Assignment 1
Ncert Exampler

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EE22BTECH11042

Question 9.3.13 Find the probability of getting 5 twice in 7 throws of a dice.
Solution:

Parameter	Value	Description
X	{0,1,2,3,4,5,6,7}	Number of 5 appearing on dice
n	7	Number of cards drawn
p	$\frac{1}{6}$	getting 5
q	$\frac{5}{6}$	getting any other number
$\mu = np$	$\frac{7}{6}$	Mean of Binomial distribution
$\sigma^2 = npq$	$\frac{35}{36}$	Varience of Binomial distribution

TABLE 0
RANDOM VARIABLE AND PARAMETER

1) *Binomial Distribution :*
The X is the random variable, the pmf of X is given by

$$p_X(k) = {}^nC_k p^k (1-p)^{n-k} \tag{1}$$

We require $\Pr(X = 2)$. Since $n = 7$,

$$p_X(2) = 0.234 \tag{2}$$

2) *Gaussian Distribution*
Let Y be gaussian variable. Using central limit theorem, we can use the gaussian distribution function:

$$p_Y(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad (x \in Y) \tag{3}$$

Using Normal distribution at X=2,

$$p_Y(2) = \frac{1}{\sqrt{2\pi\left(\frac{35}{36}\right)}} e^{-\frac{\left(2-\frac{7}{6}\right)^2}{2\left(\frac{35}{36}\right)}} \tag{4}$$

$$= \frac{1}{\sqrt{2\pi\left(\frac{35}{36}\right)}} e^{-\frac{5}{14}} \tag{5}$$

$$= 0.283 \tag{6}$$

3) *Using Q function:*
let Y be a gaussian Random variable

$$Y \sim N(\mu, \sigma) \tag{7}$$

$$\sim N(1.166, 0.972) \tag{8}$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$\begin{aligned} p_Y(x) &\approx \Pr(x - 0.5 < Y < x + 0.5) && (9)\\ &\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) && (10)\\ &\approx F_Y(x + 0.5) - F_Y(x - 0.5) && (11)\end{aligned}$$

CDF of Y is defined as:

$$\begin{aligned} F_Y(x) &= \Pr(Y < x) && (12)\\ &= \Pr\left(\frac{Y-\mu}{\sigma} < \frac{x-\mu}{\sigma}\right) && (13)\\ \implies \frac{Y-\mu}{\sigma} &\sim N(0,1) && (14)\\ &= 1 - \Pr\left(\frac{Y-\mu}{\sigma} > \frac{x-\mu}{\sigma}\right) && (15)\\ &= \begin{cases} 1 - Q\left(\frac{x-\mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu-x}{\sigma}\right) & x < \mu \end{cases} && (16)\end{aligned}$$

Then probability in terms of Q funtion is

$$\implies p_Y(x) \approx Q\left(\frac{(x-0.5)-\mu}{\sigma}\right) - Q\left(\frac{(x+0.5)-\mu}{\sigma}\right) \quad (17)$$

The Gaussian approximation for $\Pr(X = 2)$ is

$$\begin{aligned} p_Y(2) &\approx Q\left(\frac{1.5 - 1.166}{0.972}\right) - Q\left(\frac{2.5 - 1.166}{0.972}\right) && (18)\\ &\approx Q(0.343) - Q(1.371) && (19)\\ &\approx 0.282 && (20)\end{aligned}$$

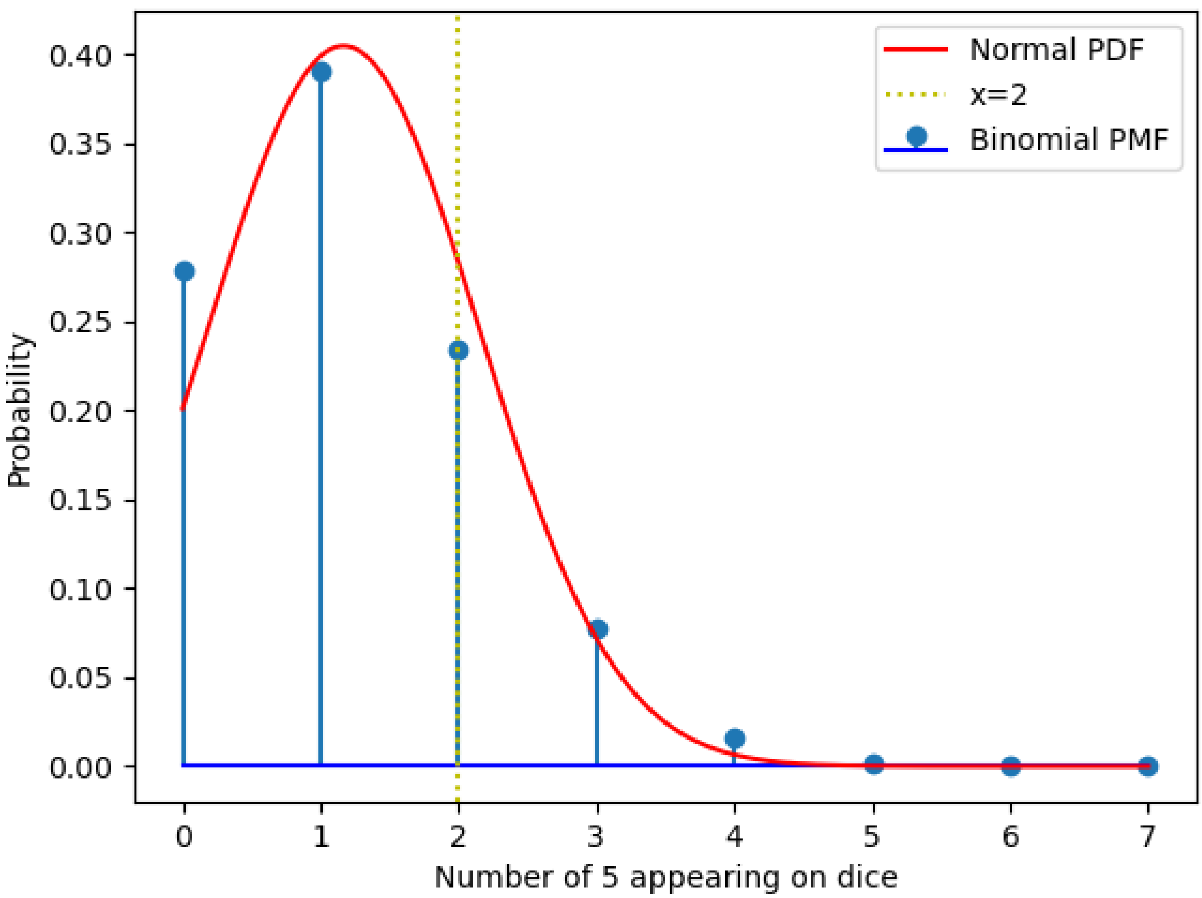

Fig. 3. Binomial and gaussian distribution