

Assignment 1

Ncert Exemplar

Rajeev Kumar
EE22BTECH11042

Question 9.3.13 Find the probability of getting 5 twice in 7 throws of a dice.

Solution:

let Y be a gaussian Random variable

Parameter	Value	Description
X	$\{0,1,2,3,4,5,6,7\}$	Number of 5 appearing on dice
n	7	Number of cards drawn
p	$\frac{1}{6}$	getting 5
q	$\frac{5}{6}$	getting any other number
$\mu = np$	$\frac{7}{6}$	Mean of Binomial distribution
$\sigma^2 = npq$	$\frac{35}{36}$	Variance of Binomial distribution

TABLE 0
RANDOM VARIABLE AND PARAMETER

$$Y \sim N(\mu, \sigma) \quad (1)$$

$$\sim N(1.166, 0.972) \quad (2)$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$p_Y(x) \approx \Pr(x - 0.5 < Y < x + 0.5) \quad (3)$$

$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \quad (4)$$

$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \quad (5)$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y < x) \quad (6)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \quad (7)$$

$$\Rightarrow \frac{Y - \mu}{\sigma} \sim N(0, 1) \quad (8)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (9)$$

$$= \begin{cases} 1 - Q\left(\frac{x - \mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu - x}{\sigma}\right) & x < \mu \end{cases} \quad (10)$$

Then probability in terms of Q funtion is

$$\Rightarrow p_Y(x) \approx Q\left(\frac{(x - 0.5) - \mu}{\sigma}\right) - Q\left(\frac{(x + 0.5) - \mu}{\sigma}\right) \quad (11)$$

The Gaussian approximation for $\Pr(X = 2)$ is

$$p_Y(2) \approx Q\left(\frac{1.5 - 1.166}{0.972}\right) - Q\left(\frac{2.5 - 1.166}{0.972}\right) \quad (12)$$

$$\approx Q(0.343) - Q(1.371) \quad (13)$$

$$\approx 0.282 \quad (14)$$

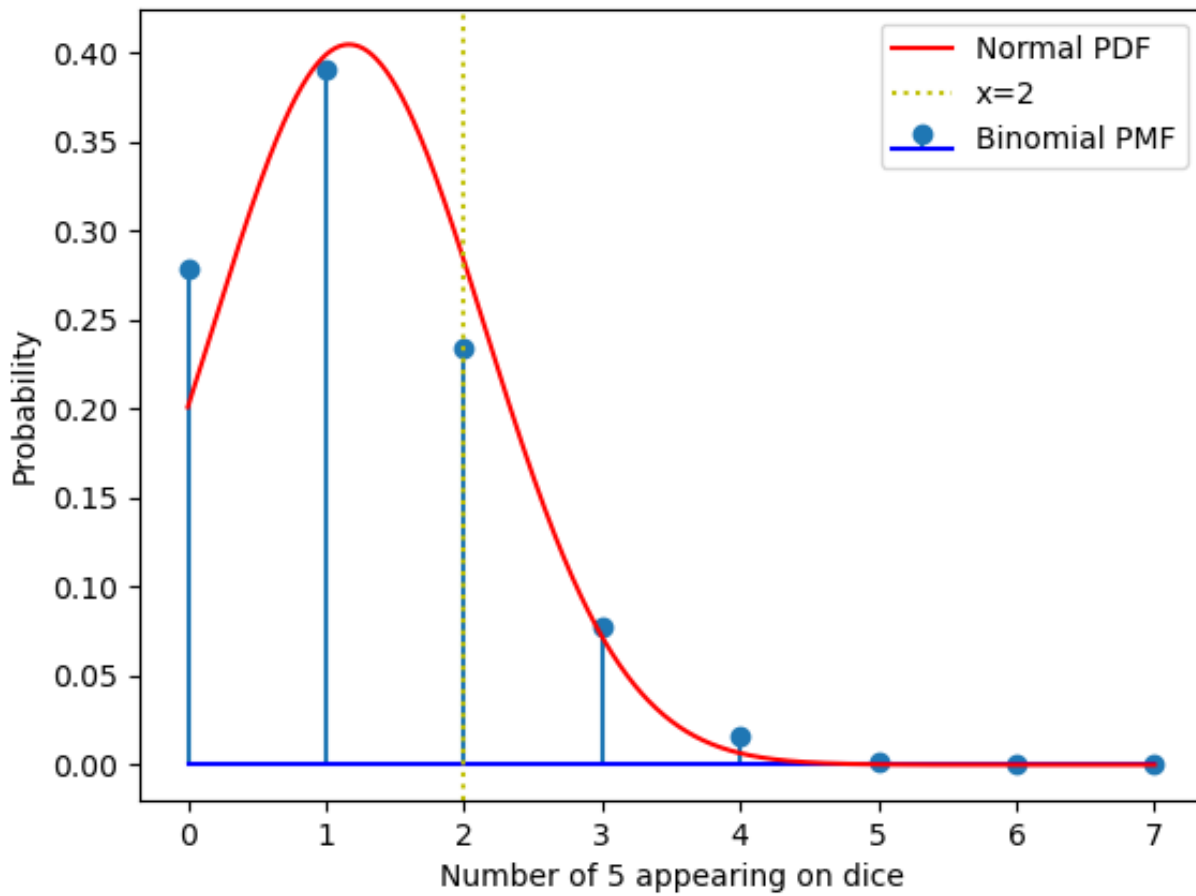


Fig. 0. Binomial and gaussian distribution