarrow \angle O = \angle BOC = 161.075355593 \quad (13)$$

Therefore $\angle BOC = 161.075355593$.

2) To find the value of $\angle BAC$:

$$\mathbf{AB} = \mathbf{B} - \mathbf{A} \quad (14)$$

$$= \begin{pmatrix} -5 \\ 7 \end{pmatrix} \quad (15)$$

$$\mathbf{AC} = \mathbf{C} - \mathbf{A} \quad (16)$$

$$= \begin{pmatrix} -4 \\ -4 \end{pmatrix} \quad (17)$$

calculating the norm of \mathbf{AB} and \mathbf{AC} , we get:

$$\|\mathbf{AB}\| = \sqrt{74} \|\mathbf{AC}\| = 4\sqrt{2} \quad (18)$$

by doing matrix multiplication, we get:

$$\mathbf{AB} \cdot \mathbf{AC} = (\mathbf{AB})(\mathbf{AC})^T \quad (19)$$

$$= -8 \quad (20)$$

to calculate the $\angle BAC$:

$$\cos A = \frac{\mathbf{AB} \cdot \mathbf{AC}}{\|\mathbf{AB}\| \|\mathbf{AC}\|} \quad (21)$$

$$= \frac{-8}{\sqrt{74} \times 4 \sqrt{2}} \quad (22)$$

$$= \frac{-8}{4 \sqrt{148}} \quad (23)$$

$$\Rightarrow \angle A = \cos^{-1} \left(\frac{-8}{4 \sqrt{148}} \right) \quad (24)$$

$$\Rightarrow \angle A = \angle BAC = 99.4623222077 \quad (25)$$

Therefore $\angle BAC = 99.4623222077$.

As we can see,

$$\angle BOC \neq 2 \times \angle BAC$$

Therefore given statement is wrong.